STRUCTURE OF SPIN-1 QCD SYSTEMS USING LIGHT-FRONT HAMILTONIAN APPROACH

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in collaboration with

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(BLFQ Collaboration)

Overview

Motivation

Basis Light-Front Quantization (BLFQ)

TMDs of ρ-meson

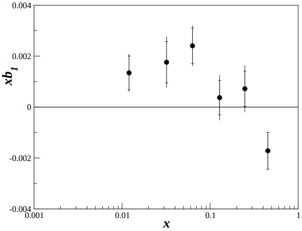
- Quark TMDs
- Gluon TMDs

PDFs of ρ-meson

Conclusion

Motivation

- ρ-meson carries the nuclear force within atomic nuclei.
- π-ρ mass splitting.
- Spin-1 composite systems provide a new spin structure through tensor structure functions:
  - absent for spin-0 or 1/2 systems
  - lattice data for GFFs and moments of the PDFs *PRD 105, 054509 (2022); PRD 56, 2743 (1997)*.
  - experimental data of tensor polarized structure function for deuteron by HERMES : a few data points *PRL 95, 242001 (2005)*
  - proposals to study the tensor structure of deuteron : JLab (approved), Fermilab (proposal in 2022), EIC, EIcC ...
  - largely unexplored field yet : can open a new field of spin physics



PR12-13-011

PR12-13-011

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-40
(Update to PR12-11-110)

FERMILAB-PUB-22-381-V

FERMILAB-PUB-22-381-V

The Transverse Structure of the Deuteron with Drell-Yan

The SpinQuest Collaboration[a]

A Letter of Intent to Jefferson Lab PAC 42

Search for Exotic Gluonic States in the Nucleus

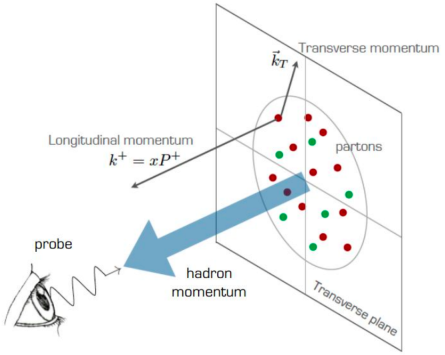
ELSEVIER Progress in Particle and Nuclear Physics Volume 119, July 2021, 103858

Review

On the physics potential to study the gluon content of proton and deuteron at NICA SPD

Transverse momentum-dependent parton distributions (TMDs)

- Information on transverse parton dynamics : SIDIS and Drell-Yan
- 3D distribution functions : $\mathrm{TMD}(x, \mathbf{k}_\perp^2)$
- Information on spin and OAM of the partons inside the hadron.



Picture Credit: A. Bacchetta

PHYSICAL REVIEW C 96, 045206 (2017)

Transverse-momentum-dependent quark distribution functions of spin-one targets: Formalism and covariant calculations

Yu Ninomiya,[1,*] Wolfgang Bentz,[1,2,†] and Ian C. Cloët[3]

NJL

NOVEL EFFECTS IN DEEP INELASTIC SCATTERING FROM SPIN-ONE HADRONS[1]

Pervez HOODBHOY[2], R.L. JAFFE and Aneesh MANOHAR[3]

PHYSICAL REVIEW D, VOLUME 62, 114004

Deep inelastic leptoproduction of spin-one hadrons

A. Bacchetta and P. J. Mulders

PHYSICAL REVIEW D 106, 014026 (2022)

BSE

Transverse momentum distributions of valence quarks in light and heavy vector mesons

Chao Shi,[1,*] Jicheng Li,[1] Ming Li,[1] Xurong Chen,[2,3] and Wenbao Jia[1]

PHYSICAL REVIEW D, VOLUME 59, 094026

Structure functions in the polarized Drell-Yan processes with spin-1/2 and spin-1 hadrons. I. General formalism

S. Hino* and S. Kumano†

PHYSICAL REVIEW D, VOLUME 60, 054018

Structure functions in the polarized Drell-Yan processes with spin-1/2 and spin-1 hadrons. II. Parton model

S. Hino* and S. Kumano†

Eur. Phys. J. C (2022) 82:1045
https://doi.org/10.1140/epjc/s10052-022-10988-5

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Light-front wavefunctions of mesons by design

Meijian Li[1,2,3,a], Yang Li[4,b], Guangyao Chen[5,6,c], Tuomas Lappi[1,2,4,d], James P. Vary[6,e]

JHEP

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PUBLISHED: January 22, 2021

LFHQCD

Light-front holographic ρ-meson distributions in the momentum space

Satvir Kaur,[a] Chandan Mondal[b,c,d] and Harleen Dahiya[a]

TRANSVERSELY POLARIZED PARTON DENSITIES, THEIR EVOLUTION AND THEIR MEASUREMENT

Xavier ARTRU
Laboratoire de Physique Théorique et Hautes Energies*
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Mustapha MEKHFI
Laboratoire de Physique Théorique
Université d'Oran Es-Senia, 31, Algéria

JHEP

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Positivity bounds on gluon TMDs for hadrons of spin ≤ 1

Sabrina Cotogno, Tom van Daal and Piet J. Mulders

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Nikhef, Science Park 105, NL-1098 XG Amsterdam, The Netherlands

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Quark TMDs of a spin-1 target

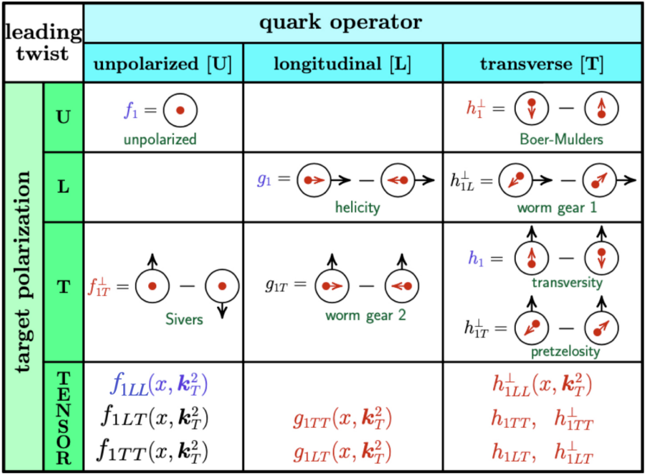


Image taken from arXiv: 2205.01249

- 18 valence quark TMDs: 9 T-even and 9 T-odd TMDs
- have tensor structure, not seen for spin< 1.

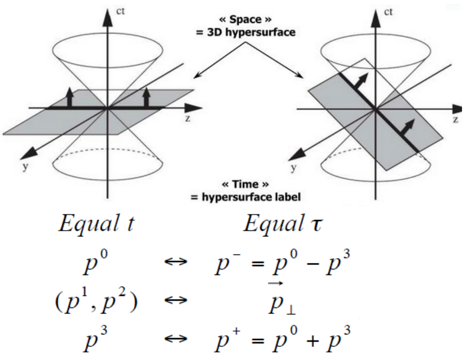
Gluon TMDs of a spin-1 target

TARGET SPIN \ PARTON SPIN	GLUONS: $-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}, \ldots$
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g $h_{1T}^{\perp g}$
LL	f_{1LL}^g		$h_{1LL}^{\perp g}$
LT	f_{1LT}^g	g_{1LT}^g	h_{1LT}^g $h_{1LT}^{\perp g}$
TT	f_{1TT}^g	g_{1TT}^g	h_{1TT}^g $h_{1TT}^{\perp g}$ $h_{1TT}^{\perp\perp g}$

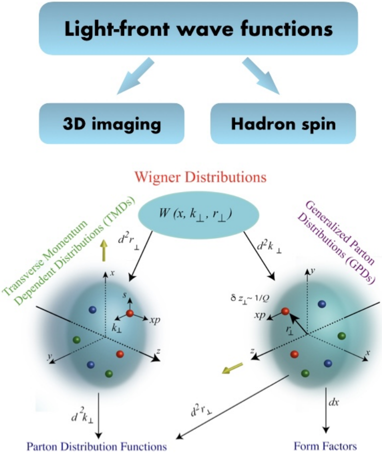
Picture Credit: P.J. Mulders

R. L. Jaffe and A. Manohar, Nuclear gluonometry, PLB 223 (1989) 218
P.J. Mulders and J. Rodrigues, PRD 63 (2001) 094021
S. Meissner, A. Metz and K. Goeke, PRD 76 (2007) 034002
D. Boer, S. Cotogno, T van Daal, et al. , JHEP 10 (2016) 013

Light-Front Coordinates



[1]

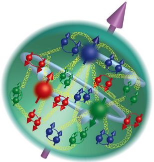


- LFWFs encode the hadronic properties in terms of their quark and gluon degrees of freedom.

[1] P. A. M. Dirac, Rev. Mod. Phys. 21, 392 (1949). ;S. J. Brodsky, G. F. de Teramond, Phys. Rev. D 77, 056007 (2008).

Basis Light-Front Quantization (BLFQ)

- Non-perturbative approach based on the Hamiltonian formalism :
  $P^+P^- |\Psi\rangle = M^2 |\Psi\rangle$
    - to solve relativistic many-body bound state problems.
    - facilitates with mass spectra and LFWFs.
    - successfully implemented to investigate the structures of spin-0 and 1/2 systems.
    - Motive : to extend the approach to investigate spin-1 hadrons.



- P^+ : longitudinal momentum of the targeted hadron
  $P^- = P^-_{\mathrm{QCD}} + P^-_{\mathrm{C}}$: LF Hamiltonian

Fock state expansion of the meson bound state

$$|\Psi\rangle = \psi_{q\bar{q}} |q\bar{q}\rangle + \psi_{q\bar{q}g} |q\bar{q}g\rangle + \psi_{q\bar{q}gg} |q\bar{q}gg\rangle + \psi_{q\bar{q}q\bar{q}} |q\bar{q}q\bar{q}\rangle + \ \ldots$$

- $\psi_{\ldots}$: LFWFs associated with the Fock components $|\ldots\rangle$.

[1] J.P.Vary, H. Honkanen, J. Li, P. Maris, S.J.Brodsky, A. Harindranath, G.F. de Teramond, PRC 81, 035205 (2010).

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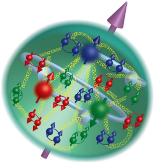

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Basis function

2D-HO basis
in transverse direction, $\Phi_{nm}(\mathbf{p}_\perp)$

Plane wave
in longitudinal direction

Fock space truncation:
$|\Psi\rangle = \psi_{q\bar{q}}\, |q\bar{q}\rangle + \psi_{q\bar{q}g}\, |q\bar{q}g\rangle$

Basis truncation:
$\sum_i (2n_i + |m_i| + 1) \leq N_{\text{max}}$
$\sum_i k_i = K$, $x = \frac{k_i}{K}$

[1] J.P.Vary, H. Honkanen, J. Li, P. Maris, S.J.Brodsky, A. Harindranath, G.F. de Teramond, PRC 81, 035205 (2010).

LF Hamiltonian

P^-_{QCD}

→ K.E. terms of quarks and gluons

→ quark-gluon vertex interaction

→ instantaneous gluon interaction

Other interactions are absorbed into the involved parameters such as coupling constant and masses.

$+$

P^-_{C} : Confining interaction term

→ For leading Fock sector

$$\kappa^4\left(x(1-x)\mathbf{r}_\perp^2 - \frac{\partial_x(x(1-x)\partial_x)}{(m_q+m_{\bar{q}})^2}\right)$$

- κ : confining strength
- $\mathbf{r}_\perp$: separation between quark and anti-quark

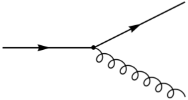
[1] BLFQ Collaboration, J. Lan et al., PLB 825 (2022) 136890
[2] BLFQ Collaboration, Z. Zhu et al., PLB 839 (2023) 137808

Light-front QCD Hamiltonian

[Brodsky et al, 1998]

$$P_{-,LFQCD} = \frac{1}{2}\int d^3x\, \bar{\psi}\gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+}\,\psi - \frac{1}{2}\int d^3x\, A_a^i (i\partial^\perp)^2 A_a^i$$

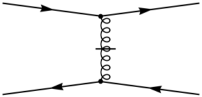
$$+g\int d^3x\, \bar{\psi}\gamma_\mu A^\mu\, \psi$$



$$+\frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma_\mu A^\mu \frac{\gamma^+}{i\partial^+}\gamma_\nu A^\nu \psi$$

$$-ig^2\int d^3x\, f^{abc}\bar{\psi}\gamma^+ T^c\psi \frac{1}{(i\partial^+)^2}(i\partial^+ A_a^\mu A_{\mu b})$$

$$+\frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma^+ T^a\psi \frac{1}{(i\partial^+)^2}\bar{\psi}\gamma^+ T^a\psi$$



$$+ig\int d^3x\, f^{abc} i\partial^\mu A^{\nu a} A_\mu^b A_\nu^c$$

$$-\frac{1}{2}g^2\int d^3x\, f^{abc} f^{ade}\, i\partial^+ A_b^\mu A_{\mu c} \frac{1}{(i\partial^+)^2}(i\partial^+ A_d^+ A_{\nu e})$$

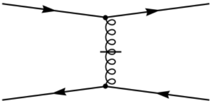
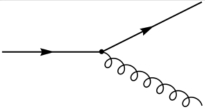
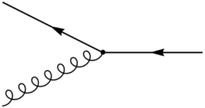
$$+\frac{1}{4}g^2\int d^3x\, f^{abc} f^{ade}\, A_b^\mu A_c^\nu A_{\mu d} A_{\nu e}.$$

6

[1] S.J. Brodsky, H.C. Pauli, S.S. Pinsky, Phys. Rep. 301, 299-486 (1998)

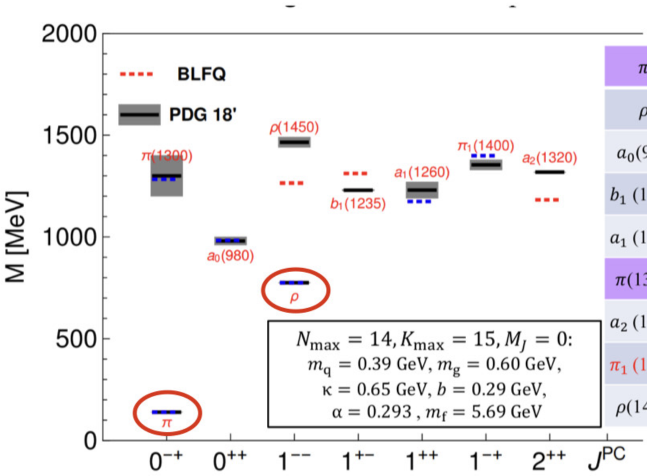
Light-front QCD Hamiltonian in this work

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

H_{int}	$\lvert q\bar{q}\rangle$	$\lvert q\bar{q}g\rangle$
$\langle q\bar{q}\rvert$		
$\langle q\bar{q}g\rvert$		0

Inclusion of higher Fock sector would make $\langle q\bar{q}g|H_{\text{int}}|q\bar{q}g\rangle \neq 0$

Light unflavored meson mass spectrum



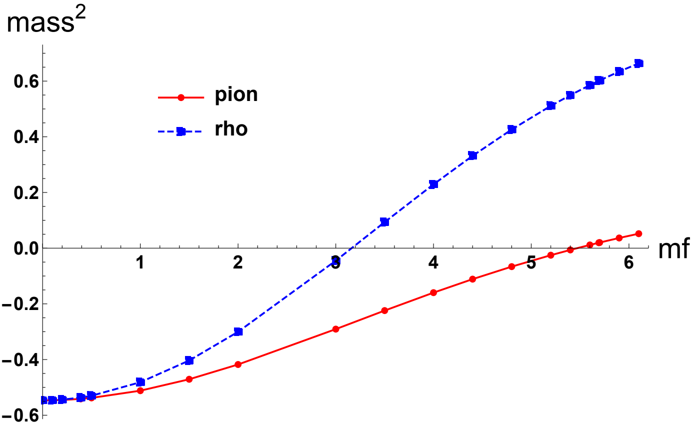
	Norm$q\bar{q}$	DC [MeV]
π	0.492	151
ρ	0.486	141
$a_0(980)$	0.370	0
b_1 (1235)	0.30	0
a_1 (1260)	0.324	13
$\pi(1300)$	0.284	59
a_2 (1320)	0.320	0
π_1 (1400)	0.002	0
$\rho(1450)$	0.312	50

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

Fix the parameters by fitting six blue states

- $\pi_1(1400)$: $|q\bar{q}g\rangle$ dominates
- $\pi(1300)$: the DC is smaller than the DC of pion

$\pi - \rho$ mass splitting



- Keeping other parameters same for π and ρ mesons

LFWFs of π and ρ mesons

- For ρ-meson, we have S, P and D wave components of LFWFs : $L_z = 0, \pm 1, \pm 2$.

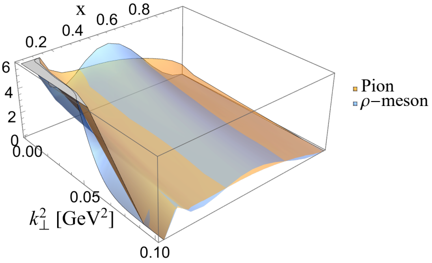
$$L_z = M_J - \sum_i^n \lambda_i$$

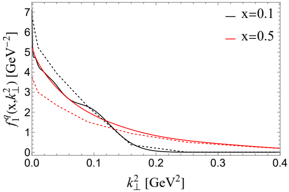
- Probability of different wave components of LFWFs:

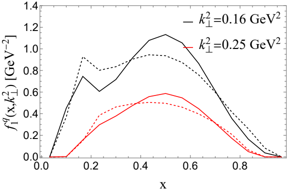
Wave Components of LFWFs	ρ-meson				Pion	
	$\lvert q\bar{q}\rangle$		$\lvert q\bar{q}g\rangle$		$\lvert q\bar{q}\rangle$	$\lvert q\bar{q}g\rangle$
	$M_J = 0$	$M_J = 1$	$M_J = 0$	$M_J = 1$	$M_J = 0$	$M_J = 0$
S-wave	48.60 %	49.14 %	50.56 %	50.12 %	49.08 %	49.99 %
P-wave	0.008 %	0.039 %	0.83 %	0.69 %	0.13 %	0.81 %
D-wave	-	$\approx 10^{-6}$ %	$\approx 10^{-5}$ %	$\approx 10^{-4}$ %	-	$\approx 10^{-5}$ %

Comparison b/w Quark TMDs in Pion and ρ-meson

$f_1^q(\mathrm{x},k_\perp^2)$ [GeV^{-2}]



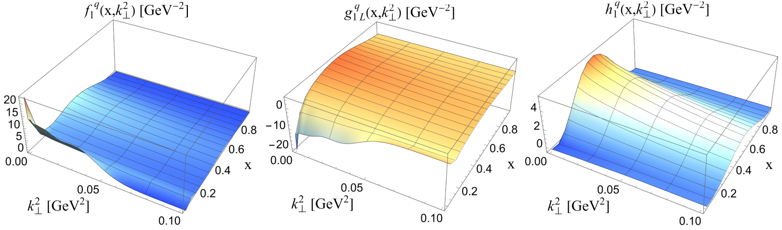




Solid-line: ρ-meson, Dashed-line: Pion

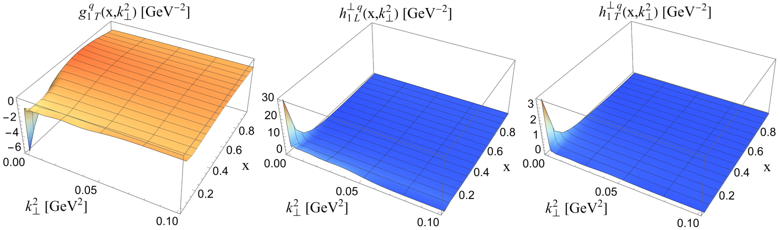
Quark TMDs of ρ-meson

(a) unpolarized (b) helicity (c) transversity

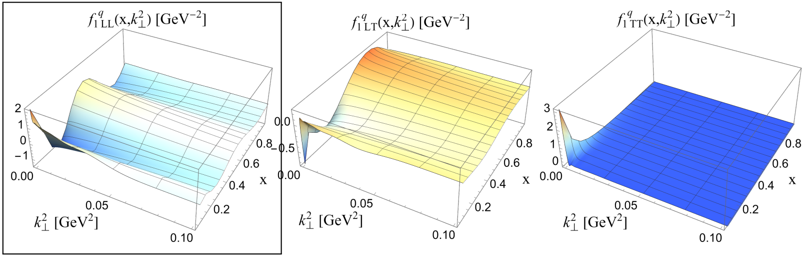


- have PDF limit : $\int d^2\mathbf{k}_\perp \mathrm{TMD}(x, \mathbf{k}_\perp^2) = \mathrm{PDF}(x)$.
- consistent with other model predictions.
- no interference between different wave compositions.

(a) worm gear 2 (b) worm gear 1 (c) pretzelosity



- do not have PDF limit.
- interference between different wave compositions.

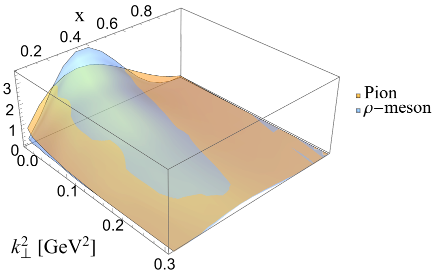


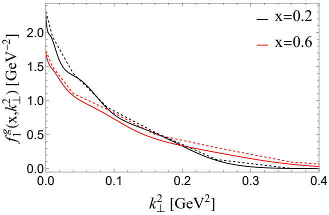
- f_{1LL} are diagonal in OAM, and has PDF limit.
- consistent with other models' predictions.

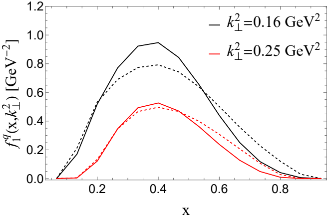
[1] S. Hino and S. Kumano, Phys. Rev. D 59, 094026 (1999).
[2] Y. Ninomiya, W. Bentz, I. C. Cloët, Phys. Rev. C 96, 045206 (2017).
[3] S. Kaur, C. Mondal, and H. Dahiya, JHEP 01, 136 (2021)

Comparison b/w gluon TMDs in Pion and ρ-meson

$f_1^g(x,k_\perp^2)$ [GeV^{-2}]

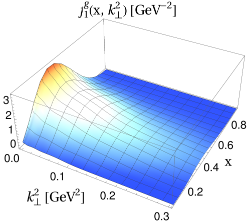




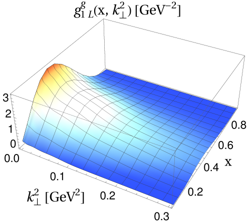


Gluon TMDs

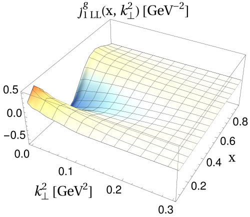
(a) unpolarized



(b) helicity



(c) tensor



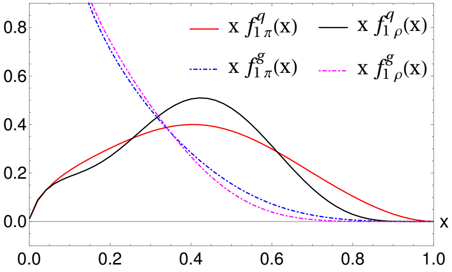
- Transversity gluon TMD $h_{1TT} = 0$
  - Reason : weak D-wave component of LFWFs
  - $h_{1TT} \neq 0$: to consider more interactions / include more sea quarks and gluons.

Contribution to the spin of ρ − meson	$\int dx\, d^2k_\perp\, g_{1L}(x, k_\perp^2)$
Valence quark	54.0%
Gluon	50.2%

Parton distribution functions (PDFs)

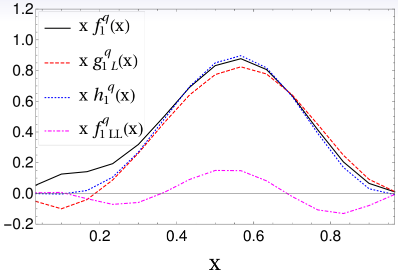
- $f_1^{q/g}(x)$, $g_1^{q/g}(x)$, $h_1^q(x)$, $f_{1LL}^{q/g}(x)$ and $h_{1TT}^g(x)$.
- h_{1TT}^g : no gluon transversity exists for spin< 1 targets [1] [2] [3].
- Sum rules are satisfied by our PDFs [4]:

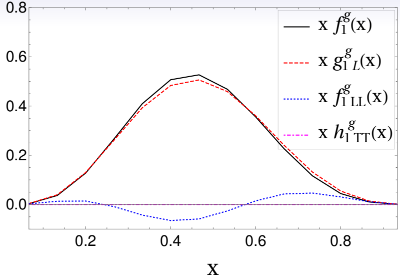
$$\int_0^1 \mathrm{d}x \; f_1^q(x) = 1 \; , \int_0^1 \mathrm{d}x \; f_{1LL}^q(x) = 0 \; .$$

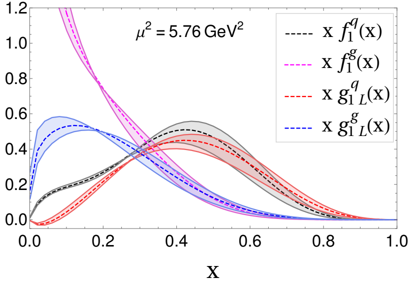


At $\mu^2 = 5.76$ GeV^2.

[1] S. Kumano, Q. -T Song, PRD 101 (2020) 5, 054011; PRD 101 (2020) 9, 094013
[2] D. Keller, The SpinQuest Collaboration, arXiv: 2205.01249
[3] A. Arbuzov, A. Bacchetta et al. Prog. Part. Nucl. Phys. 119 (2021) 103858
[4] Y. Ninomiya, W. Bentz and I. C. Cloët, Phys. Rev. C 96, 045206 (2017).


x f_1^q(x)
x g_{1L}^q(x)
x h_1^q(x)
x f_{1LL}^q(x)
x


x f_1^g(x)
x g_{1L}^g(x)
x f_{1LL}^g(x)
x h_{1TT}^g(x)
x


$\mu^2 = 5.76\ \mathrm{GeV}^2$
x f_1^q(x)
x f_1^g(x)
x g_{1L}^q(x)
x g_{1L}^g(x)
x

Moments of the PDFs

	$\langle x\rangle_{f^q_{1\rho}}$	$\langle x^2\rangle_{f^q_{1\rho}}$	$\langle x^3\rangle_{f^q_{1\rho}}$	$\langle x\rangle_{f^q_{1\pi}}$	$\langle x^2\rangle_{f^q_{1\pi}}$	$\langle x^3\rangle_{f^q_{1\pi}}$
BLFQ	0.235	0.095	0.045	0.233	0.097	0.050
BSE	0.316	0.155	0.091	-	-	-
Lattice QCD	0.334(21)	0.174(47)	0.066(39)	0.24 ± 0.01	0.09 ± 0.03	0.043 ± 0.015

	$\langle x\rangle_{f^g_{1\rho}}$	$\langle x^2\rangle_{f^g_{1\rho}}$	$\langle x^3\rangle_{f^g_{1\rho}}$	$\langle x\rangle_{f^g_{1\pi}}$	$\langle x^2\rangle_{f^g_{1\pi}}$	$\langle x^3\rangle_{f^g_{1\pi}}$
BLFQ	0.425	0.063	0.018	0.428	0.068	0.021

	$\langle x^0\rangle_{g^q_{1L\rho}}$	$\langle x\rangle_{g^q_{1L\rho}}$	$\langle x^2\rangle_{g^q_{1L\rho}}$
BLFQ	0.476	0.205	0.092
BSE	0.660	0.227	0.111
Lattice QCD	0.570(32)	0.212(17)	0.077(34)

[1] C. Best et al., PRD 56, 2743 (1997)
[2] C. Shi et al. PRD 106, 014026 (2022)

Conclusion

- Studied the structure of ρ-meson through TMDs/PDFs using LF Hamiltonian approach.
- Along with the ordinary TMDs, the tensor polarized has also been investigated : exotic hadron structure can be found.
- Qualitative consistency with other model predictions.
- Investigated gluon TMDs for ρ-meson, and made predictions.
- found $h_{1TT} = 0$, however inclusion of more interaction terms: stronger D wave (non zero h_{1TT}).
- Future plans : include multi-gluons and sea quarks.

Work in progress!

Thank you!

$$A^{\Lambda'\Lambda}_{\mathcal{N};\lambda'_q\lambda_q} \quad (13)$$

$$f_1^q = \frac{1}{3}\sum_{\mathcal{N}=2}^{3}\left[A^{++}_{\mathcal{N};++} + A^{00}_{\mathcal{N};++} + A^{--}_{\mathcal{N};++}\right] , \quad (14)$$

$$g_{1L}^q = \frac{1}{2}\sum_{\mathcal{N}=2}^{3}\left[A^{++}_{\mathcal{N};++} - A^{++}_{\mathcal{N};--}\right] , \quad (15)$$

$$h_1^q = \frac{1}{\sqrt{2}}\sum_{\mathcal{N}=2}^{3} A^{+0}_{\mathcal{N};+-} , \quad (16)$$

$$g_{1T}^q = \frac{M_V}{\sqrt{2}k_\perp^2}\sum_{\mathcal{N}=2}^{3}\Re\left[k_R\left(A^{+0}_{\mathcal{N};++} + A^{0-}_{\mathcal{N};++}\right)\right] , \quad (17)$$

$$h_{1L}^{\perp q} = -\frac{M_V}{k_\perp^2}\sum_{\mathcal{N}=2}^{3}\Re\left[k_R\ A^{--}_{\mathcal{N};-+}\right] , \quad (18)$$

$$h_{1T}^{\perp q} = \frac{\sqrt{2}M_V^2}{k_\perp^4}\sum_{\mathcal{N}=2}^{3}\Re\left[k_R^2\ A^{0-}_{\mathcal{N};-+}\right] , \quad (19)$$

$$f_{1LL}^q = \sum_{\mathcal{N}=2}^{3}\left[A^{00}_{\mathcal{N};++} - \frac{1}{2}\left(A^{++}_{\mathcal{N};++} + A^{--}_{\mathcal{N};++}\right)\right] , \quad (20)$$

$$f_{1LT}^q = \frac{M_V}{\sqrt{2}k_\perp^2}\sum_{\mathcal{N}=2}^{3}\Re\left[k_R\left(A^{+0}_{\mathcal{N};++} - A^{0-}_{\mathcal{N};++}\right)\right] , \quad (21)$$

$$f_{1TT}^q = \frac{M_V^2}{k_\perp^4}\sum_{\mathcal{N}=2}^{3}\Re\left[k_R^2\ A^{+-}_{\mathcal{N};++}\right] . \quad (22)$$

$$A^{\Lambda',\Lambda}_{2;\lambda'_q\lambda_q} = \sum_{\lambda_{\bar q}}\frac{\prod_{i=1}^{2}\mathrm{d}x'_i\mathrm{d}^2p'_{\perp i}}{[2(2\pi)^3]^2}2(2\pi)^3\ \Psi^{*\Lambda'}_{2;\lambda'_q\lambda_{\bar q}}\Psi^{\Lambda}_{2;\lambda_q\lambda_{\bar q}}\delta^3\left(\tilde p'_1 - \tilde k\right)\delta^3\left(\tilde P - \sum_i \tilde p'_i\right) ,$$

$$A^{\Lambda',\Lambda}_{3;\lambda'_q\lambda_q} = \sum_{\lambda_{\bar q},\lambda_g}\frac{\prod_{i=1}^{3}\mathrm{d}x_i\mathrm{d}^2p_{\perp i}}{[2(2\pi)^3]^3}2(2\pi)^3\ \Psi^{*\Lambda'}_{3;\lambda'_q\lambda_{\bar q}\lambda_g}\Psi^{\Lambda}_{3;\lambda_q\lambda_{\bar q}\lambda_g}\delta^3\left(\tilde p_1 - \tilde k\right)\delta^3\left(\tilde P - \sum_i \tilde p_i\right) ,$$