Raios Cósmicos e Cronologia

INSTITUTO DE FÍSICA GLEB WATAGHIN

Confirming the action of the Schwinger mechanism in QCD

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University of Campinas, São Paulo - Brazil

Based on:

A. C. A, M. N. Ferreira and J. Papavassiliou, Phys. Rev. D105, 014030 (2022)

A.C.A., F. De Soto, M. N. Ferreira, J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J.Rodríguez-Quintero, *Phys. Lett. B* 841 (2023) 137906

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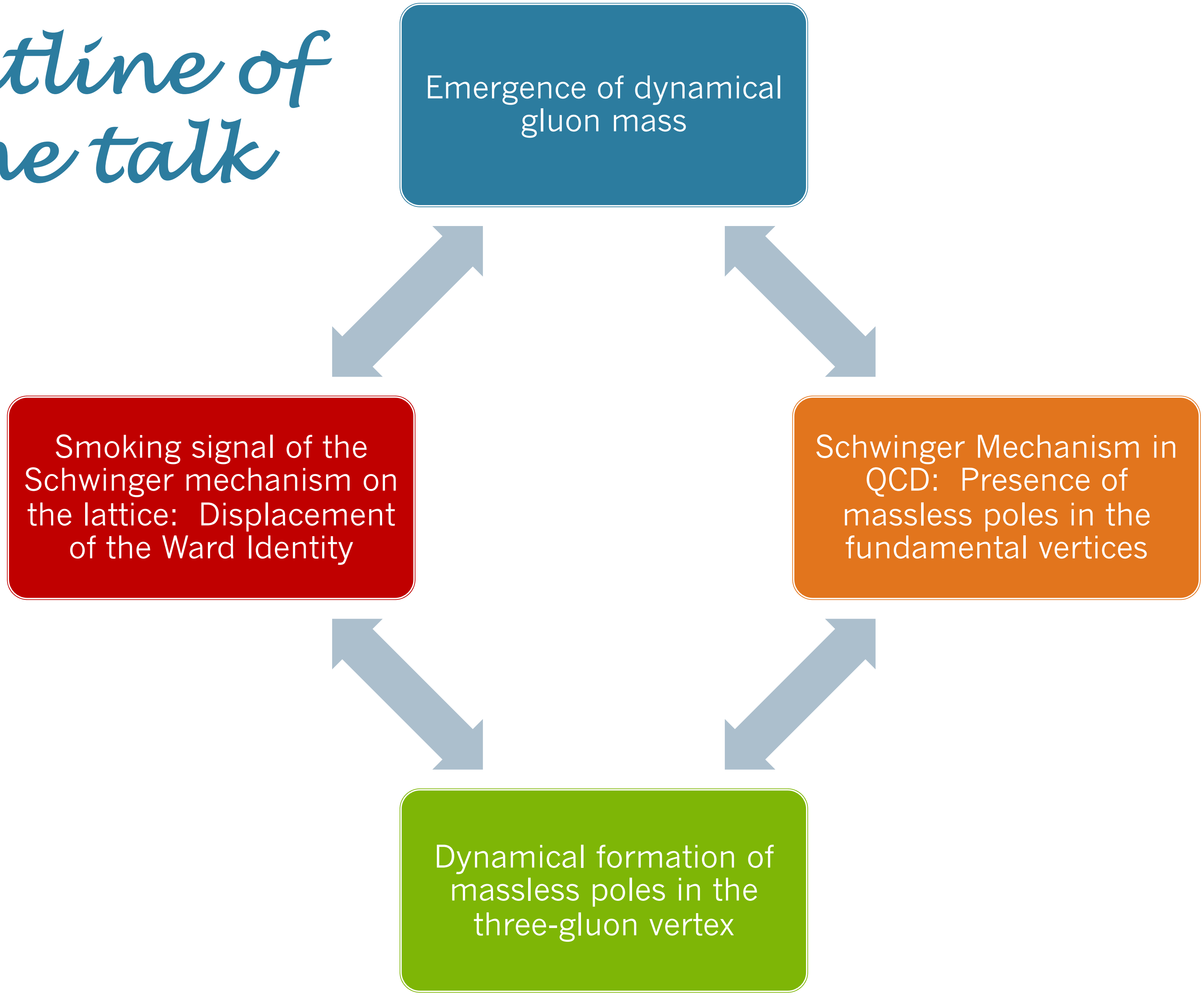
Light Cone, September 18-22, 2023, CBPF, Rio de Janeiro, Brazil

Outline of the talk



QCD Lagrangian

- The gauge fields (gluons) are masless at the level **QCD** lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}G^{\mu\nu}_a G^a_{\mu\nu} + \frac{1}{2\xi}(\partial^\mu A^\mu_a)^2 - \bar{c}^a(\partial^\mu D^{ac}_\mu)c^c + \mathcal{L}_{quarks}$$

Gauge fixing

Ghost

- Gluonic field strength tensor

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

- Covariant derivative in adjoint $D^{ab}_\mu = \partial_\mu \delta^{ac} + g f^{amb} A^m_\mu$

- A mass term ($m^2A_\mu^2$) for the gluon is forbidden by gauge and BRST symmetry

- Properly regularized perturbation theory cannot generate a gluon mass at any finite order.

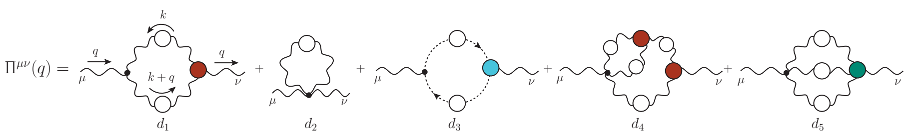
$$\int \frac{d^d k}{k^2} = 0$$

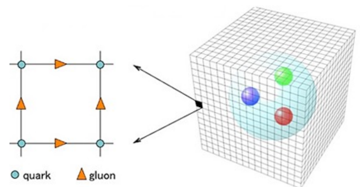
However, gluon self-interactions can generate a nonperturbative dynamical mass.

J. M. Cornwall, Phys. Rev. D26, 1453 (1982)

- First sign of emergence of a gluon mass appears in the *gluon propagator.*
- *J.M. Cornwall proposed in 82 that the gluon propagator is finite at the origin.*

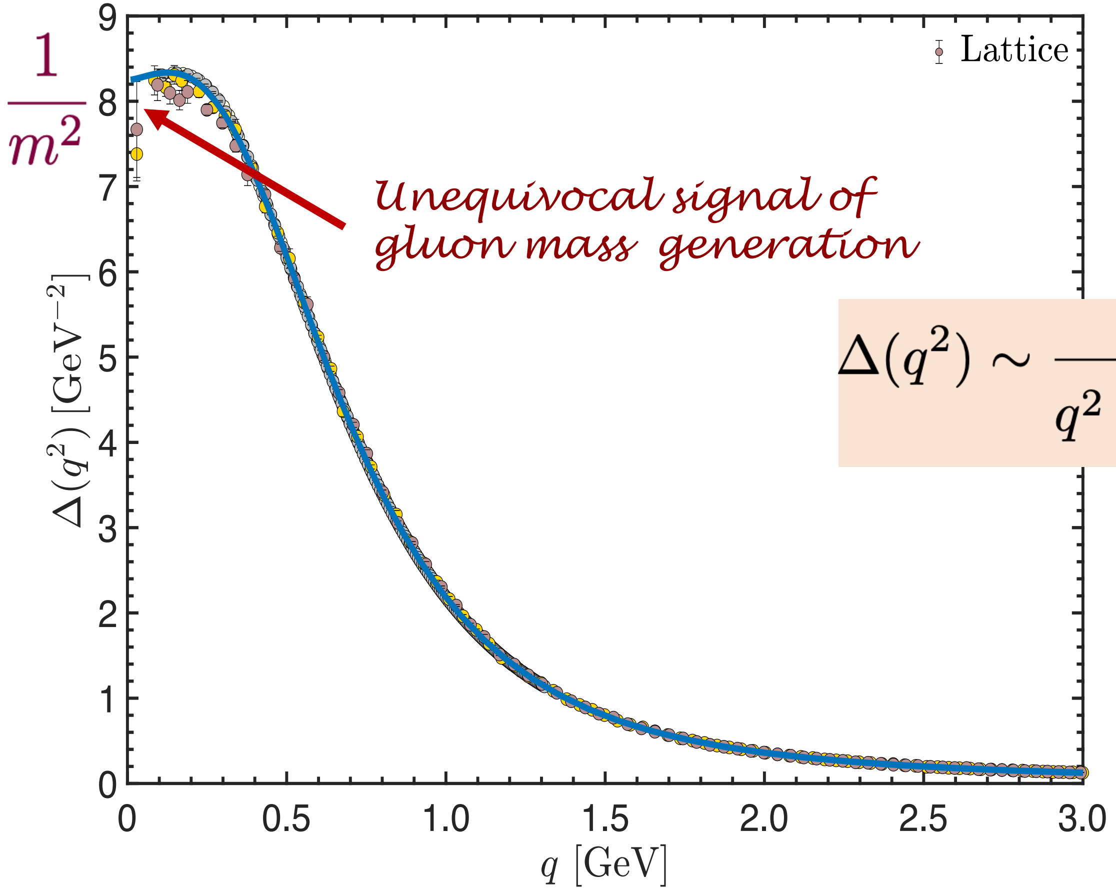
Nonperturbative methods are required
(e.g. Schwinger-Dyson equations and lattice)





The gluon propagator (in Landau gauge)

$$\Delta^{\mu\nu}(q) = \left[g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right] \Delta(q^2)$$



$$\Delta(q^2) \sim \frac{1}{q^2 + m^2 + q^2 c \ln\left(\frac{q^2+m^2}{\Lambda^2}\right) + \cdots}$$

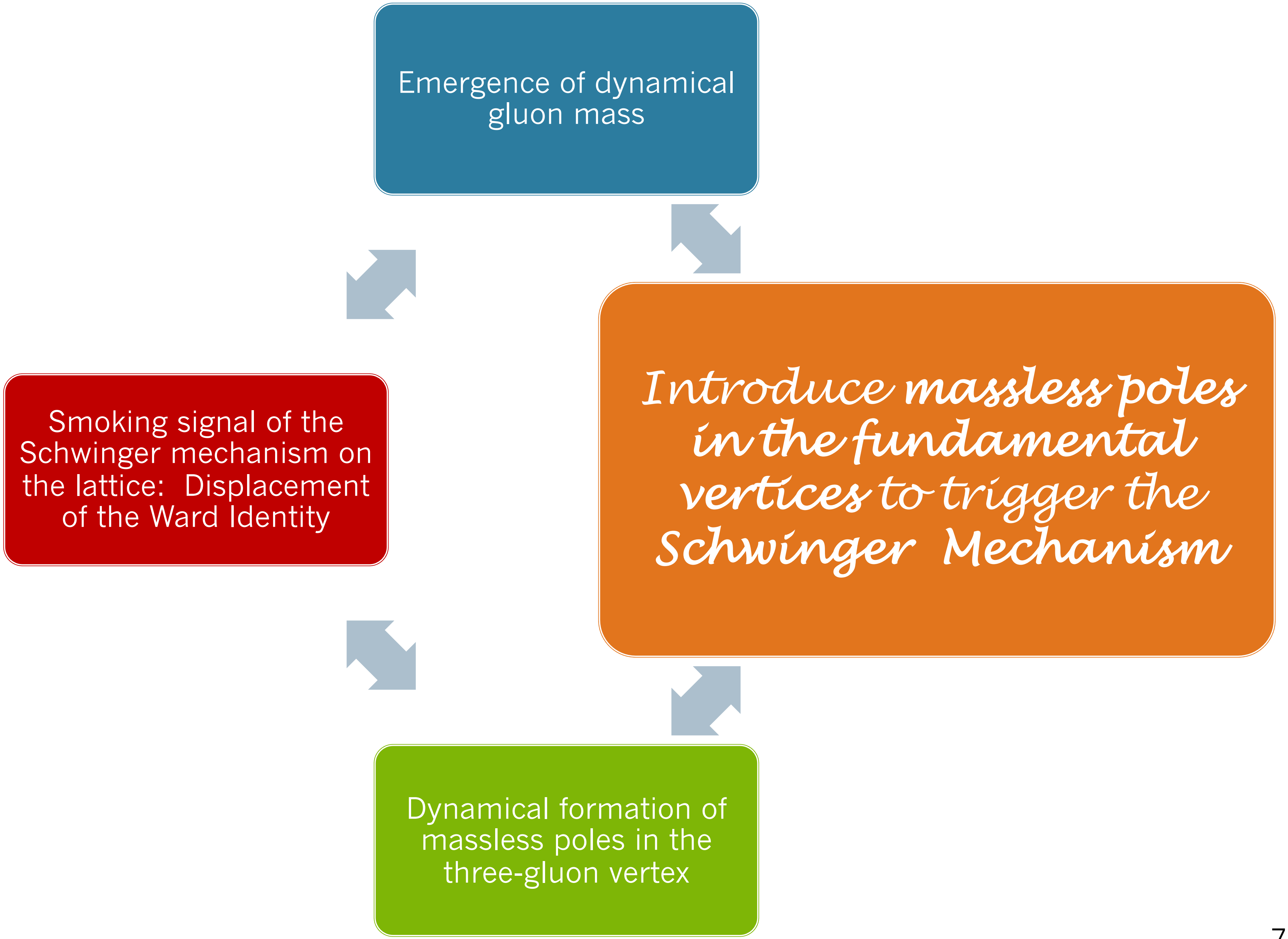
$q^2 = 0$

$$\Delta(0)^{-1} = m^2$$

I.L.Bogolubsky, et al , PoS **LAT2007**, 290 (2007)
A.Cucchieri and T.Mendes, PoS **LAT2007**, 297 (2007)
O.Oliveira and P.J.Silva, PoS **QCD-TNT09**, 033 (2009)

- *Dynamical mass generation is an emergent phenomena, it must be explained without modifying the Lagrangian*

Question 1: How can one generate a gluon mass (saturation of the gluon propagator at zero momentum) without breaking the gauge symmetry?


Emergence of dynamical gluon mass
Introduce massless poles in the fundamental vertices to trigger the Schwinger Mechanism
Dynamical formation of massless poles in the three-gluon vertex
Smoking signal of the Schwinger mechanism on the lattice: Displacement of the Ward Identity

Schwinger Mechanism

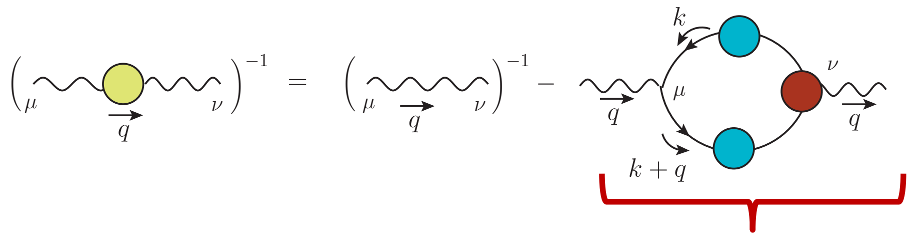
J. S. Schwinger, Phys. Rev.125, 397 (1962);
Phys.Rev.128, 2425 (1962).

Gauge invariance and mass

A gauge boson may acquire a mass, even if the gauge symmetry forbids a mass term at the level of the fundamental Lagrangian, provided that its vacuum polarization function develops a pole at zero momentum transfer.

Schwinger Mechanism

- Schwinger-Dyson Equation for the gauge boson



$\mathbf{\Pi}(q^2)$ ← Vaccum polarization

$$\Delta^{-1}(q^2) = q^2[1 + \mathbf{\Pi}(q^2)]$$

- If the vaccum polarization has a pole in $q^2 = 0$ with positive residue m^2, i.e.

$$\mathbf{\Pi}(q^2) = \frac{m^2}{q^2}$$

Massless poles

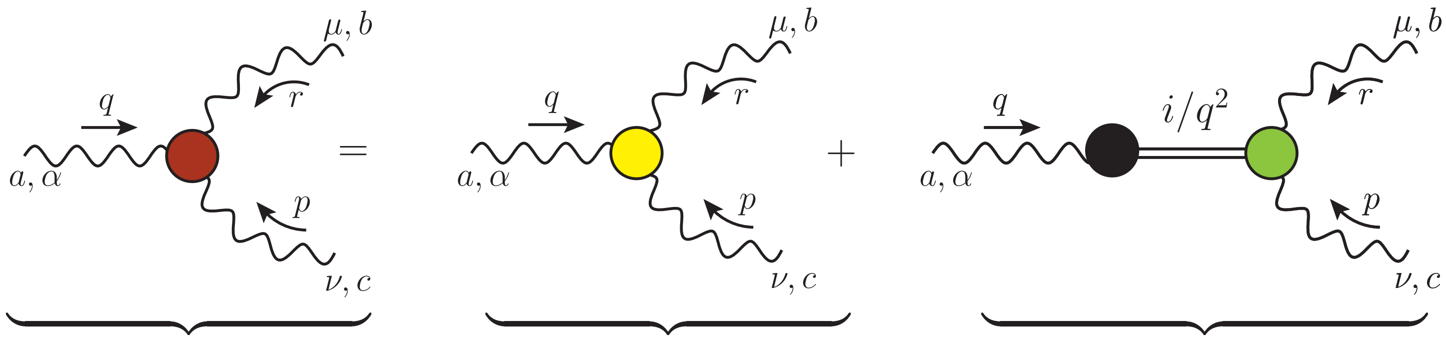
- Then

$$\Delta^{-1}(q^2) = q^2\left[1 + \frac{m^2}{q^2}\right] = q^2 + m^2 \quad \xrightarrow[q^2 \to 0]{} \quad \Delta^{-1}(0) = m^2$$

Dynamical mass generation requires the emergence of massless poles in the vaccum polarization → coming from the vertices (nonpertubative origin)

Vertices with massless poles in QCD

- To trigger the Schwinger mechanism, we need the presence of massless poles in the full three gluon vertex

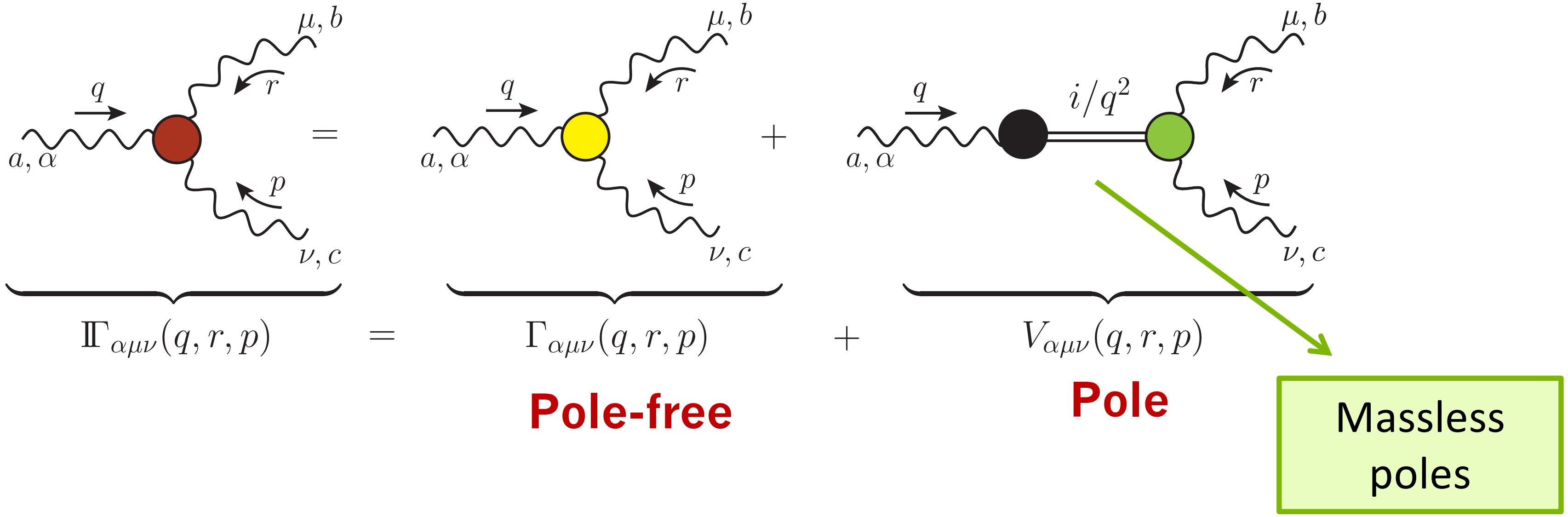


$$\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,p) = \Gamma_{\alpha\mu\nu}(q,r,p) + V_{\alpha\mu\nu}(q,r,p)$$

Pole-free

Vertices with massless poles in QCD

- To trigger the Schwinger mechanism, we need the presence of massless poles in the full three gluon vertex



Vertices with massless poles in QCD

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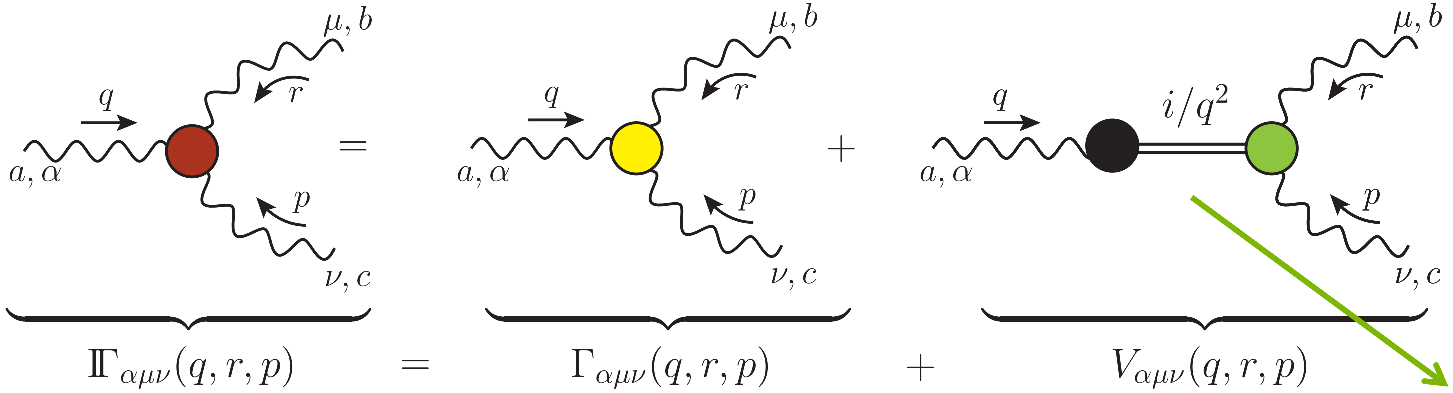


Pole-free **Pole**

Massless poles

$$V_{\alpha\mu\nu}(q,r,p) = \frac{q_\alpha}{q^2} C_{\mu\nu}(q,r,p)$$
$$= \frac{q_\alpha}{q^2}\left[C_1 g_{\mu\nu} + \cdots\right]$$

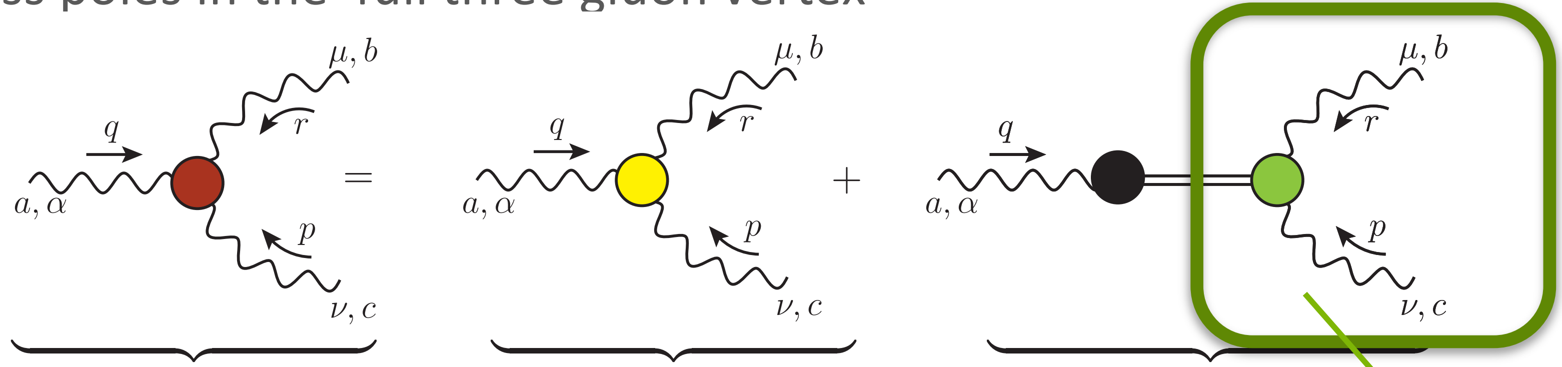
$$P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p) V_{\alpha\mu\nu}(q,r,p) = 0$$

$$P_{\mu\nu}(q) = \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right]$$

Longitudinally coupled
Drops out when embedded in a S-matrix element and also in transversely projected Green's functions

Vertices with massless poles in QCD

- To trigger the Schwinger mechanism, we need the presence of massless poles in the full three gluon vertex



$$\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,p) = \Gamma_{\alpha\mu\nu}(q,r,p) + V_{\alpha\mu\nu}(q,r,p)$$

Pole-free **Pole**

BSE amplitude
gluon-gluon massless scalar

$$V_{\alpha\mu\nu}(q,r,p) = \frac{q_\alpha}{q^2} C_{\mu\nu}(q,r,p)$$
$$= \frac{q_\alpha}{q^2}\left[C_1 g_{\mu\nu} + \cdots\right]$$

$$P^{\alpha}_{\alpha'}(q) P^{\mu}_{\mu'}(r) P^{\nu}_{\nu'}(p) V_{\alpha\mu\nu}(q,r,p) = 0$$

$$P_{\mu\nu}(q) = \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right]$$

Longitudinally coupled
Drops out when embedded in a S-matrix element and also in transversely projected Green's functions

Question 2: How the QCD dynamics generates the massless poles which appear in the fundamental vertices?

Emergence of dynamical gluon mass

Smoking signal of the Schwinger mechanism on the lattice: Displacement of the Ward Identity

Schwinger Mechanism in QCD: Presence of massless poles in the fundamental vertices

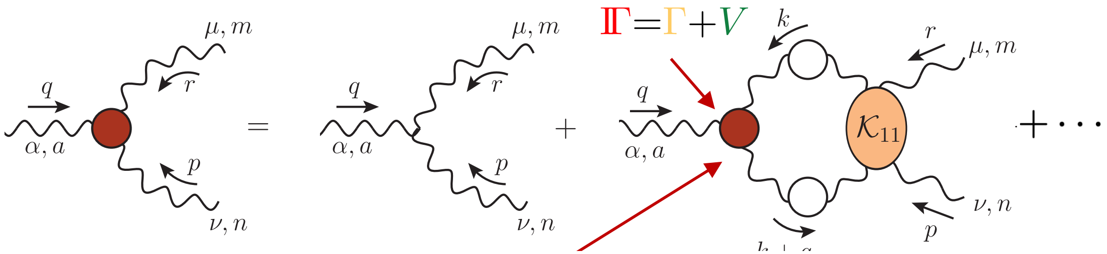
The QCD interaction should have enough strengthen to generate them.

Bethe-Salpeter equation governs their formation

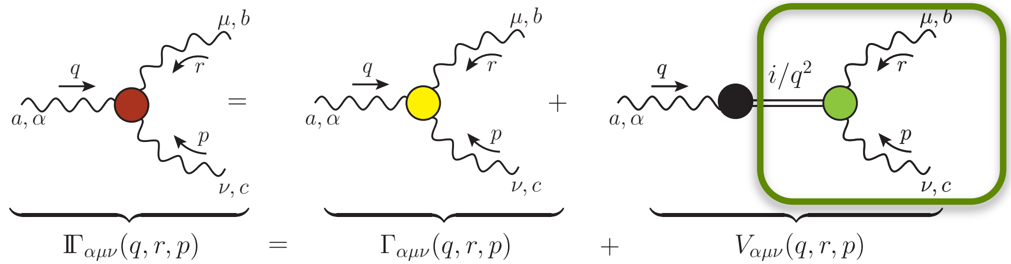
Dynamical equation for the massless pole

Equation for
the full vertex

$$\mathbb{I}\Gamma_\mu(q,r,p)$$



Substitute:



$$V_{\alpha\mu\nu}(q,r,p) = \frac{q_\alpha}{q^2}\left[C_1 g_{\mu\nu}\cdots\right]$$

Take the limit

$$q \to 0$$

$$\lim_{q\to 0} C_1(q,r,p) = 2(q\cdot r)\left[\frac{\partial C_1(q,r,p)}{\partial p^2}\right]_{q=0} + \mathcal{O}(q^2)$$

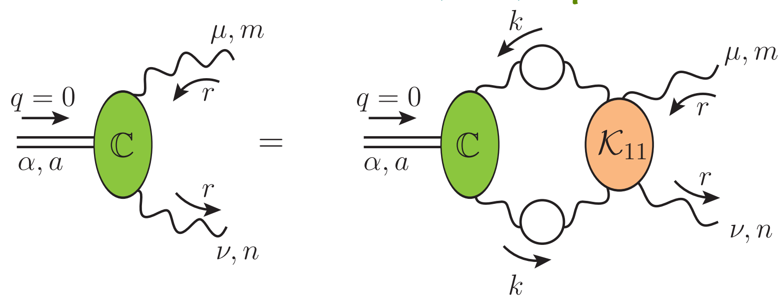
$\mathbb{C}(q^2)$ *Residue function*

Bose symmetry

$$C_1(q,r,p) = -C_1(q,p,r)$$

$$C_1(0,r,-r) = 0$$

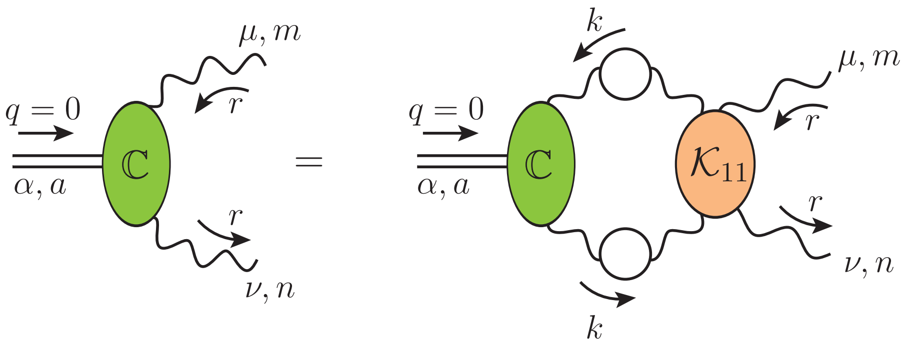
We obtain the BSE for the massless pole!



$$\mathbb{C}(r^2) = -\alpha_s \int_k \mathbb{C}(k^2)\Delta^2(k)\mathcal{K}_{11}(k,r)$$

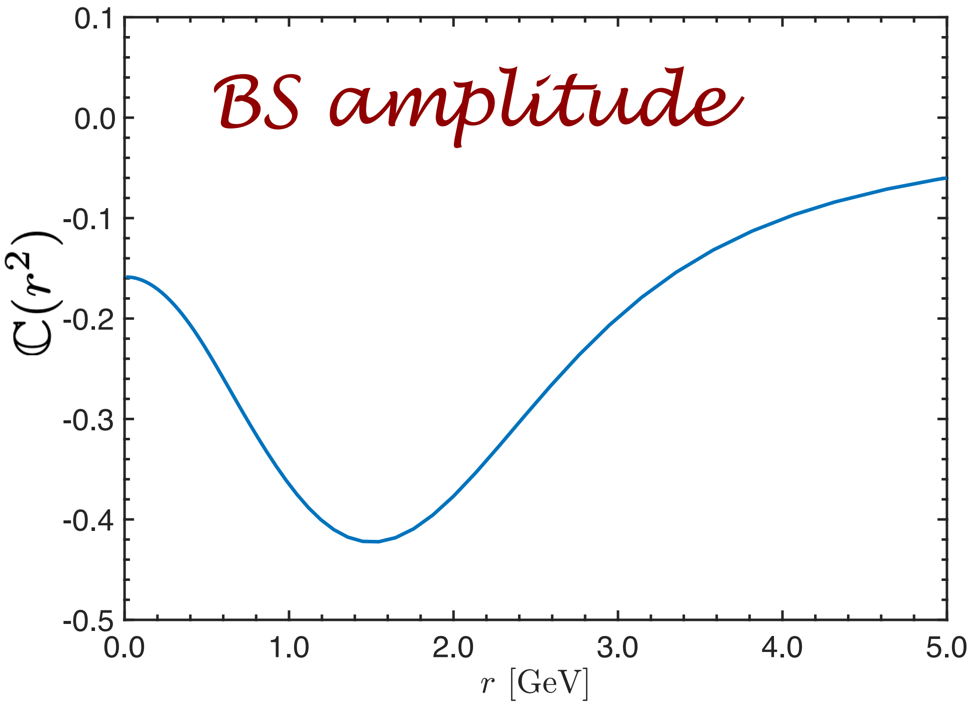
BSE describing the dynamical formation of massless pole

Dynamical BSE for the massless pole



$$\mathbb{C}(r^2) = -\alpha_s \int_k \mathbb{C}(k^2)\Delta^2(k)\mathcal{K}_{11}(k, r)$$

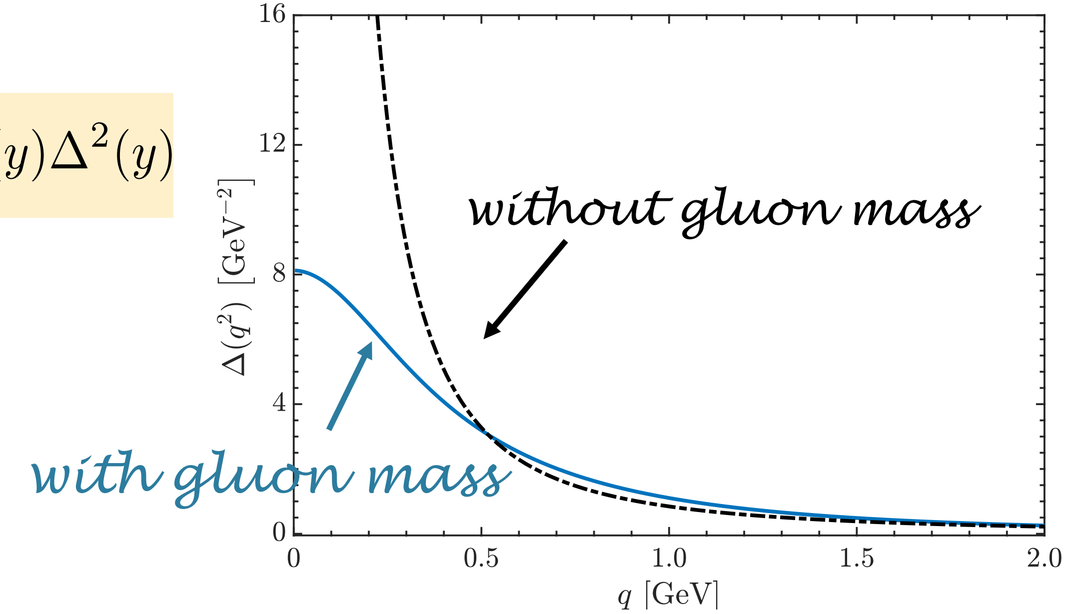
- *Eigenvalue problem*
- Solution when $\alpha_s \approx 0.3$ @ $\mu = 4.3\,\mathrm{GeV}$



A.C.A, D.Binosi, C.T.Figueiredo and J.Papavassiliou, Eur. Phys. J. C78 (2018) no.3, 181

- *The BS amplitude $\mathbb{C}(r^2)$ is directly connected with the gluon mass !*

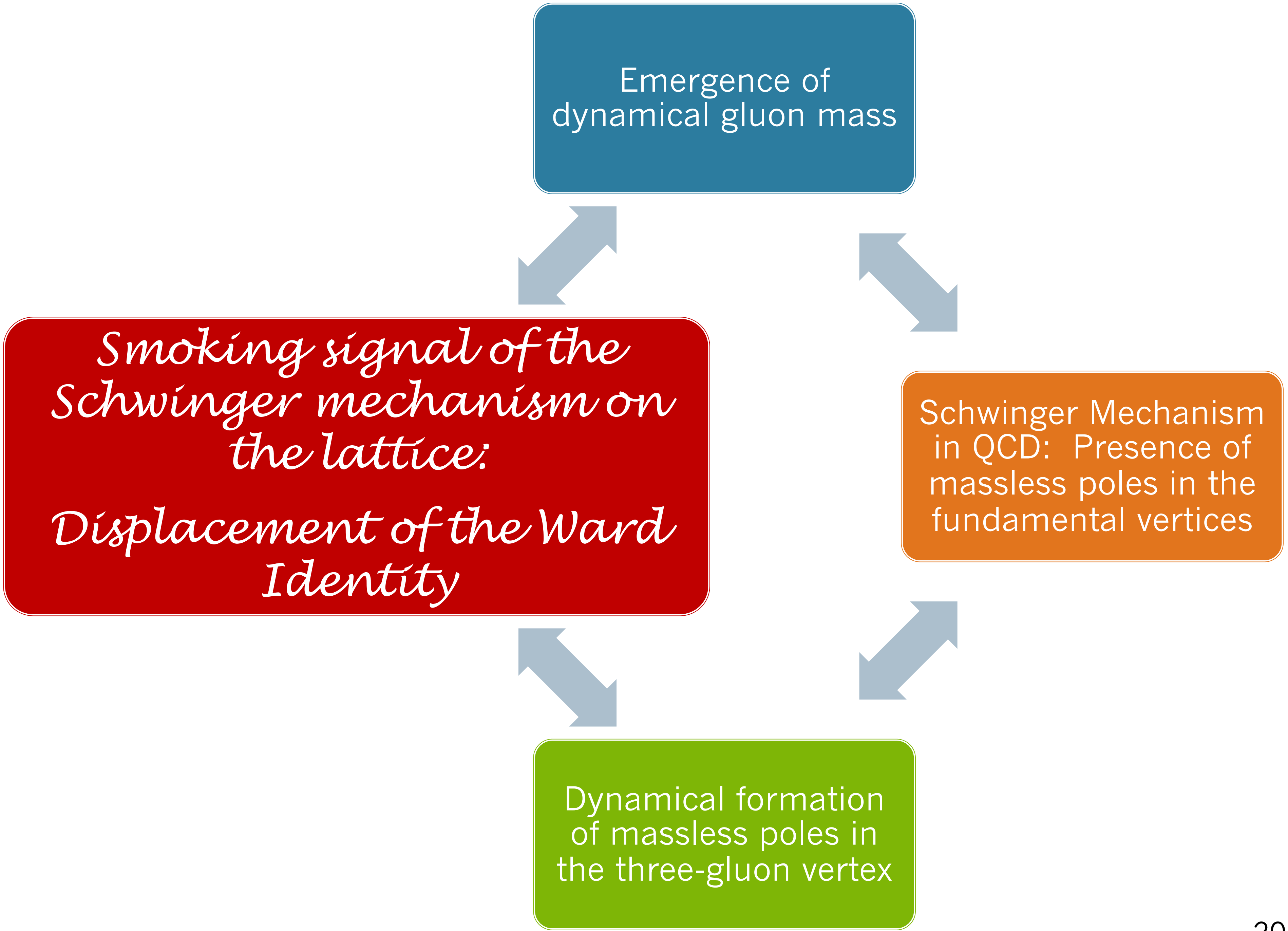
$$\Delta^{-1}(0) = m^2 = -\frac{3C_{\rm A}\alpha_s}{8\pi}\int_0^\infty dy\, y^2\mathbb{C}(y)\Delta^2(y)$$



- Gluon propagator acquires a mass – due to presence of the massless poles!!! (self-stabilizing effect).

The theory solves its infrared problems

Question 3: *Is there a way to confirm that the action of the Schwinger Mechanism in QCD using the lattice results?*


Emergence of dynamical gluon mass
Schwinger Mechanism in QCD: Presence of massless poles in the fundamental vertices
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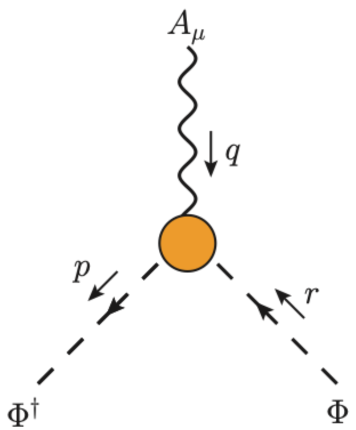
Smoking signal of the Schwinger mechanism on the lattice:
Displacement of the Ward Identity

- The Ward identities undergo *a finite displacement* with respect to the result found in the absence of the Schwinger Mechanism.
- The quantity that determines this effect is $\mathbb{C}(q^2)$
- Let's see how the *signal of the displacement of the Ward identity* emerges in a simple context: Scalar QED.

Scalar QED



$$\Gamma^\mu(q, r, -p)$$

Ward-Takahashi identity

$$q^\mu \Gamma_\mu(q, r, -p) = \mathcal{D}^{-1}(p^2) - \mathcal{D}^{-1}(r^2)$$

Full scalar propagator

Schwinger mechanism off

(without the presence of poles)

Ward-Takahashi identity

$$q^\mu \Gamma_\mu(q,r,-p) = \mathcal{D}^{-1}(p^2) - \mathcal{D}^{-1}(r^2)$$

$q \to 0$
$p \to r$

Taylor expansion

Ward identity

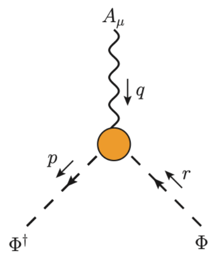
$$\Gamma_\mu(0,r,-r) = \frac{\mathcal{D}^{-1}(r^2)}{\partial r^\mu}$$

Tensorial decomposition (Soft photon limit)

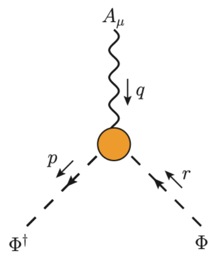
$$\Gamma^\mu(0,r,-r) = L^\star_{sg}(r^2) r^\mu$$

$$L^\star_{sg}(r^2) = 2\frac{\partial \mathcal{D}^{-1}(r^2)}{\partial r^2}$$

Scalar QED



Scalar QED



Schwinger mechanism off

(without the presence of poles)

Ward Takahashi identity

$$q^\mu \Gamma_\mu(q,r,-p) = \mathcal{D}^{-1}(p^2) - \mathcal{D}^{-1}(r^2)$$

$q \to 0$

$p \to r$

Taylor expansion

Ward identity

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$$L^\star_{sg}(r^2) = 2\frac{\partial \mathcal{D}^{-1}(r^2)}{\partial r^2}$$

Schwinger mechanism on

(with the presence of poles)

$$\mathrm{I\Gamma}_\mu(q,r,-p) = \underbrace{\Gamma_\mu(q,r,-p)}_{\text{pole-free}} + \frac{q_\mu}{q^2}\underbrace{C(q,r,-p)}_{\text{pole}}$$

The Ward-Takahashi identity **does not change**

$$q^\mu \mathrm{I\Gamma}_\mu(q,r,-p) = q^\mu \Gamma_\mu(q,r,-p) + C(q,r,-p) = \mathcal{D}^{-1}(p^2) - \mathcal{D}^{-1}(r^2)$$

$q \to 0$

$p \to r$

Taylor expansion

Ward identity

$$\Gamma_\mu(0,r,-r) = \frac{\mathcal{D}^{-1}(r^2)}{\partial r^\mu} - 2r_\mu \underbrace{\left[\frac{\partial C(q,r,-p)}{\partial p^2}\right]_{q=0}}_{\mathbb{C}(r^2)}$$

Tensorial decomposition

$$\Gamma^\mu(0,r,-r) = L_{sg}(r^2) r^\mu$$

$$L_{sg}(r^2) = 2\frac{\partial \mathcal{D}^{-1}(r^2)}{\partial r^2} - \underbrace{2\mathbb{C}(r^2)}_{\text{displacement}}$$

$L^\star_{sg}(r^2)$

displacement

Displacement of the WI of the three-gluon vertex

- Now, let us apply the same procedure to the three gluon vertex which satisfies the Slavnov-Taylor identity

$$q^\alpha \Gamma_{\alpha\mu\nu}(q,r,p) + C_1(q,r,p)g_{\mu\nu} = F(q^2)\left[\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r,q,p)\right]$$

$q \to 0$

$p \to r$

Taylor expansion

$$L_{sg}(r^2) = L^\star_{sg}(r^2) + \mathbb{C}(r^2)$$

Lattice

Computed with inputs from the lattice

displacement

It is the BS amplitude!

where $$L^\star_{sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \left(\frac{d\Delta^{-1}(r^2)}{dr^2}\right)\right]$$

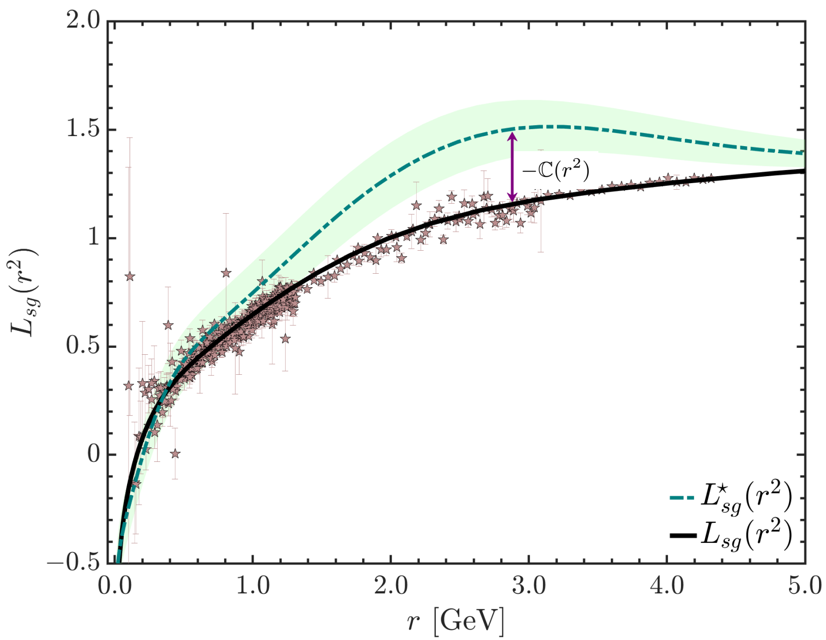
$$L_{sg}(r^2) = L^{\star}_{sg}(r^2) + \mathbb{C}(r^2)$$

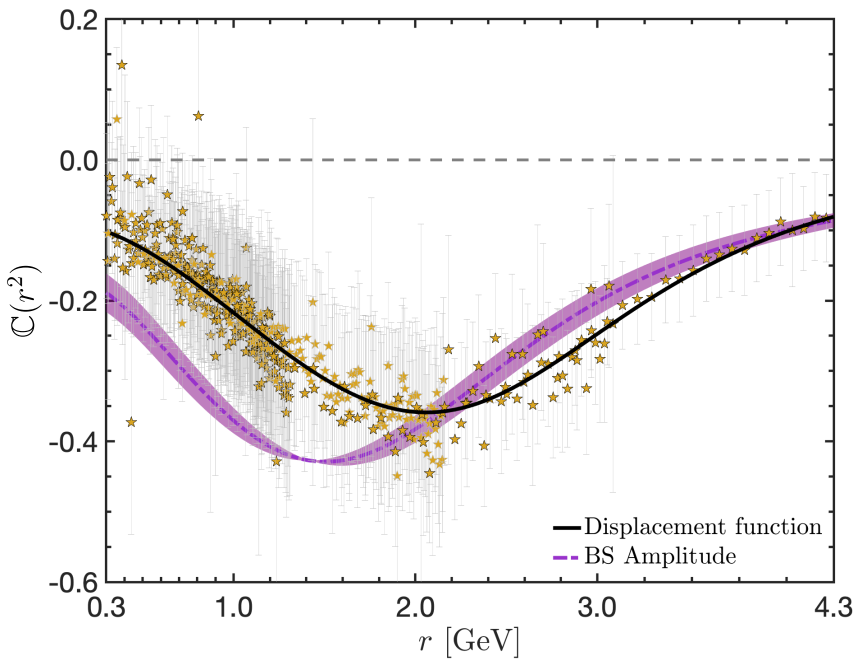
Lattice

Computed with inputs from the lattice

displacement

It is the BS amplitude!





A.C.A., D. Binosi, C.T. Figueiredo and J.Papavassiliou, Phys. Rev. D 94, no.4, 045002 (2016);
A.C.A., M.N. Ferreira, and J.Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022);
A.C.A., F.De Soto, M. N. Ferreira, J. Papavassiliou, F. Pinto-Gómez, C.D. Roberts, J. Rodríguez-Quintero, *Phys. Lett. B* 841 (2023) 137906.

Conclusions

- The apparent simplicity of the QCD Lagrangian conceals an enormous wealth of dynamical patterns, giving rise to a vast array of complex ***emergent phenomena.***

- Gluon self-interactions generate a ***dynamical mass scale*** in the gauge sector of QCD.

- Dynamics and symmetry are tightly intertwined:

  Displacement of the Ward identity ⟷ *BSE amplitude for pole formation*

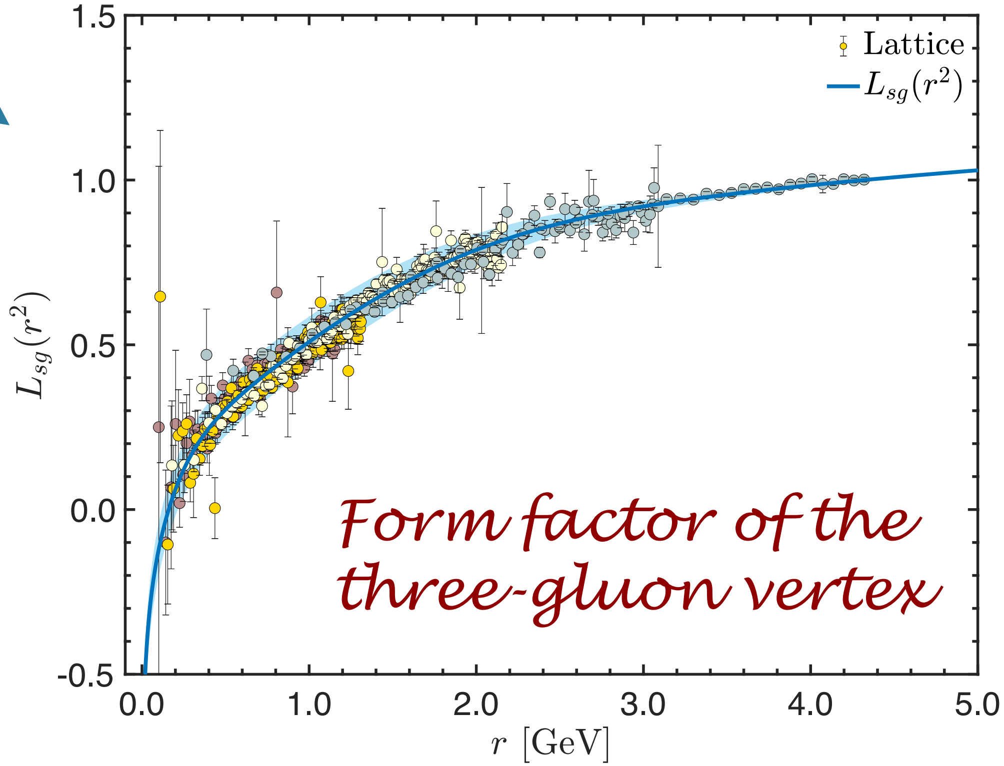
- ***Smoking gun signal*** corroborates the action of the ***Schwinger mechanism*** in QCD and the ***emergence of a dynamical gluon mass.***

- Backup

Displacement of the WI of the three-gluon vertex

$$\mathbb{C}(r^2) = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \left[\frac{d\Delta^{-1}(r^2)}{dr^2}\right]\right\},$$

displacement



A.C. A., F. De Soto, M.N.Ferreira, J.Papavassiliou, J.Rodríguez-Quintero, Phys. Lett. B818 (2021) 136352.

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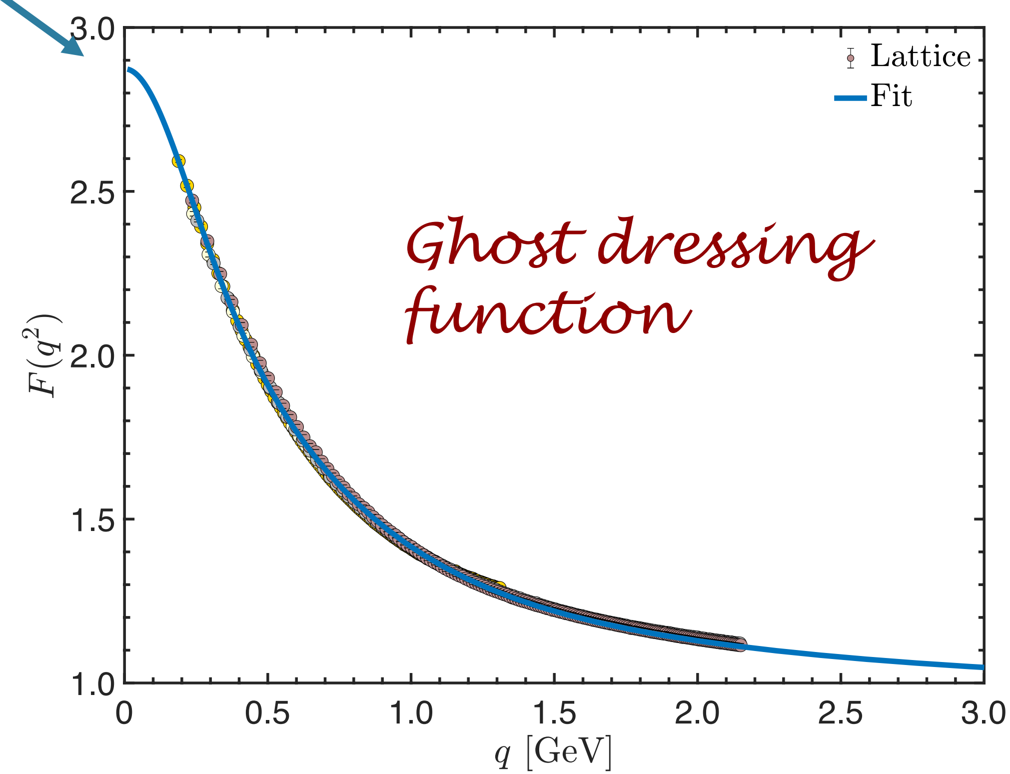
$$\underbrace{\mathbb{C}(r^2)}_{\text{displacement}} = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \left[\frac{d\Delta^{-1}(r^2)}{dr^2}\right]\right\},$$



$$D(q^2) = \frac{F(q^2)}{q^2}$$ Ghost propagator

I.L.Bogolubsky, et al , PoS **LAT2007**, 290 (2007)
A.Cucchieri and T.Mendes, PoS **LAT2007**, 297 (2007)
O.Oliveira and P.J.Silva, PoS **QCD-TNT09**, 033 (2009)

A.C.A., C.O. Ambrosio, F. De Soto, M.N. Ferreira, B.M. Oliveira, J.Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D 104 no.5, 054028, (2021)

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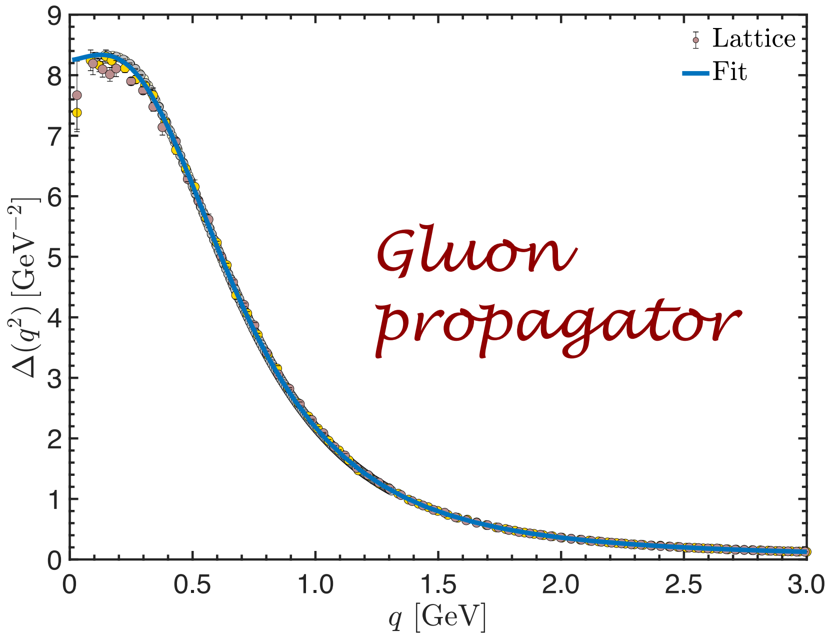

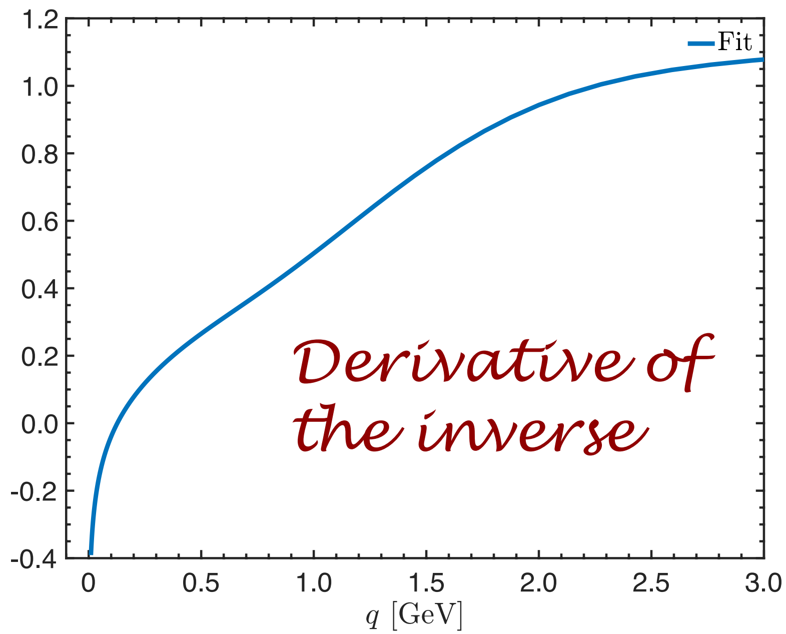

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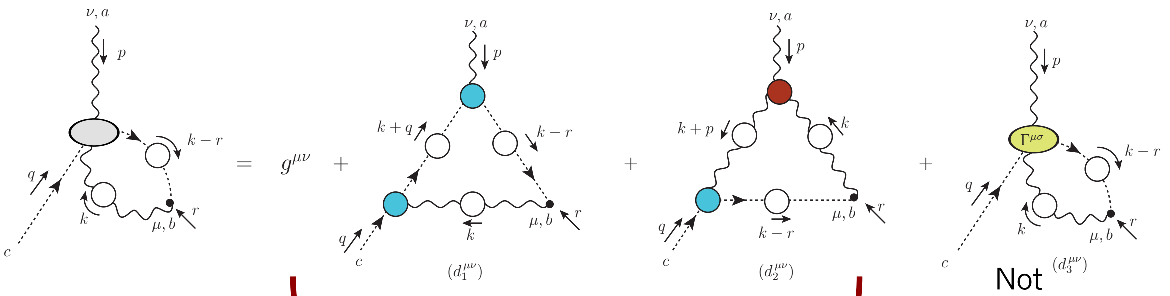
Displacement of the WI of the three-gluon vertex

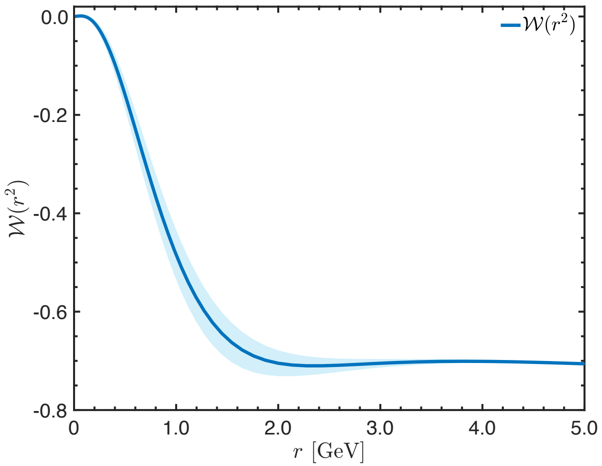
$$\mathbb{C}(r^2) = L_{sg}(r^2) - F(0)\left\{\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \left[\frac{d\Delta^{-1}(r^2)}{dr^2}\right]\right\},$$

displacement

partial derivative of the ghost-gluon kernel

- No lattice results for $\mathcal{W}(r^2)$
- Computed from its own SDE using lattice inputs





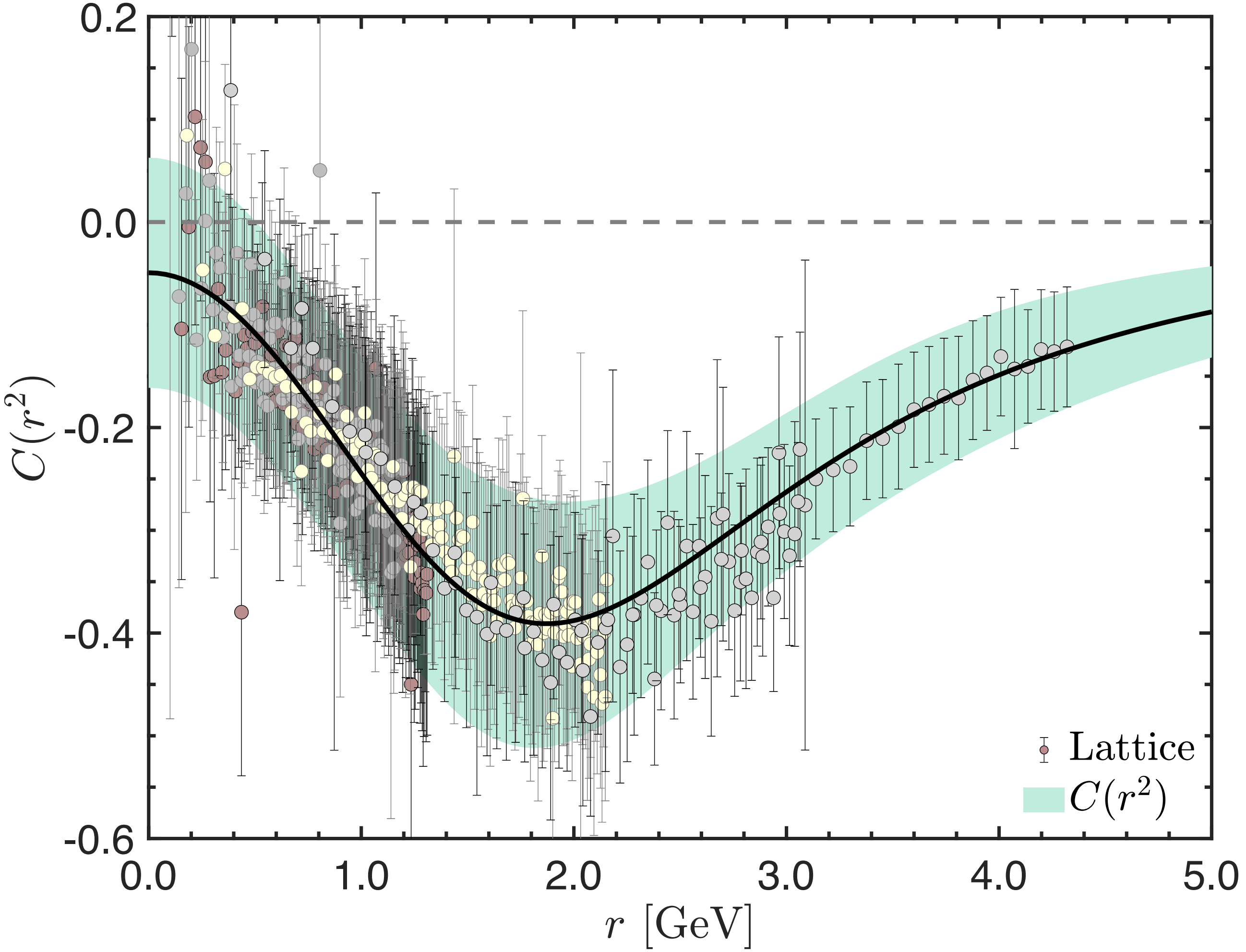
The result is dominated by a particular projection of the three-gluon vertex , evaluated on the lattice

$$\overline{\mathbb{\Gamma}}_{\alpha\mu\nu}(q,r,p) = P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)L_{sg}(s^2)$$
$$\times\left[(q-r)^{\nu'}g^{\mu'\alpha'} + (r-p)^{\alpha'}g^{\mu'\nu'} + (p-q)^{\mu'}g^{\nu'\alpha'}\right]$$

F. Pinto-Gómez, F. De Soto, M. N. Ferreira, J. Papavassiliou and J. Rodríguez-Quintero, *Phys.Lett.B* 838 (2023) 137737

Model-independent determination of the displacement function

- The lattice is "blind" to specific dynamical mechanisms



A.C.A., F.De Soto, M. N. Ferreira, J. Papavassiliou, F. Pinto-Gómez, C.D. Roberts and J. Rodríguez-Quintero, *Phys. Lett.B* 841 137906 (2023)