

Enhanced charm CP asymmetries from Final State Interactions

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Collaborators:

Ignacio Bediaga - CBPF (Rio)

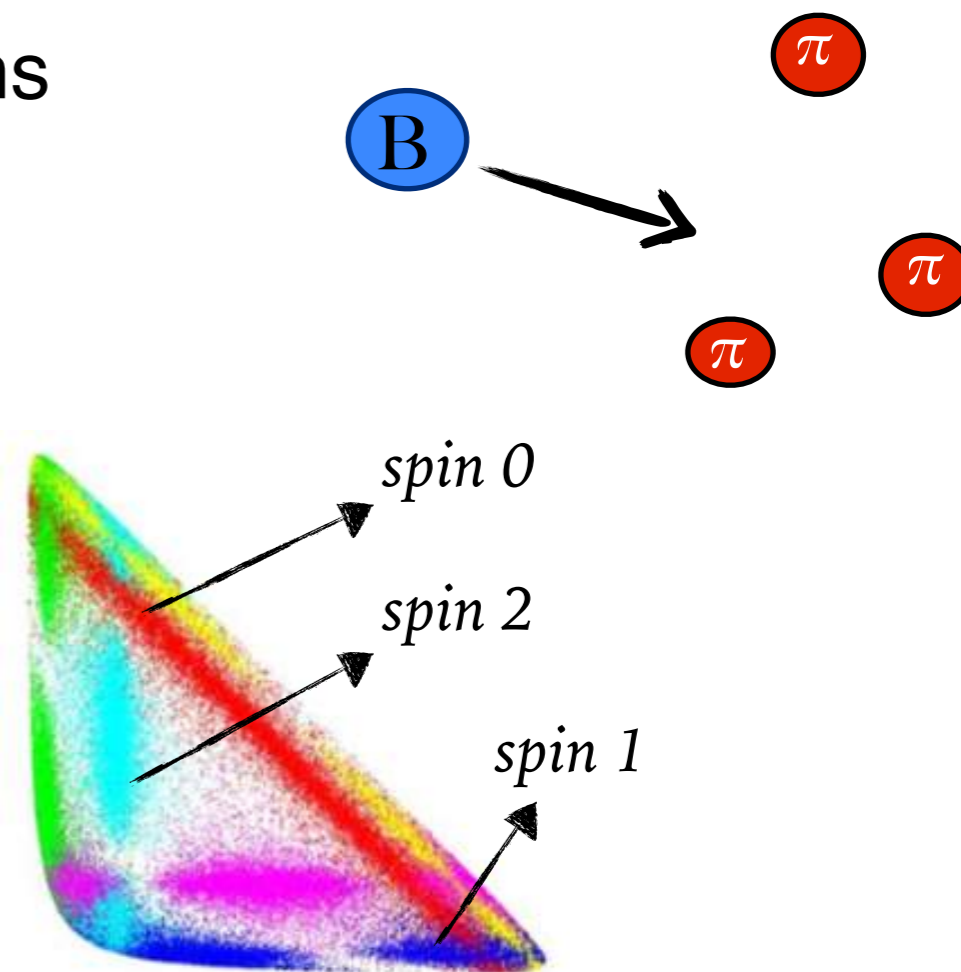
Tobias Frederico - ITA (S.J. dos Campos)

José Ramon Pelaez - UCM (Madrid)

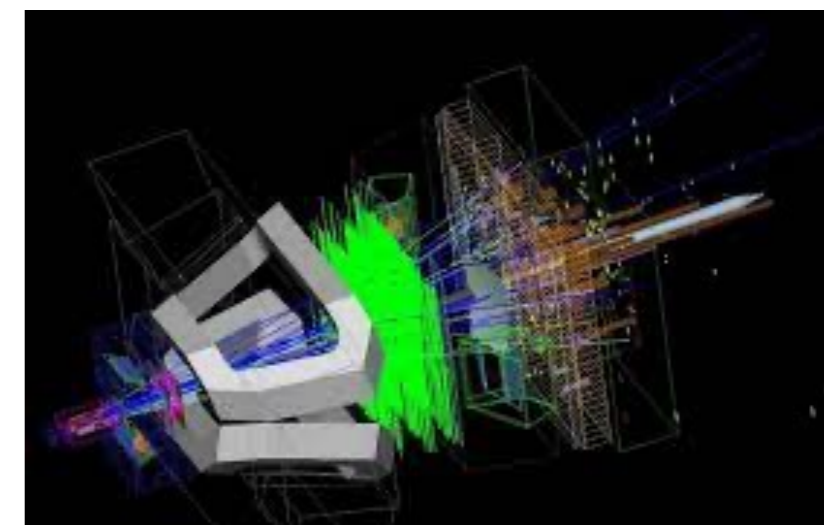


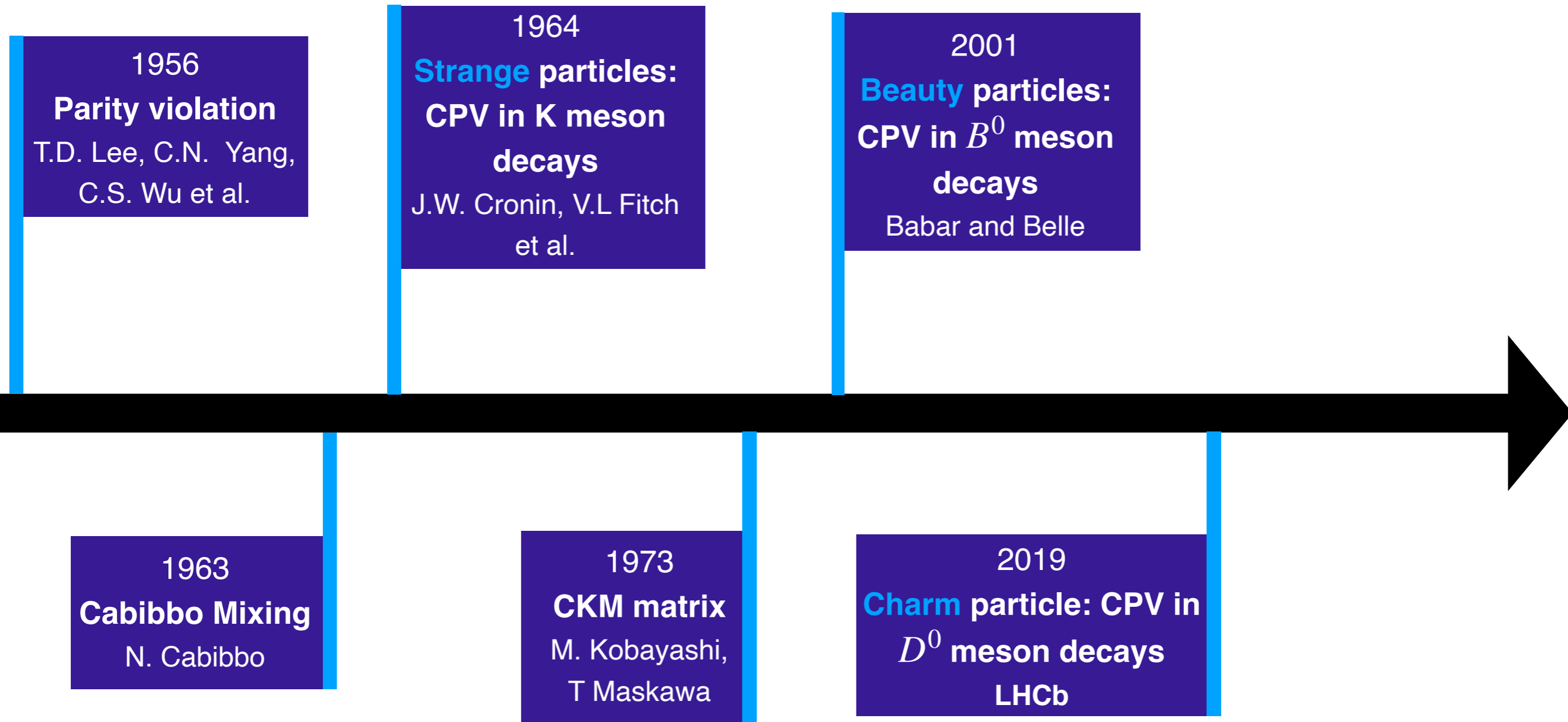
- Multi-body hadronic decays of B and D mesons
 - ↳ are sensitive to strong phases

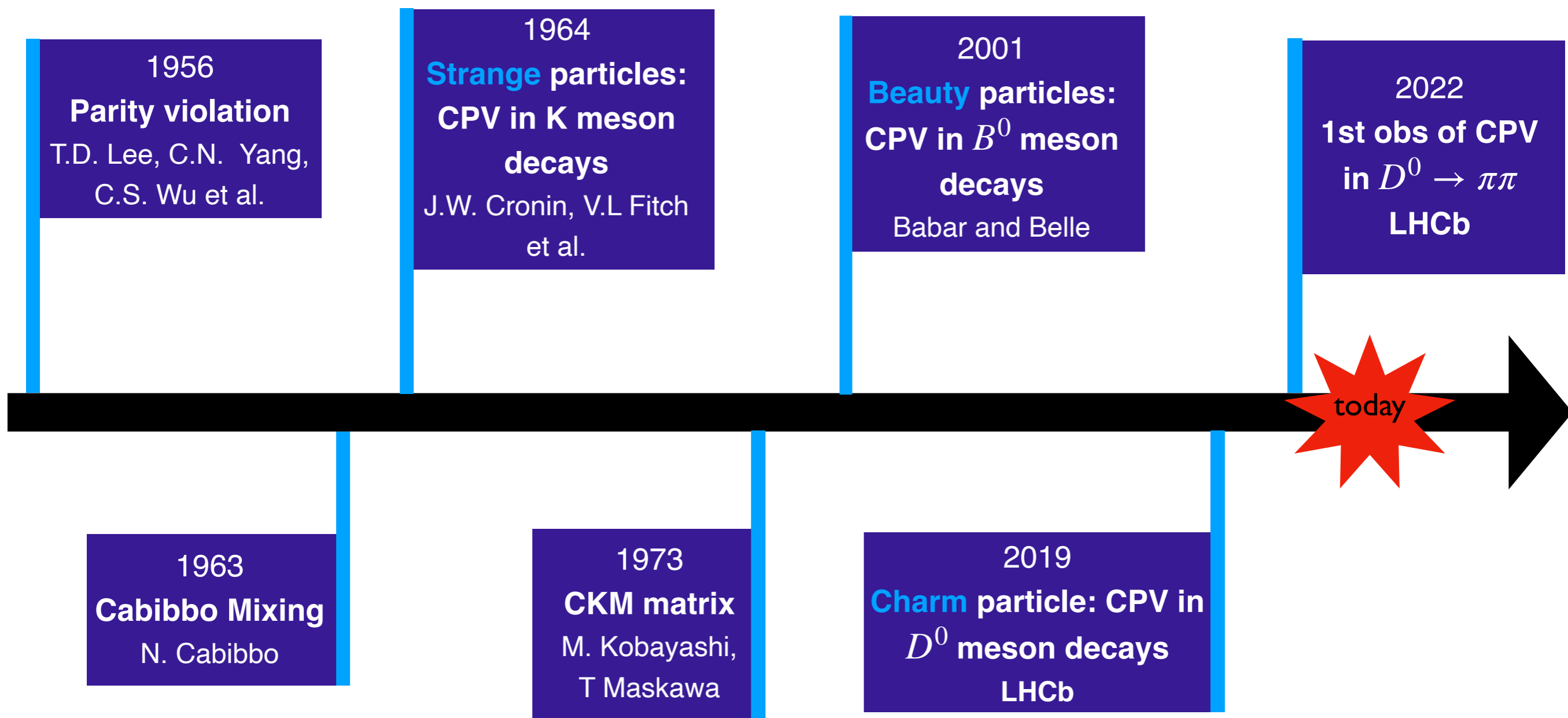
- signature of resonances on data
- spectroscopy → exotics, tetra-quarks, pentaquark, ...
- study CP violation



- high data sample from LHCb run II
 - more to come from LHCb, BelleII, BESIII
 - better models are needed (**challenge**)









LHCb
LHCb

detector

- CPV in $B^\pm \rightarrow h^\pm h^- h^+$

LHCb Run II 5.9 fb⁻¹

PRD D 108 012008 (2023)

- integrated

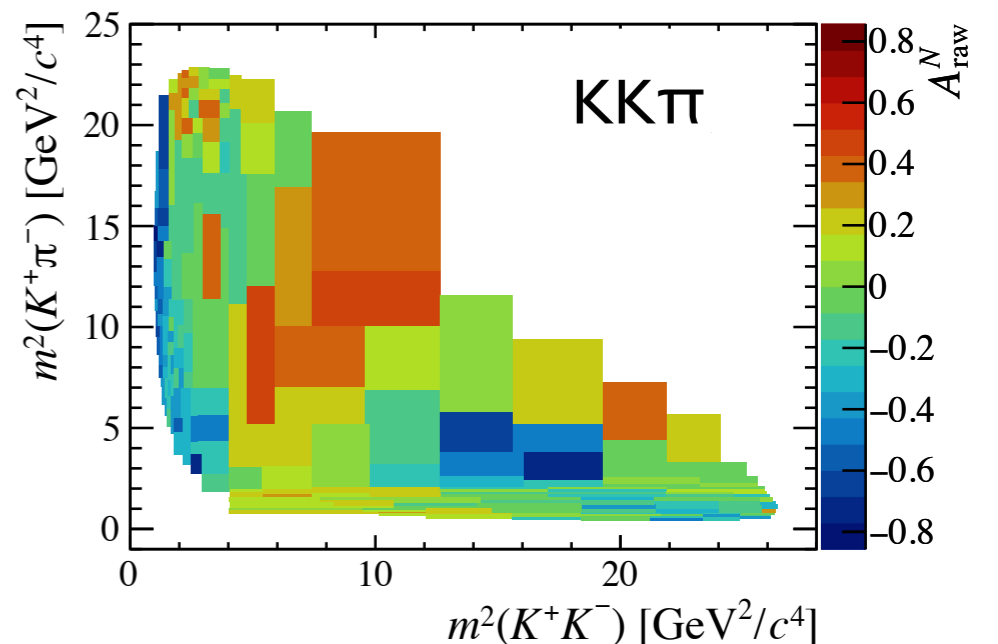
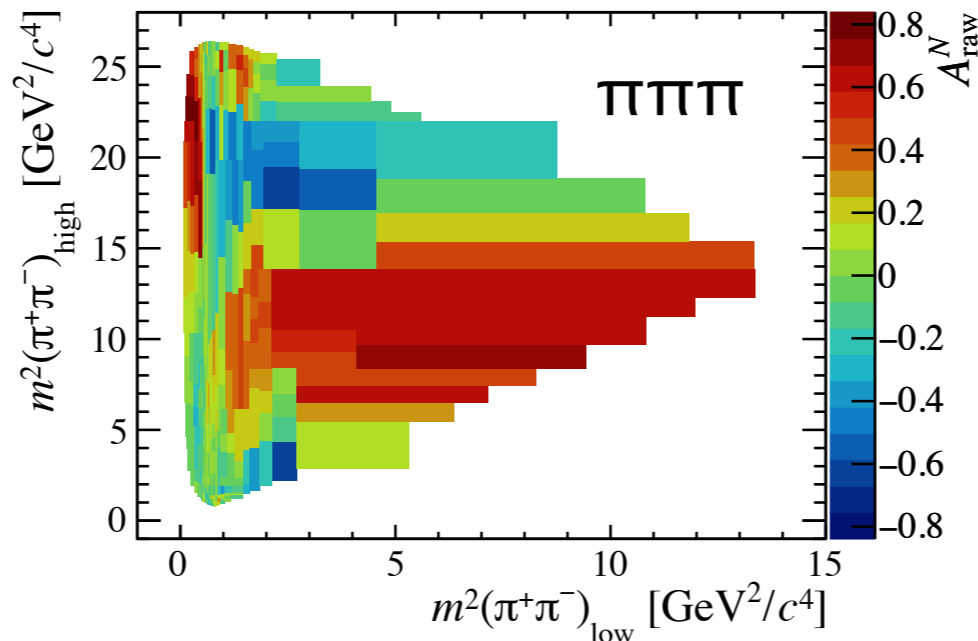
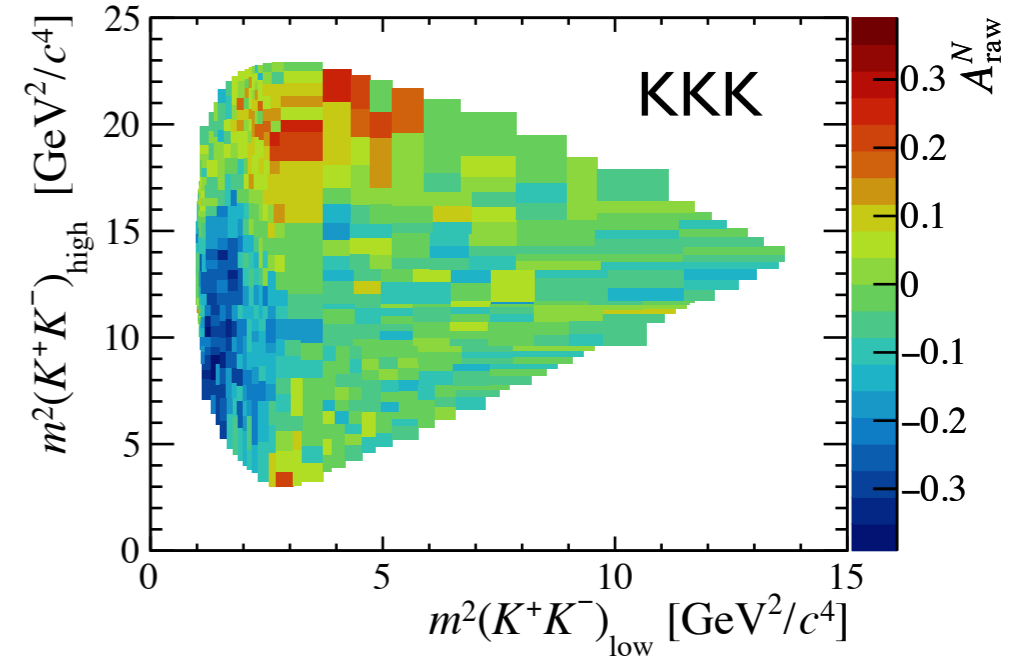
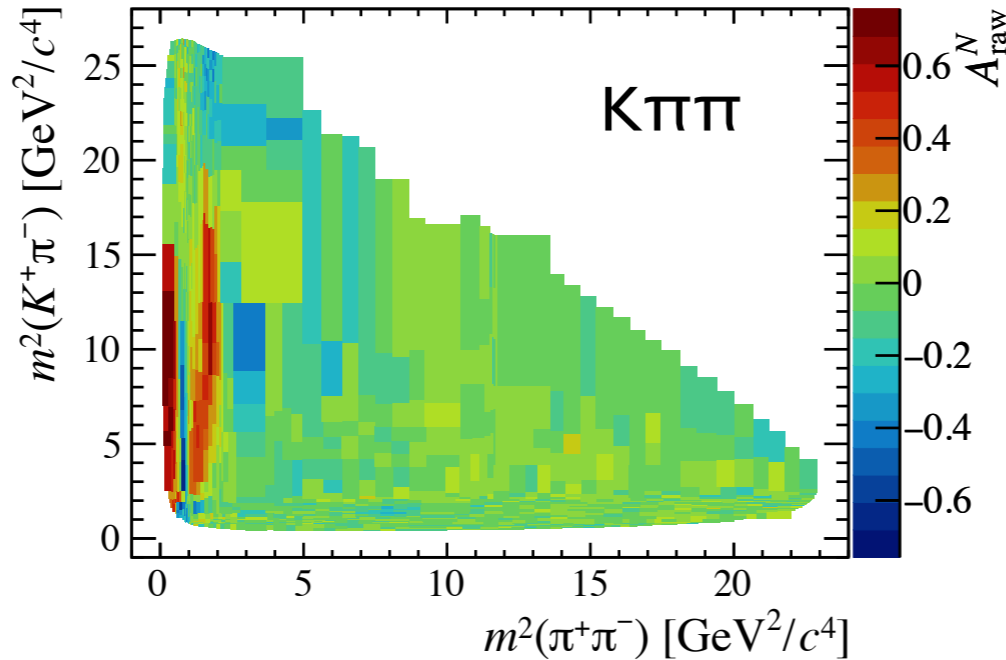
$$A_{CP}(B^\pm \rightarrow K^\pm \pi^+ \pi^-) = +0.011 \pm 0.002,$$

$$A_{CP}(B^\pm \rightarrow K^\pm K^+ K^-) = -0.037 \pm 0.002,$$

$$A_{CP}(B^\pm \rightarrow \pi^\pm \pi^+ \pi^-) = +0.080 \pm 0.004,$$

$$A_{CP}(B^\pm \rightarrow \pi^\pm K^+ K^-) = -0.114 \pm 0.007,$$

- giant CPV





- $\Delta A_{CP}^{\text{LHCb}} = A_{cp}(D^0 \rightarrow K^+K^-) - A_{cp}(D^0 \rightarrow \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$

Phys. Rev. Lett. 122, 211803 (2019)



- $$\Delta A_{CP}^{LHCb} = A_{cp}(D^0 \rightarrow K^+K^-) - A_{cp}(D^0 \rightarrow \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$$

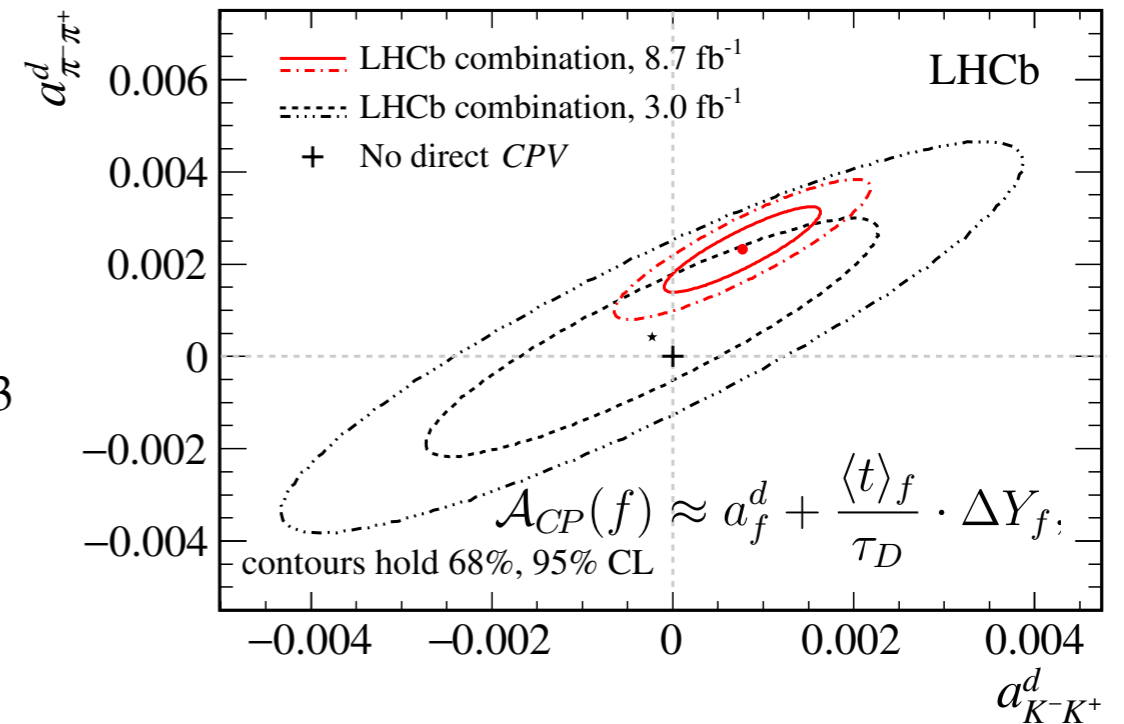
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→ direct CP asymmetry observation

- $$A_{CP}^{LHCb}(KK) = (0.77 \pm 0.57) \times 10^{-3}$$

↪
$$A_{CP}^{LHCb}(\pi\pi) = (2.32 \pm 0.61) \times 10^{-3}$$

[arXiv:2209.03179](https://arxiv.org/abs/2209.03179)





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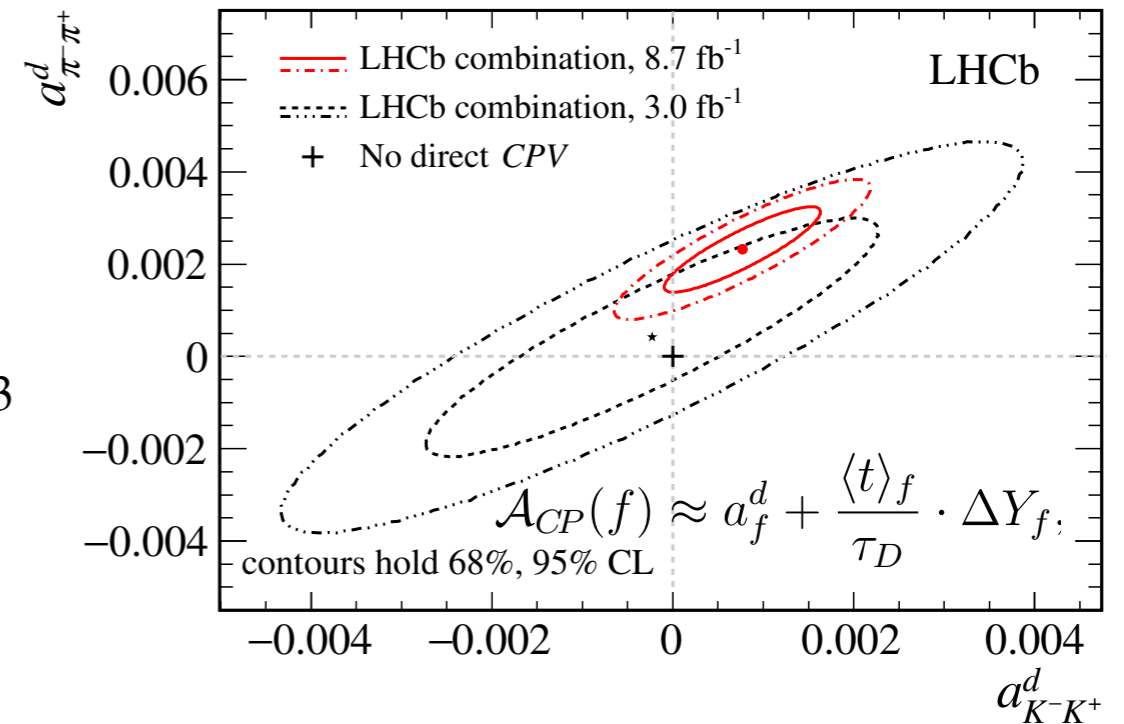
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- $$\text{QCD} \rightarrow \text{LCSR predictions } A_{CP} \approx 10^{-4} \text{ (1 order magnitude below)}$$

- $$\hookrightarrow \text{new physics? nonperturbative effects?!}$$

Khodjamirian, Petrov,
Phys. Lett. B 774, 235 (2017)



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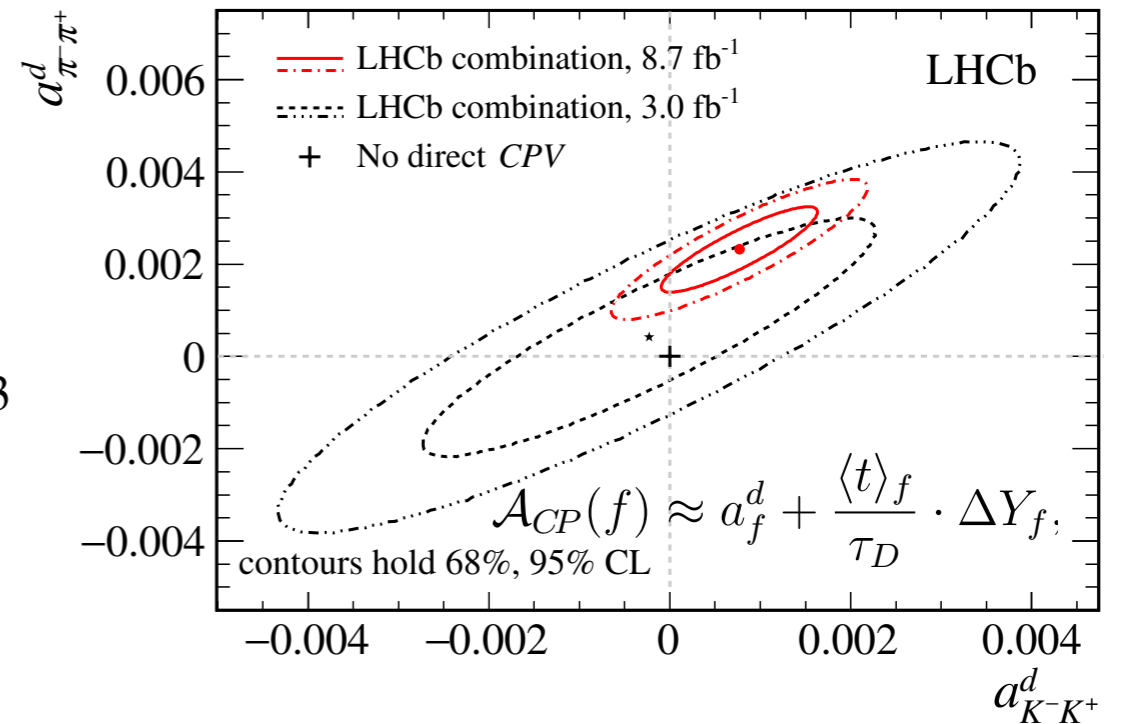
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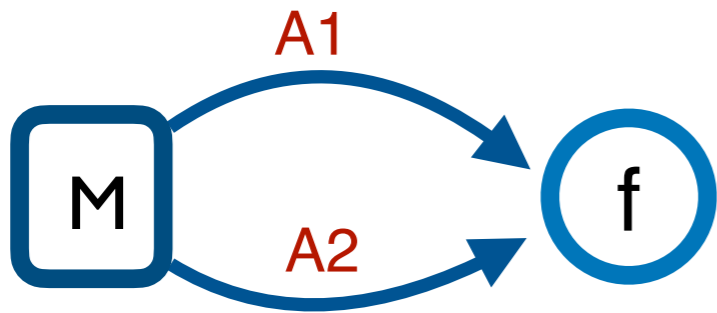
→ what about CPV on $D \rightarrow hhh$?

→ searches in many process at LHCb, BESIII, BelleII

↳ is expected soon with LHCb run II

- Interference effect

2 amplitudes: SAME final state, \neq strong (δ_i) and weak (ϕ_i) phases



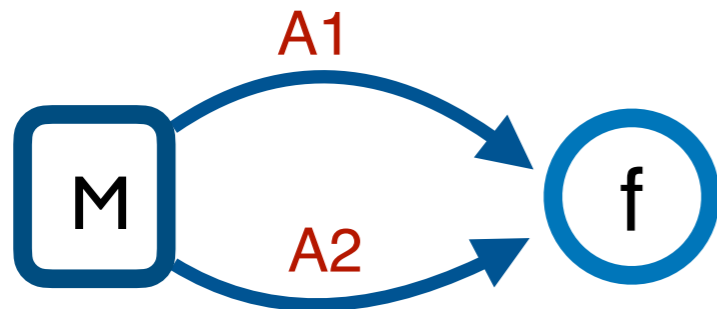
$$\langle f | T | M \rangle = A_1 e^{i(\delta_1 + \phi_1)} + A_2 e^{i(\delta_2 + \phi_2)}$$

CP \rightarrow

$$\langle \bar{f} | T | \bar{M} \rangle = A_1 e^{i(\delta_1 - \phi_1)} + A_2 e^{i(\delta_2 - \phi_2)}$$

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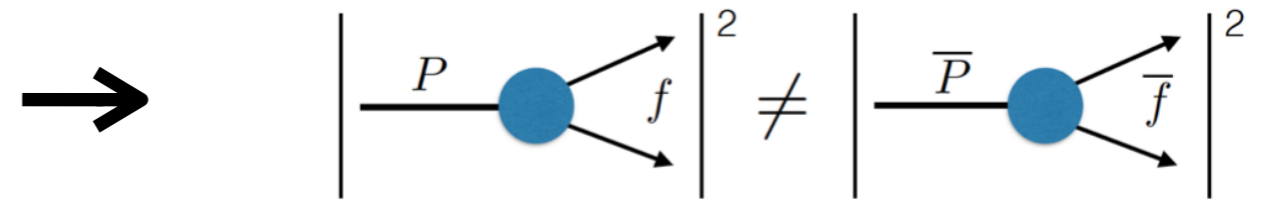


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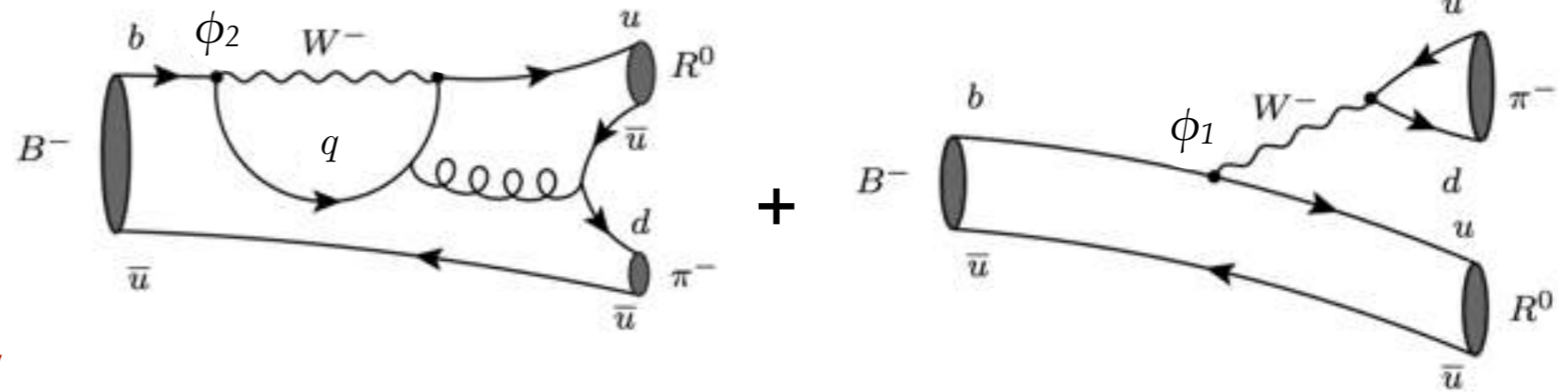


- $$\Gamma(M \rightarrow f) - \Gamma(\bar{M} \rightarrow \bar{f}) = |\langle f | T | M \rangle|^2 - |\langle \bar{f} | T | \bar{M} \rangle|^2 = -4A_1 A_2 \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2)$$

- weak phase $\phi \rightarrow$ CKM

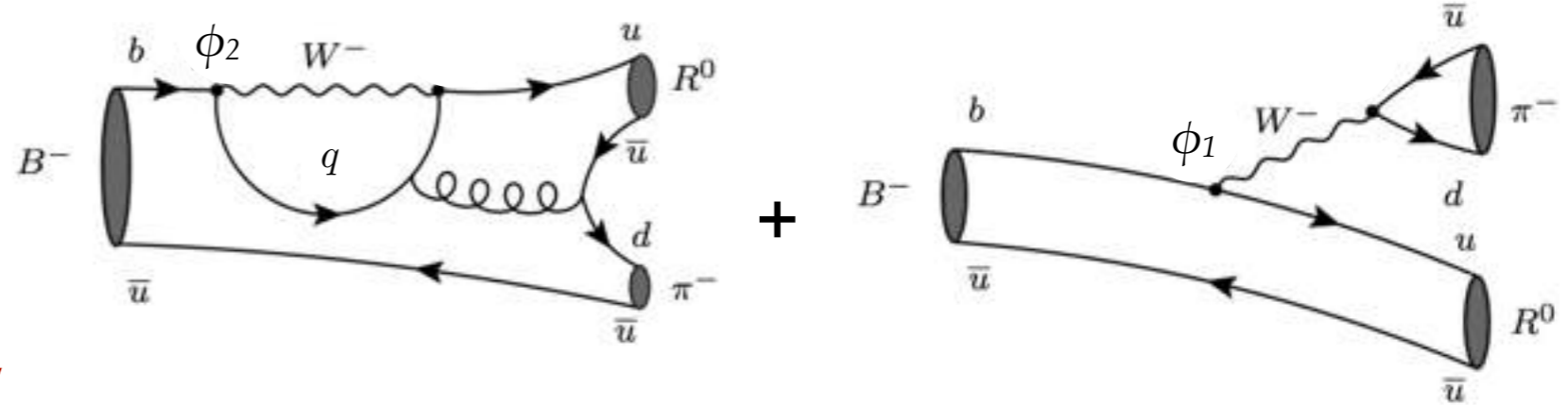
- strong phase $\delta \rightarrow$ QCD

- CPV at quark level: BSS model Bander Silverman & Soni PRL 43 (1979) 242



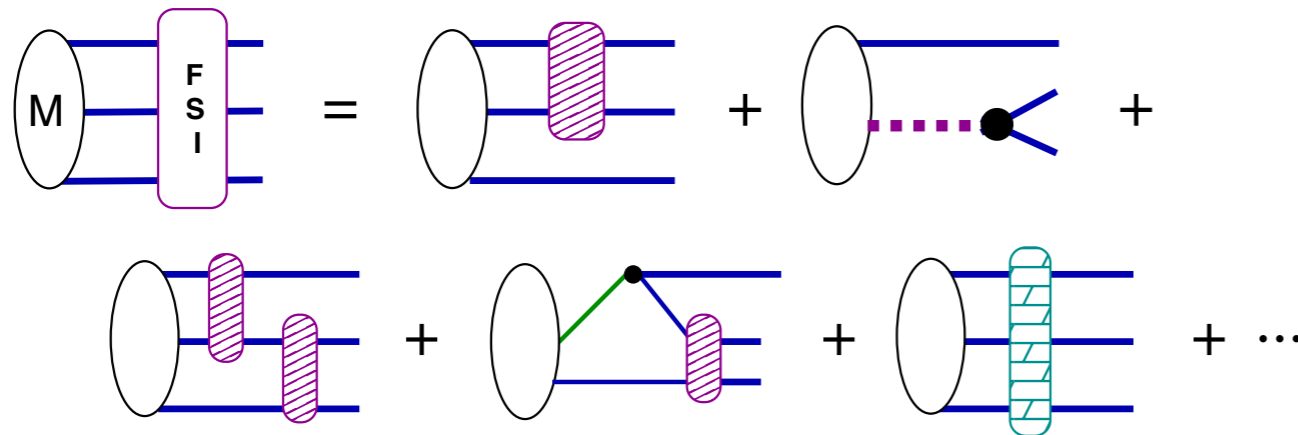
- not enough for CPV

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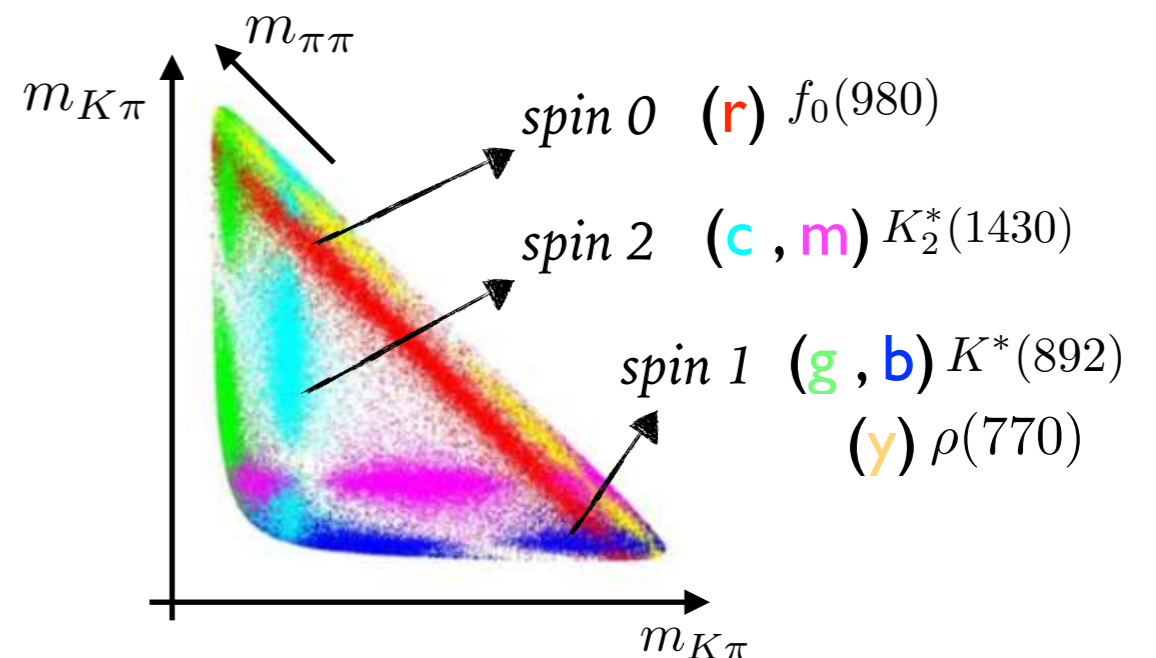


- not enough for CPV

- hadronic (not quark) interactions are natural sources of strong phase



- exemplo: $D^0 \rightarrow K_s \pi^- \pi^+$



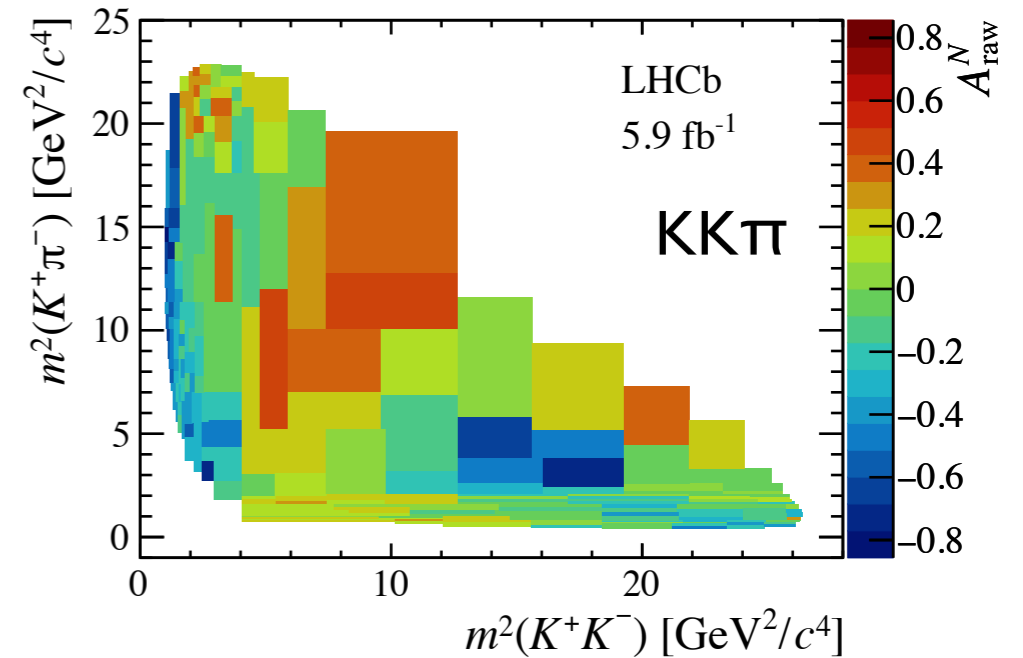
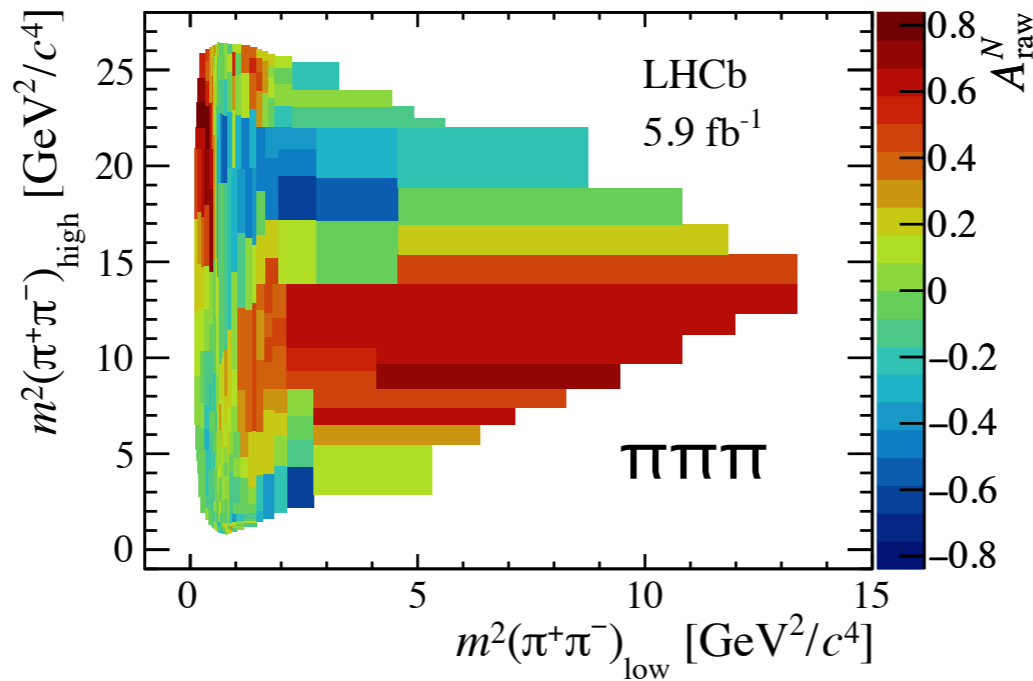
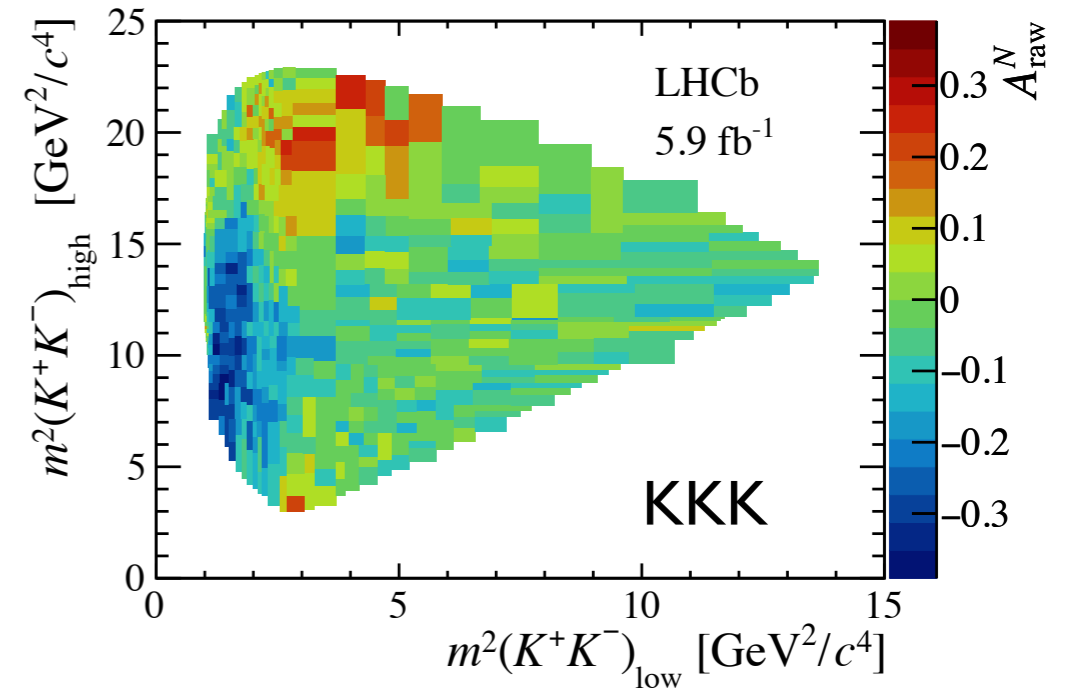
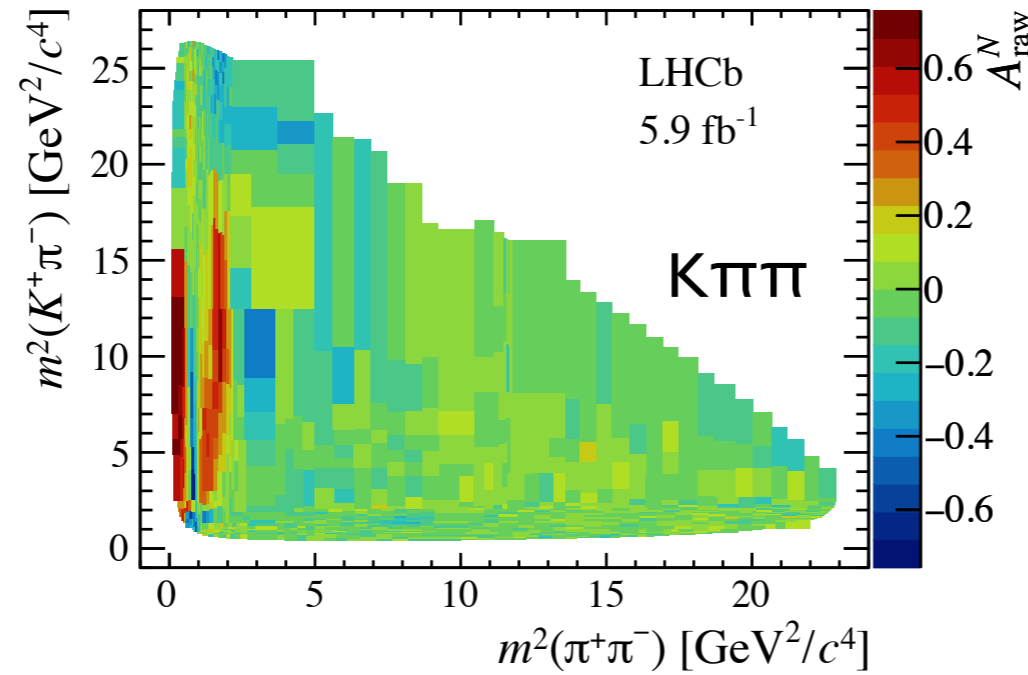
● $B^\pm \rightarrow h^\pm h^- h^+$



giant localized A_{CP}

$$A_{CP} = \frac{\Gamma(M \rightarrow f) - \Gamma(\bar{M} \rightarrow \bar{f})}{\Gamma(M \rightarrow f) + \Gamma(\bar{M} \rightarrow \bar{f})}$$

● suggest dynamic effect



- CPT must be preserved

$$\Delta\Gamma_f = \Gamma(M \rightarrow f) - \Gamma(\bar{M} \rightarrow \bar{f}) = |\langle f | T | M \rangle|^2 - |\langle \bar{f} | T | \bar{M} \rangle|^2$$

$$\rightarrow \sum \Delta\Gamma_{CP} = 0$$

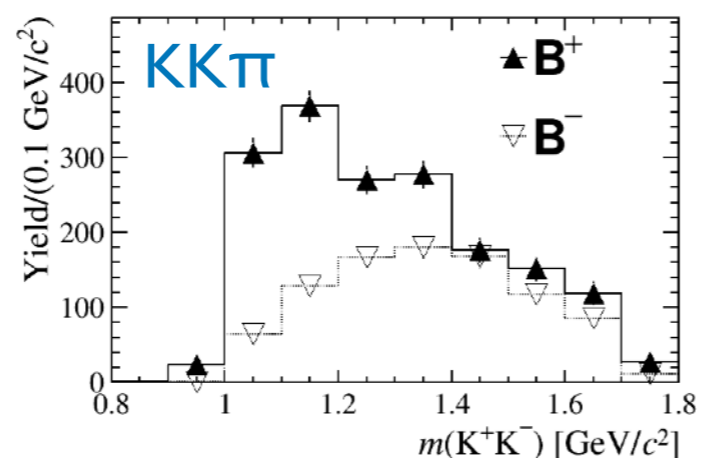
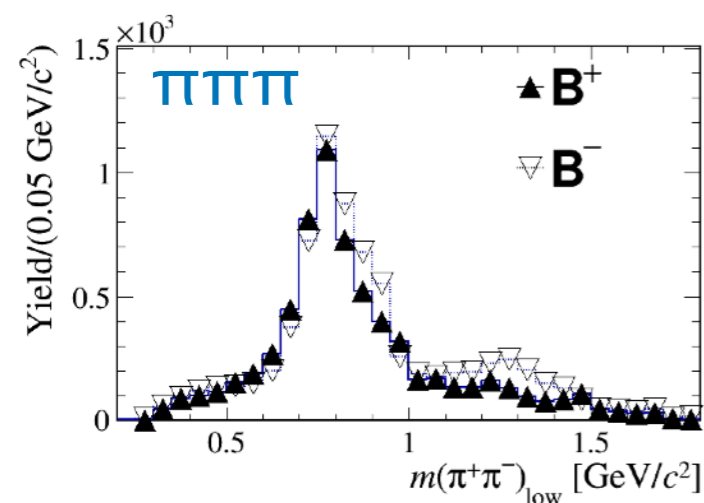
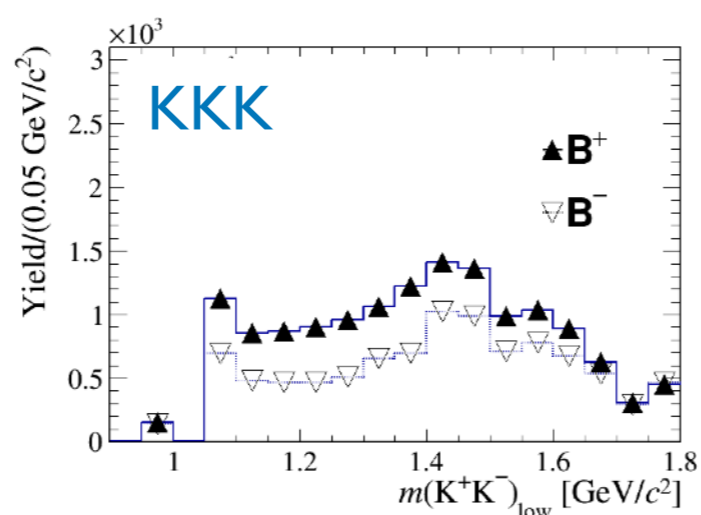
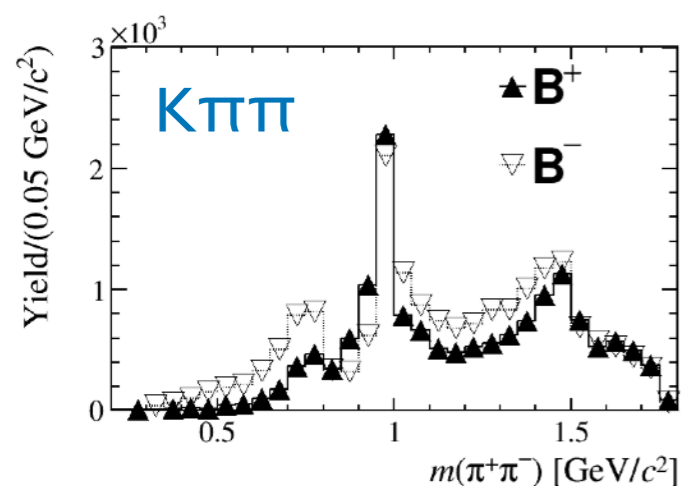
CPV in one channel should be compensated by another, same quantum #, with opposite sign

Lifetime $\tau = 1 / \Gamma_{\text{total}} = 1 / \bar{\Gamma}_{\text{total}}$

$$\Gamma_{\text{total}} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \dots$$

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- LHCb run 1 projections



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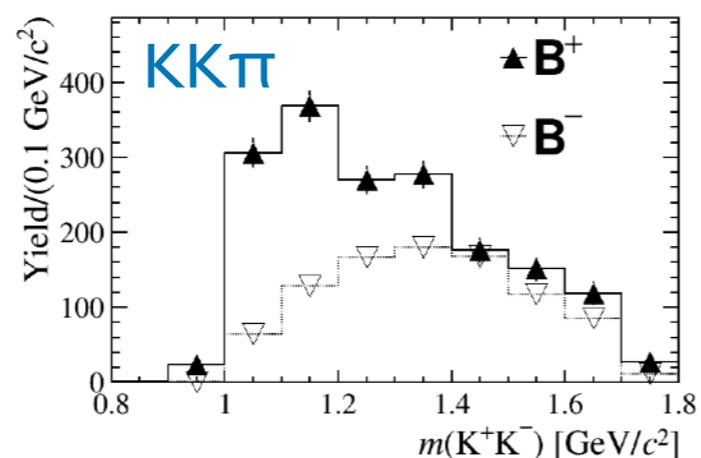
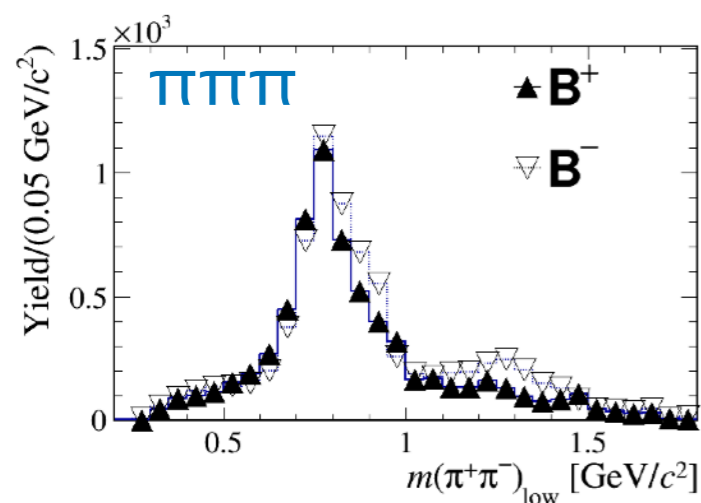
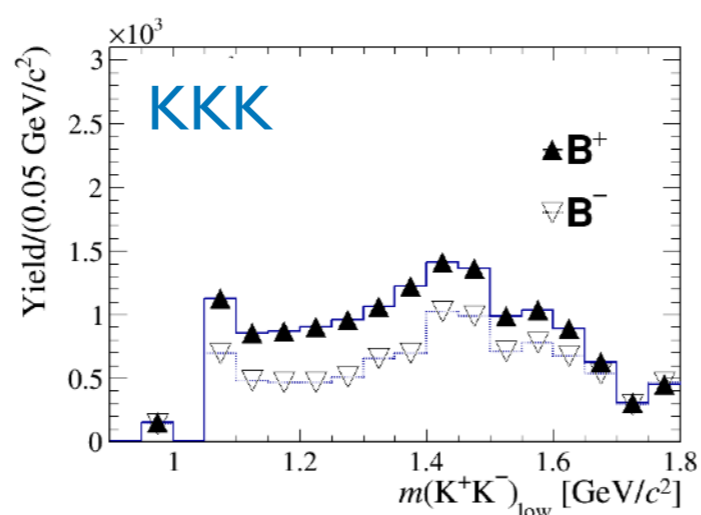
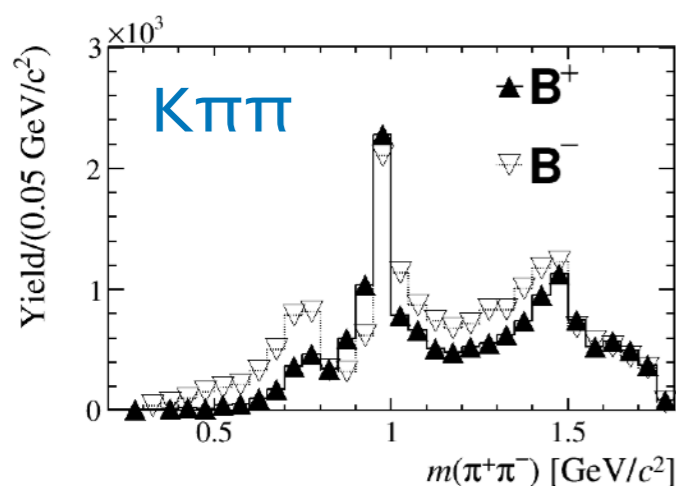
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- LHCb run 1 projections



- rescattering $\pi\pi \rightarrow KK$

→ CPV at [1 -1.6] GeV
 Frederico, Bediaga, Lourenço
 PRD89(2014)094013

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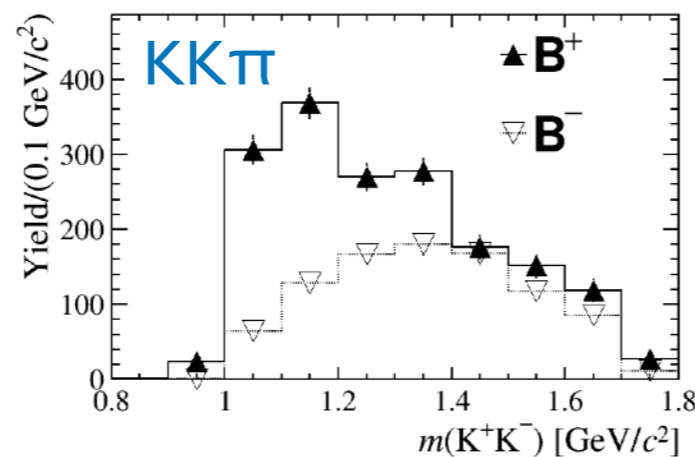
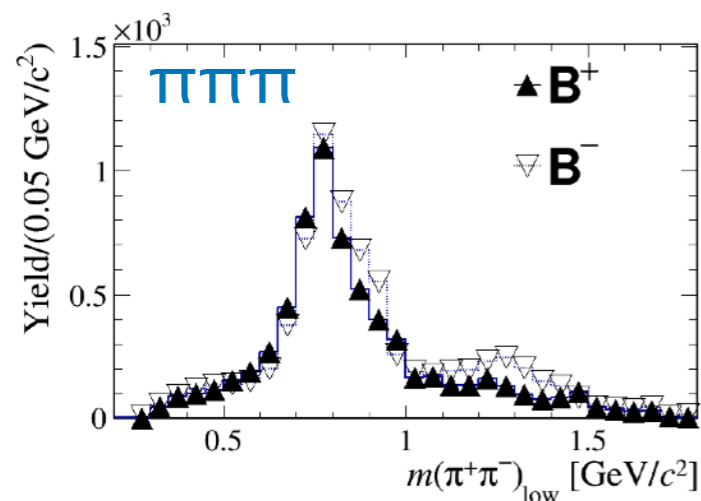
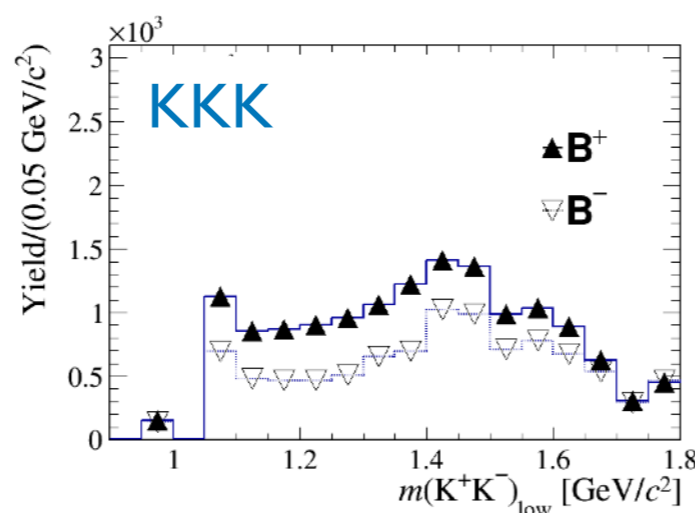
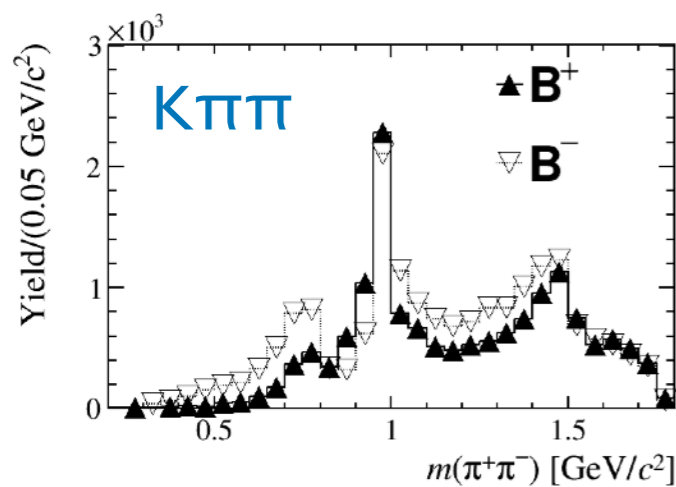
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- LHCb run 1 projections



- rescattering $\pi\pi \rightarrow KK$

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 Frederico, Bediaga, Lourenço
 PRD89(2014)094013

- implemented in LHCb amplitude analysis:

↳ $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ PRD101 (2020) 012006;
 PRL 124 (2020) 031801

↳ $B^\pm \rightarrow \pi^\pm K^- K^+$ PRL 123 (2019) 231802

- $B \rightarrow K\pi\pi$ - LHCb full run1 data

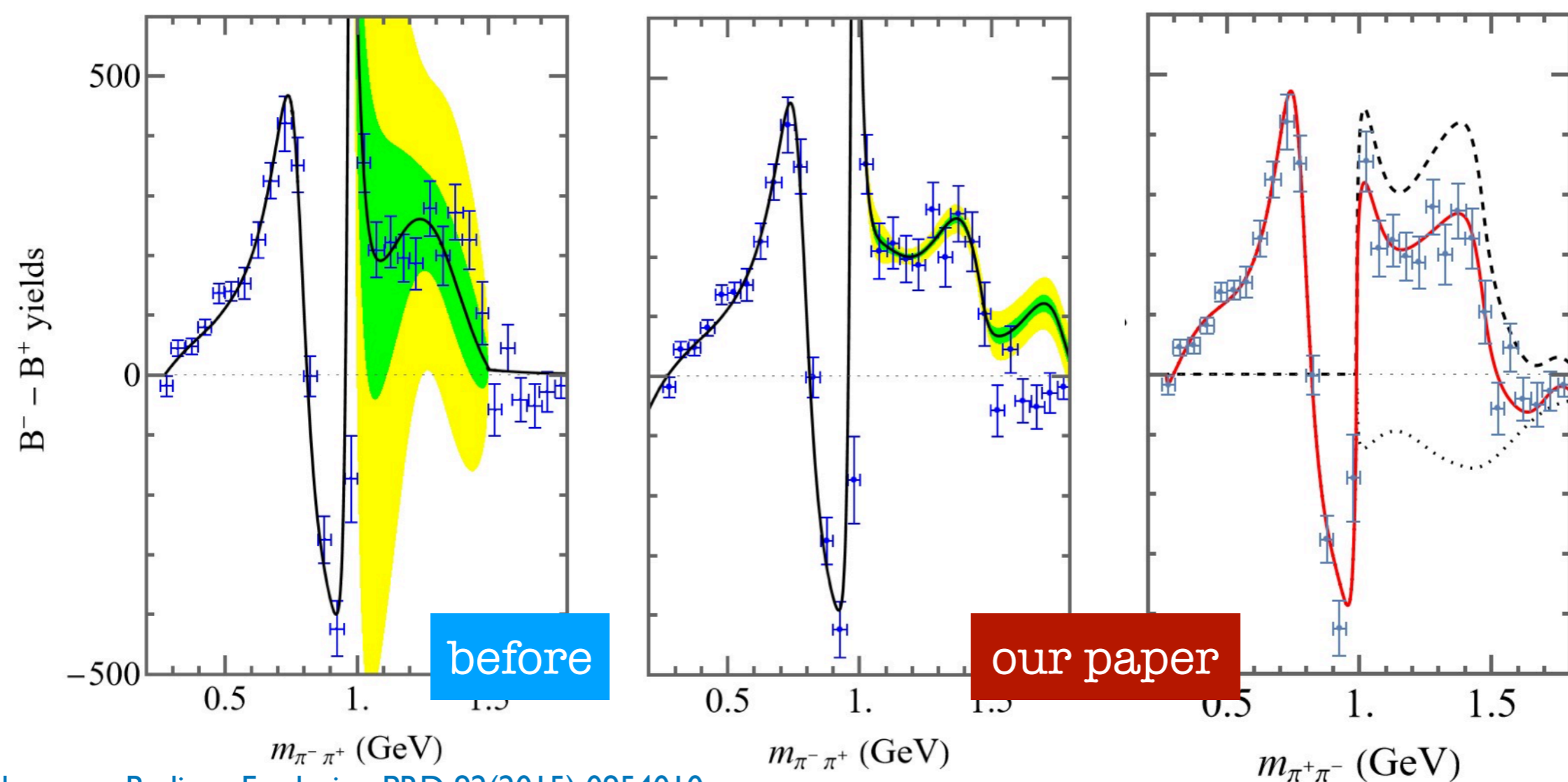
PCM, Pelaez PRL130 201901 2023

→ revisit previous work, improving FSI amplitudes

$$\delta_{\pi\pi KK} = 2\delta_{\pi\pi} \text{ and } |S_{\pi\pi KK}| \rightarrow \sqrt{1 - \eta^2}$$

CFD $\pi\pi \rightarrow K\bar{K}$ dispersive analysis

$KK \rightarrow \pi\pi$ and $\pi\pi \rightarrow \pi\pi$ dispersive analysis



Alvarenga, Bediaga, Frederico PRD 92(2015) 0954010

● $B \rightarrow K\pi\pi$ - LHCb full run1 data

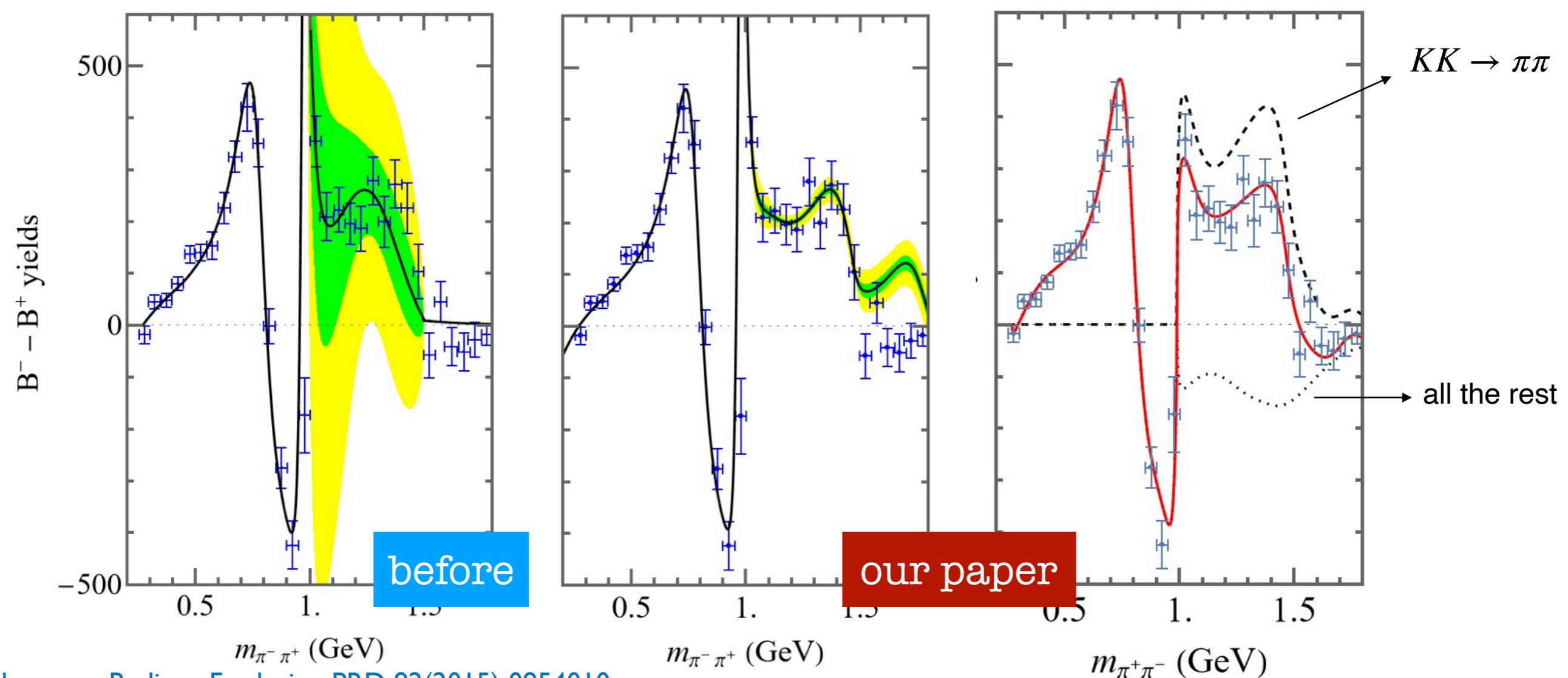
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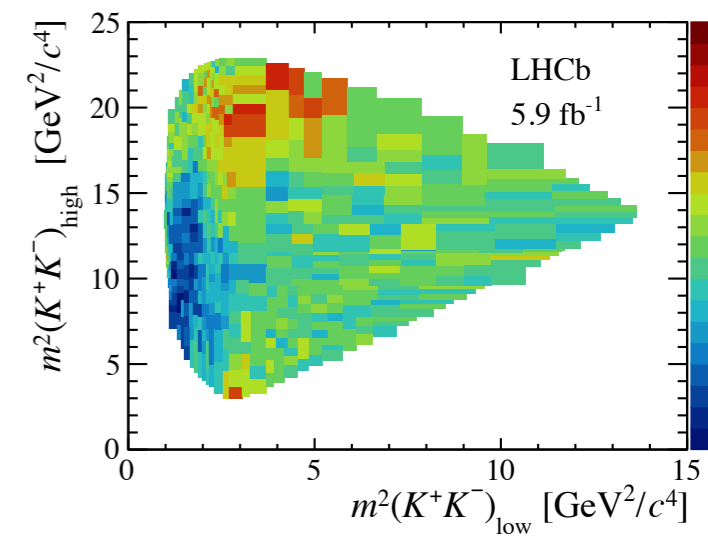
$KK \rightarrow \pi\pi$ and $\pi\pi \rightarrow \pi\pi$ dispersive analysis



Alvarenga, Bediaga, Frederico PRD 92(2015) 0954010

➔ conclusion remains the same: $KK \rightarrow \pi\pi$ dominates between 1-1.5 GeV

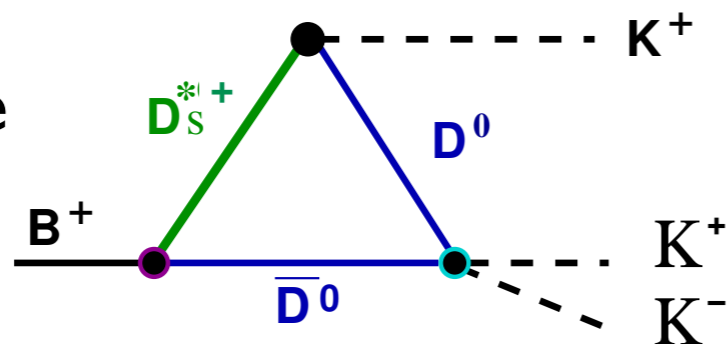
- $B^+ \rightarrow K^- K^+ K^+$ Bediaga, Frederico, MAGALHAES, PLB 780 (2018) 357
- analogue of $\pi\pi \leftrightarrow KK$ but with $D\bar{D} \rightarrow K^+ K^- (\pi^+ \pi^-)$



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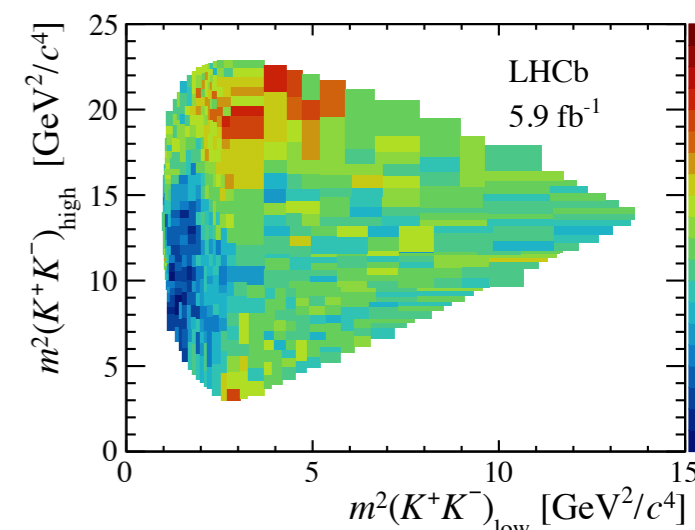
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- charm triangle



$$Br [B \rightarrow DD_s^*] \sim 1\%$$

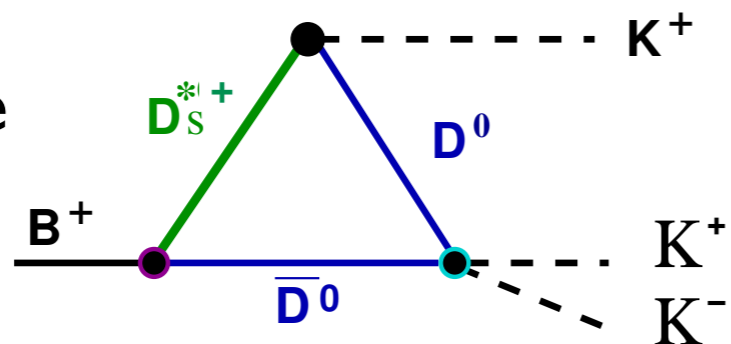
$$\rightarrow 1000 \times Br [B \rightarrow KKK]$$



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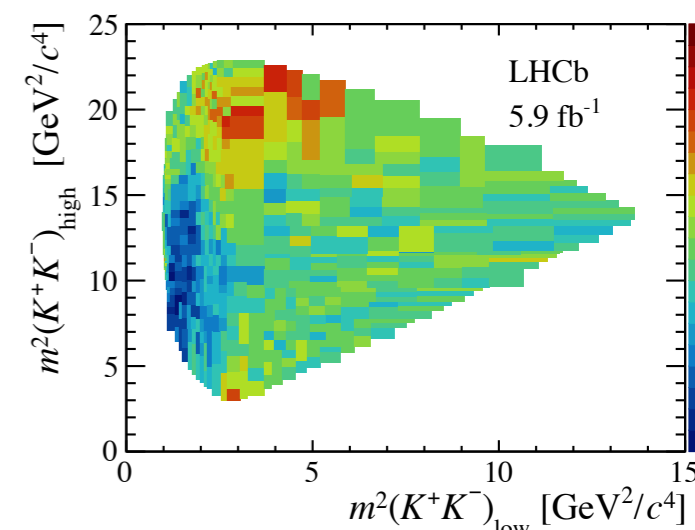
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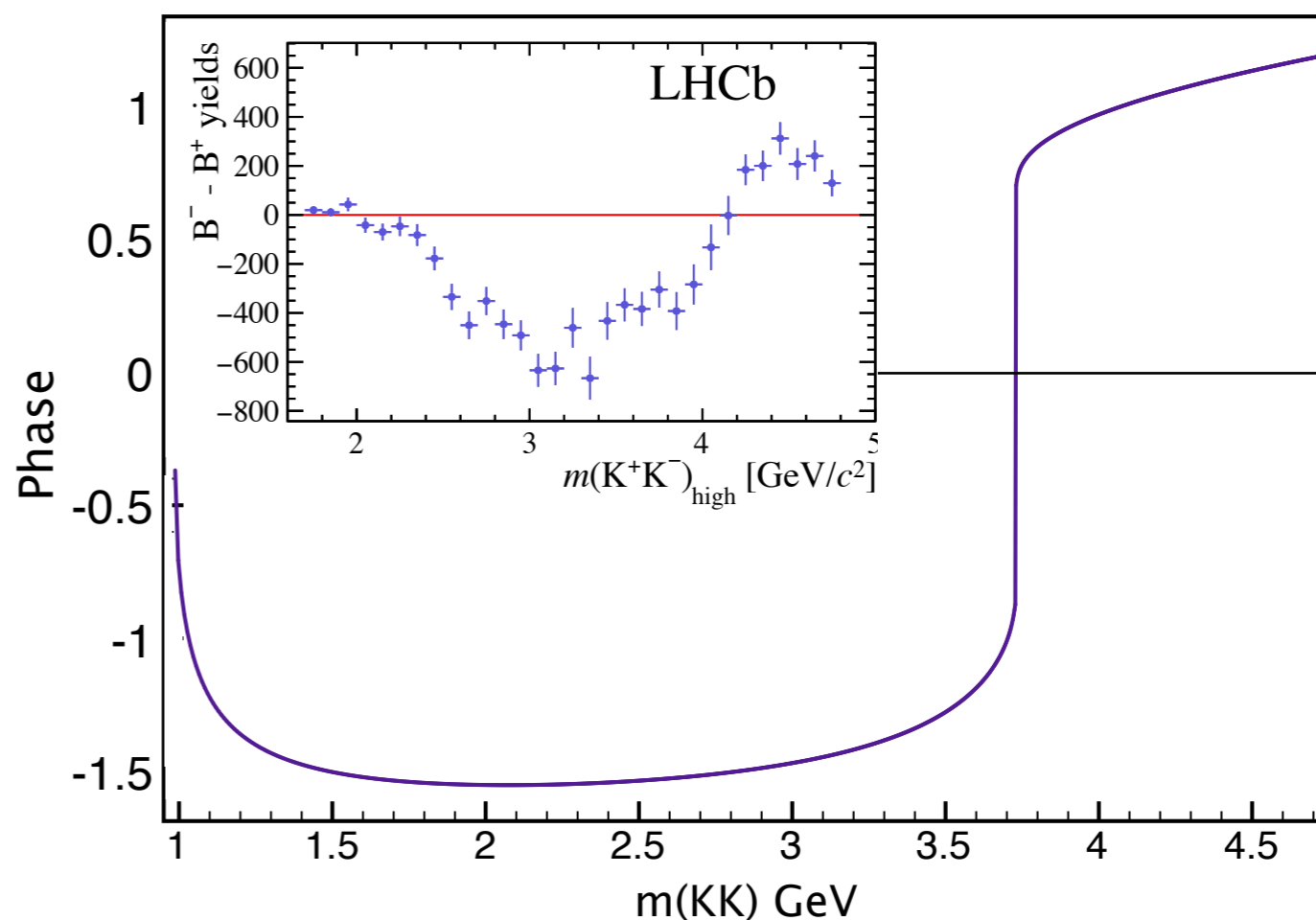


$$Br [B \rightarrow DD_s^*] \sim 1\%$$

$$\rightarrow 1000 \times Br [B \rightarrow KKK]$$



- change sign $\sim D\bar{D}$ open channel

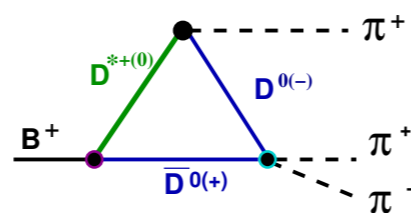


● $A_{B^\pm \rightarrow \pi^- \pi^+ \pi^\pm}(s_{12}, s_{23}) =$
 $+ a_0 e^{\pm i\gamma}$

$$\gamma = 70^\circ$$

$$a_0 = 2 e^{i(\delta_s = 45^\circ)}$$

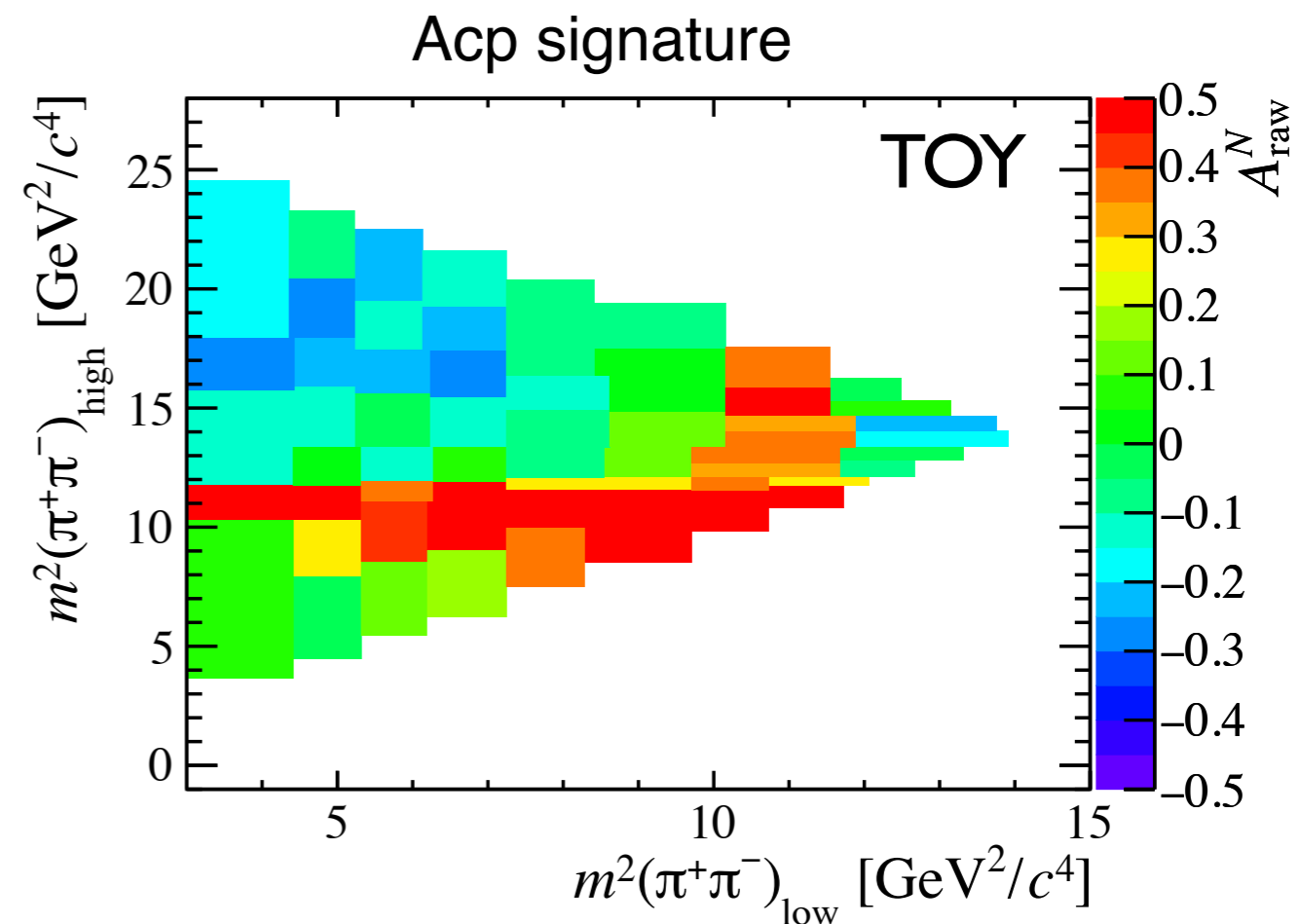
- charm rescattering as a source of strong phase

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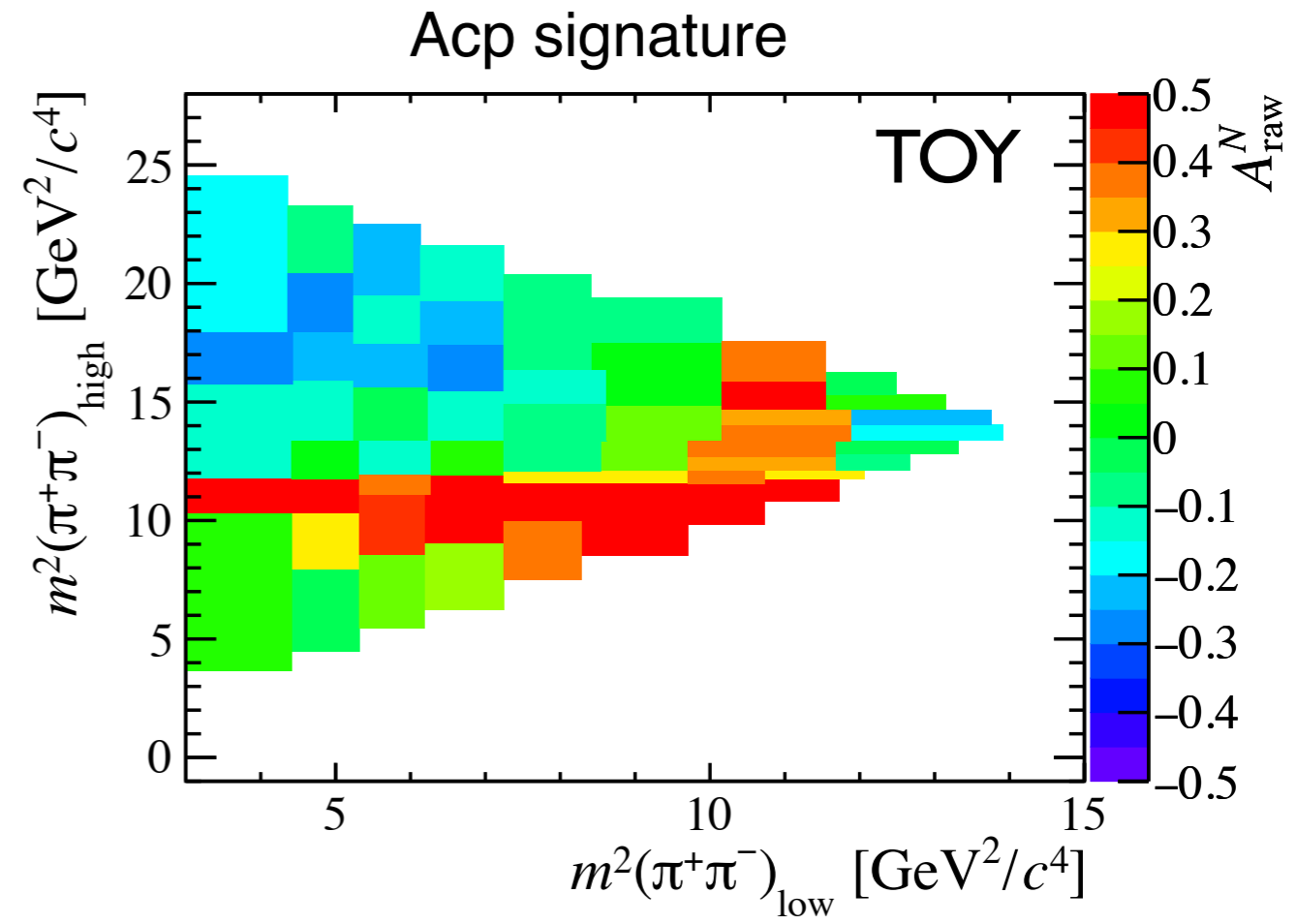
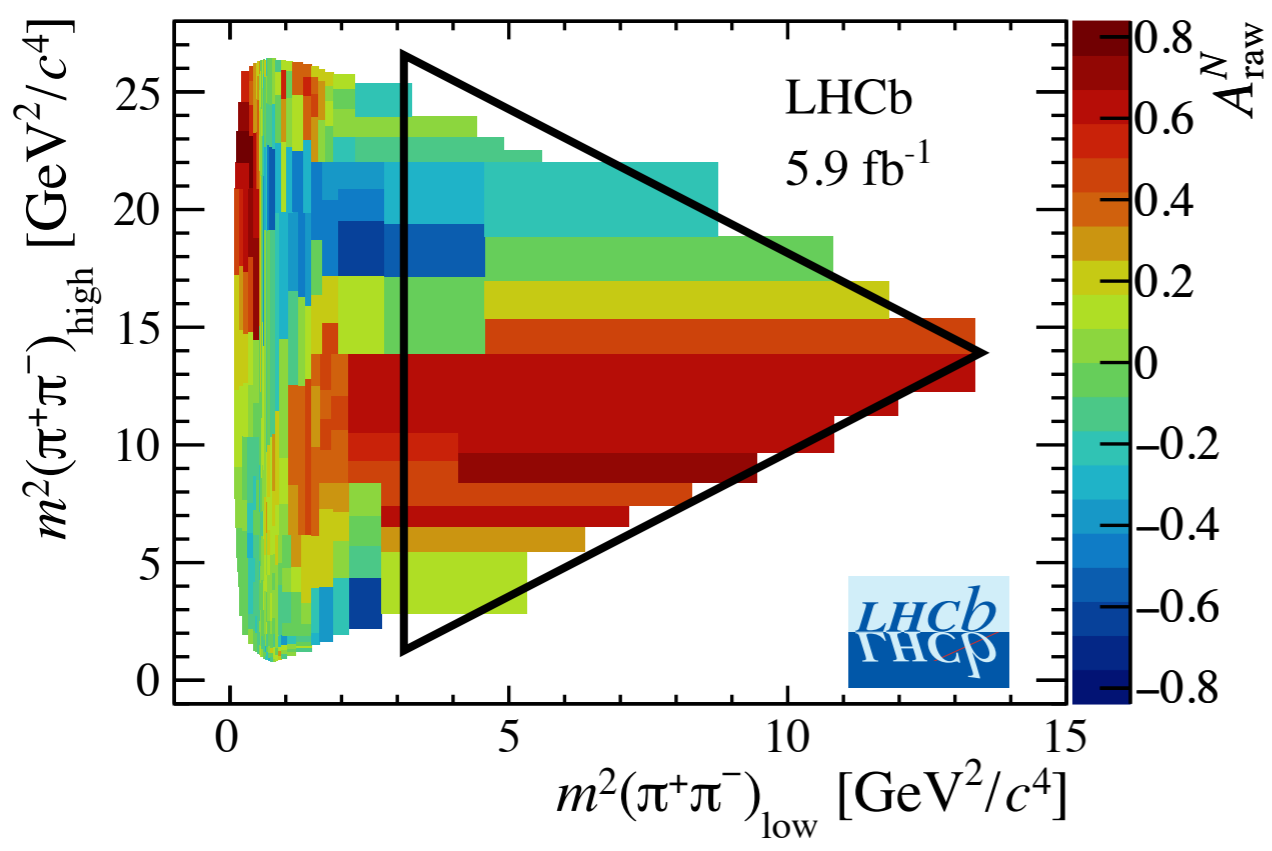


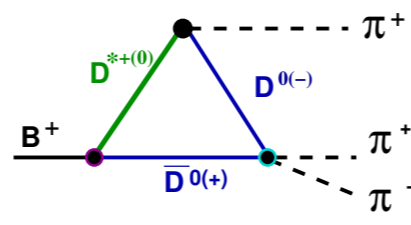
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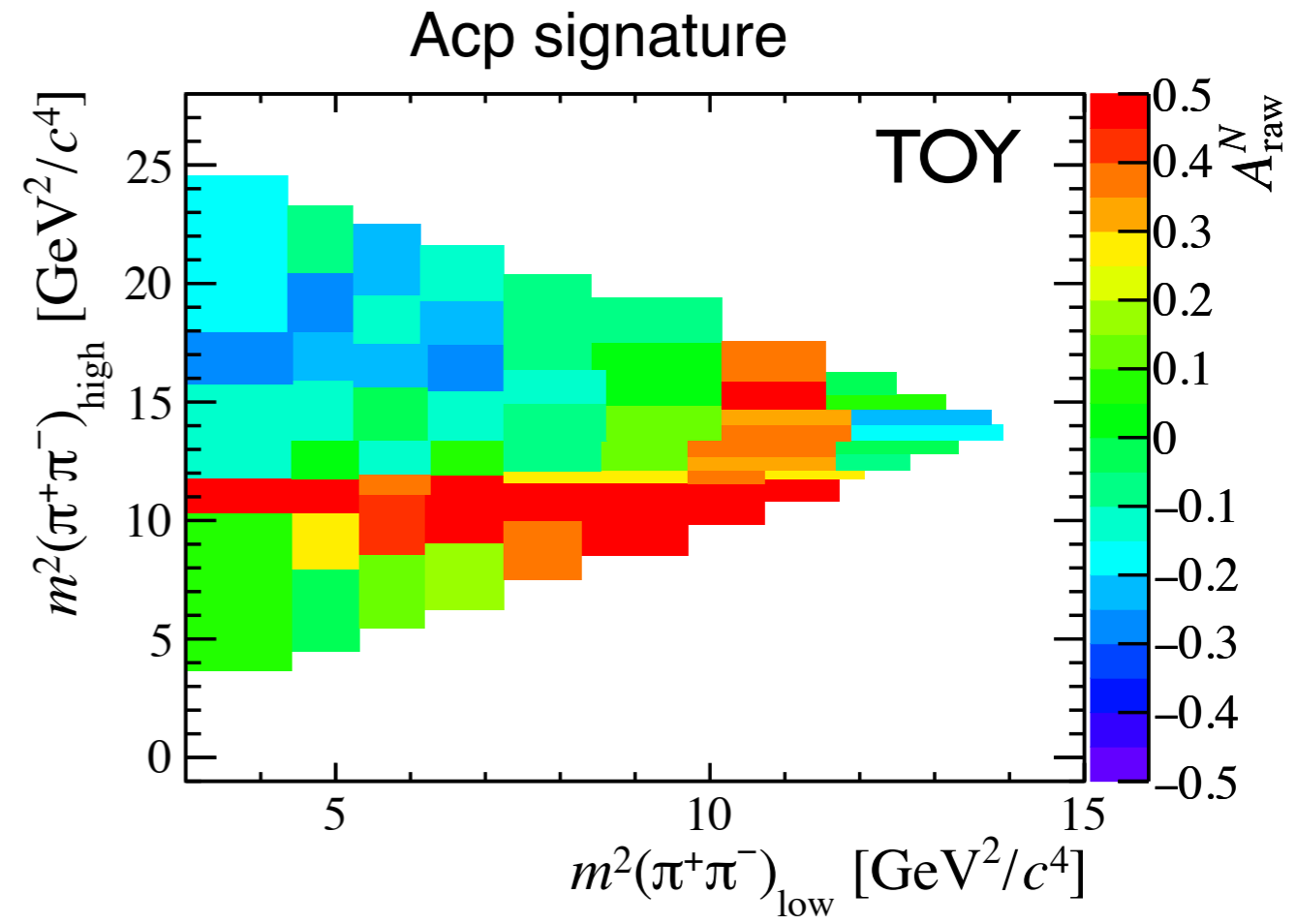
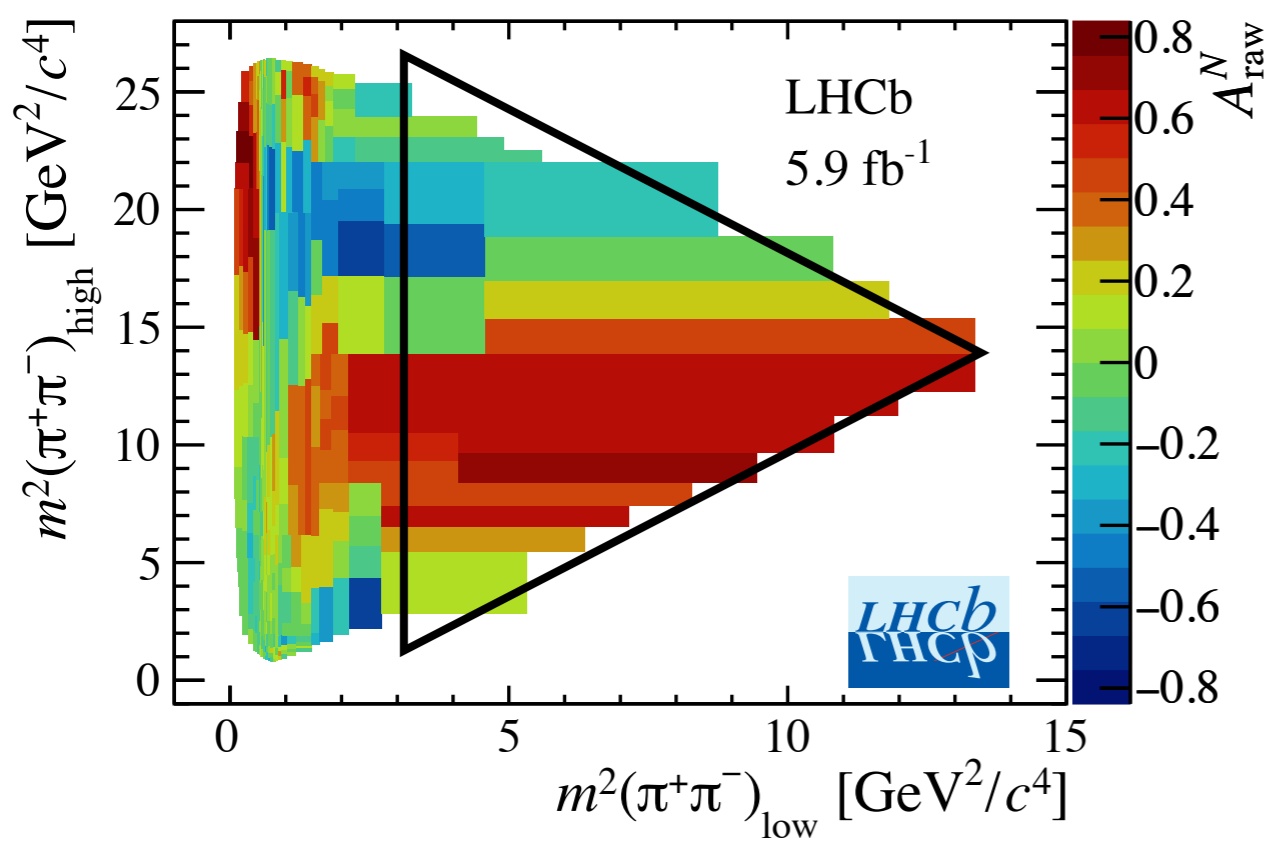


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$$\gamma = 70^\circ$$

$$a_0 = 2 e^{i(\delta_s = 45^\circ)}$$

- charm rescattering as a source of strong phase



- ➔ mimic CPV pattern at high mass
- ➔ New mechanism of CPV!

➔ to be tested on data!

- how to explain the CPV in charm?



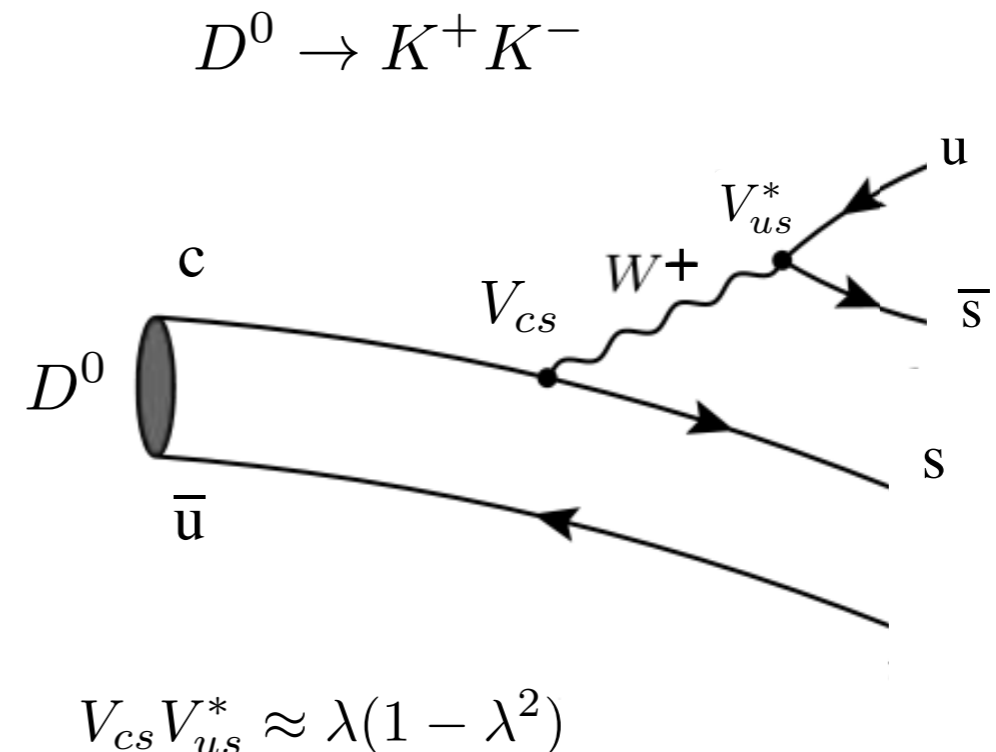
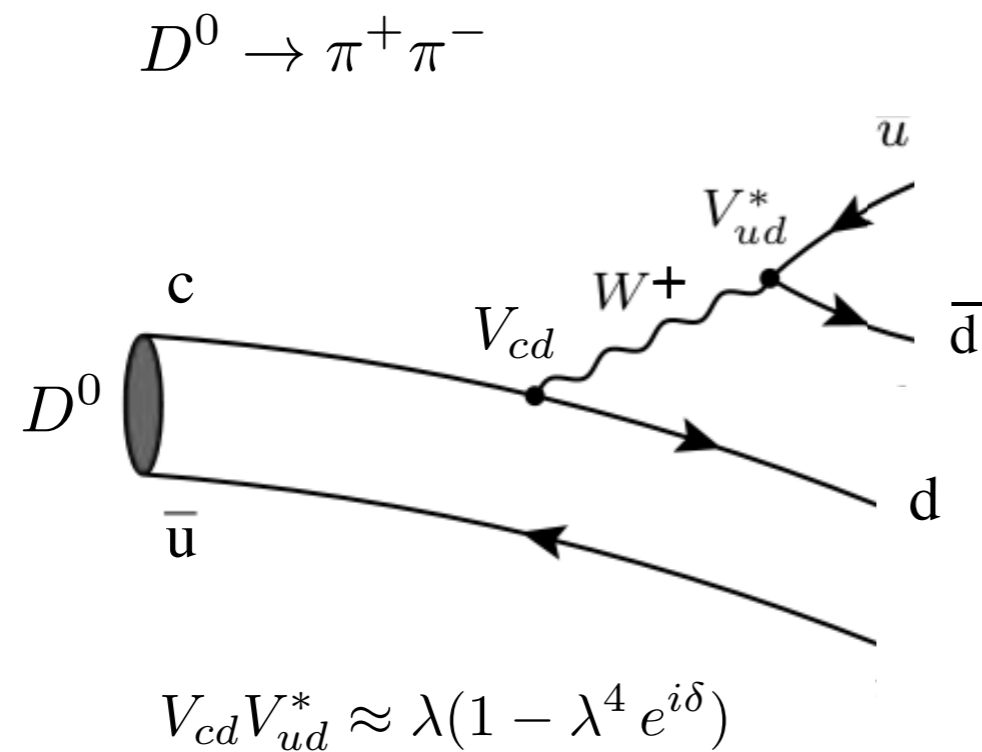
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- how to explain the CPV in charm?



$$\Delta A_{CP}^{\text{LHCb}} = A_{cp}(D^0 \rightarrow K^+K^-) - A_{cp}(D^0 \rightarrow \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$$

- single cabibbo suppressed decays



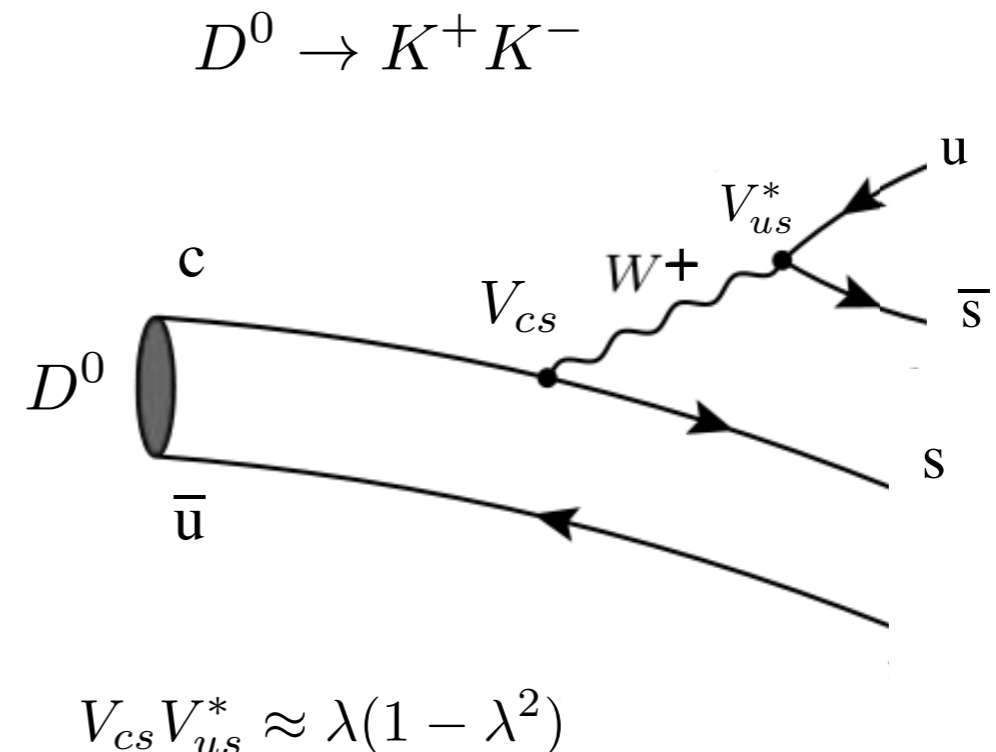
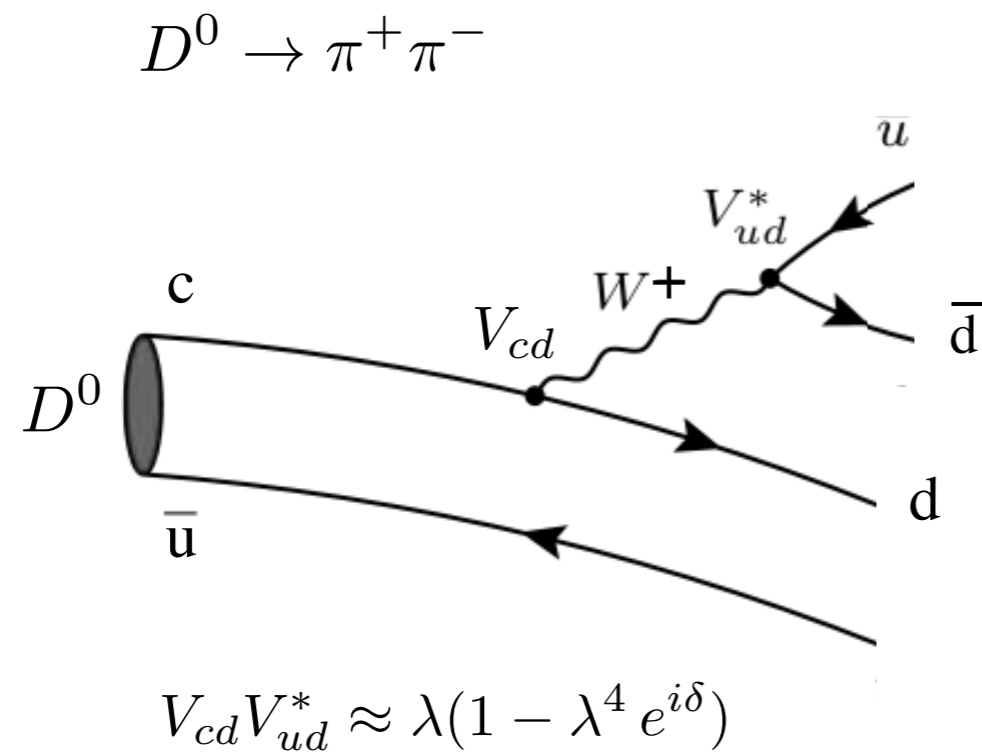
- weak phase in KK is 20 times smaller [Lenz, Wilkinson, Annu. Rev. Nucl. Part. Sci. 71, 59 \(2021\)](#)

- how to explain the CPV in charm?



$$\Delta A_{CP}^{\text{LHCb}} = A_{cp}(D^0 \rightarrow K^+K^-) - A_{cp}(D^0 \rightarrow \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$$

- single cabibbo suppressed decays



- weak phase in KK is 20 times smaller [Lenz, Wilkinson, Annu. Rev. Nucl. Part. Sci. 71, 59 \(2021\)](#)

➔ what about strong phases if not from penguin? **hadronic FSI**

[Bediaga, Frederico, PCM PRL 131 051802 \(2023\)](#) ; [Grossman, Schacht JHEP07 20 \(2019\)](#); [Schacht, Soni PLB825 136855 \(2022\)](#).

QCD short-distance

- QCDF → how to calculate penguin contributions? call BSM effects

Pich, Solomonidi, Silva
arXiv:2305.11951

Chala, Lenz, Rusov, Scholtz,
JHEP 07, 161 (2019).

- LCSR → QCD, model independent but predictions are 1 order magnitude below

Khodjamirian, Petrov,
Phys. Lett. B 774, 235 (2017)

long distance effect:

- topological and group symmetry approach

- with SU(3) breaking through FSI (fit agrees)

H.-Y. Cheng and C.-W. Chiang, PRD 100, 093002 (2019).
F. Buccella, A. Paul and P. Santorelli, PRD 99, 113001 (2019)
Franco, Mishima, Silvestrini JHEP05, 140 (2012)

- with resonances (fit agrees)

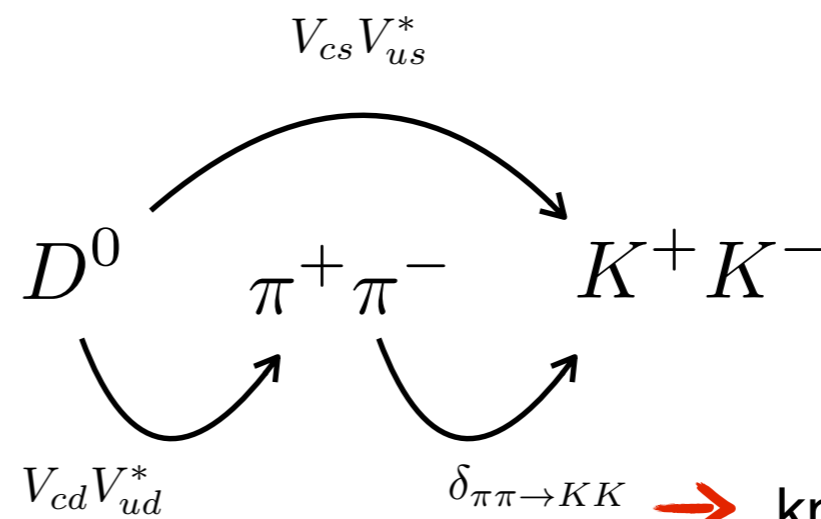
Schacht and A. Soni, Phys. Lett. B 825, 136855 (2022).
Y. Grossman and S. Schacht, JHEP 07, 20 (2019)

- FSI with CPT (prediction agrees)

Bediaga, Frederico, Magalhaes
PRL131, 051802 (2023)

[arxiv 2203.04056v2](https://arxiv.org/abs/2203.04056v2)

- D and \bar{D} can decay to $\pi\pi$ and KK

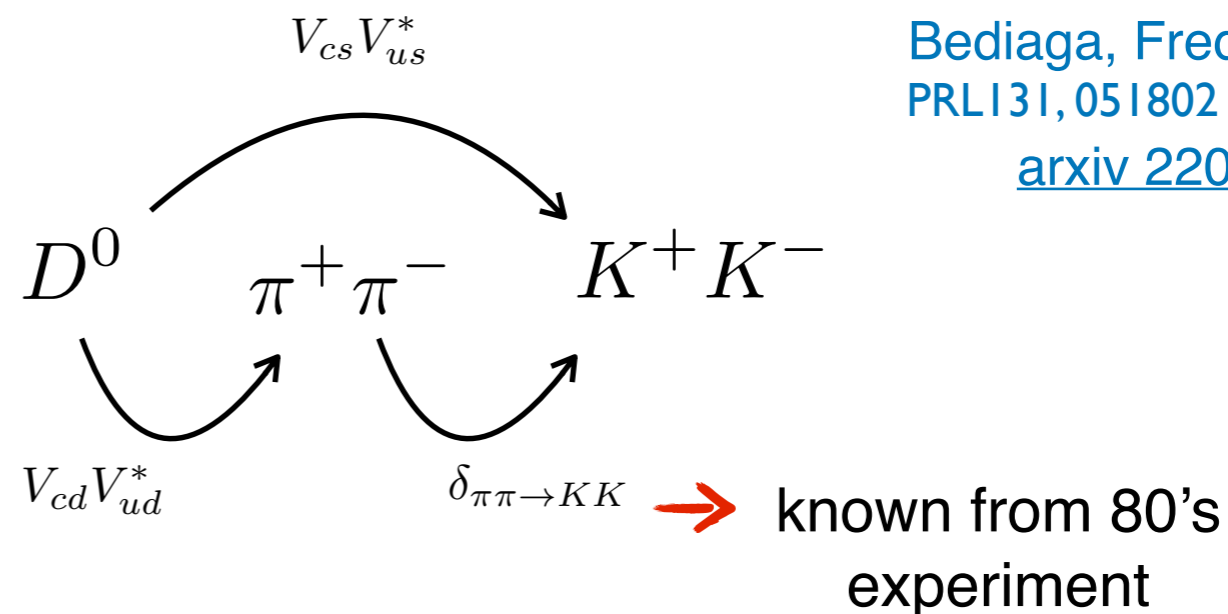


Bediaga, Frederico, PCM
PRL131, 051802 (2023)

[arxiv 2203.04056v3](https://arxiv.org/abs/2203.04056v3)

→ known from 80's
experiment

- D and \bar{D} can decay to $\pi\pi$ and KK



Bediaga, Frederico, PCM
 PRL131, 051802 (2023)
[arxiv 2203.04056v3](https://arxiv.org/abs/2203.04056v3)

- build amplitudes decays implying three constraints:

- CPT invariance relates channels with same quantum numbers

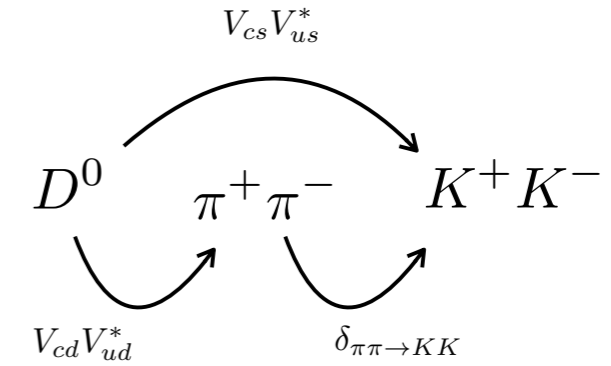
$$\rightarrow \sum \Delta\Gamma_{CP} = 0$$

- Watson theorem relates the strong phase from the rescattering process to the decay amplitudes

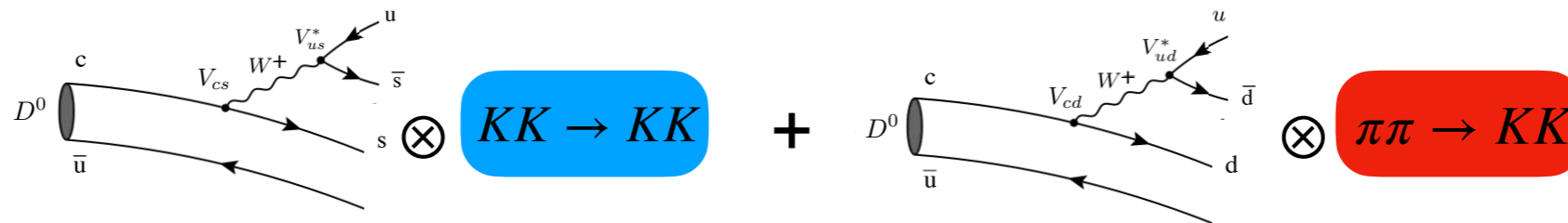
- the unitarity of the strong S-matrix - assume only 2 channels ie the dominant ones $\pi\pi, K\bar{K}$

- dressing the weak tree topology with FSI

→ penguin are suppressed



- $D^0 \rightarrow KK$

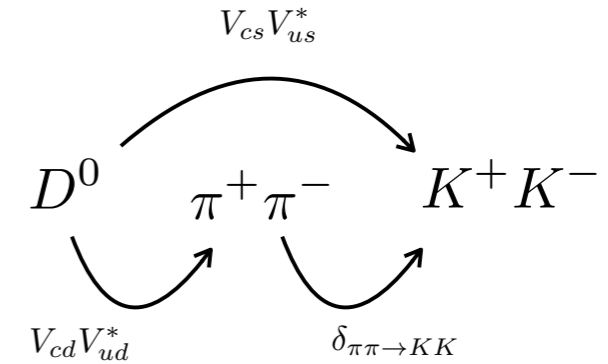


→ $\mathcal{A}_{D^0 \rightarrow KK} = \eta e^{2i\delta_{KK}} V_{cs}^* V_{us} a_{KK} + i\sqrt{1-\eta^2} e^{i(\delta_{\pi\pi} + \delta_{KK})} V_{cd}^* V_{ud} a_{\pi\pi}$

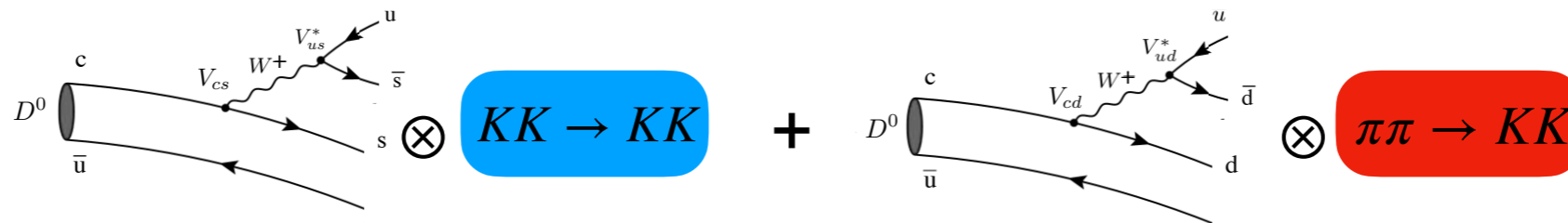
→ $\mathcal{A}_{\bar{D}^0 \rightarrow f}$ same with CKM cc.

- dressing the weak tree topology with FSI

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- $D^0 \rightarrow KK$



$$\rightarrow \mathcal{A}_{D^0 \rightarrow KK} = \eta e^{2i\delta_{KK}} V_{cs}^* V_{us} a_{KK} + i\sqrt{1-\eta^2} e^{i(\delta_{\pi\pi} + \delta_{KK})} V_{cd}^* V_{ud} a_{\pi\pi}$$

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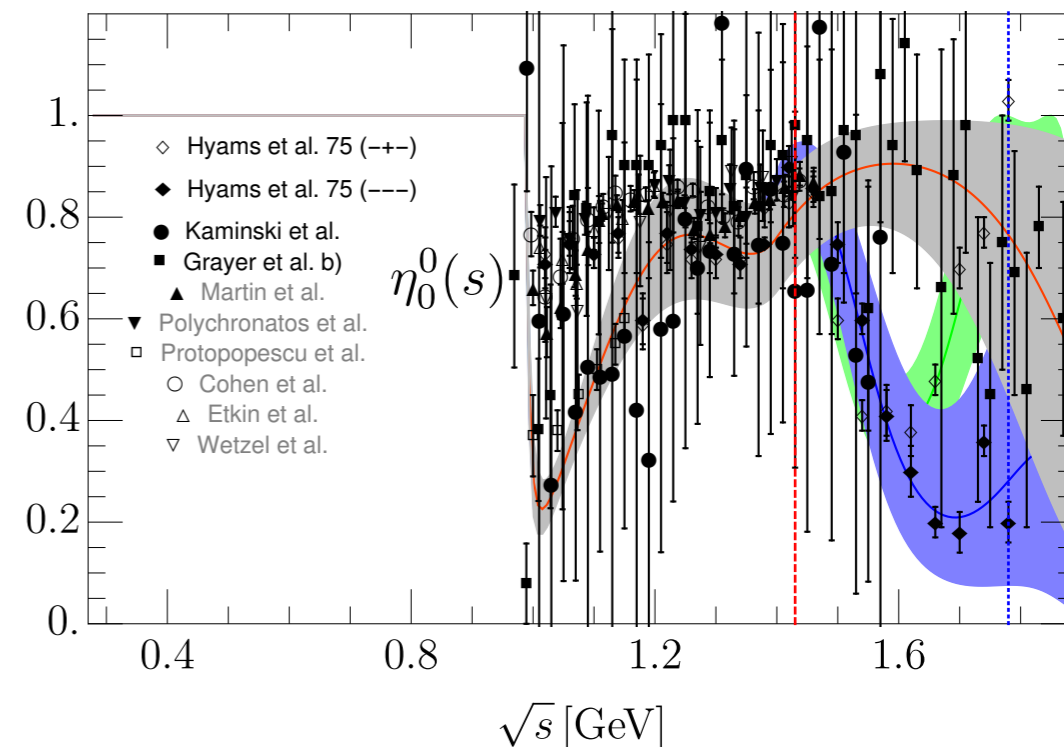
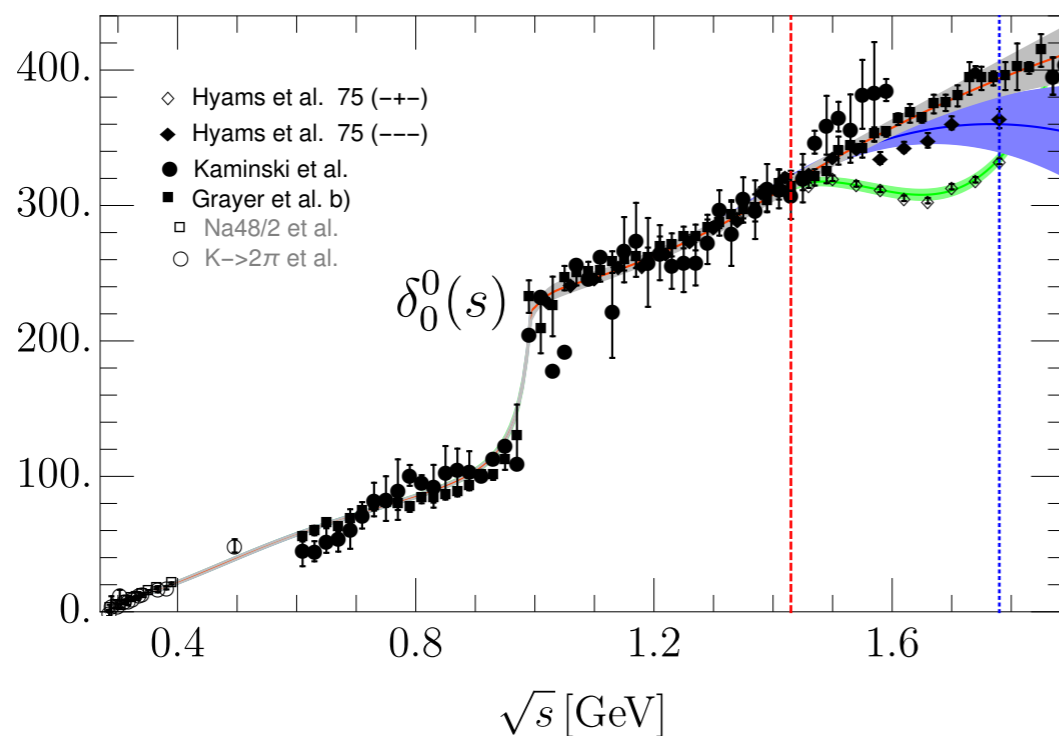
- $D^0 \rightarrow \pi\pi$



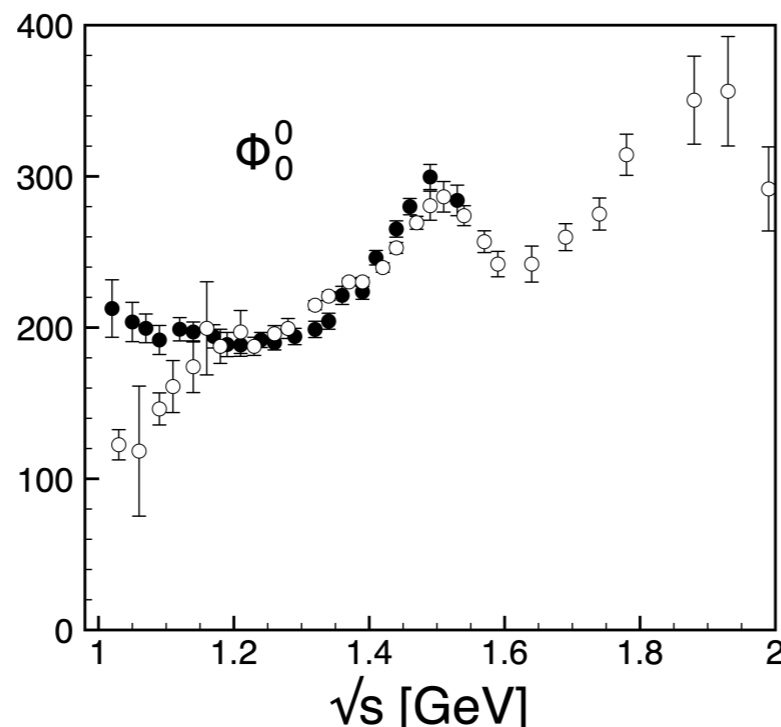
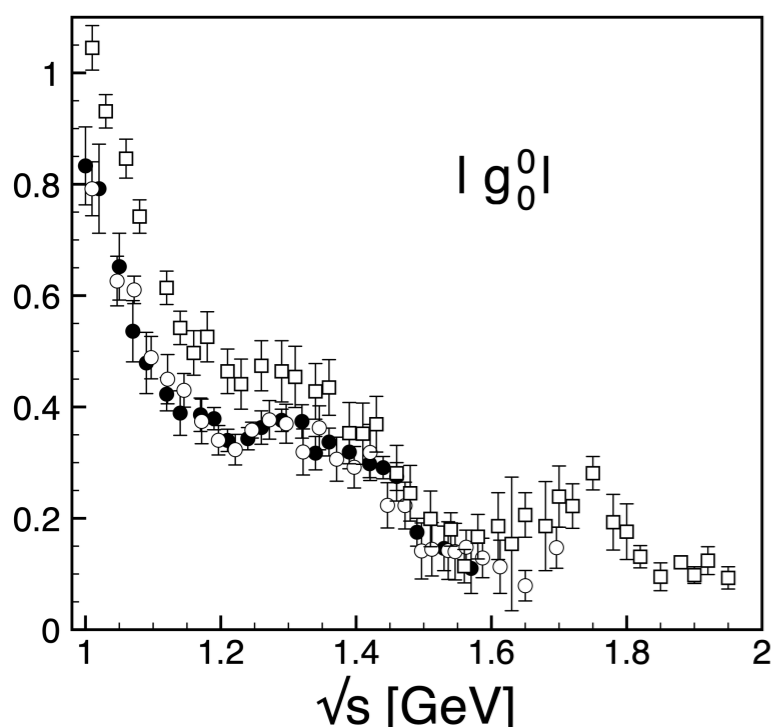
$$\rightarrow \mathcal{A}_{D^0 \rightarrow \pi\pi} = \eta e^{2i\delta_{\pi\pi}} V_{cd}^* V_{ud} a_{\pi\pi} + i\sqrt{1-\eta^2} e^{i(\delta_{\pi\pi} + \delta_{KK})} V_{cs}^* V_{us} a_{KK}$$

- a_{KK} and $a_{\pi\pi}$ do not carry any strong phases (real)

● $\pi\pi \rightarrow \pi\pi$ Pelaez, Rodas, Elvira *Eur.Phys.J.C* 79 (2019) 12, 1008



● $\pi\pi \rightarrow KK$ Pelaez and Rodas, *Eur. Phys. J. C* 78, 897 (2018)



$$\begin{aligned} \Rightarrow S_{\pi\pi, KK}(s) &= i\sqrt{1-\eta^2} e^{i(\delta_{\pi\pi} + \delta_{KK})} \\ &= i4\sqrt{\frac{q_\pi q_K}{s}} |g_0^0(s)| e^{i\phi_0^0(s)} \Theta(s - 4m_K^2) \end{aligned}$$

$$|g_0^0(M_D^2)| \approx 0.125 \pm 0.025$$

$$\phi_0^0 = \delta_{\pi\pi} + \delta_{KK} \approx 343^\circ \pm 8^\circ$$

$$\Rightarrow \eta \approx 0.973$$

- $\Delta\Gamma_f = \Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow f)$

$$\mathcal{A}_{D^0 \rightarrow \pi\pi} = \eta e^{2i\delta_{\pi\pi}} V_{cd}^* V_{ud} a_{\pi\pi} + i\sqrt{1-\eta^2} e^{i(\delta_{\pi\pi} + \delta_{KK})} V_{cs}^* V_{us} a_{KK}$$

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→ $\Delta\Gamma_{\pi\pi} = -\Delta\Gamma_{KK} = 4 \operatorname{Im}[V_{cs} V_{us}^* V_{cd}^* V_{ud}] a_{\pi\pi} a_{KK} \eta \sqrt{1-\eta^2} \cos \phi$

- $\phi = \delta_{KK} - \delta_{\pi\pi}$

- the sign of $\Delta\Gamma_f$ is determined by the CKM elements and the S-wave phase-shifts

- $\Delta\Gamma_f = \Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow f)$

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- need to quantify $a_{\pi\pi}$ and a_{KK} :

at D^0 mass $\sqrt{1-\eta^2} \ll 1$ →

$$\Gamma_{\pi\pi} \approx \eta^2 |V_{cd}^* V_{ud}|^2 a_{\pi\pi}^2$$

$$\Gamma_{KK} \approx \eta^2 |V_{cs}^* V_{us}|^2 a_{KK}^2$$

$$\text{Br}[D \rightarrow f] = \Gamma_f / \Gamma_{total}$$

we can use
experimental input

- $\Delta\Gamma_f = \Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow f)$

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- $A_{CP}(f) = \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow f)}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow f)} = \Delta\Gamma_f / 2\Gamma_f$

- $$A_{CP}(f) \approx \pm 2 \frac{\text{Im}[V_{cs} V_{us}^* V_{cd}^* V_{ud}]}{|V_{cs} V_{us}^* V_{cd}^* V_{ud}|} \eta^{-1} \sqrt{1 - \eta^2} \cos \phi \left[\frac{\text{Br}(D^0 \rightarrow K^+ K^-)}{\text{Br}(D^0 \rightarrow \pi^+ \pi^-)} \right]^{\pm \frac{1}{2}}$$

- $$\text{Br}(D^0 \rightarrow \pi^+ \pi^-) = (1.455 \pm 0.024) \times 10^{-3}$$
- $$\text{Br}(D^0 \rightarrow K^+ K^-) = (4.08 \pm 0.06) \times 10^{-3}$$

PDG



- $$\frac{\text{Im}[V_{cs} V_{us}^* V_{cd}^* V_{ud}]}{|V_{cs} V_{us}^* V_{cd}^* V_{ud}|} = (6.02 \pm 0.32) \times 10^{-4}$$

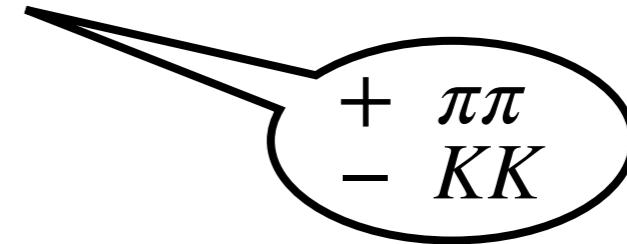
PDG

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 ϕ_0^0

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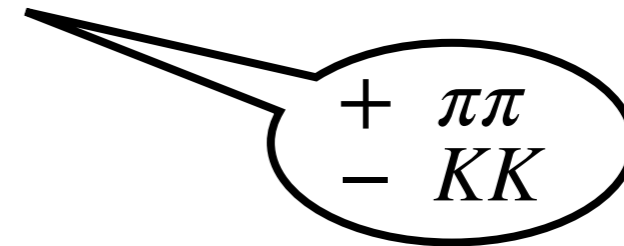
from $\pi\pi$ and $\pi\pi \rightarrow KK$ data: $\cos \phi = 0.99 \pm 0.18$.

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 ϕ_0^0

from $\pi\pi$ and $\pi\pi \rightarrow KK$ data: $\cos \phi = 0.99 \pm 0.18$.

- $$A_{CP}(\pi\pi) = (1.99 \pm 0.37) \times 10^{-3} \sqrt{\eta^{-2} - 1}$$

$$A_{CP}(KK) = -(0.71 \pm 0.13) \times 10^{-3} \sqrt{\eta^{-2} - 1}$$

as a function of inelasticity

- $\Delta A_{CP}^{th} = -(2.70 \pm 0.50) \times 10^{-3} \sqrt{\eta^{-2} - 1}$

$$\Delta A_{CP}^{LHCb} = -(1.54 \pm 0.29) \times 10^{-3}$$

- $\Delta A_{CP}^{th} = -(2.70 \pm 0.50) \times 10^{-3} \sqrt{\eta^{-2} - 1}$

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- from $\pi\pi \rightarrow KK$ data (only one set) $\rightarrow \eta \approx 0.973 \pm 0.011$

$$\Delta A_{CP}^{th} = -(0.64 \pm 0.18) \times 10^{-3} \quad 3\sigma$$

\rightarrow largest theoretical prediction within SM without relying on fitting parameters

\rightarrow systematic uncertainties are unknown in $\eta \rightarrow$ error is underestimated

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\rightarrow largest theoretical prediction within SM without relying on fitting parameters

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- Alternatively one can assume all inelasticity in $\pi\pi \rightarrow \pi\pi$ is due to KK

\rightarrow more precise data (Grayer) $\rightarrow \eta = 0.78 \pm 0.08^*$

$$\Delta A_{CP}^{th} = -(1.31 \pm 0.20) \times 10^{-3} \quad 1\sigma$$

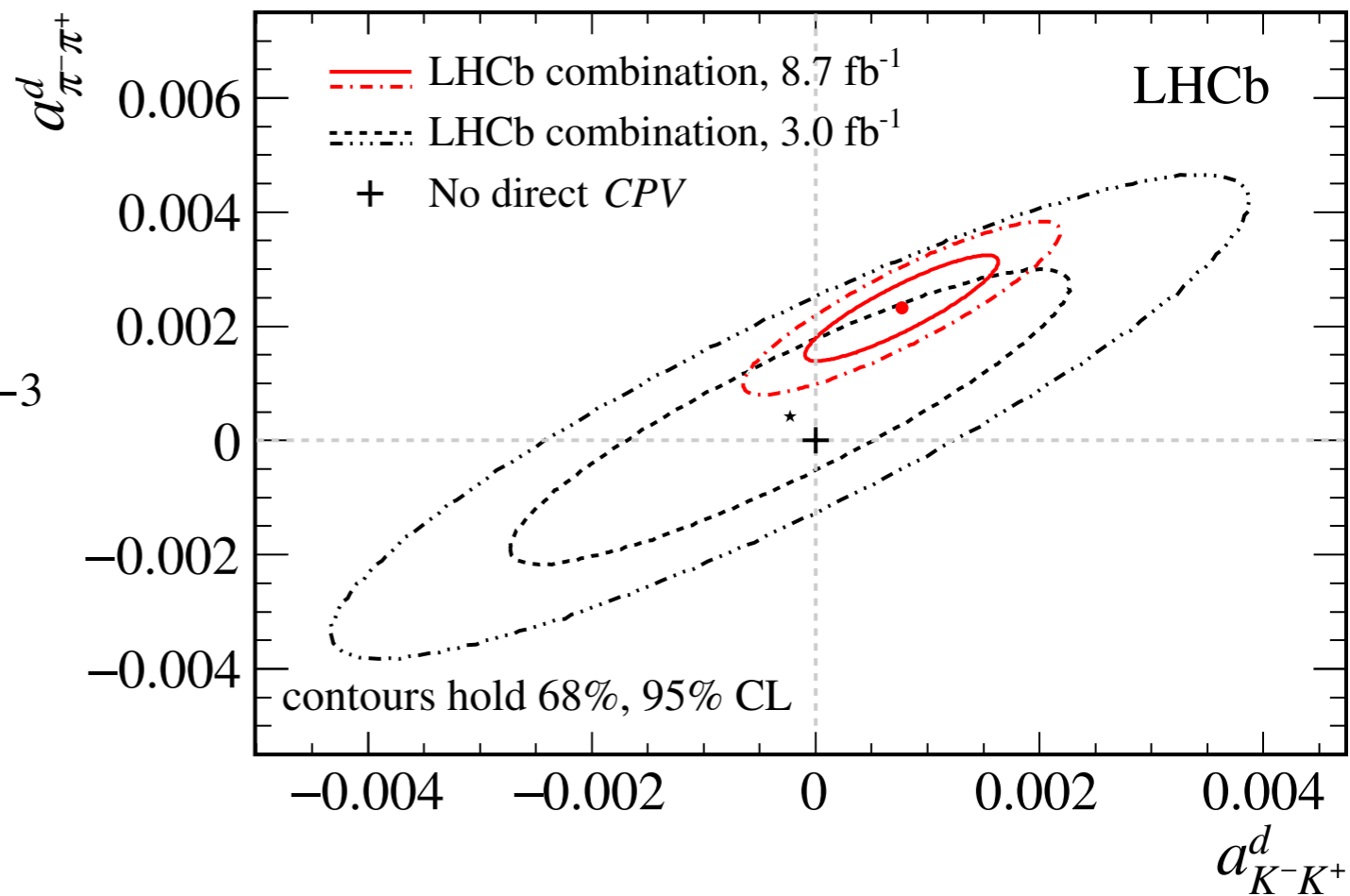
* $\sqrt{1 - \eta^2} \approx 1$ not valid

- direct CP asymmetry observation

$$A_{CP}^{LHCb}(KK) = (0.77 \pm 0.57) \times 10^{-3}$$

$$\hookrightarrow A_{CP}^{LHCb}(\pi\pi) = (2.32 \pm 0.61) \times 10^{-3}$$

[arXiv:2209.03179](https://arxiv.org/abs/2209.03179)

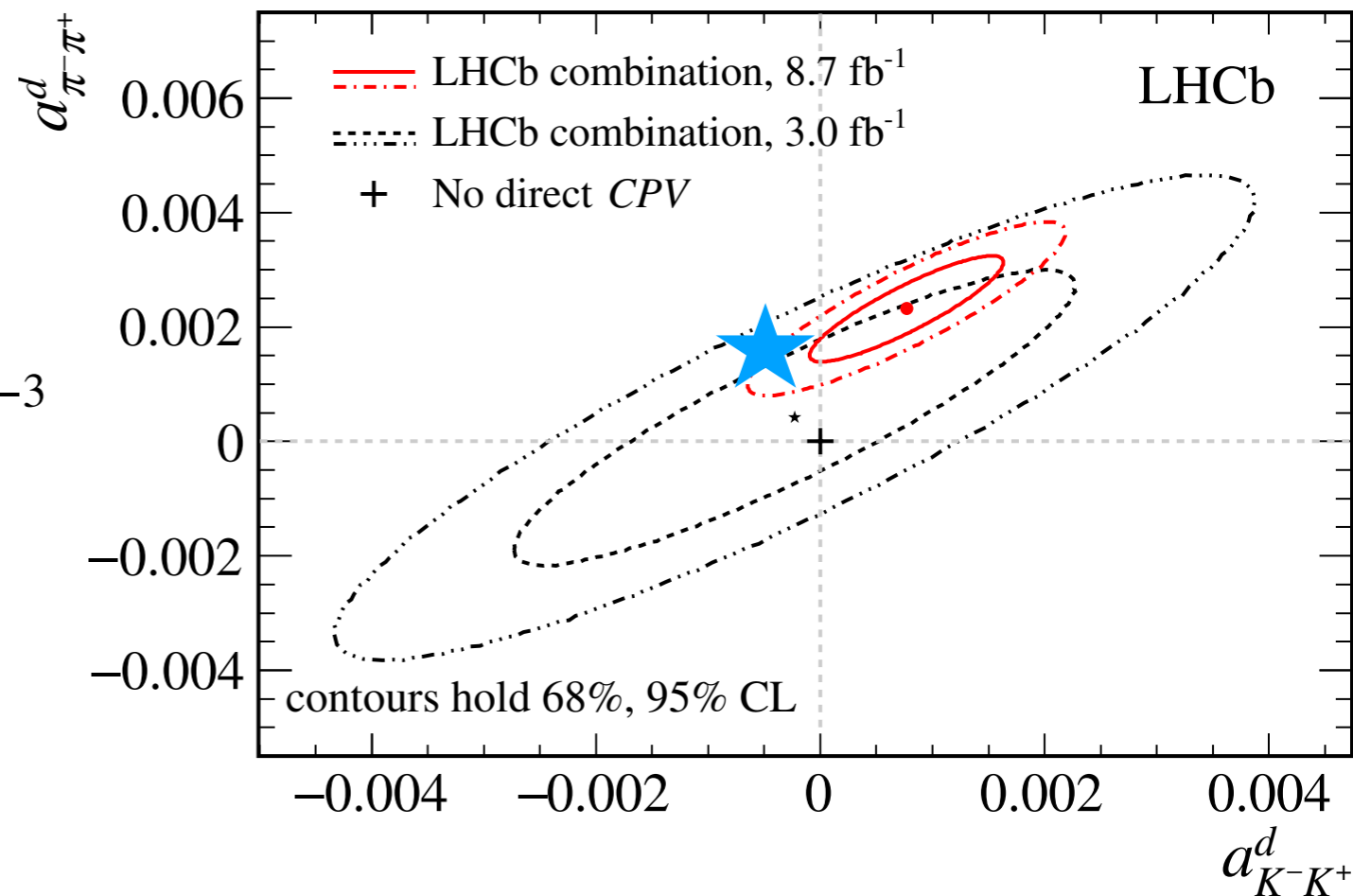


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- with $\eta = 0.78 \pm 0.08$

$$A_{CP}(KK) = - (0.34 \pm 0.15) \times 10^{-3}$$

$$A_{CP}(\pi\pi) = (0.97 \pm 0.05) \times 10^{-3}$$

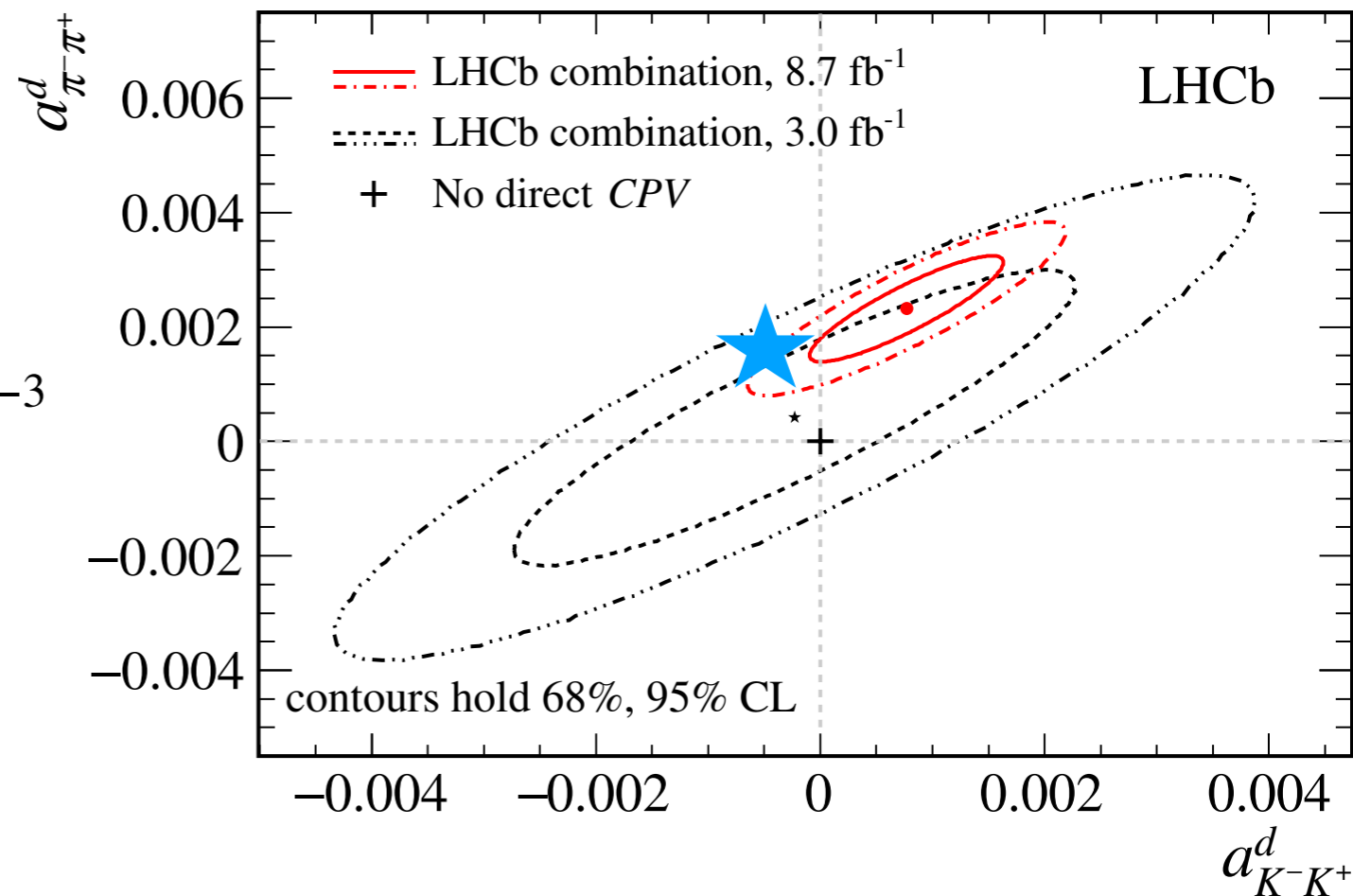
2σ

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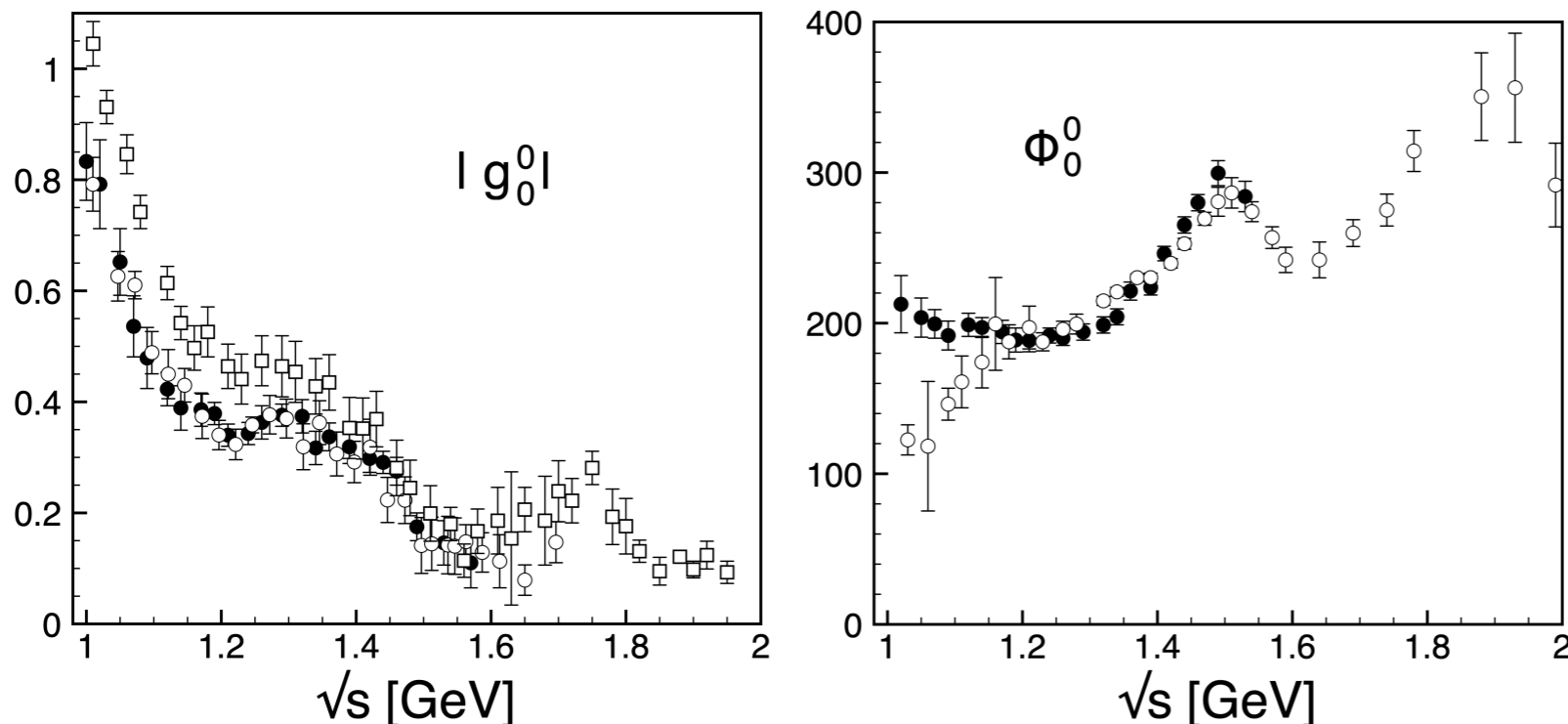
$$A_{CP}(\pi\pi) = (0.97 \pm 0.05) \times 10^{-3}$$

2σ

\hookrightarrow we still need more data to fully understand it

- In 3-body decays this effect will be bigger and phase-space distributed
 - ↳ $D^+ \rightarrow \pi^+ \pi^- \pi^+$ and $D^+ \rightarrow \pi^+ K^- K^+$ have exactly the same Weak vertex
- expected CPV in run II analysis

50 million events...



- **hadronic FSI** (and their strong phases) are crucial to explain CP violation in B and D decays at low and high mass regions
- by including the **FSI $\pi\pi \rightarrow KK$** we are able to identify:
 - one of the GIANT source of CPV observed in $B^\pm \rightarrow K^\pm \pi^+ \pi^-$
 - explain the large CPV observed in $D^0(\bar{D}^0) \rightarrow \pi\pi$ and $K\bar{K}$

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thank you!

Obrigada!

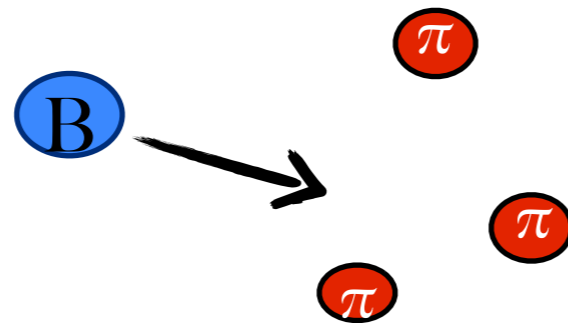


Backup slides

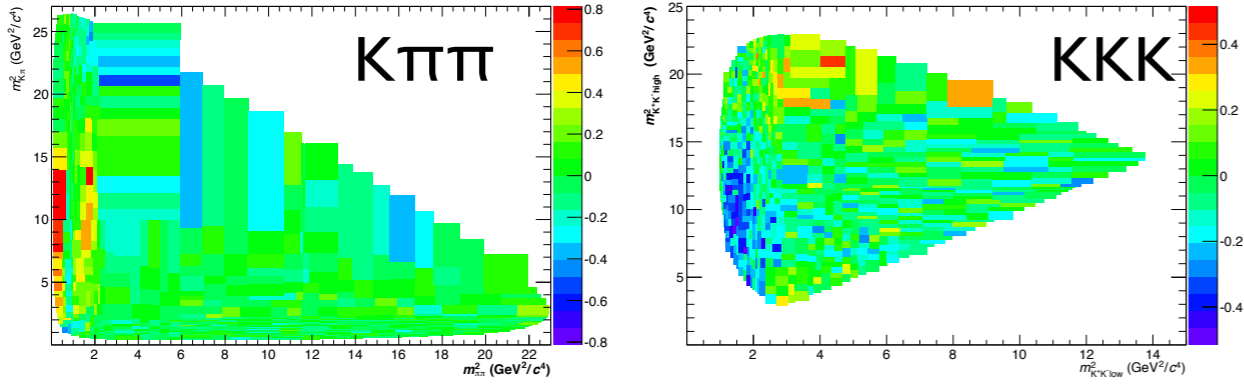
● 2-body decays: 

- energy is fixed by the decay particle
- FSI is relevant at the fix energy (ie is a number)
- CPV is also a number

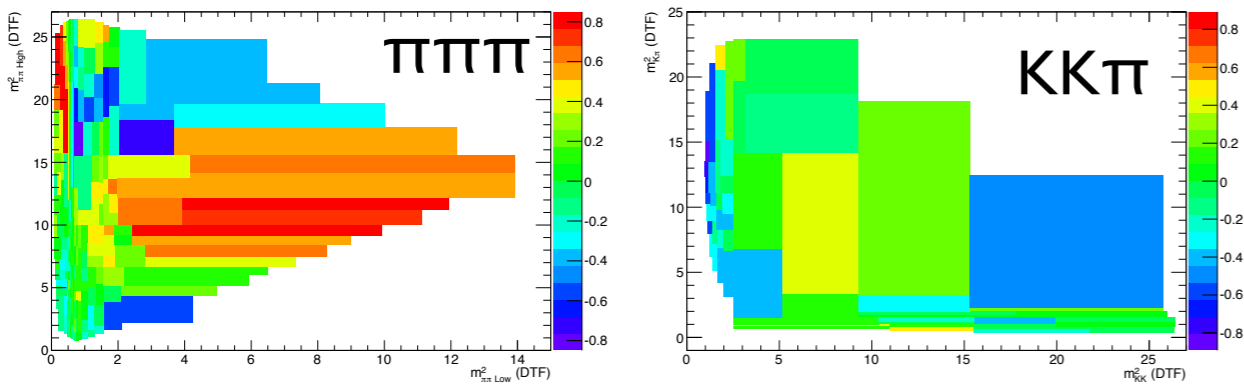
● 3-body decays:



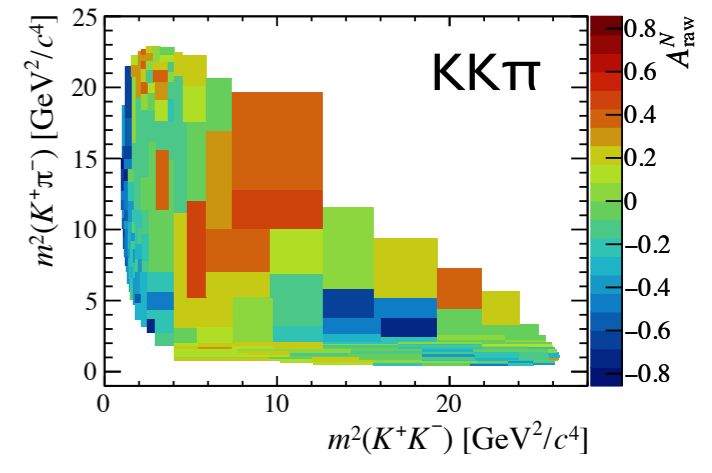
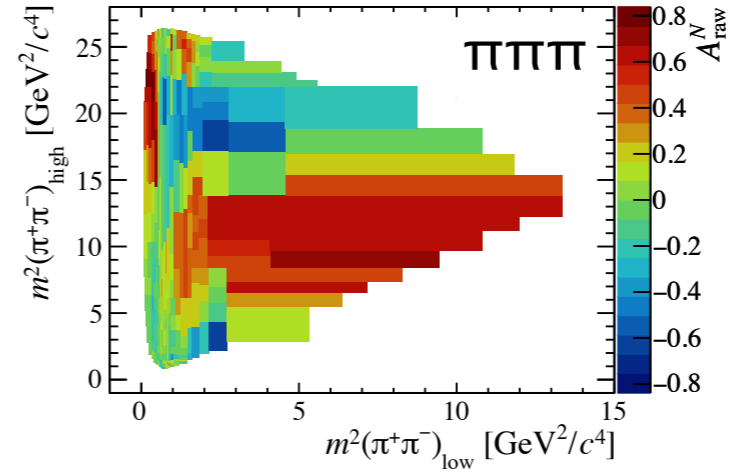
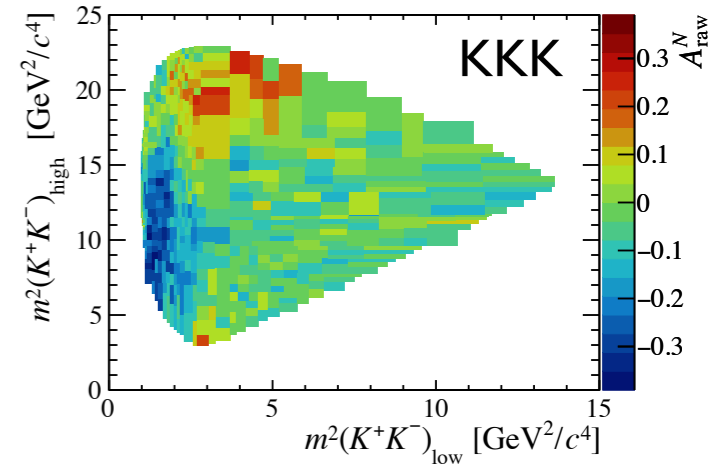
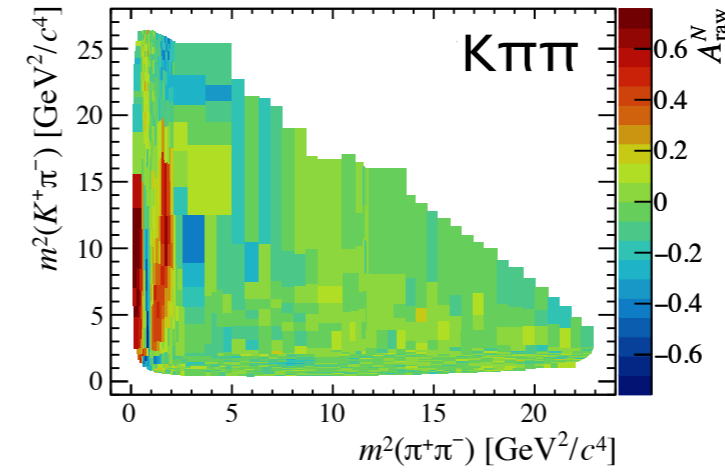
- energy is distributed by particle momenta and $\sum p_i = M_B^2$
 - FSI is a function that depend on the invariant moment of each pair
 - the strong phase contributing to CPV will by a distribution in energy
- ➔ FSI affects more drastically CPV in 3-body



● run I ($3 fb^{-1}$ luminosity)
PRD90 (2014) 112004



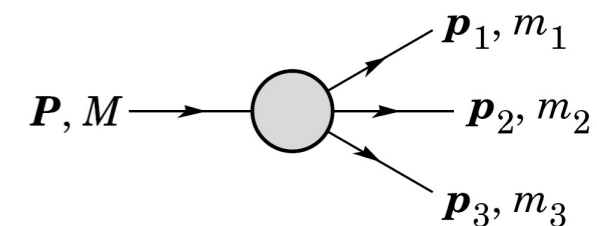
● run II ($5.9 fb^{-1}$ luminosity)
arXiv:2206.07622 PRD accepted



$K\pi\pi$: 2.8
 KKK : 3.3
 $\pi\pi\pi$: 4.1
 $KK\pi$: 5.3

- In three-body decay phase-space is **NOT** one-dimension!

↪ bi-dimension phase-space information



- DALITZ PLOT : proposed by Richard Dalitz (1925-2006) in 1953

Mandelstam variables for 3-body

$$s_{12} = (p_1 + p_2)^2 = m_{12}^2$$

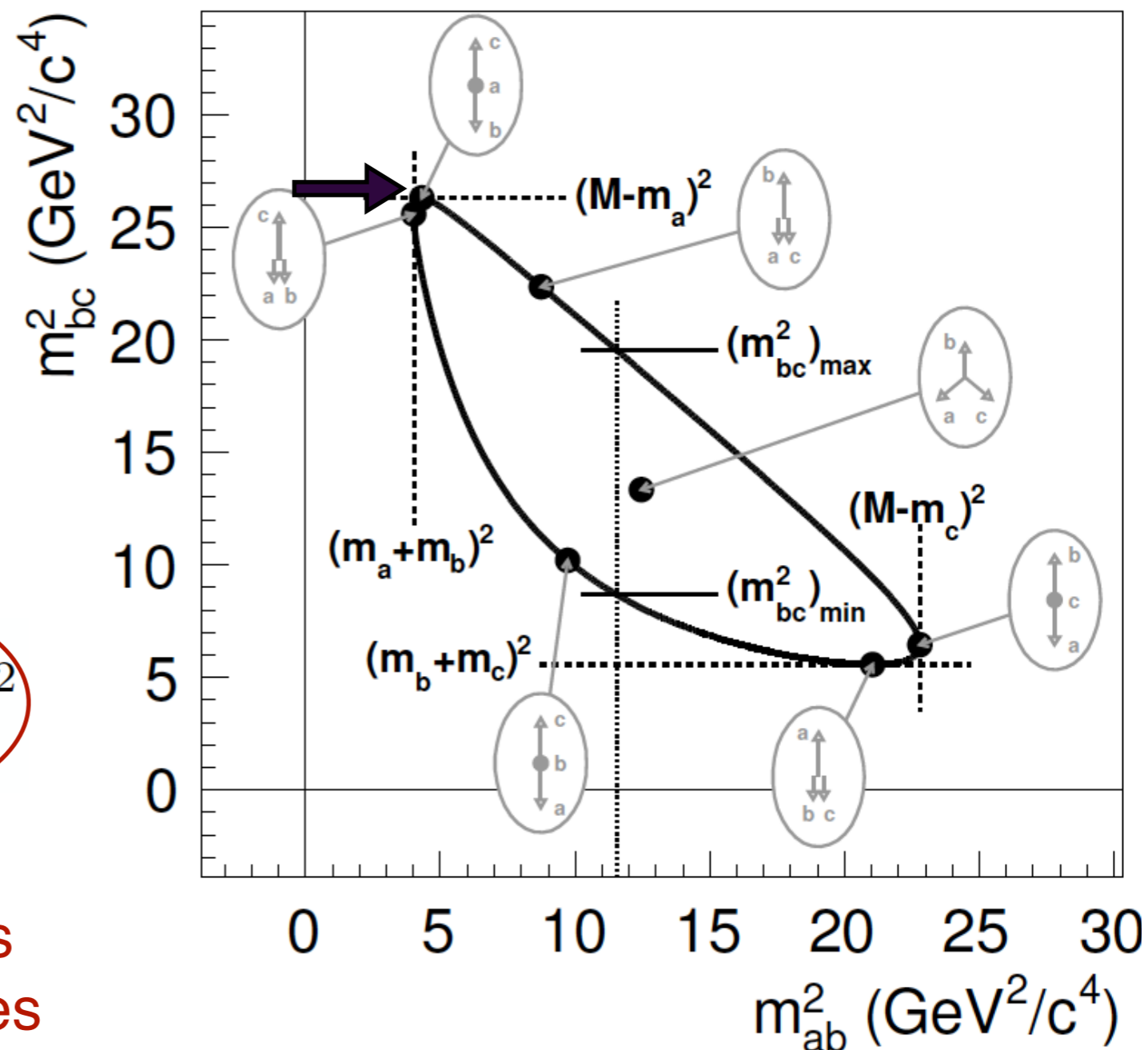
$$s_{13} = (p_1 + p_3)^2 = m_{13}^2$$

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$$s_{12} + s_{13} + s_{12} = M^2 + m_1^2 + m_2^2 + m_3^2$$

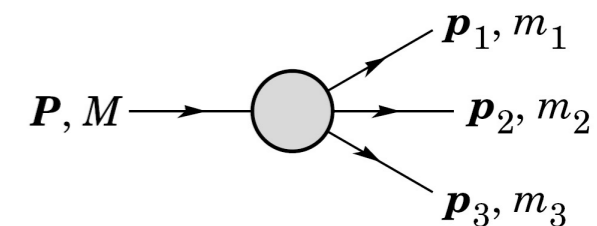
$$\frac{d\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3} \frac{1}{32M^3} |\mathcal{A}(s_{12}, s_{23})|^2$$

dynamics
resonances



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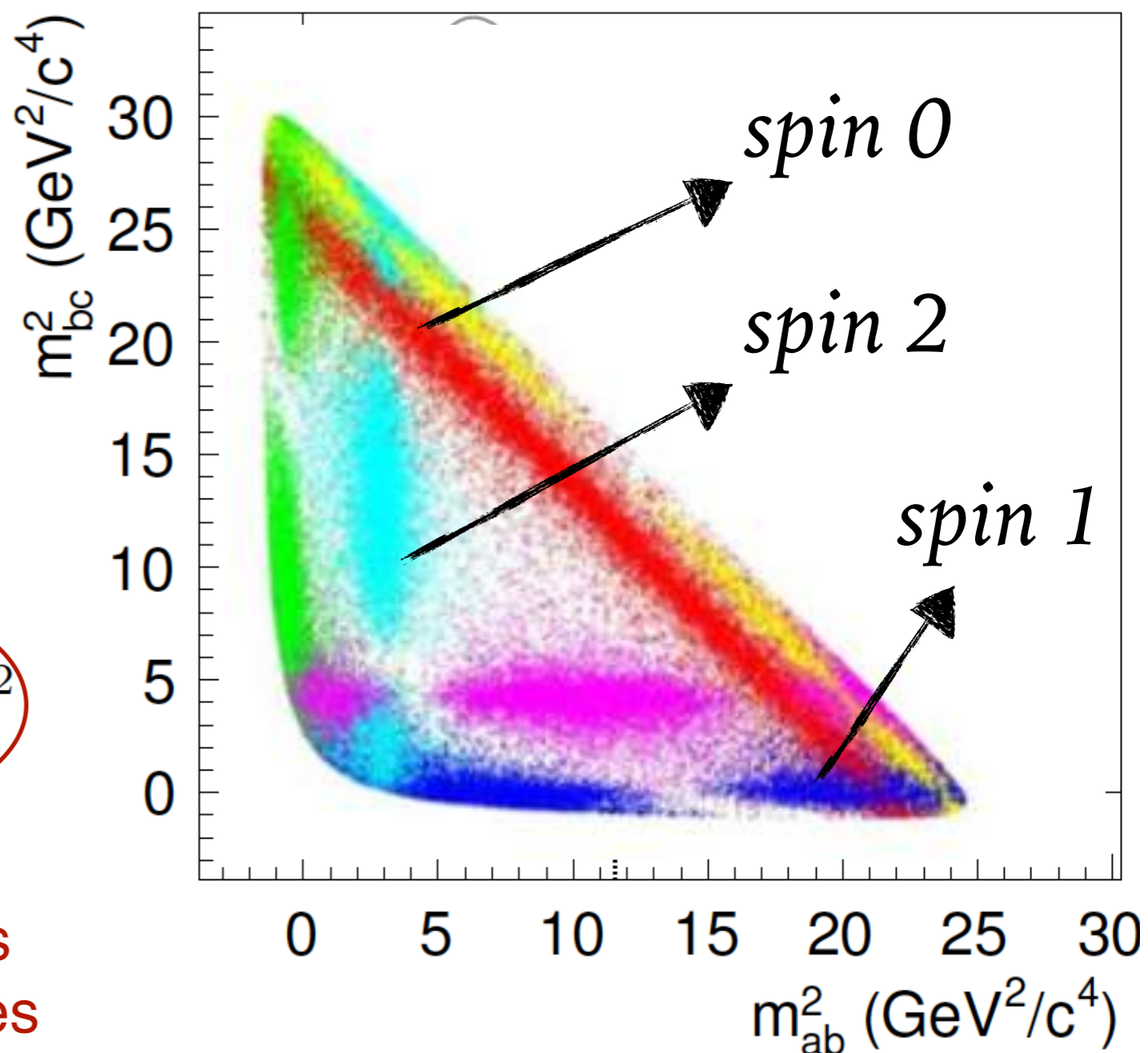
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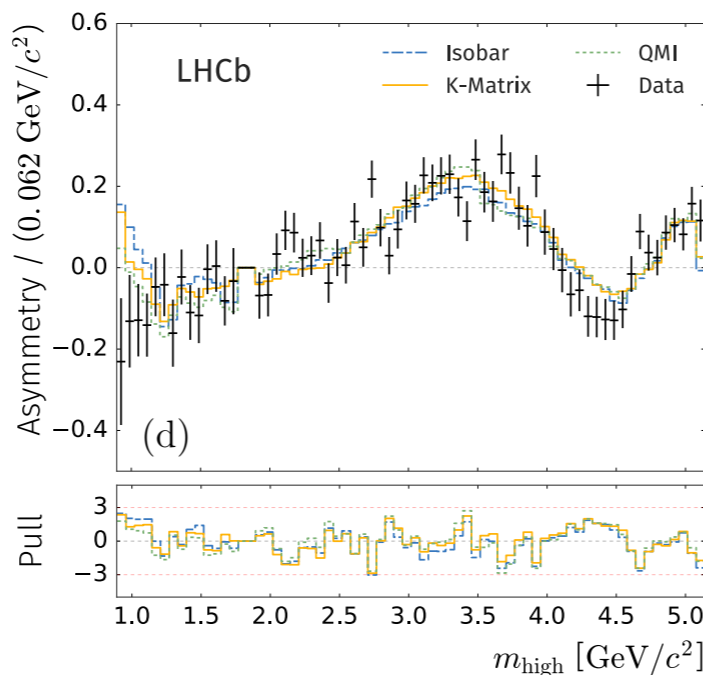
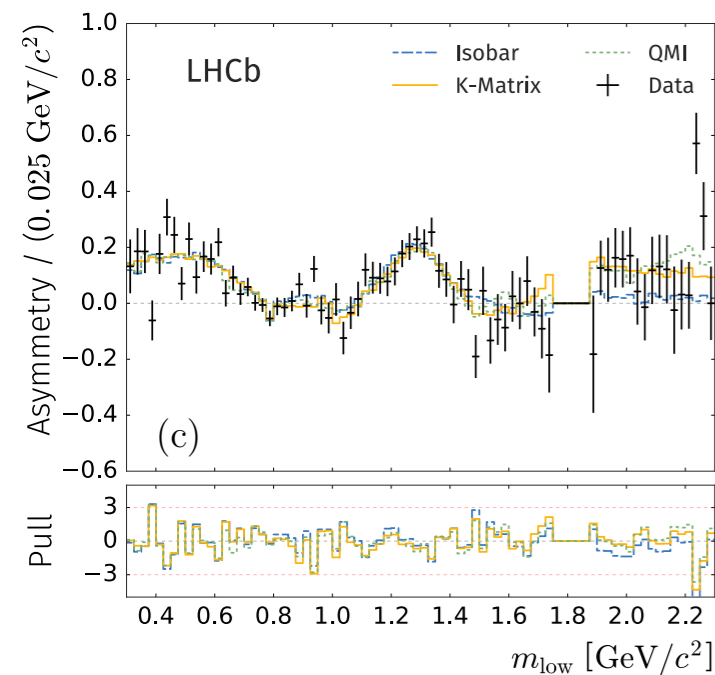
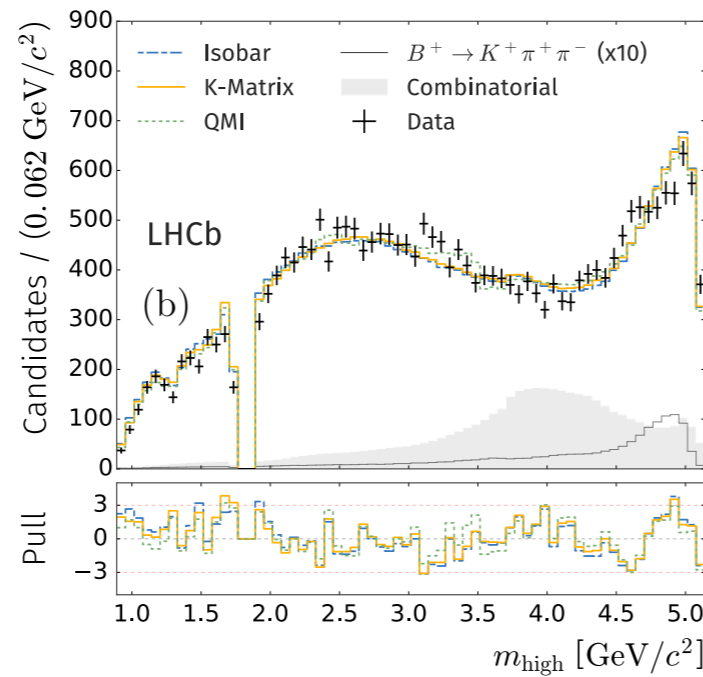
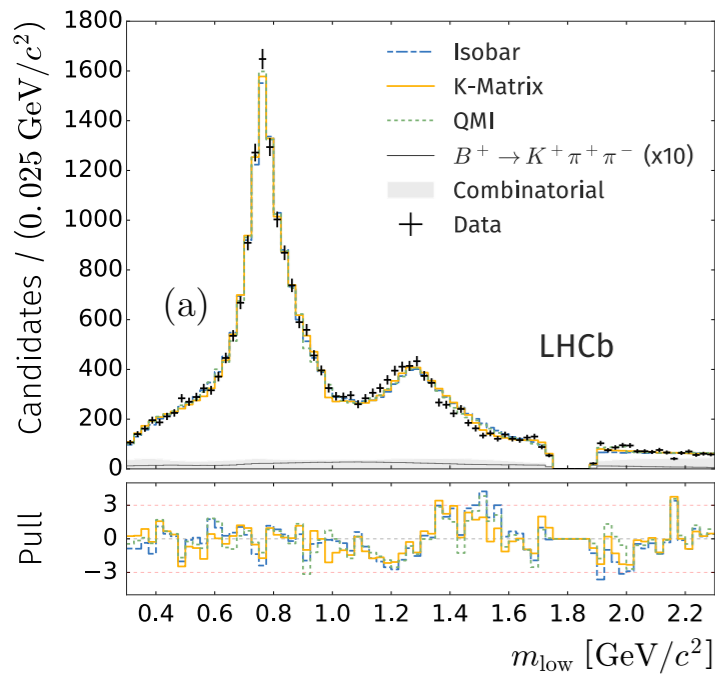


recent Amplitude analysis $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$

PRD101 (2020) 012006; PRL 124 (2020) 031801

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| Contribution | Fit fraction (10^{-2}) | A_{CP} (10^{-2}) | B^+ phase ($^\circ$) | B^- phase ($^\circ$) |
|---------------------|----------------------------|---------------------------|--------------------------|--------------------------|
| Isobar model | | | | |
| $\rho(770)^0$ | $55.5 \pm 0.6 \pm 2.5$ | $+0.7 \pm 1.1 \pm 1.6$ | — | — |
| $\omega(782)$ | $0.50 \pm 0.03 \pm 0.05$ | $-4.8 \pm 6.5 \pm 3.8$ | $-19 \pm 6 \pm 1$ | $+8 \pm 6 \pm 1$ |
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| $\rho(1450)^0$ | $5.2 \pm 0.3 \pm 1.9$ | $-12.9 \pm 3.3 \pm 35.9$ | $+127 \pm 4 \pm 21$ | $+154 \pm 4 \pm 6$ |
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| S-wave | $25.4 \pm 0.5 \pm 3.6$ | $+14.4 \pm 1.8 \pm 2.1$ | — | — |
| Rescattering | $1.4 \pm 0.1 \pm 0.5$ | $+44.7 \pm 8.6 \pm 17.3$ | $-35 \pm 6 \pm 10$ | $-4 \pm 4 \pm 25$ |
| σ | $25.2 \pm 0.5 \pm 5.0$ | $+16.0 \pm 1.7 \pm 2.2$ | $+115 \pm 2 \pm 14$ | $+179 \pm 1 \pm 95$ |
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| $\rho(770)^0$ | $56.5 \pm 0.7 \pm 3.4$ | $+4.2 \pm 1.5 \pm 6.4$ | — | — |
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ANA for $B^\pm \rightarrow \pi^\pm K^- K^+$ PRL 123 (2019) 231802

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| $K_0^{*}(1430)^0$ | $4.5 \pm 0.7 \pm 1.2$ | $+10.4 \pm 14.9 \pm 8.8$ | $0.74 \pm 0.09 \pm 0.09$ | $-176 \pm 10 \pm 16$ |
| Single pole | $32.3 \pm 1.5 \pm 4.1$ | $-10.7 \pm 5.3 \pm 3.5$ | $2.19 \pm 0.13 \pm 0.17$ | $-138 \pm 7 \pm 5$ |
| $\rho(1450)^0$ | $30.7 \pm 1.2 \pm 0.9$ | $-10.9 \pm 4.4 \pm 2.4$ | $2.14 \pm 0.11 \pm 0.07$ | $-175 \pm 10 \pm 15$ |
| $f_2(1270)$ | $7.5 \pm 0.8 \pm 0.7$ | $+26.7 \pm 10.2 \pm 4.8$ | $1.92 \pm 0.10 \pm 0.07$ | $140 \pm 13 \pm 20$ |
| Rescattering | $16.4 \pm 0.8 \pm 1.0$ | $-66.4 \pm 3.8 \pm 1.9$ | $1.91 \pm 0.09 \pm 0.06$ | $-56 \pm 12 \pm 18$ |
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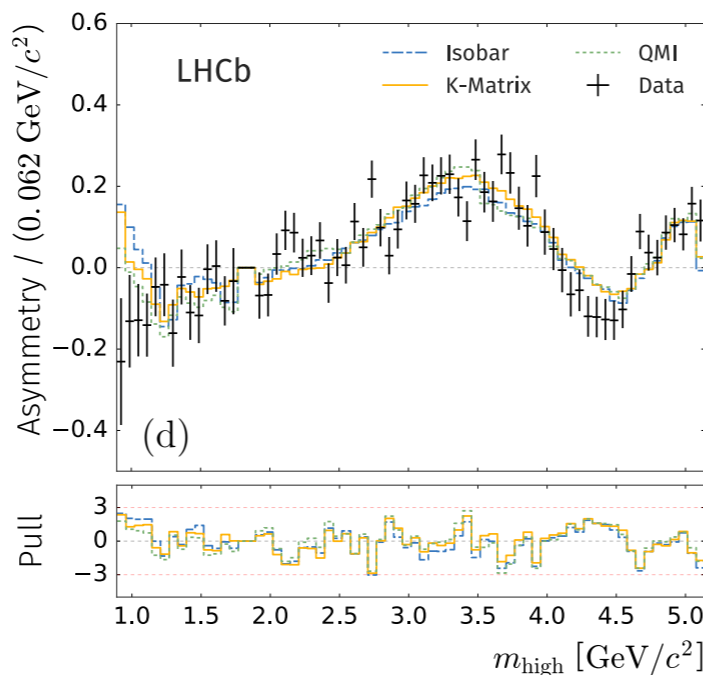
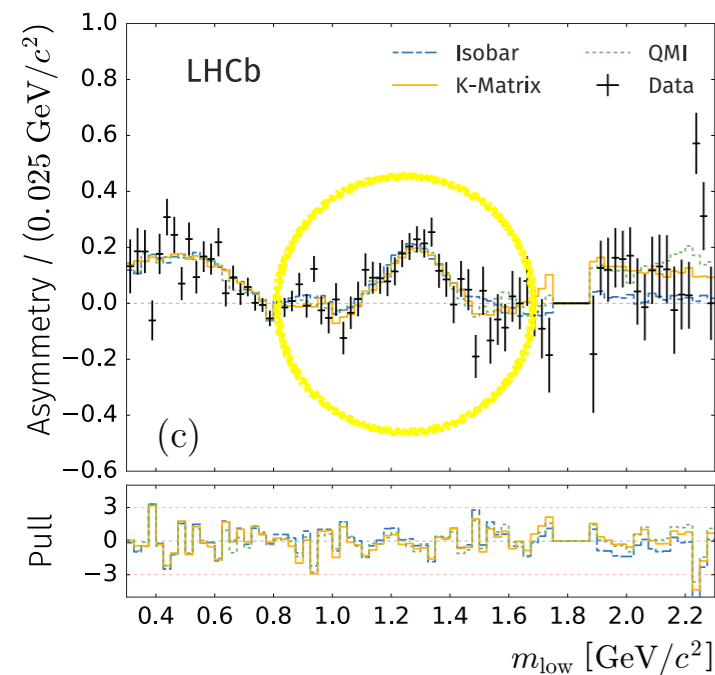
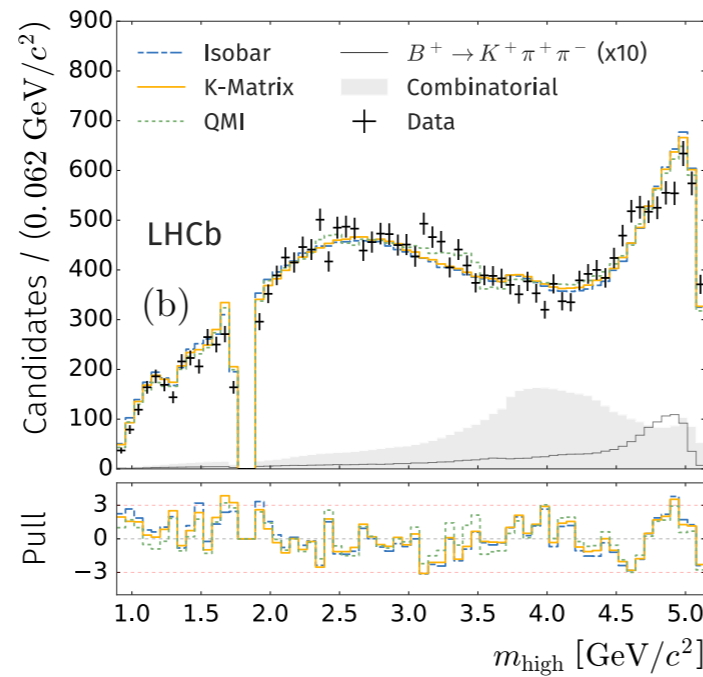
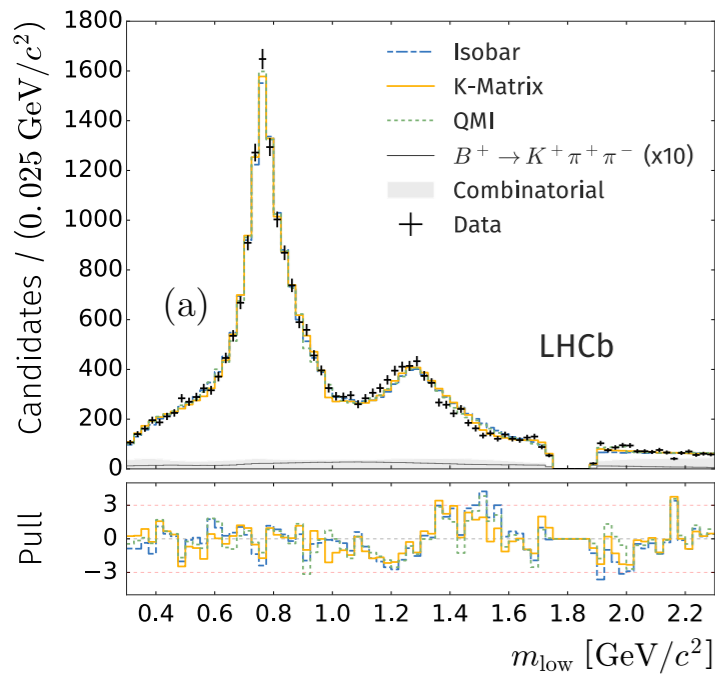


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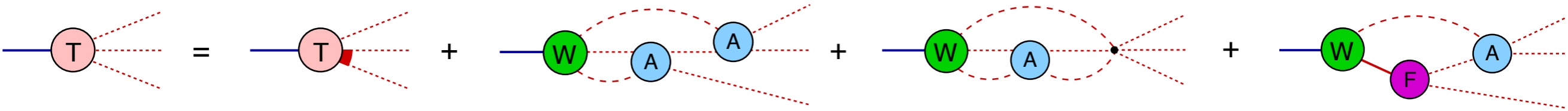


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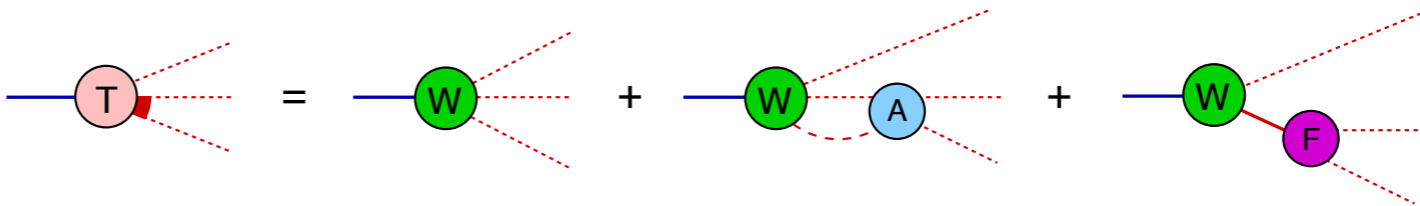
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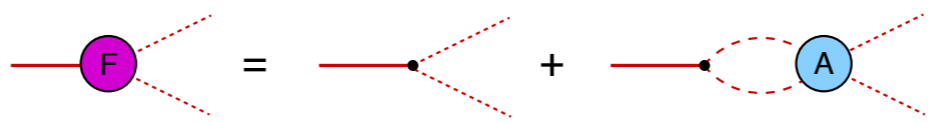
- Any 3-body decay amplitude



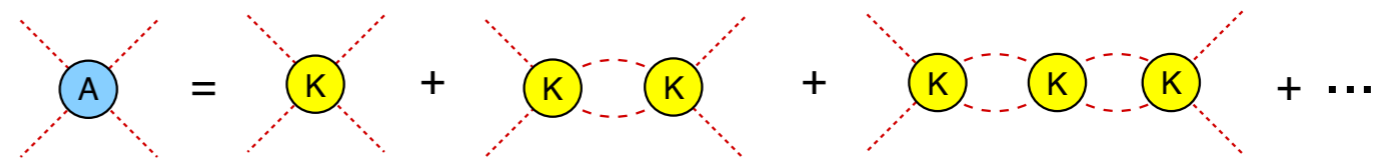
(2+1) approach



Form factor

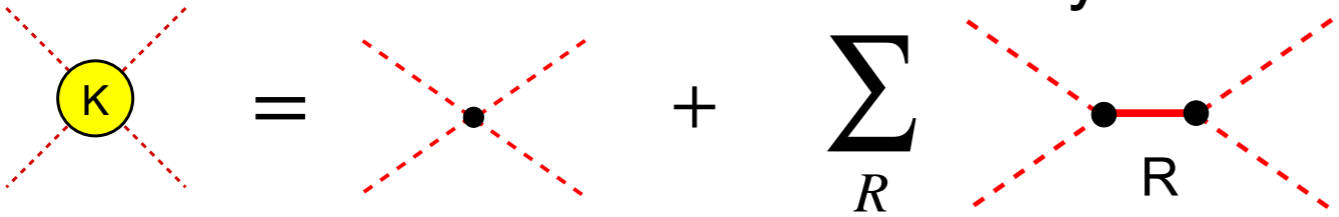


meson-meson

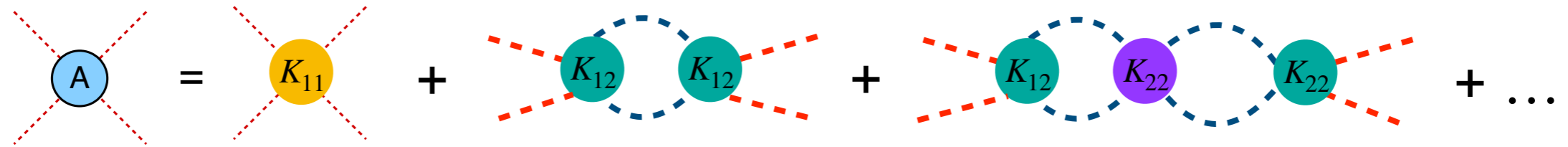


MAGALHAES, A. dos Reis, Robilotta
PRD 102, 076012 (2020)

- kernel should includes all the mm dynamics



- Unitarized amplitude should includes all channels with the same (J,I)



- FSI in $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^+K^-$ can include multiple mesons

- general S-matrix can mix many FSI states
$$S = \begin{pmatrix} S_{2M,2M} & S_{2M,3M} & S_{2M,4M} & \cdots \\ S_{3M,2M} & S_{3M,3M} & S_{3M,4M} & \cdots \\ S_{4M,2M} & S_{4M,3M} & S_{4M,4M} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

- assume only 2 couple-channels to FSI, ie the dominant ones $\pi\pi, K\bar{K}$

$$\rightarrow S_{2M,2M} = \begin{pmatrix} S_{\pi\pi,\pi\pi} & S_{\pi\pi,KK} \\ S_{KK,\pi\pi} & S_{KK,KK} \end{pmatrix}$$

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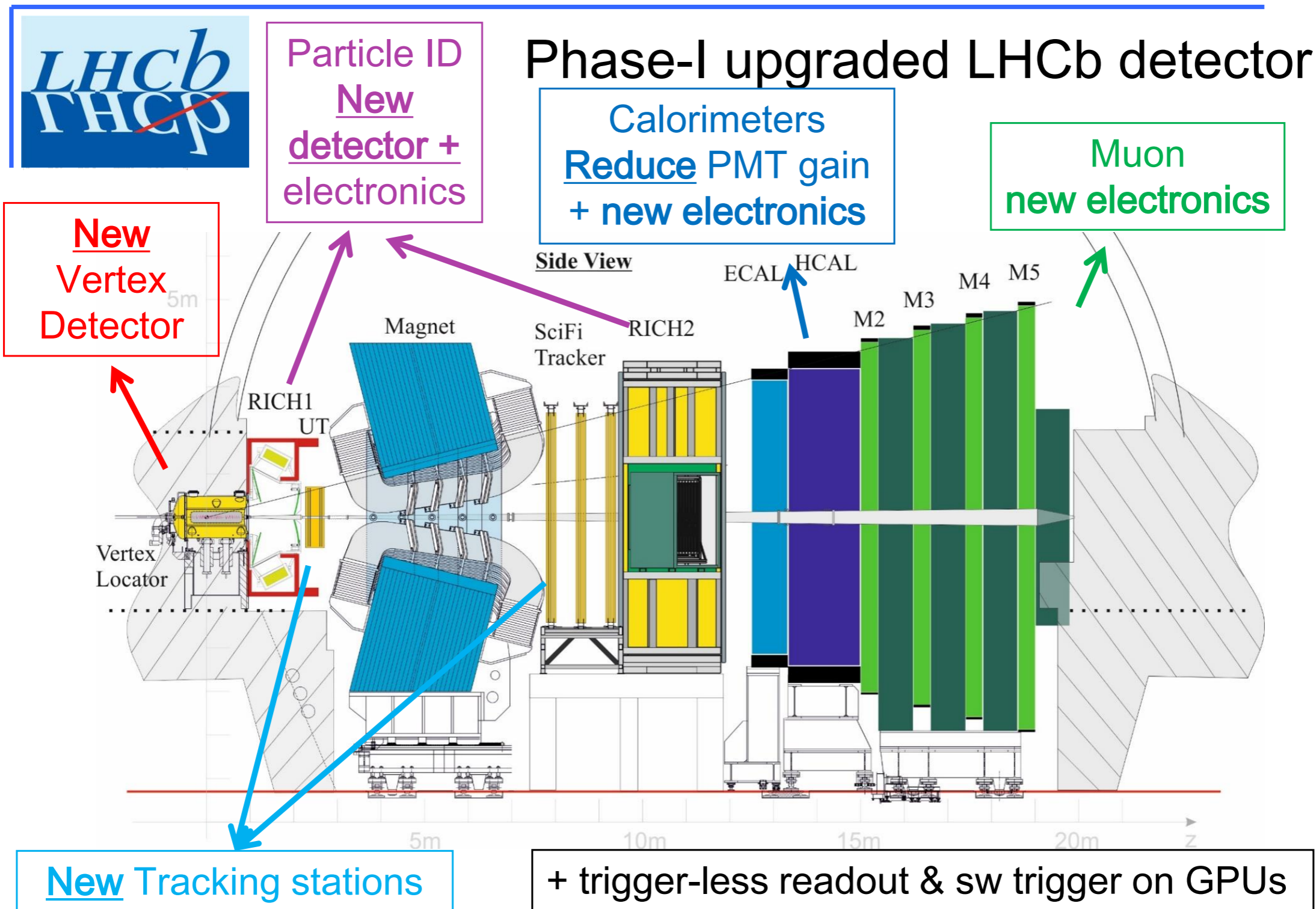
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- CPT constraint restricted to the two-channels:
$$\sum_{f=(\pi\pi,KK)} (|\mathcal{A}_{D^0 \rightarrow f}|^2 - |\mathcal{A}_{\bar{D}^0 \rightarrow f}|^2) = 0$$



[back](#)

- R1, R2 make 2 approximations:

1 only 2-channel can interact $(S_{\lambda\lambda'}) = \begin{pmatrix} \eta e^{2i\delta_{11}} & i\sqrt{1-\eta^2} e^{i(\delta_{11}+\delta_{22})} \\ i\sqrt{1-\eta^2} e^{i(\delta_{11}+\delta_{22})} & \eta e^{2i\delta_{22}} \end{pmatrix}$

↪ $\delta_{\pi\pi KK} \simeq \delta_{\pi\pi\pi\pi} + \delta_{KKKK}$

↪ $|S_{\pi\pi KK}| \rightarrow \sqrt{1-\eta^2}$

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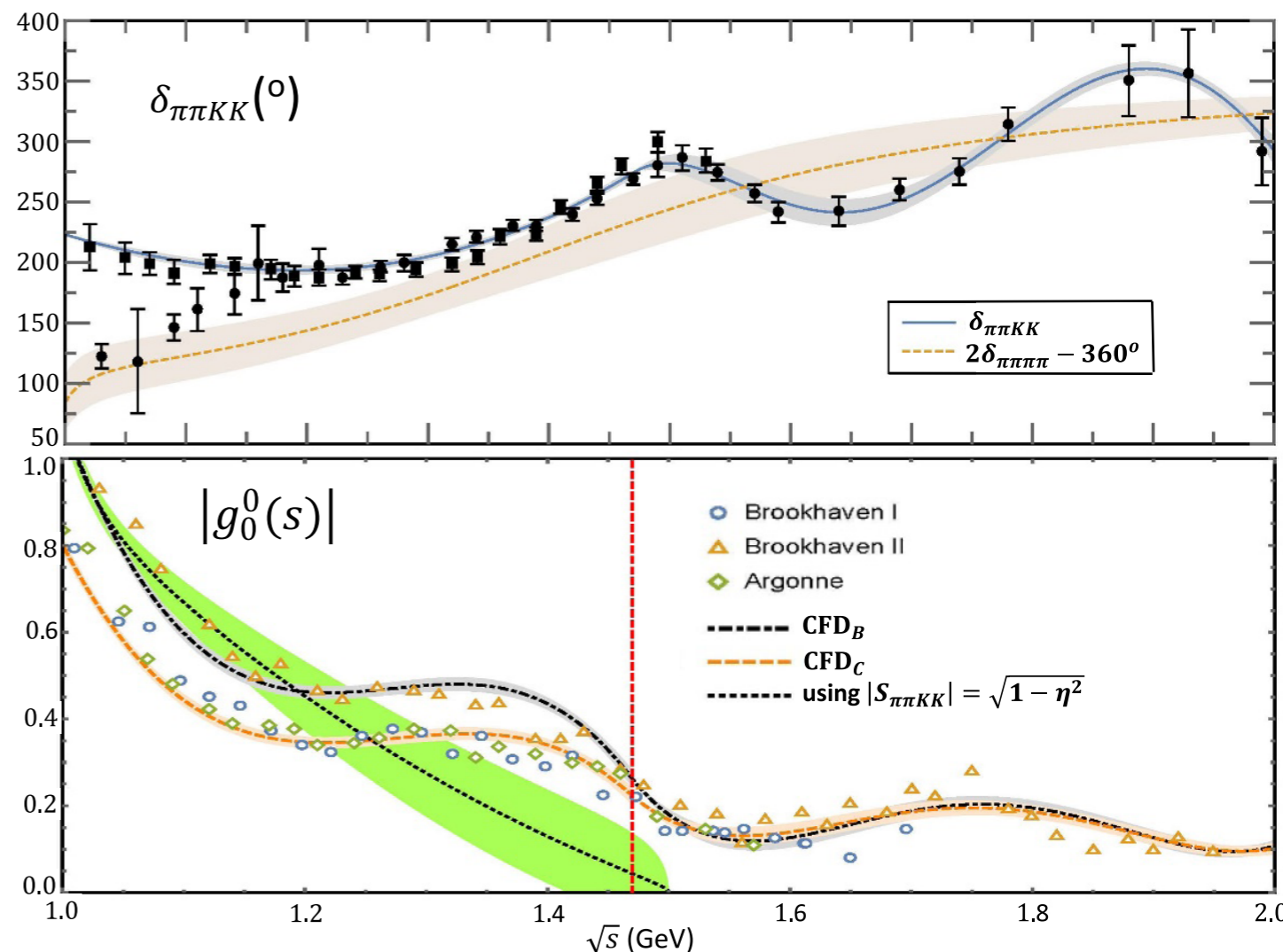
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- Are the conclusions still valid?
We can do better!

