Physics 526 Particle and Nuclear Physics
Lecture 12
Relativistic Quark Model for Mesons and Baryons (BLFQ)

1

Basis Light-Front Quantization:
Foundations, Recent Results and Plans

James P. Vary

Department of Physics and Astronomy
Iowa State University
Ames, USA

Light-Cone 2023: Hadrons and Symmetries
Rio de Janeiro, Brazil
November 18-22, 2023

2

General starting point: QCD Lagrangian

- Lattice Gauge theory
- Feynman Covariant Perturbation Theory
- Hamiltonian Field Theory
- Covariant Wave Equations

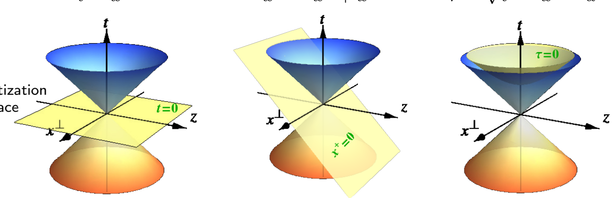
Steps to set up non-perturbative calculational framework	Specific selections for these applications
Make Legendre Transform	L --> $T^{\mu\nu}$ Energy-Momentum tensor
Select a coordinate system	Light-Front coordinates & identify generators
Fix the Gauge	Light-Front Gauge ($A^+ = 0$)
Eliminate dependent fields	Dirac -> Pauli fields
Select basis for the fields	2D HO + Jacobi Polynomials (or DLCQ)
Quantize the theory	Basis Light-Front Quantization (BLFQ)
Select cutoffs	N_{max}, L or K, Fock space limits
Evaluate/store Hamiltonian matrix	High-Performance Computers (HPCs)
Solve for eigenpairs (mass states & Light-Front Wavefunctions (LFWFs)	High-Performance Computers (HPCs)
Solve for experimental observables	Matrix elements of operators with LFWFs

3

Dirac's forms of relativistic dynamics [Dirac, Rev. Mod. Phys. **21**, 392 1949]
Instant form is the well-known form of dynamics starting with $x^0 = t = 0$
$K^i = M^{0i}$, $J^i = \frac{1}{2}\varepsilon^{ijk}M^{jk}$, $\varepsilon^{ijk} = (+1,-1,0)$ for (cyclic, anti-cyclic, repeated) indeces
Front form defines relativistic dynamics on the light front (LF): $x^+ = x^0 + x^3 = t+z = 0$

$P^{\pm} \triangleq P^0 \pm P^3$, $\vec{P}^{\perp} \triangleq (P^1, P^2)$, $x^{\pm} \triangleq x^0 \pm x^3$, $\vec{x}^{\perp} \triangleq (x^1, x^2)$, $E^i = M^{+i}$, $E^+ = M^{+-}$, $F^i = M^{-i}$

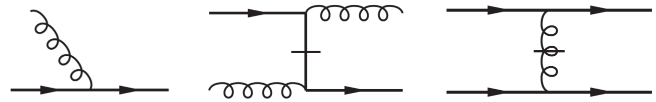
	instant form	front form	point form
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$
quantization surface			
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^μ
kinematical	$\vec{P}, \vec{J}$	$\vec{P}^{\perp}, P^+, \vec{E}^{\perp}, E^+, J^-$	$\vec{J}, \vec{K}$
dynamical	$\vec{K}, P^0$	$\vec{F}^{\perp}, P^-$	$\vec{P}, P^0$
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_{\perp}^2 + m^2)/p^+$	$p^\mu = mv^\mu \ (v^2 = 1)$



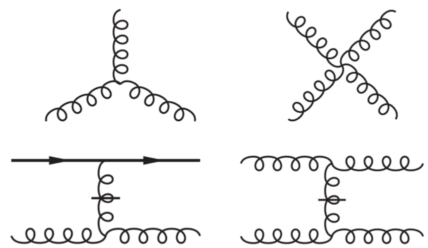
Adapted from talk by Yang Li

4

Light Front (LF) Hamiltonian Defined by its Elementary Vertices in LF Gauge



QED & QCD



QCD

5

Discretized Light Cone Quantization

[H.C. Pauli & S.J. Brodsky, PRD32 (1985)]

⇩

Basis Light Front Quantization

[J.P. Vary, et al., PRC81 (2010)]

$$\phi(\vec{k}_\perp, x) = \sum_\alpha \left[f_\alpha(\vec{k}_\perp, x) a_\alpha + f_\alpha^*(\vec{k}_\perp, x) a_\alpha^\dagger \right]$$

where $\{a_\alpha\}$ satisfy usual (anti-) commutation rules.

Furthermore, $f_\alpha(\vec{x})$ are arbitrary except for conditions:

Orthonormal: $$\int f_\alpha(\vec{k}_\perp, x) f_{\alpha'}^*(\vec{k}_\perp, x) \frac{d^2 k_\perp dx}{(2\pi)^3 2x(1-x)} = \delta_{\alpha\alpha'}$$

Complete: $$\sum_\alpha f_\alpha(\vec{k}_\perp, x) f_\alpha^*(\vec{k}'_\perp, x') = 16\pi^3 \sqrt{x(1-x)} \delta^2(\vec{k}_\perp - \vec{k}'_\perp) \delta(x - x')$$

For mesons we adopt (later extended to baryons): [Y. Li, et al., PLB758 (2016)]

$$f_{\alpha=\{nml\}}(\vec{k}_\perp, x) = \phi_{nm}\left(\vec{k}_\perp / \sqrt{x(1-x)}\right) \chi_l(x)$$

ϕ_{nm} 2D-HO functions as in AdS/QCD

χ_l Jacobi polynomials times $x^a(1-x)^b$

6

BLFQ
Symmetries & Constraints

Baryon number $\sum_i b_i = B$

Charge $\sum_i q_i = Q$

Angular momentum projection (M-scheme) $\sum_i (m_i + s_i) = J_z$

Longitudinal momentum (Bjorken sum rule) $\sum_i x_i = \sum_i \frac{k_i}{K} = 1$

Longitudinal mode regulator (Jacobi) $\sum_i l_i \leq L$

Transverse mode regulator (2D HO) $\sum_i (2n_i + |m_i| + 1) \leq N_{max}$

"Internal coordinates" $\vec{k}_{i\perp} = \vec{p}_{i\perp} - x_i \vec{P}_\perp \Rightarrow \sum_i \vec{k}_{i\perp} = 0$

$H \rightarrow H + \lambda H_{CM}$

Global Color Singlets (QCD)

Light Front Gauge

Optional Fock-Space Truncation

All $J \geq J_z$ states in one calculation

Finite basis regulators

Preserve transverse boost invariance

7

Light-Front Regularization and Renormalization Approaches

1. Regulators in BLFQ (N_{max}, L or K)
2. Additional Fock space truncations (if any)
3. Counterterms identified/tested*
4. Sector-dependent renormalization**
5. Renormalization Group Project for Effective Particles (RGPEP)***
6. Similarity Renormalization Group (SRG) & Okubo-Lee-Suzuki (OLS) in No Core Shell Model**** - adapted to BLFQ as a non-perturbative EFT approach (future)

* D. Chakrabarti, A. Harindranath and J.P. Vary, Phys. Rev. D **69**, 034502 (2004)
* P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D **91**, 105009 (2015)
** R.J. Perry, A. Harindranath and K. G. Wilson, Phys. Rev. Lett. **65**, 2959 (1990)
** V. A. Karmanov, J.-F. Mathiot, and A. V. Smirnov, Phys. Rev. D **77**, 085028 (2008); Phys. Rev. D **86**, 085006 (2012)
** Y. Li, V.A. Karmanov, P. Maris and J.P. Vary, Phys. Letts. B. 748, **278** (2015)
** **X. Zhao, LC2023: Wednesday at 11:00**
*** S. Glazek, Phys. Rev. D 103, 014021 (2021); **K. Serafin, LC2023: Wednesday at 11:30**
**** B.R. Barrett, P. Navratil and J.P. Vary, Prog. Part. Nucl. Phys. **69**, 131 (2013)

8

Light-Front Wavefunctions (LFWFs)

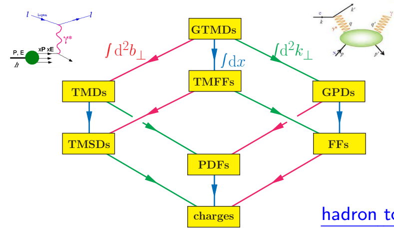
$$|\psi_h(P,j,\lambda)\rangle = \sum_n \int [\mathrm{d}\mu_n]\, \psi_{n/h}(\{\vec{k}_{i\perp}, x_i, \lambda_i\}_n) |\{\vec{p}_{i\perp}, p_i^+, \lambda_i\}_n\rangle$$

LFWFs are *frame-independent* (boost invariant) and depend only on the relative variables: $x_i \equiv p_i^+/P^+, \vec{k}_{i\perp} \equiv \vec{p}_{i\perp} - x_i \vec{P}_\perp$

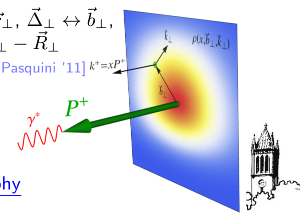
LFWFs provide intrinsic information of the structure of hadrons, and are indispensable for exclusive processes in DIS [Lepage '80]

- Overlap of LFWFs: structure functions (e.g. PDFs), form factors, ...
- Integrating out LFWFs: light-cone distributions (e.g. DAs)

"Hadron Physics without LFWFs is like Biology without DNA!"
— Stanley J. Brodsky



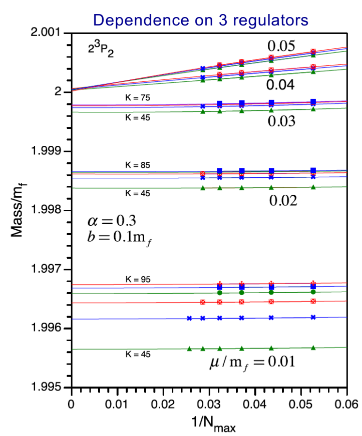
$\vec{k}_\perp \leftrightarrow \vec{r}_\perp$, $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$,
$\vec{b}_\perp = \vec{r}_\perp - \vec{R}_\perp$
[Lorcé & Pasquini '11]

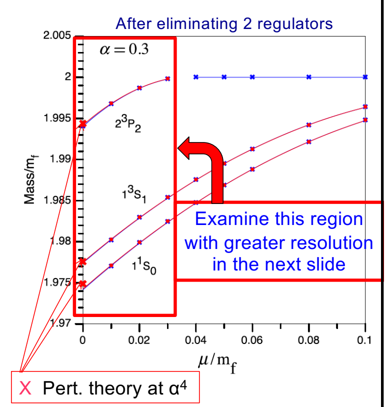


hadron tomography

9

Basis Light-Front Quantization (BLFQ)
Positronium in QED at Strong Coupling (α = 0.3) in e+e- space with V_{eff}
Systematic removal of regulators (b = HO momentum scale)

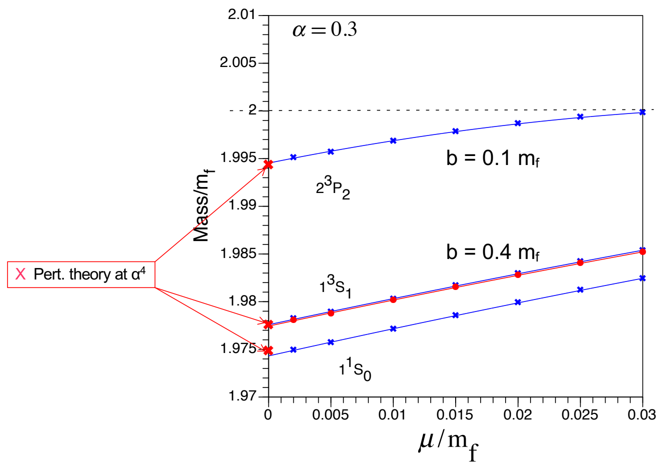




P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D **91**, 105009 (2015)

10

Positronium in QED at Strong Coupling
Covariant Basis Light-Front Quantization (BLFQ)



P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D **91**, 105009 (2015)

11

X. Zhao, LC2023
Wednesday at 11:00

Positronium with one dynamical photon

- QED Lagrangian $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m_e)\Psi$

- Light-front QED Hamiltonian from standard Legendre transformation

$$P^- = \int d^2x^\perp dx^- \; F^{\mu+}\partial_+ A_\mu + i\bar{\Psi}\gamma^+\partial_+\Psi - \mathcal{L}$$

Light-cone gauge: $(A^+ = 0)$

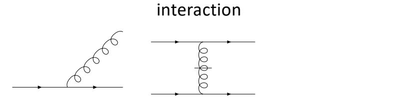
$$= \int d^2x^\perp dx^- \; \frac{1}{2}\bar{\Psi}\gamma^+\frac{m_e^2 + (i\partial^\perp)^2}{i\partial^+}\Psi + \frac{1}{2}A^j(i\partial^\perp)^2A^j$$

kinetic energy terms

$$+ e j^\mu A_\mu + \frac{e^2}{2}j^+\frac{1}{(i\partial^+)^2}j^+$$

vertex interaction

instantaneous photon interaction



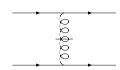
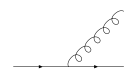
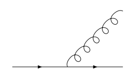
12

X. Zhao, LC2023
Wednesday at 11:00

Interaction Part of Hamiltonian

$|\mathbf{Ps}\rangle = a|ee\rangle + b|ee\gamma\rangle + c|\gamma\rangle + d|eeee\rangle + \ldots$

$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$

H_{int}	$\lvert e\bar{e}\rangle$	$\lvert e\bar{e}\gamma\rangle$
$\langle e\bar{e}\rvert$		
$\langle e\bar{e}\gamma\rvert$		0 excluded by *gauge principle* [Tang et al, 1991]

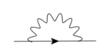
Kaiyu Fu, et al., in preparation

13

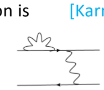
X. Zhao, LC2023
Wednesday at 11:00

Mass Renormalization

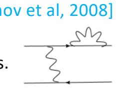
- Mass counterterm $\Delta_m = m_{bare} - m_{phys}$ is needed for fermion self-energy correction



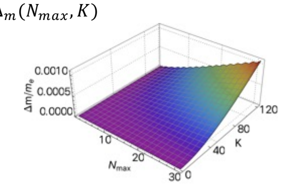
- Mass renormalization needs to be performed on single physical electron
  - Prediction power on positronium mass
- Mass counterterm is determined by fitting single electron mass
  - Complication: Δ_m depends on UV cutoff and thus is basis dependent.
  - An extension of sector-dependent renormalization is needed: $\Delta_m(N_{max}, K)$ [Karmanov et al, 2008]



vs.



Here at α = 1/137



[Kaiyu Fu, et al, in preparation]

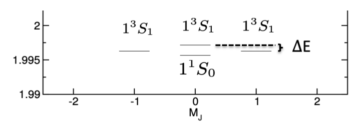
- Mass counterterm is at higher order: $\Delta_m \propto \alpha m\, E_B \propto \alpha^2 m$

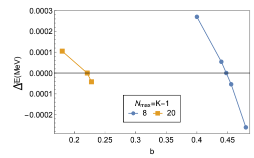
14

X. Zhao, LC2023
Wednesday at 11:00

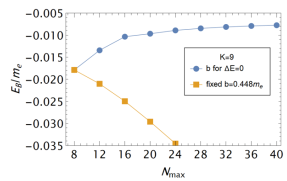
Basis Scale and Rotational Symmetry

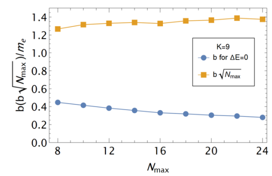
- Adjust the 2d harmonic oscillator basis scale parameter b to minimize the energy difference within the triplet 1^3S_1





- Maintaining rotational symmetry leads to a corresponding UV cutoff



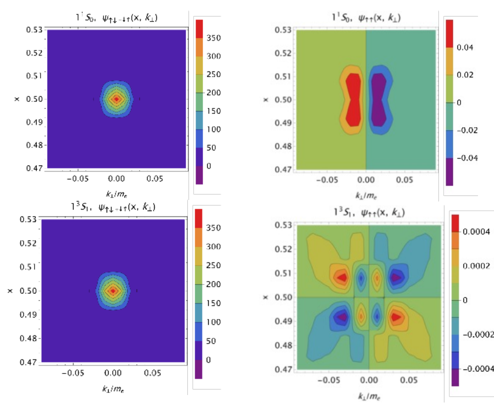


[Kaiyu Fu et al, in preparation]

15

X. Zhao, LC2023
Wednesday at 11:00

Wave Functions for S-Wave States



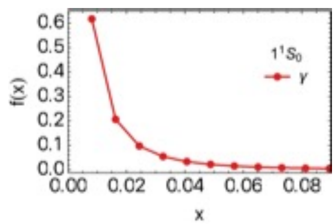
- Wave functions in $|e^+e^-\rangle$ Fock sector, dominant and non-dominant helicity component
- Nodal structure visible in non-dominant helicity component

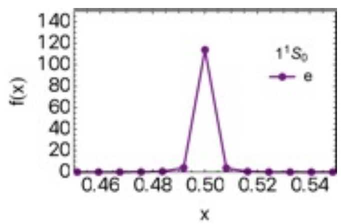
[Kaiyu Fu, et al, in preparation]

16

X. Zhao, LC2023
Wednesday at 11:00

Parton Distribution Functions (PDFs)
of the electron and photon





- $|e^+e^-\rangle$ Fock sector carries 99.1% probability.
- The peak of photon PDF is at small x region.

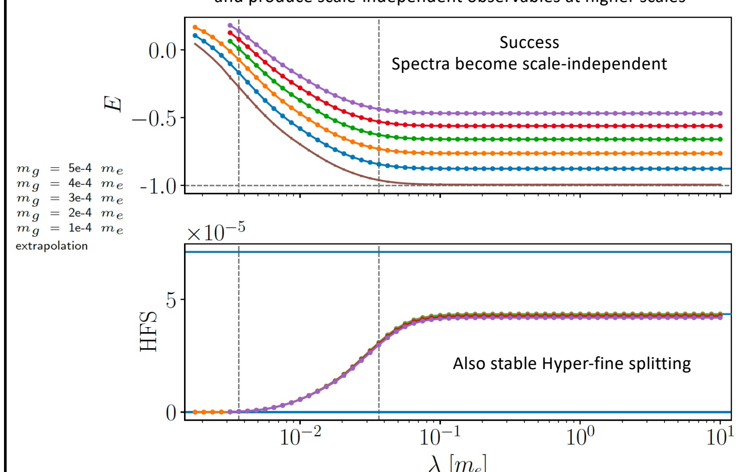
[Kaiyu Fu, et al, in preparation]

17

17

K. Serafin, LC2023
Wednesday at 11:30

Renormalization Group Project for Effective Particles (RGPEP)
Coupling and Mass parameters for positronium run with scale (λ)
and produce scale-independent observables at higher scales

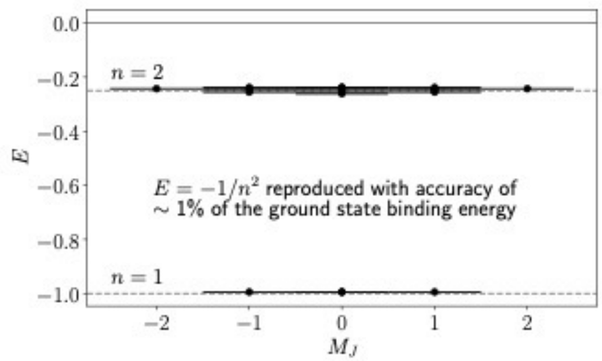


18

K. Serafin, LC2023
Wednesday at 11:30

Renormalization Group Project for Effective Particles (RGPEP)

Positronium spectrum, effective interactions, $\alpha = 1/137$



Each point is an $m_g \to 0$ extrapolation obtained for $N_{\max} = 16$, $K = 5793$, and $K_n = 91$.

19

Overview of BLFQ/tBLFQ applications to mesons and baryons

Common features (except for some recent developments)

Transverse confinement from 2D HO (in common with LF Holography)
Longitudinal confinement (Y. Li, et al, PLB 2016, PRD 2017)
Basis states from exact solutions of a reference Hamiltonian
Compare results with experiment, lattice, DSE/BSE, . . .

Distinct features

For Veff
1) one-gluon exchange (Y. Li, et al, PLB 2016, PRD 2017)
2) NJL model for light meson applications (S. Jia, et al, PRC 2019)

For Fock space truncation
1) Valence sector
2) Valence sector plus dynamical gluon (plus sea quarks, plus . . .)

For observables
1) Static properties and decays of many states
2) Transitions between states
3) Non-perturbative probes (tBLFQ) (X. Zhao, et al, PRD 2013)

Next Generation Methods

BLFQ on Quantum Computers (W. Qian, et al., Phys. Rev. Res. 2022)

20

Heavy Quarkonia [Y.Li,PLB758,2016; PRD96,2017]

- Effective Hamiltonian in the $q\bar{q}$ sector

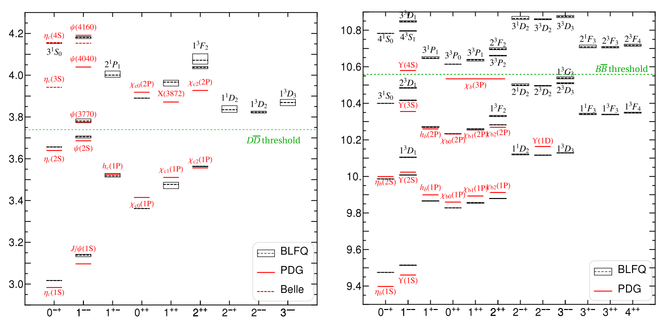
$$H_{\text{eff}} = \underbrace{\overbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}}}^{H_0} + \underbrace{V_g}_{\text{one-gluon exchange}}$$

where $x = p_q^+/P^+$, $\vec{k}_\perp = \vec{k}_{q\perp} = \vec{p}_{q\perp} - x\vec{P}_\perp = -\vec{k}_{\bar{q}\perp} = -\left(\vec{p}_{\bar{q}\perp} - (1-x)\vec{P}_\perp\right)$, $\vec{r}_\perp = \vec{r}_{q\perp} - \vec{r}_{\bar{q}\perp}$.

  - Confinement
    transverse holographic confinement [S.J.Brodsky,PR584,2015]
    longitudinal confinement [Y.Li,PLB758,2016]
  - One-gluon exchange with running coupling
    $V_g = -\frac{4}{3}\frac{4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_{\sigma'}\gamma^\mu u_\sigma \bar{v}_s \gamma_\mu v_{s'}$
- Basis representation
  - valence Fock sector: $|q\bar{q}\rangle$
  - basis functions: eigenfunctions of H_0 (LF kinetic energy+ confinement)

21

Spectroscopy [Y. Li, et al., Phys. Letts. B 758, 118 (2016); Phys. Rev. D 96, 016022 (2017)]



	κ (GeV)	m_q (GeV)	rms (MeV)	$\overline{\delta_J M}$ (MeV)	$N_{\max}$	basis dim.
$c\bar{c}$	0.966	1.603	31	17	32	1812
$b\bar{b}$	1.389	4.902	38	8	32	1812

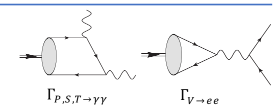
κ determined from fits to spectrum follows the HQET trajectory $\kappa_h \propto \sqrt{M_h}$, in agreement with recent LFH result [Dosch et al, PRD**95** (2017)]

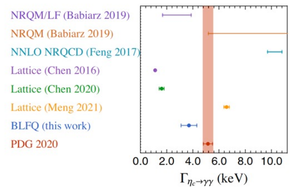
22

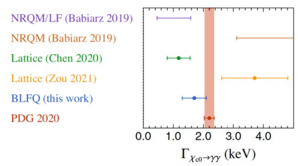
Yang Li, Meijian Li and James P. Vary, PRD 105, L071901 (2022)

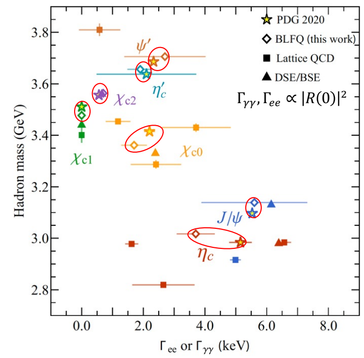
Diphoton width $\Gamma_{\gamma\gamma}$ of charmonia in BLFQ

- ✓ Notoriously challenging
- ✓ BLFQ predictions are very competetive!
  - ✓ No parameters were adjusted!







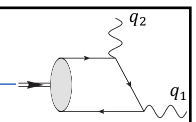


Lattice: Dudek '06, Chen '16, Chen '20, Meng '21, Zou '21;
NRQCD: Feng '15 & '17
NRQM: Babiarz '19 & '20

Comparison of theoretical prediction of masses and dilepton/diphoton widths combined

23

Transition form factor: η_c

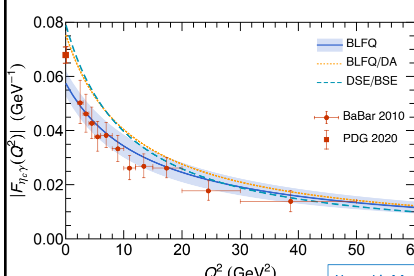


$$\mathcal{M}^{\mu\nu} = 4\pi\alpha_{em}\varepsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}F_{P\gamma\gamma}(q_1^2, q_2^2)$$

- ✓ Diphoton width $\Gamma_{\gamma\gamma} = \frac{\pi}{4}\alpha_{em}^2 M_P^3 |F_{P\gamma\gamma}(0,0)|^2$
- ✓ Single-tag TFF $F_{P\gamma}(Q^2) = F_{P\gamma\gamma}(Q^2, 0) = F_{P\gamma\gamma}(0, Q^2)$

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_c}\int \frac{dx}{2\sqrt{x(1-x)}}\int \frac{d^2k_\perp}{(2\pi)^3}\frac{\psi_{\uparrow\downarrow-\downarrow\uparrow}(x,\vec{k}_\perp)}{k_\perp^2 + m_f^2 + x(1-x)Q^2}$$

Lepage '80, Feldman '97, Babiarz, '19



- BABAR data: described by monopole form with pole mass $\Lambda^2 = 8.5 \pm 0.6 \pm 0.7$ GeV^2, and width 5.12 (53) keV
- BLFQ: using N_{max}=8 wave function corresponding to $\mu \approx 2m_c$. Basis sensitivity band is taken as the difference between the N_{max}=8,16 results.
- BLFQ/DA: prediction using the LCDA obtained from the LFWF
- Theoretical prediction in good agreement with both the width and the form factor.

DSE/BSE: Chen PRD 2017

Yang Li, Meijian Li and James P. Vary, PRD 105, L071901 (2022)

24

BLFQ Basis States

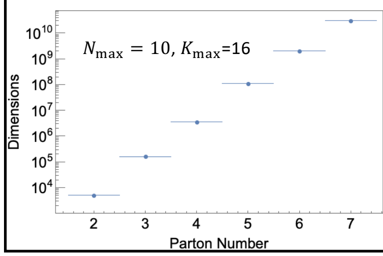
➢ BLFQ basis: expansion in Fock space

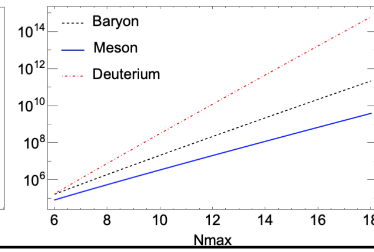
$$|\beta_{\mathrm{meson}}\rangle = |qq\rangle + |qqg\rangle + |gg\rangle + |qq\ qq\rangle + |qq\ gg\rangle + |qq\ qq\ g\rangle + |qqqqgg\rangle + \cdots$$

$$|\beta_{\mathrm{baryon}}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\ qq\rangle + |qqq\ gg\rangle + |qqq\ qqg\rangle + |qqqqqgg\rangle + \cdots$$

$$|\beta_{\mathrm{deuterium}}\rangle = |qqq\ qqq\rangle + |qqq\ qqq\ g\rangle + |qqq\ qqq\ qq\rangle + |qqq\ qqq\ gg\rangle + \cdots$$

➢ Dimension of basis states increases with number of Fock sectors
=> motivation for quantum computing

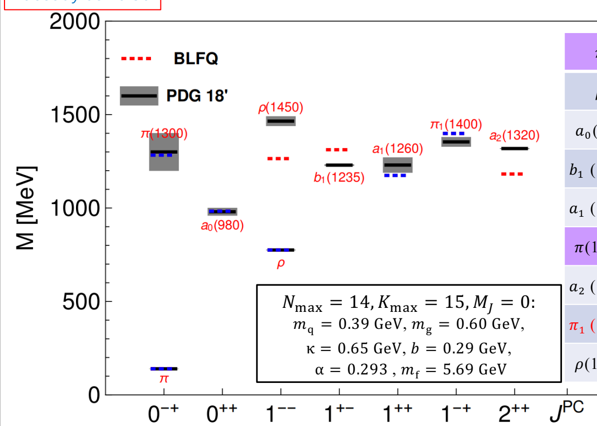




25

J. Lan, LC2023
Tuesday at 16:30

Light Meson Mass Spectrum



	Normqq	DC [MeV]
π	0.492	151
ρ	0.486	141
$a_0(980)$	0.370	0
b_1 (1235)	0.30	0
a_1 (1260)	0.324	13
$\pi(1300)$	0.284	59
a_2 (1320)	0.320	0
π_1 (1400)	0.002	0
$\rho(1450)$	0.312	50

$|\mathrm{meson}\rangle = a|qq\rangle + b|qqg\rangle + \cdots$

Fix the parameters by fitting six blue states

- $\pi_1(1400)$: $|qqg\rangle$ dominates
- $\pi(1300)$: the DC is smaller than the DC of pion

[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]

26

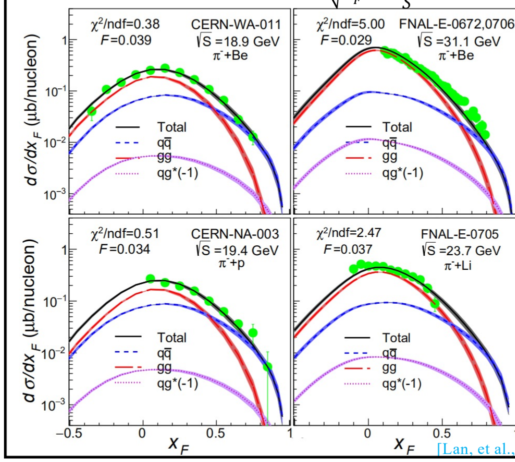
J. Lan, LC2023
Tuesday at 16:30

J/ψ production cross section

$\pi^{\pm} N \rightarrow J/\psi\ X$

$$\frac{d\sigma}{dx_F}[J/\psi] = F \sum_{i,j=q,q,g} \int_{2m_c}^{2m_D} dM_{cc} \frac{2M_{cc}}{S\sqrt{x_F^2 + \frac{4M_{cc}^2}{S}}} \hat{\sigma}_{ij}(s, m_c^2, \mu_R^2, \mu_F^2) f_i^{\pi^{\pm}}(x_1, \mu_F^2) f_j^N(x_2, \mu_F^2)$$

[nCTEQ 2015]



CEM
[Chang, et al, PRD 102 (2020) 054024];
[Nason, et al, NPB 303 (1988) 607];
[Mangano, et al, NPB 405 (1993) 507]

- significant gg contribution
- multiple pion energies
- multiple nuclear targets

Agree with experimental data (FNAL E672, E706, E705, CERN NA3,WA11).

[Lan, et al., PLB 825, 136890 (2022)]

27

S. Kaur, LC2023
Thursday at 16:30

Vector meson Structure

Quark TMDs of a spin-1 target

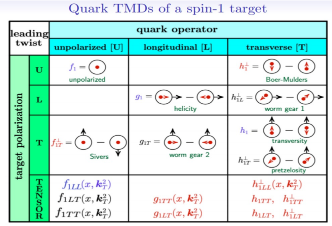
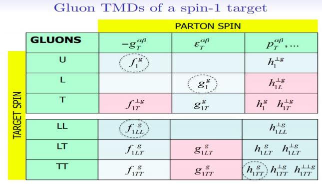


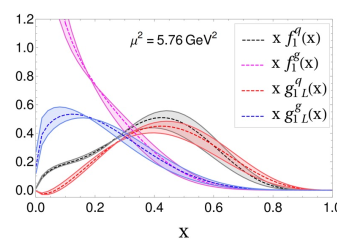
Image taken from arXiv: 2205.01249

Gluon TMDs of a spin-1 target



Picture Credit: P.J. Mulders

$|\rho\rangle = a|qq\rangle + b|qqg\rangle + \cdots$



Preliminary

[S. Kaur, et al, in preparation]

28

Baryons with one dynamical gluon

$$|P_{baryon}\rangle = \Psi_1|qqq\rangle + \Psi_2|qqqg\rangle$$

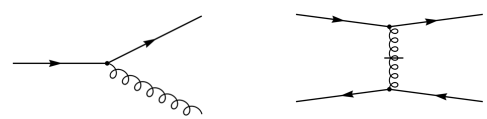
$$\boldsymbol{P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

$H_{trans} \sim \kappa_T^4 r^2$ -- Brodsky, Teramond arXiv: 1203.4025

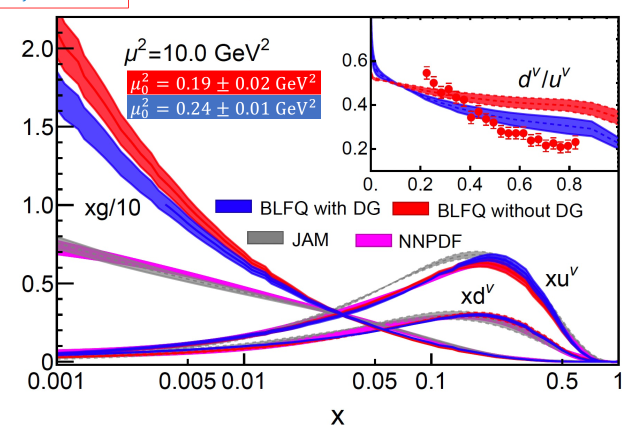
$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i}\left(x_i x_j \partial_{x_j}\right)$ ---Y Li, X Zhao , P Maris , J Vary, PLB 758(2016)

$$H_{Interact} = H_{Vertex} + H_{inst} = g\psi\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+\,\frac{1}{(i\partial^+)^2}j^+$$



29

Unpolarized PDFs



JAM and NNPDF are global fits to experimental data

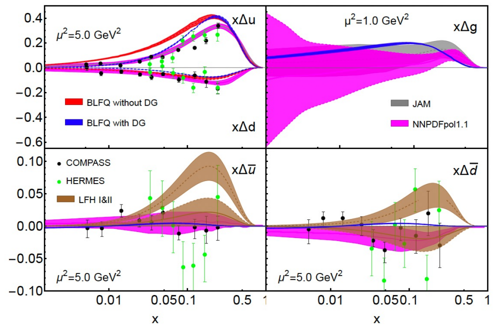
With the dynamical gluon Fock sector, the gluon distribution is closer to the global fits.

S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph]. [EPJC 77 (2017) 663]

30

C. Mondal, LC2023
Tuesday at 11:00

Helicity PDFs



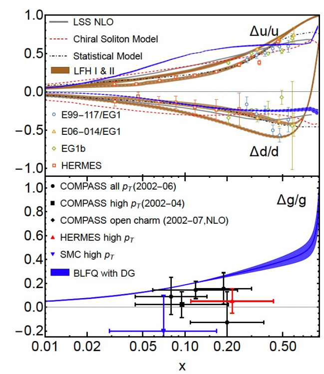
- Quark spin: $\frac{1}{2}\Sigma_u = 0.438 \pm 0.004$, $\frac{1}{2}\Delta\Sigma_d = -0.080 \pm 0.002$.
- Gluon spin: $\Delta G = 0.131 \pm 0.003$, sizeable to the proton spin.
- PHENIX Collaboration: $\Delta G^{[0.02,0.3]} = 0.2 \pm 0.1$.
- Sea quarks: solely generated from the QCD evolution.

[1] BLFQ: 2209.08584 [hep-ph], LFH: 124 (2020), 082003; PHENIX: PRL 103 (2009) 012003].

31

C. Mondal, LC2023
Tuesday at 11:00

Helicity Asymmetries



- Experimentally, the expected increase of $\Delta u/u$ is observed.
- For d quark: remains negative in the experimentally covered region.
- Global analyses favor negative values of $\Delta d/d$ at large-x.

[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].

32

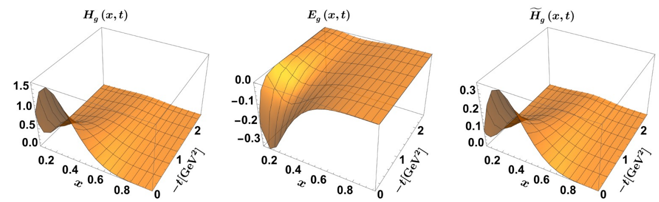
C. Mondal, LC2023
Tuesday at 11:00

Gluon GPDs

$$F^g(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\,\bar{u}(p',\lambda')\left(\gamma^+ H^g(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M} E^g(x,\xi,t)\right) u(p,\lambda)\,,$$

$$\tilde{F}^g(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\,\bar{u}(p',\lambda')\left(\gamma^+\gamma_5 \tilde{H}^g(x,\xi,t) + \frac{\Delta^+\gamma_5}{2M} \tilde{E}^g(x,\xi,t)\right) u(p,\lambda)\,.$$

Non-skewed GPDs



- Total Angular Momentum: $J = \frac{1}{2}\int \mathrm{d}x\, x[H(x,0) + E(x,0)]$;
  $J_g = 0.066,\ 13.2\%$ of the proton TAM.

[1] B. Lin, S. Nair, S.Xu, CM, X. Zhao, J. P. Vary, 2308.08275 [hep-ph].

33

BLFQ Prediction on Spin Decomposition

C. Mondal, LC2023
Tuesday at 11:00

Quark and gluon helicities :

$$\Delta\Sigma_q = \int \mathrm{d}x\, \Delta q(x)$$

$$\Delta\Sigma_g = \int \mathrm{d}x\, \Delta G(x)$$

Total AM :

$$J_i = \int \mathrm{d}x\, x\,[H_i(x,0,0) + E_i(x,0,0)]$$

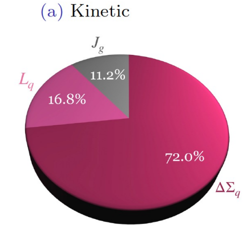
Kinetic OAM :

$$L_q = \int \mathrm{d}x\, [x\,\{H_q(x,0,0) + E_q(x,0,0)\} - \widetilde{H}_q(x,0,0)]$$

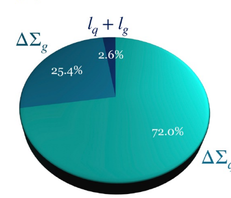
Canonical OAM :

$$l_i^z = -\int \mathrm{d}x\, \mathrm{d}^2\vec{p}_\perp\, \frac{\vec{p}_\perp^{\,2}}{M^2}\, F_{1,4}^i(x,0,\vec{p}_\perp^{\,2},0,0)$$

(a) Kinetic



(b) Canonical



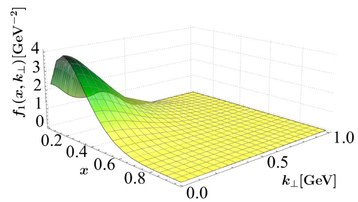
[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].

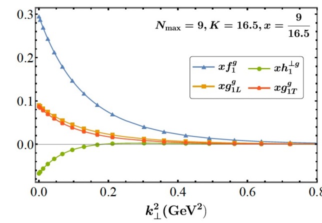
7 / 12

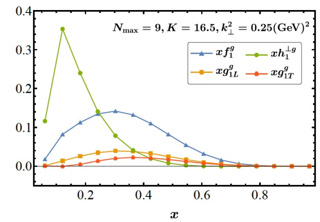
34

C. Mondal, LC2023
Tuesday at 11:00

Gluon TMDs







- Positivity bounds

$$f_1^g(x, \boldsymbol{k}_\perp^2) > 0, \quad f_1^g(x, \boldsymbol{k}_\perp^2) \geq |g_{1L}^g(x, \boldsymbol{k}_\perp^2)|,$$

$$f_1^g(x, \boldsymbol{k}_\perp^2) \geq \frac{|\boldsymbol{k}_\perp|}{M}|g_{1T}^g(x, \boldsymbol{k}_\perp^2)|,$$

$$f_1^g(x, \boldsymbol{k}_\perp^2) \geq \frac{|\boldsymbol{k}_\perp|^2}{2M^2}|h_1^{\perp g}(x, \boldsymbol{k}_\perp^2)|$$

- Satisfies Mulders-Rodrigues relations

[1] Hongyao Yu, *et. al.* in preparation

35

How do baryons distribute their mass and couple to gravity?

36

X. Cao LC2023
Tuesday at 17:00

Quantum Stress on the Light Front

For Scalar systems:

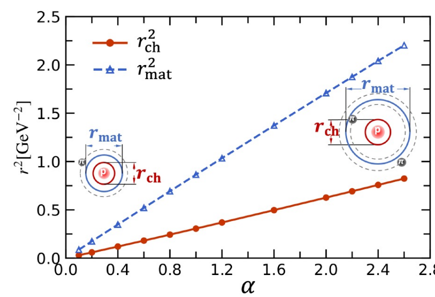
$$\langle p'|T^{\mu\nu}(0)|p\rangle = 2P^\mu P^\nu A(q^2) + \frac{1}{2}(q^\mu q^\nu - q^2 g^{\mu\nu})D(q^2)$$

Model problem: Scalar nucleon interacting via scalar meson field

$$\mathscr{L} = \partial_\mu \chi^\dagger \partial^\mu \chi - m_0^2 \chi^\dagger \chi + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2}\mu_0^2 \varphi^2 + g_0 \chi^\dagger \chi \varphi.$$

Renormalize so that m = 0.94 GeV and μ = 0.14 GeV with $\alpha = g^2/16\pi m^2$. Work in quenched approximation through Fock sectors with 4 mesons In light-front quantized approach.

Matter radius is more than double the charge radius for all couplings haracteristic of a neutral pion cloud.



37

S. Nair, LC2023
Tuesday at 16:00

Nucleon scattering by the classical gravitational field is described by the gravitational (energy momentum tensor) form factors (GFFs).

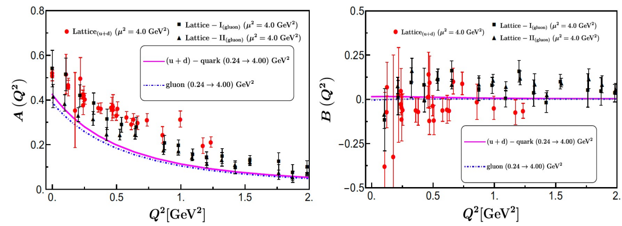
$$\langle P'|\theta_i^{\mu\nu}(0)|P\rangle = \bar{U}'\Big[-B(q^2)\frac{\bar{P}^\mu\bar{P}^\nu}{M} + (A(q^2)+B(q^2))\frac{1}{2}(\gamma^\mu\bar{P}^\nu + \gamma^\nu\bar{P}^\mu) + C(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}(q^2)Mg^{\mu\nu}\Big]U$$

- $A(Q^2)$ and $B(Q^2)$ are obtained from the $(++)$ component.
- $C(Q^2)$ and $\bar{C}(Q^2)$ are extracted from the transverse (i,j) components where $(i,j) \in (1,2)$.
- The GFF $D(Q^2)$ also called the D-term is related to the $C(Q^2)$ as $D(Q^2) = 4C(Q^2)$.

38

S. Nair, LC2023
Tuesday at 16:00

GFF of the Proton included dynamical gluon and scale evolution



$Q^2 = 0$	$i = u$	$i = d$	$i = u + d$	$i = g$
A_i	0.305	0.120	0.425	0.389
B_i	0.175	−0.160	0.148	−0.005

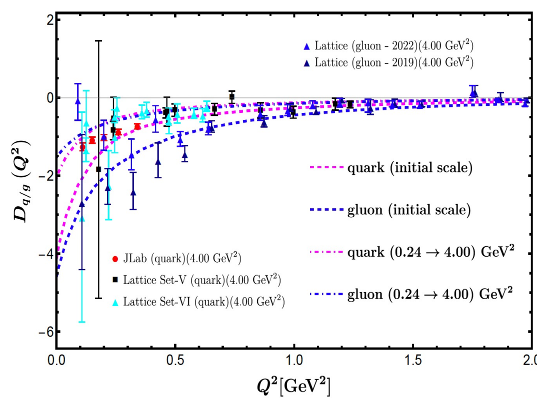
—[Lattice - I]$_{(gluon)}$ P. E. Shanahan, W. Detmold, Phys. Rev. D 99, 014511 (2019).

—[Lattice - II]$_{(gluon)}$ D. A. Pefkou, D. C. Hackett, P. E. Shanahan, Phys. Rev. D 105, 054509 (2022).

39

S. Nair, LC2023
Tuesday at 16:00

GFF of the Proton included dynamical gluon and scale evolution



—[Lattice $_{(gluon)}$ P. E. Shanahan, W. Detmold, Phys. Rev. D 99, 014511 (2019).

—[Lattice]$_{(gluon)}$ D. A. Pefkou, D. C. Hackett, P. E. Shanahan, Phys. Rev. D 105, 054509 (2022).

40

S. Xu, LC2023
Wednesday at 9:30

Next Step Forward

$|P_{baryon}\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqquu\rangle + |qqqdd\rangle + |qqqss\rangle + |qqqgg\rangle$

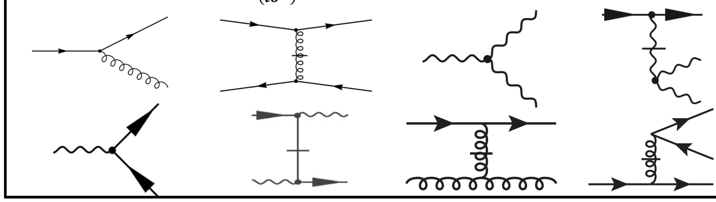
$$P^{-} = H_{K.E.} + H_{Interact}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

No explicit confinement!

CM factorization with Lagrange multiplier term

$$H_{Interact} = g\psi\,\gamma^{\mu}T^{a}\,\psi\,A_{\mu}^{a} + \frac{g^2 C_F}{2}\,j^{+}\frac{1}{(i\partial^{+})^2}j^{+} + \frac{g^2 C_F}{2}\,\psi\gamma^{\mu}A_{\mu}\frac{\gamma^{+}}{i\partial^{+}}A_{\nu}\gamma^{\nu}\psi$$
$$-g^2 C_F\psi\gamma^{+}\psi\frac{1}{(i\partial^{+})^2}i\partial^{+}A_{\mu}^{a}A_{b}^{\mu} + igf^{abc}i\partial^{\mu}A_{a}^{\nu}A_{\mu}^{b}A_{\nu}^{c}$$



41

S. Xu, LC2023
Wednesday at 9:30

Fock Sector Decomposition

$|P_{baryon}\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqquu\rangle + |qqqdd\rangle + |qqqss\rangle + |qqqgg\rangle$

$|qqq\ qq\rangle \sim$ 3 color singlet states

1 singlet × singlet

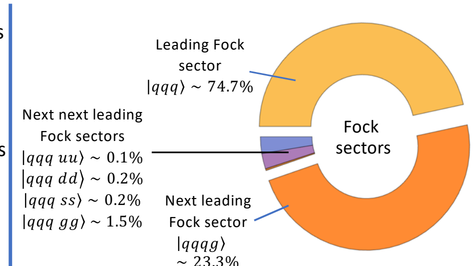
2 octet × octet

$|qqq\ gg\rangle \sim$ 6 color singlet states

1 singlet × singlet

4 octet × octet

1 decuplet × octet × octet



N_{max} = 7, K = 10, matrix dimension = 379,708

m_u	m_d	m_s	m_{f1}	m_{f2}	g	b
0.9 GeV	0.85 GeV	1.0 GeV	1.7 GeV	3.4 GeV	3.0	0.6GeV

42

S. Xu, LC2023
Wednesday at 9:30

Parton Distribution Functions

- Parton distribution functions includes the five-particle Fock sectors

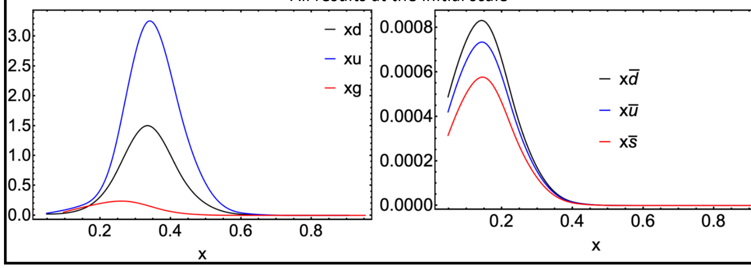
As we include $qqqgg$ Fock sector, the endpoint behavior can be improved

Due to Fock sector truncation (no $qqq\,qq\,g$, $qqq\,ggg$), our five-particle contribution is small

Our results provide qualitative agree with experimental results

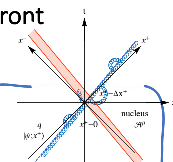
Preliminary results

All results at the initial scale



43

Forward quark jet-nucleus scattering in a light-front Hamiltonian approach



Time-dependent Basis Light-Front Quantization (tBLFQ)

- First-principles:

In the light-front Hamiltonian formalism, the state obeys the time-evolution equation, and the Hamiltonian is derived from the QCD Lagrangian

$$\frac{1}{2}P^-(x^+)|\psi(x^+)\rangle = i\frac{\partial}{\partial x^+}|\psi(x^+)\rangle$$

- Nonperturbative treatment:

The time evolution operator is divided into many small timesteps, each timestep is evaluated numerically and intermediate states are accessible,

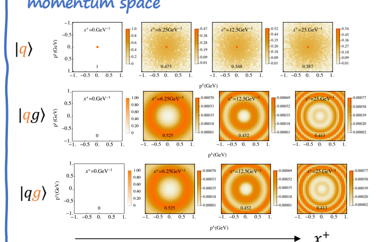
$$|\psi(x^+)\rangle = \mathcal{T}_+ \exp\left[-\frac{i}{2}\int_0^{x^+} dz^+ P^-(z^+)\right]|\psi(0)\rangle$$
$$= \lim_{n\to\infty}\prod_{k=1}^{n}\mathcal{T}_+ \exp\left[-\frac{i}{2}\int_{x_{k-1}^+}^{x_k^+} dz^+ P^-(z^+)\right]|\psi(0)\rangle$$

- Basis representation:

Optimal basis has the same symmetries of the system, and it is the key to numerical efficiency

We consider scattering of a high-energy quark moving in the positive z direction, on a high-energy nucleus moving in the negative z direction.

Time evolution of a quark state in the $|q\rangle + |qg\rangle$ Fock space observed from the transverse momentum space



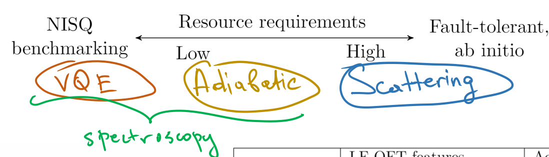
M. Li, T. Lappi and X. Zhao, PRD 104, 056104 (2021); M. Li, T. Lappi, X. Zhao and C. Salgado, PRD 108, 036016 (2023)

44

Wenyang Qian, et al

Quantum Simulation of QFT in the Front Form

2002.04016, 2105.10941, 2011.13443, 2009.07885



$$\mathfrak{Q}_{\text{Direct}} = O(K \log K)$$

$$\mathfrak{Q}_{\text{Compact}} = O(\sqrt{K} \log K)$$

	Trotter	Oracle
Direct	✓	✓
Compact	✗	✓

$$O_F|x,i\rangle = |x, y_i\rangle \ ,$$

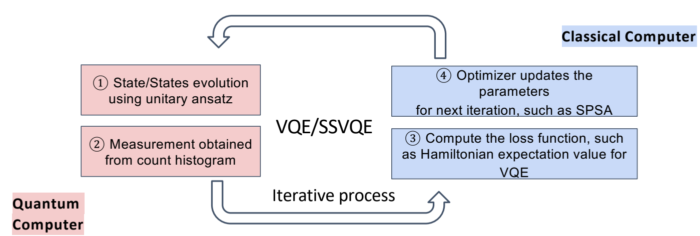
$$O_H|x,y,0\rangle = |x,y,H_{xy}\rangle$$

	LF QFT features	Advantages for QC
Resources	No ghost fields Linear EoM	Low qubit count
	LF momentum > 0	Efficient encoding
Evolution	Sparse Hamiltonians	Using sparsity-based methods
Measurement	LF wavefunction $\rightarrow$ $\rightarrow$ static quantities; Simple form of operators in the second-quantized formalism	Simple form of measurement operators
Other	Trivial vacuum, fewer cut-offs, no fermion doubling, form invariance of H	

45

Light front approach to hadrons on quantum computers

- Quantum computers: New tool to simulate many-body quantum system. (quantum mechanical nature and high scalability)
- In the Noisy Intermediate-Scale Quantum (NISQ) era, the Variational Quantum Eigensolver (VQE) and Subspace-search VQE (SSVQE) [Nakanishi, 1810.09434] approaches are promising tools to solve nuclear physics problems.
- Advantages of light front Hamiltonian formalism are directly applicable
- We first formulate the problem on the light front and then map the Hamiltonian to qubits (quantum bits)



Wenyang Qian, et al. Phys. Rev. Research 4, 043193 (2022)

46

Formulating the problem on qubits

- We adopt the Hamiltonian used in a previous work: [Qian, 2005.13806]

$$H_{\mathrm{eff},\gamma_5} = \overbrace{\underbrace{\frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\mathbf{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}}}^{H_{\mathrm{eff}}} + V_g + H_{\gamma_5}$$

[Vary, 0905.1411]

- Basis representation (BLFQ) is key to represent the Hamiltonian on qubits.
- Small-size Hamiltonians (4-by-4 and 16-by-16) are used.
- Direct encoding and compact encoding are compared.

[Seeley, 1208.5986]
[Kreshchuk, 2002.04016]

	N_f	$\alpha_s(0)$	κ (MeV)	m_q (MeV)	$N_{\max}$	$L_{\max}$	Matrix dimension
$H_{\mathrm{eff}}^{(1,1)}$	3	0.89	560 ± 10	300 ± 10	1	1	4 by 4
$H_{\mathrm{eff}}^{(4,1)}$					4	1	16 by 16

$$H_{\mathrm{direct}}^{(1,1)} = 2269462\,\mathrm{IIII} - 284243\,(\mathrm{ZIII} + \mathrm{IIZI}) - 850488\,(\mathrm{IZII} + \mathrm{IIIZ}) + 12714\,(\mathrm{XZXI} + \mathrm{YZYI}) - 7883\,(\mathrm{IXZX} + \mathrm{IYZY}),$$

$$H_{\mathrm{compact}}^{(1,1)} = 1134731\,\mathrm{II} - 566245\,\mathrm{IZ} + 4831\,\mathrm{XI} + 20598\,\mathrm{XZ}$$

Wenyang Qian, et al. Phys. Rev. Research 4, 043193 (2022)

47

LC2023 Talks on BLFQ, extensions and future directions

Tuesday
11:00 - 11:30 Chandan Mondal - Glue and sea inside proton: A light-front Hamiltonian approach
• 16:00 - 16:30 Sreeraj Nair - Proton gravitational form factors with basis light-front quantization
• 16:30 - 17:00 Jiangshan Lan - Beyond Valence Distributions in meson with Basis Light-Front Quantization
• 17:00 - 17:20 Xianghui Cao - Quantum stress on the light front

Wednesday
• 9:30 - 10:00 Siqi Xu (Awardee) -Towards a Hamiltonian first principle approach for baryons
• 11:00 - 11:30 Xingbo Zhao - Positronium structure from a basis light-front approach
• 11:30 - 12:00 Kamil Serafin - Positronium in quantum electrodynamics of effective particles

Thursday
• 16:30 - 16:50 Satvir Kaur - Structure of spin-1 QCD systems using lightfront Hamiltonian approach

48

48

Summary and Outlook

Basis Light Front Quantization approach to mesons and baryons yields competitive descriptions and predictions

- Positronium test applications found successful
- Bound states and transitions of hadrons are described
- Time-dependent scattering applications are advancing
- Plan: continue to expand the Fock spaces (e.g. more gluons)
- Plan: continue to develop renormalization & counterterms
- Efficient utilization of supercomputing resources
- Well-positioned to exploit advances in quantum computing

Thank you for your attention

49