Proton gravitational form factors with basis light-front quantization

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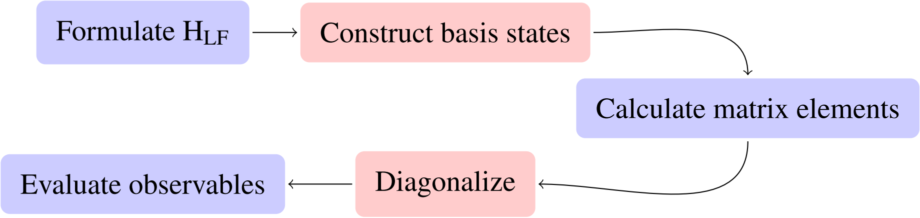
Basis Light-Front Quantization (BLFQ)

- Nonperturbative approach
- based on the Hamiltonian formalism in light-front dynamics
- Solve the light-front eigenvalue equation : $\mathrm{H}_{\mathrm{LF}} \mid \psi\rangle = M^2 \mid \psi\rangle$
- General BLFQ algorithm :



——J. P. Vary, H. Honkanen, J. Li, P. Maris, S. J. Brodsky, A. Harindranath, G. F. de Teramond, P. Sternberg, E. G. Ng, and C. Yang, Phys. Rev. C 81, 035205 (2010).
——P. Wiecki, Y. Li, X. Zhao, P. Maris, and J. P. Vary, Phys. Rev. D 91, 105009 (2015).

Effective Hamiltonian : $H_{\text{eff}} = P^{-}_{\text{eff}} P^{+}$

Valence Fock sector ($| qqq\rangle$)

$$H_{\text{eff}} = \sum_a \frac{\vec{k}^2_{\perp a} + m_a^2}{x_a} + \frac{1}{2}\sum_{a \neq b} V^{\text{conf}}_{ab} + \frac{1}{2}\sum_{a \neq b} V^{\text{OGE}}_{ab}$$

- 3D confining potential V^{conf}_{ab}
  - transverse[1] : $\kappa^4 \left[x_a x_b (\vec{r}_{\perp a} - \vec{r}_{\perp b})^2 \right]$
  - longitudinal[2] : $\kappa^4 \left[\frac{\partial_{x_a}(x_a x_b \partial_{x_b})}{(m_a + m_b)^2} \right]$
- One-gluon exchange interaction

$$V^{\text{OGE}}_{ab} = \frac{4\pi C_F \alpha_s}{Q^2_{ab}} \bar{u}(k'_a, s'_a)\gamma^\mu u(k_a, s_a)\bar{u}(k'_b, s'_b)\gamma^\nu u(k_b, s_b) g_{\mu\nu}$$

—[1] Brodsky, Teramond, Dosch and Erlich, Phys. Rep. 584, 1 (2015).
—[2] Li, Maris, Zhao and Vary, Phys. Lett. B (2016).

Proton wavefunction

Basis in BLFQ approach

- longitudinal direction $\rightarrow$ discretized plane-wave basis
- transverse direction $\rightarrow$ 2D harmonic oscillator function ($\phi_{n,m}$)

Basis truncation

- $\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$
- $K = \sum_i k_i$ with the longitudinal momentum fraction
  $x_i = p_i^+/P^+ = k_i/K$

LFWF in the BLFQ basis

$$\Psi^{\Lambda}_{\{x_i,\vec{k}_{i\perp},\lambda_i\}} = \sum_{\{n_i,m_i\}} \psi^{\Lambda}_{\{x_i,n_i,m_i,\lambda_i\}} \prod_i \phi_{n_i,m_i}(\vec{k}_{i\perp}; b)$$

Nucleon gravitational form factors (GFFs)

gauge invariant symmetric energy-momentum tensor

$$\theta^{\mu\nu} = \frac{1}{2}\overline{\psi}\, i\left[\gamma^{\mu}D^{\nu} + \gamma^{\nu}D^{\mu}\right]\psi - F^{\mu\lambda a}F^{\nu}_{\lambda a} + \frac{1}{4}g^{\mu\nu}\left(F_{\lambda\sigma a}\right)^{2} - g^{\mu\nu}\overline{\psi}\left(i\gamma^{\lambda}D_{\lambda} - m\right)\psi$$

$F^{\mu\nu}_{a}$ is the field strength tensor.

fermionic contribution to the EMT

$$\theta^{\mu\nu}_{q} = \frac{1}{2}\overline{\psi} i\left[\gamma^{\mu}D^{\nu} + \gamma^{\nu}D^{\mu}\right]\psi$$

The GFFs are linked to the matrix elements of the EMT

Nucleon gravitational form factors

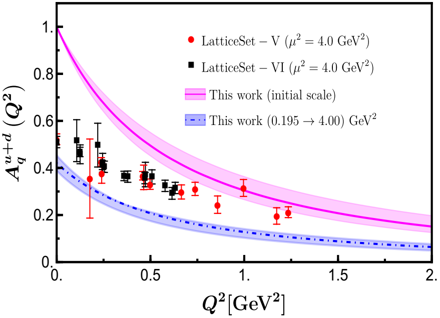
For a spin 1/2 composite system, the standard parametrization of the symmetric EMT reads [1,2]

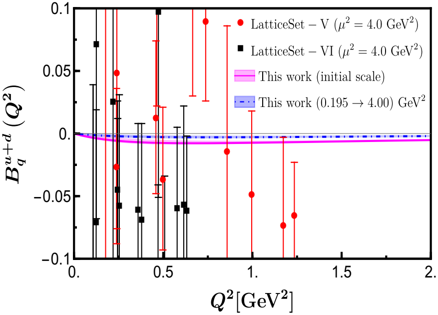
$$\langle P'|\theta_i^{\mu\nu}(0)|P\rangle = \bar{U}'\Big[-B(q^2)\frac{\bar{P}^\mu\bar{P}^\nu}{M} + (A(q^2)+B(q^2))\frac{1}{2}(\gamma^\mu\bar{P}^\nu + \gamma^\nu\bar{P}^\mu) + C(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}(q^2)Mg^{\mu\nu}\Big]U$$

- $A(Q^2)$ and $B(Q^2)$ are obtained from the $(++)$ component.
- $C(Q^2)$ and $\bar{C}(Q^2)$ are extracted from the transverse (i,j) components where $(i,j)\in(1,2)$.

—[1] X. Ji, X. Xiong, F. Yuan, Phys. Lett. B 717 (2012) 214?218
—[2] A. Harindranath, R. Kundu, A. Mukherjee, Phys. Lett. B 728 (2014) 63?67

Results : $A_q(Q^2)$ and $B_q(Q^2)$





- At initial scale obey the sum rule $\sum_i A_i = 1$ and $\sum_i B_i = 0$.
- The bands reflect a 10% uncertainty in the α_s coupling constant.
- DGLAP evolution done using HOPPET[2] toolkit to compare with lattice[1] results.

—[1] P. Hagler, et al., Phys. Rev. D 77 (2008) 094502.

—[2] G. P. Salam, J. Rojo, Comput. Phys. Commun. 180 (2009) 120-156.

The D-term ($D = 4C$)

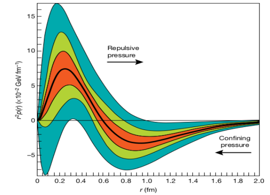
First measurement of the pressure distribution experienced by the quarks in the proton

Letter | Published: 16 May 2018

The pressure distribution inside the proton

V. D. Burkert, L. Elouadrhiri & F. X. Girod

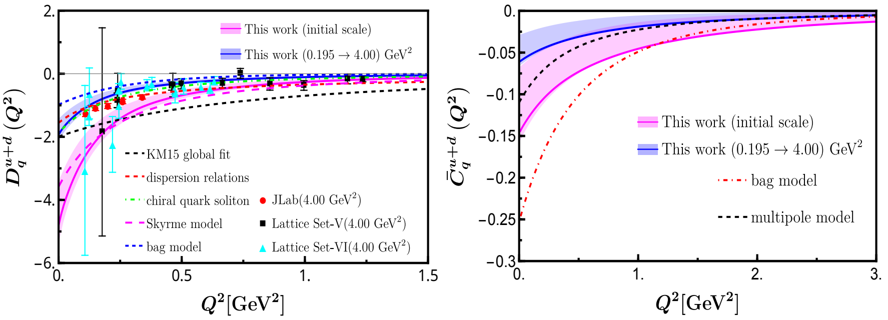
Nature **557**, 396–399(2018) | Cite this article



"The average peak pressure near the center is about 10^{35} pascals which is about 10 times greater than the pressure in the heart of a neutron star".

- $\sum_a A_a(0) = 1 \qquad \sum_a B_a(0) = 0$
- $D(0)$ is not constrained by general principles.
- $D(q^2)$ is related to the stress tensor and internal forces.

Results : $D_q(Q^2)$ and $\bar{C}_q(Q^2)$

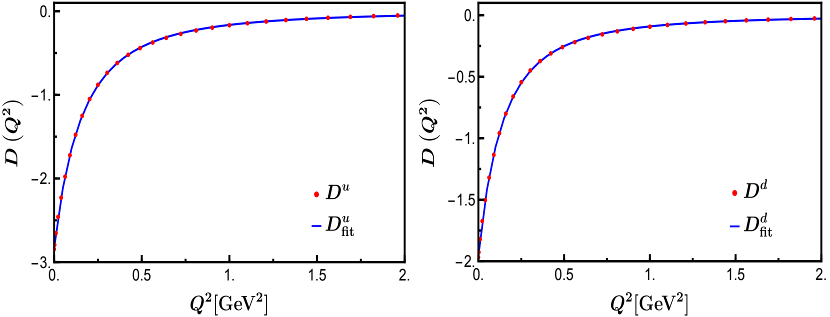


- A negative D-term is a sign of a stable bound system.
- D-term aligns well with lattice results, JLab experimental data, and a myriad of other theoretical projections
- $\bar{C}^{u+d}(Q^2)$ is negative and consistent with the bag model and multipole model

Fitting GFFs

We fit GFFs with following three parameter function:

$$f(Q^2) = \frac{a_0}{(1 + a_1 Q^2)^{a_2}}$$



GFF	a_0	a_1	a_2
$D^u(Q^2)$	-2.8332	3.1475	2.0046
$D^d(Q^2)$	-1.9579	3.3592	2.0830

Mechanical properties

The D-term can be directly related to the pressure in the center of the nucleon , mechanical radius and energy density

$$p_0 = -\frac{1}{24\pi^2 M_n}\int_0^\infty dQ^2\, Q^3\, D(Q^2), \qquad \langle r^2_{\rm mech}\rangle = D(0)\left[\int_0^\infty dQ^2\, D(Q^2)\right]^{-1}$$

$$\mathcal{E} = \frac{M_n}{4\pi^2}\int_0^\infty dQ^2 \left(A(Q^2) + \frac{Q^2}{4M_n^2}D(Q^2)\right)$$

Approaches/Models	p_0 [GeV/fm^3]	$\mathcal{E}$ [GeV/fm^3]	$\langle r^2_{\rm mech}\rangle$ [fm^2]
Our work ($\sqrt{0.195}$ GeV $\to$ 2 GeV)	0.42	1.08	0.76
QCDSR set-I (1 GeV)[1]	0.67	1.76	0.54
Skyrme model[2]	0.47	2.28	-
modified Skyrme model[3]	0.26	1.45	-
χQSM[4]	0.23	1.70	-
Soliton model[5]	0.58	3.56	-
LCSM-LO[6]	0.84	0.92	0.54

—[1] K. Azizi and U. Özdem, Eur. Phys. J. C 80, 104 (2020).
—[2] C. Cebulla et. al. , Nucl. Phys. A 794, 87–114 (2007).
—[3] H.-C. Kim, et. al. , Phys. Lett. B 718, 625–631 (2012).
—[4] K. Goeke et. al. , Phys. Rev. D 75, 094021 (2007).
—[5] J.-H. Jung, et. al. , Phys. Rev. D 89, 114021 (2014).
—[6] I. V. Anikin, Phys. Rev. D 99, 094026 (2019).

Pressure and Shear forces

The 2D pressure and shear in the impact parameter space[1]

$$p(b) = \frac{1}{2M}\frac{1}{b}\frac{d}{d\,b}\left[b\frac{d}{d\,b}\tilde{D}_q(b)\right], \quad s(b) = -\frac{1}{M}b\frac{d}{d\,b}\left[\frac{1}{b}\frac{d}{db}\tilde{D}_q(b)\right]$$

The FT of the D-term expressed using the Bessel function (J_0)

$$\tilde{D}(b) = \frac{1}{(2\pi)^2}\int d^2\boldsymbol{q}^{\perp}\, e^{-i\boldsymbol{q}^{\perp}\boldsymbol{b}^{\perp}} D(q^2) = \frac{1}{2\pi}\int_0^{\infty} d\boldsymbol{q}^{\perp 2} J_0\left(\boldsymbol{q}^{\perp}\boldsymbol{b}^{\perp}\right) D(q^2)$$

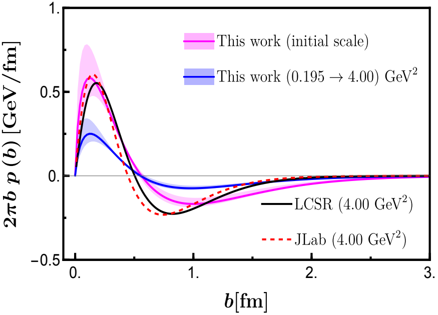
where, $b = |\vec{b}_{\perp}|$ represents the impact parameter.

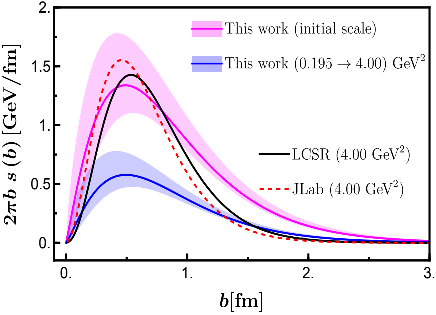
A spherical shell of radius b experiences normal and tangential forces[2]

$$F_n(b) = \left(p(b) + \frac{1}{2}s(b)\right), \quad F_t(b) = \left(p(b) - \frac{1}{2}s(b)\right)$$

—[1] A. Freese and G. A. Miller, Phys. Rev. D 103, 094023 (2021).
—[2] M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A 33, 1830025 (2018).

Results : pressure and shear

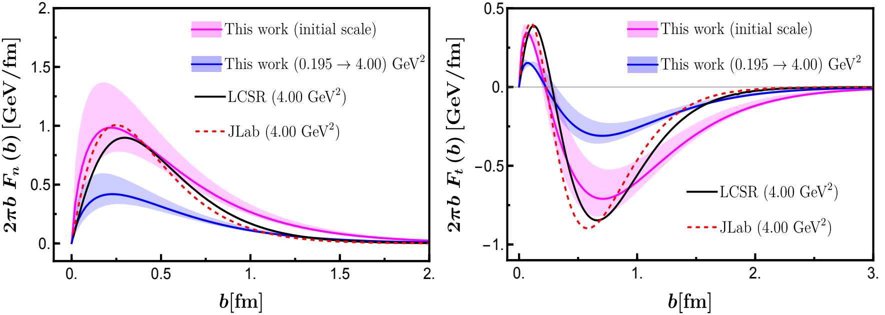




- $p(b)$ must follow the von Laue condition $\int_0^\infty db\, b^2\, p(b) = 0$
- presure → positive region followed by a negative region
- shear → typically positive in stable hydrostatic systems
- compared with the light-cone sum rule (LCSR)[1] and data from JLab[2]

—[1] I. V. Anikin, Phys. Rev. D 99, 094026 (2019).
—[2] V. D. Burkert, L. Elouadrhiri, and F. X. Girod, Nature 557, 396–399 (2018)

Results : normal and tangential force



- positive nature of $F_n(b)$
- $F_t(b)$ showcases a dual character: a positive core, indicative of repulsive forces, and a subsequent negative domain, signifying attractive forces
- It's worth noting that this binding force exhibits a greater magnitude than the repulsive counterpart.

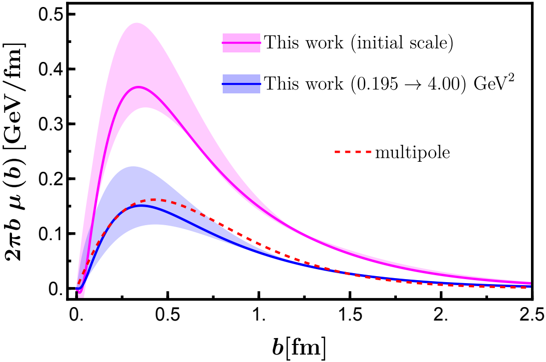
2D Galilean distributions

Galilean energy density $(\mu(b))$, radial pressure $(\sigma^r(b))$, tangential pressure $(\sigma^t(b))$, isotropic pressure $(\sigma(b))$, and pressure anisotropy $(\Pi(b))$ [1]

$$
\begin{aligned}
\mu(b) &= M\left[\frac{A(b)}{2} + \overline{C}(b) + \frac{1}{4M^2}\frac{1}{b}\frac{d}{db}\left(b\frac{d}{db}\left[\frac{B(b)}{2} - 4C(b)\right]\right)\right] \\
\sigma^r(b) &= M\left[-\overline{C}(b) + \frac{1}{M^2}\frac{1}{b}\frac{dC(b)}{db}\right] \\
\sigma^t(b) &= M\left[-\overline{C}(b) + \frac{1}{M^2}\frac{d^2C(b)}{d(b)^2}\right] \\
\sigma(b) &= M\left[-\overline{C}(b) + \frac{1}{2}\frac{1}{M^2}\frac{1}{b}\frac{d}{db}\left(b\frac{d\,C(b)}{db}\right)\right] \\
\Pi(b) &= M\left[-\frac{1}{M^2}b\frac{d}{db}\left(\frac{1}{b}\frac{dC(b)}{db}\right)\right]
\end{aligned}
$$

—[1] C. Lorcé, H. Moutarde, and A. P. Trawi?ski, Eur. Phys. J. C 79, 89 (2019)

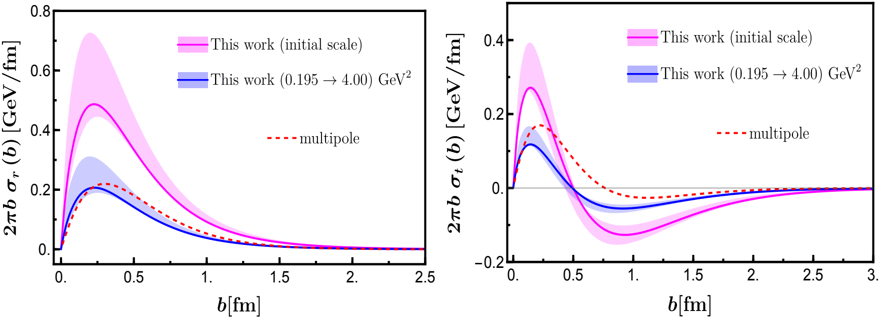
Results : energy density



- positive energy density
- compared with the result in a multipole model (red dashed line)[1]

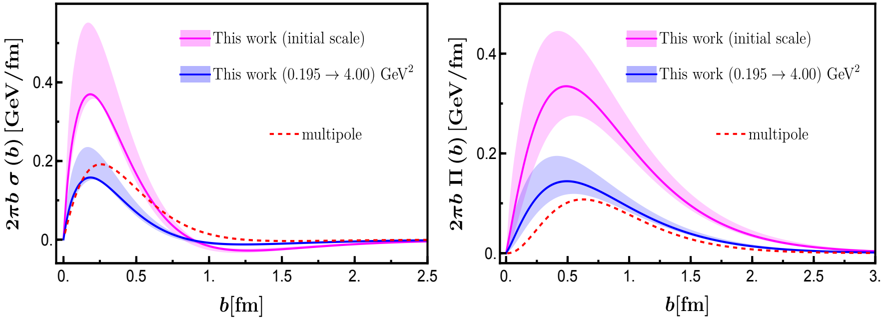
—[1] C. Lorcé, H. Moutarde, and A. P. Trawi?ski, Eur. Phys. J. C 79, 89 (2019)

Results : radial and tangential pressure



- radial pressure : consistently positive, indicating a repulsive nature
- tangential pressure : positive (repulsive) region is centered in the impact parameter space, while the negative (attractive) region spans towards the larger values of b.

Results : isotropic pressure and pressure anisotropy



- $\sigma = \dfrac{(\sigma_r + \sigma_t)}{2}$ and $\Pi = \sigma_r - \sigma_t$
- the radial pressure consistently exceeds the tangential one

Effective Hamiltonian with One Dynamical Gluon

$$|\,\text{proton}\rangle = a\,|\,qqq\rangle + b\,|\,qqqg\rangle + c\,|\,qqqq\bar{q}\rangle + \ldots$$

kinetic energy

transverse confining potential[2]

$$H_{\text{eff}} = \sum_a \frac{\vec{p}_{\perp a}^{\,2} + m_a^2}{x_a} + \frac{1}{2}\sum_{a\neq b}\kappa^4\left[x_a x_b(\vec{r}_{\perp a} - \vec{r}_{\perp b})^2\right]$$

$$-\frac{1}{2}\sum_{a\neq b}\kappa^4\left[\frac{\partial_{x_a}(x_a x_b \partial_{x_b})}{(m_a+m_b)^2}\right] + H_{\text{vertex}} + H_{\text{inst}}$$

longitudinal confining potential[3]

QCD interactions [4]

—[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].
—[2] Brodsky,Teramond,Dosch and Erlich, Phys.Rep.584,1(2015).
—[3] Li, Maris, Zhao and Vary, Phys. Lett. B (2016).
—[4] Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998).

QCD interactions and parameters

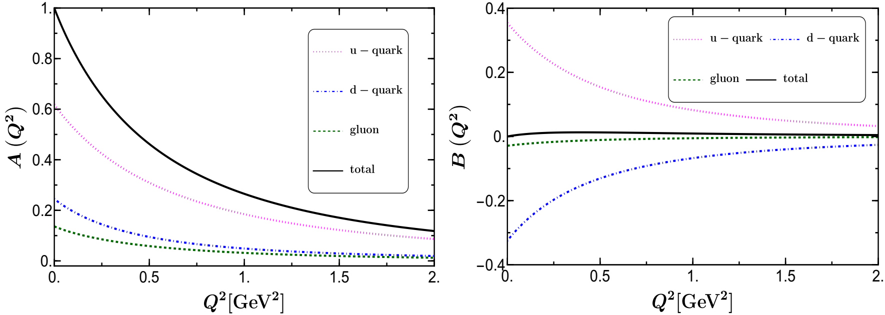
vertex interaction and the instantaneous interaction

$$H_{\text{vertex}} + H_{\text{inst}} = g\bar{\psi}\gamma_\mu T^a A_a^\mu \psi + \frac{1}{2} g^2 \bar{\psi}\gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi}\gamma^+ T^a \psi$$

- The transverse and longitudinal truncation parameters are set to $N_{\max} = 9$ and $K = 16.5$
- harmonic oscillator scale parameter $b = 0.70$ GeV and the UV cutoff for the instantaneous interaction $b_{\text{inst}} = 3.00$ GeV
- The model parameters are $\{m_u, m_d, m_g, \kappa, m_f, g\} = \{0.31, 0.25, 0.50, 0.54, 1.80, 2.40\}$, with all values in GeV unit, except for g
- These values are derived by fitting the proton mass (M), its electromagnetic properties, and flavor FFs[1]

—[1] S. Xu, et. al., 2209.08584 [hep-ph]

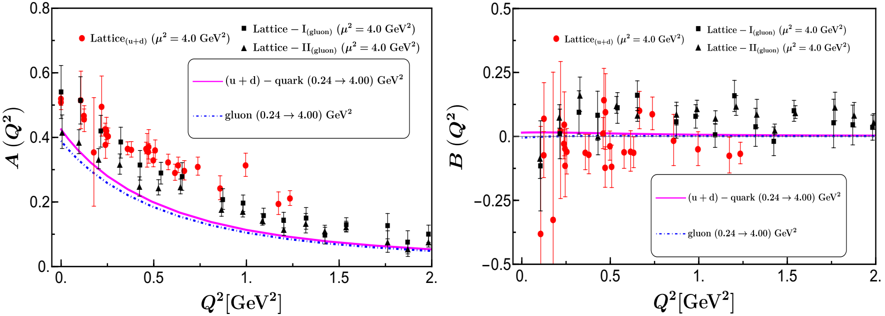
Results : $A(Q^2)$ and $B(Q^2)$ with dynamical gluon at initial scale



- $A(Q^2)$ and $B(Q^2)$ obey the sum rule $\sum_i A_i = 1$ and $\sum_i B_i \approx 0$.

$Q^2 = 0$	$i = u$	$i = d$	$i = g$	$u + d + g$
A_i	0.619	0.245	0.136	1.00
B_i	0.354	−0.325	−0.0289	3.534×10^{-11}

Results : Evolved $A(Q^2)$ and $B(Q^2)$ with dynamical gluon

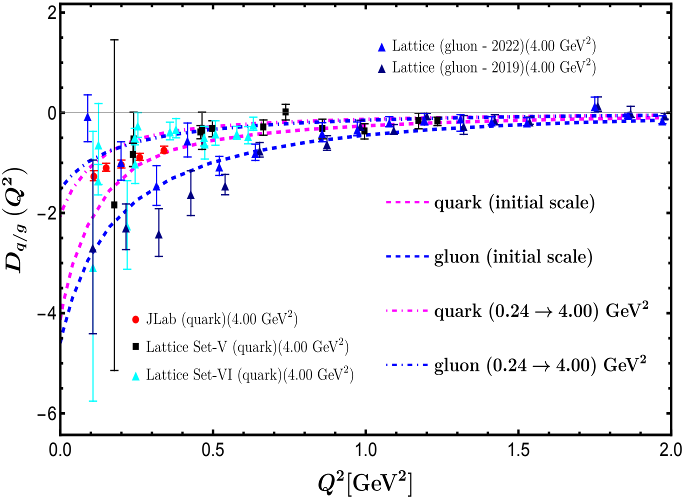


$Q^2 = 0$	$i = u$	$i = d$	$i = u + d$	$i = g$
A_i	0.305	0.120	0.425	0.389
B_i	0.175	−0.160	0.148	−0.005

—[Lattice - I]$_{(\text{gluon})}$ P. E. Shanahan, W. Detmold, Phys. Rev. D 99, 014511 (2019).

—[Lattice - II]$_{(\text{gluon})}$ D. A. Pefkou, D. C. Hackett, P. E. Shanahan, Phys. Rev. D 105, 054509 (2022).

Preliminary result : $D(Q^2)$ with dynamical gluon



—[Lattice] $_{\text{(gluon)}}$ P. E. Shanahan, W. Detmold, Phys. Rev. D 99, 014511 (2019).

—[Lattice]$_{\text{(gluon)}}$ D. A. Pefkou, D. C. Hackett, P. E. Shanahan, Phys. Rev. D 105, 054509 (2022).

Mechanical properties : Preliminary result

The D-term can be directly related to the pressure in the center of the nucleon , mechanical radius and energy density[1]

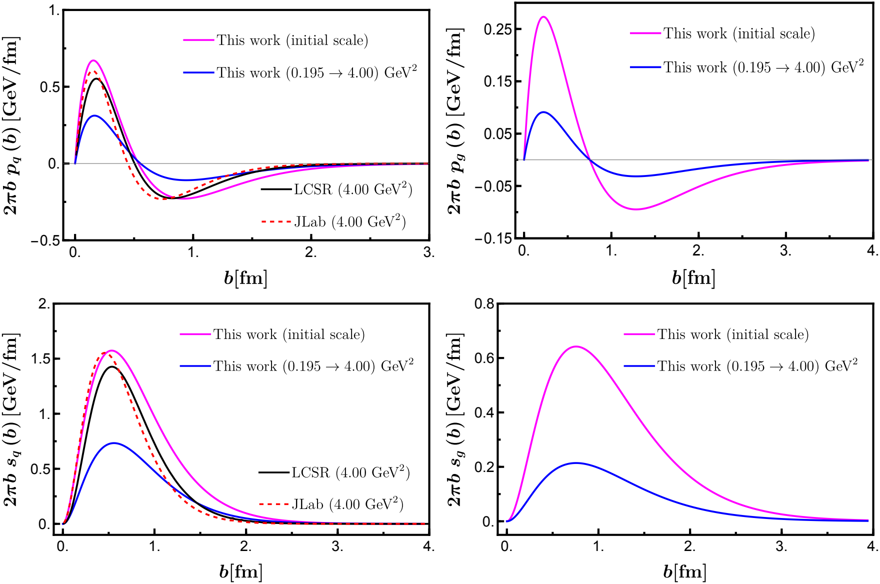
$$p_0 = -\frac{1}{24\pi^2 M_n}\int_0^\infty \mathrm{d}Q^2\, Q^3\, D(Q^2), \qquad \langle r^2_{\mathrm{mech}}\rangle = D(0)\left[\int_0^\infty \mathrm{d}Q^2\, D(Q^2)\right]^{-1}$$

$$\mathcal{E} = \frac{M_n}{4\pi^2}\int_0^\infty \mathrm{d}Q^2 \left(A(Q^2) + \frac{Q^2}{4M_n^2} D(Q^2)\right)$$

Our work	p_0 [GeV/fm^3]	$\mathcal{E}$ [GeV/fm^3]	$\langle r^2_{\mathrm{mech}}\rangle$ [fm^2]
for quark ($\sqrt{0.24}$ GeV $\to$ 2 GeV)	0.53	0.85	0.69
for gluon ($\sqrt{0.24}$ GeV $\to$ 2 GeV)	0.61	0.69	0.57

—[1] Polyakov, Maxim V. and Schweitzer, Peter, Int.J.Mod.Phys.A 33 (2018) 26.

Preliminary Results : Pressure and shear



—[1] I. V. Anikin, Phys. Rev. D 99, 094026 (2019).
—[2] V. D. Burkert, L. Elouadrhiri, and F. X. Girod, Nature 557, 396–399 (2018)

Conclusion

- GFFs for proton's valence quarks, evaluated using BLFQ, align with lattice QCD and JLab findings.
- Positive core and negative tail for internal pressure and positive shear force were observed.
- The gluon D-term is negative and consistent with lattice data.
- Work in progress on the calculation of GFF $\widetilde{C}_g$ that adheres to the sum rule $\sum_{i=q,g} \widetilde{C}_i(Q^2) = 0$.

Upcoming talk from team BLFQ

- Beyond Valence Distributions in meson with Basis Light-Front Quantization - Jiangshan Lan - Tuesday - 16:30
- Quantum stress on the light front - Xianghui Cao - Tuesday - 17:00
- Towards a Hamiltonian first principle approach for baryons - Siqi Xu - Wednesday - 09:30
- Positronium structure from a basis light-front approach - Xingbo Zhao - Wednesday - 11:00
- Positronium in quantum electrodynamics of effective particles - Kamil Serafin - Wednesday - 11:30
- Structure of spin-1 QCD systems using light-front Hamiltonian approach - Satvir Kaur - Thursday - 16:30

Thank you!

Back Up Slides

Comparison of GFFs

Approaches/Models	$A_q^{u+d}(0)$	$J_q(0) = \frac{1}{2}[A_q^{u+d}(0) + B_q^{u+d}(0)]$	$D^{u+d}(0) = 4C^{u+d}(0)$	$\bar{C}_q^{u+d}(0)$
This work ($\sqrt{0.195}$ GeV $\to$ 2 GeV)	0.420	0.210	-1.925	-0.061
IP [**?**]	-	-	-	1.4×10^{-2}

Nucleon gravitational form factors

The GFFs $A_q(Q^2)$ and $B_q(Q^2)$ can be written in terms of the overlap of LFWFs

$$
\begin{aligned}
A_q(Q^2) &= \frac{1}{2}\sum_{\{\lambda_i\}}\int [\mathrm{d}\mathcal{X}\,\mathrm{d}\mathcal{P}_\perp]\, x_1 \Big\{ \Psi^{\uparrow *}_{\{x_i',\vec{k}_{i\perp}',\lambda_i'\}} \Psi^{\uparrow}_{\{x_i,\vec{k}_{i\perp},\lambda_i\}} + \\
&\quad \Psi^{\downarrow *}_{\{x_i',\vec{k}_{i\perp}',\lambda_i'\}} \Psi^{\downarrow}_{\{x_i,\vec{k}_{i\perp},\lambda_i\}} \Big\}, \\
B_q(Q^2) &= \frac{M}{iq^{(2)}}\sum_{\{\lambda_i\}}\int [\mathrm{d}\mathcal{X}\,\mathrm{d}\mathcal{P}_\perp]\, x_1 \Big\{ \Psi^{\uparrow *}_{\{x_i',\vec{k}_{i\perp}',\lambda_i'\}} \Psi^{\downarrow}_{\{x_i,\vec{k}_{i\perp},\lambda_i\}} + \\
&\quad \Psi^{\downarrow *}_{\{x_i',\vec{k}_{i\perp}',\lambda_i'\}} \Psi^{\uparrow}_{\{x_i,\vec{k}_{i\perp},\lambda_i\}} \Big\},
\end{aligned}
$$

Nucleon gravitational form factors

The GFFs $C_q(Q^2)$ and $\bar{C}_q(Q^2)$ can be written in terms of the overlap of LFWFs

$$
\begin{aligned}
C_q(Q^2) &= \frac{1}{8\,q^{(1)}q^{(2)}} \sum_{\{\lambda_i\}} \int [\mathrm{d}\mathcal{X}\,\mathrm{d}\mathcal{P}_\perp]\,\mathcal{O}^1\mathcal{O}^2 \Big\{ \Psi^{\uparrow *}_{\{x_i', \vec{k}_{i\perp}', \lambda_i'\}} \Psi^{\uparrow}_{\{x_i, \vec{k}_{i\perp}, \lambda_i\}} + \\
&\quad \Psi^{\downarrow *}_{\{x_i', \vec{k}_{i\perp}', \lambda_i'\}} \Psi^{\downarrow}_{\{x_i, \vec{k}_{i\perp}, \lambda_i\}} \Big\}, \\
\overline{C}_q(Q^2) &= \frac{1}{8\,M^2} \Bigg[-\frac{1}{2} \sum_{\{\lambda_i\}} \int [\mathrm{d}\mathcal{X}\,\mathrm{d}\mathcal{P}_\perp] \left(\mathcal{O}^1\mathcal{O}^1 + \mathcal{O}^2\mathcal{O}^2 \right) \\
&\quad \Big\{ \Psi^{\uparrow *}_{\{x_i', \vec{k}_{i\perp}', \lambda_i'\}} \Psi^{\uparrow}_{\{x_i, \vec{k}_{i\perp}, \lambda_i\}} + \\
&\quad \Psi^{\downarrow *}_{\{x_i', \vec{k}_{i\perp}', \lambda_i'\}} \Psi^{\downarrow}_{\{x_i, \vec{k}_{i\perp}, \lambda_i\}} \Big\} - 4q^2 C_q(Q^2) \Bigg],
\end{aligned}
$$

where the operator $\mathcal{O}^j = 2k_\perp^{(j)} + (1-x)q_\perp^{(j)}$