Spectra, form factors and hadronic structure functions from a deformed AdS model

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Work done in collaboration with:

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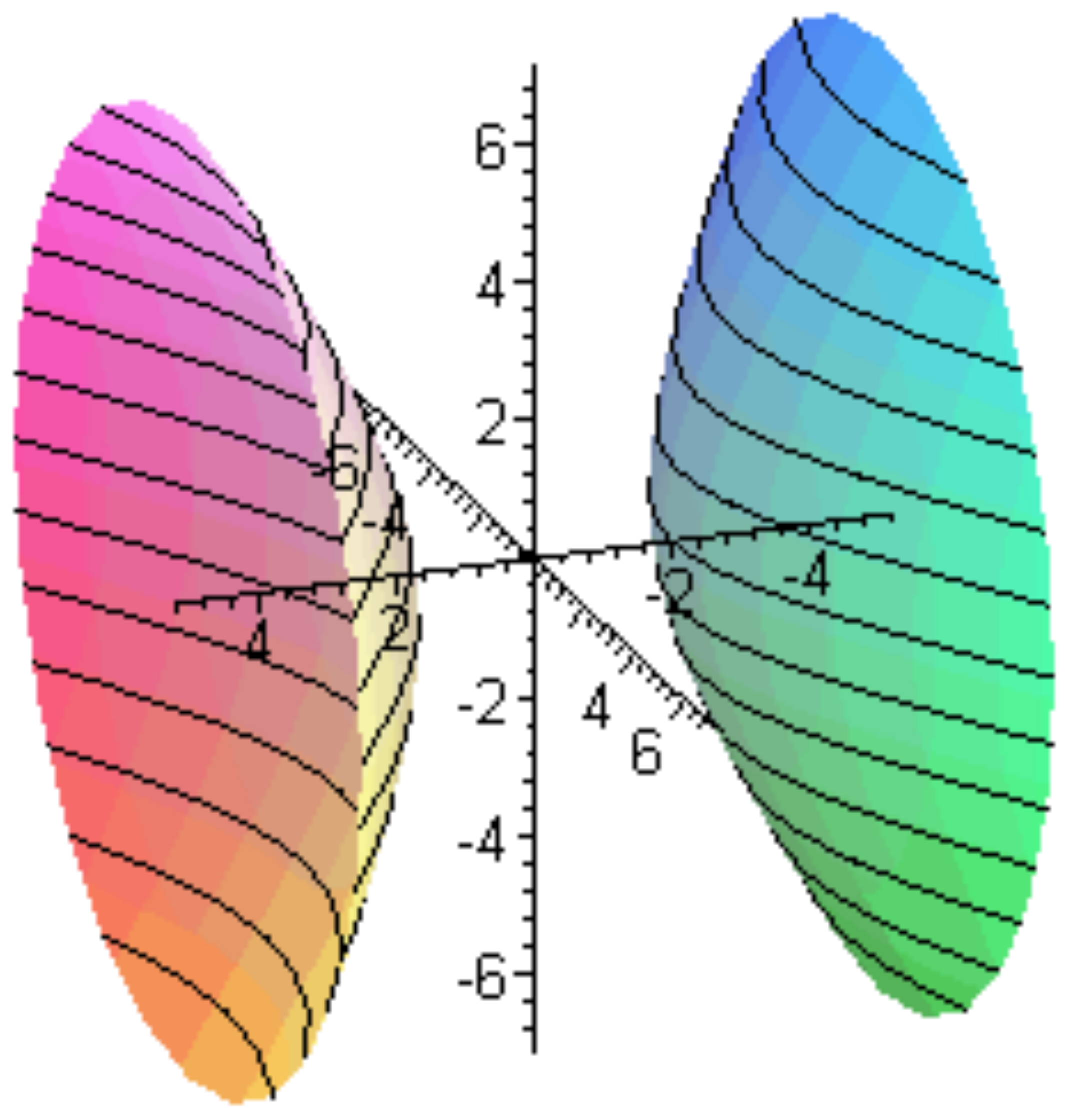
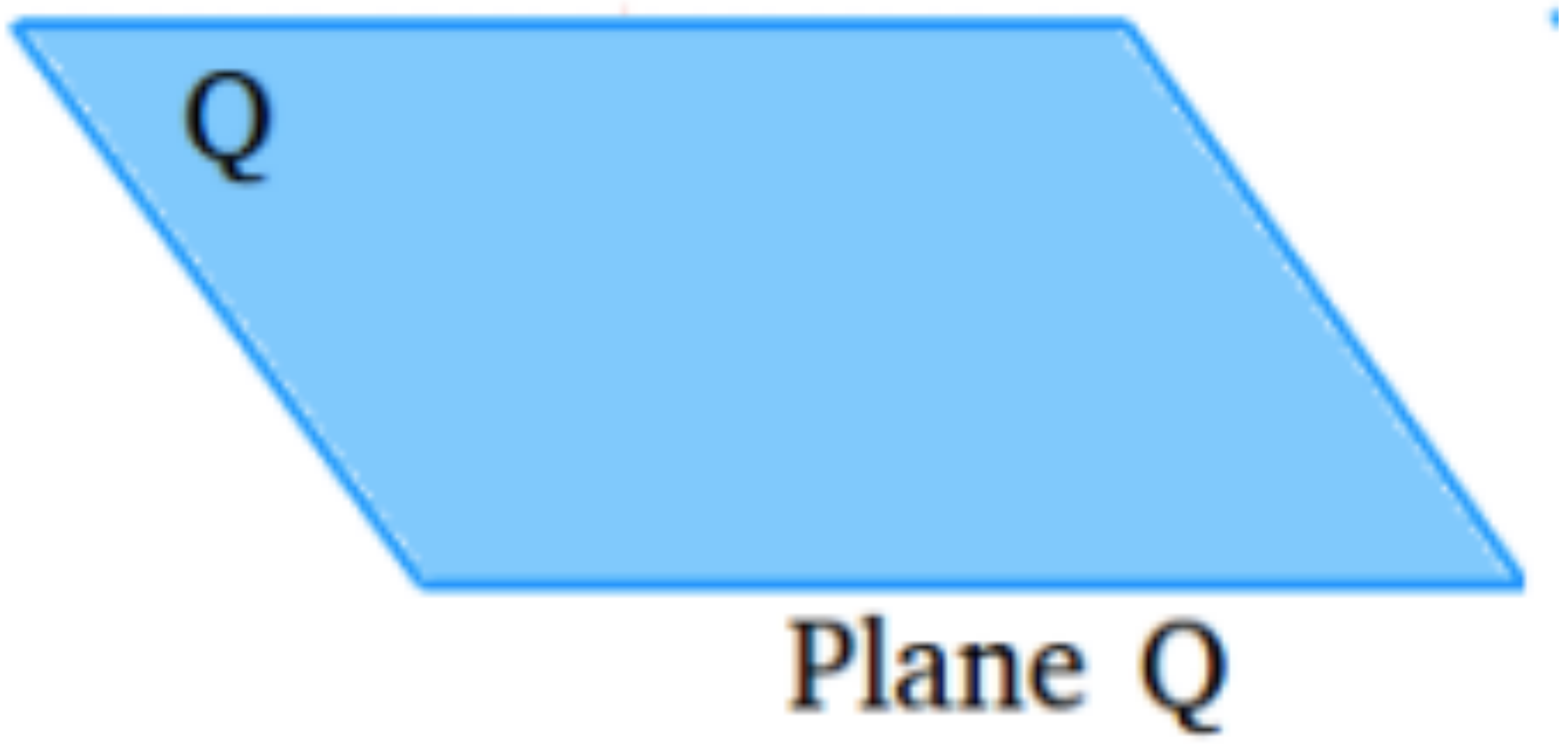
- Alfredo Vega (Valparaíso, Chile)

Holography from AdS/CFT Correspondence



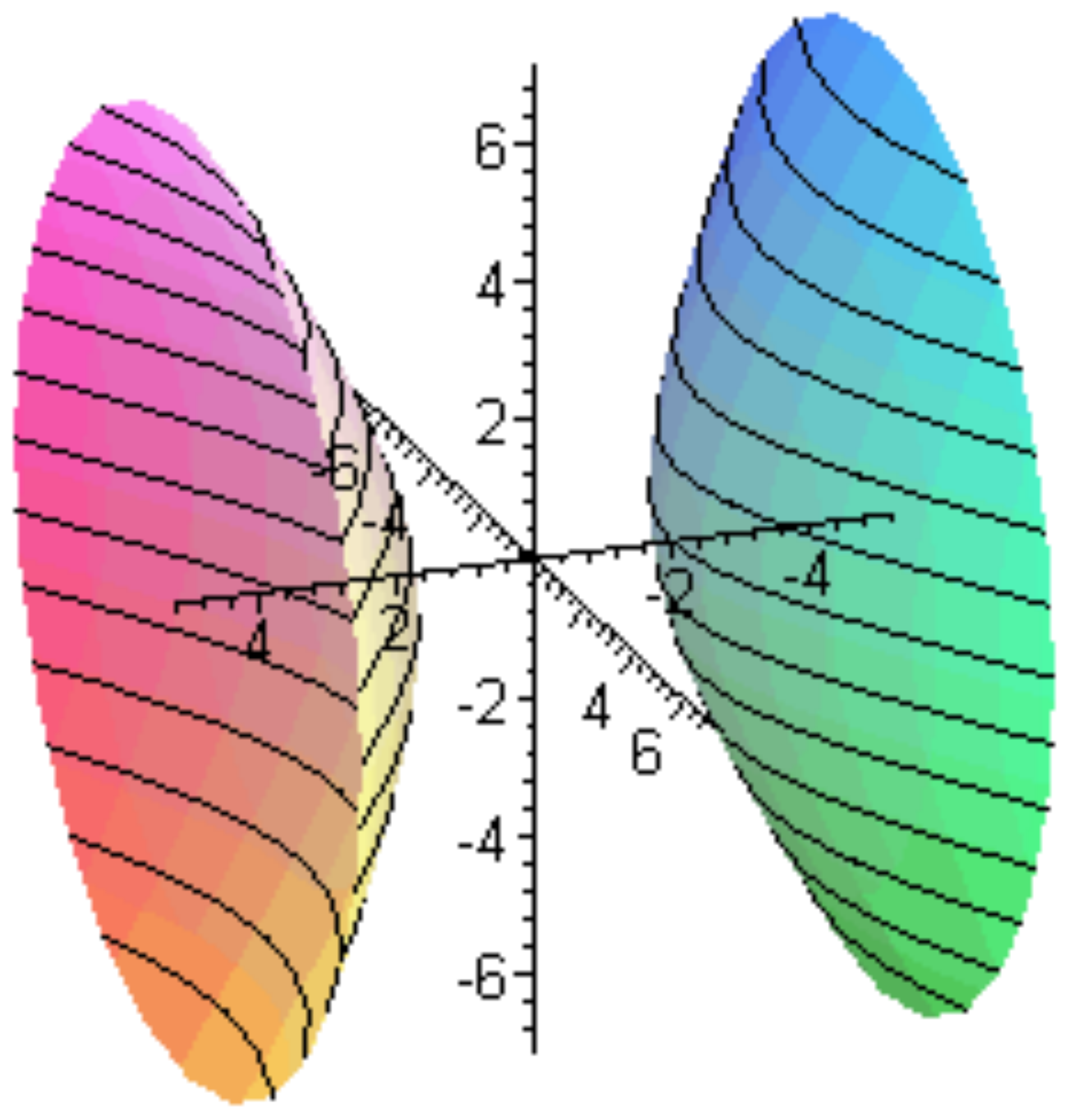


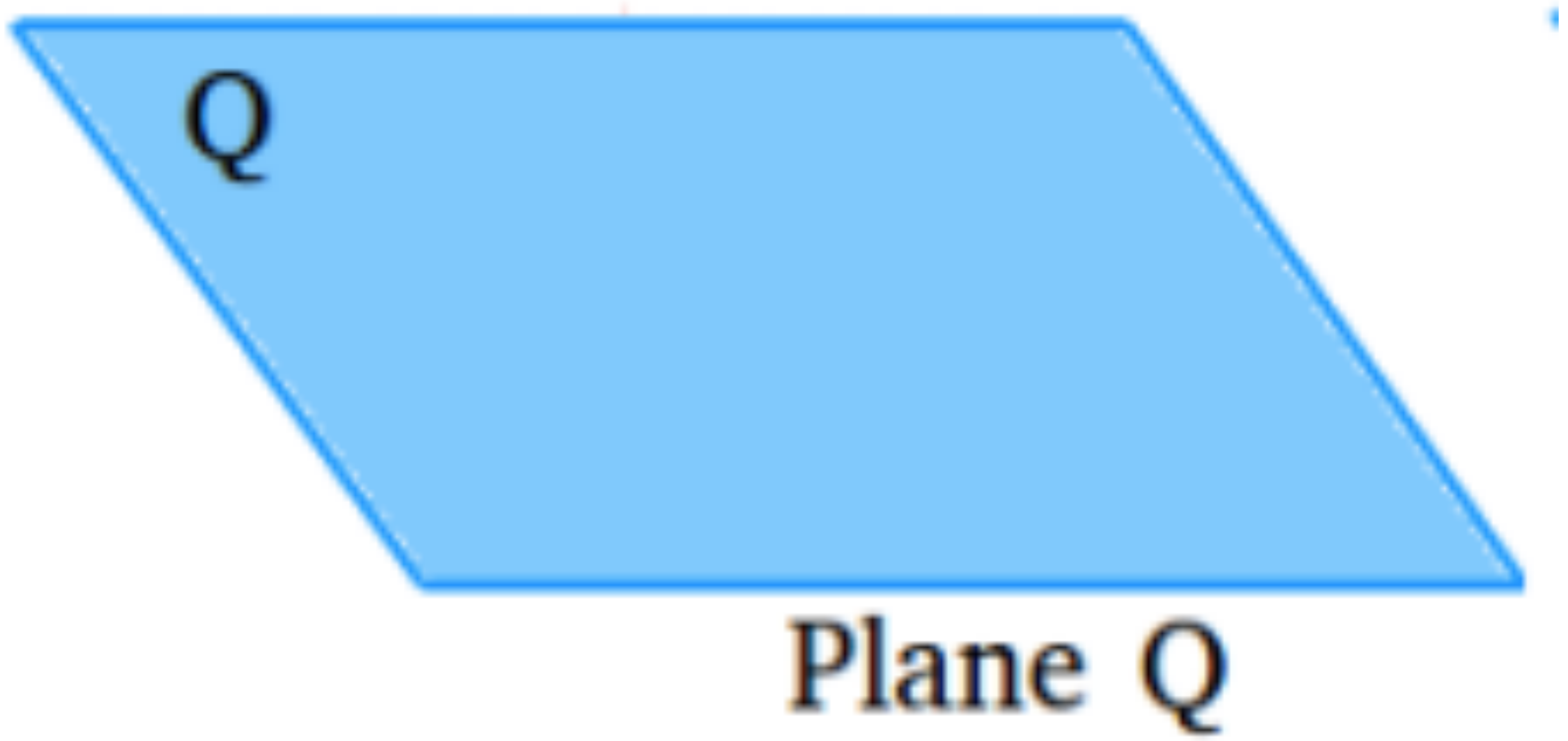
Gravity (Classical)		Field Theory (Quantum)
AdS space (d+1) dim. Curved space	↔	Minkowski / Euclidean d-dim. Flat spaces
AdS_5	↔	$\mathbf{R}^{3,1}$ / $\mathbf{R}^4$
AdS_4	↔	$\mathbf{R}^{2,1}$ / $\mathbf{R}^3$
⋮	↔	⋮ / ⋮

Gravity
(Classical)

Field Theory
(Quantum)

Holography from AdS/CFT Correspondence





AdS space
(d+1) dim. Curved space

Minkowski / Euclidean
d-dim. Flat spaces

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)$$

$$ds^2 = -\,dt^2 + d\vec{x}^2$$

$$ds^2 = dt^2 + d\vec{x}^2$$

L = AdS radius

AdS/QCD Holographic models for SU(N) interactions

AdS space with an IR cut off ⟷ Field Theory with an IR cut off

Examples:

Hard Wall model

Polchinski & Strassler '02
HBF and N Braga'03

Action $= \int d^d x \, dz \sqrt{-g} \, \mathscr{L}$

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)$$

$$0 \leq z \leq z_{max}$$

IR scale: $z_{max} \sim 1/\Lambda_{QCD}$

Applications: Glueball masses and for any other hadron.

Soft Wall model

KKSS (Karch et al'06)

Action $= \int d^d x \, dz \sqrt{-g} \, e^{-kz^2} \mathscr{L}$

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)$$

$$0 \leq z \leq \infty$$

IR scale: $k \sim \Lambda_{QCD}^2$

Application: Vector meson masses. Does not work with other hadrons.

Hadronic Spectra from Deformed AdS backgrounds

Capossoli, Contreras, Li, Vega, HBF CPC 2020

$$\text{Action} = \int d^d x\, dz \sqrt{-g}\, \mathscr{L}$$

AdS metric

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)$$

Deformed AdS metric

$$ds^2 = \frac{L^2}{z^2} e^{kz^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)$$

$$\equiv e^{2A(z)}\left(-dt^2 + d\vec{x}^2 + dz^2\right)$$

$$A(z) = \log\frac{L}{z} + \frac{k}{2}z^2$$

Hadronic spectra for glueballs states

$$S = \int \mathrm{d}^5 x \sqrt{-g}\, [g^{mn}\partial_m X \partial^n X + M_5^2 X^2].$$

$$-\psi''(z) + \left[\frac{9}{4}A'^2(z) + \frac{3}{2}A''(z) + e^{2A(z)}M_5^2\right]\psi(z) = -\, q^2\psi(z),$$

$$M_5^2 = J(J+4)\,; \qquad (\text{even}\, J)\,.$$

$$X(z, x^\mu) = v(z)\mathrm{e}^{\mathrm{i}q_\mu x^\mu}. \qquad v(z) = \psi(z)e^{-\frac{3}{2}A(z)}$$

$$M_5^2 = (J+6)(J+2)\,; \qquad (\text{odd}\, J)\,,$$

Hadronic Spectra from Deformed AdS backgrounds

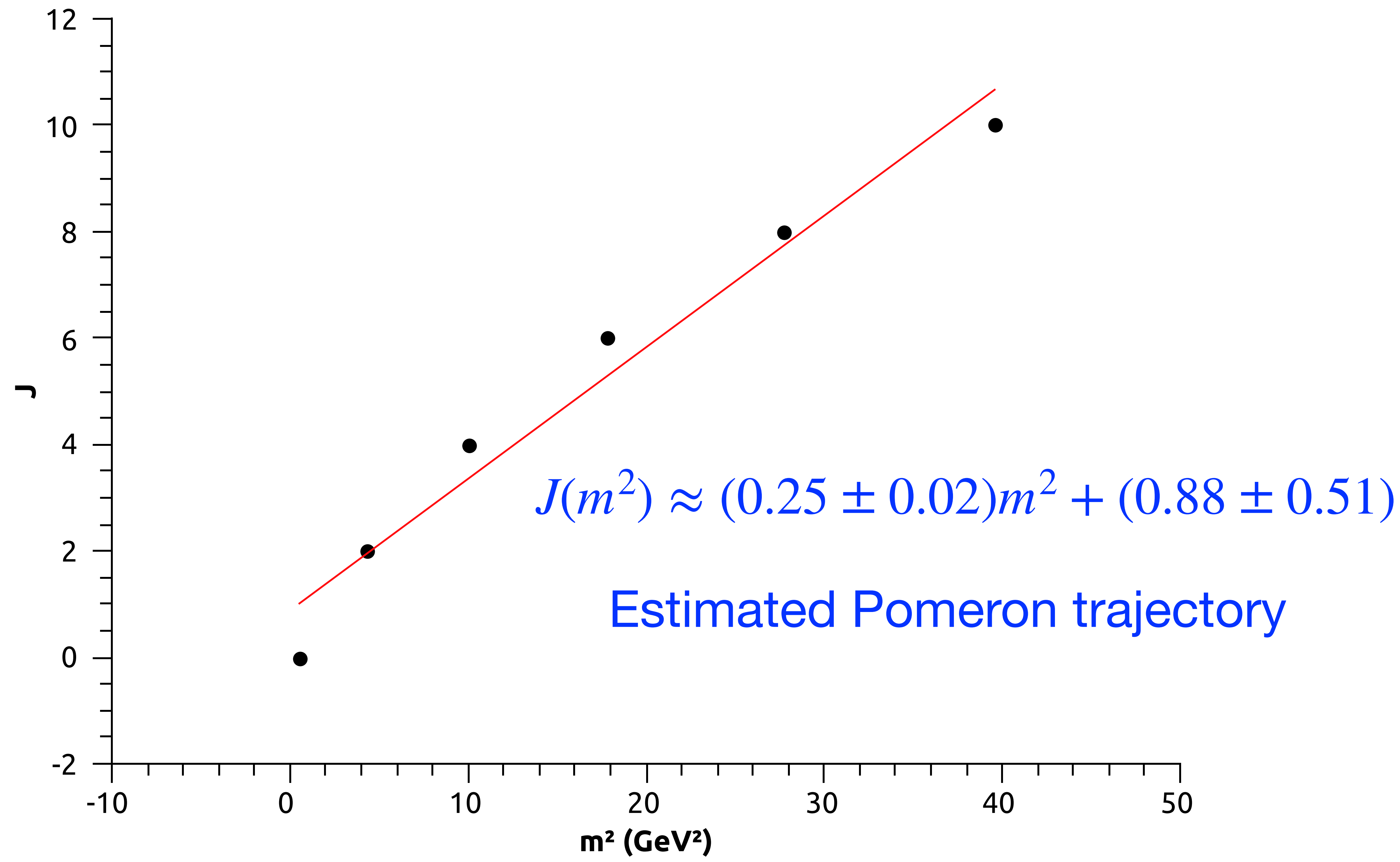
Hadronic spectra for glueballs states

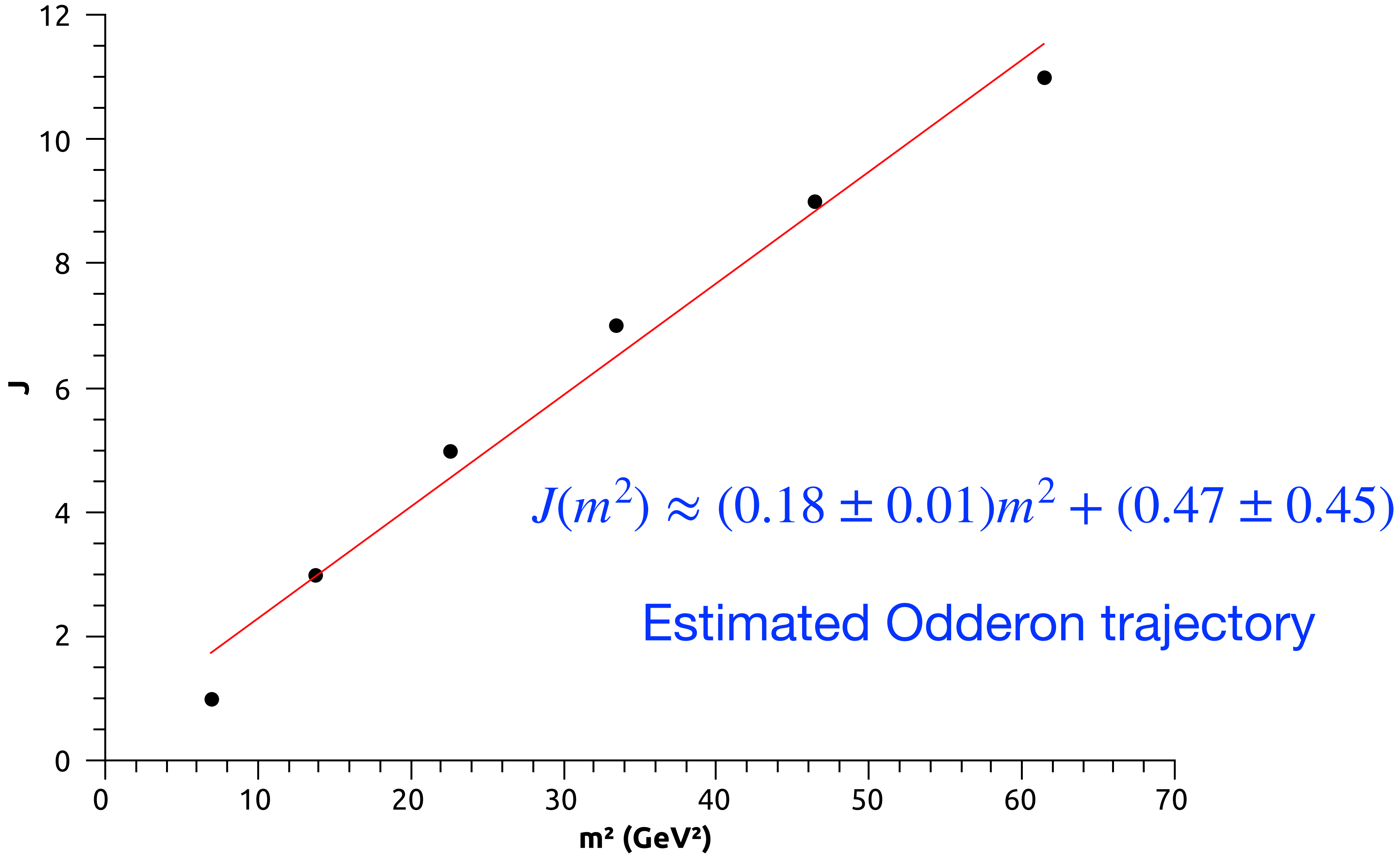
$k_{gbe} = 0.31^2\ \text{GeV}^2.$

	Even glueball states J^{PC}					
	0^{++}	2^{++}	4^{++}	6^{++}	8^{++}	10^{++}
Masses	0.76	2.08	3.17	4.22	5.26	6.30

$k_{gbo} = 0.31^2\ \text{GeV}^2.$

	Odd glueball states J^{PC}					
	1^{--}	3^{--}	5^{--}	7^{--}	9^{--}	11^{--}
Masses	2.63	3.70	4.74	5.78	6.81	7.84





Hadronic Spectra from Deformed AdS backgrounds

Hadronic spectra for scalar mesons

$$S = \int d^5x \sqrt{-g}\, [g^{mn}\partial_m X \partial^n X + M_5^2 X^2].$$

AdS/CFT prescription $M_5^2 = (\Delta - p)(\Delta + p - 4)$

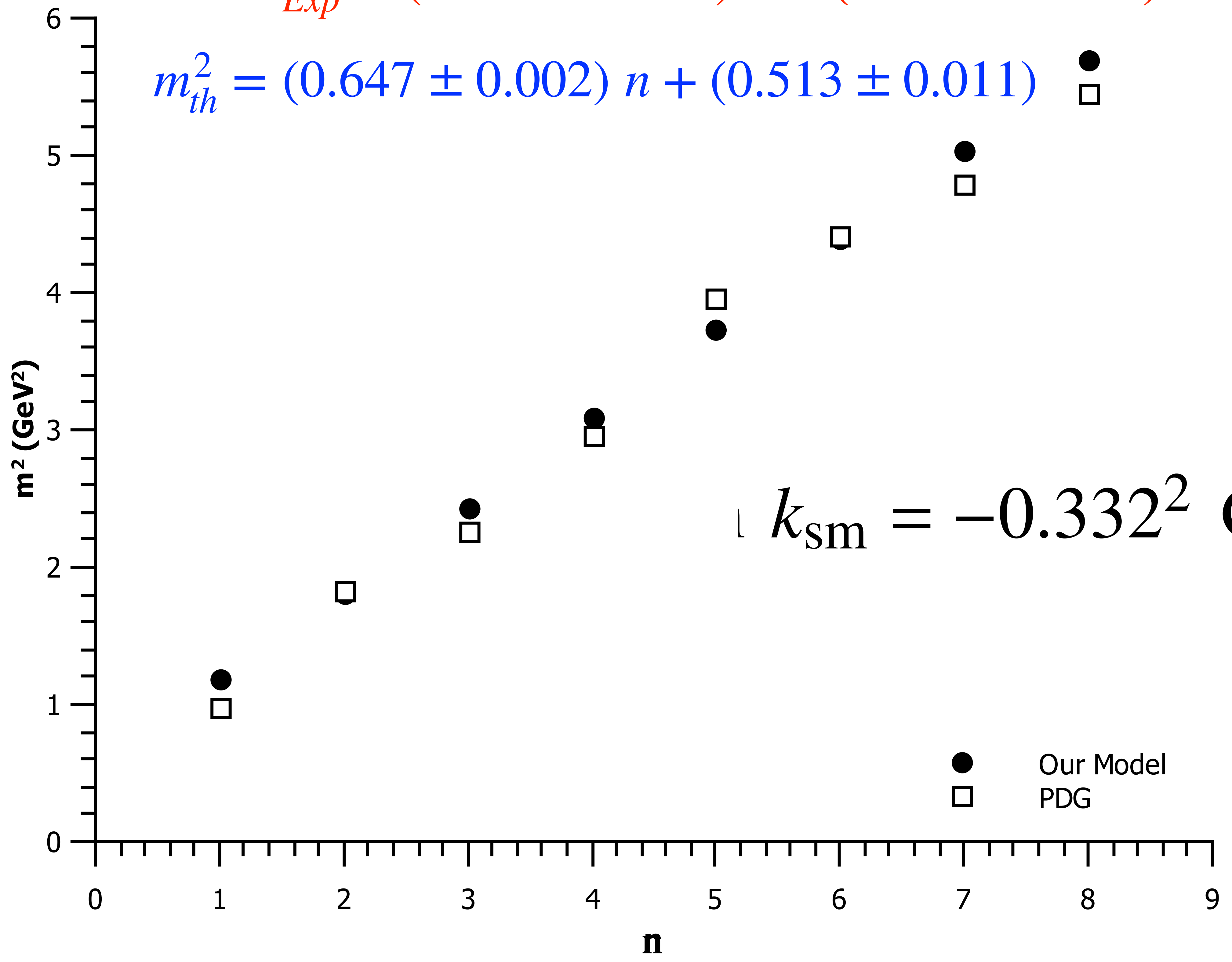
$p = J = 0$

$\Delta_{\text{quark}} = 3/2$ $\Delta_{\text{meson}} = 3$ $M_5^2 = -3$

	Scalar meson f_0 $(0^+(0^{++}))$			
	f_0 meson	M_{exp}/GeV [69]	M_{th}/GeV	$\%M$
$n = 1$	$f_0(980)$	0.990 ± 0.02	1.089	9.97
$n = 2$	$f_0(1370)$	1.2 to 1.5	1.343	0.54
$n = 3$	$f_0(1500)$	1.504 ± 0.006	1.562	3.87
$n = 4$	$f_0(1710)$	$1.723^{+0.006}_{-0.005}$	1.757	1.96
$n = 5$	$f_0(2020)$	1.992 ± 0.016	1.933	2.96
$n = 6$	$f_0(2100)$	2.101 ± 0.007	2.095	0.27
$n = 7$	$f_0(2200)$	2.189 ± 0.013	2.246	2.61
$n = 8$	$f_0(2330)$	2.337 ± 0.014	2.388	2.17

$m^2_{Exp} = (0.639 \pm 0.027)\, n + (0.458 \pm 0.135)$

$m^2_{th} = (0.647 \pm 0.002)\, n + (0.513 \pm 0.011)$



$k_{\text{sm}} = -0.332^2\ \text{GeV}^2.$

[69] PDG 2018

Hadronic Spectra from Deformed AdS backgrounds

Hadronic spectra for vector mesons

AdS/CFT prescription $M_5^2 = (\Delta - p)(\Delta + p - 4)$

$p = J = 1$

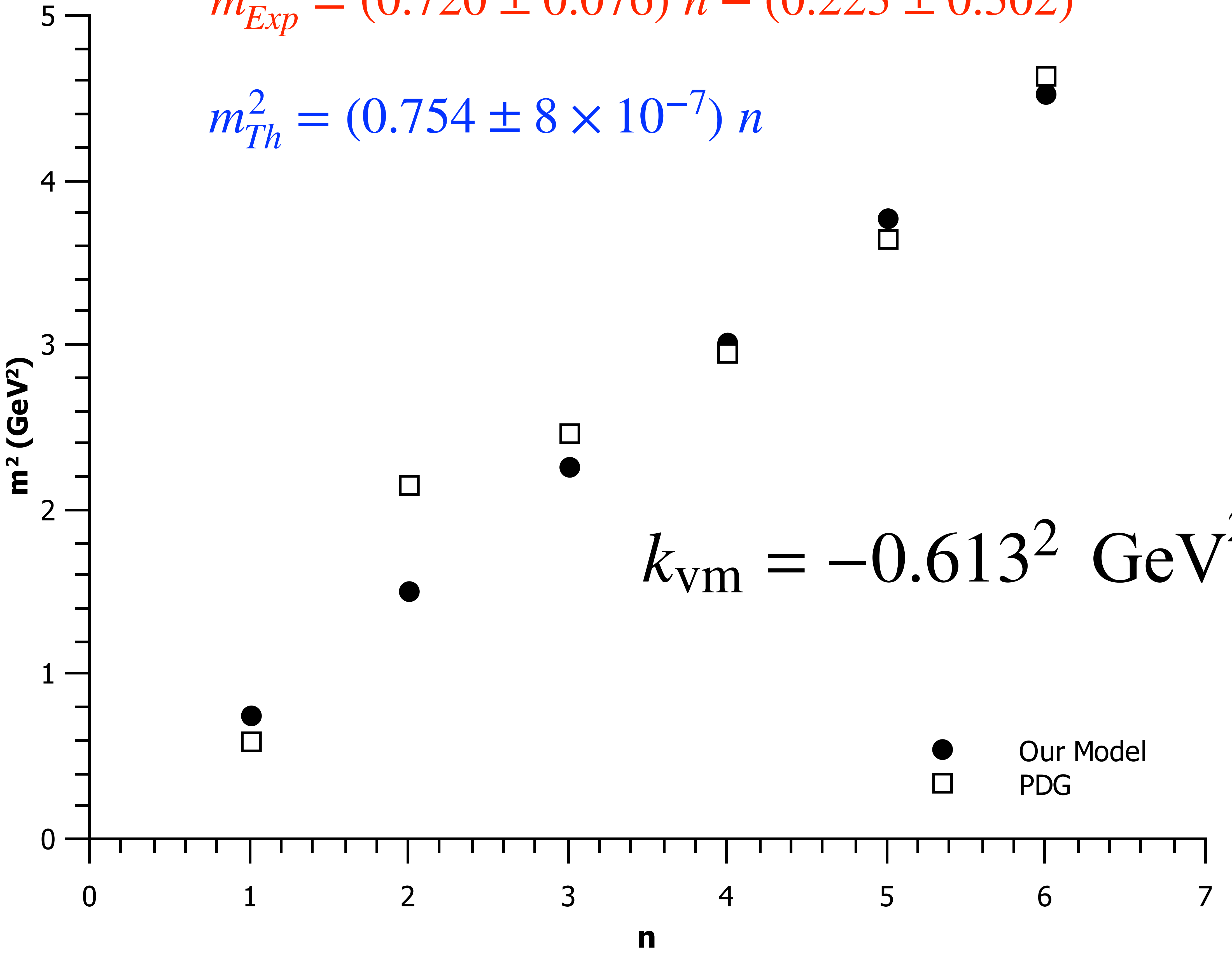
$$S = -\frac{1}{2}\int d^5x\,\sqrt{-g}\left[\frac{1}{2}g^{pm}g^{qn}F_{mn}F_{pq} + M_5^2 g^{pm}A_pA_m\right],$$

$\Delta_{\text{quark}} = 3/2$ $\Delta_{\text{meson}} = 3$ $M_5^2 = 0$

	Vector meson ρ $(1^+(1^{--}))$			
	ρ meson	M_{exp}/GeV [69]	M_{th}/GeV	$\%M$
$n = 1$	$\rho(770)$	0.77526 ± 0.00025	0.868327	12.0422
$n = 2$	$\rho(1450)$	1.465 ± 0.025	1.228	16.1775
$n = 3$	$\rho(1570)$	1.570 ± 0.070	1.50399	4.20467
$n = 4$	$\rho(1700)$	1.720 ± 0.020	1.73665	0.968271
$n = 5$	$\rho(1900)$	1.909 ± 0.042	1.94164	1.70972
$n = 6$	$\rho(2150)$	2.155 ± 0.021	2.12696	1.30123

$m^2_{Exp} = (0.720 \pm 0.076)\, n - (0.223 \pm 0.302)$

$m^2_{Th} = (0.754 \pm 8 \times 10^{-7})\, n$



$k_{\text{vm}} = -0.613^2\ \text{GeV}^2.$

[69] PDG 2018

Hadronic Spectra from Deformed AdS backgrounds

Hadronic spectra for baryons

$$S = \int_{\mathrm{AdS}} \mathrm{d}^5 x \sqrt{g}\, \bar{\Psi}(\not{D} - m_5)\Psi.$$

Deformed

One gets a Schrödinger-like equation written for both right and left sectors:

$$-\psi''_{R/L}(z) + \left[m_5^2 e^{2A(z)} \pm m_5 e^{A(z)} A'(z)\right] \psi_{R/L}(z) = M_n^2 \psi^n_{R/L}(z),$$

where M_n are the four-dimensional fermion masses.

$$A(z) = \log \frac{L}{z} + \frac{k}{2} z^2$$

Hadronic Spectra from Deformed AdS backgrounds

AdS/CFT prescription (fermions)

$$|m_5| = \Delta - 2 .$$

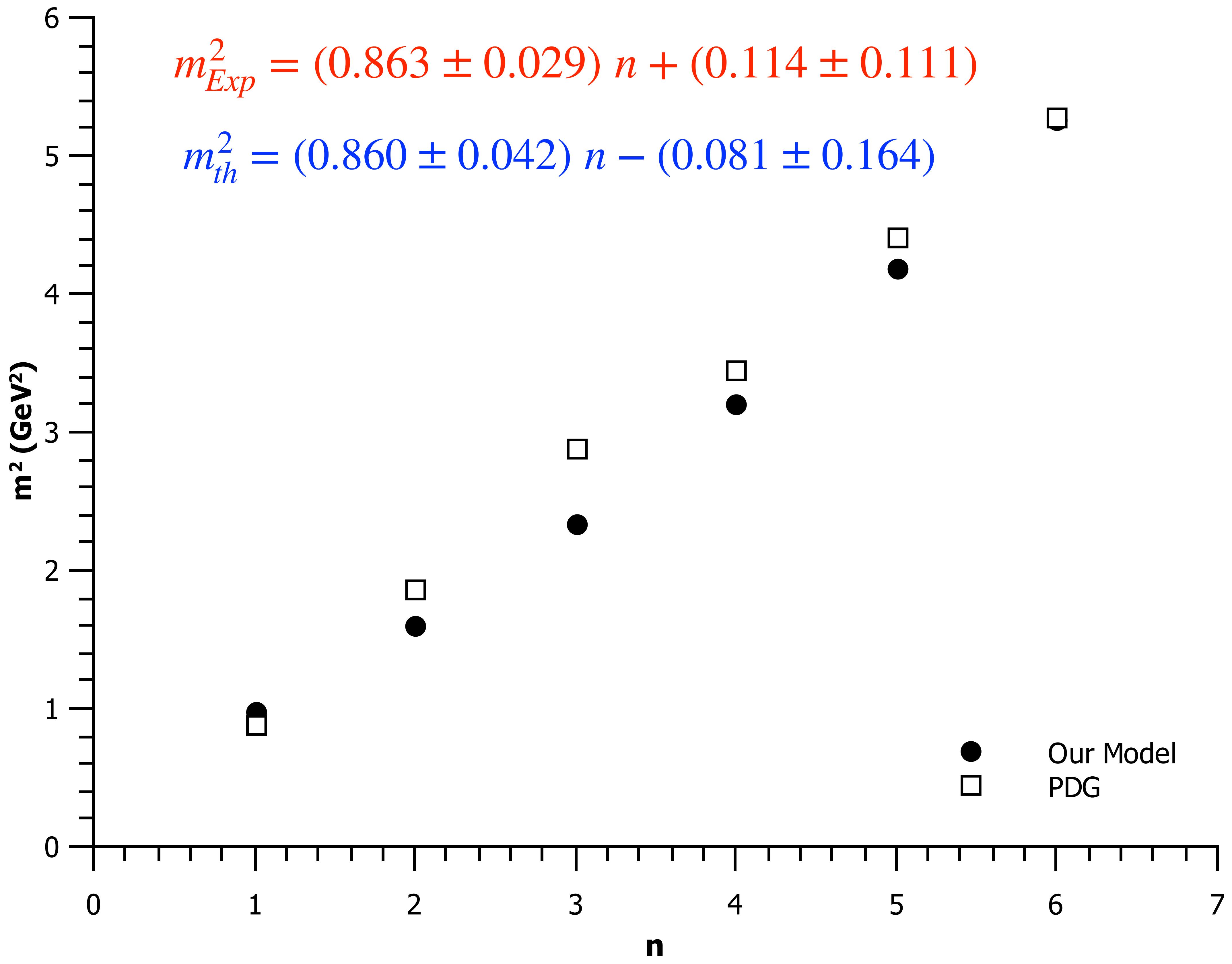
$\Delta_{\text{quark}} = 3/2$ $\Delta_{\text{Baryon}} = 9/2$ $m_5 = 5/2 .$

Results for spin 1/2 baryons spectra

$$k_{1/2} = 0.205^2 \ \text{GeV}^2,$$

	Baryons $N(1/2^+)$			
	N baryon	M_{exp}/GeV [69]	M_{th}/GeV	$\%M$
$n=1$	$N(939)$	0.93949 ± 0.00005	0.98683	5.04
$n=2$	$N(1440)$	1.360 to 1.380	1.264	7.76
$n=3$	$N(1710)$	1.680 to 1.720	1.531	9.94
$n=4$	$N(1880)$	1.820 to 1.900	1.791	3.70
$n=5$	$N(2100)$	2.050 to 2.150	2.046	2.58
$n=6$	$N(2300)$	$2.300^{+0.006\ +0.1}_{-0.005\ -0}$	2.296	0.19

[69] PDG 2018



Hadronic Spectra from Deformed AdS backgrounds

Results for higher spin baryons spectra

$$\Delta_{3/2} = 9/2 + 1 = 11/2$$

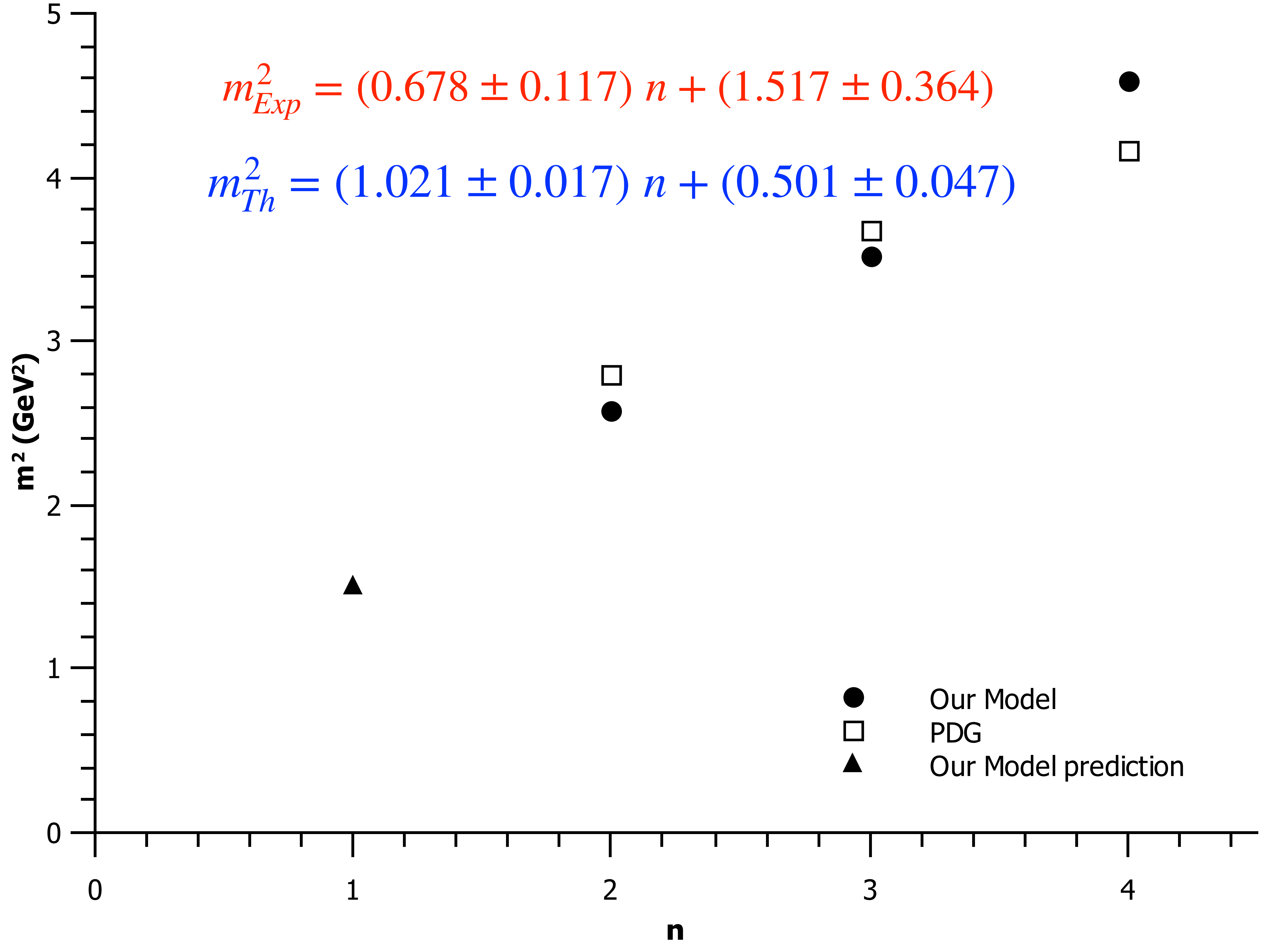
$$m_5 = 7/2.$$

$$k_{3/2} = 0.205^2 \ \text{GeV}^2,$$

	Baryons $N(3/2^+)$			
	N baryon	M_{exp}/GeV [69]	M_{th}/GeV	$\%M$
$n=1$	$N(1720)$	1.660 to 1.690	1.326	23.05
$n=2$	$N(1900)$	1.900 to 1.940	1.606	12.27
$n=3$	$N(2040)$	$2.040^{+0.003}_{-0.004} \pm 0.025$	1.878	8.72

	Baryons $N(3/2^+)$			
	N baryon	M_{exp}/GeV [69]	M_{th}/GeV	$\%M$
$n=1$			1.326	
$n=2$	$N(1720)$	1.660 to 1.690	1.606	4.14
$n=3$	$N(1900)$	1.900 to 1.940	1.878	2.19
$n=4$	$N(2040)$	$2.040^{+0.003}_{-0.004} \pm 0.025$	2.144	5.09

[69] PDG 2018



Hadronic Spectra from Deformed AdS backgrounds

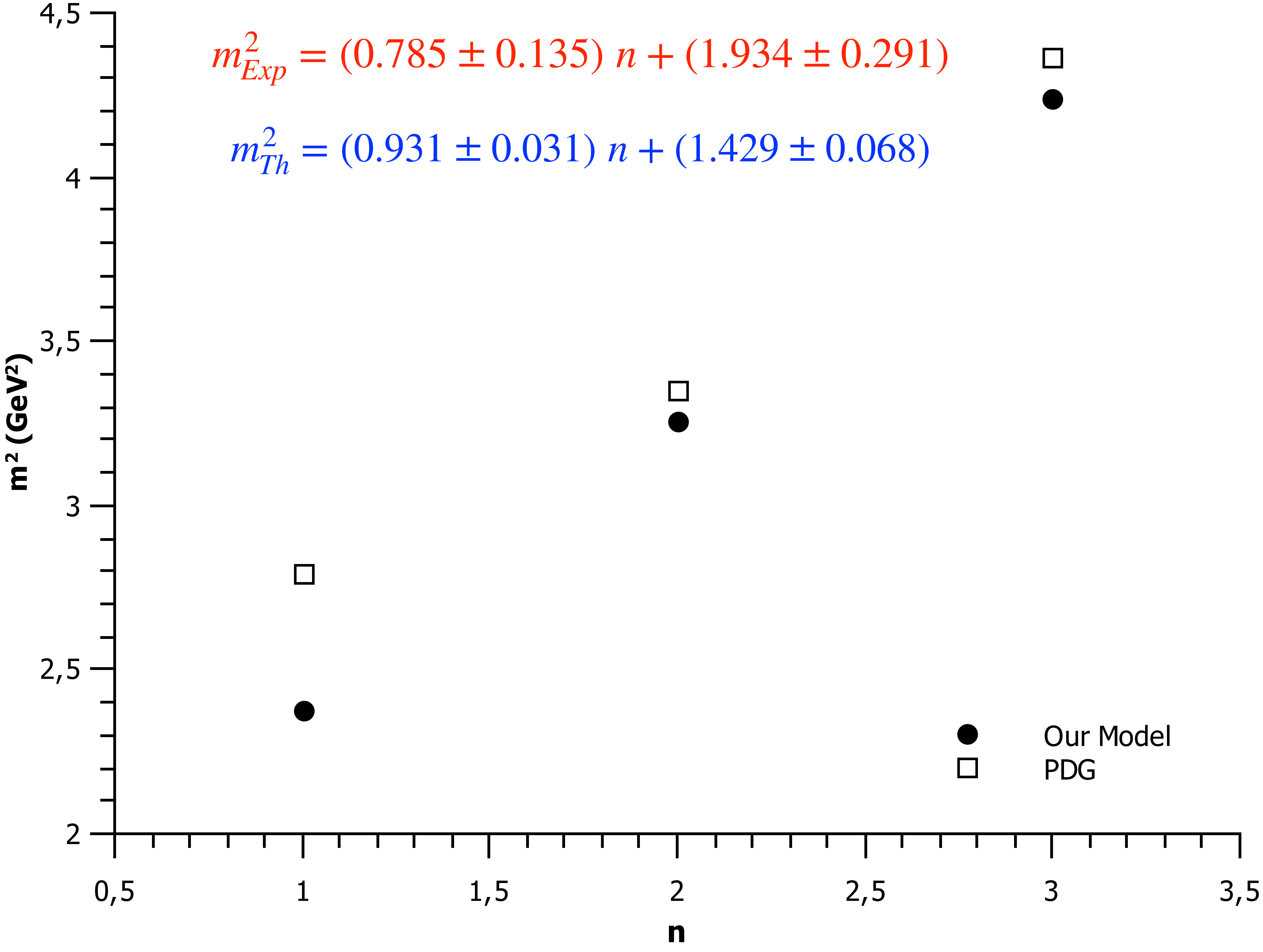
Results for higher spin baryons spectra

$$k_{5/2} = 0.190^2 \ \text{GeV}^2,$$

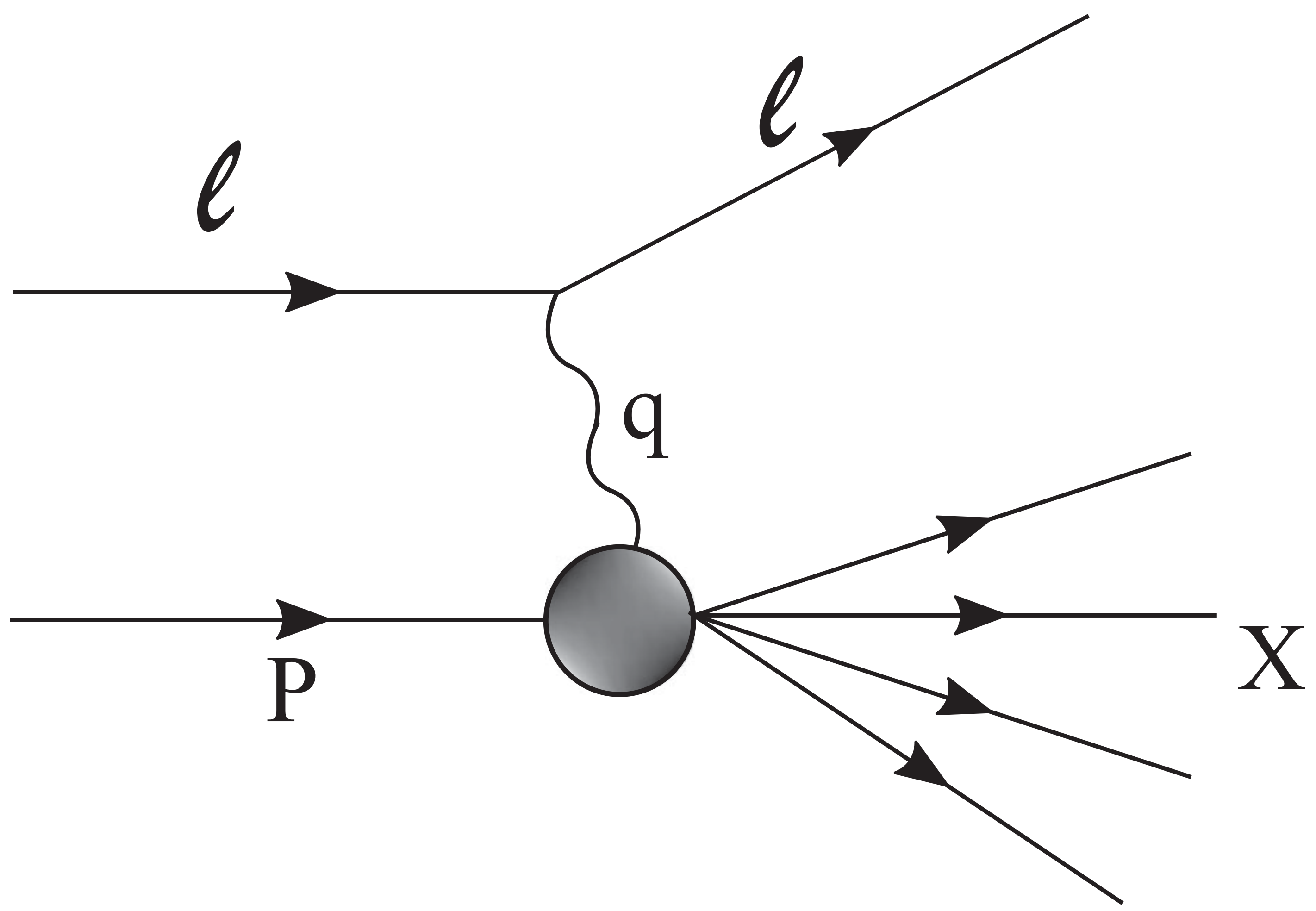
$$\Delta_{5/2} = 13/2 \qquad m_5 = 9/2.$$

	Baryons $N(5/2^+)$			
	N baryon	M_{exp}/GeV [69]	M_{th}/GeV	$\%M$
$n=1$	$N(1680)$	1.665 to 1.680	1.542	7.78
$n=2$	$N(1860)$	1.830^{+120}_{-60}	1.804	1.44
$n=3$	$N(2000)$	2.090 ± 120	2.059	1.49

[69] PDG 2018



Deep Inelastic Scattering (DIS)



The so-called *Bjorken variable* parametrizes this fragmentation according to:

$$x = -\frac{q^2}{2P \cdot q}, \quad (1)$$

where q^2 is the transferred momentum from the lepton to the proton by a virtual photon and P is the initial proton momentum, with mass defined as $P^2 = -M^2$.

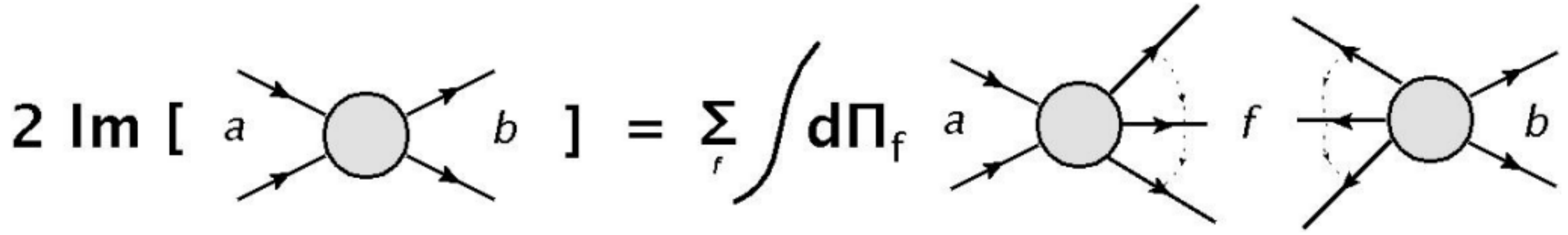
Scattering Amplitude

$$i\mathcal{M}_{lp\to lX} = (iQ)\bar{u}\gamma_\mu u\left(\frac{i}{q^2}\right)(ie)\int d^4y e^{iq\cdot y}\langle X|J^\mu(y)|P\rangle,$$

Deep Inelastic Scattering (DIS)

the optical theorem

$$\sum_X \int d\Pi_X |\mathcal{M}_{\gamma p \to X}|^2 = 2\mathbb{I}\mathrm{m}\mathcal{M}_{\gamma p \to \gamma p}$$



Hadronic Tensor

$$W^{\mu\nu} = \frac{i}{4\pi}\sum_s \int d^4y e^{iq.y}\langle P, s|\mathcal{T}\{J^\mu(y)J^\nu(0)\}|P, s\rangle.$$

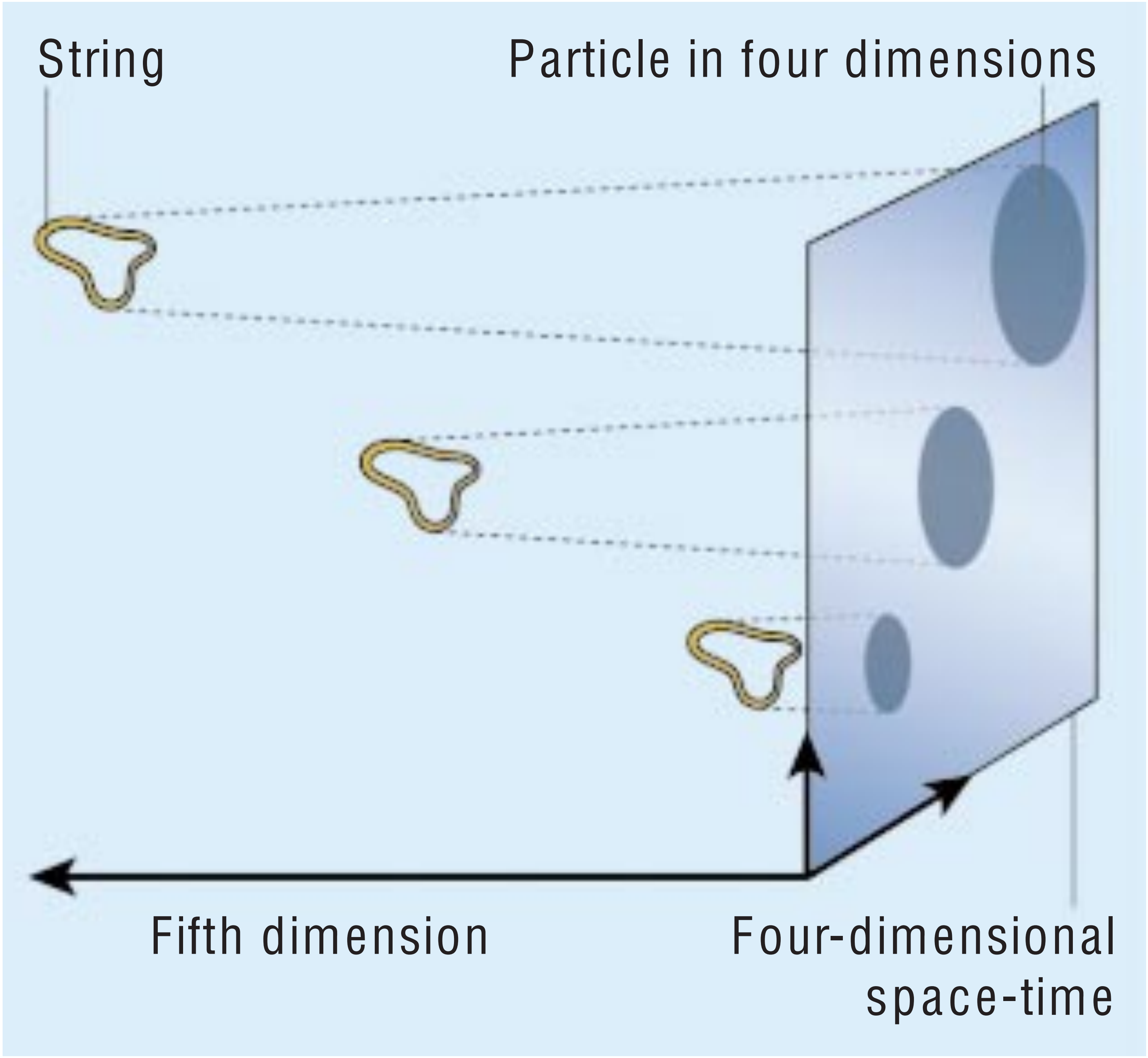
$$W^{\mu\nu} = F_1\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \frac{2x}{q^2}F_2\left(P^\mu + \frac{q^\mu}{2x}\right)\left(P^\nu + \frac{q^\nu}{2x}\right).$$

$F_{1,2} \equiv F_{1,2}(x, q^2),$ Structure functions.

DIS from AdS/QCD

- Polchinski & Strassler'03 Scalar and Fermionic (S=1/2) DIS within the Hard Wall Model, for three regimes: large, small and very small x;

- Maldacena, Nature'03: "Particles such as the proton can be imagined as vibrating strings. We also know that protons contain smaller, point-like particles, going against the string theory. But in five dimensions, the contradiction disappears"



Proton Structure functions from AdS/QCD with deformed background

Capossoli, Contreras, Li, Vega, HBF PRD 2020

$$S = \int d^5x \sqrt{-g}\mathcal{L}$$

$$ds^2 = g_{mn}dx^m dx^n = e^{2A(z)}(dz^2 + \eta_{\mu\nu}dy^\mu dy^\nu).$$

$$A(z) = \log\frac{L}{z} + \frac{k}{2}z^2$$

Proton Structure functions from AdS/QCD with deformed background

A. Computing the electromagnetic field

$$S = -\int d^5x \sqrt{-g}\frac{1}{4}F^{mn}F_{mn}, \qquad F^{mn} = \partial^m \phi^n - \partial^n \phi^m.$$

$$\phi_\mu = -\frac{1}{2}\eta_\mu e^{iq\cdot y} B(z,q)$$

$$B(z,q) = kz^2\Gamma\left[1-\frac{q^2}{2k}\right] U\left(1-\frac{q^2}{2k}; 2; -\frac{kz^2}{2}\right)$$

Proton Structure functions from AdS/QCD with deformed background

B. Computing the baryonic states

$$S = \int d^5x \sqrt{g}\bar{\Psi}(\not{D} - m_5)\Psi,$$

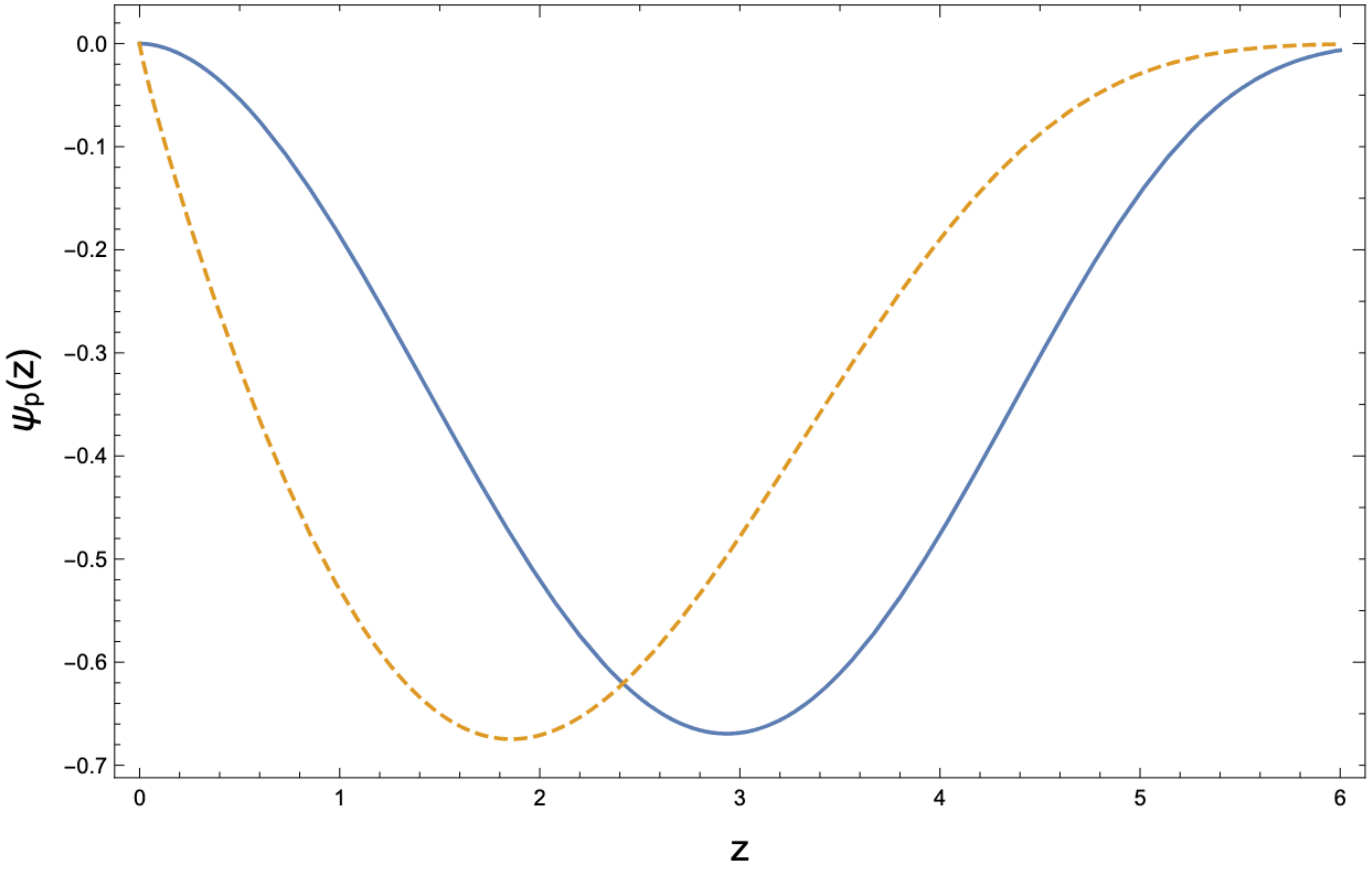
$$|m_5| = \Delta_{\text{can}} + \gamma - 2.$$

anomalous dimension

$$\Psi_i = e^{i P\cdot y} z^2 e^{-k z^2} \left[\left(\frac{1+\gamma_5}{2} \right) \psi_L^i(z) + \left(\frac{1-\gamma_5}{2} \right) \psi_R^i(z) \right] u_{s_i}(P)$$

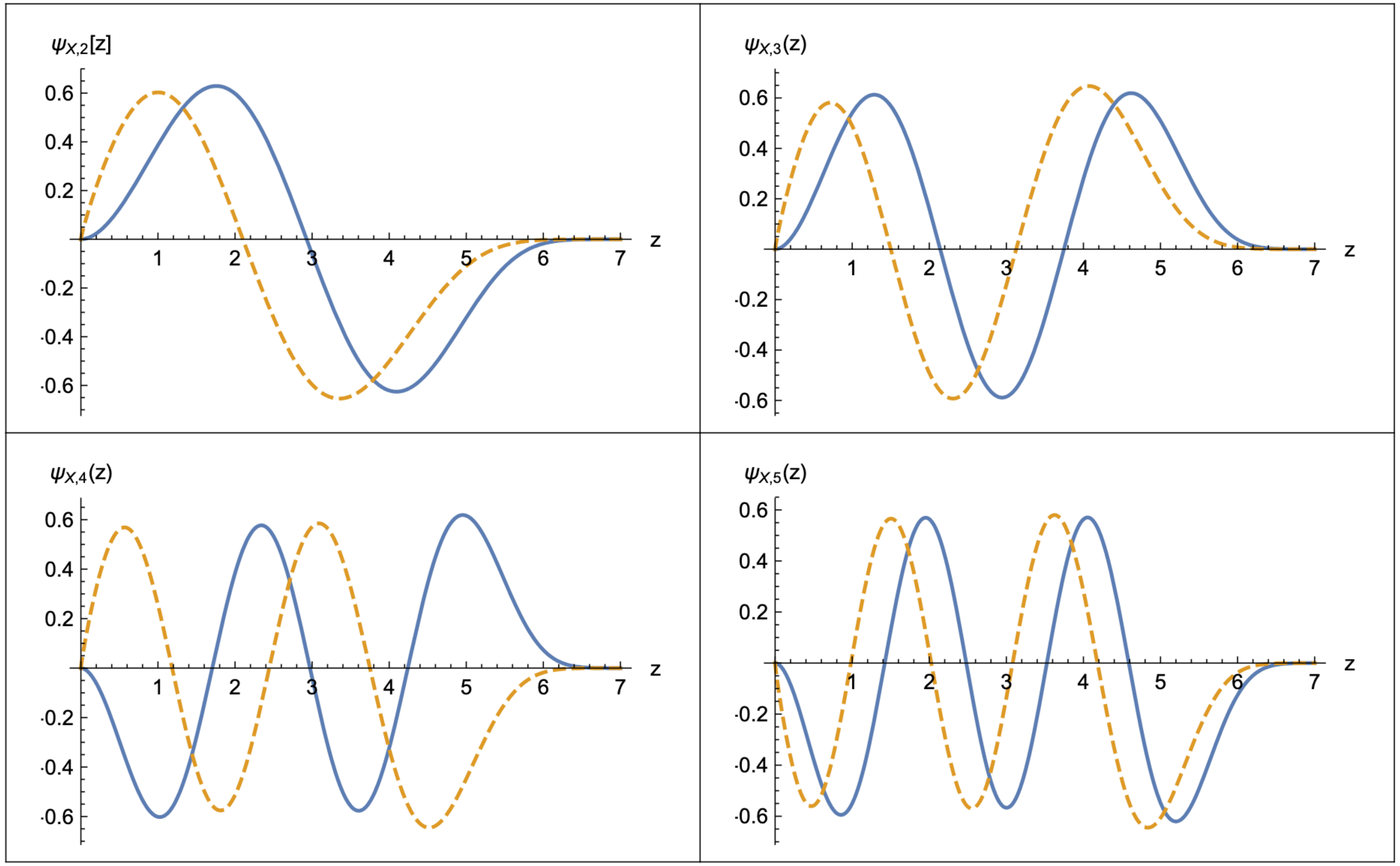
$$\Psi_X = e^{i P_X\cdot y} z^2 e^{-k z^2} \left[\left(\frac{1+\gamma_5}{2} \right) \psi_L^X(z) + \left(\frac{1-\gamma_5}{2} \right) \psi_R^X(z) \right] u_{s_X}(P_X)$$

Proton Structure functions from AdS/QCD with deformed background



Chiral wave functions for left (solid line) and right (dashed line) for the proton ($M_p \equiv M_1 = 0.938$ GeV) using $k = 0.443^2$ GeV^2 and $m_5 = 0.878$ GeV.

Proton Structure functions from AdS/QCD with deformed background



Chiral wave functions for some excited states with n=2, 3, 4, 5 using $k = 0.443^2$ GeV^2 and $m_5 = 0.878$ GeV. In each panel, the left chirality is represented by a solid line, and the right chirality by a dashed line.

Proton Structure functions from AdS/QCD with deformed background

$$\eta_\mu \langle P+q, s_X | J^\mu(q) | P, s\rangle = S_{\rm int}$$

$$= g_V \int dz d^4y \sqrt{-g}\phi^\mu \bar{\Psi}_X \Gamma_\mu \Psi_i .$$

$$= \frac{g_V}{2}(2\pi)^4 \delta^4(P_X - P - q)\eta^\mu [\bar{u}_{s_X}\gamma_\mu \hat{P}_R u_{s_i}\mathcal{I}_L + \bar{u}_{s_X}\gamma_\mu \hat{P}_L u_{s_i}\mathcal{I}_R],$$

with

$$\mathcal{I}_{R/L} = \int dz B(z,q)\psi^X_{R/L}(z,P_X)\psi^i_{R/L}(z,P).$$

and

$$B(z,q) = k z^2 \Gamma\left[1 - \frac{q^2}{2k}\right] U\left(1 - \frac{q^2}{2k}; 2; -\frac{k z^2}{2}\right)$$

Proton Structure functions from AdS/QCD with deformed background

$$\eta_\mu \eta_\nu W^{\mu\nu} = \frac{g_{\mathrm{eff}}^2}{4} \sum_{M_X^2} \left\{ (I_L^2 + I_R^2) \left[(P\cdot\eta)^2 - \frac{1}{2}\eta\cdot\eta (P^2 + P\cdot q) \right] + I_L I_R M_X^2 M_0^2 \eta\cdot\eta \right\}$$

$$\times \delta(M_X^2 - (P+q)^2)$$

$$= \eta^2 F_1(q^2, x) + \frac{2x}{q^2} (\eta\cdot P)^2 F_2^2(q^2, x).$$

$$\delta(M_X^2 - (P+q)^2) \propto \left(\frac{\partial M_n^2}{\partial n} \right)^{-1}$$

Proton Structure functions from AdS/QCD with deformed background

$$F_1(q^2,x) = \frac{g_{\text{eff}}^2}{4}\left[M_0\sqrt{M_0^2+q^2\left(\frac{1-x}{x}\right)}\mathcal{I}_L\mathcal{I}_R + (\mathcal{I}_L^2+\mathcal{I}_R^2)\left(\frac{q^2}{4x}+\frac{M_0^2}{2}\right)\right]\frac{1}{M_X^2}$$

$$M_X^2(q^2,x) = M_0^2 + q^2\left(\frac{1-x}{x}\right)$$

$$F_2(q^2,x) = \frac{g_{\text{eff}}^2}{8}\frac{q^2}{x}(\mathcal{I}_L^2+\mathcal{I}_R^2)\frac{1}{M_X^2},$$

$$\mathcal{I}_{R/L} = \int dz B(z,q)\psi^X_{R/L}(z,P_X)\psi^i_{R/L}(z,P).$$

in the limit of $M_X \gg M_0$, $q \gg M_0$, and $x \to 1$,

$$F_1(q^2,x) \approx \frac{1}{2}F_2(q^2,x),$$

which behaves like the Callan-Gross relation $2xF_1 = F_2$, for $x \to 1$.

Proton Structure functions from AdS/QCD with deformed background

Numerical fit of experimental data. These parameters provide the proton mass as 0.938 GeV and the structure $F_2(x, q^2)$ in next slide

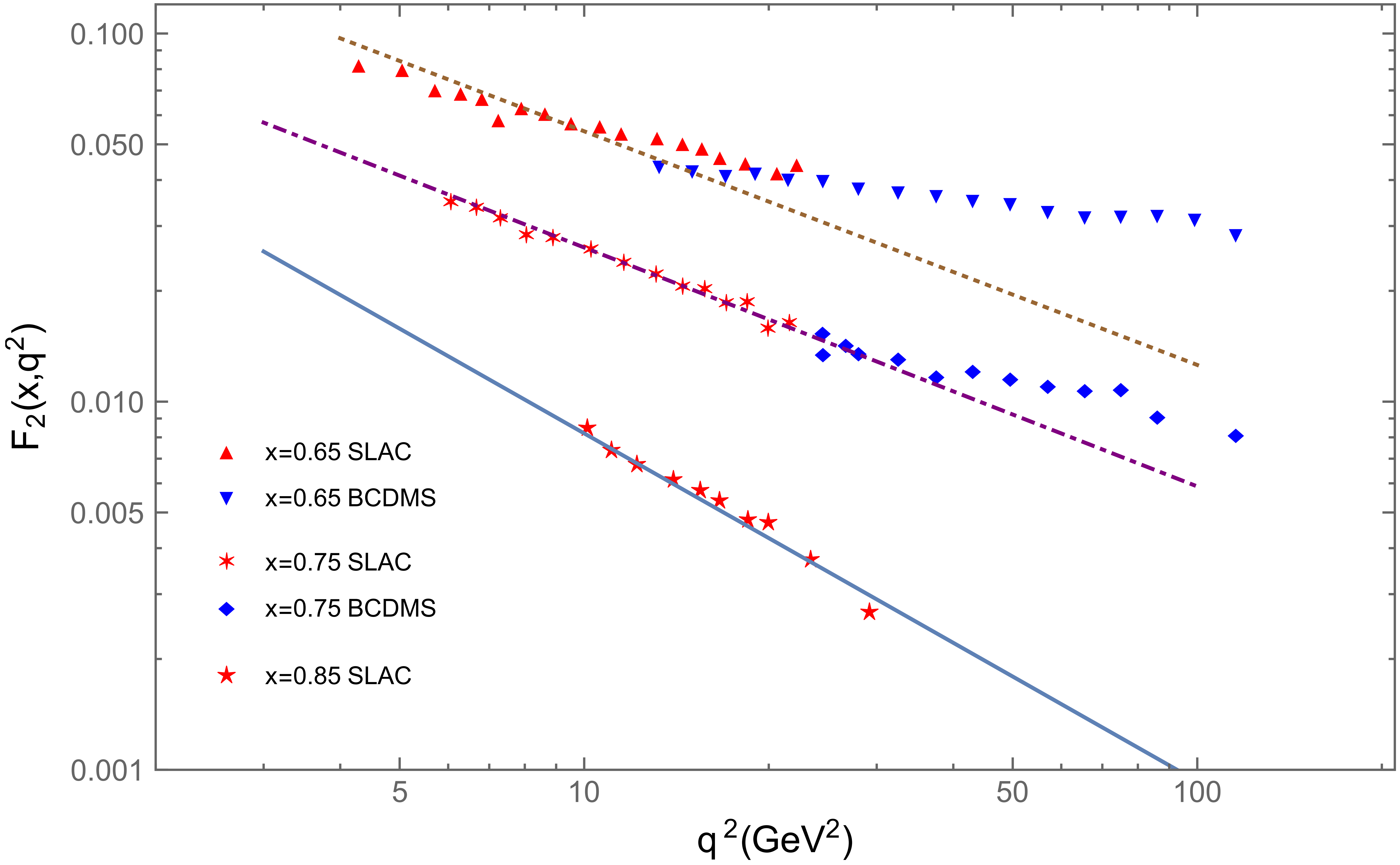
x	m_5 (GeV)	k (GeV^2)	$g_{\rm eff}^2$	γ
0.85	0.878	0.443^2	1.83	0.378
0.75	0.565	0.583^2	1.65	0.065
0.65	0.505	0.612^2	3.65	0.005

$$|m_5| = \Delta_{\rm can} + \gamma - 2.$$

anomalous dimension

Proton Structure functions from AdS/QCD with deformed background

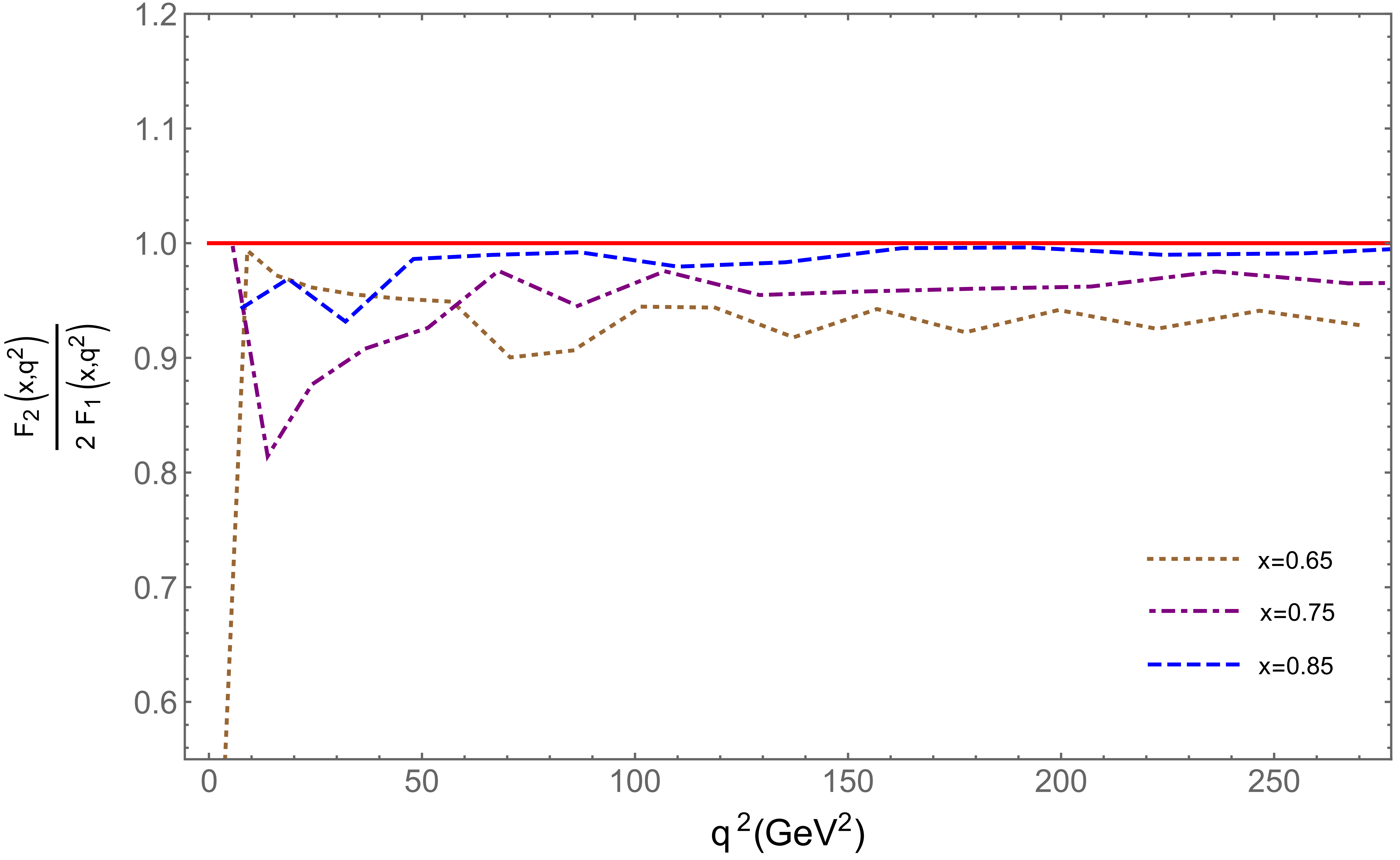
Our results for $F_2(x, q^2)$ (lines) compared with experimental data



Proton Structure functions from AdS/QCD with deformed background

Our results for the ratio $F_2(x, q^2)/2F_1(x, q^2)$

(dotted, dotted-dashed, and dashed lines) compared with the value 1



Pion form factor from an AdS deformed background

Contreras, Capossoli, Li, Vega, HBF, NPB 2022

$$S = \int d^5x \sqrt{-g}\, \mathcal{L}\,, \qquad ds^2 = g^I_{mn} dx^m dx^n = \frac{e^{k_I z^2}}{z^2}\left(dz^2 + \eta_{\mu\nu} dy^\mu dy^\nu\right),$$

where the index $I = \pi, \gamma$ is associated with the pion and the photon, respectively.

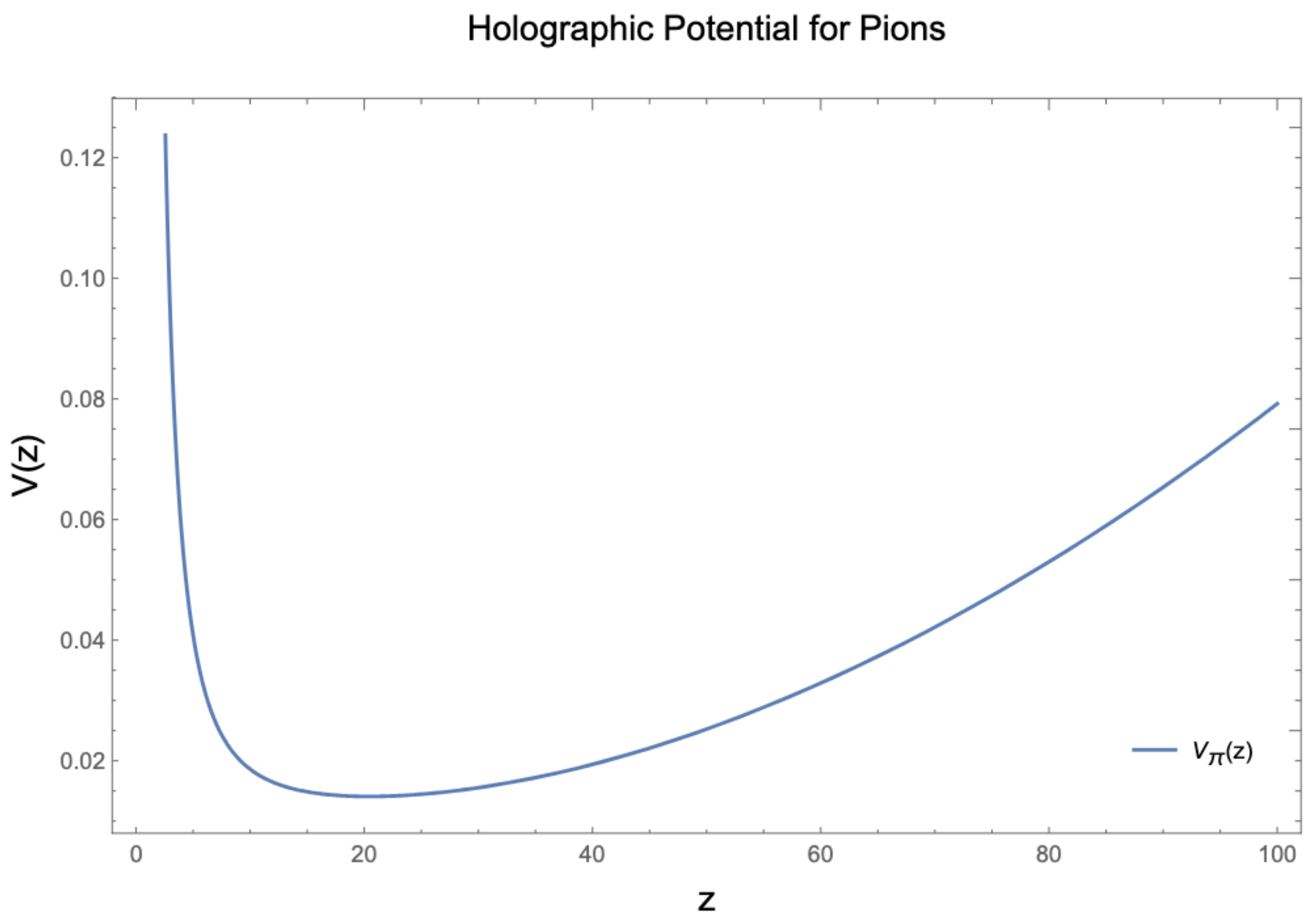
2.1. Scalar field in the deformed AdS/QCD model

$$S = \int d^5x \sqrt{-g_\pi}\, [g_\pi^{mn} \partial_m X \partial^n X + M_5^2 X^2]\,, \qquad A_\pi(z) = \log \frac{L}{z} + \frac{k}{2} z^2$$

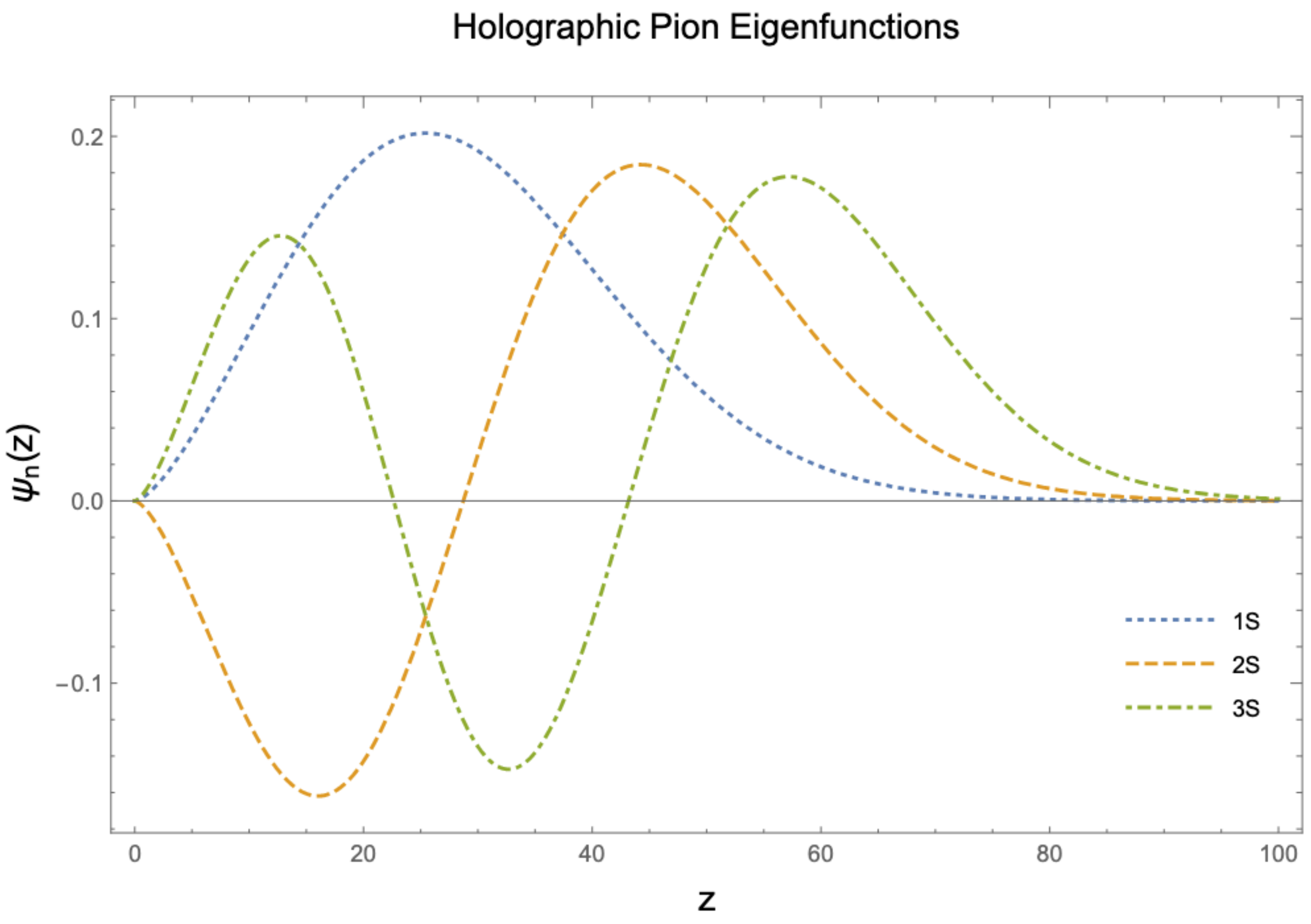
$$-\psi''(z) + \left[\frac{9}{4}A'^2(z) + \frac{3}{2}A''(z) + e^{2A(z)} M_5^2\right]\psi(z) = -\,q^2\psi(z), \qquad M_5^2 = -3.$$

This equation does not have analytic solutions. Solving it numerically with $k_\pi = -\,0.0425^2$ GeV^2 we get $m_\pi = 0.139$ GeV compatible with the meson π mass.

Pion form factor from an AdS deformed background



Holographic potential for bulk eigenmodes dual to pions.



Ground and first two excited bulk eigenmode states dual to pions.

Pion form factor from an AdS deformed background

2.2. Gauge boson field in the deformed AdS/QCD model

$$S = -\frac{1}{c_\gamma^2}\int d^5x\sqrt{-g_\gamma}\,\frac{1}{4}F^{mn}F_{mn}\,,\qquad F^{mn} = \partial^m\phi^n - \partial^n\phi^m\,.$$

$$A(z) = \log\frac{L}{z} + \frac{k}{2}z^2$$

$$\phi_\mu(z,q) = -\frac{\eta_\mu e^{iq\cdot y}}{2}\,k_\gamma\, z^2\,\Gamma\left[1-\frac{q^2}{2k_\gamma}\right]\mathcal{U}\left(1-\frac{q^2}{2k_\gamma};\;2;\;-\frac{k_\gamma\, z^2}{2}\right)$$

$$\equiv -\frac{\eta_\mu e^{iq\cdot y}}{2}\,\mathcal{B}(z,q)\,,$$

Pion form factor from an AdS deformed background

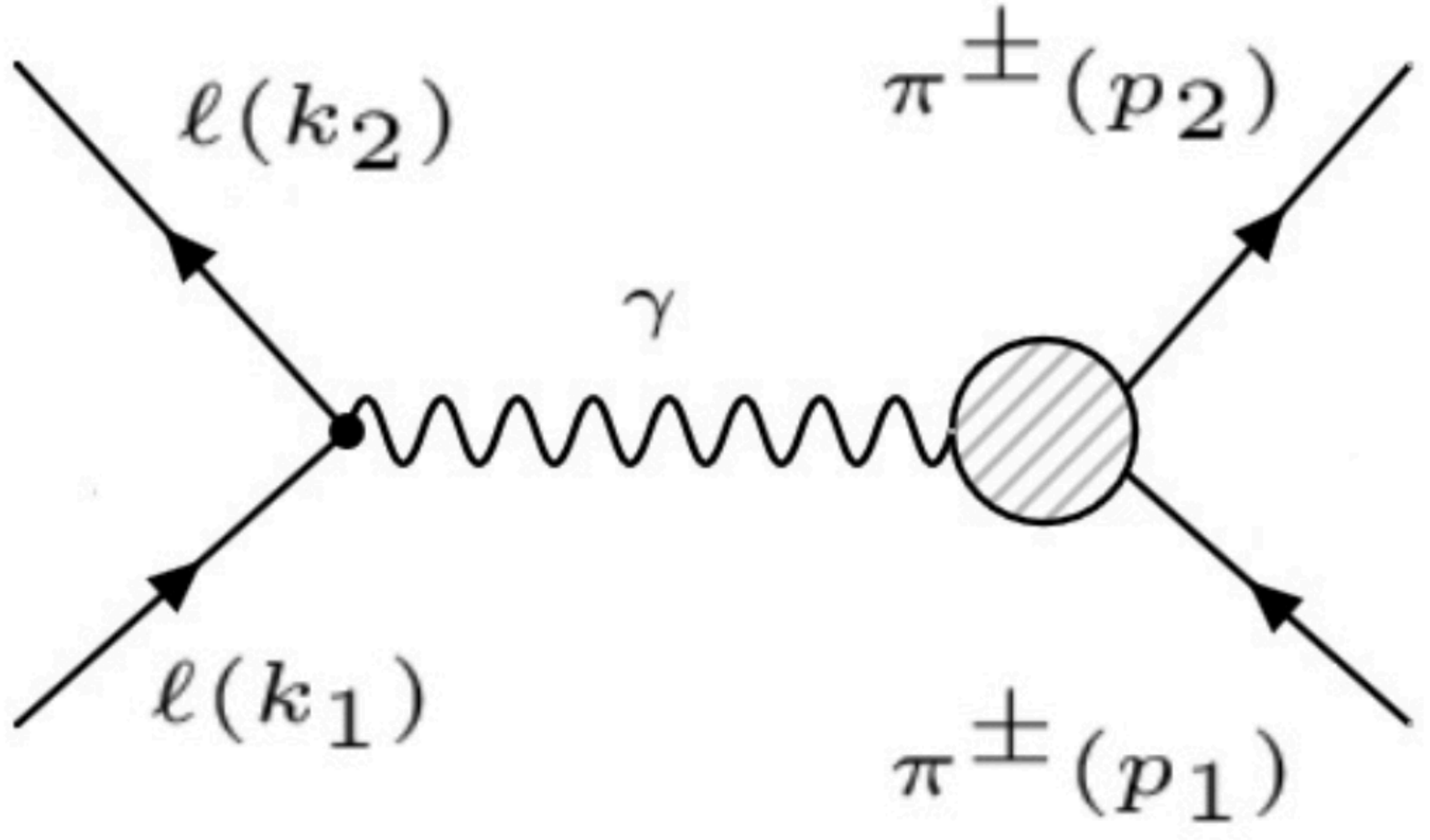
3. Pion form factor

$$S_{\rm eff} = g_{\rm eff} \int d^5x \sqrt{-g_\pi}\, g_\pi^{mn}\, \phi_m(x,z) \Big[X_{p_1}(x,z)\, \partial_m X^*_{p_2}(x,z) - X^*_{p_2}(x,z)\, \partial_m X_{p_1}(x,z) \Big],$$

$$F_\pi(q^2) = \int dz\, \psi_1(z)\, \mathcal{B}(z,q^2)\, \psi_1(z).$$

$$\langle r_\pi^2 \rangle = -6\, \frac{dF_\pi(q^2)}{dq^2}\bigg|_{q^2=0}.$$

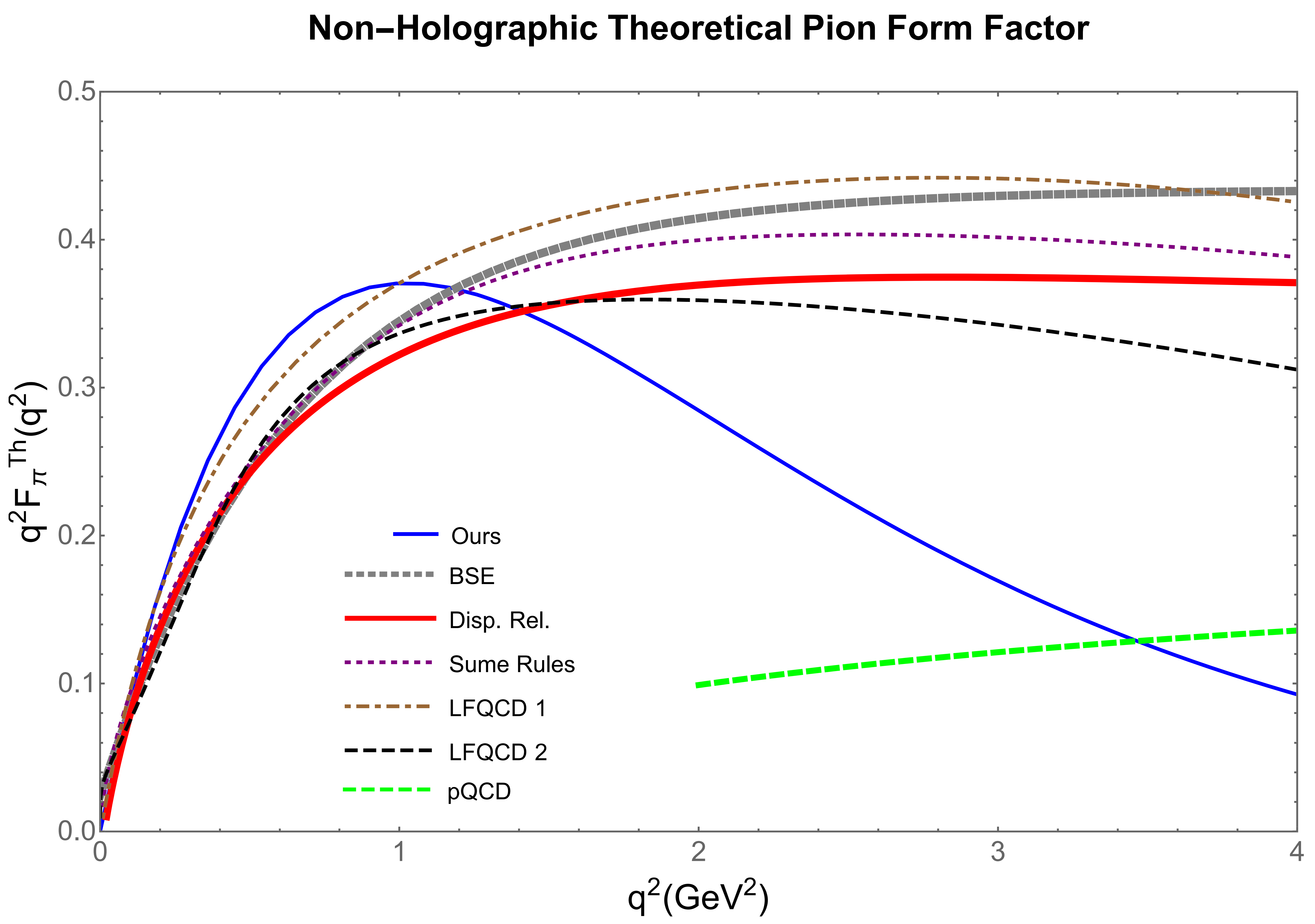
$$g_{\rm eff} = 1$$

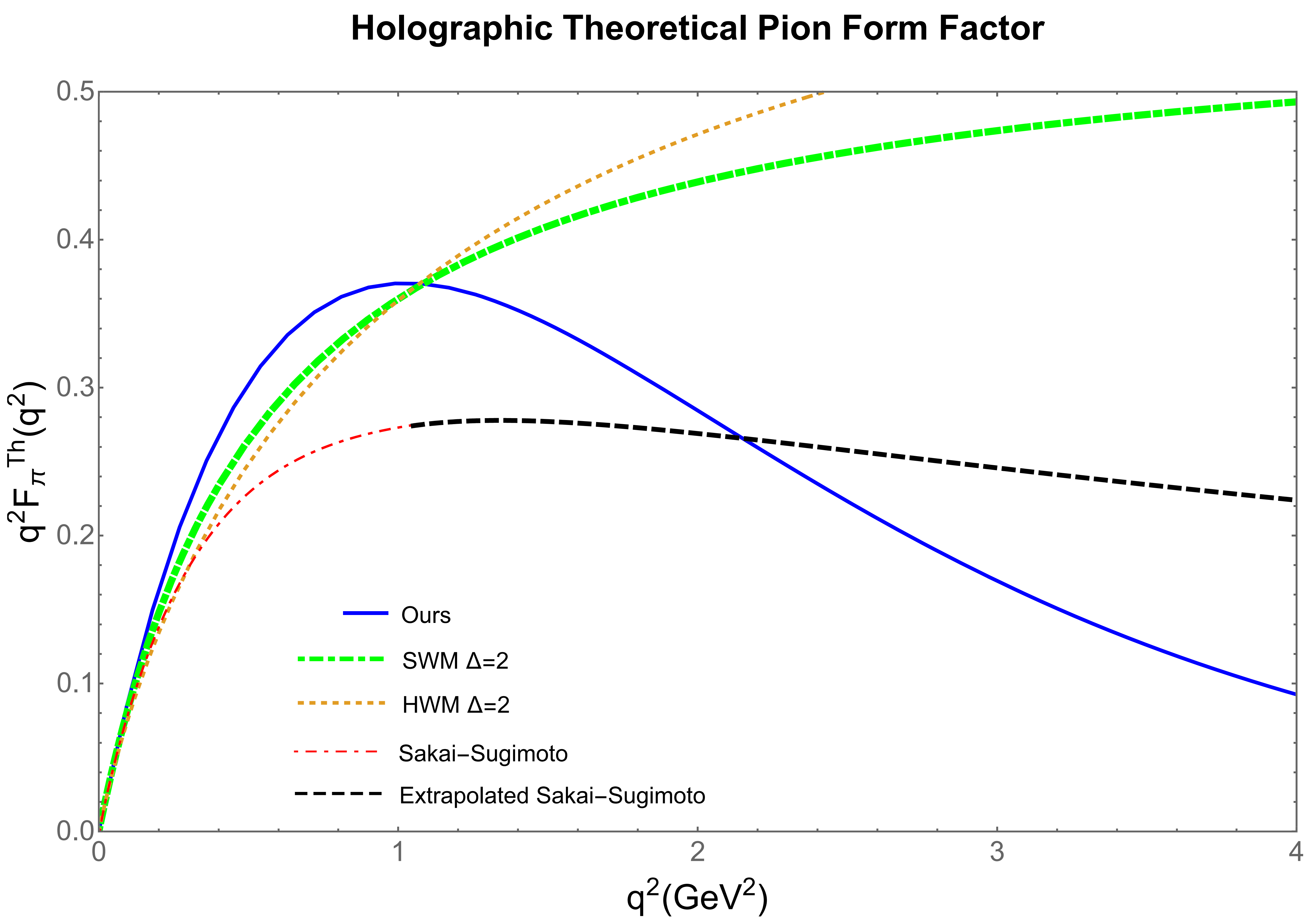


Scattering pions and leptons via the exchange of a virtual photon. The shaded blob represents the effective vertex used to define the electromagnetic pion form factor.

Pion form factor from an AdS deformed background

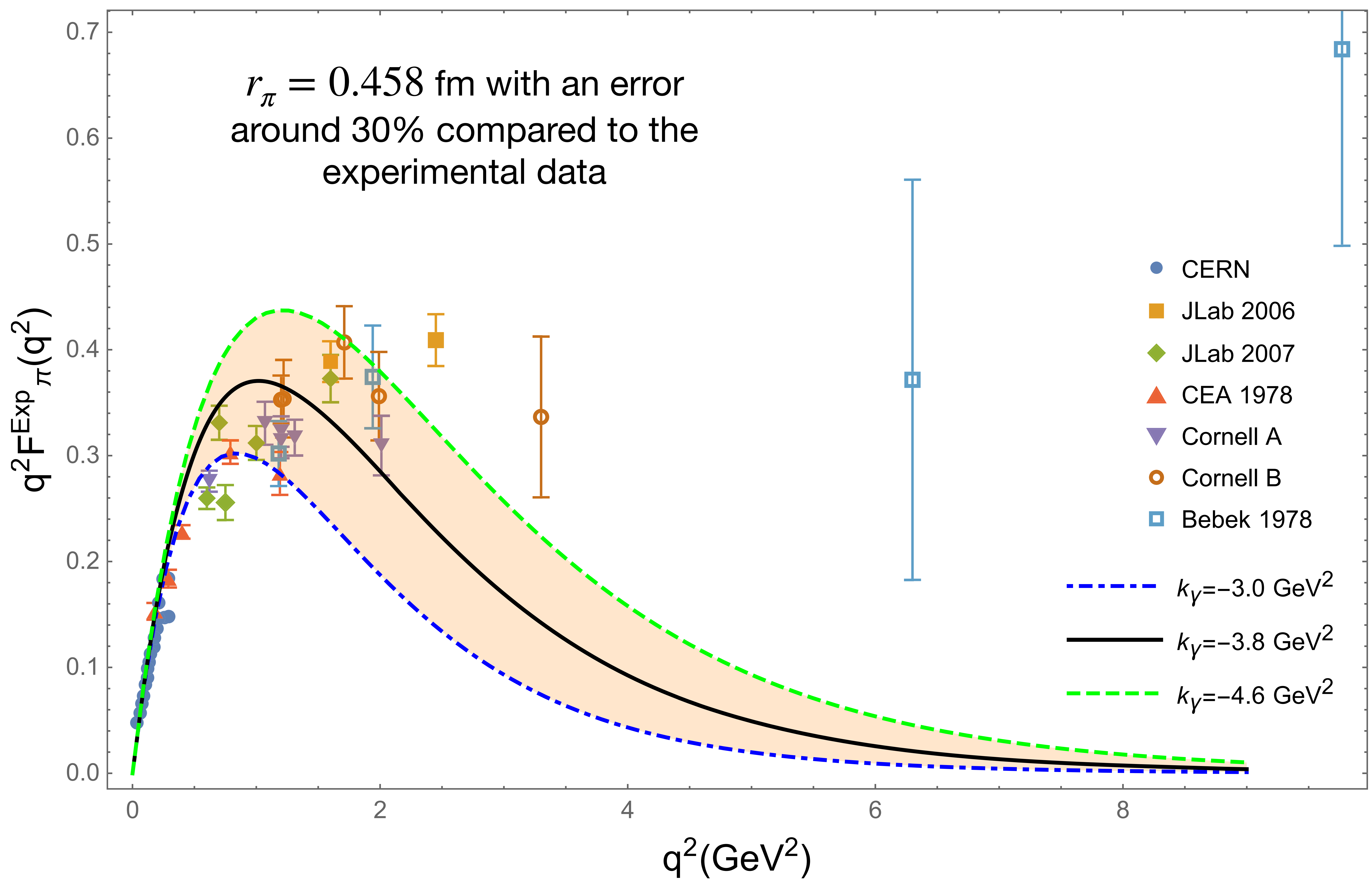
4. Numerical results for the pion form factor *4.1. Pion form factor and pion radius for* $\Delta = 3$





Pion form factor from an AdS deformed background

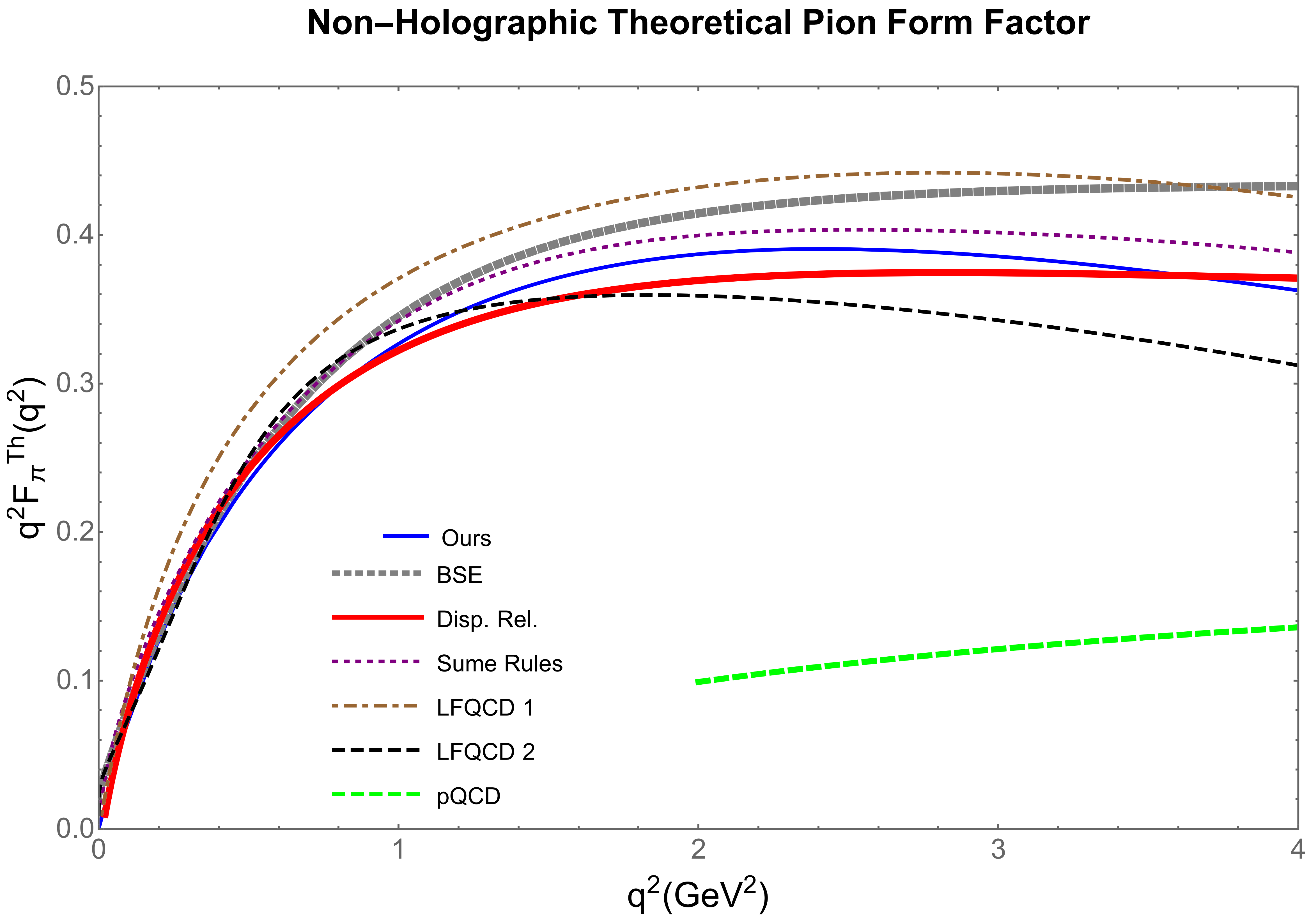
Our results for the Pion Form Factor with Δ=3

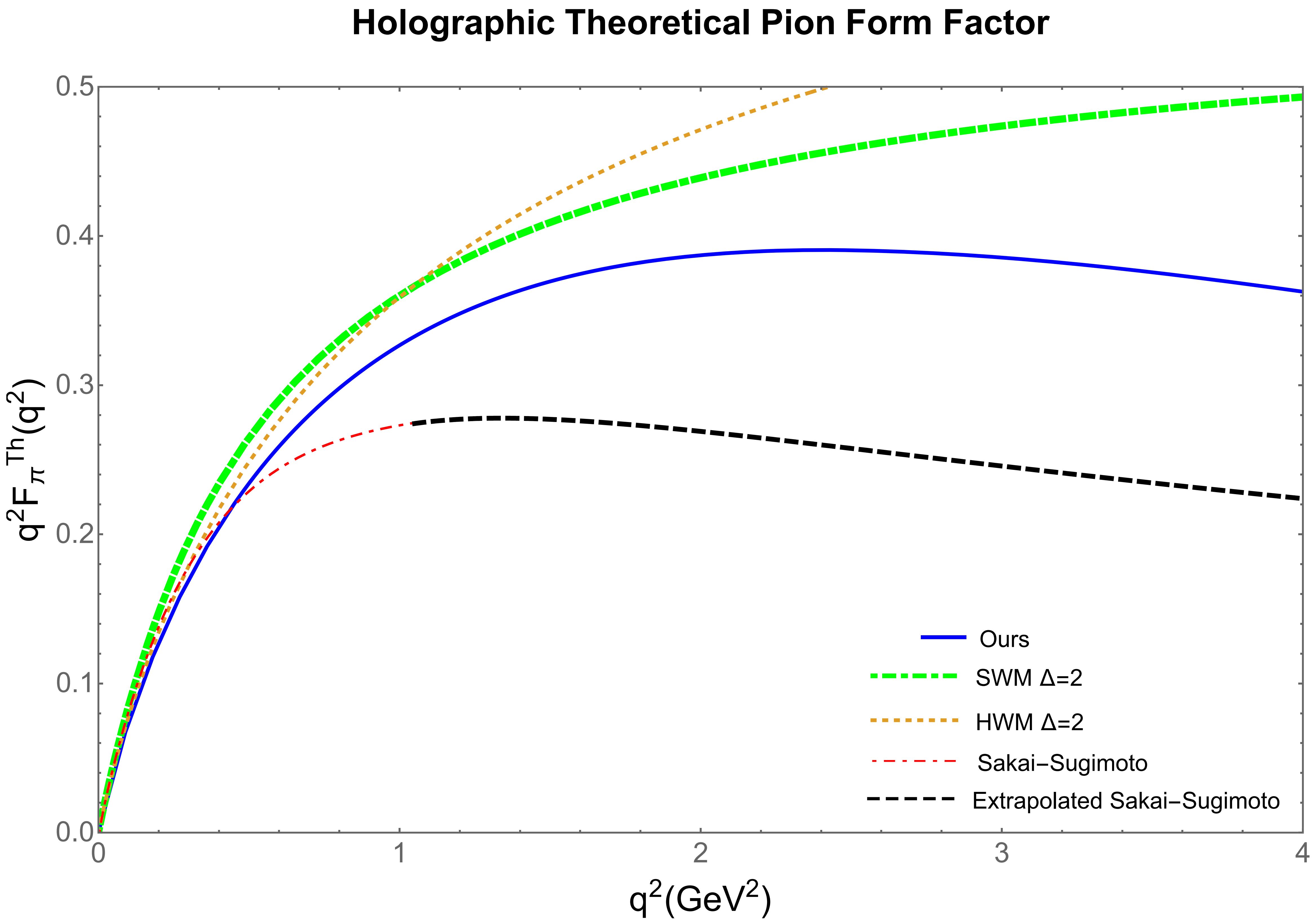


Pion form factor from an AdS deformed background

4.2. Pion form factor and pion radius for $\Delta = 3$ and k dependent of the momentum

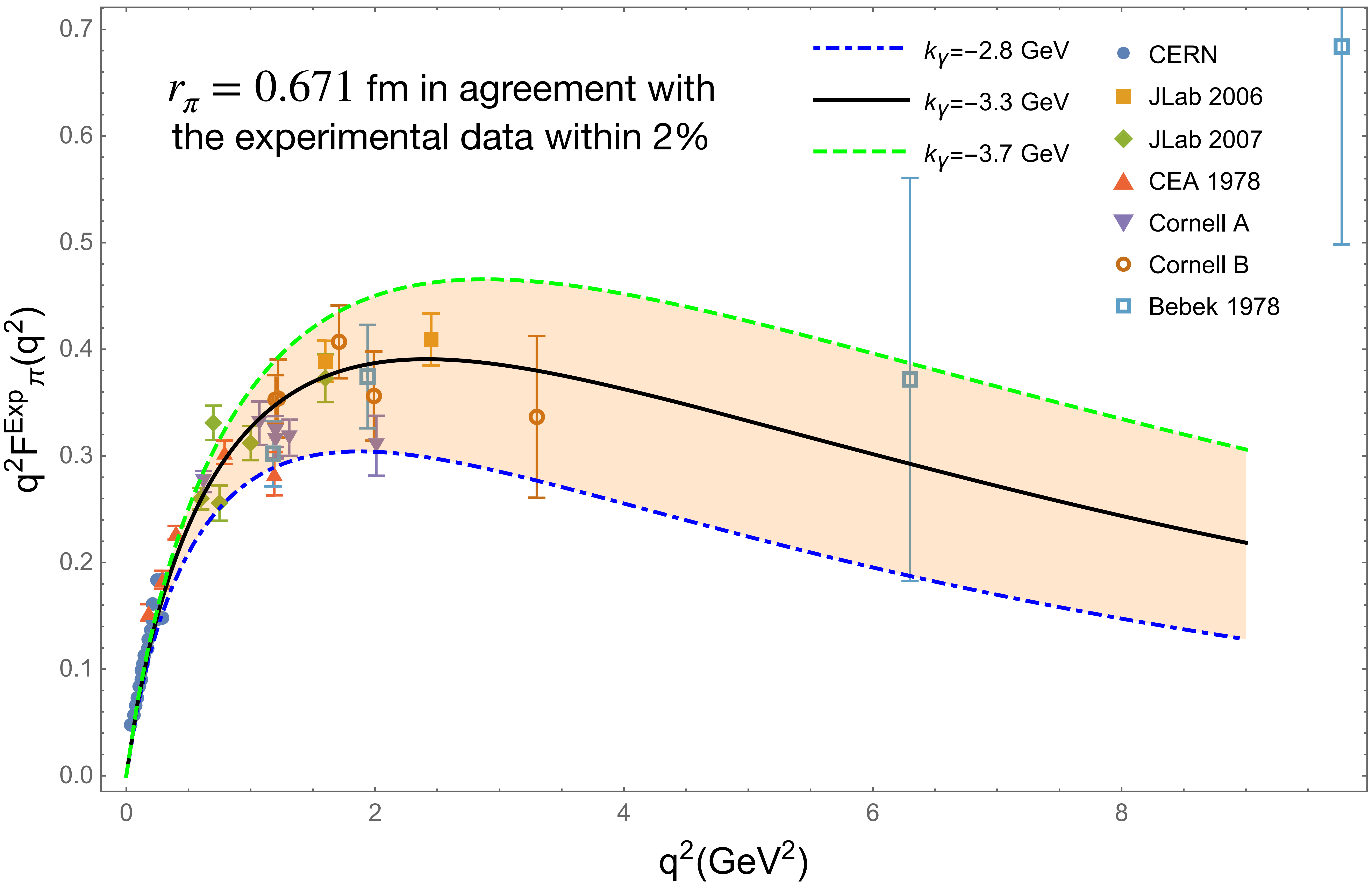
$$k_\gamma \rightarrow k_\gamma(q) = q\, k_\gamma\,.$$





Pion form factor from an AdS deformed background

Our results for the Pion Form Factor with Δ=3 and $k_\gamma \to k_\gamma(q) = q\, k_\gamma$



Pion form factor from an AdS deformed background

Appendix A. Large q^2 analysis in the AdS deformed background

$$F_\pi(q^2)\Big|_{q\to\infty} \to \left(\frac{1}{q^2}\right)^{\Delta-1}$$

$\Delta = 3$, our case;

$\Delta = 2$ in light-front softwall model.

Brodsky, de Teramond PRD 2008

For a review, see: Dosch, de Teramond, Brodsky, *Nucl. Part. Phys. Proc.* 2022

Appendix B. Pion form factor in the original softwall model

$$F_\pi(q) = \frac{32\, k_\gamma^2}{\left(q^2 + 4\,|k_\gamma|\right)\left(q^2 + 8\,|k_\gamma|\right)},$$

$$k_\gamma \to k_\gamma(q) = q\, k_\gamma$$

$$F_\pi(q^2) \sim \frac{1}{q^2}\,,$$

fulfilling the expected scaling law even considering $\Delta = 3$.

Conclusions

- We have found reasonable results for the hadronic spectra, DIS structure functions and form factors within the deformed AdS/QCD model with a quadratic exponential in the holographic coordinate.

- We are considering improving theses results and the deformed model.

Thank you!!!

Backup slides

Proton and Neutron form factors from an AdS deformed background

Contreras, Capossoli, Li, Vega, HBF, PLB 2021

$$I_{\text{int}} = \int d^5x \sqrt{-g^B} \left\{ \bar{\psi}_f \, \Gamma^m \, \phi_m \, \psi_i + \frac{i\,\eta_N}{2} \bar{\psi}_f \, \left[\Gamma^m, \Gamma^n\right] F_{mn} \, \psi_i \right\},$$

$$C_1(q) = \frac{1}{2} \int dz \left[\psi_L(z)^2 + \psi_R(z)^2\right] B(z,q)$$

$$C_2(q) = \frac{1}{2} \int dz \, e^{A_B(z)} \, \partial_z \, B(z,q) \left[\psi_L(z)^2 - \psi_R(z)^2\right]$$

$$C_3(q) = \int dz \, e^{A_B(z)} \, 2 \, M_n \, \psi_L(z) \, \psi_R(z) \, B(z,q).$$

These functions will define the form factors for nucleons as

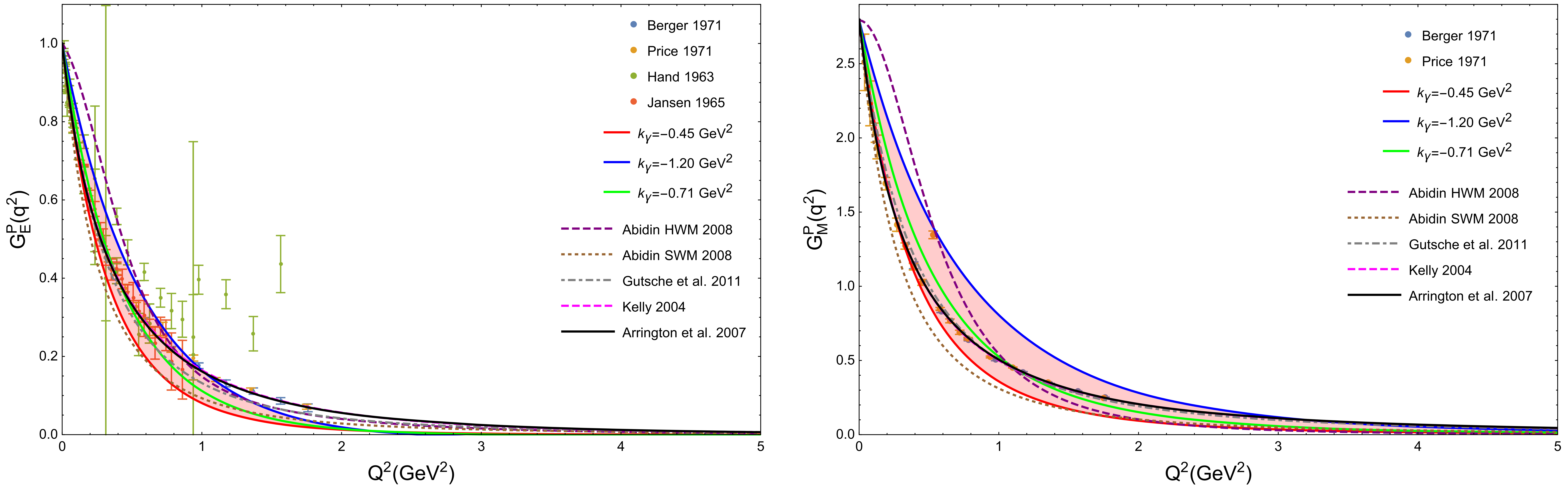
$$F_1^N(q) = C_1(q) + \eta_N \, C_2(q), \quad F_2^N(q) = \eta_N \, C_3(q).$$

Another set of form factors that we can describe are the Sachs electric and magnetic ones, defined for nucleons as

$$G_E^N(q) = F_1^N(q) - \frac{q^2}{4\,M_N^2} \, F_2^N(q), \quad G_M^N(q) = F_1^N(q) + F_2^N(q),$$

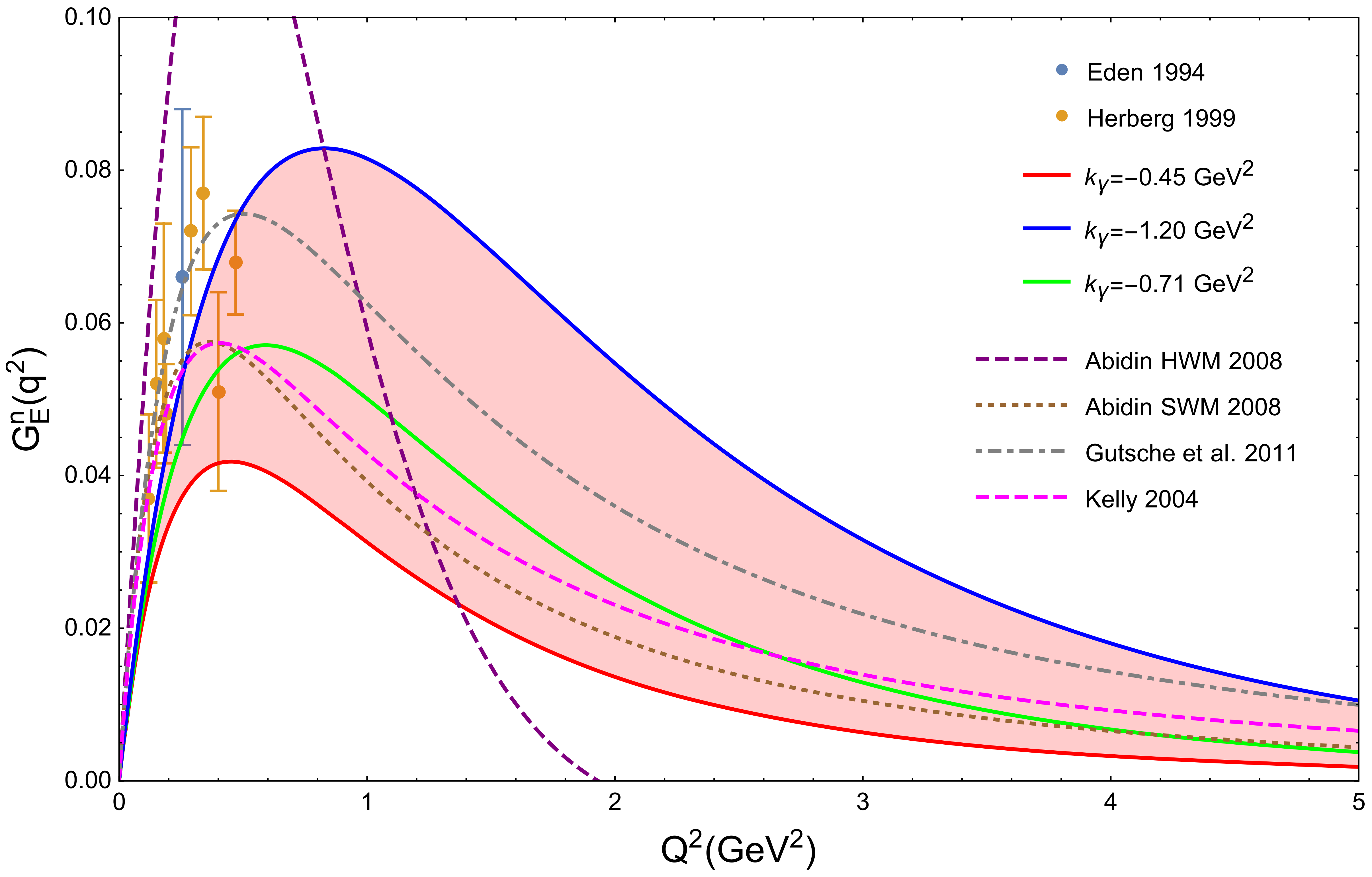
Proton and Neutron form factors from an AdS deformed background

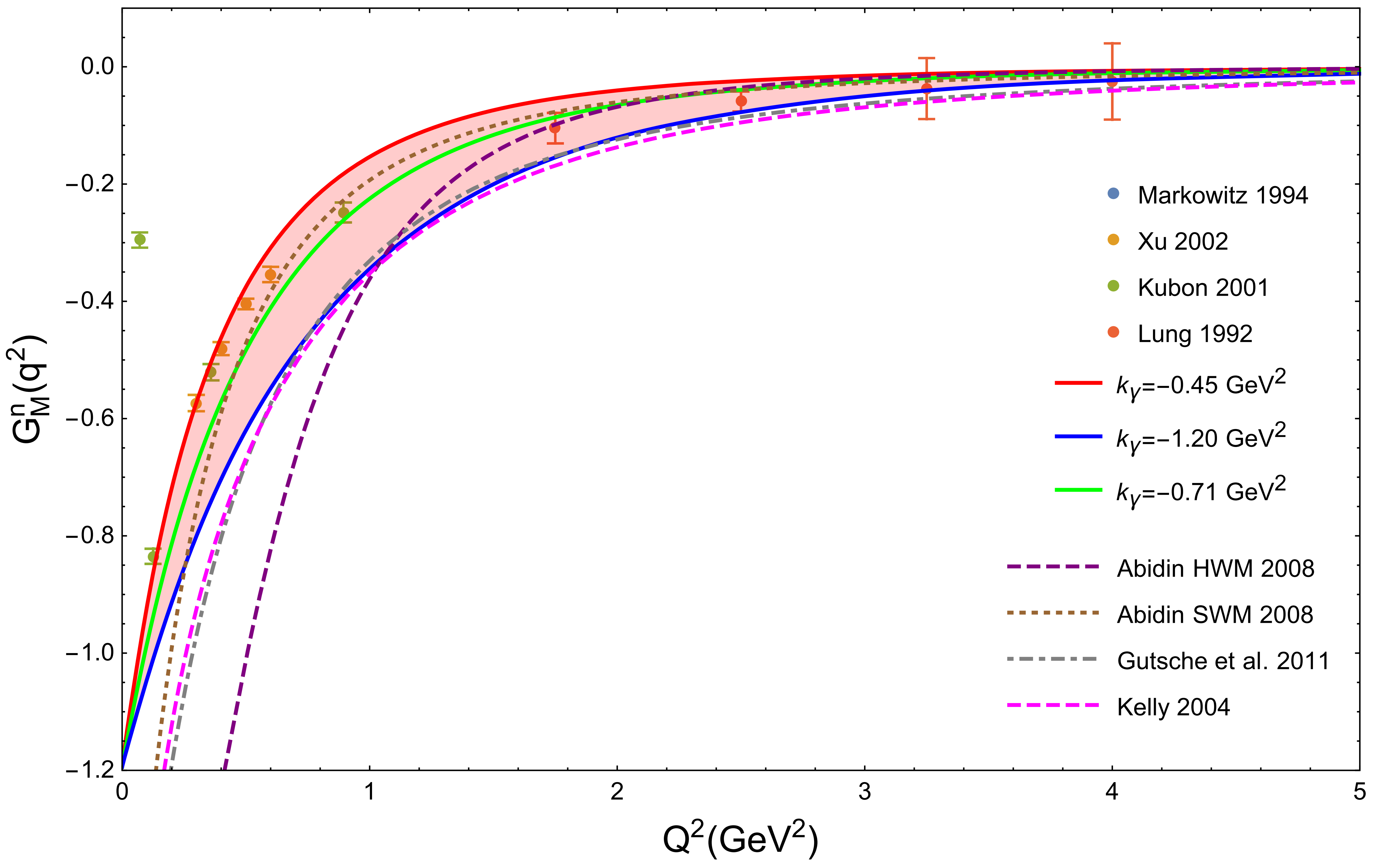
Proton



Proton and Neutron form factors from an AdS deformed background

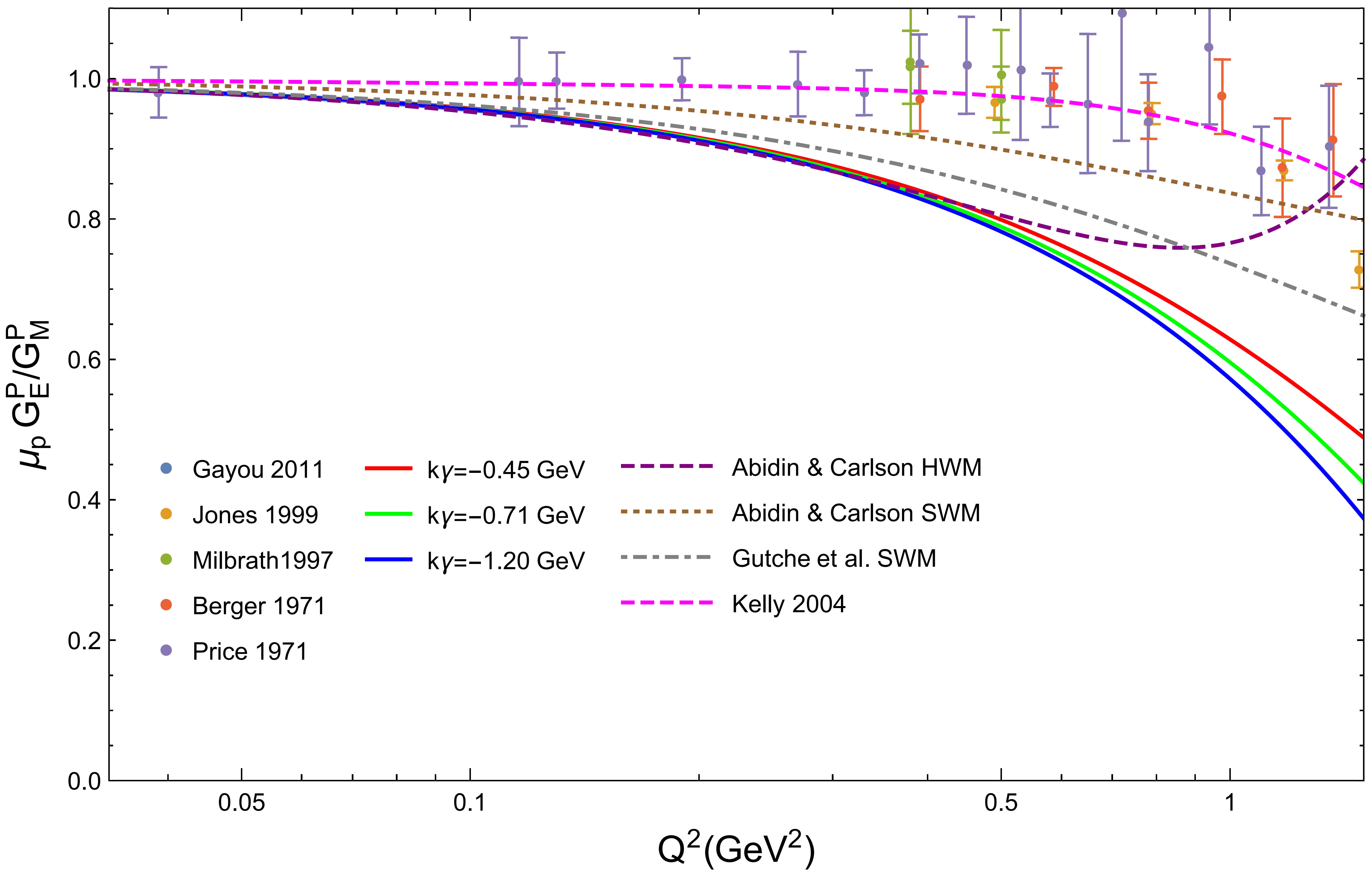
Neutron

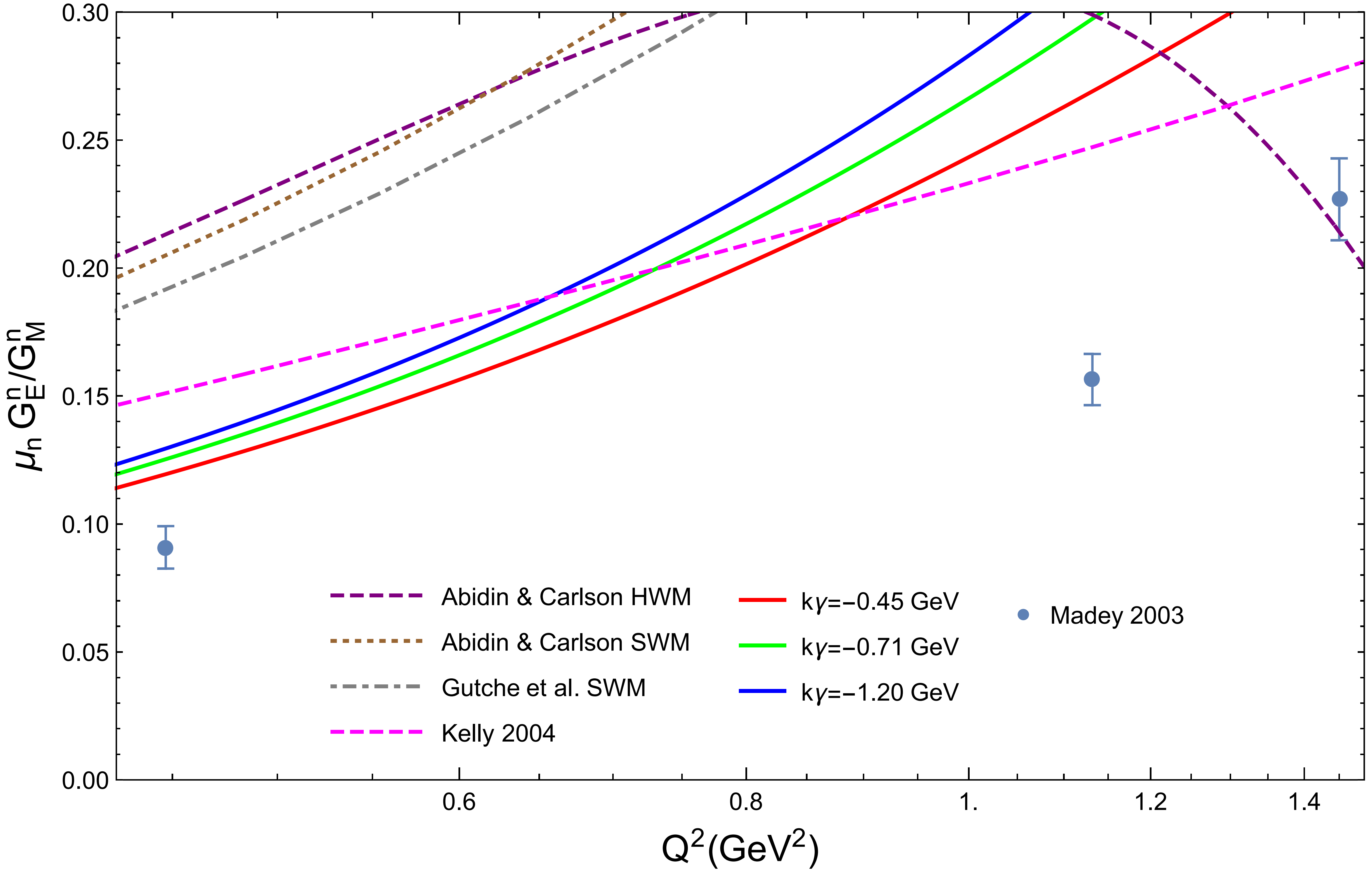




Proton and Neutron form factors from an AdS deformed background

Ratios





nucleon magnetic moments $\quad G_M^p(0) = \mu_p$ and $G_M^n(0) = \mu_n$,

Proton and Neutron form factors from an AdS deformed background

Table 1
Holographic results are calculated with $\kappa_\gamma = -0.450$ GeV^2, in Eq. (19). Experimental data is taken from PDG [45]. For the neutron charge radius, the mean square charge radius, given in fm^2, is considered.

Nucleon	Experimental (fm)	This work (fm)
Proton charge radius	0.8409 ± 0.0004	0.8576
Proton magnetic radius	0.851 ± 0.026	0.7929
Neutron charge radius*	-0.1161 ± 0.0022	-0.0668
Neutron magnetic radius	$0.864^{+0.009}_{-0.008}$	0.7933

$$\langle r^2_{N,E} \rangle = -6 \left. \frac{d\,G^N_E(q^2)}{d\,q^2} \right|_{q^2 \to 0}, \qquad \langle r^2_{N,M} \rangle = -\frac{6}{G^N_M(0)} \left. \frac{d\,G^N_M(q^2)}{d\,q^2} \right|_{q^2 \to 0}.$$

AdS/CFT Correspondence

Maldacena '97:

- String Theory defined in $AdS_5 \times S^5$ is dual to Conformal $\mathcal{N} = 4$ Super $SU(N)$ Yang-Mils Theory with $N \to \infty$ in $d = 4$ Minkowski (or Euclidean) space (**Strong Statement = Conjecture**);
- Supergravity fields (low energy limit of string theory) in $AdS_5 \times S^5$ are dual to operators in the Hilbert Space of Conformal $\mathcal{N} = 4$ Super $SU(N)$ Yang-Mils Theory with $N \to \infty$ in $d = 4$ Minkowski (or Euclidean) space (**Weak Statement = Proven**).
- Other forms of the correspondence in other spacetimes, Strong and Weak, were also proposed or proven.

Confinement

- Kinar, Schreiber, Sonnenschein '00: General Criteria for Confining theories from the AdS/CFT correspondence defined in diagonal metrics
$ds^2 = -g_{00}dt^2 + g_{ii}dx_i^2 + g_{zz}dz^2 + g_{TT}dx_T^2$;
- Hard Wall is confining at zero and finite temperatures (HBF, N. Braga, C. N. Ferreira'06);
- Original Soft Wall is not confining $\Rightarrow$ Solution: Modified metric
$ds^2 = \frac{L^2}{z^2}e^{kz^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)$ instead of exponential in the Action, confining at zero and finite temperatures (Andreev, Zakharov'06);
- This solution implies the same spectrum for vector mesons as in the Original Soft Wall model.

Glueballs

AdS/CFT prescription (scalars) $M_5^2 = (\Delta)(\Delta - 4),$

Scalar Glueballs $\Delta = 4$

$$\mathcal{O}_4 = \text{Tr}\left(F^2\right) = \text{Tr}\left(F^{\mu\nu}F_{\mu\nu}\right);$$

Higher Spin J Glueballs

$$\mathcal{O}_{4+J} = \text{Tr}\left(FD_{\{\mu 1} \ldots D_{\mu J\}}F\right)$$

de Teramond-Brodsky Prescription

$$\Delta = 4 + J$$

$$M_5^2 = J(J+4)\,; \qquad (\text{even } J)$$

Oddballs   Capossoli, HBF 2013

$$\mathcal{O}_6 = \text{Sym Tr}\left(\tilde{F}_{\mu\nu}F^2\right), \qquad \Delta = 6$$

Higher Spin J Oddballs

$$\mathcal{O}_{6+J} = \text{Sym Tr}\left(\tilde{F}_{\mu\nu}FD_{\{\mu 1} \ldots D_{\mu J\}}F\right),$$

$$\Delta = 6 + J$$

$$M_5^2 = (J+6)(J+2)\,; \qquad (\text{odd } J)\,,$$

Partial Summary: Hadronic Spectra from Deformed AdS backgrounds

- Forkel, Beyer, Tobias JHEP 2007:
  Different warp factors (functions) for different family particles

- Capossoli, Contreras, Li, Vega, HBF CPC 2020:

One warp factor with different scales k for different family particles

Glueballs (even/odd) $k_{\rm gbe} = k_{\rm gbo} = 0.31^2 {\rm GeV}^2$

Scalar/Vector mesons $k_{\rm sm} = -0.332^2 \ {\rm GeV}^2.$; $k_{\rm vm} \approx 3\, k_{\rm sm}$

Baryons $k_{1/2} = 0.205^2 \ {\rm GeV}^2,$; $k_{1/2} = k_{3/2} \approx k_{5/2}$

Previous works of UFRJ Group on DIS from AdS/QCD

- Ballon Bayona, HBF, Braga,’08a DIS for scalars within the Soft Wall Model for large and small x, and for Fermions in a hybrid (soft + hard wall) model for large x.
- Ballon Bayona, HBF, Braga’08b DIS for mesons within the D3-D7 Brane Model for large and intermediate values of x, and elastic form factors;
- Ballon Bayona, HBF, Braga’08c DIS for scalars within Supergravity for small x $\Rightarrow$ Geometric Scaling;
- Ballon Bayona, HBF, Braga’10 DIS off a **plasma** with flavor from the D3-D7 brane model;
- Ballon Bayona, HBF, Braga, Torres’10 DIS for vector mesons in holographic D4-D8 model;
- Ballon Bayona, HBF, Braga, Ihl, Torres'13 Generalized baryon form factors and proton structure functions in the Sakai–Sugimoto model;
- Capossoli, HBF’15 DIS in the exponentially small x from the holographic softwall model $\Rightarrow$ Saturation;
- …