Multigluon Correlation Functions from lattice QCD

The four gluon vertex

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Motivation

QCD Dynamics at a fundamental level

importance of higher order corrections (two loop contributions to gluon DSE, etc.)

Test our understanding of QCD Green functions (ghost dominance at IR)

Effective charge $$\frac{g^2}{4\,\pi}\;\Gamma^{(4g)}(p^2)\;p^2\,D(p^2)$$

Landau gauge

(pure gauge) Lattice point of view

Lattice Approach

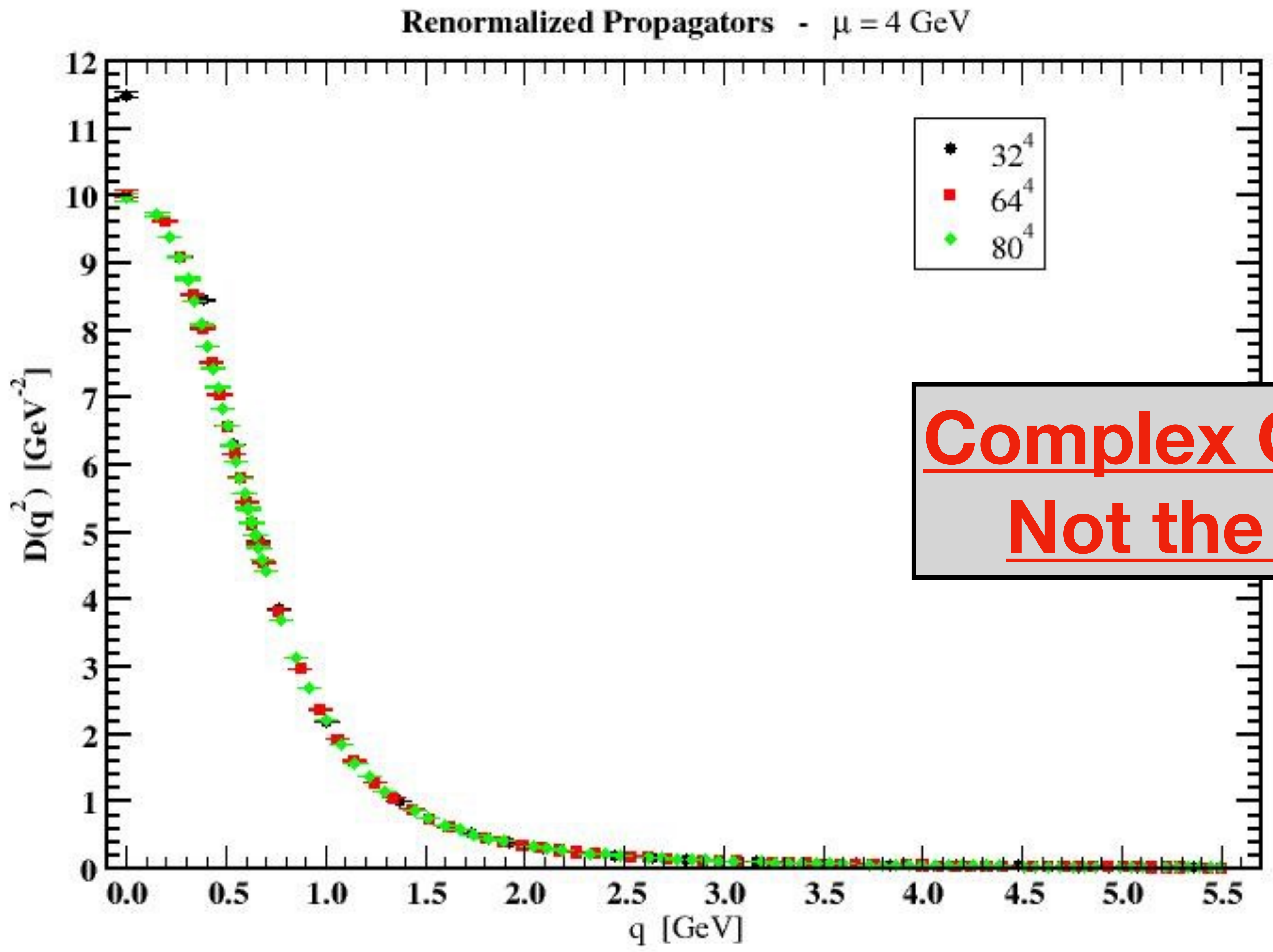
Importance Sampling **allows to access** the **gluon field** $A_\mu^a(x)$

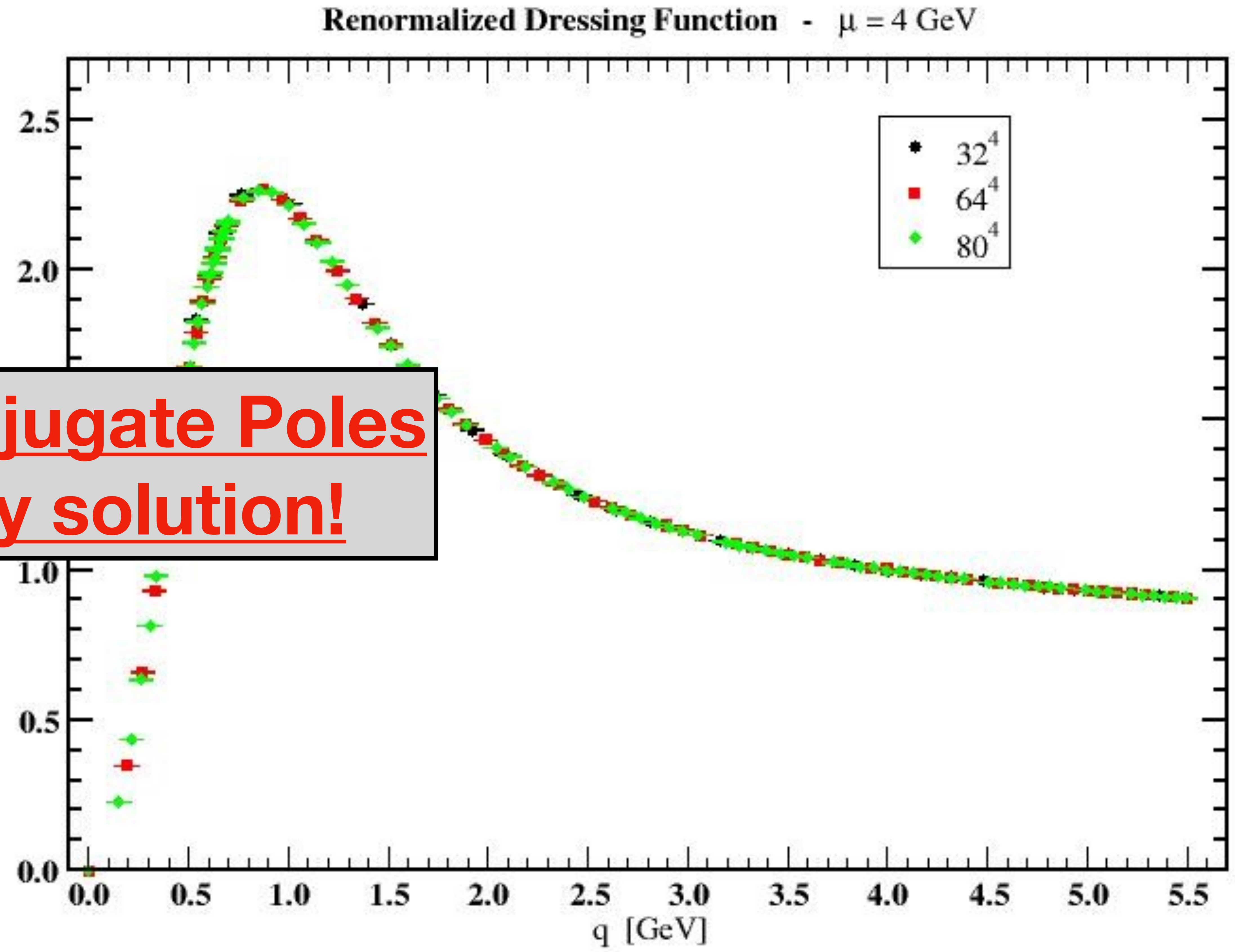
$$\mathcal{G}^{(n)}(x_1, \ldots, x_n) = \langle 0 | \, T \, \left(A_{\mu_1}^{a_1}(x_1) \cdots A_{\mu_n}^{a_n}(x_n) \right) \, | 0 \rangle$$

Larger n: → Noisy → Larger sets of configurations

Disconnected parts → kinematical configurations or Combinations of components

Gluon Propagator



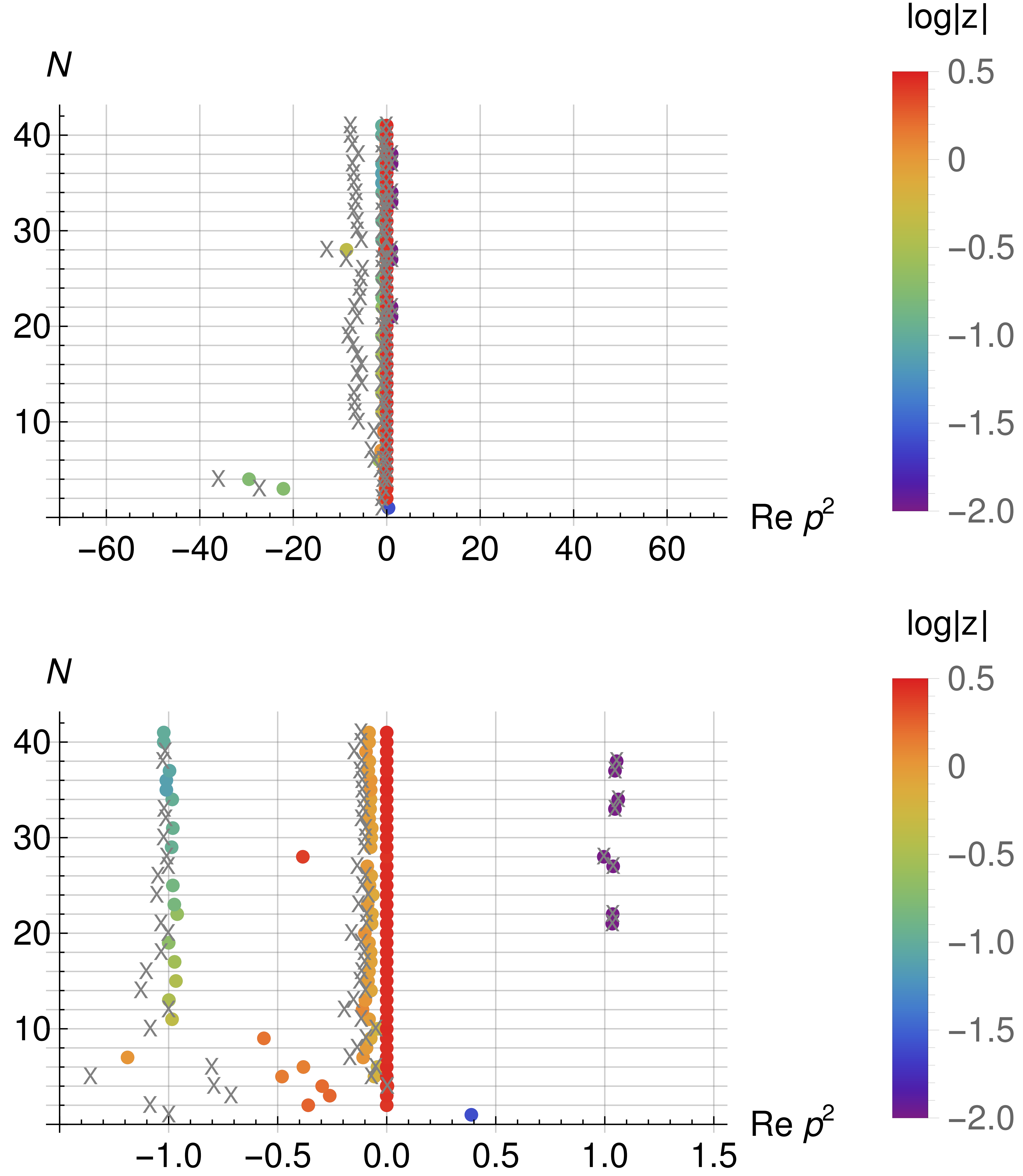


Complex Conjugate Poles
Not the only solution!

Massless Ghost

$$D^{ab}(p^2) = -\,\delta^{ab}\,\frac{d_G(p^2)}{p^2}$$

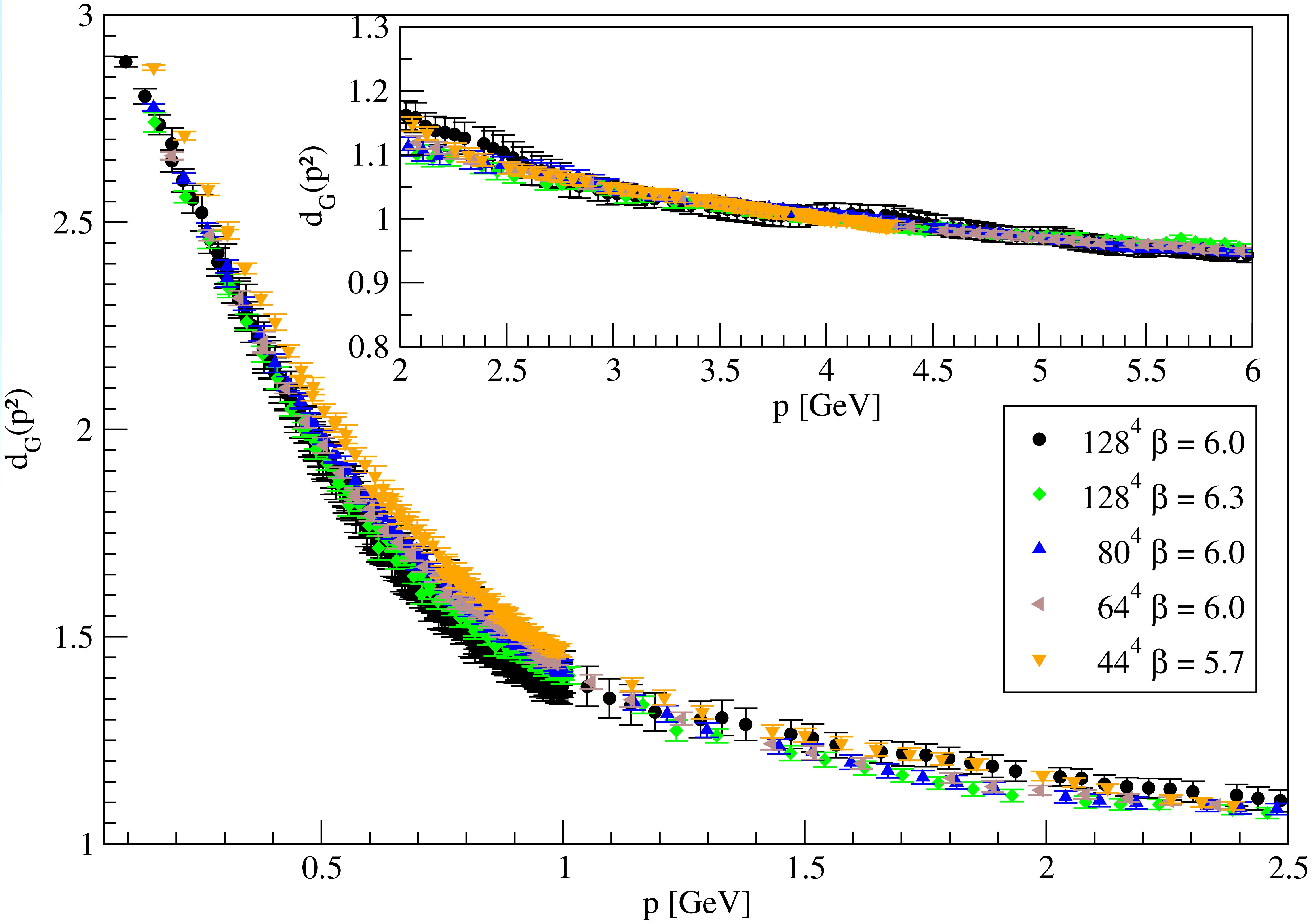
lattice studies support a massless ghost



A F Falcão et al, *Phys. Rev. D* **102** (2020) 11, 114518

IR Ghost Dominance / Log Divergence

$$D^{ab}(p^2) = -\,\delta^{ab}\,\frac{d_G(p^2)}{p^2}$$

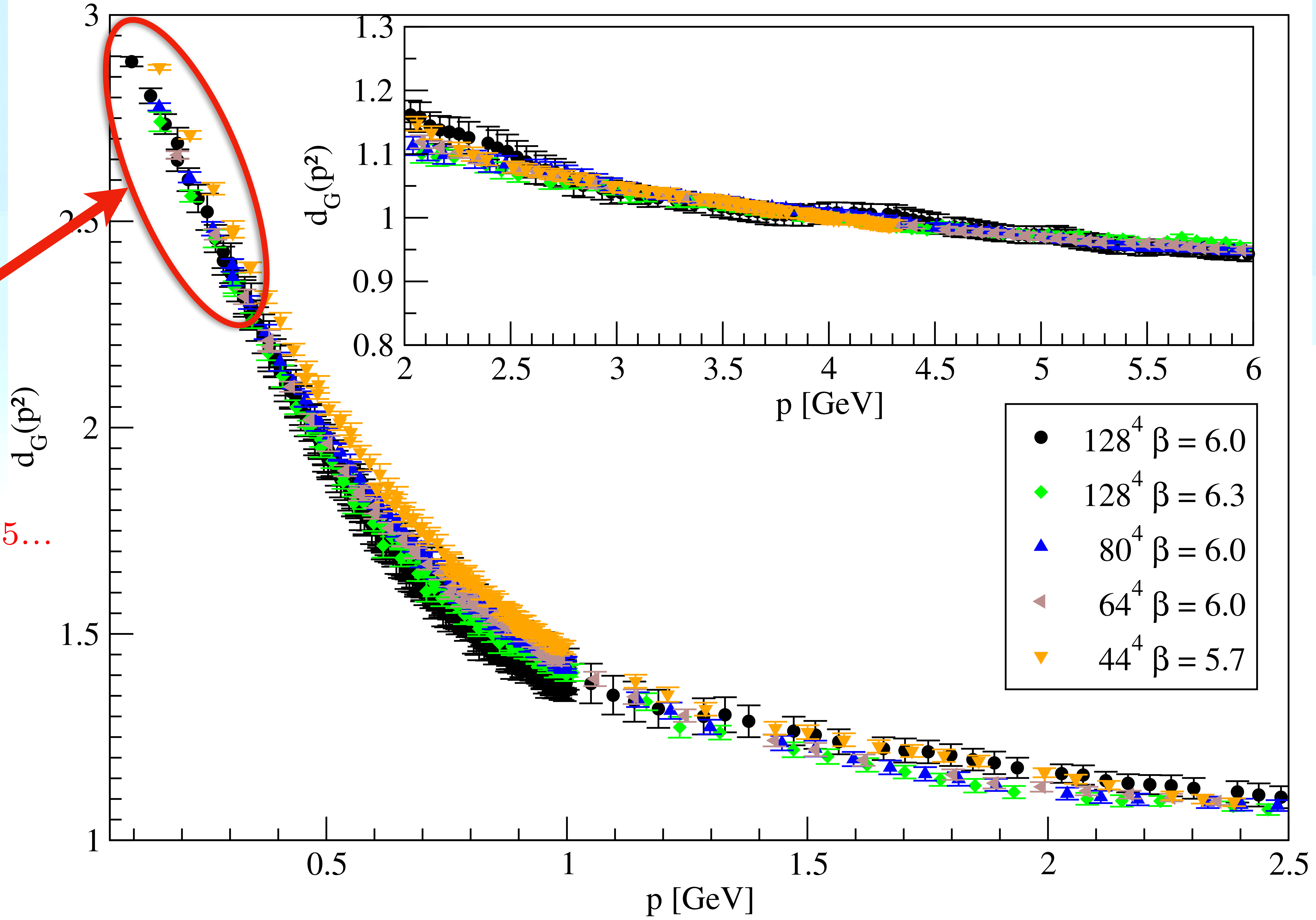


A G Duarte et al, *Phys. Rev. D* **94** (2016) 1, 014502

IR Ghost Dominance / Log Divergence

$$D^{ab}(p^2) = -\,\delta^{ab}\,\frac{d_G(p^2)}{p^2}$$

$$d_G(p^2) \propto \left(\ln\frac{p^2 + (1.7\ \mathrm{GeV})^2}{(0.4\ \mathrm{GeV})^2}\right)^{-0.2045\ldots}$$



A G Duarte et al, *Phys. Rev. D* **94** (2016) 1, 014502

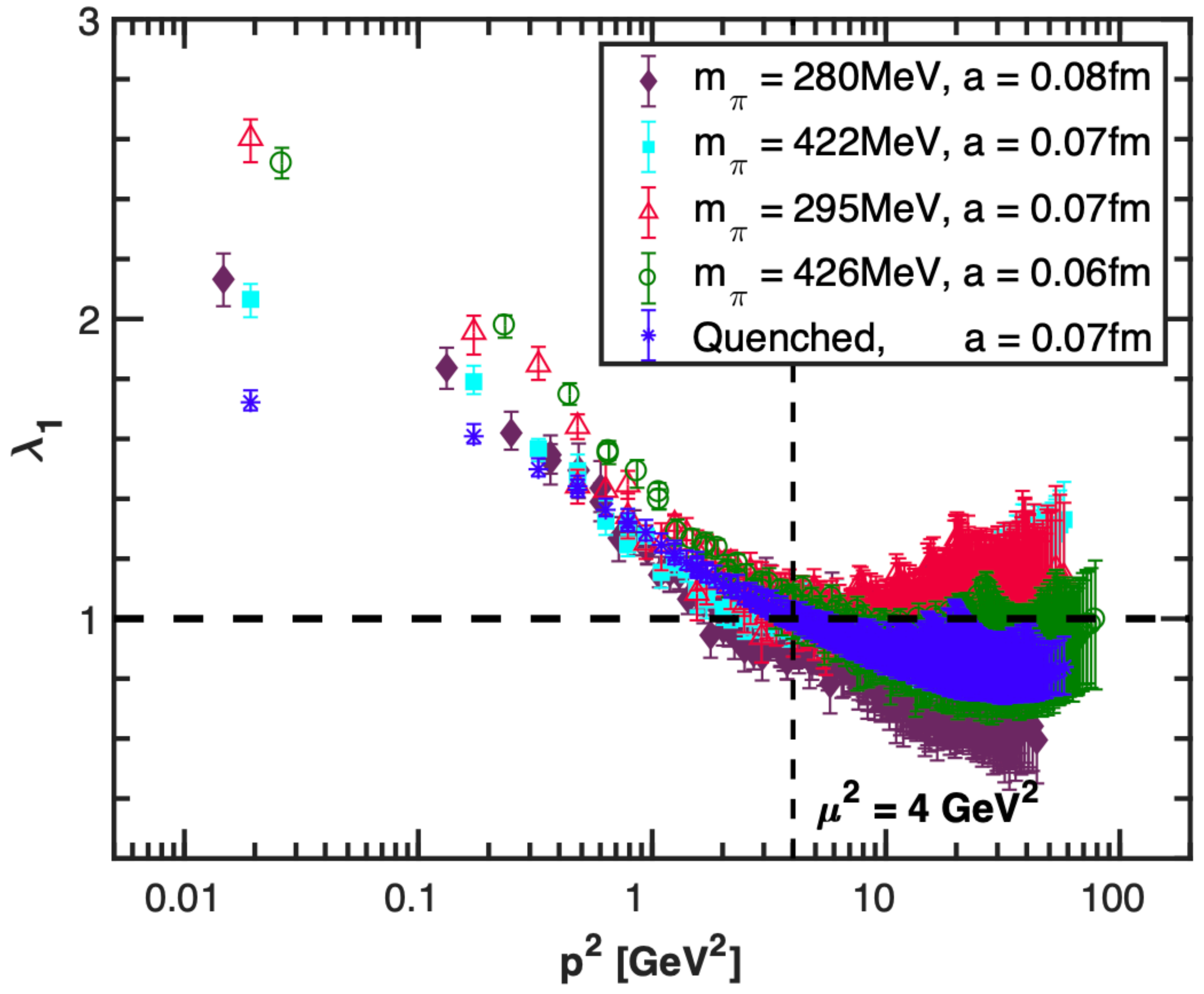
IR Ghost Dominance / Log Divergence

Full QCD

$$\lambda_1(p_1,p_2,p_3) = \frac{F(p_3^2)}{2}\Big\{ A(p_1^2)\Big[X_0 + \left(p_1^2 - (p_1p_2)\right)\ X_3\Big] + A(p_2^2)\Big[\overline{X}_0 + \left(p_2^2 - (p_1p_2)\right)\ \overline{X}_3\Big] + B(p_1^2)\Big[X_1 + X_2\Big] + B(p_2^2)\Big[\overline{X}_1 + \overline{X}_2\Big]\Big\}$$

IR Ghost Dominance / Log Divergence

Full QCD $\lambda_1(p_1, p_2, p_3) =$



A. Kizilersu et al, Phys.Rev.D **103** (2021) 11, 114515

4-gluon correlation function

$$\langle\, A_{\mu_1}(p_1)\; A_{\mu_2}(p_2)\; A_{\mu_3}(p_3)\; A_{\mu_4}(p_4)\,\rangle =$$

$$= \int \mathcal{D}A\; A_{\mu_1}(p_1)\; A_{\mu_2}(p_2)\; A_{\mu_3}(p_3)\; A_{\mu_4}(p_4)\; e^{-S}$$

4-gluon correlation function

$\vec{p}_i \propto \vec{p}$

Single momentum scale

In Landau gauge simplifies the tensor analysis

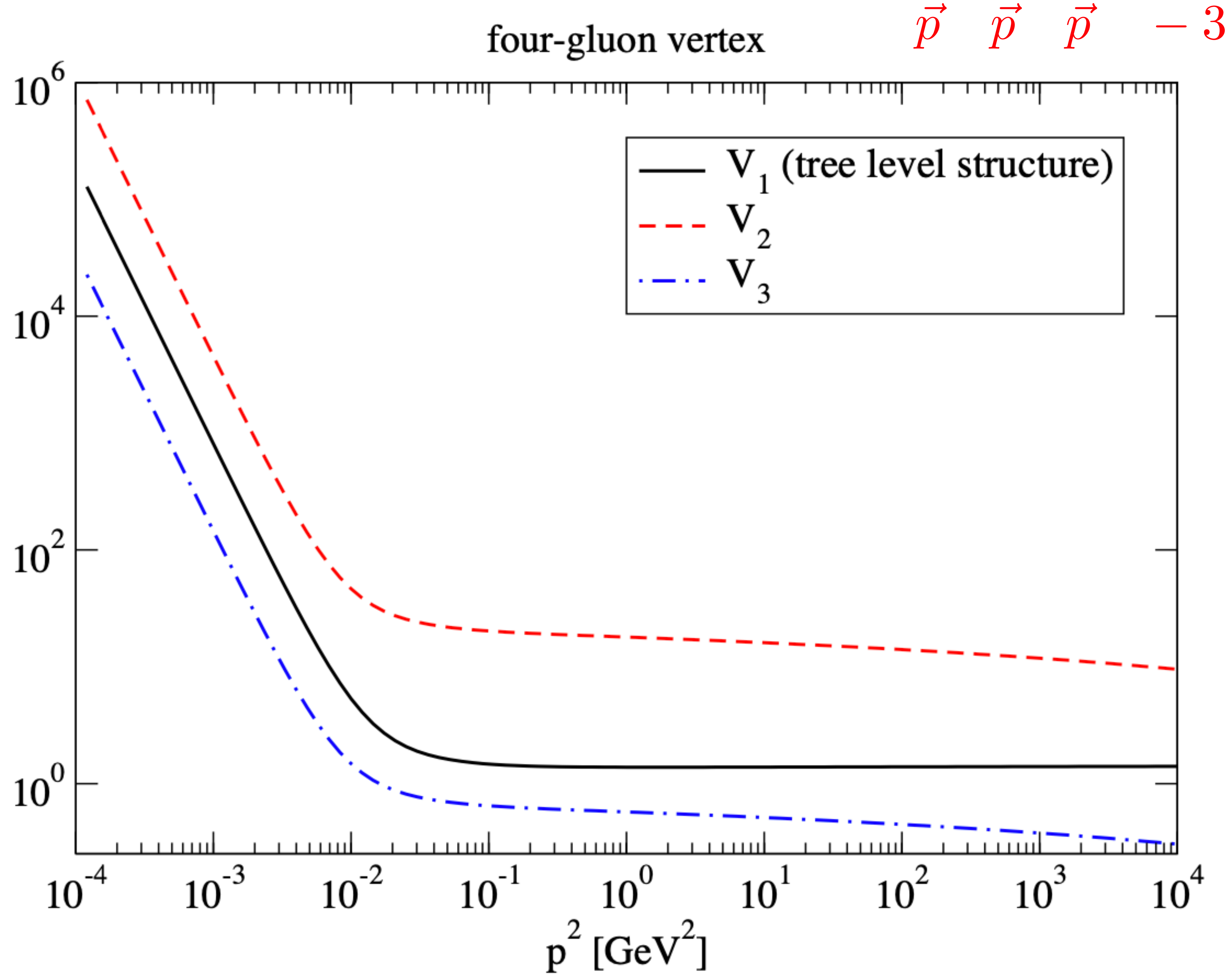
Contributions only from the tensors proportional to $\delta_{\mu\nu}$

Recent Continuum Calculations

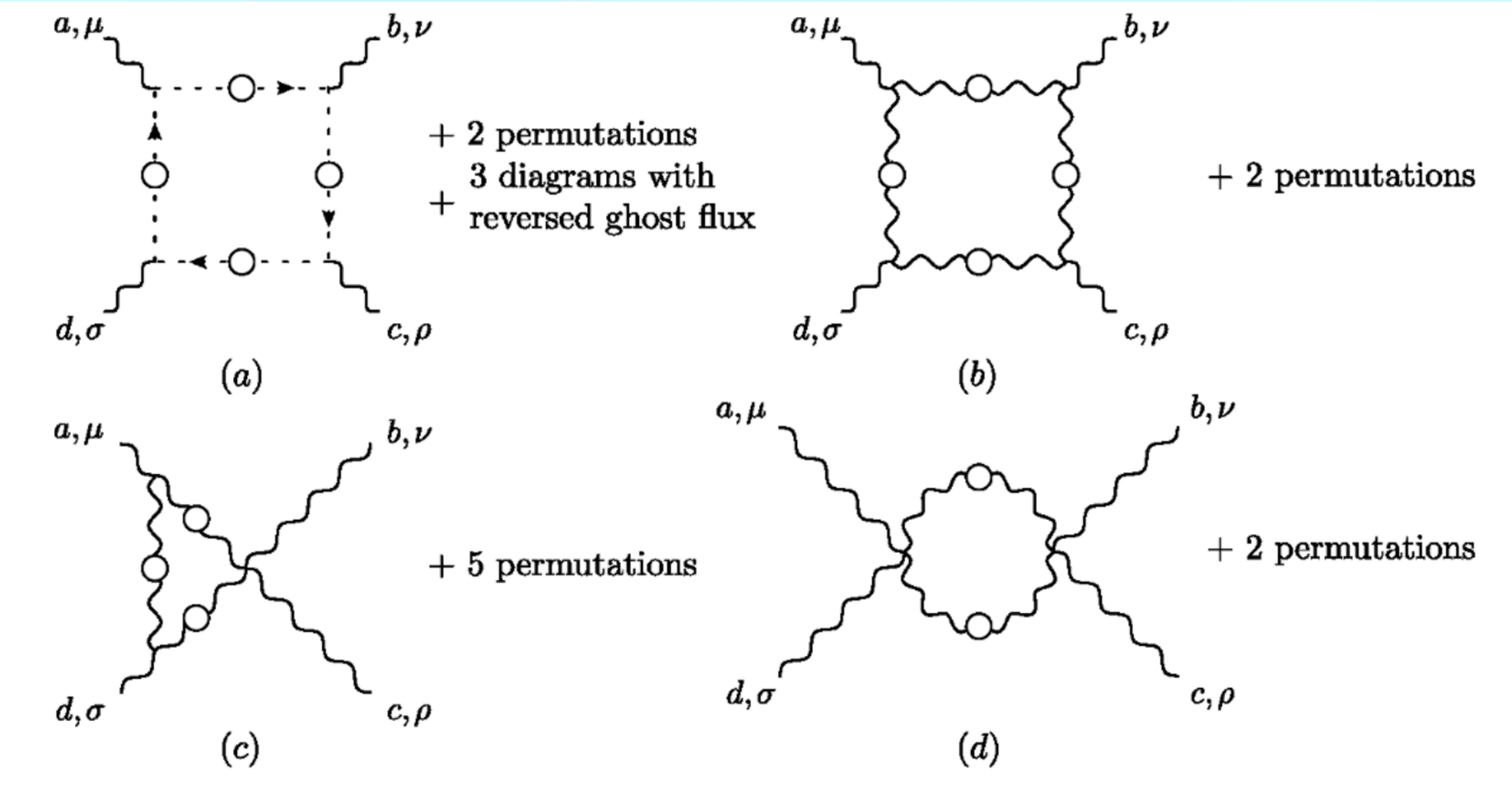
C Kellermann, C S Fischer, *Phys. Rev. D* ***78*** *(2008) 025015*
D Binosi, D Ibañez, J Papavassiliou, *JHEP* ***1409*** *(2014) 059*
A K Cyrol, M Q Huber, L von Smekal, *Eur Phys J C* ***75*** *(2015) 102*

C Kellermann, C S Fischer, *Phys. Rev. D* **78** *(2008) 025015*

$\vec{p} \quad \vec{p} \quad \vec{p} \quad -3\,\vec{p}$

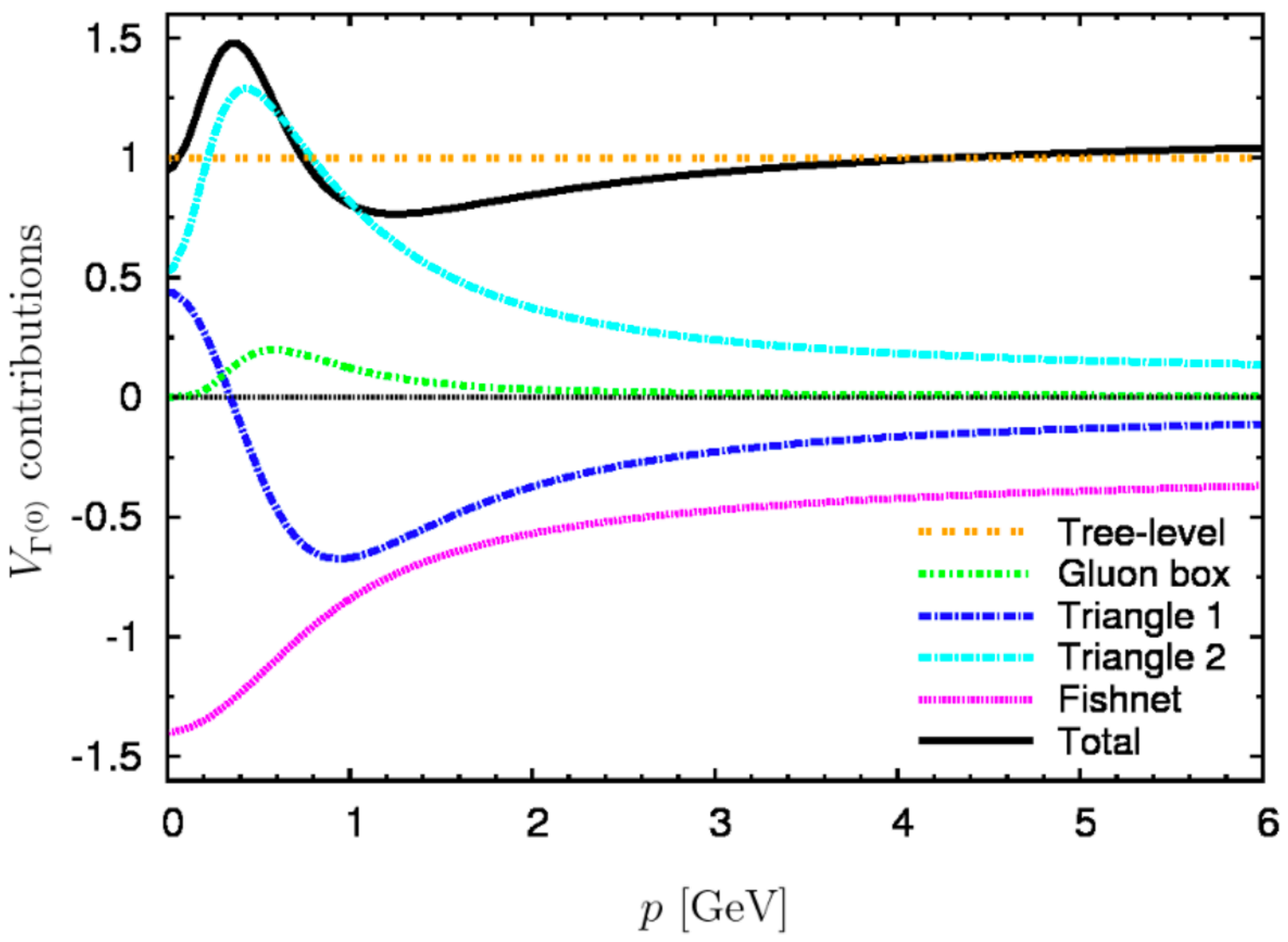


D Binosi, D Ibañez, J Papavassiliou, *JHEP* **1409** *(2014) 059*

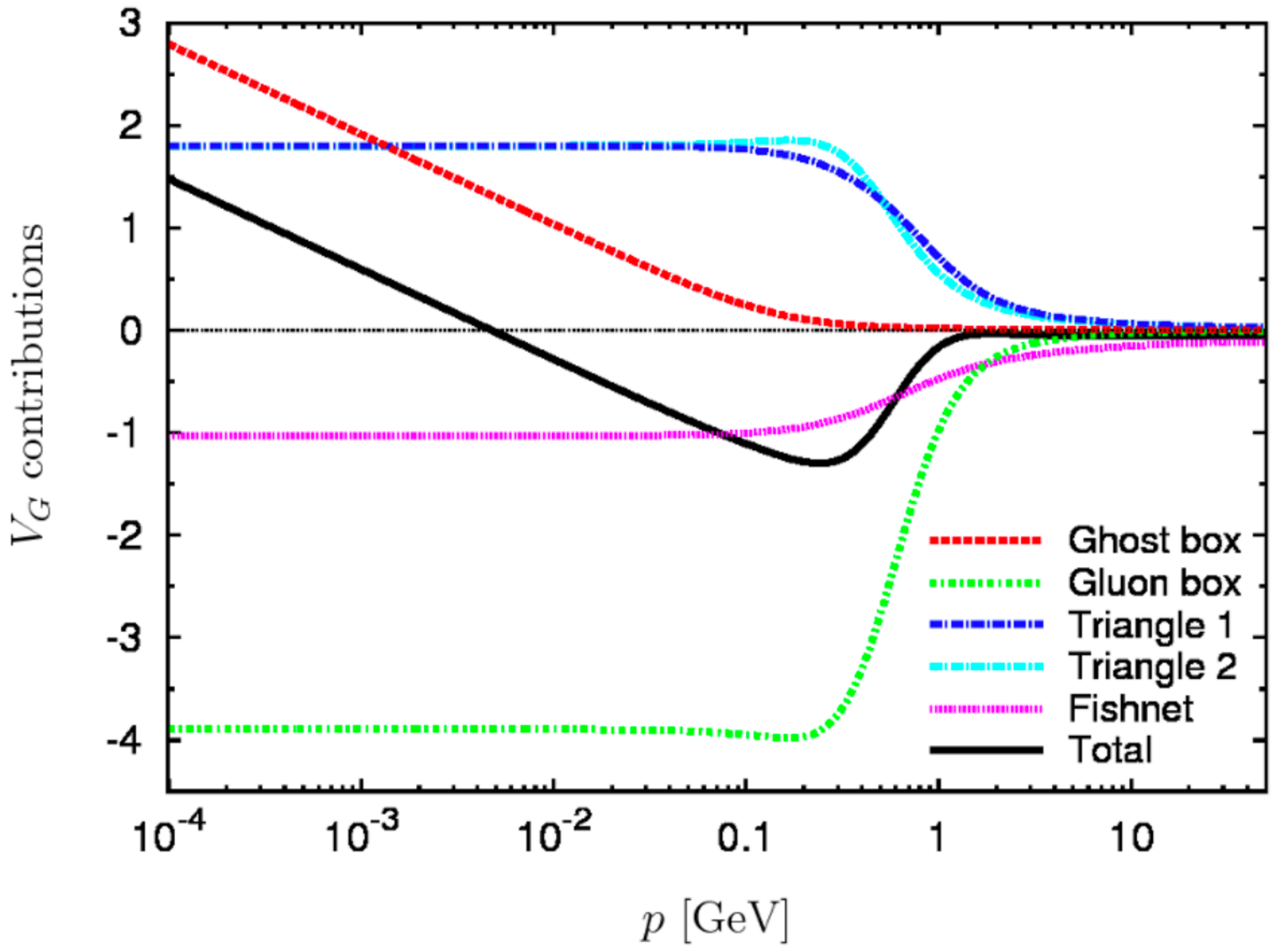


$$\Gamma^{abcd}_{\mu\nu\rho\sigma}(p,p,p,-3p)\Big|_{gg} = V_{\Gamma^{(0)}}(p^2)\Gamma^{abcd(0)}_{\mu\nu\rho\sigma} + V_G(p^2)G^{abcd}_{\mu\nu\rho\sigma},$$

$$G^{abcd}_{\mu\nu\rho\sigma} = (\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})\underbrace{(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho})}_{R_{\mu\nu\rho\sigma}}$$

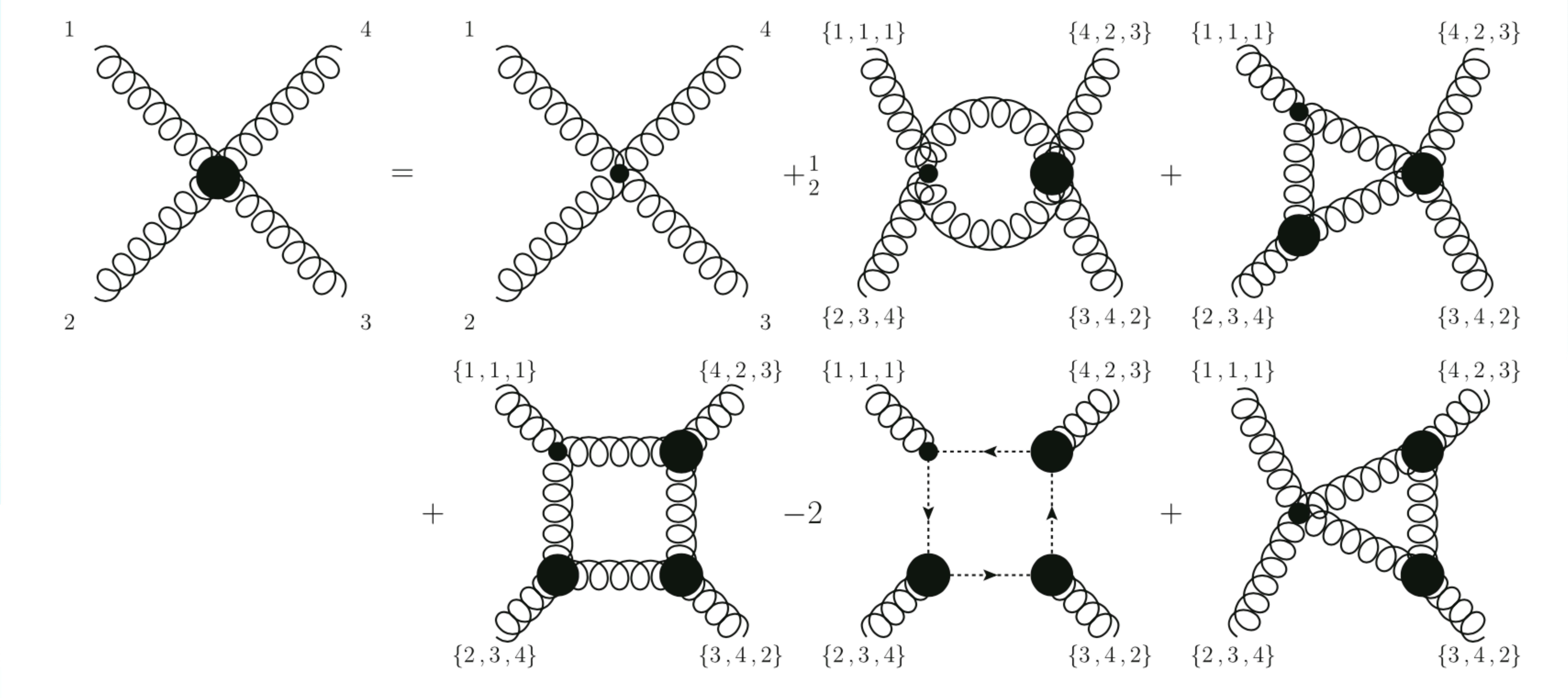


$V_{\Gamma^{(0)}}$ contributions
1.5
1
0.5
0
-0.5
-1
-1.5
0
1
2
3
4
5
6
p [GeV]
Tree-level
Gluon box
Triangle 1
Triangle 2
Fishnet
Total

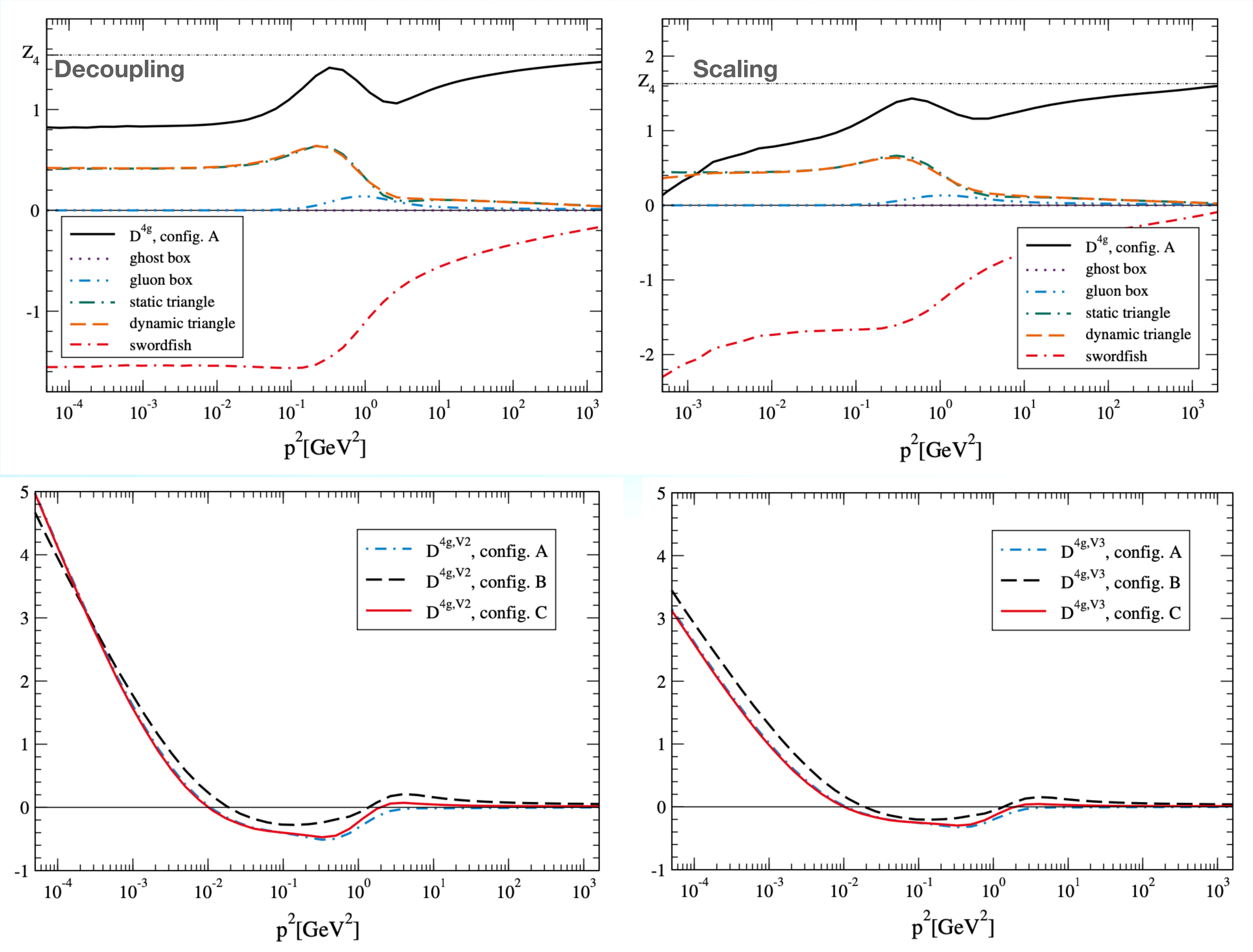


V_G contributions
3
2
1
0
-1
-2
-3
-4
10^{-4}
10^{-3}
10^{-2}
0.1
1
10
p [GeV]
Ghost box
Gluon box
Triangle 1
Triangle 2
Fishnet
Total

A K Cyrol, M Q Huber, L von Smekal, *Eur Phys J C* **75** *(2015) 102*



IR dominant (primitively divergent diagrams)
One-loop contributions (disregard contributions with five-point diagrams and ghost-gluon five-point functions)
Take tree level tensor structure for three-gluon and four-gluon diagrams



Decoupling
Z4
D^{4g}, config. A
ghost box
gluon box
static triangle
dynamic triangle
swordfish
p²[GeV²]
Scaling
Z4
D^{4g}, config. A
ghost box
gluon box
static triangle
dynamic triangle
swordfish
p²[GeV²]
$D^{4g,V2}$, config. A
$D^{4g,V2}$, config. B
$D^{4g,V2}$, config. C
p²[GeV²]
$D^{4g,V3}$, config. A
$D^{4g,V3}$, config. B
$D^{4g,V3}$, config. C
p²[GeV²]

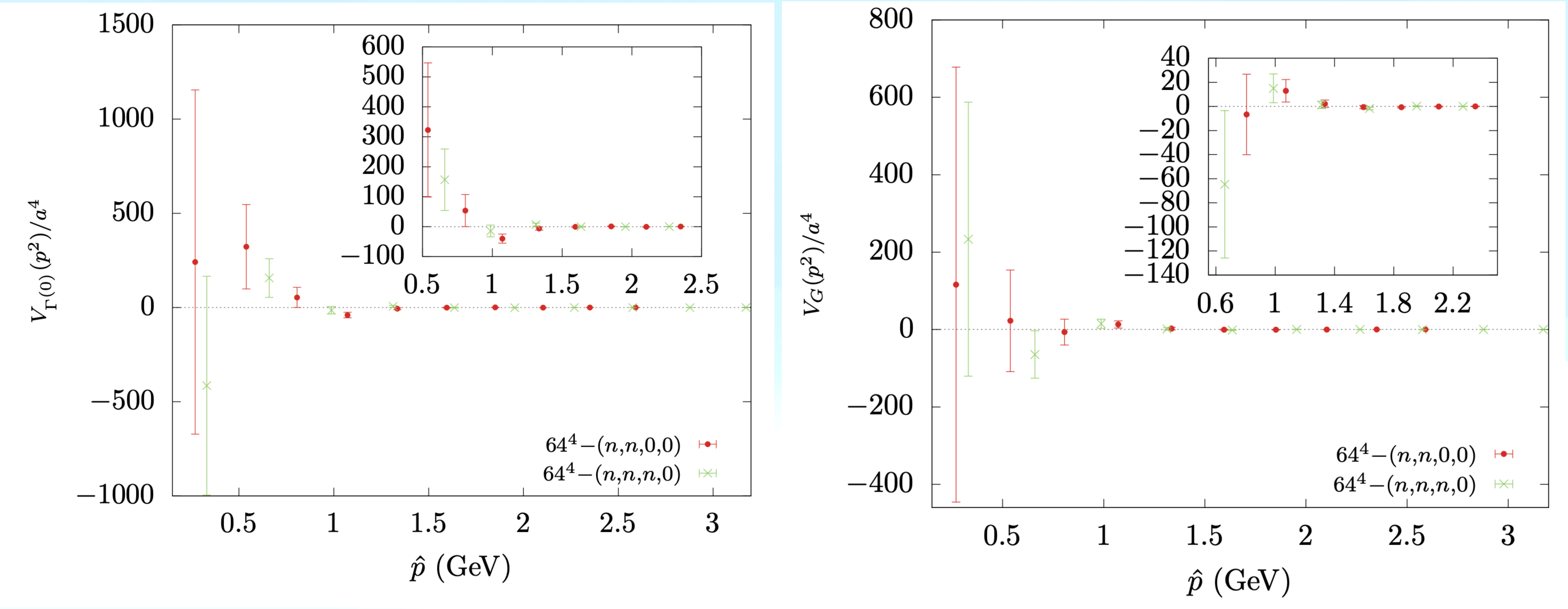
First tentative of a lattice calculation can be found in
G T R Catumba Master Thesis ArXiv: 2101.06074

D. Binosi, D. Ibañez, J. Papavassiliou JHEP 9, 059 (2014) arXiv:1407.3677

p, p, p, -3p Two tensor structures

$$\widetilde{\Gamma}^{(0)\,abcd}_{\mu\nu\eta\zeta} = f_{abr} f_{cdr} \left(\delta_{\mu\eta}\delta_{\nu\zeta} - \delta_{\mu\zeta}\delta_{\nu\eta}\right) + f_{acr} f_{bdr} \left(\delta_{\mu\nu}\delta_{\eta\zeta} - \delta_{\mu\zeta}\delta_{\nu\eta}\right) + f_{adr} f_{bcr} \left(\delta_{\mu\nu}\delta_{\eta\zeta} - \delta_{\mu\eta}\delta_{\nu\zeta}\right)$$

$$G^{abcd}_{\mu\nu\eta\zeta} = \left(\delta^{ab}\,\delta^{cd} + \delta^{ac}\,\delta^{bd} + \delta^{ad}\,\delta^{bc}\right)\left(\delta_{\mu\nu}\,\delta_{\eta\zeta} + \delta_{\mu\eta}\,\delta_{\nu\zeta} + \delta_{\mu\zeta}\,\delta_{\nu\eta}\right)$$



$V_{\Gamma(0)}(p^2)/a^4$
$\hat{p}$ (GeV)
$64^4-(n,n,0,0)$
$64^4-(n,n,n,0)$
$V_G(p^2)/a^4$
$\hat{p}$ (GeV)
$64^4-(n,n,0,0)$
$64^4-(n,n,n,0)$

For **tensor analysis** see:

J A Gracey, Phys Rev D90, 025011 (2014) arXiv: 1406.1618
G Eichmann, C S Fischer, W Heupel, Phys Rev D92, 056006 (2015) arXiv: 1505.06336

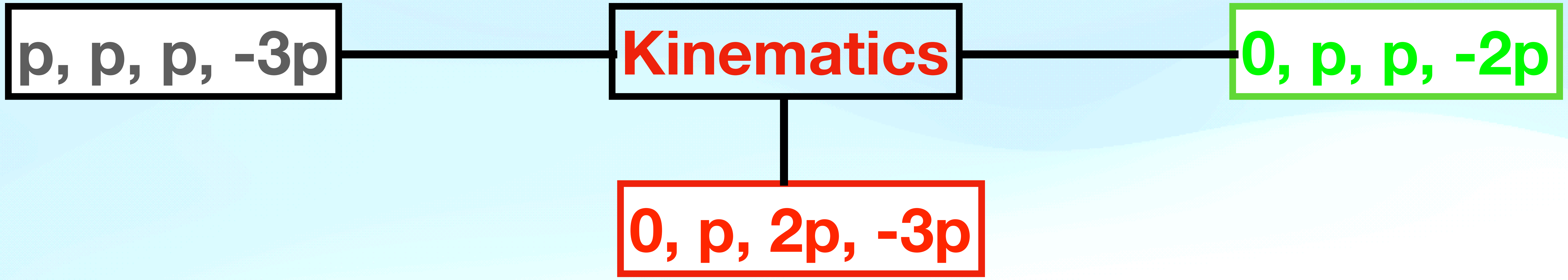
$$\widetilde{\Gamma}^{(0)\,abcd}_{\;\;\mu\nu\eta\zeta} = f_{abr} f_{cdr} \left(\delta_{\mu\eta}\delta_{\nu\zeta} - \delta_{\mu\zeta}\delta_{\nu\eta}\right) + f_{acr} f_{bdr} \left(\delta_{\mu\nu}\delta_{\eta\zeta} - \delta_{\mu\zeta}\delta_{\nu\eta}\right) + f_{adr} f_{bcr} \left(\delta_{\mu\nu}\delta_{\eta\zeta} - \delta_{\mu\eta}\delta_{\nu\zeta}\right)$$

$$\widetilde{\Gamma}^{(1)\,abcd}_{\;\;\mu\nu\eta\zeta} = d_{abr} d_{cdr} \left(g_{\mu\eta} g_{\nu\zeta} + g_{\mu\zeta} g_{\nu\eta}\right) + d_{acr} d_{bdr} \left(g_{\mu\zeta} g_{\nu\eta} + g_{\mu\nu} g_{\eta\zeta}\right) + d_{adr} d_{bcr} \left(g_{\mu\nu} g_{\eta\zeta} + g_{\mu\eta} g_{\nu\zeta}\right)$$

$$\widetilde{\Gamma} = F(p^2)\,\widetilde{\Gamma}^{(0)} + G(p^2)\,\widetilde{\Gamma}^{(1)} + H(p^2)\,\widetilde{\Gamma}^{(2)}$$

Not an orthogonal basis

$$\mathcal{G}^{(4)} = \widetilde{\Gamma} \left(P^{\perp}(p) D(p^2) \right)^3 \left(P^{\perp}(3p) D(9p^2) \right)$$



Measure the three form factors

$F^{(0)}(p^2)$ $F^{(1)}(p^2)$ $F^{(2)}(p^2)$

Tree Level Tensor

$$\beta = 6.0 \qquad a = 0.102 \text{ fm} \qquad a^{-1} = 1.943 \text{ GeV}$$

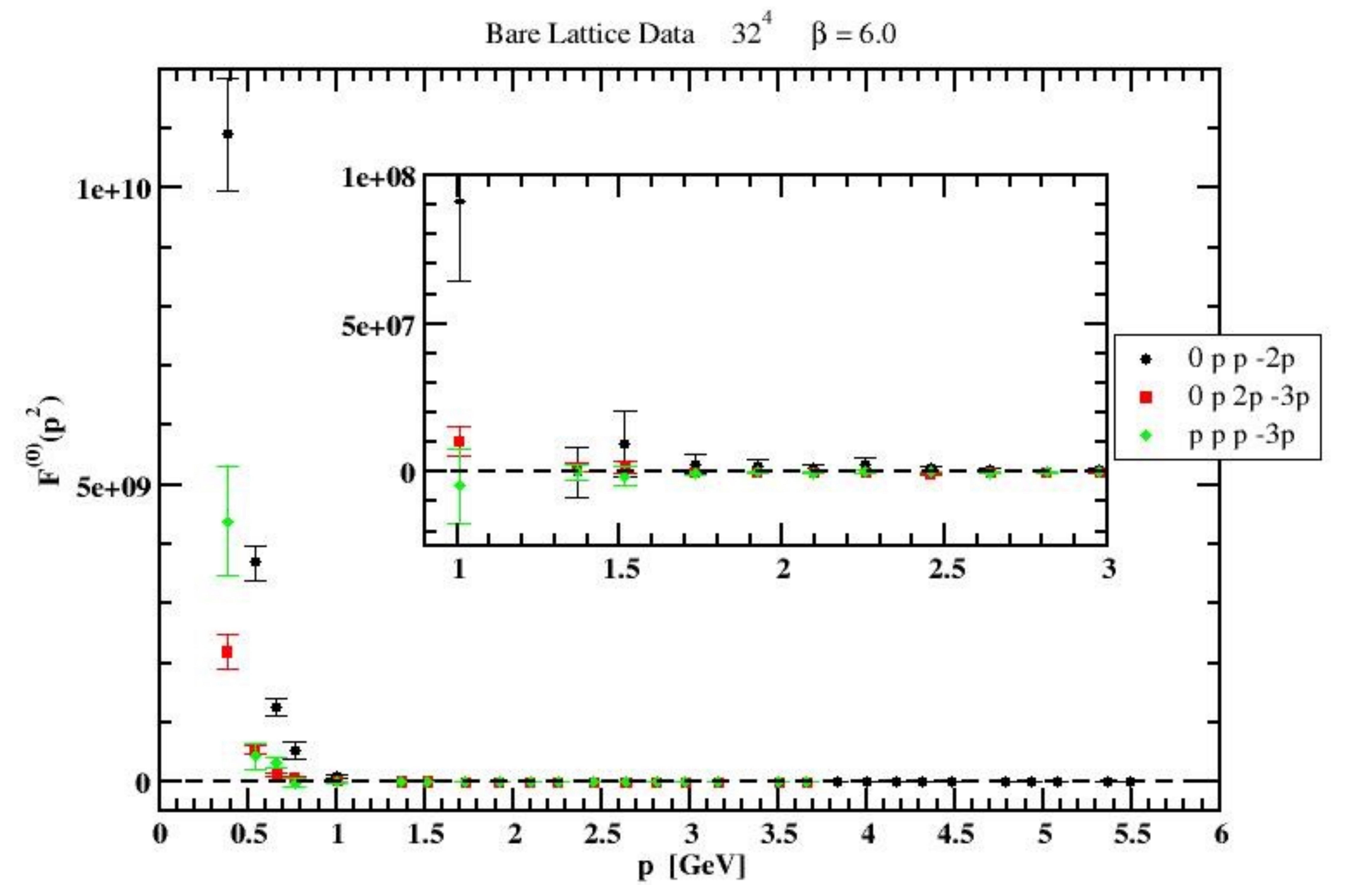
Lattice	Configs	p_{min}
32^4	4620	381 MeV
64^4	2000	191 MeV
80^4	1801	153 MeV

Averaged over equivalent momenta, including the negative momenta !

(1 1 1 1)

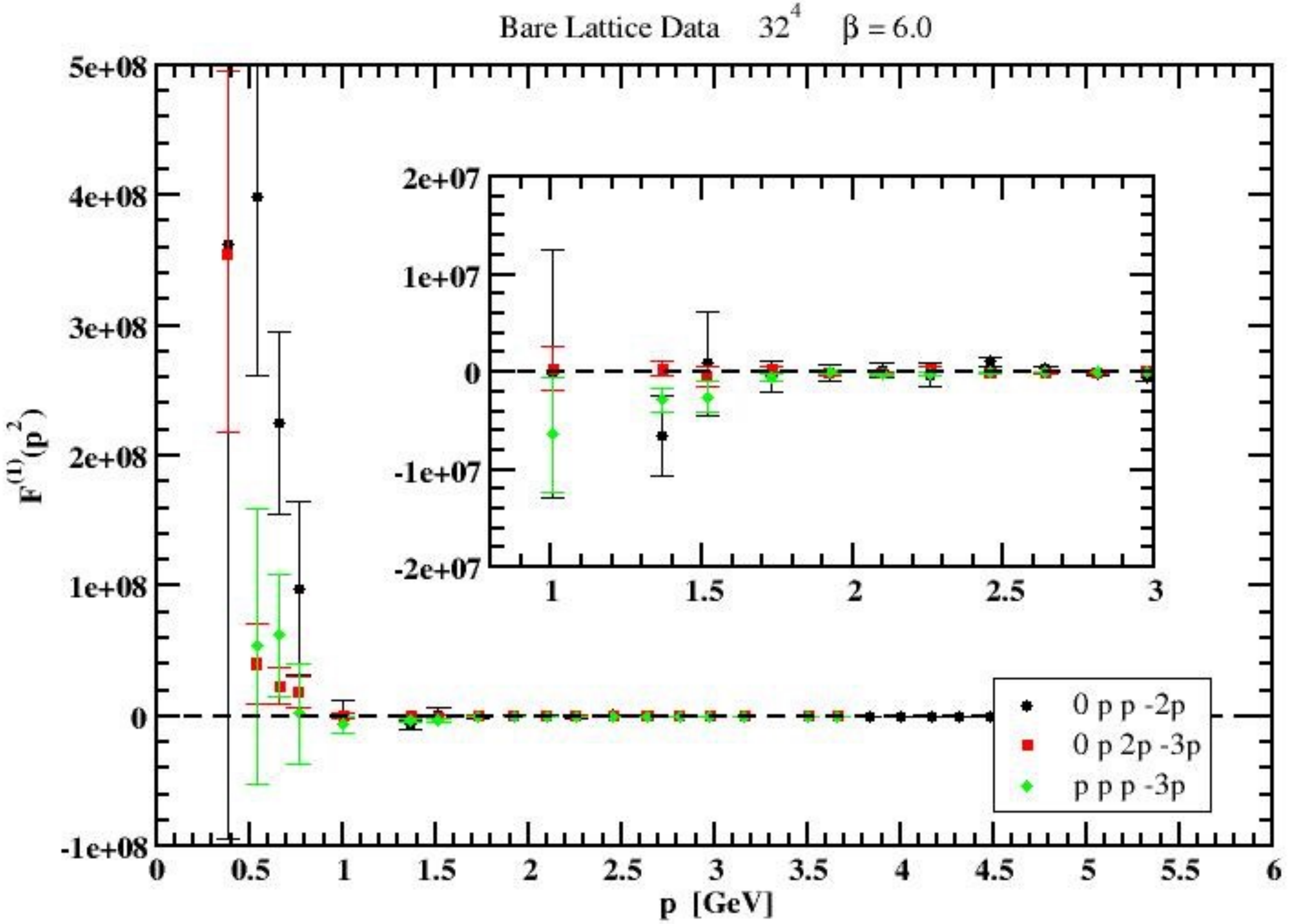
(-1 1 1 1) (1 -1 1 1) (1 1 -1 1) (1 1 1 -1)
(-1 -1 1 1) (-1 1 -1 1) (-1 1 1 -1) (1 -1 -1 1) (1 -1 1 -1) (1 1 -1 -1)
(1 -1 -1 -1) (-1 1 -1 -1) (-1 -1 1 -1) (-1 -1 -1 1)
(-1 -1 -1 -1)

Bare Green Function



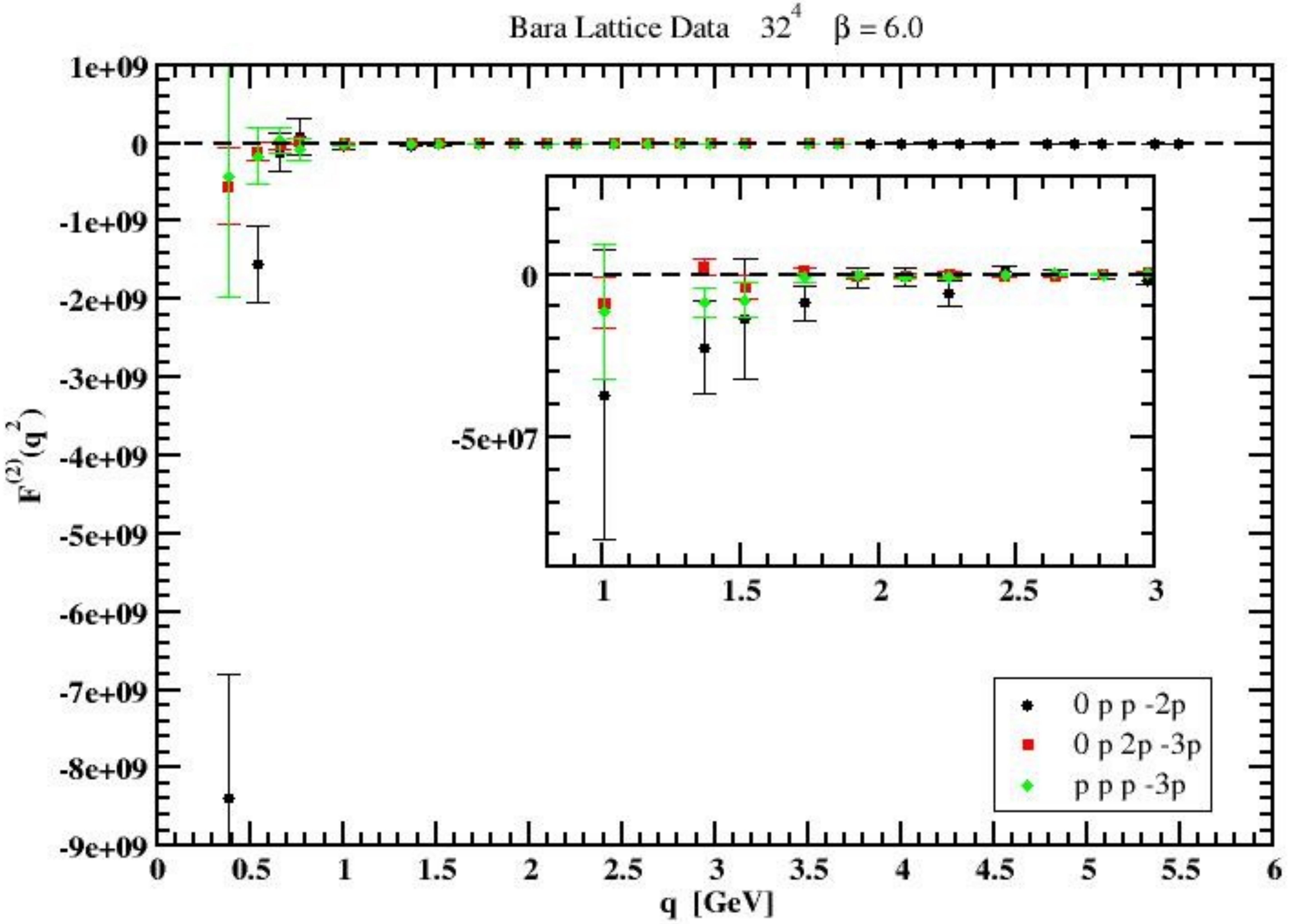
Bare Green Function

Bare Green Function

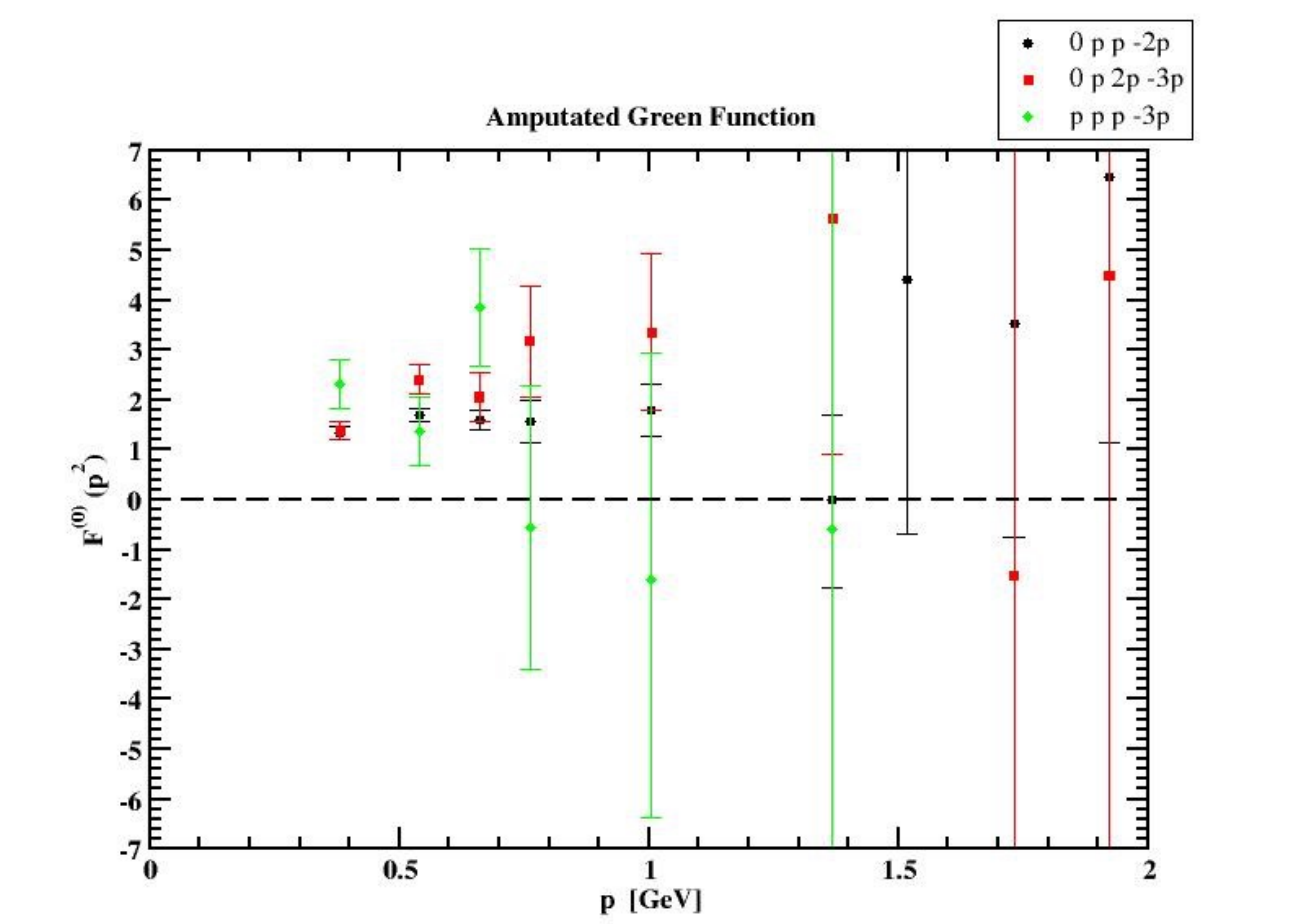


Bare Green Function

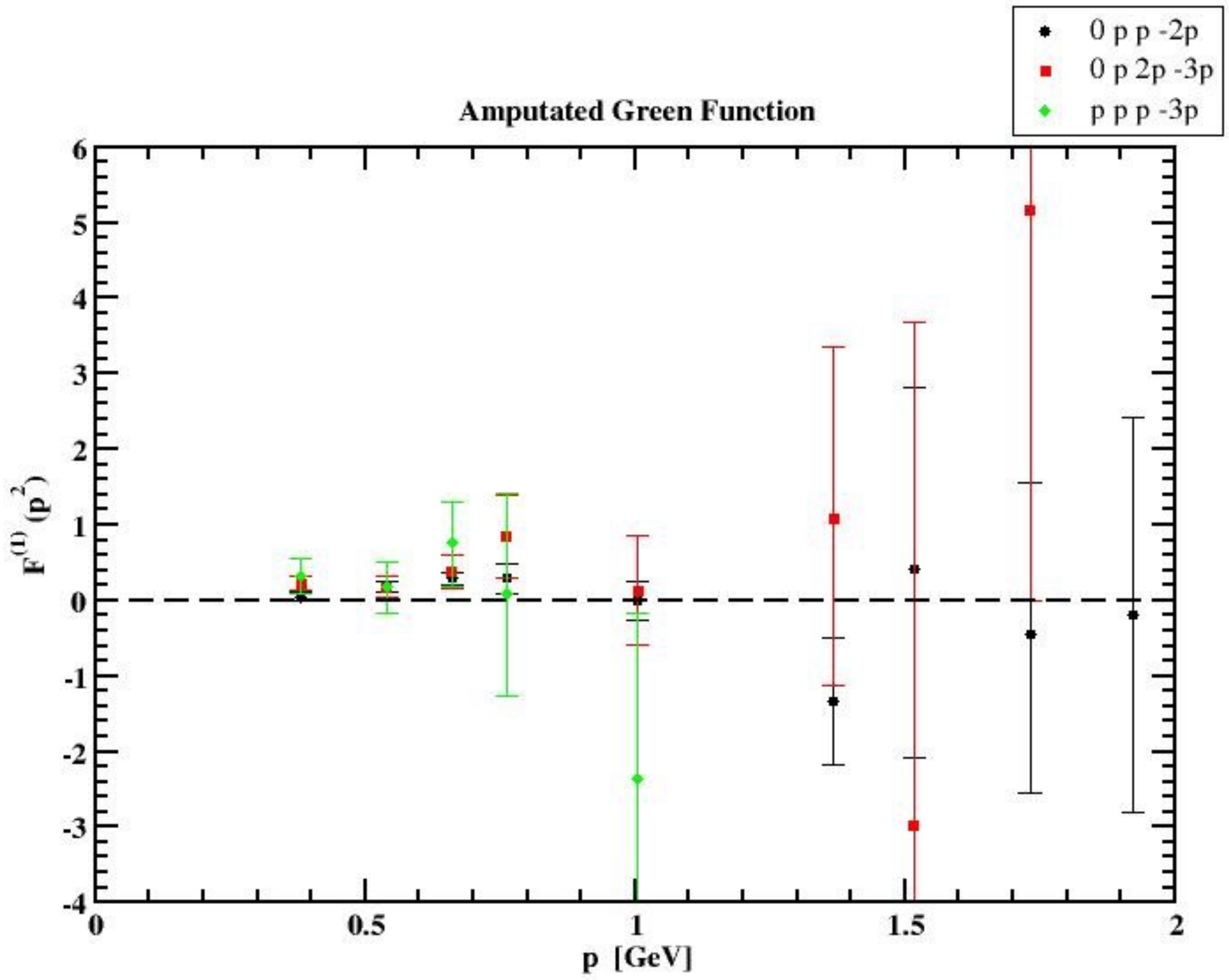
Bare Green Function



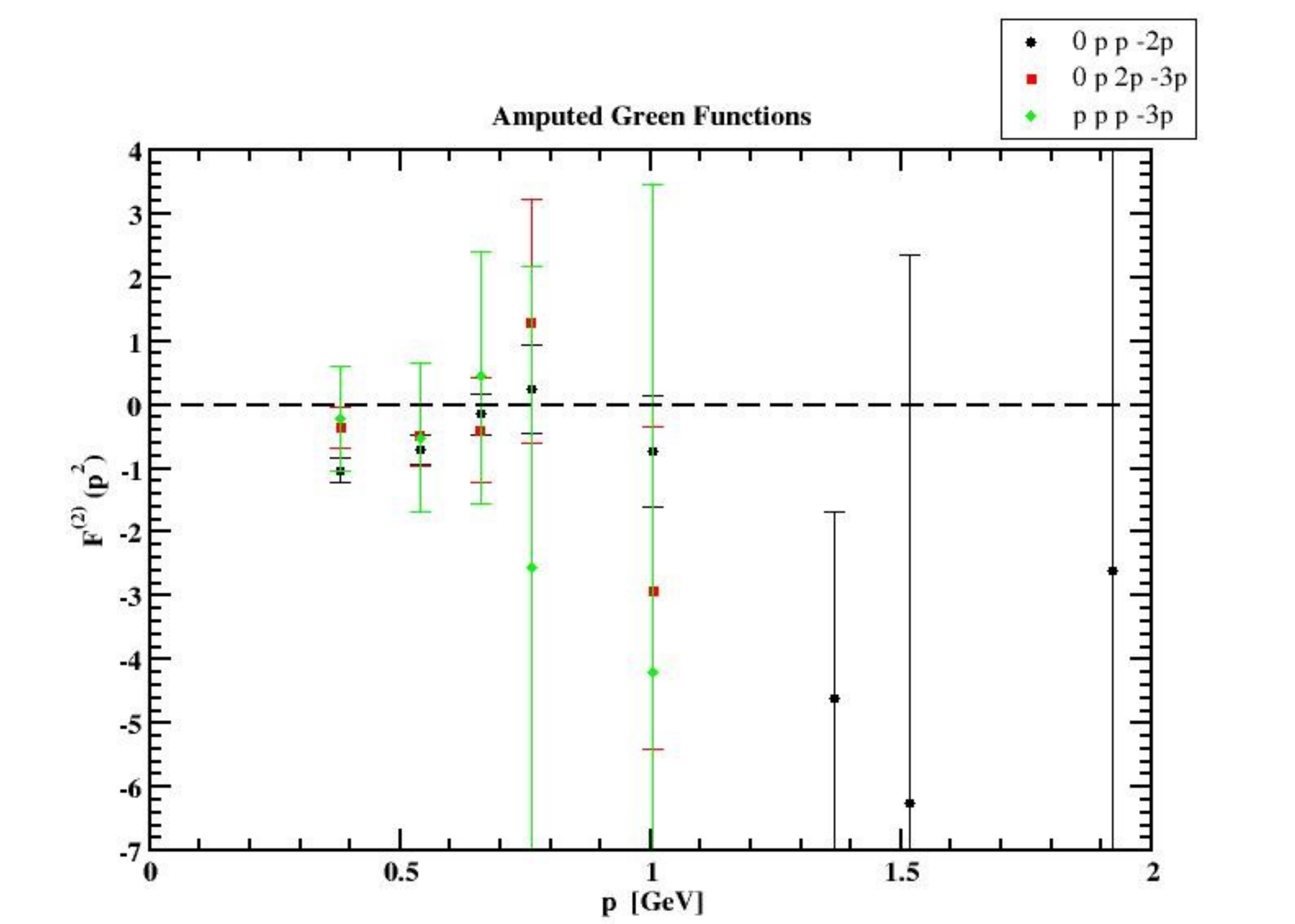
Bare Amputated Green Function



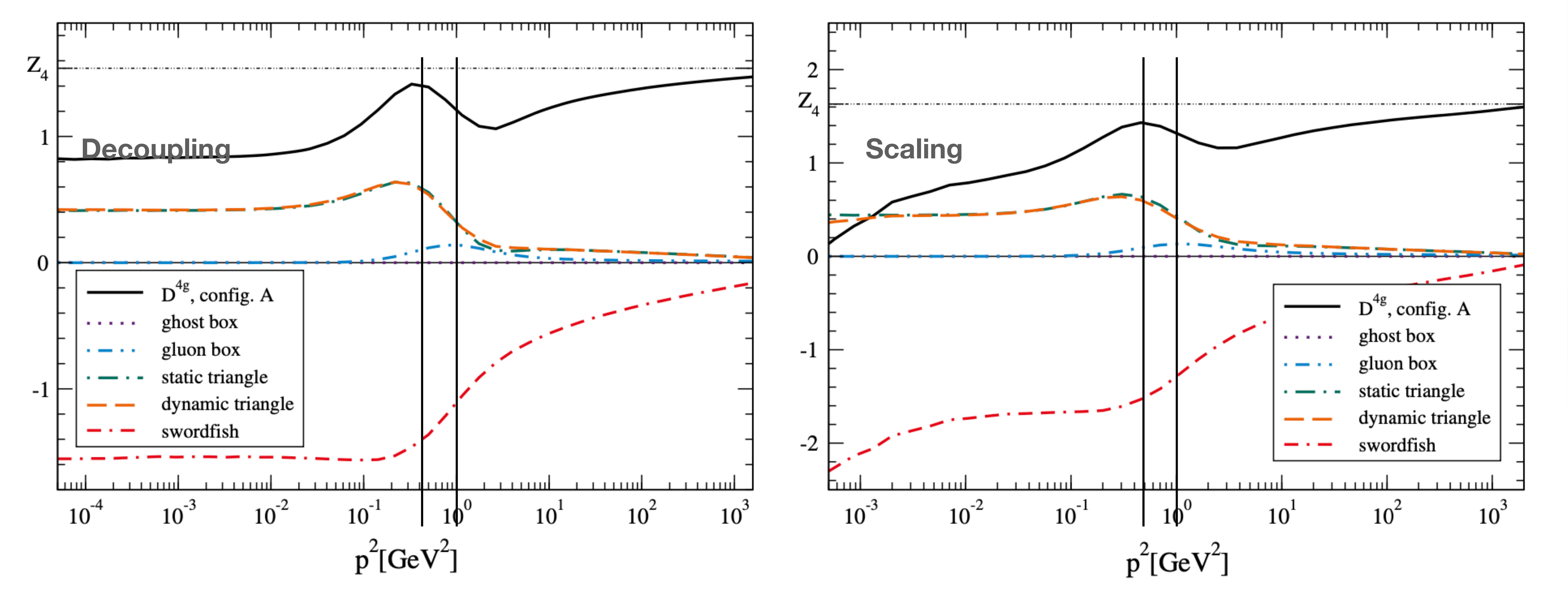
Bare Amputated Green Function



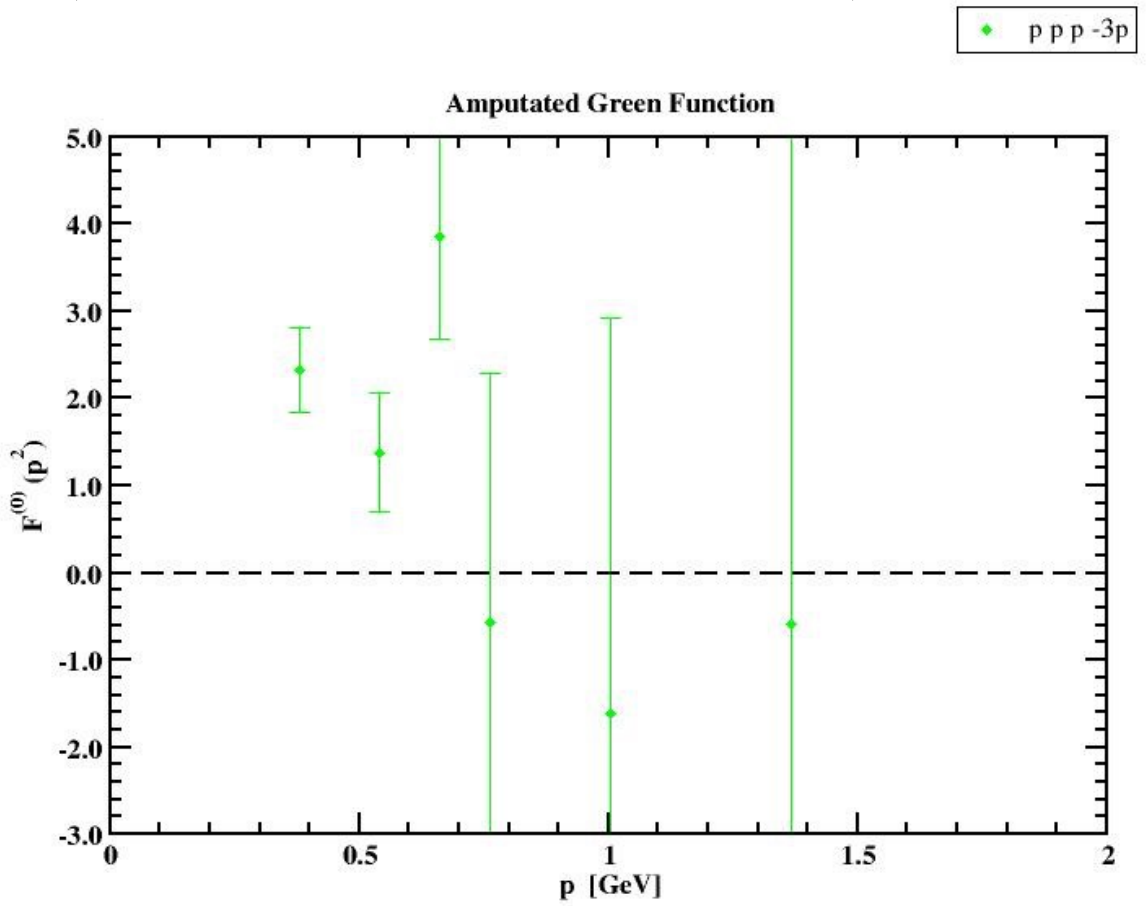
Bare Amputated Green Function



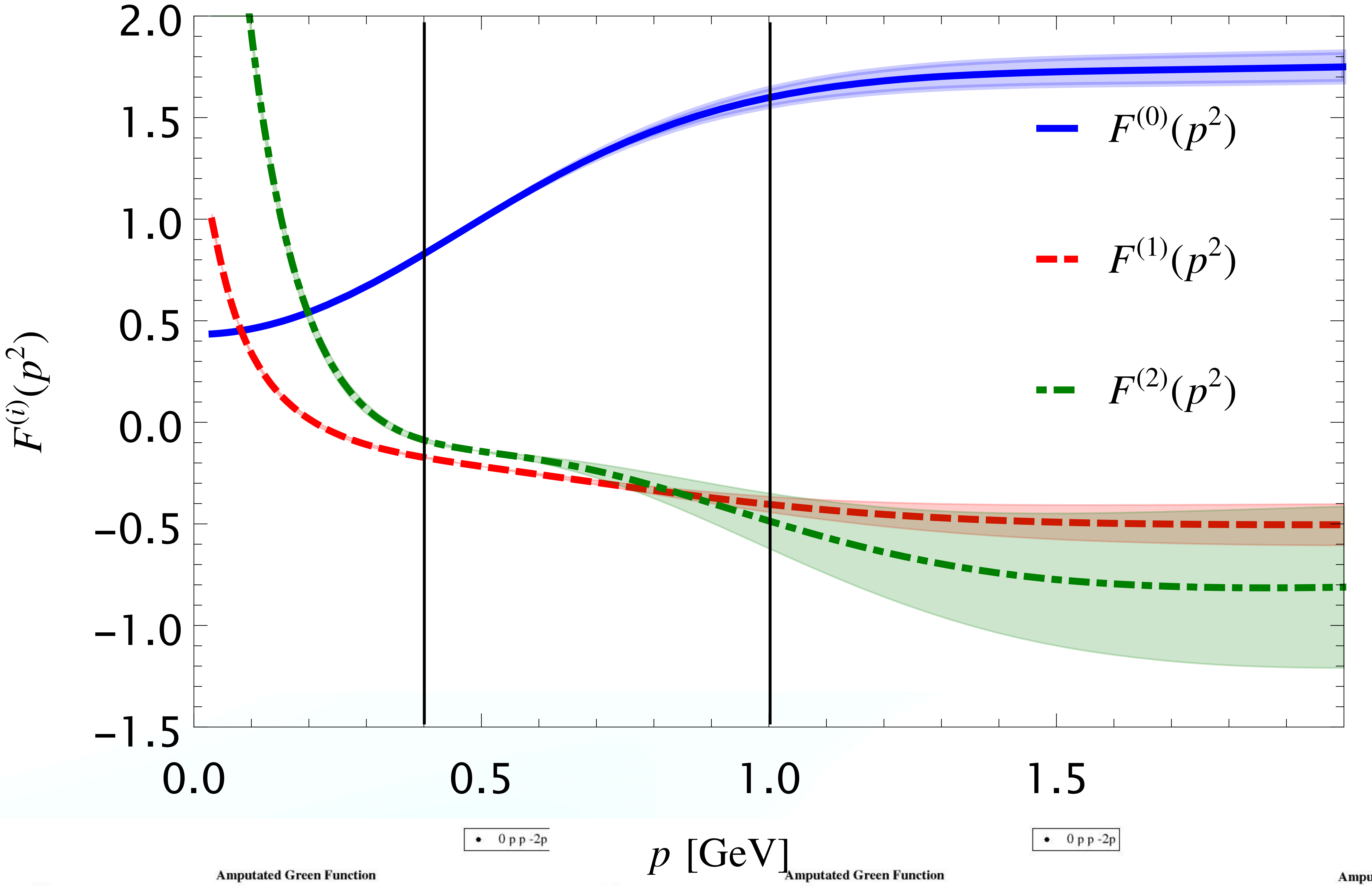
Continuum Calculations



A K Cyrol, M Q Huber, L von Smekal, *Eur Phys J C* ***75*** *(2015) 102*

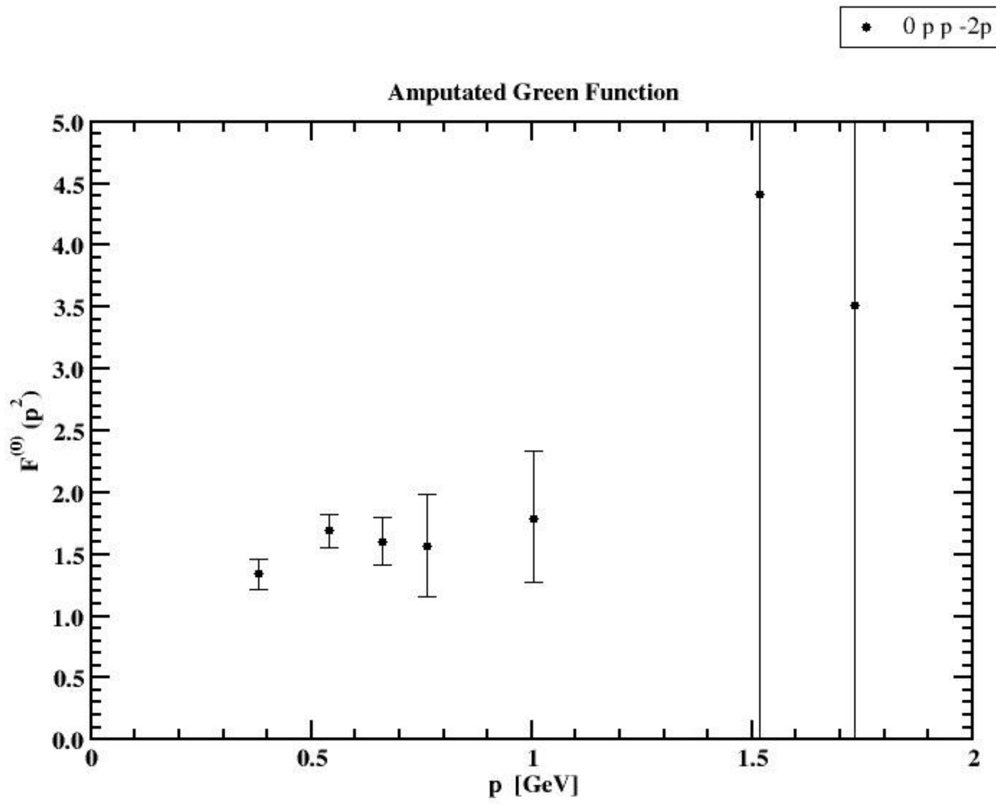


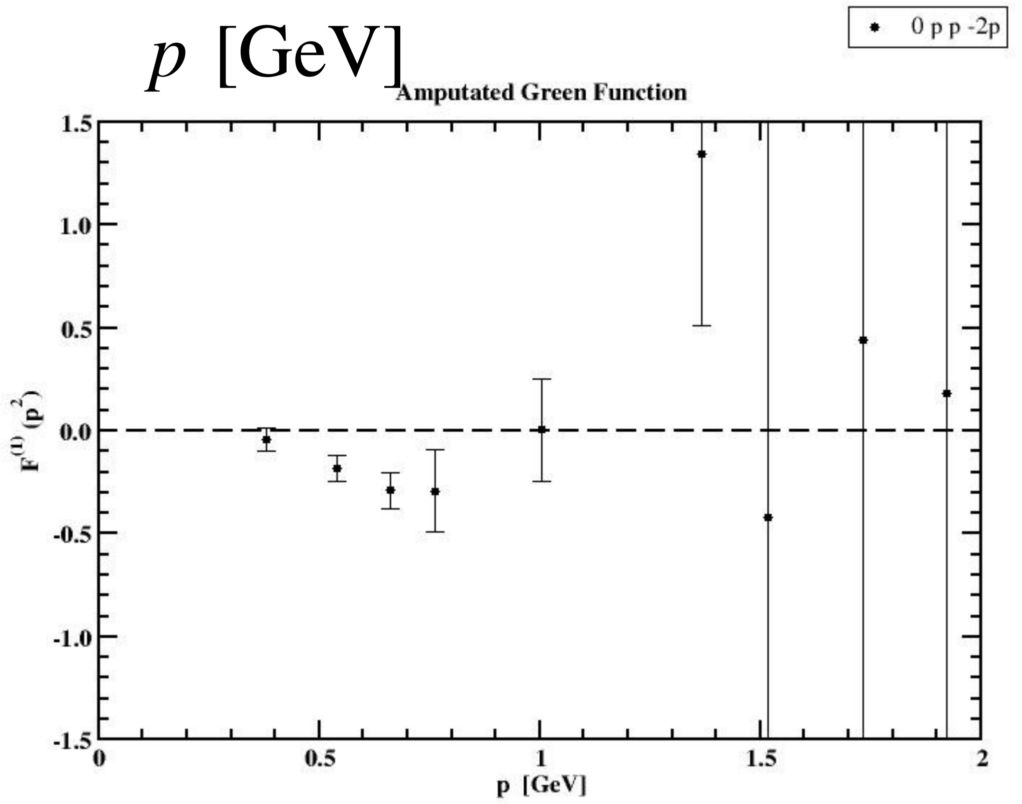
Continuum Calculations

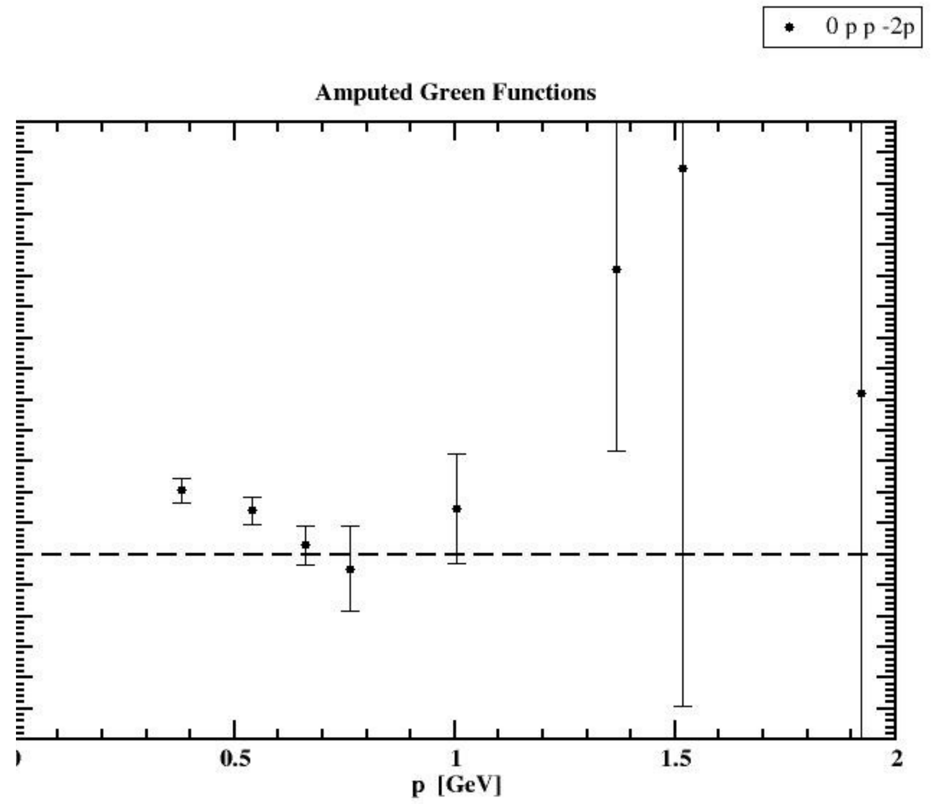


One loop truncated Dyson-Schwinger equation by

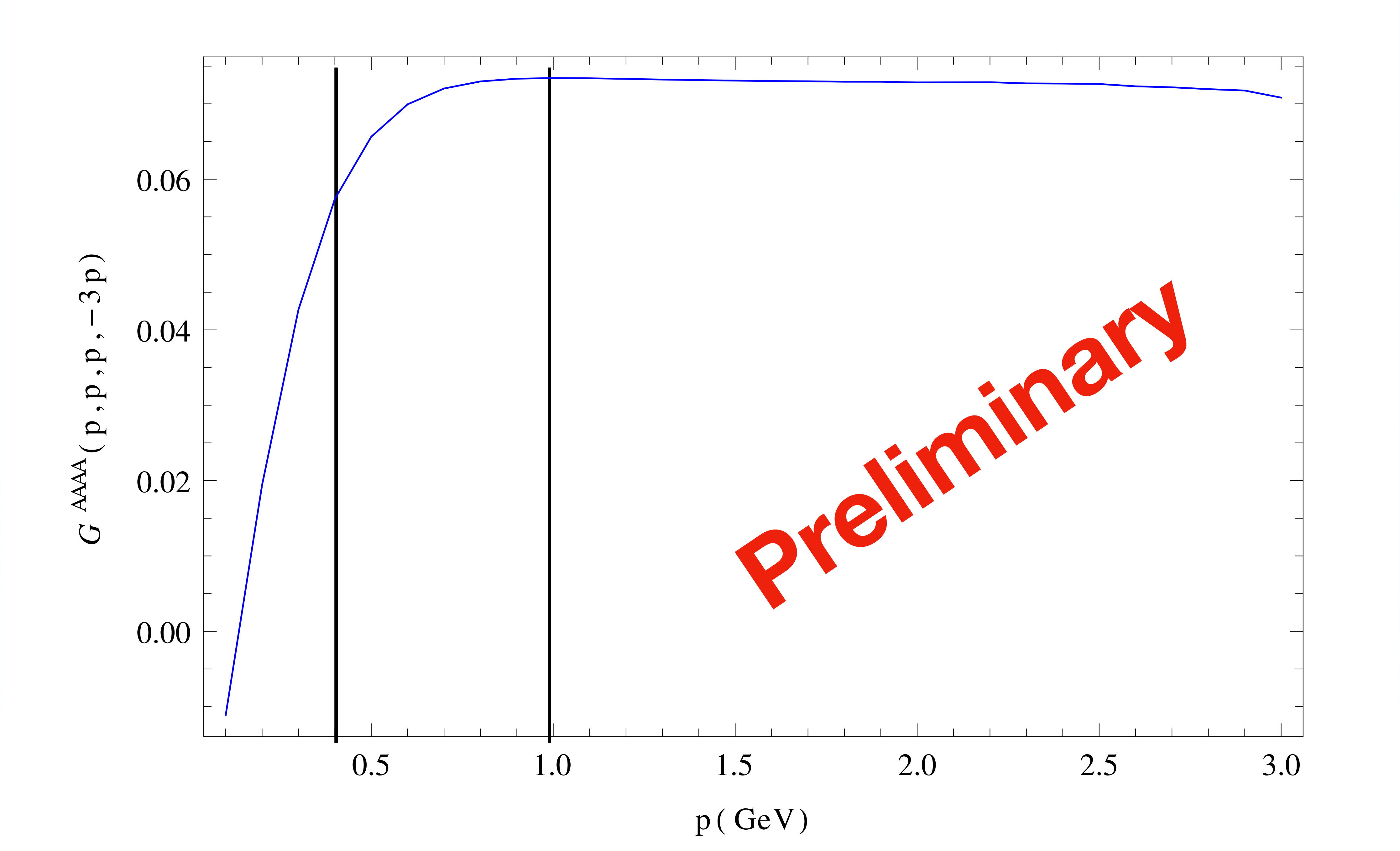
A C Aguilar, M N Ferreira, J Papavassiliou, L R Santos





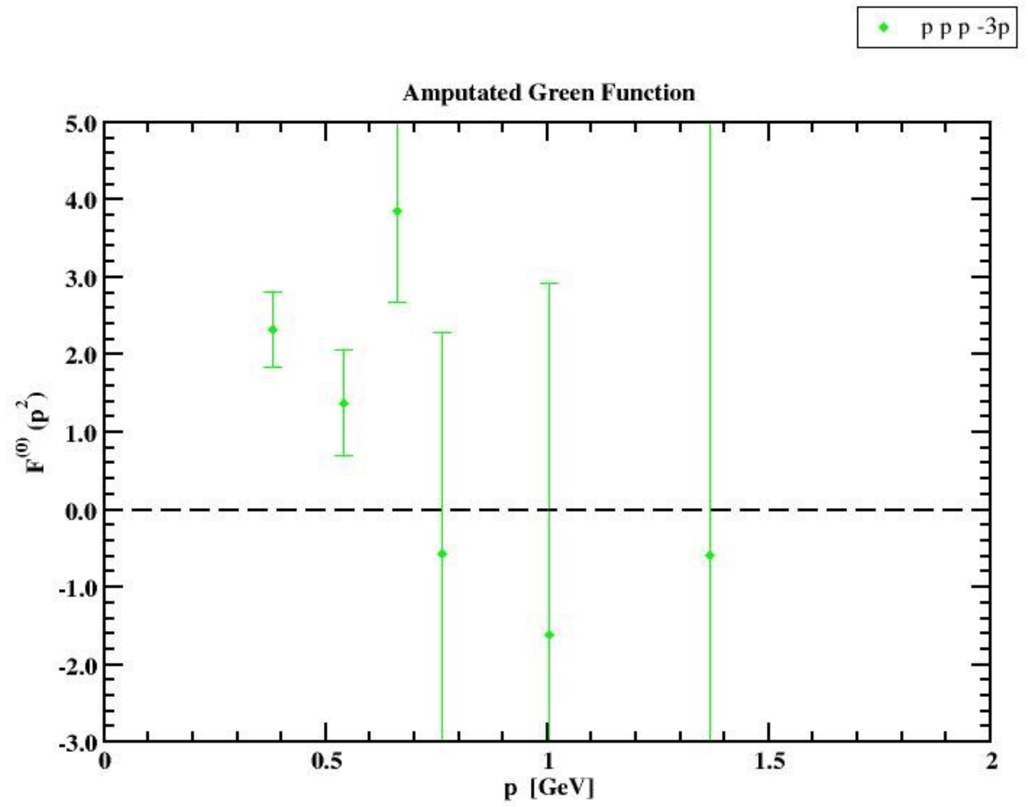


Continuum Calculations



Curci-Ferrari Model (one-loop)

M Peláez, P Pais



Summary and Conclusions

- gluon correlation functions up to 4-external legs are possible with standard lattice methods (require large statistical ensembles)

- increase the statistics and improve the tensor analysis

- At mid range momentum or larger tree level structure dominates

- Study better the good (at least) qualitative agreement between Lattice and DSE approach

- Look at other kinematics and extend collaboration to get a better picture

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