Light-Cone 2023: Hadrons and Symmetries

Rio de Janeiro, September 18-22, 2023

CBPF CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Beyond valence distributions in meson with Basis Light-front Quantization

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Rio de Janeiro · September 19, 2023


06/04/2023 Excursion

Liujiaxia Reservoir
Elevation ~ 2050m

Outline

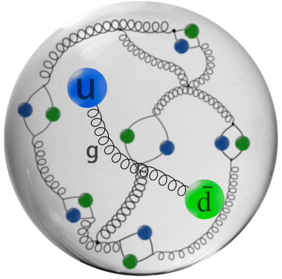
- Basis Light-front Quantization approach
- Application to Light Mesons
  - A review of BLFQ-NJL
  - Light mesons with one dynamical gluon (LFWF, EMFF, PDA, PDF, GPD & TMD)
  - The pion $q\bar{q}g$ BLFQ .vs. BSE
- Application to Strangeonia and Strange mesons
- Conclusion & Outlook

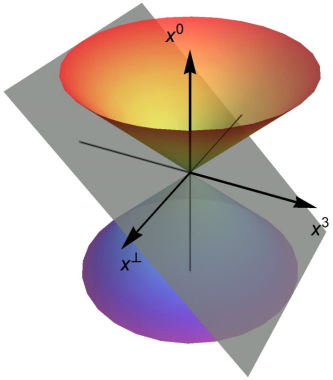
Basis Light-front Quantization

- Nonperturbative eigenvalue problem [Vary et al, 2008]

$$P^{-}|\beta\rangle = P_{\beta}^{-}|\beta\rangle$$

  - P^{-}: light-front Hamiltonian
  - $|\beta\rangle$: mass eigenstate
  - P_{β}^{-}: eigenvalue for $|\beta\rangle$

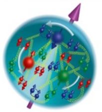




- Evaluate observables for eigenstate

$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$

- Fock sector expansion
  - Eg. $|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + d|q\bar{q}gg\rangle + \cdots$



$|\text{Baryon}\rangle = a|qqq\rangle + b|qqqg\rangle + c|qqq\bar{q}q\rangle + d|qqqgg\rangle + \cdots$

Siqi Xu, Wednesday at 9:30

- Discretized basis
  - Transverse: 2D harmonic oscillator basis: $\Phi_{n,m}^{b}(\vec{p}_{\perp})$.
  - Longitudinal: plane-wave basis, labeled by k.
  - Basis truncation:

$$\sum_i (2n_i + |m_i| + 1) \leq N_{max},$$
$$\sum_i k_i = K.$$

N_{max}, K are basis truncation parameters.

Large N_{max} and K : High UV cutoff & low IR cutoff

Take a review

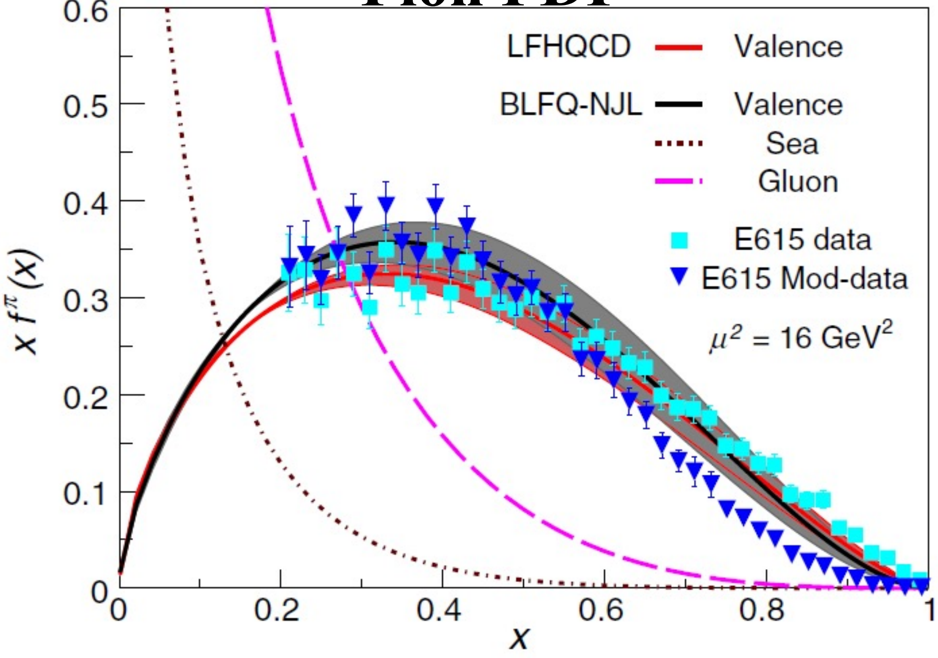
PDFs for Light Mesons

$|\mathrm{meson}\rangle = |q\bar{q}\rangle + \cdots$

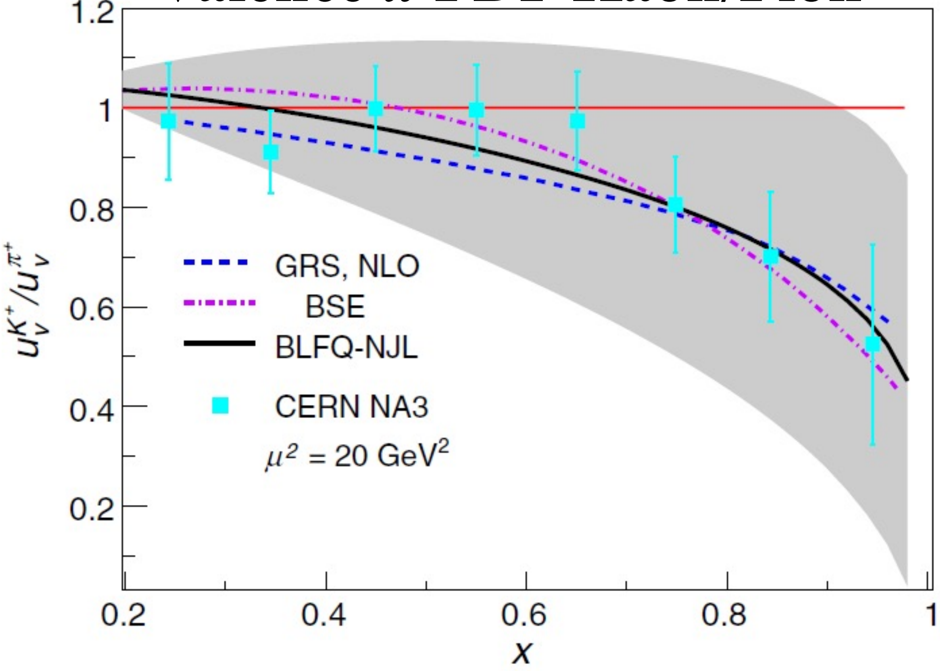
$$H_{\mathrm{eff}} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x\left(x(1-x)\partial_x\right) + H_{\mathrm{eff}}^{\mathrm{NJL}}$$

Diagonalizing H_{eff} → LF wave function → Initial PDFs → Scale evolution

Pion PDF



Valence *u* PDF Kaon/Pion

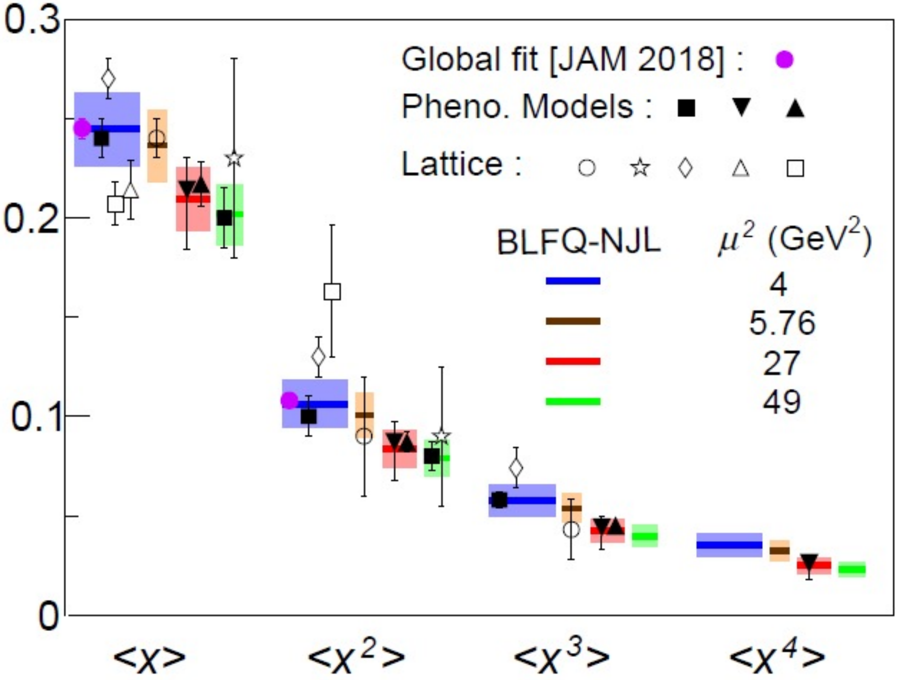


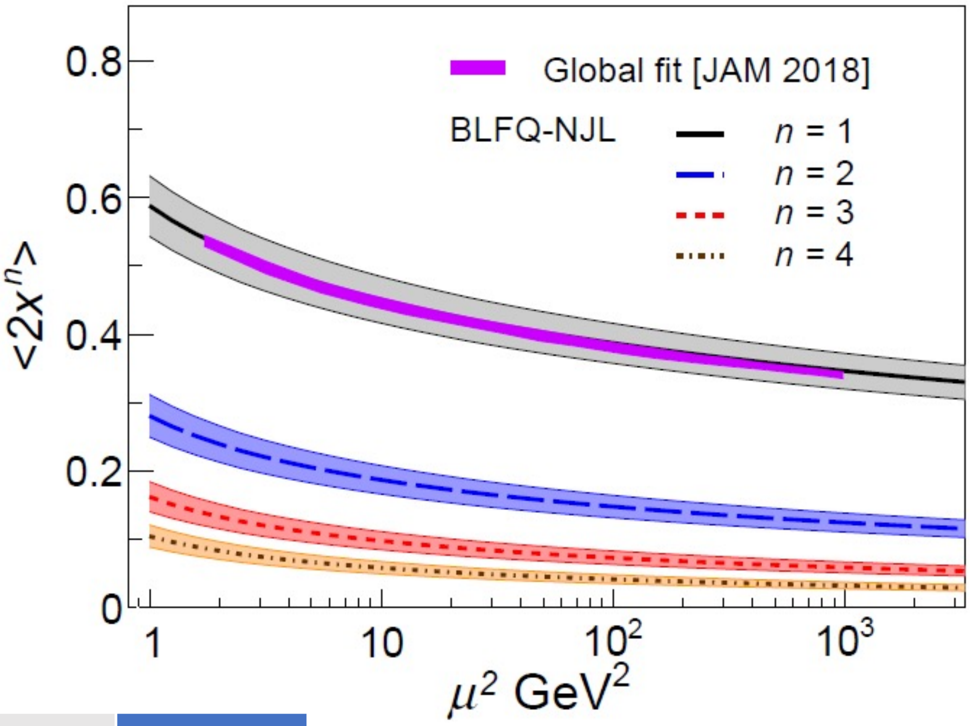
[Lan, Mondal, Jia, Zhao, Vary, PRL122(2019)172001]

Agree with experimental results

The Moments of Pion Valence Quark PDF

$$\langle x^n \rangle = \int_0^1 dx\ x^n f_v^{\pi/K}(x,\mu^2),\ \ n = 1, 2, 3, 4.$$





$\langle x \rangle$ @ 4 GeV²	Valence	Gluon	Sea
BLFQ-NJL	**0.489**	**0.398**	**0.113**
[Ding *et. al.*, BSE model 2019']	0.48(3)	0.41(2)	0.11(2)

Agree with other results

[Lan, Mondal, Jia, Zhao, Vary, PRD101(2020)034024]

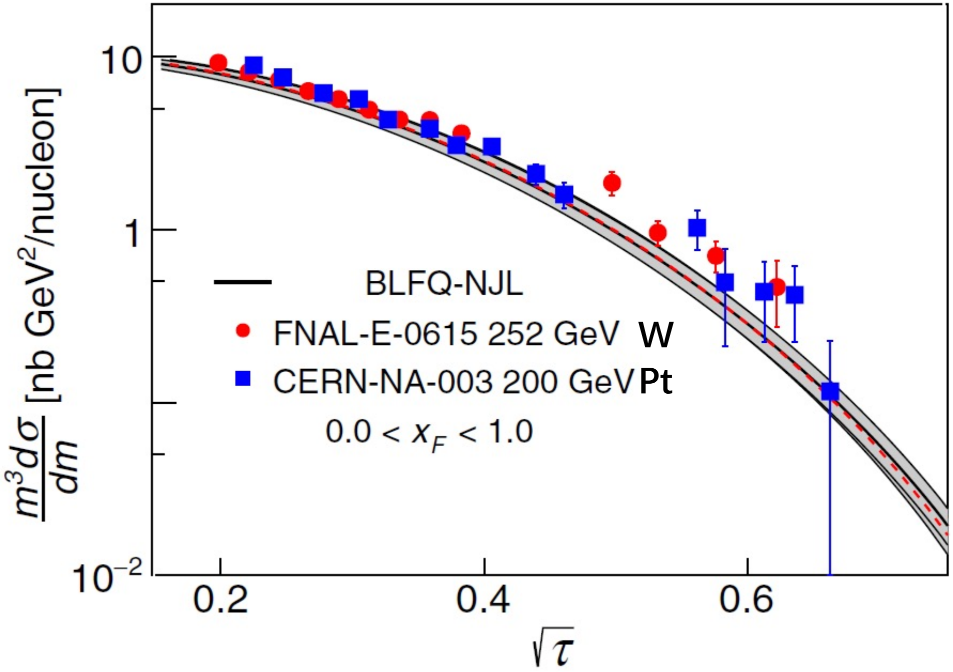
Drell-Yan Cross Section

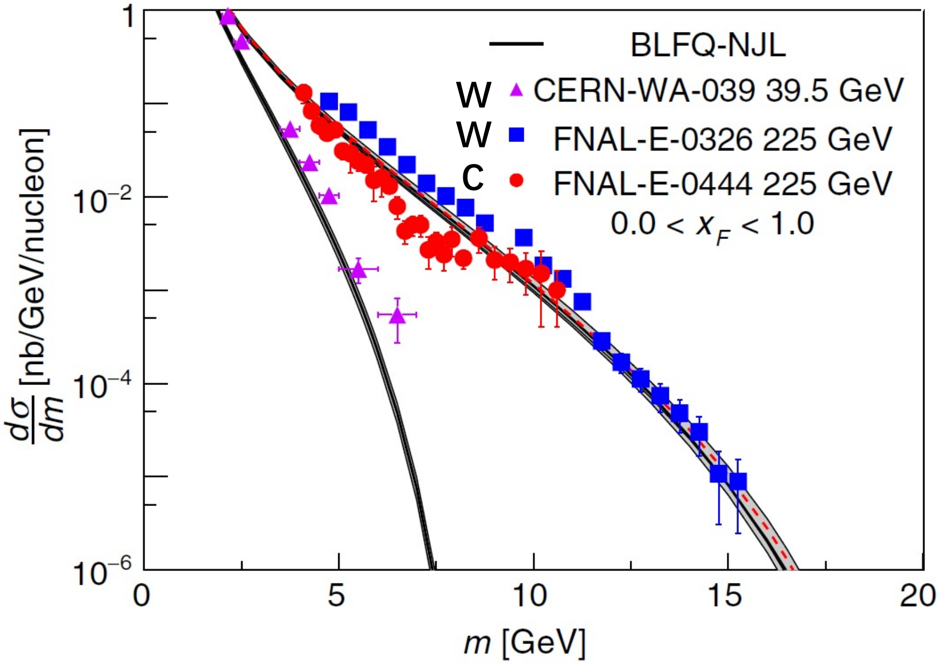
$$\pi^- \ \text{Nucleus} \rightarrow \mu^+ \ \mu^- \ X$$

[S. D. Drell and T.-M. Yan, PRL (1970)]
[T. Becher et al, JKEP07(2008)030]; [P. C. Barry et al, PRL121(2018)152001]
[C. Anastasiou et al, PRL91(2003)182002]

[nCTEQ 2015]

$$\frac{m^3 d^2\sigma}{dmdY} = \frac{8\pi\alpha^2}{9}\frac{m^2}{s}\sum_{ij} dx_1 dx_2 \tilde{C}_{ij}(x_1, x_2, s, m, \mu_f) f_{i/\pi}(x_1, \mu_f) f_{j/N}(x_2, \mu_f)$$





Agree with experimental data (**FNAL E615**, **326**, **444**, & **CERN NA3**, **WA-039**).

[Lan, Mondal, Jia, Zhao, Vary, PRD101(2020)034024]

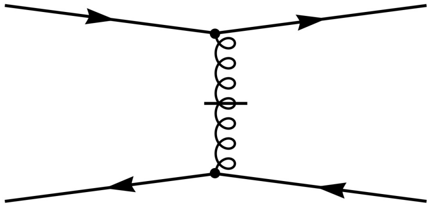
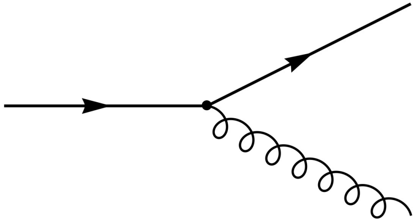
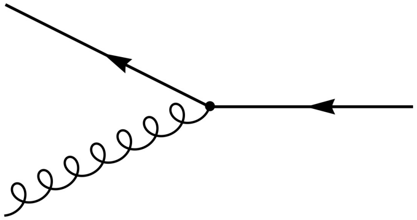
One step forward

[Lan, et al, PRL 19']

$$|\text{meson}\rangle = |q\bar{q}\rangle + \cdots$$

[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$\boldsymbol{H}_{\textbf{int}}$	$\lvert q\bar{q}\rangle$	$\lvert q\bar{q}g\rangle$
$\langle q\bar{q}\rvert$		
$\langle q\bar{q}g\rvert$		0

$$H_{\text{eff}} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x(x(1-x)\partial_x) + H_{\text{eff}}^{\text{NJL}}$$

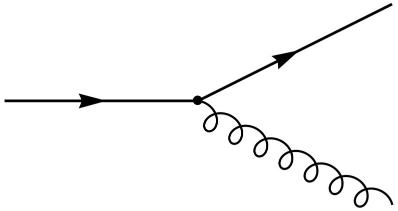
$$P^- = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x(x(1-x)\partial_x) + \boldsymbol{H}_{\textbf{int}}$$

Light-front QCD Hamiltonian

[Brodsky et al, 1998]

$$P_{-,LFQCD} = \frac{1}{2}\int d^3x\, \bar{\psi}\gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+}\psi + \frac{1}{2}\int d^3x\, A^i_a (i\partial^\perp)^2 A^i_a$$

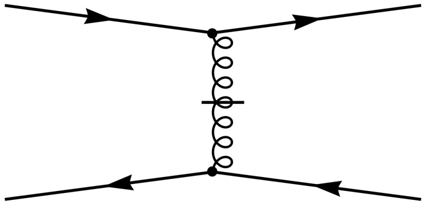
$$\boxed{+g\int d^3x\, \bar{\psi}\gamma_\mu A^\mu\, \psi}$$



$$+\frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma_\mu A^\mu \frac{\gamma^+}{i\partial^+}\gamma_\nu A^\nu \psi$$

$$-ig^2\int d^3x\, f^{abc}\bar{\psi}\gamma^+ T^c\psi \frac{1}{(i\partial^+)^2}(i\partial^+ A^\mu_a A_{\mu b})$$

$$\boxed{+\frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma^+ T^a\psi \frac{1}{(i\partial^+)^2}\bar{\psi}\gamma^+ T^a\psi}$$

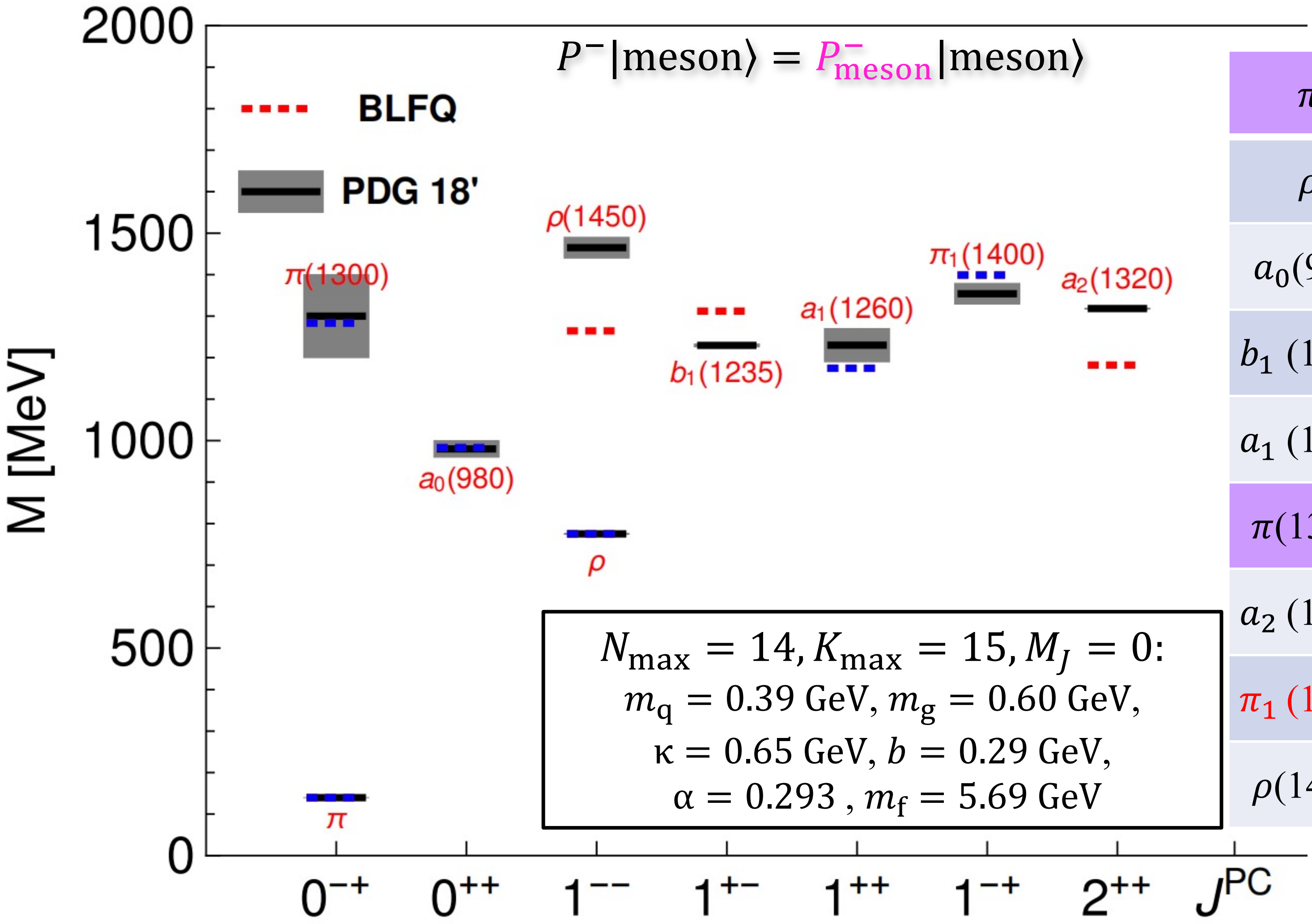


$$+ig\int d^3x\, f^{abc} i\partial^\mu A^{\nu a} A^b_\mu A^c_\nu$$

$$-\frac{1}{2}g^2\int d^3x\, f^{abc}\, f^{ade}\, i\partial^+ A^\mu_b A_{\mu c}\frac{1}{(i\partial^+)^2}(i\partial^+ A^+_d A_{\nu e})$$

$$+\frac{1}{4}g^2\int d^3x\, f^{abc}\, f^{ade}\, A^\mu_b A^\nu_c A_{\mu d} A_{\nu e}.$$

Light Meson Mass Spectrum



$$P^-|\text{meson}\rangle = P^-_{\text{meson}}|\text{meson}\rangle$$

$N_{\max} = 14, K_{\max} = 15, M_J = 0$:
$m_q = 0.39$ GeV, $m_g = 0.60$ GeV,
$\kappa = 0.65$ GeV, $b = 0.29$ GeV,
$\alpha = 0.293$, $m_f = 5.69$ GeV

	Norm$q\bar{q}$	**DC [MeV]**
π	0.492	151
ρ	0.486	141
$a_0(980)$	0.370	0
b_1 (1235)	0.30	0
a_1 (1260)	0.324	13
$\pi(1300)$	0.284	59
a_2 (1320)	0.320	0
π_1 (1400)	0.002	0
$\rho(1450)$	0.312	50

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

Fix the parameters by fitting six blue states

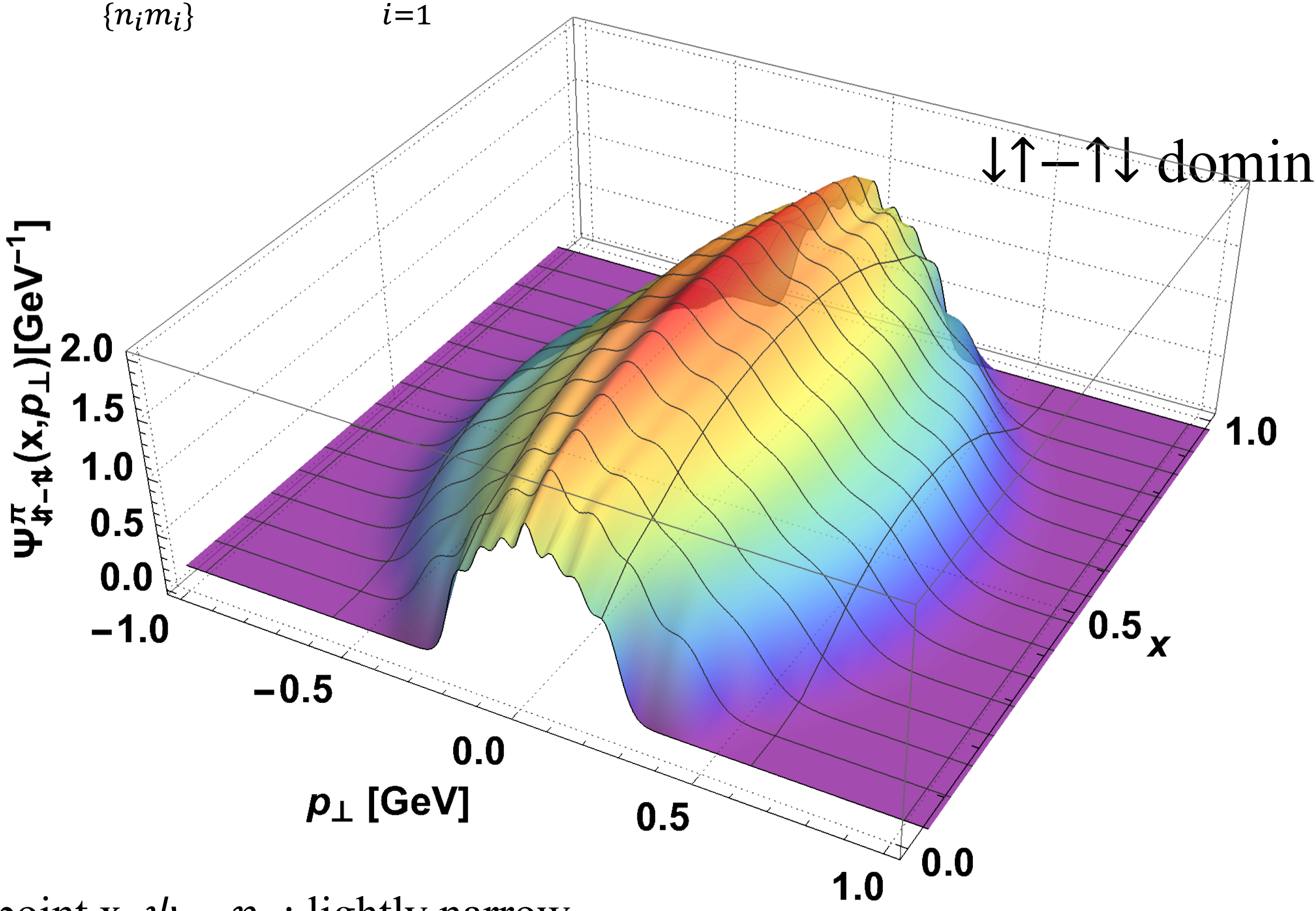
- $\pi_1(1400)$: $|q\bar{q}g\rangle$ dominates
- $\pi(1300)$: the DC is smaller than the DC of pion

[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]

The Wave Function in Leading Fock Sector

$$\Psi^{\mathcal{N},M_J}_{\{x_i,\vec{p}_{\perp i}^{\,2},\lambda_i\}} = \sum_{\{n_i m_i\}} \psi^{\mathcal{N}}(\{\bar{\alpha}_i\}) \prod_{i=1}^{\mathcal{N}} \phi_{n_i m_i}(\vec{p}_{\perp i}, b)$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$



$\downarrow\uparrow - \uparrow\downarrow$ dominant in $|q\bar{q}\rangle$

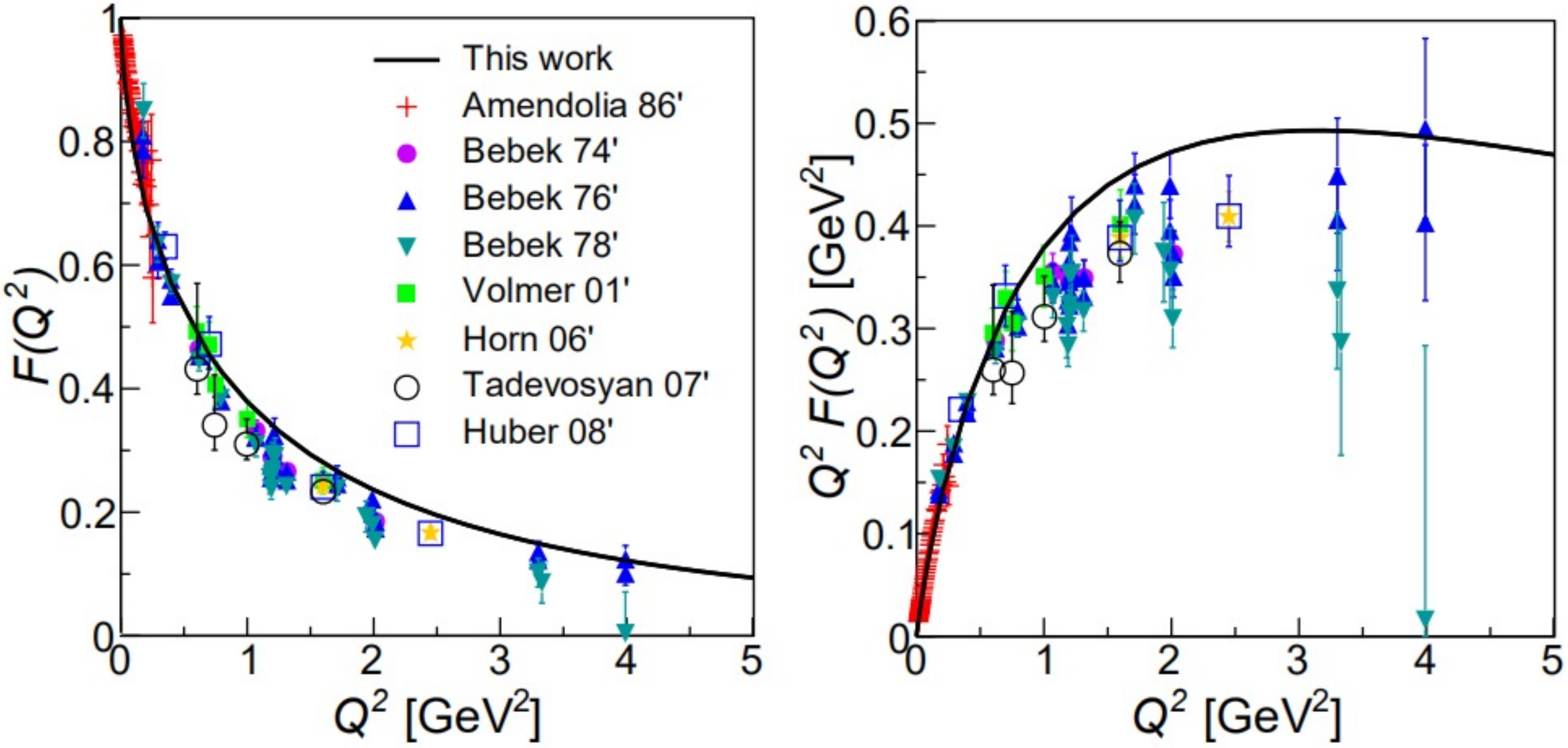
- At endpoint x, $\psi \sim p_\perp$: lightly narrow
- At middle x, $\psi \sim p_\perp$: a little bit wide

Pion Electromagnetic Form Factor

[Brodsky & de Teramond, PRD 77(2008)056007]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$$\langle\Psi(p')|J^{+}_{EM}(0)|\Psi(p)\rangle = (\mathrm{p}+\mathrm{p}')^{+}\,\mathrm{F}(\mathrm{Q}^2)$$



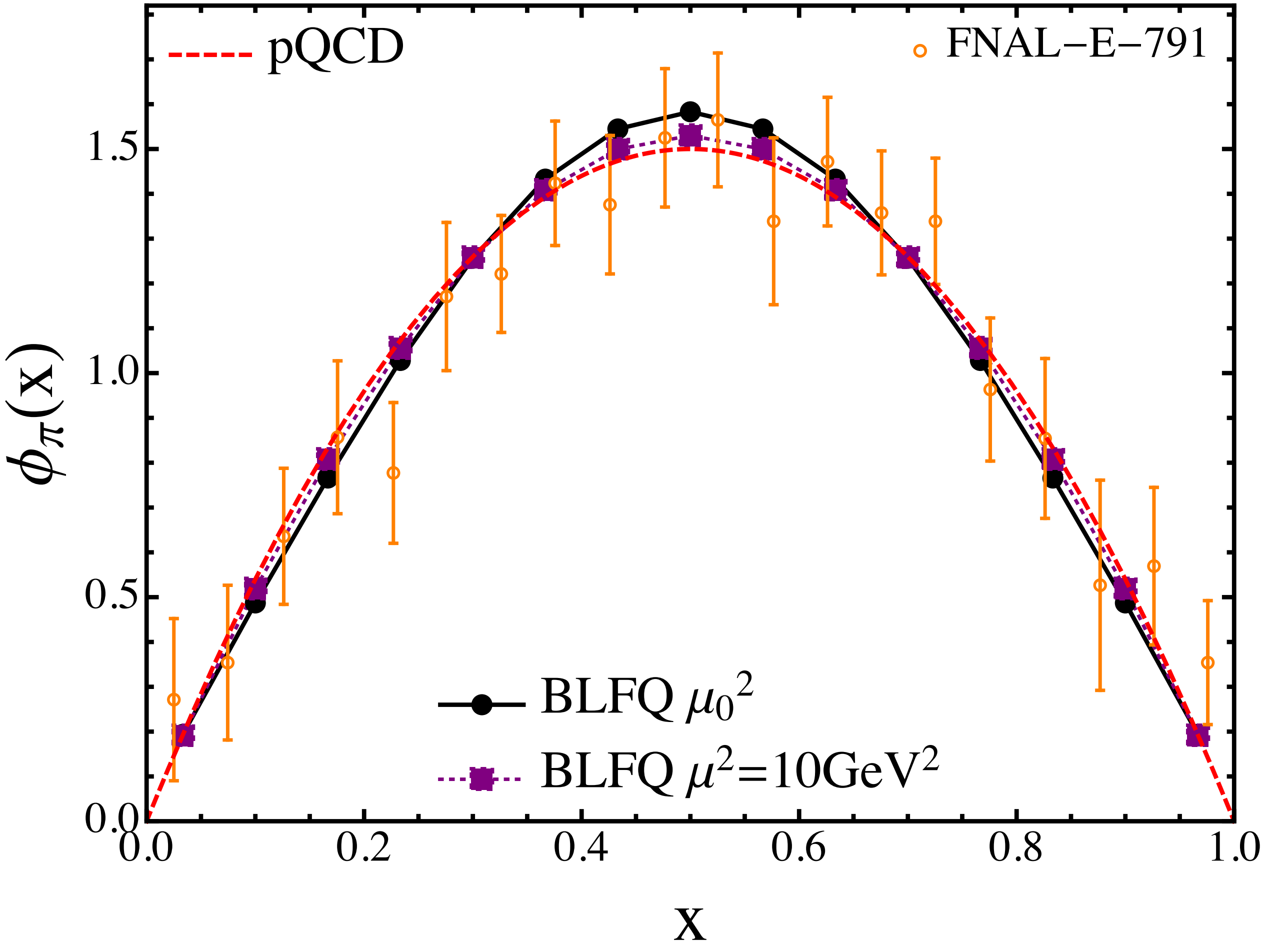
- FF is in reasonable agreement with experimental data
- $\mathrm{F}(\mathrm{Q}^2) \propto 1/\,\mathrm{Q}^2$ for large Q^2

[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]

Pion PDA

$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

[Ruiz Arriola & Broniowski, PRD 66(2002)094016]



- Endpoint behavior almost agrees with pQCD
- Consistent with FNAL-E-791 experiment

Preliminary

Pion PDF at Model Scale

$$f_i(x) = \sum_{\mathcal{N},\lambda_i} \int [d\mathcal{X} d\mathcal{P}^{\perp}]_{\mathcal{N}} \left| \psi^{\mathcal{N},M_j=0}_{\{x_i,\vec{p}^{\,2}_{\perp i},\lambda_i\}} \right|^2 \delta(x - x_i)$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

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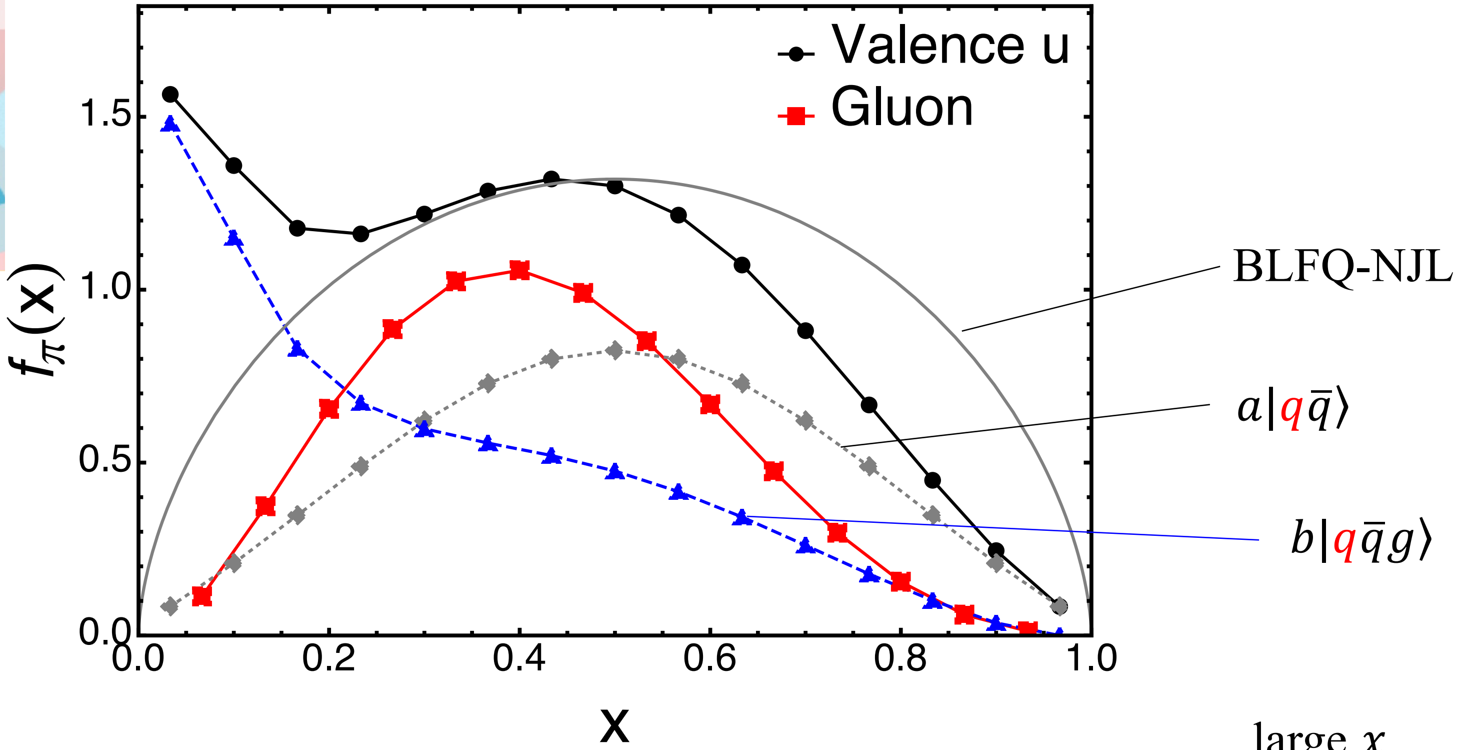
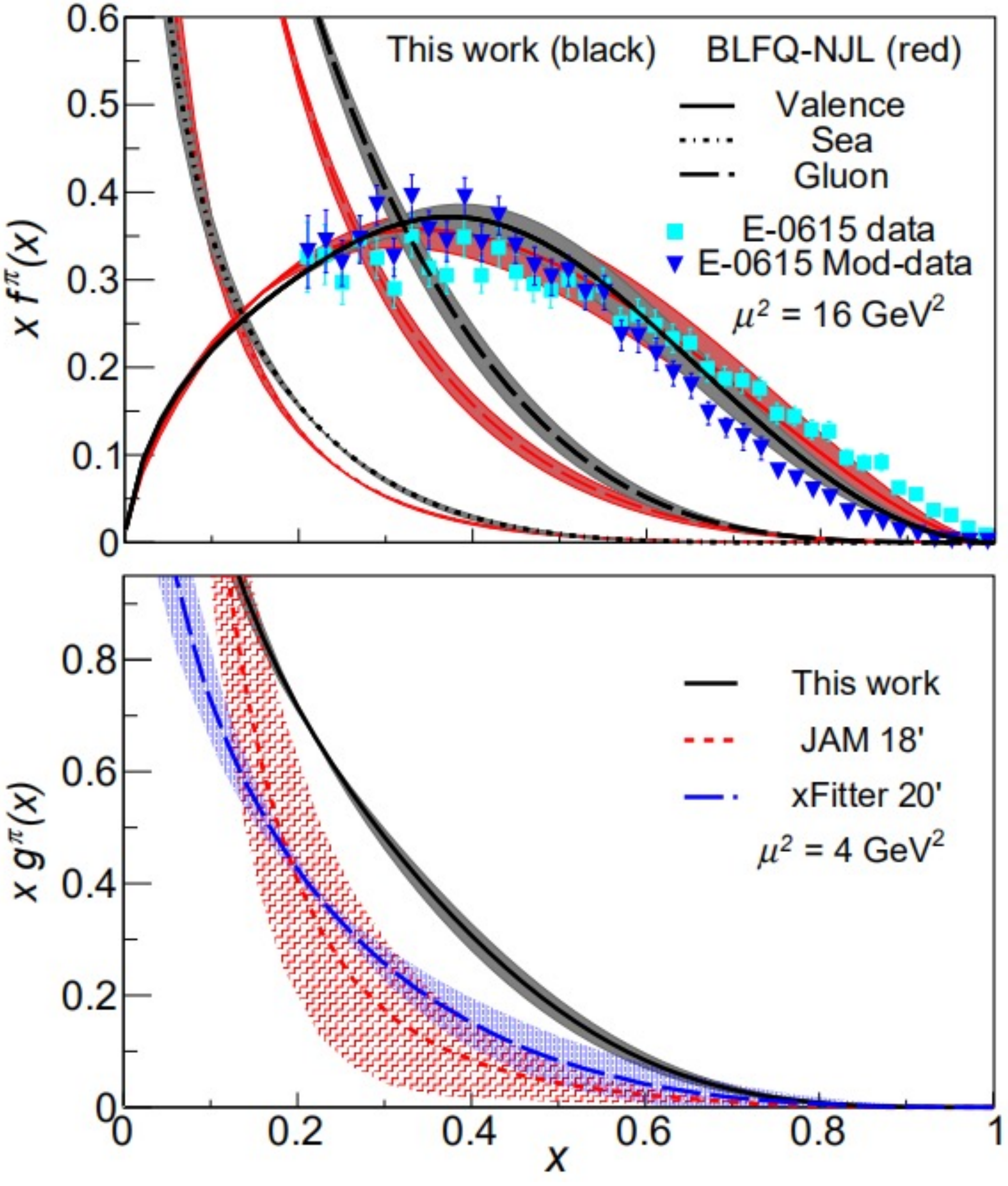


			large x
$\mu^2_{0\mathrm{BLFQ-NJL}} = 0.24\ \mathrm{GeV}^2$	$\langle x\rangle_{\mathbf{gluon}} = \mathbf{0}$;	$\langle x\rangle_{\mathbf{valence}\ u} = \mathbf{0.5}$	$(1-x)^{0.596}$
$\mu^2_{0\mathrm{BLFQ}} = 0.34\ \mathrm{GeV}^2$	$\langle x\rangle_{\mathbf{gluon}} = \mathbf{0.216}$;	$\langle x\rangle_{\mathbf{valence}\ u} = \mathbf{0.392}$	$(1-x)^{1.4}$

[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]

Pion PDF with QCD Evolution



$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

- Large-x behavior $(1-x)^{1.77}$ closer to pQCD
- The gluon distribution significantly increases

$\langle x\rangle$ @ 4 GeV^2	Valence	Gluon	Sea
BLFQ	**0.483**	**0.421**	**0.096**
BLFQ-NJL	0.489	0.398	0.113
[BSE 2019’]	0.48(3)	0.41(2)	0.11(2)

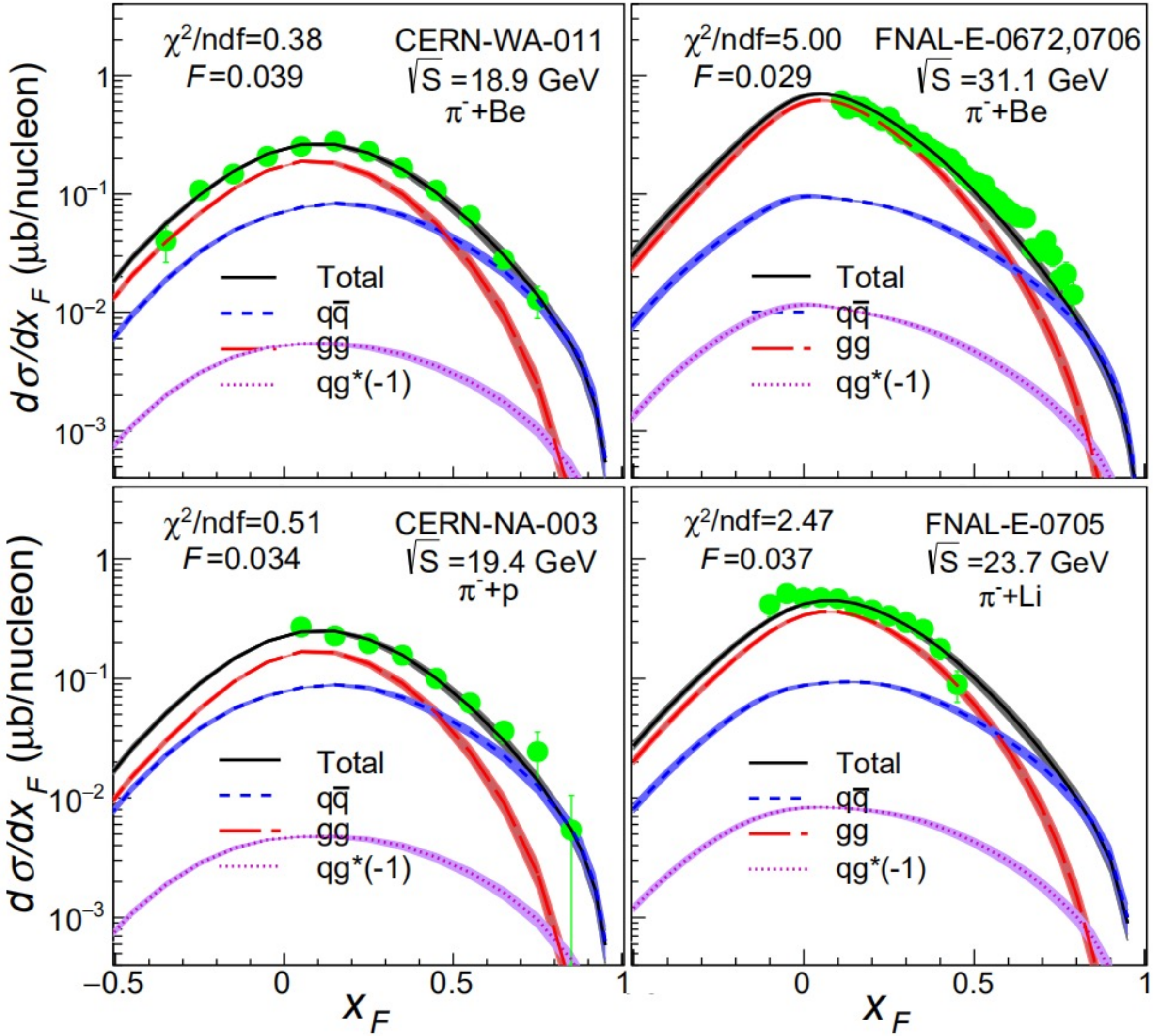
[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]

J/ψ production cross section

$\pi^{\pm} N \to J/\psi\ X$

$$\frac{d\sigma}{dx_F}|J/\psi = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{S\sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{S}}} \hat{\sigma}_{ij}\,(s, m_c^2, \mu_R^2, \mu_F^2) f_i^{\pi^{\pm}}(x_1, \mu_F^2) f_j^N(x_2, \mu_F^2)$$

[nCTEQ 2015]



CEM

[Chang, et al, PRD 102 (2020) 054024];
[Nason, et al, NPB 303 (1988) 607];
[Mangano, et al, NPB 405 (1993) 507]

- significantly gg contribution
- various energies of pions
- different target

Agree with experimental data (FNAL E672, E706, E705, CERN NA3,WA11).

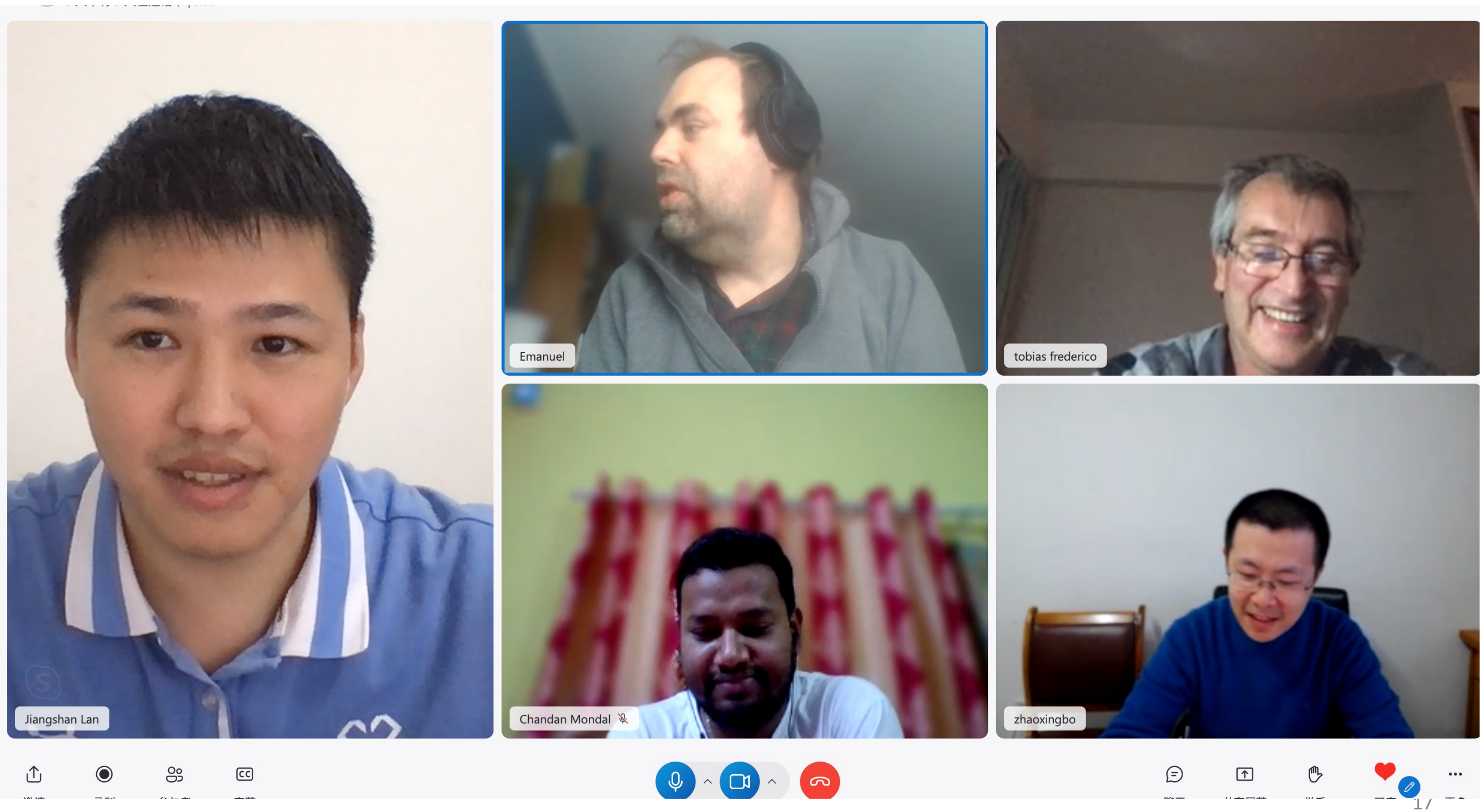
[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]

The pion $q\bar{q}g$ BLFQ .vs. BSE

BLFQ
Phys. Lett. B 825 (2022) 136890

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

BSE in Minkowski space
Phys. Rev. D 103 (2021) 014002
Phys. Rev. D 105 (2022) L071505
Phys. Lett. B 820 (2021) 136494



The pion $q\bar{q}g$ BLFQ .vs. BSE

$$P^-P^+|\Psi\rangle = M^2|\Psi\rangle$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$$|\Psi\rangle =$$
$$\sum_{i_q s_q}\sum_{i_q s_q}\int\left[\prod_{j=q,\bar{q}}\frac{d^3p_j}{(2\pi)^3 2p_j^+}\right]2P^+(2\pi)^3\delta^3(\boldsymbol{p}_q+\boldsymbol{p}_{\bar{q}}-\boldsymbol{P})\,\psi^{s_q s_q}_{q\bar{q},i_q i_q}(x_q,\boldsymbol{p}_{q\perp},x_{\bar{q}},\boldsymbol{p}_{\bar{q}\perp})b^\dagger_{i_q s_q}(\boldsymbol{p}_q)d^\dagger_{i_q s_q}(\boldsymbol{p}_{\bar{q}})|0\rangle$$
$$+\sum_{i_q s_q}\sum_{i_q s_q}\sum_{\lambda a}\int\left[\prod_{j=q,\bar{q},g}\frac{d^3p_j}{(2\pi)^3 2p_j^+}\right]2P^+(2\pi)^3\delta^3(\boldsymbol{p}_q+\boldsymbol{p}_{\bar{q}}+\boldsymbol{p}_g-\boldsymbol{P})$$
$$\times\psi^{s_q s_q \lambda}_{q\bar{q}g,i_q i_q a}(x_q,\boldsymbol{p}_{q\perp},x_{\bar{q}},\boldsymbol{p}_{\bar{q}\perp},x_g,\boldsymbol{p}_{g\perp})b^\dagger_{i_q s_q}(\boldsymbol{p}_q)d^\dagger_{i_q s_q}(\boldsymbol{p}_{\bar{q}})a^\dagger_{\lambda a}(\boldsymbol{p}_g)|0\rangle$$
$$+\cdots$$

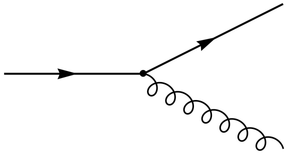
$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle$$

$$\psi^{s_q s_q \lambda}_{q\bar{q}g,i_q i_q a} = \frac{1}{M_\pi^2 - M_{0,qqg}^2}\left[V\psi^{s_q s_q}_{q\bar{q},i_q i_q}\right]$$

V: the interaction connecting the $q\bar{q}$ and $q\bar{q}g$ sector

$$\left[V\psi^{s_q s_q}_{q\bar{q},i_q i_q}\right] =$$
$$\frac{g_s\sqrt{2}}{x_q+x_g}\sum_{i_1 s_1}T^a_{i_q,i_1}\,W_\lambda^{s_q s_1}(p_q,p_g)\psi^{s_1 s_q}_{q\bar{q},i_1 i_q}(x_{\bar{q}},\boldsymbol{p}_{\bar{q}\perp})$$
$$-\frac{g_s\sqrt{2}}{x_{\bar{q}}+x_g}\sum_{i_1 s_1}T^a_{i_1,i_q}\,\overline{W}_\lambda^{s_1 s_q}(p_{\bar{q}},p_g)\psi^{s_q s_1}_{q\bar{q},i_q i_1}(x_q,\boldsymbol{p}_{q\perp})$$

$$M^2_{0,q\bar{q}g} = \sum_{j=q\bar{q}g}\frac{\boldsymbol{p}_{j\perp}^2+m_j^2}{x_j}$$



$$\boldsymbol{p}_{q\perp}+\boldsymbol{p}_{\bar{q}\perp}+\boldsymbol{p}_{g\perp} = 0,\quad x_q+x_{\bar{q}}+x_g = 1$$

For clarity, we write the spin-flip matrix elements:

[Brodsky et al. Phys. Rep. 301 (1998) 299–486]

$$W_\lambda^{+\,-}(p_q,p_g) = m_f\frac{x_g(1-x_q)}{\sqrt{x_1 x_q}}\delta_{\lambda,-}$$
$$W_\lambda^{-\,+}(p_q,p_g) = -m_f\frac{x_g(1-x_q)}{\sqrt{x_1 x_q}}\delta_{\lambda,+}$$
$$\overline{W}_\lambda^{+\,-}(p_{\bar{q}},p_g) = \lambda m_f\frac{x_g(1-x_{\bar{q}})}{\sqrt{x_1 x_{\bar{q}}}}$$
$$\overline{W}_\lambda^{-\,+}(p_{\bar{q}},p_g) = 0$$

[Emanuel Ydrefors, Jiangshan Lan, et al. in progress]

The pion $q\bar{q}g$ BLFQ .vs. BSE

The contribution from $q\bar{q}g$ sector to the quark and the gluon PDF:

$$\Delta u_q(x_q) = \frac{1}{(2\pi)^6}\sum_{i_q s_q}\sum_{i_{\bar{q}} s_{\bar{q}}}\sum_{\lambda a}\int \frac{d^2\boldsymbol{p}_{q\perp} d^2\boldsymbol{p}_{g\perp} dx_g}{4x_q x_g(1-x_q-x_g)}\left|\psi^{s_q s_{\bar{q}}\lambda}_{q\bar{q}g, i_q i_{\bar{q}} a}(x_q, \boldsymbol{p}_{q\perp}, x_{\bar{q}}, \boldsymbol{p}_{\bar{q}\perp}, x_g, \boldsymbol{p}_{g\perp})\right|^{\mathbf{2}}$$

$$\Delta u_g(x_g) = \frac{1}{(2\pi)^6}\sum_{i_q s_q}\sum_{i_{\bar{q}} s_{\bar{q}}}\sum_{\lambda a}\int \frac{d^2\boldsymbol{p}_{q\perp} d^2\boldsymbol{p}_{g\perp} dx_q}{4x_q x_g(1-x_q-x_g)}\left|\psi^{s_q s_{\bar{q}}\lambda}_{q\bar{q}g, i_q i_{\bar{q}} a}(x_q, \boldsymbol{p}_{q\perp}, x_{\bar{q}}, \boldsymbol{p}_{\bar{q}\perp}, x_g, \boldsymbol{p}_{g\perp})\right|^{\mathbf{2}}$$

[Phys. Rev. D 50 (1994) 6895]

Take a power-law form for simplicity:

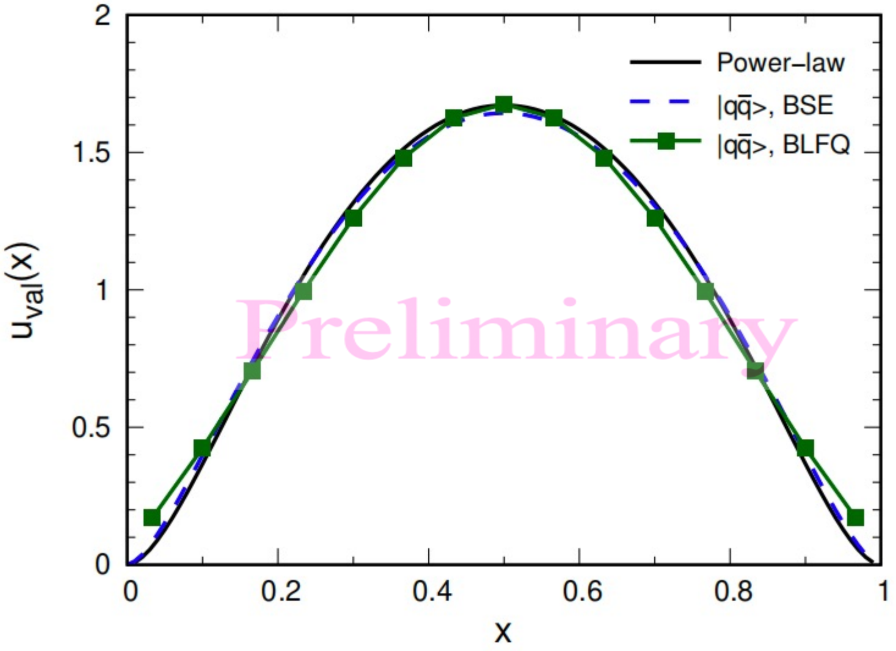
$$\psi_{pl}(x_q, \boldsymbol{p}_{q\perp}) = N\left[1 + \frac{A_{0,eff}(x_q, \boldsymbol{p}_{q\perp})/4 - m_q^2}{\beta^2}\right]^{-s}$$

$$A_{0,eff}(x_q, \boldsymbol{p}_{q\perp}) = \frac{\boldsymbol{p}_{q\perp}^2 + m_q^2}{x_q} + \frac{\boldsymbol{p}_{\bar{q}\perp}^2 + m_{\bar{q}}^2}{x_{\bar{q}}}$$

Fitting the valence PDF of both the BLFQ and the BSE, $s = 1.4$ and $\beta/m_q = 1.16$

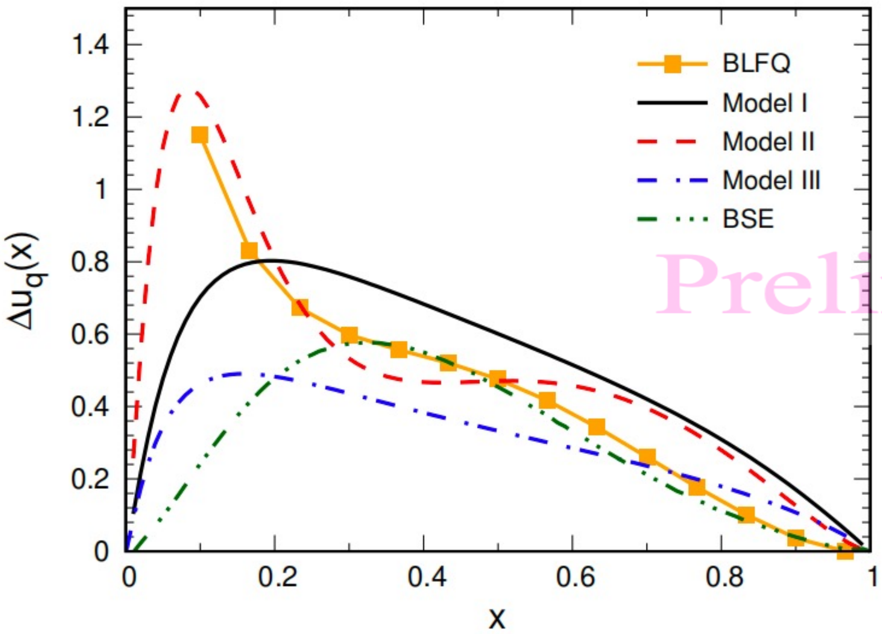
BLFQ: Phys. Lett. B 825 (2022) 136890

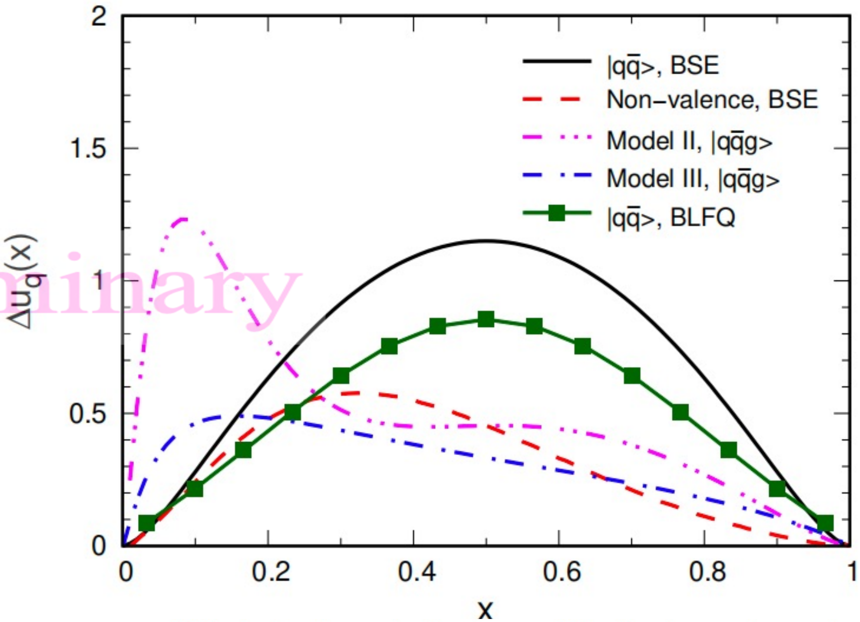
BSE: Phys. Rev. D 103 (2021) 014002



[Emanuel Ydrefors, Jiangshan Lan, et al. in progress]

The pion $q\bar{q}g$ BLFQ .vs. BSE





BLFQ: Phys. Lett. B 825 (2022) 136890
BSE: Phys. Rev. D 103 (2021) 014002

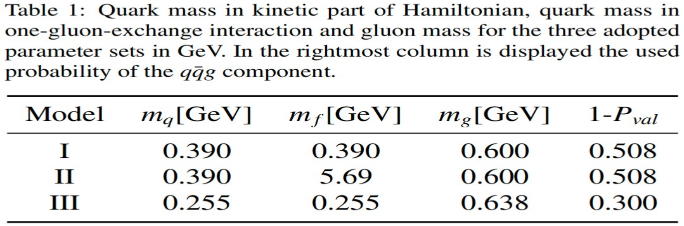


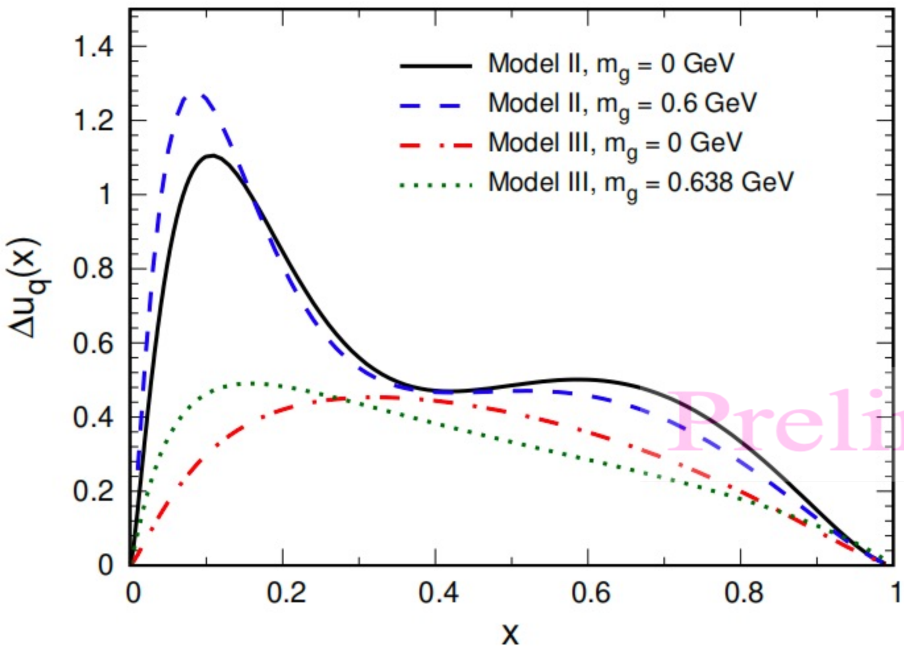
Table 1: Quark mass in kinetic part of Hamiltonian, quark mass in one-gluon-exchange interaction and gluon mass for the three adopted parameter sets in GeV. In the rightmost column is displayed the used probability of the $q\bar{q}g$ component.

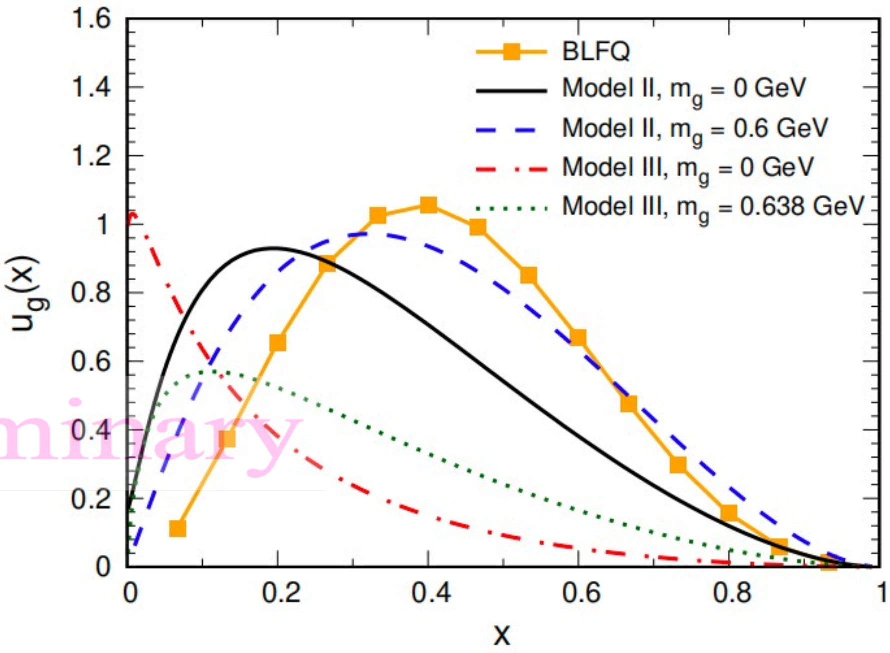
Model	m_q[GeV]	m_f[GeV]	m_g[GeV]	1-P_{val}
I	0.390	0.390	0.600	0.508
II	0.390	5.69	0.600	0.508
III	0.255	0.255	0.638	0.300

- BLFQ ~ discrete x .vs. Model ~ continuous x
- The Model II agrees with the BLFQ: a large bump at low-x is reproduced
- Model I .vs. II: the bump is related to the large value of m_f
- The BSE result differs from Model III: the BSE result contains the contributions from $q\bar{q}ng$, where $n = 1,2, \dots, \infty$
- The second Fock sector is mostly important at small- x

[Emanuel Ydrefors, Jiangshan Lan, et al. in progress]

The pion $q\bar{q}g$ BLFQ .vs. BSE





BLFQ: Phys. Lett. B 825 (2022) 136890
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I	0.390	0.390	0.600	0.508
II	0.390	5.69	0.600	0.508
III	0.255	0.255	0.638	0.300

The impact of the m_g on the pion PDFs:

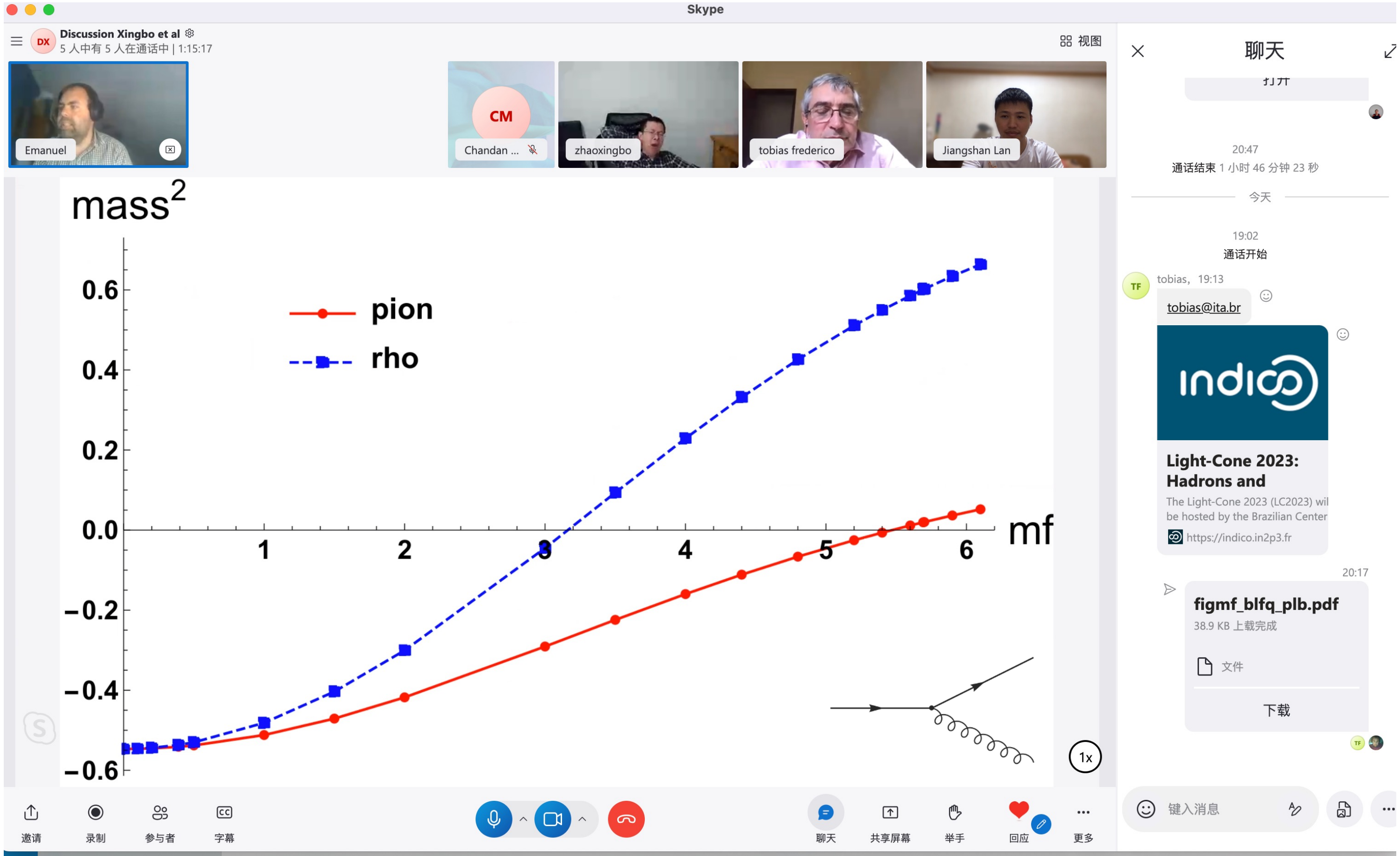
- An increase of the m_g leads to a shift of the quark PDF to lower values of x
- A larger m_g gives a gluon PDF shifted towards larger- x
- The perturbative results agree qualitatively with the BLFQ

[Emanuel Ydrefors, Jiangshan Lan, et al. in progress]

The pion $q\bar{q}g$ BLFQ .vs. BSE

We explicitly show an enhancement of the low-x contribution in the quark PDF is directly associated with the large spin-flip matrix element, necessary to provide the $\pi - \rho$ mass splitting. [M. Burkardt, *Dynamical vertex mass generation and chiral symmetry breaking on the light front*, Phys. Rev. D 58 (1998) 096015].

It was proposed to enhance the spin flip matrix element of the effective quark-gluon coupling QCD LF-Hamiltonian by introducing a large effective quark vertex mass (mf).



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通话结束 1 小时 46 分钟 23 秒

19:02
通话开始

tobias，19:13
tobias@ita.br

Light-Cone 2023: Hadrons and

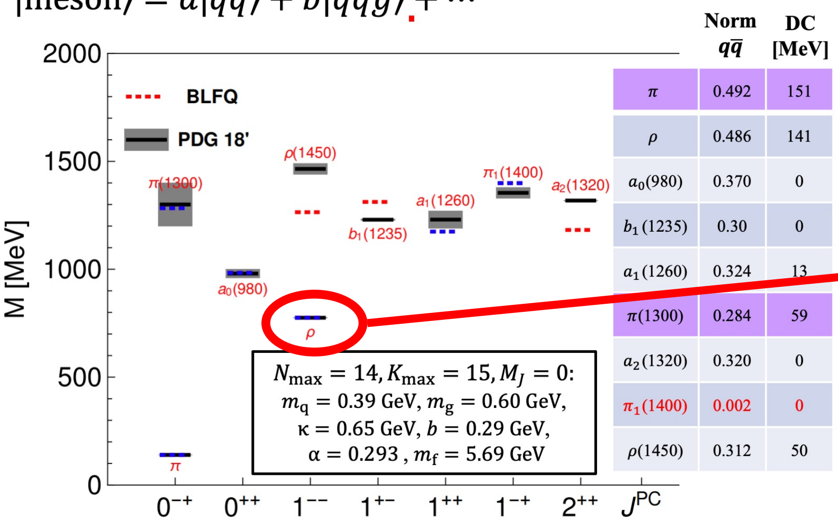
The Light-Cone 2023 (LC2023) wil be hosted by the Brazilian Center

https://indico.in2p3.fr

20:17
figmf_blfq_plb.pdf
38.9 KB 上载完成

Vector meson Structure

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$



$N_{\max} = 14, K_{\max} = 15, M_J = 0$:
$m_q = 0.39$ GeV, $m_g = 0.60$ GeV,
$\kappa = 0.65$ GeV, $b = 0.29$ GeV,
$\alpha = 0.293$, $m_f = 5.69$ GeV

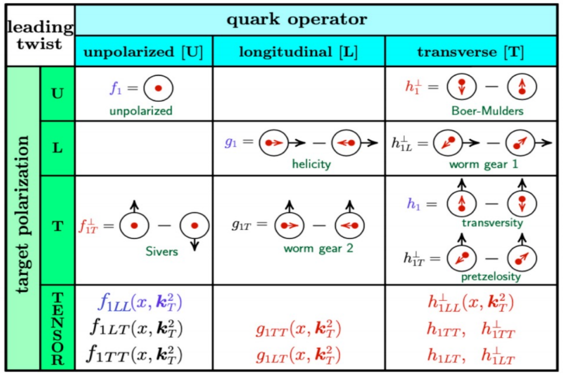
	Norm $q\bar{q}$	DC [MeV]
π	0.492	151
ρ	0.486	141
$a_0(980)$	0.370	0
$b_1(1235)$	0.30	0
$a_1(1260)$	0.324	13
$\pi(1300)$	0.284	59
$a_2(1320)$	0.320	0
$\pi_1(1400)$	0.002	0
$\rho(1450)$	0.312	50

"Structure of spin-1 QCD systems using light-front Hamiltonian approach"

Satvir Kaur

Thursday 4:30pm-4:50pm

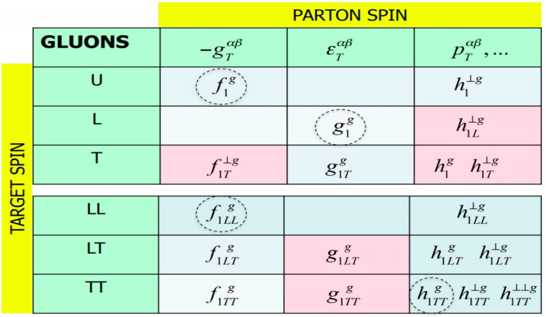
Quark TMDs of a spin-1 target



leading twist		quark operator		
		unpolarized [U]	**longitudinal [L]**	**transverse [T]**
target polarization	U	$f_1 =$ unpolarized		$h_1^\perp =$ − Boer-Mulders
	L		$g_1 =$ − helicity	$h_{1L}^\perp =$ − worm gear 1
	T	$f_{1T}^\perp =$ − Sivers	$g_{1T} =$ − worm gear 2	$h_1 =$ − transversity $h_{1T}^\perp =$ − pretzelosity
	TENSOR	$f_{1LL}(x, \boldsymbol{k}_T^2)$ $f_{1LT}(x, \boldsymbol{k}_T^2)$ $f_{1TT}(x, \boldsymbol{k}_T^2)$	$g_{1TT}(x, \boldsymbol{k}_T^2)$ $g_{1LT}(x, \boldsymbol{k}_T^2)$	$h_{1LL}^\perp(x, \boldsymbol{k}_T^2)$ h_{1TT}, $h_{1TT}^\perp$ h_{1LT}, $h_{1LT}^\perp$

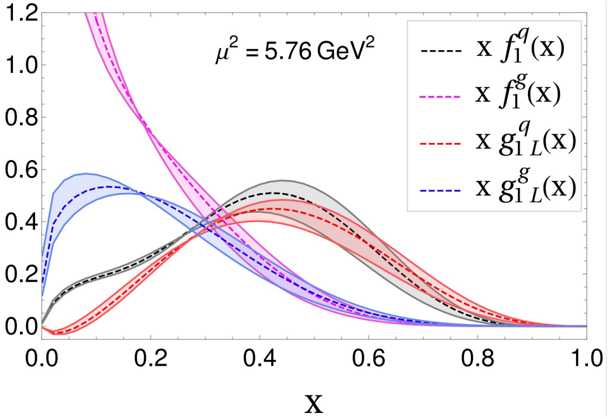
Image taken from arXiv: 2205.01249

Gluon TMDs of a spin-1 target



		PARTON SPIN		
	GLUONS	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}, \ldots$
TARGET SPIN	U	f_1^g		$h_1^{\perp g}$
	L		g_1^g	$h_{1L}^{\perp g}$
	T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g $h_{1T}^{\perp g}$
	LL	f_{1LL}^g		$h_{1LL}^{\perp g}$
	LT	f_{1LT}^g	g_{1LT}^g	h_{1LT}^g $h_{1LT}^{\perp g}$
	TT	f_{1TT}^g	g_{1TT}^g	h_{1TT}^g $h_{1TT}^{\perp g}$ $h_{1TT}^{\perp\perp g}$

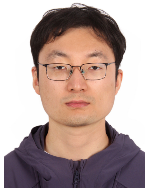
Picture Credit: P.J. Mulders



Preliminary

[S. Kaur et al, in preparation]

Pion TMD

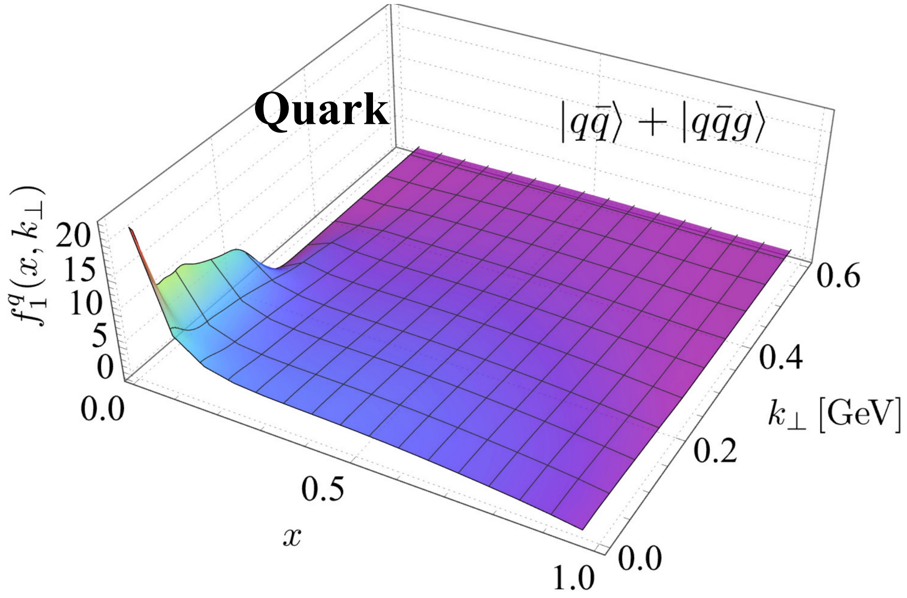


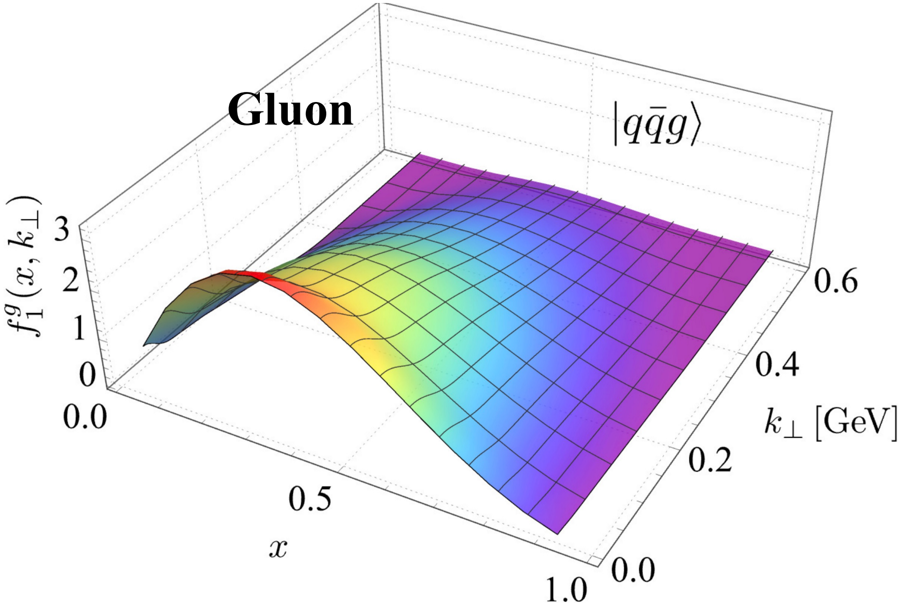
[Boer & Mulders PRD 57 (1998) 5780]
[Pasquini et al, PRD 90 (2014) 014050]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$$f_1^q(x,k_\perp) = \frac{1}{2}\int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(z^- k^+ - z_\perp k_\perp)} \left\langle \pi, P \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P \right\rangle_{z^+=0}$$

$$f_1^g(x,k_\perp) = \frac{1}{xP^+}\int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(z^- k^+ - z_\perp k_\perp)} \left\langle \pi, P \middle| G^{+\mu}\left(-\frac{z}{2}\right) G_\mu^+\left(\frac{z}{2}\right) \middle| \pi, P \right\rangle_{z^+=0}$$





- The TMD decreases with $k_\perp$
- Vanishes after $k_\perp \sim 0.6$ GeV

Preliminary

[K. Fu, J. Lan et al, in preparation]

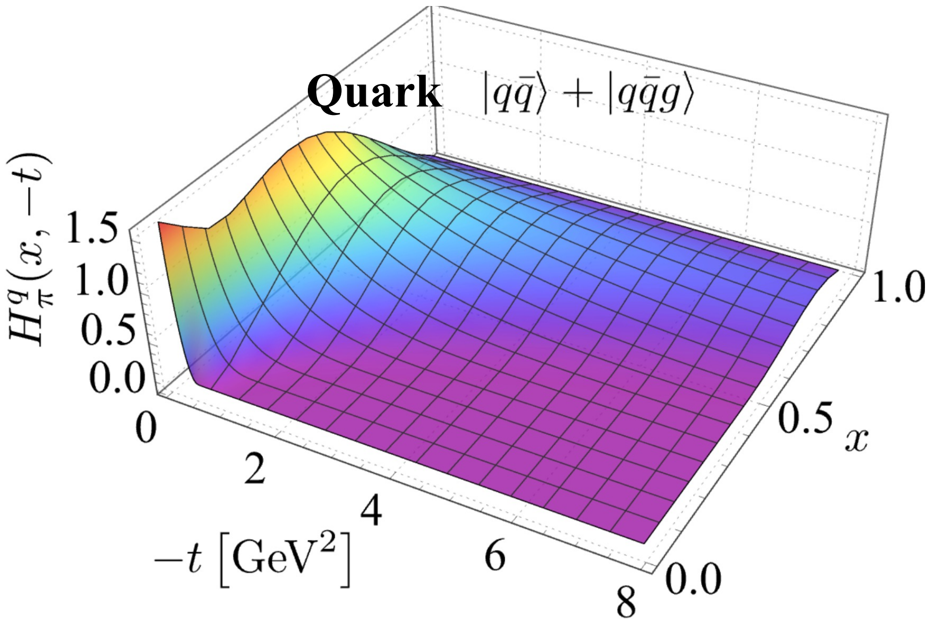
Pion GPD

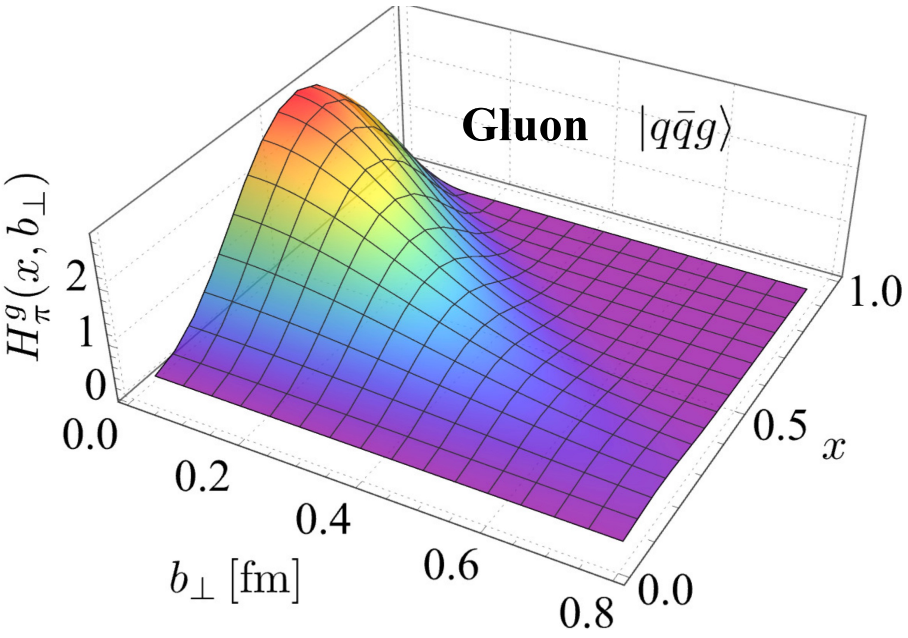
[M. Diehl, Phys. Rep. 388 (2003) 41-277]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$$H_\pi^q(x, \xi = 0, t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \right| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \left| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_\perp=0}}$$

$$H_\pi^g(x, \xi = 0, t) = \frac{1}{P^+}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \right| G^{+\mu}\left(-\frac{z}{2}\right) G_\mu^+\left(\frac{z}{2}\right) \left| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_\perp=0}}$$





- Quark content enhanced at small x with $|q\bar{q}g\rangle$
- Falls slowly at larger x
- Emerge at larger x range for larger $-t$

Preliminary

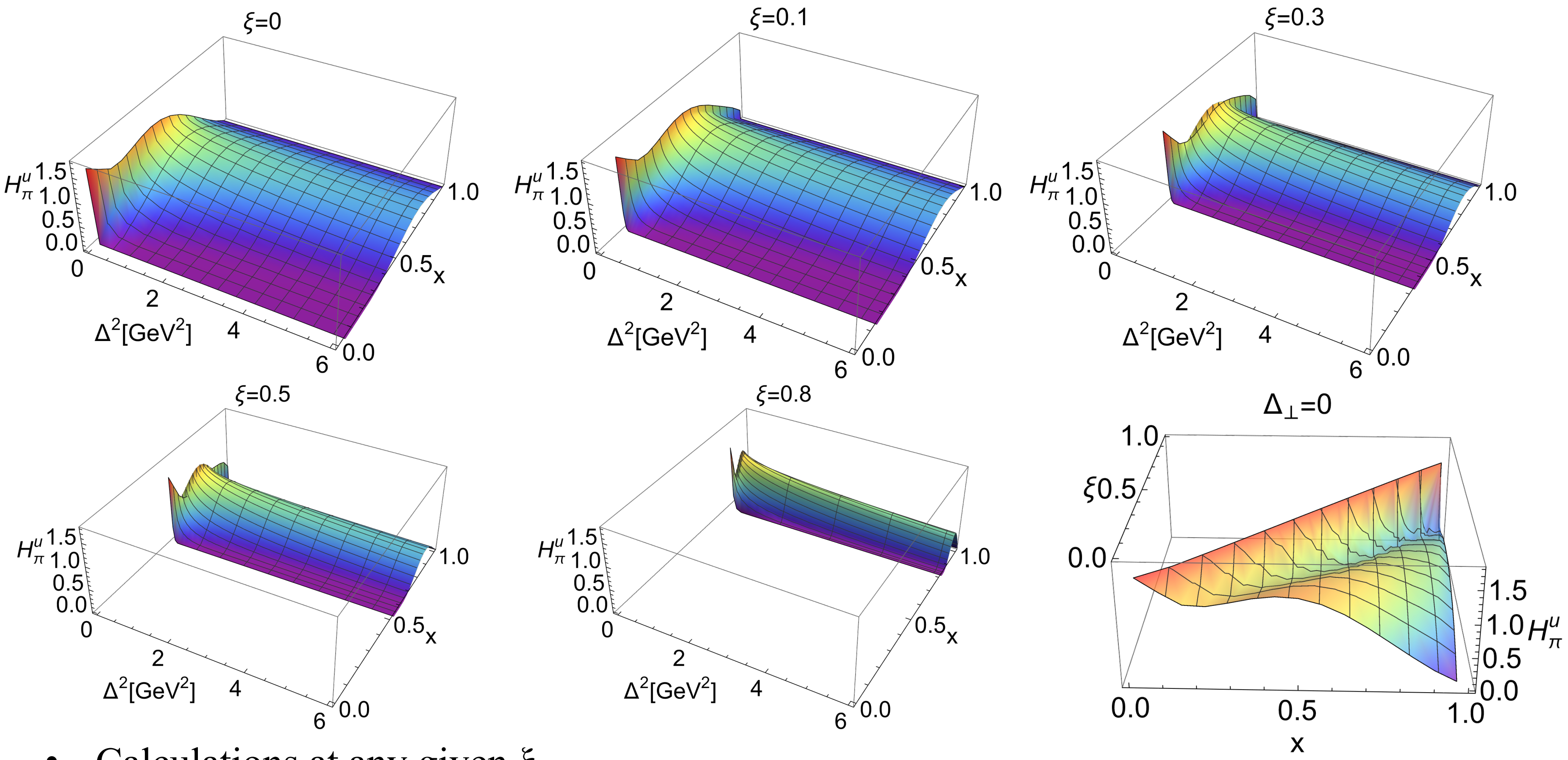
[K. Fu, J. Lan et al, in preparation]

Pion nonzero-skewness GPD

[M. Diehl, Phys. Rep. 388 (2003) 41-277]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$$H_\pi^q(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2}\right| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)\left|\pi, P-\frac{\Delta}{2}\right\rangle_{z^+=0;z_\perp=0}$$



- Calculations at any given ξ
- Emerge at larger x range for larger $-t = \Delta^2$

Preliminary

[Z. Zhang, J. Lan et al, in preparation]

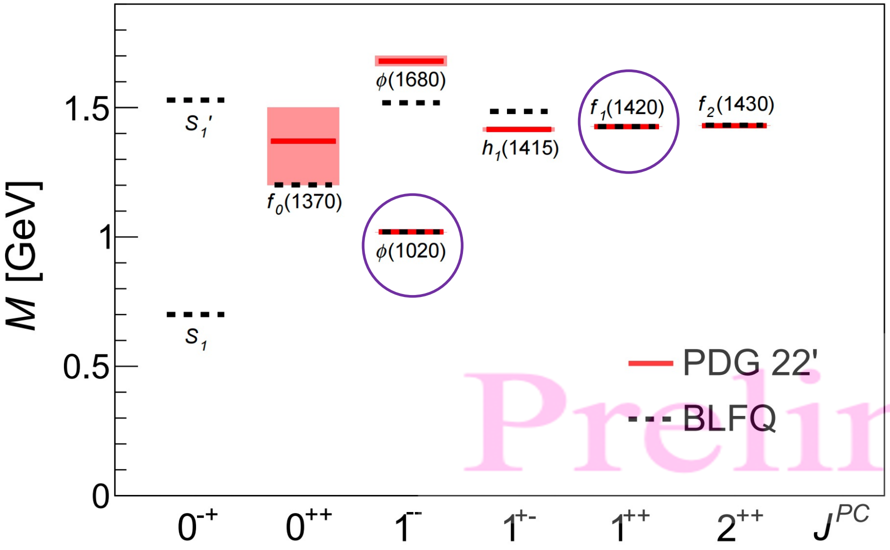
Light Meson Mass Spectrum

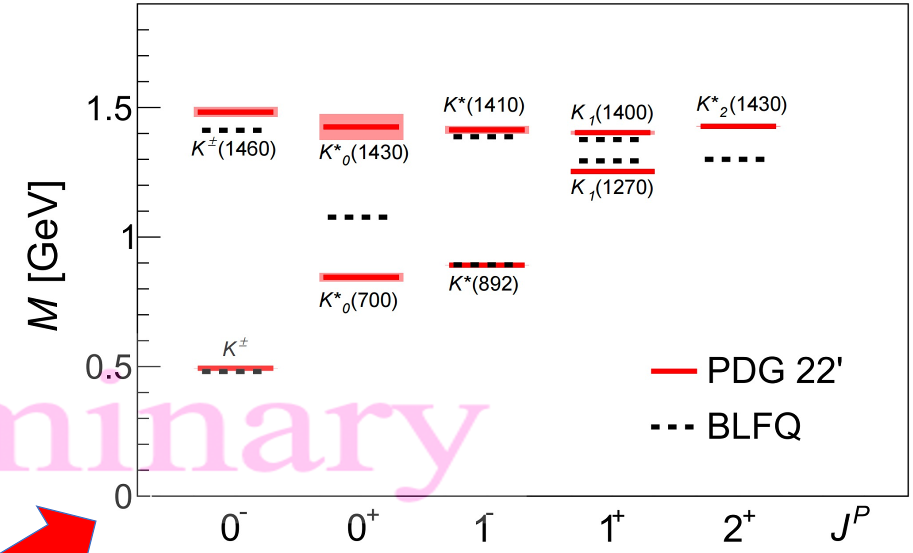
$$P^-|\text{meson}\rangle = P^-_{\text{meson}}|\text{meson}\rangle$$

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

[Lan, et al, PLB 825 (2022) 136890]

$$|\text{meson}\rangle = a|s\bar{s}\rangle + b|s\bar{s}g\rangle + \cdots \quad \Rightarrow \quad |\text{meson}\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \cdots$$





In units of GeV except g_s [Lan, et al, PLB 22']

m_q	m_{f_q}	m_g	b	κ	g_s	m_s	m_{f_s}
0.39	5.69	0.60	0.29	0.65	1.92	0.55	7.13

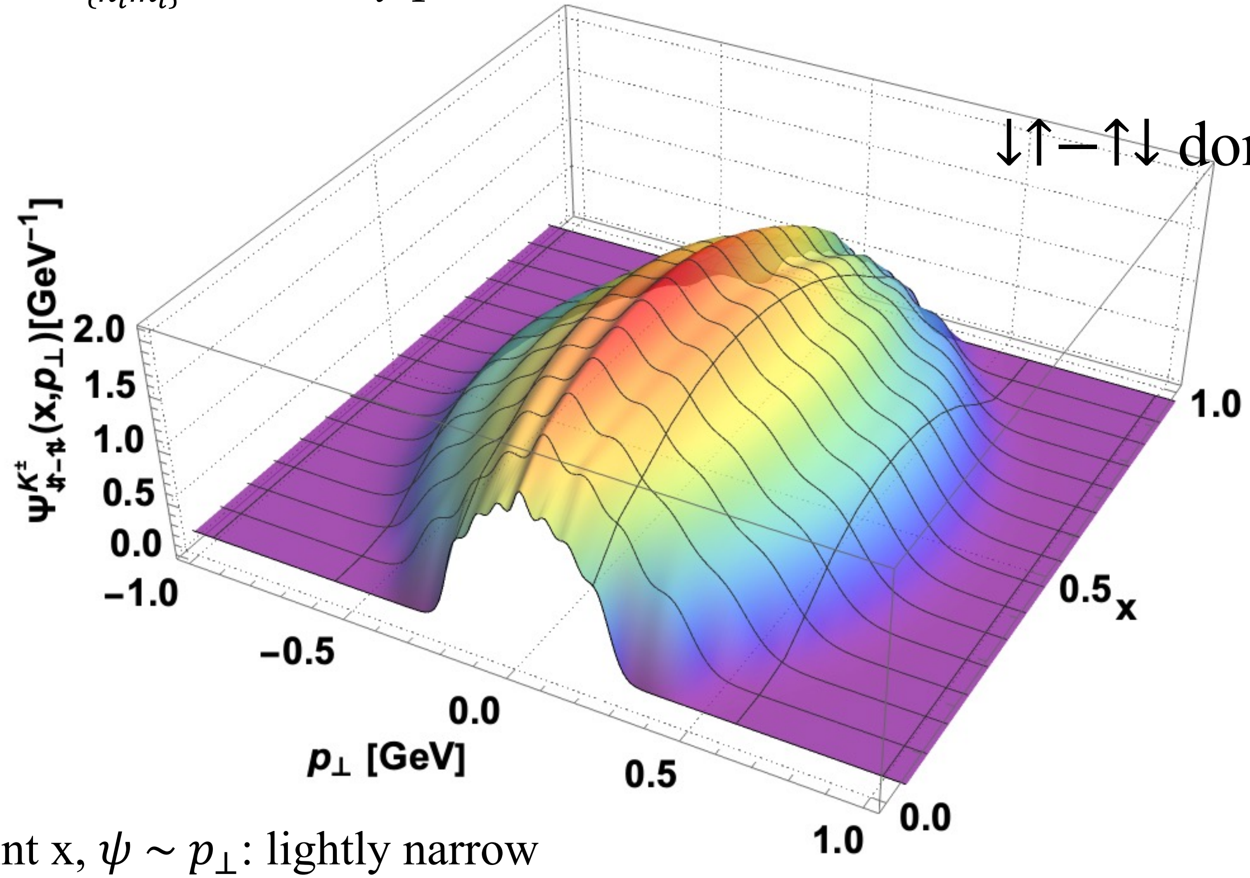
- Agree with experimental data (K, $K^*(892)$, $K^*(1410)$, $K_1(1270)$, $K_1(1400)$)
- The DC of kaon, 156.9 MeV, agrees with the result from the PDG 18', 155.6(4) MeV

[Jiangshan Lan, Jialin Chen et al, in preparation]

The Wave Function in Leading Fock Sector

$$\Psi^{\mathcal{N},M_J}_{\{x_i,\vec{p}^{\,2}_{\perp i},\lambda_i\}} = \sum_{\{n_i m_i\}} \psi^{\mathcal{N}}(\{\bar{\alpha}_i\}) \prod_{i=1}^{\mathcal{N}} \phi_{n_i m_i}(\vec{p}_{\perp i}, b)$$

$$|K\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \cdots$$



$\downarrow\uparrow-\uparrow\downarrow$ dominant in $|q\bar{s}\rangle$

- At endpoint x, $\psi \sim p_\perp$: lightly narrow
- At middle x, $\psi \sim p_\perp$: a little bit wide
- The peak slightly less than x=1/2

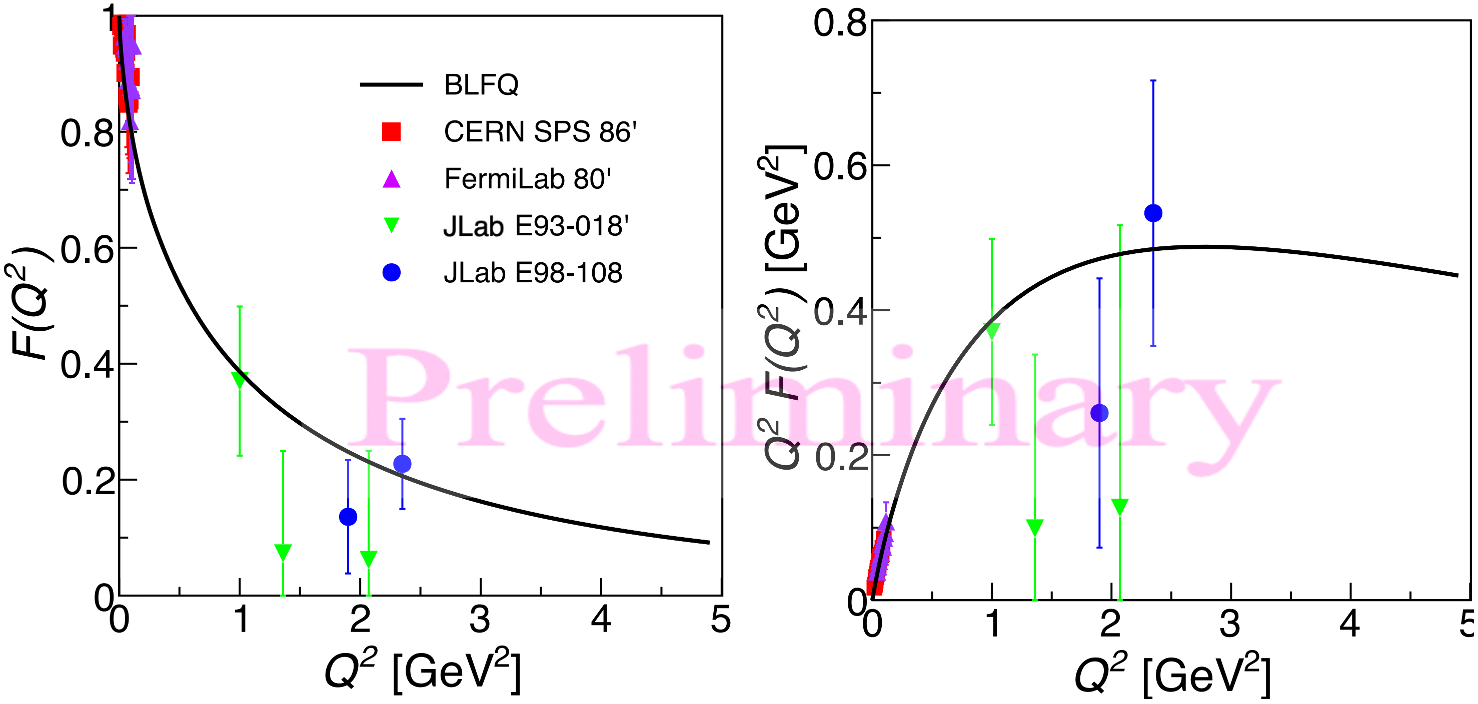
[Jiangshan Lan, Jialin Chen et al, in preparation]

Kaon Electromagnetic Form Factor

[Brodsky & de Teramond, PRD 77:056007 (2008)]

$$\langle \Psi(p') | J^{+}_{EM}(0) | \Psi(p) \rangle = (\mathrm{p} + \mathrm{p}')^{+}\ \mathrm{F}(\mathrm{Q}^2)$$

$$|\boldsymbol{K}\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \cdots$$

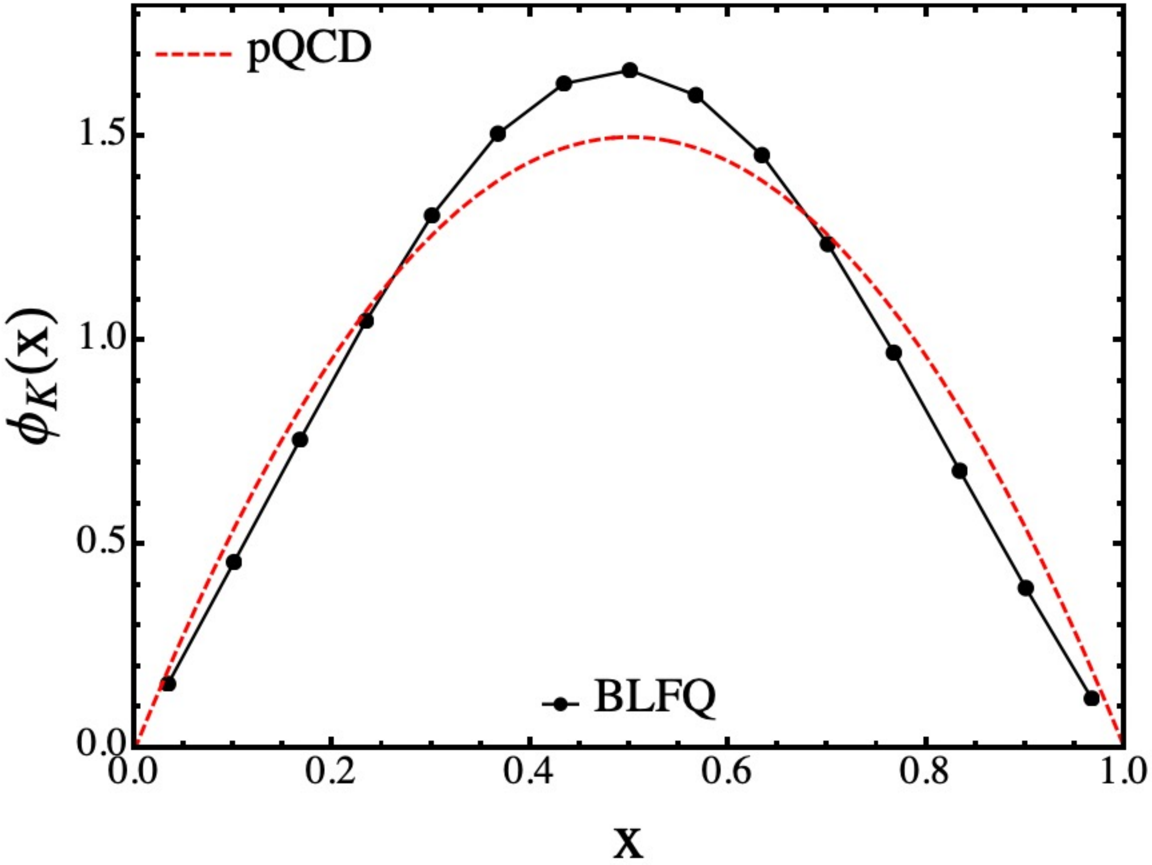


- FF is in reasonable agreement with experimental data
- $F(Q^2) \propto 1/\,Q^2$ for large Q^2

[Jiangshan Lan, Jialin Chen et al, in preparation]

Kaon PDA

$$|\boldsymbol{K}\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \cdots$$



- Endpoint behavior at small x almost agrees with pQCD

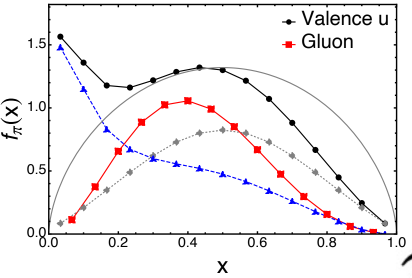
Preliminary

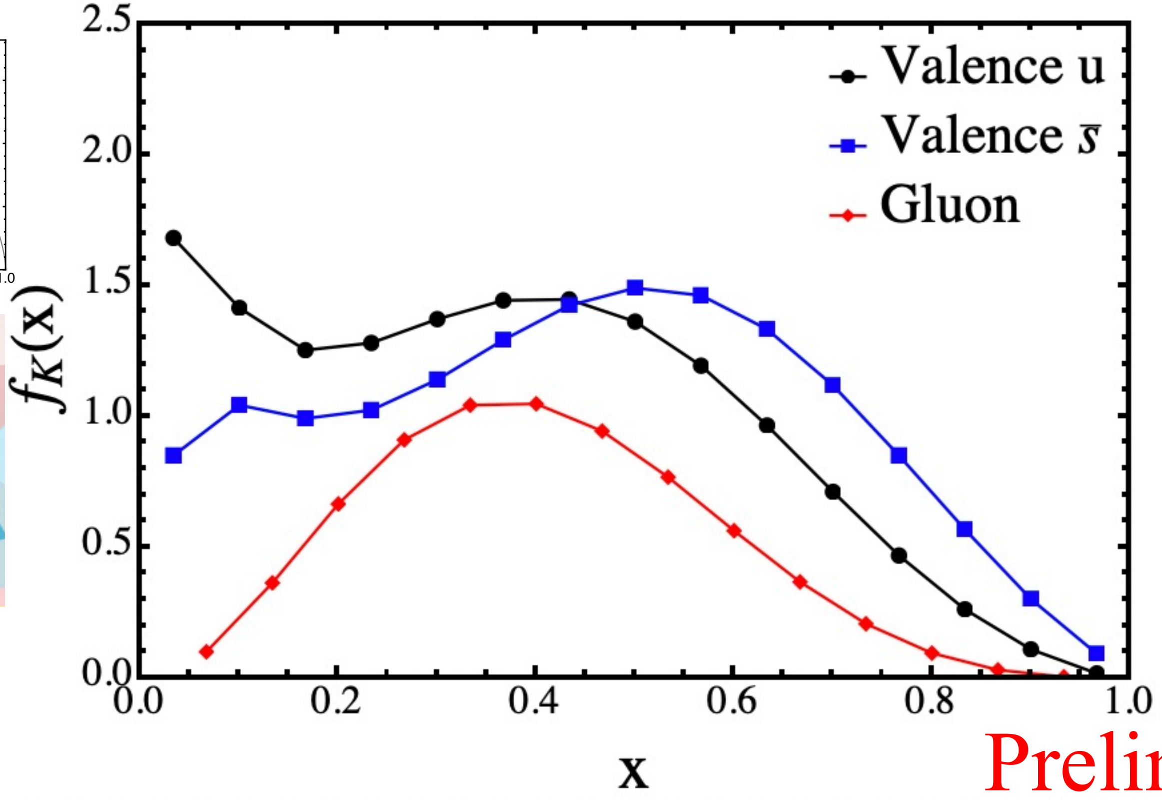
[Jiangshan Lan, Jialin Chen et al, in preparation]

Kaon PDF at Model Scale

$$f_i(x) = \sum_{\mathcal{N},\lambda_i} \int [d\mathcal{X} d\mathcal{P}^{\perp}]_{\mathcal{N}} \left|\psi^{\mathcal{N},M_j=0}_{\{x_i,\vec{p}^2_{\perp i},\lambda_i\}}\right|^2 \delta(x - x_i)$$

$$|\boldsymbol{K}\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \cdots$$





Preliminary

π: $\mu^2_{0\text{BLFQ}} = 0.34\ \text{GeV}^2$ $\langle x\rangle_{\text{gluon}} = 0.216$; $\langle x\rangle_{\text{val.}\,u} = 0.392$

K: $\mu^2_{0\text{BLFQ}} = 0.42\ \text{GeV}^2$ $\langle x\rangle_{\text{gluon}} = 0.193$; $\langle x\rangle_{\text{val.}\,u} = 0.363$; $\langle x\rangle_{\text{val.}\,\bar{s}} = 0.444$

[Jiangshan Lan, Jialin Chen et al, in preparation]

Kaon PDF with QCD Evolution

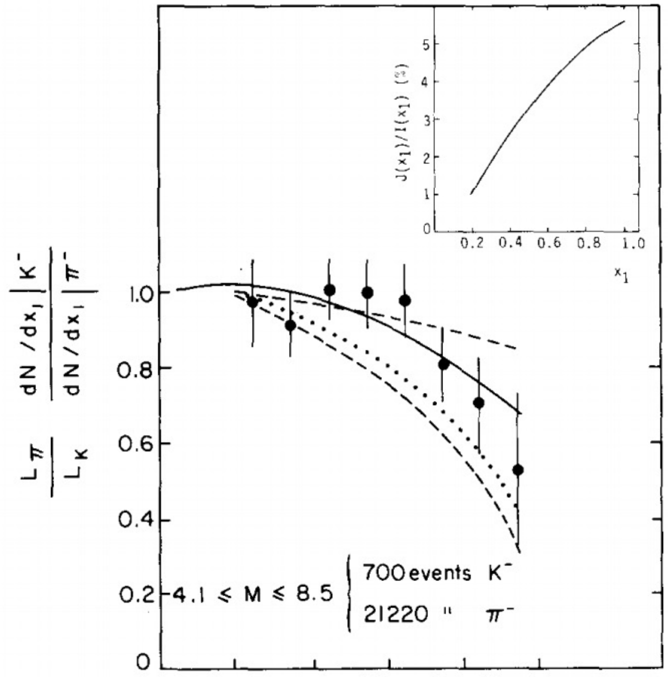
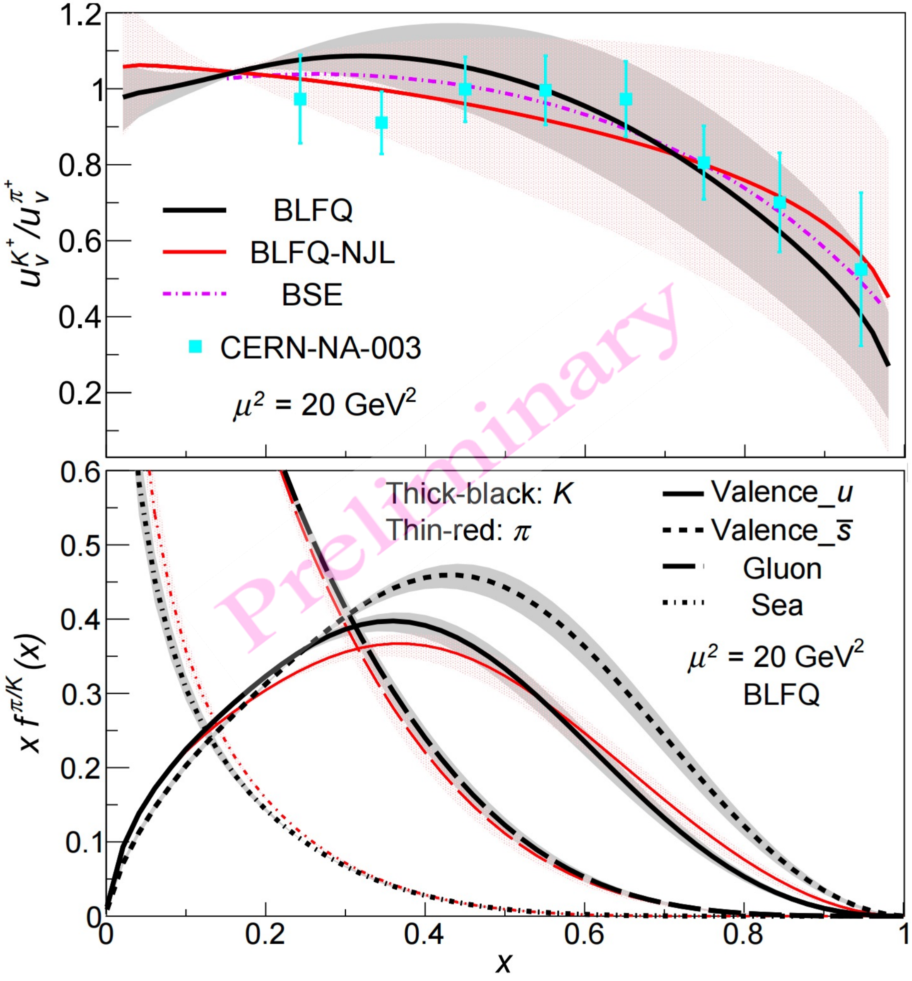


Fig. 2. The data points represent $(L_\pi/L_K)(dN/dx_1)_K/(dN/dx_1)_\pi$ as defined by eq. (4). The dashed curves represent the limits of the ratio $[\bar{u}_K(x_1)/\bar{u}_\pi(x_1)]C(x_1)^{-1}$ where $C(x_1)$ is defined in eq. (3), $\bar{u}_K/\bar{u}_\pi$ and $s_K/\bar{u}_K$ are taken from ref. [5], and the ratio $J(x_1)/I(x_1)$ is shown in the insert. The upper (lower) curve corresponds to $A = 1/8$ $(A = 1/2)$. The dotted and solid curves represent the ratio $\bar{u}_K/\bar{u}_\pi$ from refs. [6] and [7], respectively.

[Phys. Lett. B 93 (1980) 354]

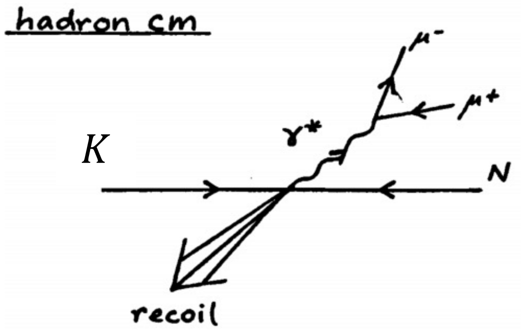
$$|\boldsymbol{K}\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \cdots$$



Agree with experimental results

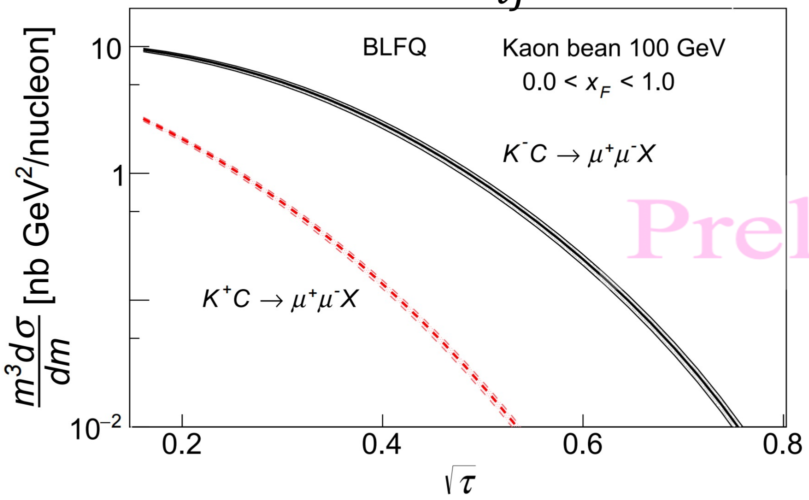
[Jiangshan Lan, Jialin Chen et al, in preparation]

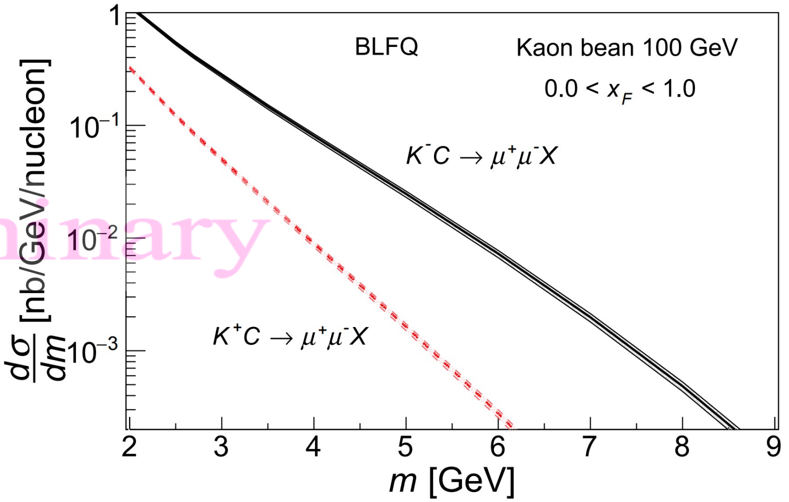
Drell-Yan Cross Section



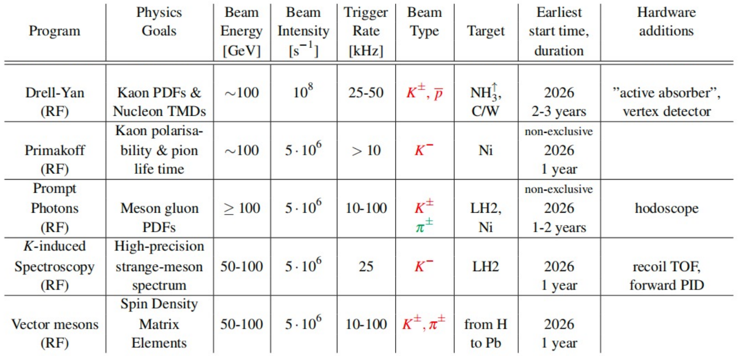
[S. D. Drell and T.-M. Yan, PRL (1970)]
[T. Becher et al, JKEP07(2008)030]; [P. C. Barry et al, PRL121(2018)152001]
[C. Anastasiou et al, PRL91(2003)182002]

$$\frac{m^3 d^2\sigma}{dm\,dY} = \frac{8\pi\alpha^2}{9}\frac{m^2}{s}\sum_{ij} dx_1 dx_2 \tilde{C}_{ij}(x_1,x_2,s,m,\mu_f) f_{i/K}(x_1,\mu_f) f_{j/N}(x_2,\mu_f)$$

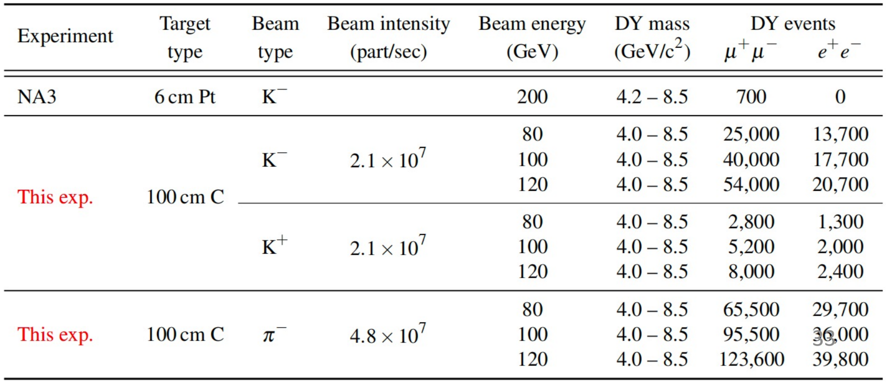




CERN-AMBER [CERN-SPSC-2019–022 /SPSC-P-360]



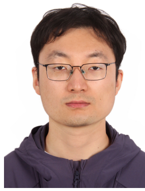
Program	Physics Goals	Beam Energy [GeV]	Beam Intensity [s^{-1}]	Trigger Rate [kHz]	Beam Type	Target	Earliest start time, duration	Hardware additions
Drell-Yan (RF)	Kaon PDFs & Nucleon TMDs	$\sim$100	10^8	25-50	$K^\pm$, $\overline{p}$	$NH_3^\uparrow$, C/W	2026 2-3 years	"active absorber", vertex detector
Primakoff (RF)	Kaon polarisability & pion life time	$\sim$100	$5 \cdot 10^6$	> 10	K^-	Ni	non-exclusive 2026 1 year	
Prompt Photons (RF)	Meson gluon PDFs	≥ 100	$5 \cdot 10^6$	10-100	$K^\pm$ $\pi^\pm$	LH2, Ni	non-exclusive 2026 1-2 years	hodoscope
K-induced Spectroscopy (RF)	High-precision strange-meson spectrum	50-100	$5 \cdot 10^6$	25	K^-	LH2	2026 1 year	recoil TOF, forward PID
Vector mesons (RF)	Spin Density Matrix Elements	50-100	$5 \cdot 10^6$	10-100	$K^\pm, \pi^\pm$	from H to Pb	2026 1 year	



Experiment	Target type	Beam type	Beam intensity (part/sec)	Beam energy (GeV)	DY mass (GeV/c^2)	DY events	
						$\mu^+\mu^-$	e^+e^-
NA3	6 cm Pt	K^-		200	4.2 – 8.5	700	0
This exp.	100 cm C	K^-	2.1×10^7	80	4.0 – 8.5	25,000	13,700
				100	4.0 – 8.5	40,000	17,700
				120	4.0 – 8.5	54,000	20,700
		K^+	2.1×10^7	80	4.0 – 8.5	2,800	1,300
				100	4.0 – 8.5	5,200	2,000
				120	4.0 – 8.5	8,000	2,400
This exp.	100 cm C	π^-	4.8×10^7	80	4.0 – 8.5	65,500	29,700
				100	4.0 – 8.5	95,500	36,000
				120	4.0 – 8.5	123,600	39,800

[Jiangshan Lan, Jialin Chen et al, in preparation]

Kaon GPD



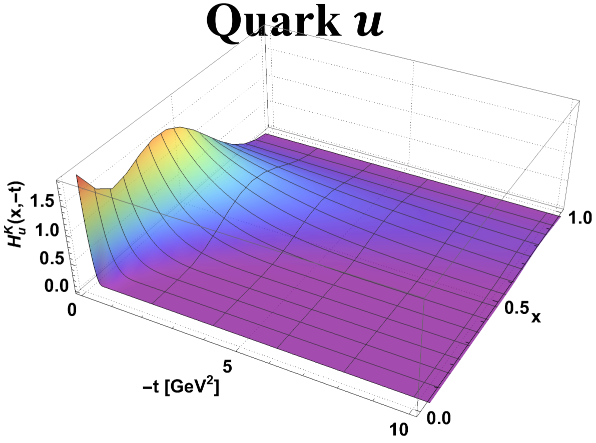
[M. Diehl, Phys. Rep. 388 (2003) 41-277]

$$|\boldsymbol{K}\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \cdots$$

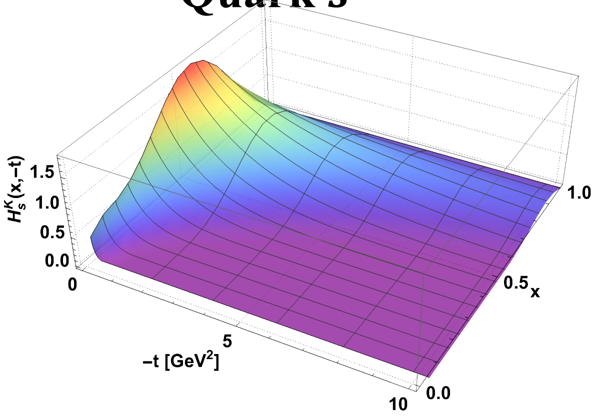
$$H_K^q(x,\xi=0,t) = \frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\left\langle K,P+\frac{\Delta}{2}\middle|\bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)\middle|K,P-\frac{\Delta}{2}\right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

$$H_K^g(x,\xi=0,t) = \frac{1}{P^+}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\left\langle K,P+\frac{\Delta}{2}\middle|G^{+\mu}\left(-\frac{z}{2}\right)G_\mu^+\left(\frac{z}{2}\right)\middle|K,P-\frac{\Delta}{2}\right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

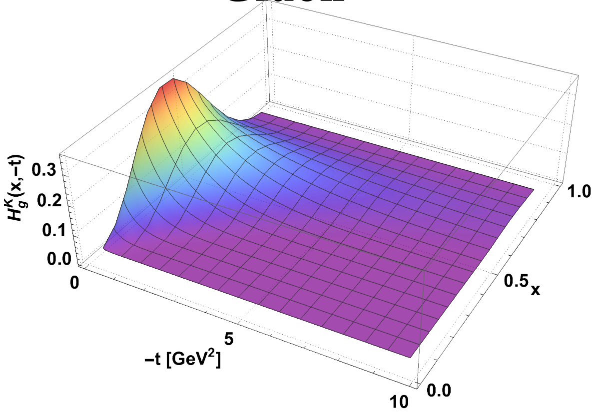
Quark u



Quark $\bar{s}$



Gluon

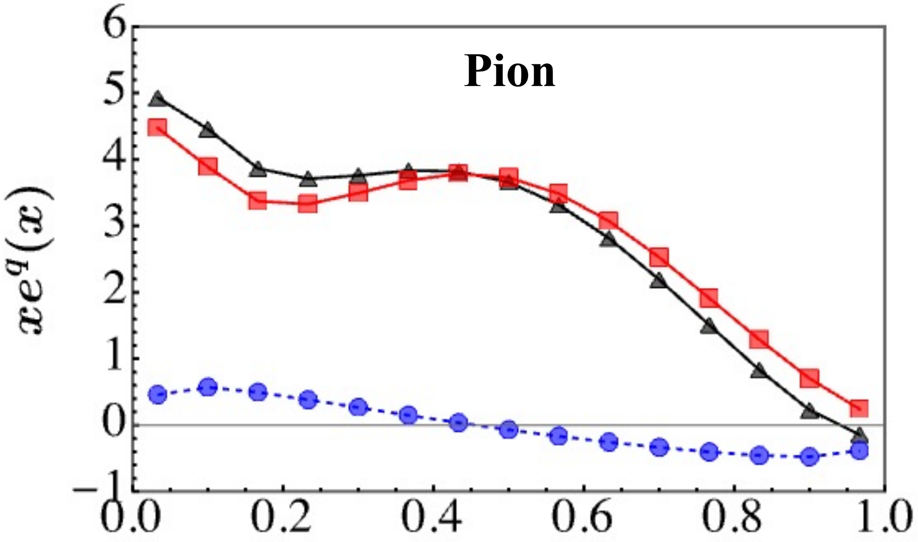


- Quark u content enhanced at small x with $|q\bar{s}g\rangle$
- Falls slowly at larger x
- Emerge at larger x range for larger $-t$

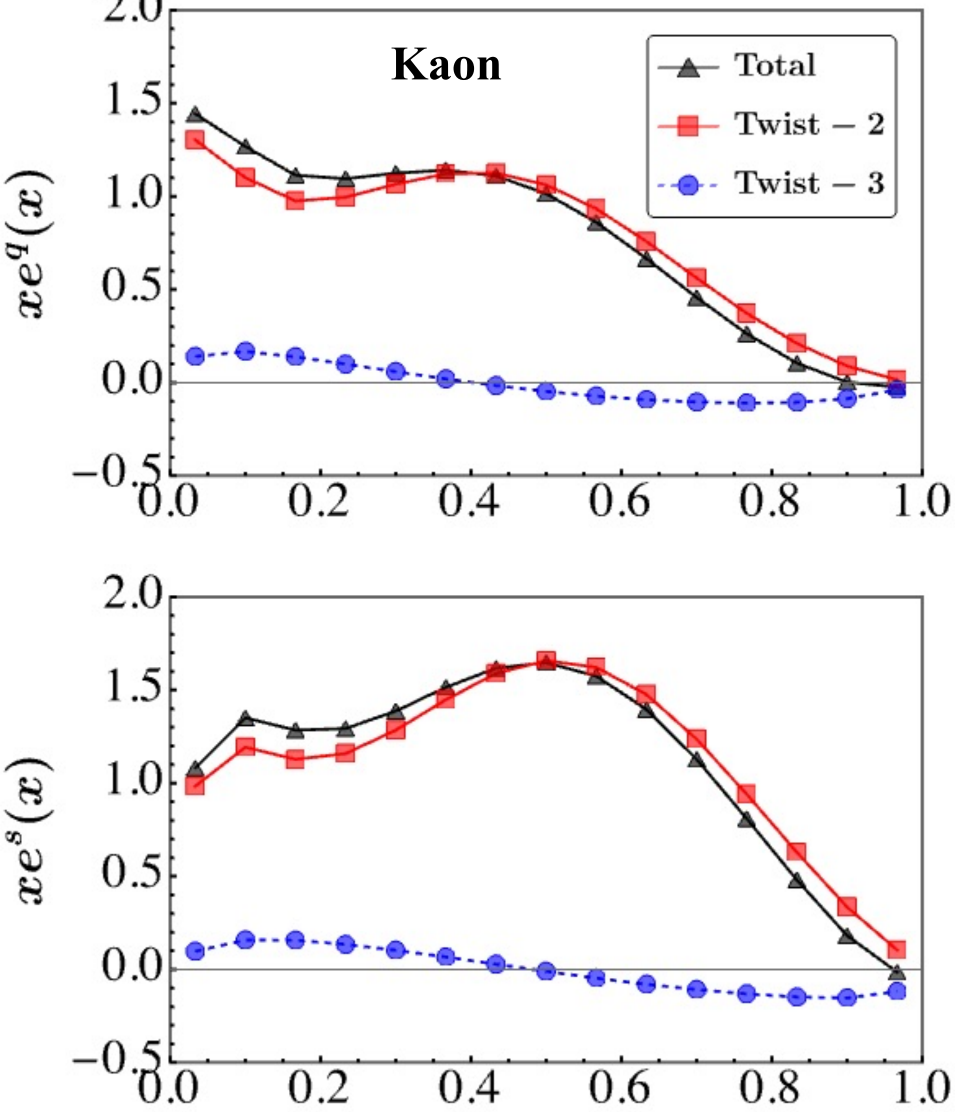
Preliminary

[K. Fu, et al, in preparation]

Twist-3 xPDFs of Pion & Kaon



[Z. Zhu et al, PLB 839 (2023) 137808]



Sum rules

$$\int \mathrm{d}x\, x\, \tilde{e}(x) = 0$$

$$\int \mathrm{d}x x e^q(x) = \frac{m_q}{M} N_q$$

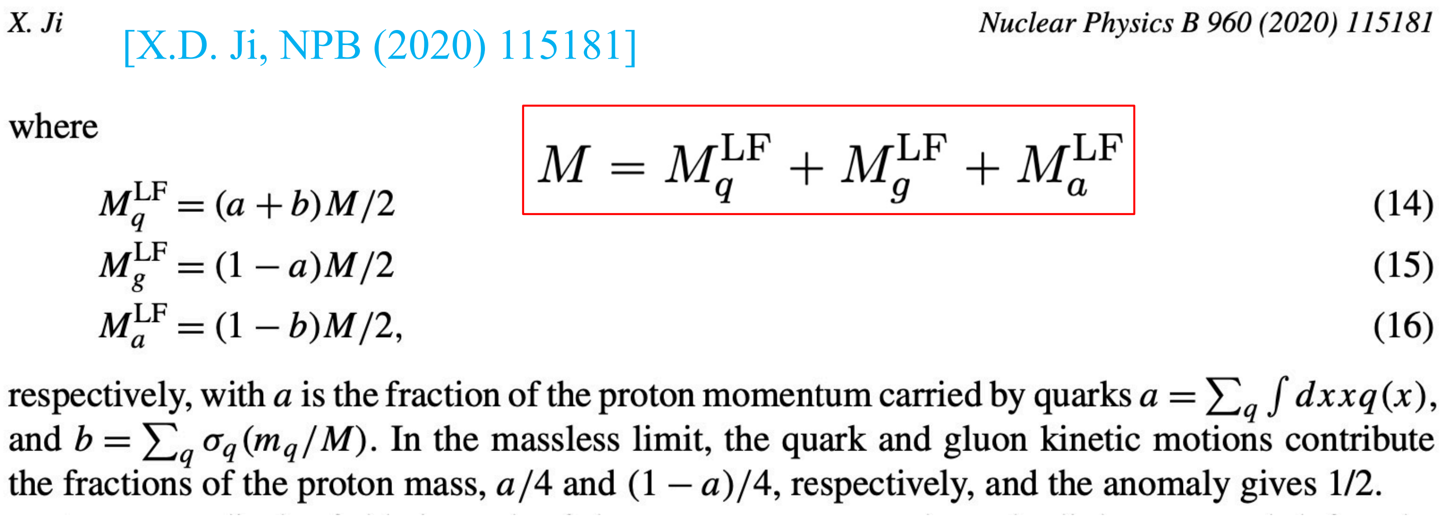
$$\int \mathrm{d}x f_1^q(x) = N_q$$

$$e\left(x, k_\perp\right) = \frac{m}{M} \frac{f_1\left(x, k_\perp\right)}{x} + \tilde{e}\left(x, k_\perp\right),$$

- ➢ BLFQ can provide twist-3 structures which satisfy QCD sum rules

Twist-3 xPDFs of Pion & Kaon

[X.D. Ji, NPB (2020) 115181]



X. Ji *Nuclear Physics B 960 (2020) 115181*

where

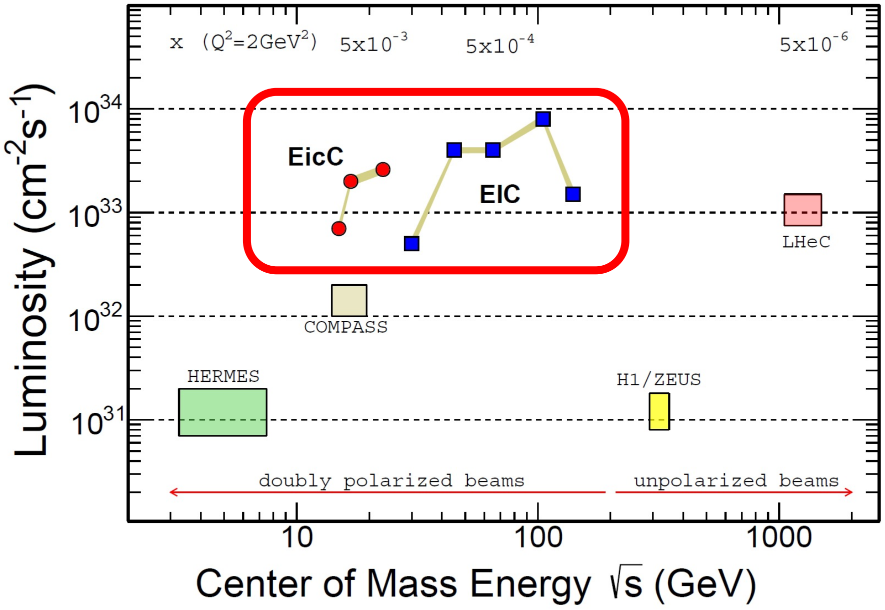
$$\boxed{M = M_q^{\rm LF} + M_g^{\rm LF} + M_a^{\rm LF}}$$

$$M_q^{\rm LF} = (a+b)M/2 \quad (14)$$

$$M_g^{\rm LF} = (1-a)M/2 \quad (15)$$

$$M_a^{\rm LF} = (1-b)M/2, \quad (16)$$

respectively, with a is the fraction of the proton momentum carried by quarks $a = \sum_q \int dx x q(x)$, and $b = \sum_q \sigma_q (m_q/M)$. In the massless limit, the quark and gluon kinetic motions contribute the fractions of the proton mass, $a/4$ and $(1-a)/4$, respectively, and the anomaly gives 1/2.



[Front.Phys.(Beijing) 16 (2021) 6, 64701]

Low twist

$$a = \sum_q \int dx x f_1^q(x)$$

High twist

$$b = \sum_q \sigma_q (m_q/M) \qquad \sigma_q = \int_{-1}^{1} dx e_q(x,\mu)$$

- ➢ High twist structures contribute to hadron structure in the EIC era.

Conclusion & Outlook

- Light-front Hamiltonian approach: **Mass Spectrum** ⟺ **Structure**
- Compared to NJL interaction, dynamical gluon in light meson:
  - ✓ Explains the properties of excited/exotic states such as $\pi(1300)$, $\pi_1(1400)$
  - ✓ Describes EMFF, $F(Q^2) \propto 1/Q^2$ for large Q^2
  - ✓ Improves endpoint behavior in PDF/PDA
  - ✓ Generates more gluon at moderate x/less gluon at small x
  - ✓ Improves agreement on J/ψ production cross section with experimental data
- $\pi - \rho$ mass splitting ~ a large spin-flip matrix element ~ a peak at low-x for quark PDF
- Preliminary results on **gluon** GPDs and TMDs of light mesons
- Light meson => Strangeonia and strange mesons
- Beyond leading twist structure needs to be studied in the EIC era

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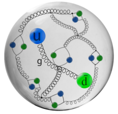
- Systematically expandable by including higher Fock sectors



$$|\text{Meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + d|q\bar{q}gg\rangle + e|gg\rangle + \cdots$$

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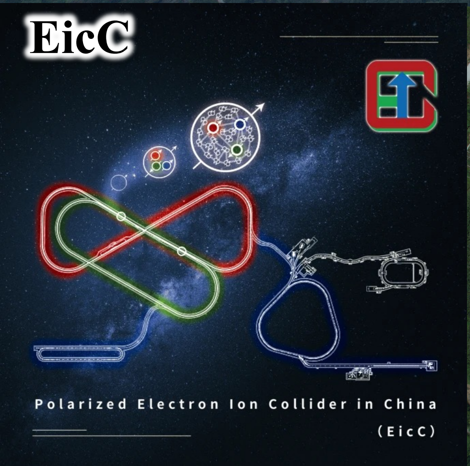
1-2 Postdoctoral Positions

✉ xbzhao@impcas.ac.cn

2023



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