A confining holographic QCD model for vector mesons and nucleons

Alfonso Ballon Bayona

Instituto de Física
Universidade Federal do Rio de Janeiro

In collaboration with **Adão S. da Silva Junior (IF-UFRJ)**

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Summary

1. Introduction

Quantum Chromodynamics (QCD): The theory of strong interactions

$$L_{QCD} = \bar{\psi}_f \left[\, i\,\gamma^{\mu} D_{\mu} - m_f \,\right] \psi_f - \frac{1}{2} \mathrm{Tr}\left[\mathrm{F}_{\mu\nu} \mathrm{F}^{\mu\nu} \right]$$

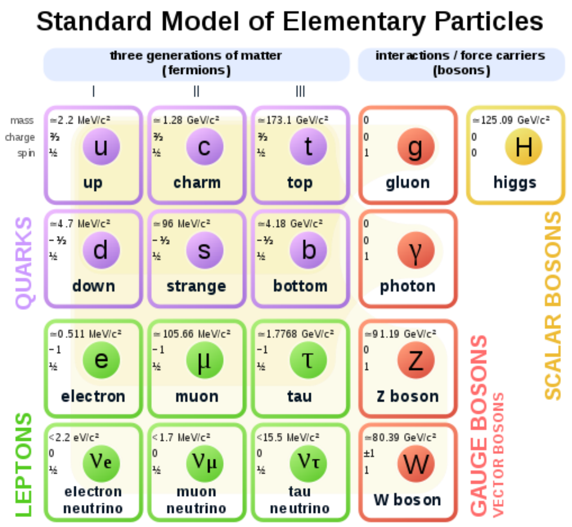
$$D_{\mu} = \partial_{\mu} - igA_{\mu} \;\;,\;\; F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} - ig\left[A_{\mu}, A_{\nu}\right]$$

Quarks are Dirac spinors ψ_f

Gluons are non-Abelian gauge fields $A_{\mu} = A_{\mu}^{a} T^{a}$

QCD is invariant under the local $SU(3)$ colour symmetry

Quarks and gluons are the elementary particles of the **standard model**



Asymptotic freedom and confinement

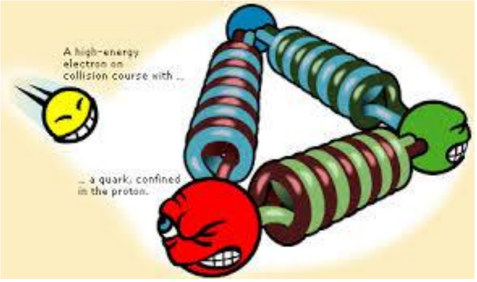
The QCD beta function

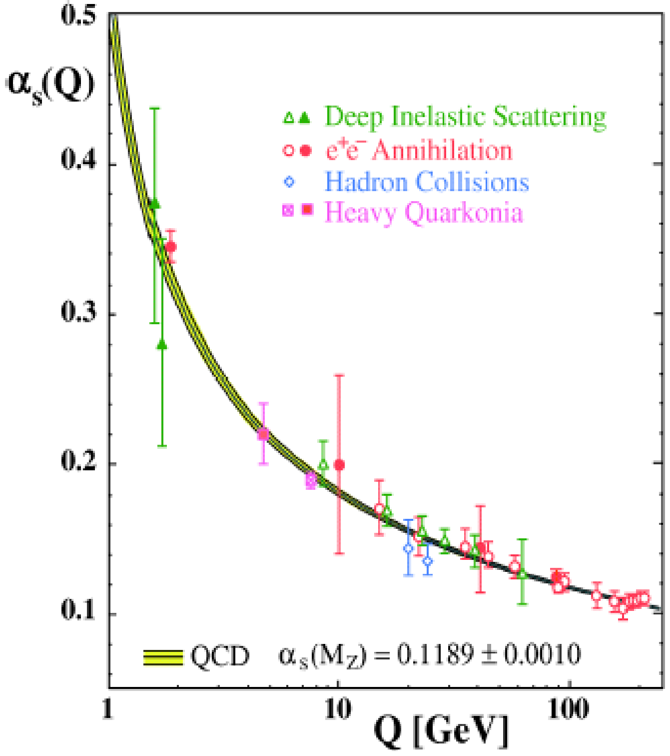
1- loop result

$$\beta_g \equiv \frac{dg}{d\,\mathrm{Log}\,\mu} = -\frac{g^3}{(4\pi)^2}\left[\frac{11}{3}N_c - \frac{2}{3}N_f\right]$$

$N_f = 6$ e $N_C = 3 \rightarrow \beta_g < 0$

Asymptotic freedom in the UV and confinement in the IR

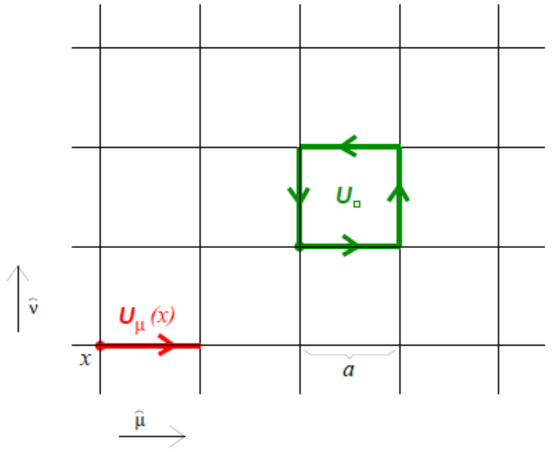




Bethke hep-ex/0606035

Non-perturbative approaches to QCD

- Lattice QCD:



Wilson 1974

- Dyson-Schwinger equations:

Dyson 1949,
Schwinger 1951

$$\left(\frac{\delta S[\phi]}{\delta \phi}\Big|_{\phi=\frac{\delta}{\delta J}} - \mathbf{J}\right)\mathbf{Z}[\mathbf{J}] = \mathbf{0}$$

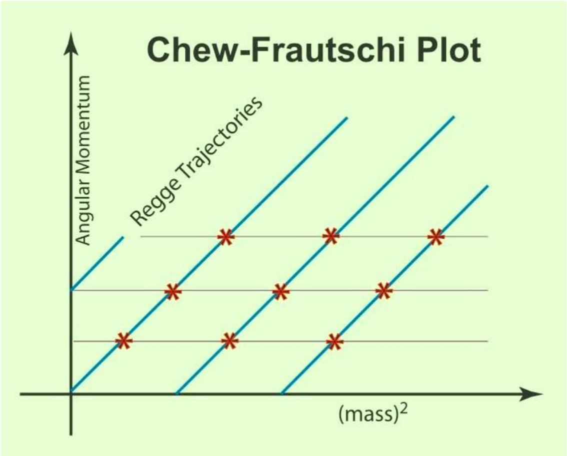
- Other approaches: *Chiral lagrangians, Nambu-Jona-Lasinio model, Light Front QCD, QCD sum rules, RG flow, …*

Hadrons and string theory

The spectrum of **baryons and mesons** organize approximately into **Regge trajectories**

$$J = J_0 + \alpha' M^2$$

Chew & Frautschi 1962



J : spin , M : mass α' : Regge slope

This can be obtained from **1d** objects (**strings**)

Nambu, Nielsen & Susskind 1969-1970

Problem: **massless spin 2 particle** → **string theory is a theory of gravity**

Veneziano scattering amplitude

$$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$$

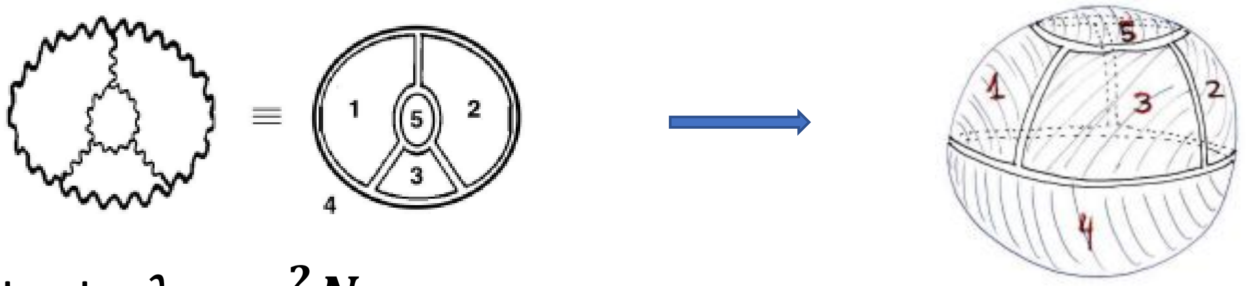
This simple amplitude satisfies the **s—t** duality and has the asymptotic behaviour $s^{J(t)}$ in the Regge limit , expected for hadronic scattering.

This amplitude is obtained from **scattering of strings** *Veneziano & many others 1968-1970*

Yang-Mills/string duality

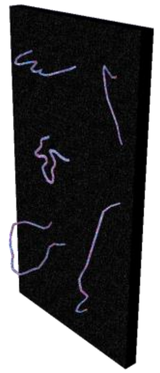
Feynman diagrams of $SU(N_c)$ gauge theories in the large N_c limit can be thought in terms of string theory

't Hooft 1974



't Hooft constant: $\lambda = g^2 N_C$

The AdS/CFT correspondence



D-branes relate the physics of open strings with the physics of closed strings

Polchinski 1995

AdS/CFT is a concrete realization of the **Yang-Mills/string duality**

Maldacena 1997

$SU(N_c)$ theory in the **large** N_c limit with **conformal symmetry** in d dimensions ⟷ string theory in **Anti-de-Sitter** space in d+1 dimensions

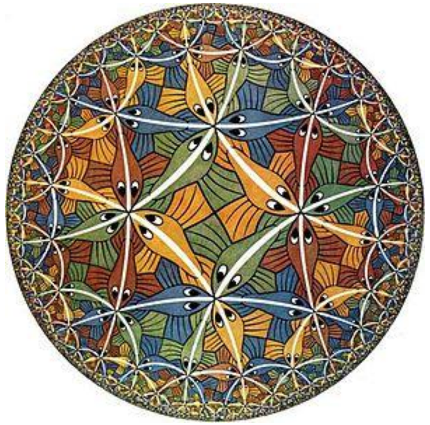
Gauge/gravity duality

In the regime $\lambda \gg 1$ string theory becomes **a classical gravitational theory**

E.g: **4-d** $N = 4$ super Yang-Mills ⟷ **supergravity IIB** in $AdS_5 \times S^5$

The AdS/CFT dictionary

Conformal symmetry group $\boldsymbol{SO(2,4)}$ becomes the $\boldsymbol{AdS_5}$ isometry group



$\boldsymbol{AdS_5}$ in **Poincaré** coordinates

$$\boldsymbol{ds^2 = \frac{R^2}{z^2}[dz^2 - dt^2 + d\,\bar{x}^2]}$$

Scale transformation $\boldsymbol{x^\mu \to \lambda\, x^\mu\, ,\, z \to z\,\lambda}$

Fields $\boldsymbol{\phi_{\dots}}$ in $\boldsymbol{AdS_5}$ couple on the **boundary** with operators $\boldsymbol{O_{\dots}}$ of the $\boldsymbol{CFT_4}$

$\boldsymbol{CFT_4}$ **partition function** ↔ gravitational **path integral** in $\boldsymbol{AdS_5}$

$$\boldsymbol{Z_{CFT}\left[\phi^0_{\dots}, g^0_{\mu\nu}\right] = Z_{AdS}[\phi_{\dots}, g_{\mu\nu}]}$$

Gubser-Klebanov-Polyakov 1998,
Witten 1998

Holographic QCD

Top-down approach

$\boldsymbol{N=4}$ super Yang-Mills in 4d	⟷	IIB string theory in $\boldsymbol{AdS_5 \times S^5}$
↕		↕
QCD-like theories	⟷	deformed string theories

E.g. Klebanov-Witten (1998), Klebanov-Strassler (2000), Maldacena-Nunez (2000), Sakai-Sugimoto (2004), Kuperstein-Sonnenschein (2004)

Bottom-up approach

CFT in 4d	⟷	Einstein gravity in $\boldsymbol{AdS_5}$
↕		↕
QCD as a **deformed CFT**	⟷	**Gravity** in a **deformed** $\boldsymbol{AdS_5}$ space

E.g. Polchinski-Strassler (2000), Erlich-Katz-Son-Stephanov (2005), Karch-Katz-Son-Stephanov (2006), Gursoy-Kiritsis-Nitti (2007), Gubser-Nellore (2008)

Confinement in Einstein-dilaton holographic QCD

Einstein-dilaton gravity in 5d:

$$S = \sigma \int d^5x \sqrt{-g} \left[R - \frac{4}{3} (\partial_m \Phi)^2 + \mathrm{V}(\Phi) \right]$$

5d ansatz in holographic QCD:

$$ds^2 = \frac{1}{\zeta(z)^2} \left[-dt^2 + d\vec{x}^2 + dz^2 \right] \;, \qquad \Phi = \Phi(z)$$

Independent field equations:

$$\zeta'' - \frac{4}{9} \zeta \Phi'^2 = 0$$

$$V - \zeta^5 \left(\zeta^{-3} \right)'' = 0$$

Warp factor in the Einstein frame

$$A(z) = -ln\, \zeta$$

Linear confinement is guaranteed by a quadratic dilaton asymptotic behaviour

$$\Phi(z \to \infty) = kz^2$$

Karch-Katz-Son-Stephanov 2006, Gursoy-Kiritsis-Nitti 2007

In this work we consider the following **analytical solutions:**

$$\Phi_I = kz^2 \quad , \qquad \zeta_I(z) = \Gamma\left(\frac{5}{4}\right)\left(\frac{3}{k}\right)^{1/4} \sqrt{z}\, I_{\frac{1}{4}}\left(\frac{2}{3}kz^2\right)$$ (model I)

$$\Phi_{II} = \frac{1}{2}\sqrt{k}\, z\sqrt{9+4kz^2} + \frac{9}{4}\sinh^{-1}\left(\frac{2}{3}\sqrt{kz}\right) \quad , \qquad \zeta_{II}(z) = z\, exp\left(\frac{2}{3}kz^2\right)$$ (model II)

Both models leads to a quadratic dilaton at large z and AdS asymptotics at small z

Warp factor in the string frame:

$$A_s(z) = -ln\, \zeta + \frac{2}{3}\Phi$$

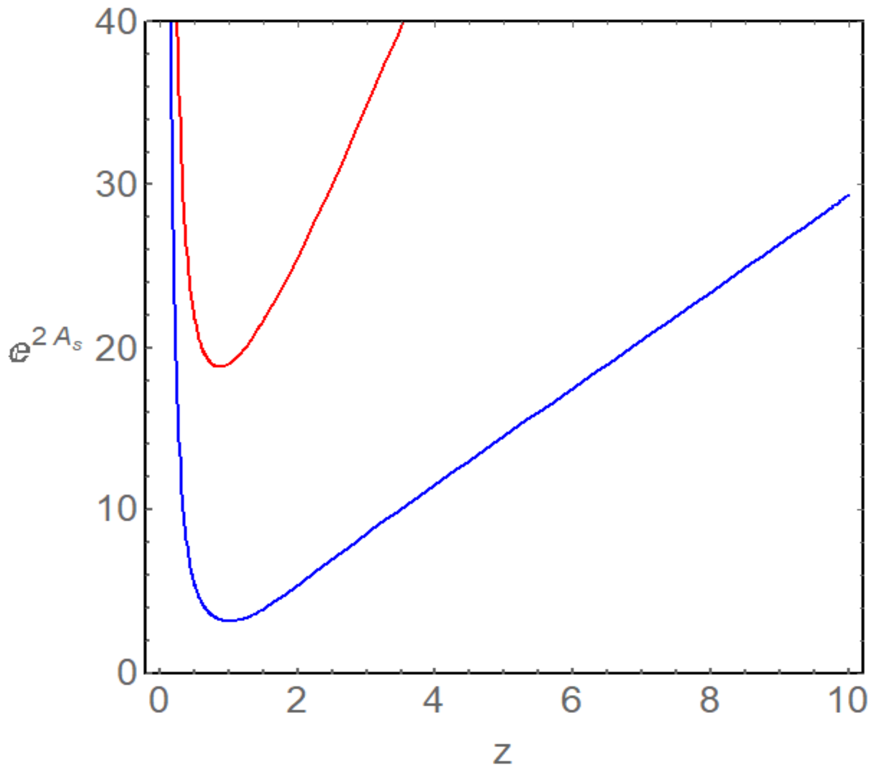
Confinement criterion

The function $\boldsymbol{f(z) = \sqrt{g_{tt}g_{xx}}}$ defined in the string frame should have a minimum $\boldsymbol{f(z^*) > 0}$

Kinar, Schreiber and Sonnenschein 1998

In terms of the warp factor we have $\boldsymbol{f(z) = exp(2A_s)}$

As shown in the figure, both models satisfy the confinement criterion



2. Vector mesons in confining holographic QCD

Pioneer works in the bottom-up approach:

Erlich-Katz-Son-Stephanov 2005, Grigoryan-Radyushkin 2007

Consider the vectorial currents associated with $\boldsymbol{SU(2)}$ isospin symmetry

$$\boldsymbol{< J^{\mu,c} > = < \bar{q}(x)\gamma^{\mu}T^{c}q(x) > =< J_R^{\mu,c} >+< J_L^{\mu,c} >}$$

These currents are responsible for the creation of vector mesons

In holographic QCD the $SU(2)_L \times SU(2)_R$ chiral symmetry is described by the Yang-Mills action

$$\boldsymbol{S = -\frac{1}{4g_5^2}\int d^4x\, dz\, \sqrt{-g}e^{-\Phi}\mathrm{Tr}\left({F_{mn}^{R}}^{2} + {F_{mn}^{L}}^{2}\right)}$$

Expanding this action at second order in the perturbations, we find in the **vectorial sector**

$$\boldsymbol{S_V = -\frac{1}{4g_5^2}\int d^4x\, dz\, \sqrt{-g}e^{-\Phi}{v_{mn}^{c}}^{2} = -\frac{1}{4g_5^2}\int d^4x\, dz\, e^{A_s-\Phi}{v_{\hat{m}\hat{n}}^{c}}^{2}}$$

where $v_{\hat{m}\hat{n}}^{c} = \partial_{\hat{m}}V_{\hat{n}}^{c} - \partial_{\hat{n}}V_{\hat{m}}^{c}$ and the indices $\hat{m}, \hat{n}$ are contracted with a 5d Minkowski metric

The 5d gauge coupling is fixed as

$$\boldsymbol{g_5^2 = \frac{12\pi^2}{N_c}}$$

to reproduce the large N QCD perturbative result for the current correlator at large energies

Varying the action one finds the field equation

$$\boldsymbol{\partial_m\left(e^{A_s-\Phi} v_c^{\hat{m}\hat{n}}\right) = 0}$$

as well as the surface term

$$\boldsymbol{\delta S_V = -\frac{1}{g_5^2}\int d^4x\, dz\, \partial_{\hat{m}}(e^{A_s-\Phi} v_c^{\hat{m}\hat{n}}\, \delta V_{\hat{n}}^c)}$$

The 5d vectorial field can be decomposed as

$$\boldsymbol{V_{\hat{m}}^c = \left(V_{\hat{z}}^c, V_{\hat{\mu}}^c\right) \quad , \quad V_{\hat{\mu},c} = V_{\hat{\mu},c}^{\perp} + \partial_{\hat{\mu}}\xi^c}$$

We can use the gauge symmetry to fix $\boldsymbol{V_z^c = 0}$ and it turns out that $\boldsymbol{\xi^c = 0}$ is the only consistent solution for the longitudinal part

The equation for the transverse sector takes the form (in momentum space)

$$\left[(\partial_z + A_s' - \Phi')\partial_z - q^2\right]V_\perp^{\hat{\mu},c} = 0$$

Taking the ansatz

$$V_\perp^{\hat{\mu},c}(q,z) = e^{-B_V(z)}\eta^{\hat{\mu}}\psi_V(q,z) \quad , \qquad B_V = \frac{1}{2}(A_s - \Phi)$$

the equation takes the Schrödinger form

$$\left[\partial_z^2 - q^2 - V_V\right]\psi_V = 0$$

with the potential given by

$$V_V = B_V'' + B_V'^2$$

The **VEV of the current operator** can be obtained from the surface term in the action variation

$$< J^{\hat{\mu},c}(x) > = \frac{\delta S_V}{\delta V_{\hat{\mu},c}^{\perp,0}(x)} = \frac{1}{g_5^2}\left[e^{A_s-\Phi}\partial_z V_\perp^{\hat{\mu},c}\right]_{z=\epsilon}$$

and we have introduced a UV regulator $z = \epsilon$ for the AdS boundary

The bulk to boundary propagator and the current correlator

The vectorial field in 5 dimensions can be mapped to the 4d source using the **bulk to boundary propagator**

$$V^{\perp}_{\hat{\mu},c}(z,x) = \int d^4y\, K^{cd}_{\hat{\mu}\hat{\nu}}(z,x;y) V^{\hat{\nu},d}_{\perp,0}(y)$$

The on-shell action takes the form

$$S^{o-s}_{V} = \frac{1}{2g_5^2}\int d^4x \int d^4y\, V^{\hat{\mu},c}_{\perp,0}(x)\left[e^{A_s-\Phi}\partial_z\, K^{cd}_{\hat{\mu}\hat{\nu}}(z,x;y)\right] V^{\hat{\nu},d}_{\perp,0}(y)$$

Using the AdS/CFT dictionary we obtain the **current correlator**

$$G^{cd}_{\hat{\mu}\hat{\nu}}(x-y) =< J_{\hat{\mu},c}(x) J_{\hat{\nu},d}(y) > = \frac{\delta S^{o-s}_{V}}{\delta V^{\hat{\mu},c}_{\perp,0}(x)\delta V^{\hat{\nu},d}_{\perp,0}(y)} = \frac{1}{g_5^2}\left[e^{A_s-\Phi}\partial_z\, K^{cd}_{\hat{\mu}\hat{\nu}}(z,x;y)\right]$$

The VEV and current correlator are related by $< J_{\hat{\mu},c}(x) > = \int d^4y\, G^{cd}_{\hat{\mu}\hat{\nu}}(x-y) V^{\nu,d}_{\perp,0}(y)$

The Sturm-Liouville equation and the spectral decomposition

The bulk to boundary propagator can be written in momentum space as

$$\boldsymbol{K^{cd}_{\hat{\mu}\hat{\nu}}(z,q) = \left(\eta_{\hat{\mu}\hat{\nu}} - \frac{q_{\hat{\mu}}q_{\hat{\nu}}}{q^2}\right)\delta^{cd}V(z,q)}$$

The field $V(z,q)$ satisfy the differential equation

$$\boldsymbol{\left[(\partial_z + A_s' - \Phi')\partial_z - q^2\right]V(z,q) = 0}$$

which can be written in **the Sturm-Liouville form**

$$\boldsymbol{[\partial_z(p(z)\partial_z) - s(z) + \lambda\, r(z)]V(z,q) = 0}$$

where

$$\boldsymbol{p(z) = r(z) = e^{A_s - \Phi}\ , \qquad s(z) = 0\,, \qquad \lambda = -q^2}$$

Then we can define the Green's function by

$$\boldsymbol{[\partial_z(p(z)\partial_z) - s(z) + \lambda\, r(z)]G(z;z') = \delta(z - z')}$$

Following Sturm-Liouville theory we obtain the spectral decomposition

$$G(z;z') = -\sum_n \frac{v^n(z)v^n(z')}{q^2 + m_{vn}^2}$$

where the Sturm-Liouville modes satisfy

$$[(\partial_z + A_s' - \Phi')\partial_z + m_{v_n}^2]v^n(z) = 0$$

and are normalised as

$$\int dz\, e^{A_s - \Phi} v^m(z) v^n(z) = \delta^{mn}$$

The field $V(q,z)$ can be obtained from the Green's function as

$$V(z',q) = -\left[e^{A_s-\Phi}\partial_z G(z;z')\right]_{z=\epsilon} = -\sum_n \frac{\left[e^{A_s-\Phi}\partial_z\, v^n(z)\right]_{z=\epsilon} v^n(z')}{q^2 + m_{v\_n}^2}$$

and the current correlator takes the form

$$G_{\mu\nu}^{cd}(q) = \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\delta^{cd}\sum_n \frac{F_{v_n}^2}{q^2 + m_{v_n}^2}\,, \qquad F_{v^n} = \frac{1}{g_5}\left[e^{A_s-\Phi}\partial_z v^n(z)\right]_{z=\epsilon}$$

The coefficients F_{v^n} **are the vector meson decay constants** consistent with large N_c QCD

3. Nucleons in confining holographic QCD

Pioneer works in the bottom-up approach:

Brodsky-Teramond 2005, Hong-Inami-Yee 2006, Abidin-Carlson 2009

Nucleons are usually described by interpolating fields. For a proton we use the Ioffe operator

$$< O(x) > = < \epsilon_{abc} \left(u_a^T(x) C \gamma_\mu u_b(x) \right) \gamma_5 \gamma^\mu d_c(x) >$$

Ioffe 1981

In holographic QCD we map this fermionic operator to a 5d Dirac spinor described by the action

$$S_F = N \int d^4x \, dz \sqrt{-g_s} e^{-\Phi} \left(\frac{i}{2} \bar{\psi} \Gamma^n D_n \psi + c.c. - i \tilde{m} \bar{\psi} \psi \right)$$

where $\Gamma^n = e^n_{\hat{a}} \Gamma^{\hat{a}}$, $D_n = \partial_n + \frac{1}{8} \omega_n^{\hat{a}\hat{b}} [\Gamma_{\hat{a}}, \Gamma_{\hat{b}}]$

Redefining the Dirac field as $\psi \to e^{\Phi/2} \psi$

the Dirac action becomes

$$S_F = N \int d^4x \, dz \sqrt{-g_s} \left(\frac{i}{2} \bar{\psi} \Gamma^n D_n \psi + c.c. - i \tilde{m} \bar{\psi} \psi \right)$$

In holographic QCD the vielbein takes the form $e^{n}_{\hat{a}} = e^{-A_s}\delta^{n}_{\hat{a}}$

and the non-vanishing components of the spin connection are $\omega^{\hat{z}\hat{\nu}}_{\hat{\mu}} = -\omega^{\hat{\nu}\hat{z}}_{\hat{\mu}} = -A_s'\,\delta^{\hat{\nu}}_{\hat{\mu}}$

so that

$$\Gamma^n D_n\psi = e^{-A_s}\left(\Gamma^{\hat{a}}\partial_{\hat{a}} + 2A_s'\Gamma^{\hat{z}}\right)\psi$$

with $\hat{a} = (\hat{z}, \hat{\mu})$

The Dirac action becomes

$$S_F = N\int d^4x\, dz\, e^{4A_s}\left(\frac{i}{2}\bar{\psi}\,\Gamma^{\hat{a}}\partial_{\hat{a}}\psi - \frac{i}{2}(\partial_{\hat{a}}\bar{\psi})\Gamma^{\hat{a}}\psi. - i\, e^{A_s}\tilde{m}\,\bar{\psi}\psi\right)$$

Varying this action we obtain the field equations

$$\left(\Gamma^{\hat{a}}\partial_{\hat{a}} + 2A_s'\Gamma^{\hat{z}} - e^{A_s}\tilde{m}\right)\psi = 0$$

$$\bar{\psi}\left(\overleftarrow{\partial_{\hat{a}}}\Gamma^{\hat{a}} + 2A_s'\Gamma^{\hat{z}} + e^{A_s}\tilde{m}\right) = 0$$

and the surface term

$$\delta S_F = N\int d^4x\left(\frac{i}{2}e^{4A_s}\delta\bar{\psi}\,\Gamma^{\hat{z}}\psi\right)_{z=\epsilon} + c.c.$$

Left and right decomposition:

$$\psi = \psi_R + \psi_L$$

where

$$\psi_{R/L} = P_{R/L}\psi \qquad \bar{\psi}_{R/L} = P_{L/R}\bar{\psi} \qquad P_{R/L} = \frac{1}{2}(1 \pm \Gamma^{\hat{z}})$$

The Dirac equation decomposes in two equations

$$\Gamma^{\hat{\mu}}\partial_{\hat{\mu}}\psi_{R/L} = \pm\left(\partial_z + 2A'_s \pm e^{A_s}\tilde{m}\right)\psi_{L/R}$$

and the surface term becomes

$$\delta S_F = N\int d^4x \left(\frac{i}{2}e^{4A_s}\delta\bar{\psi}_L\,\psi_R - \frac{i}{2}e^{4A_s}\delta\bar{\psi}_R\,\psi_L\right)_{z=\epsilon} + c.c.$$

Since ψ_R and ψ_L are not independent we need to correct the Dirac action as

$$S'_F = S_F + N\int d^4x \left(\sqrt{-\gamma}\frac{i}{2}\bar{\psi}\psi\right)_{z=\epsilon} = S_F + N\int d^4x \left(\frac{i}{2}e^{4A_s}(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)\right)_{z=\epsilon}$$

so that

$$\delta S'_F = N\int d^4x \left(\frac{i}{2}e^{4A_s}\delta\bar{\psi}_L\psi_R\right)_{z=\epsilon} + c.c.$$

At small z (near the boundary) the left and right spinor fields have the asymptotic behaviour

$$\boldsymbol{\psi_L(x,z) = \alpha_L(x) z^{2-m} + \cdots + \beta_L(x) z^{3+m} + \ \dots}$$
$$\boldsymbol{\psi_R(x,z) = \alpha_R(x) z^{3-m} + \cdots + \beta_R(x) z^{2+m} + \ \dots}$$

In our framework the independent source is $\alpha_L(x)$ that couples to the operator O_R so that

$$\boldsymbol{< O_R > = \frac{\delta S'_F}{\delta \bar{\alpha}_L} = iN \left(z^{2-m} e^{4A_s} \psi_R\right)_{z=\epsilon}}$$
$$\boldsymbol{= iN \frac{\Gamma^{\hat{\mu}} \partial_{\hat{\mu}}}{\partial^2} \left(z^{2-m} e^{4A_s} \left(\partial_z + 2A'_s + e^{A_s} \widetilde{m}\right) \psi_L\right)_{z=\epsilon}}$$

We finally note that combining the left and right coupled spinor equations we obtain

$$\boldsymbol{\left[\left(\partial_z + 2A'_s \pm e^{A_s} \widetilde{m}\right)\left(\partial_z + 2A'_s \mp e^{A_s} \widetilde{m}\right) + \partial^2\right] \psi_{R/L} = 0}$$

Using a plane-wave ansatz for the x directions we have

$$\boldsymbol{\psi_{R/L}(x,z) = \int d^4q \; e^{iq\cdot x} F_{R/L}(q,z) \alpha_{R/L}(q)}$$

and we obtain

$$\boldsymbol{\left[\left(\partial_z + 2A'_s \pm e^{A_s} \widetilde{m}\right)\left(\partial_z + 2A'_s \mp e^{A_s} \widetilde{m}\right) + Q^2\right] F_{R/L} = 0}$$

with $Q = \sqrt{-q^2}$

The Schrödinger equation

Using a Bogoliubov transformation $$F_{R/L}(q,z) = e^{-2A_s(z)}\xi_{R/L}(q,z)$$

we obtain the Schrödinger equations $$\left[\partial_z^2 + Q^2 - V_{R/L}\right]\xi_{R/L} = 0$$

with the potentials given by $$V_{R/L} = \pm\partial_z\left(e^{A_s}\tilde{m}\right) + \left(e^{A_s}\tilde{m}\right)^2$$

Inspired by the soft wall model we propose the following ansatz for the mass term:

$$\tilde{m} = e^{-A_s}\left(-mA_s' + \frac{1}{2}\Phi'\right)$$

The bulk to boundary propagator and the nucleon correlator

$$\psi_L(z,x) = \int d^4y\, F_L(z,x;y)\alpha_L(y)$$

The on-shell action takes the form

$$S_F^{\prime o-s} = N\int d^4x \int d^4y \frac{\Gamma^{\hat{\mu}}\partial_{\hat{\mu}}}{\partial^2}\left(\frac{i}{2}\bar{\alpha}_L(x)\left(z^{2-m}e^{4A_s}\left(\partial_z + 2\,A_s' + e^{A_s}\tilde{m}\right)F_L(z,\mathrm{x};\mathrm{y})\right)_{z=\epsilon}\alpha_L(y) + c.c.\right)$$

where $\partial_{\hat{\mu}} = \partial/\partial(x-y)^{\hat{\mu}}$

The 2-point nucleon correlator takes the form

$$\Gamma_R(x-y) = < O_R(x)\bar{O}_R(y) \succ = P_R \frac{\delta < \bar{O}_R(y) >}{\delta \bar{\alpha}_L(x)}$$

$$= iP_R \frac{\Gamma^{\hat{\mu}}\partial_{\hat{\mu}}}{\partial^2}\left(z^{2-m} e^{4A_s}\left(\partial_z + 2\,A_s' + e^{A_s}\tilde{m}\right)F_L(z,\mathrm{x};\mathrm{y})\right)_{z=\epsilon}$$

The bulk to boundary propagator, in momentum space, satisfies the differential equation

$$\left[(\partial_z + 4A_s')\partial_z + \Theta_\mathrm{L} + Q^2\right]F_L(z,q) = 0$$

where

$$\theta_L(z) = 2A_s'' + 4A_s'^2 + \partial_Z\left(e^{A_s}\tilde{m}\right) - e^{2A_s}\tilde{m}^2$$

This equation can be written in the Sturm-Liouville form

$$[\partial_z(p(z)\partial_z) - s(z) + \lambda\, r(z)]F_L(z,q) = 0$$

where

$$p(z) = r(z) = e^{4A_s}, \qquad s(z) = -e^{4A_s}\Theta_L\,, \qquad \lambda = Q^2$$

Following Sturm-Liouville theory we obtain the spectral decomposition

$$G(z;z') = -\sum_n \frac{f_{L,n}(z) f_{L,n}(z')}{q^2 + m_n^2}$$

where the Sturm-Liouville modes satisfy

$$\left[(\partial_z + 4A_s')\partial_z + \Theta_L + m_n^2\right] f_{L,n}(z) = 0$$

and are normalised as

$$\int dz\, e^{4A_s} f_{L,m}(z) f_{L,n}(z) = \delta^{mn}$$

We bulk to boundary propagator takes the form

$$F_L(q,z') = -\left[e^{4A_s}\left(F_L(z)\partial_z G_L(z;z') - G_L(z;z')\partial_z F_L(z)\right)\right]_{z=\epsilon}$$
$$= \sum_n \frac{f_n m_n f_{L,n}(z')}{q^2 + m_n^2}$$

We obtain the following spectral decomposition for the nucleon correlator

$$\Gamma_R(q) = -P_R \Gamma^\mu q_\mu \left(\frac{1}{Q^2}\sum_n f_n^2 + \sum_n \frac{f_n^2}{q^2 + m_n^2}\right), \qquad f_n = \frac{1}{g_5}\left[z^{-2-m} f_{R,n}(z)\right]_{z=\epsilon}$$

consistent with large N_c QCD

The coefficients f_n are the nucleon "decay constants"

The first term is a UV divergence that can be subtracted using holographic renormalisation

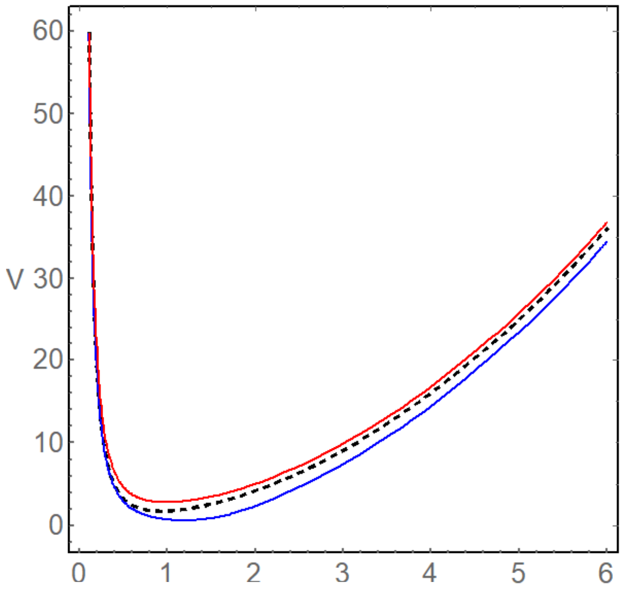
4. Results

Spectrum of vector mesons

For vector mesons the conformal dimension $\Delta = 3$ is protected (conserved current)

We solve the Schrödinger equation for the normalisable modes

In the figure we compare the Schrödinger potentials for model I (blue), model II (red) and the soft wall model (black dashed)



The mass of the first vector meson (the ρ meson) can be used to fix the infrared parameter $\boldsymbol{k}$, **which is the only parameter in the model**

The mass ratios presented below are independent of the choice of $\boldsymbol{k}$

Ratio	Einstein-dilaton I	Einstein-dilaton II	Soft wall	Hard wall	Experimental
m_{ρ_1}/m_{ρ_0}	1.591	1.34	1.414	2.295	1.652 ± 0.048
m_{ρ_2}/m_{ρ_0}	2.015	1.611	1.732	3.598	1.888 ± 0.032
m_{ρ_3}/m_{ρ_0}	2.365	1.843	2	4.903	2.216 ± 0.026
m_{ρ_4}/m_{ρ_0}	2.67	2.049	2.236	6.209	2.46 ± 0.039
m_{ρ_5}/m_{ρ_0}	2.944	2.236	2.45	7.514	2.769 ± 0.022

Spectrum of nucleons

For nucleons the conformal dimension can vary with the energy scale (anomalous dimension)

We consider two cases: $\boldsymbol{\Delta = 7/2}$ and $\boldsymbol{\Delta = 9/2}$

We solve the Schrödinger equation for the normalisable modes

The mass ratios presented below are independent of the choice of $\boldsymbol{k}$

Ratio	Einstein-dilaton I	Einstein-dilaton II	Soft wall	Hard wall	Experimental
m_{N_0}/m_{ρ_0}	0.987	0.988	1.414	1.593	1.209 ± 0.002
m_{N_1}/m_{ρ_0}	1.623	1.339	1.732	2.917	1.856 ± 0.039
m_{N_2}/m_{ρ_0}	2.053	1.613	2	4.23	2.204 ± 0.039
m_{N_3}/m_{ρ_0}	2.403	1.847	2.236	5.54	2.423 ± 0.065
m_{N_4}/m_{ρ_0}	2.707	2.054	2.449	6.849	2.706 ± 0.065

$\Delta = 7/2$

Ratio	Einstein-dilaton I	Einstein-dilaton II	Soft wall	Hard wall	Experimental
m_{N_0}/m_{ρ_0}	0.896	0.952	1.732	2.136	1.209 ± 0.002
m_{N_1}/m_{ρ_0}	1.593	1.314	2	3.5	1.856 ± 0.039
m_{N_2}/m_{ρ_0}	2.04	1.595	2.236	4.832	2.204 ± 0.039
m_{N_3}/m_{ρ_0}	2.399	1.833	2.449	6.153	2.423 ± 0.065
m_{N_4}/m_{ρ_0}	2.708	2.043	2.646	7.468	2.706 ± 0.065

$\Delta = 9/2$

Decay constants

Vector meson decay constants:

Ratio	Einstein-dilaton I	Einstein-dilaton II	Soft wall	Hard wall	Experimental
$\sqrt{F_{\rho 0}}/m_{\rho 0}$	0.3719	0.283	0.3355	0.4246	0.446 ± 0.0019
$\sqrt{F_{\rho 1}}/m_{\rho 0}$	0.4704	0.3407	0.3989	0.7946	0.5588 ± 0.017
$\sqrt{F_{\rho 2}}/m_{\rho 0}$	0.5298	0.3798	0.4415	1.114	-
$\sqrt{F_{\rho 3}}/m_{\rho 0}$	0.5741	0.41	0.4744	1.405	-
$\sqrt{F_{\rho 4}}/m_{\rho 0}$	0.61	0.4351	0.5017	1.677	-

Nucleon "decay constants":

$\Delta = 7/2$

Ratio	Einstein-dilaton I	Einstein-dilaton II	Soft wall	Hard wall	Experimental
f_{N_0}/m_{ρ_0}	1.248	1.532	0.707	2.797	-
f_{N_1}/m_{ρ_0}	1.466	1.803	1	6.874	-
f_{N_2}/m_{ρ_0}	1.71	1.999	1.225	11.98	-
f_{N_3}/m_{ρ_0}	1.923	2.162	1.414	17.94	-
f_{N_4}/m_{ρ_0}	2.114	2.303	1.581	24.65	-

$\Delta = 9/2$

Ratio	Einstein-dilaton I	Einstein-dilaton II	Soft wall	Hard wall	Experimental
f_{N_0}/m_{ρ_0}	1.916	5.495	0.5	5.708	-
f_{N_1}/m_{ρ_0}	2.18	6.626	0.866	19.19	-
f_{N_2}/m_{ρ_0}	2.656	7.552	1.225	42.7	-
f_{N_3}/m_{ρ_0}	3.124	8.376	1.581	77.92	-
f_{N_4}/m_{ρ_0}	3.283	9.136	1.937	126.3	-

5. Spontaneous chiral symmetry breaking in confining holographic QCD

B-B, Frederico, Mamani and de Paula 2023

Spontaneous chiral symmetry breaking described by a 5d Yang-Mills-Higgs action

$$S = -\int d^5x \sqrt{-g}\, e^{-\Phi}\{\mathrm{Tr}\,[\,|D_m X|^2 + f(\Phi)V(|X|)] + \frac{1}{4g_5^2}\mathrm{Tr}[F_{mn}^{(L)\,2} + F_{mn}^{(R)\,2}]\}$$

where

$$F_{mn}^{(L/R)} = \partial_m A_n^{(L/R)} - \partial_n A_m^{(L/R)} - i[A_m^{(L/R)}, A_n^{(L/R)}],$$
$$D_m X = \partial_m X - iA_m^{(L)} X + i\, X\, A_m^{(R)}$$

and

$$V(|X|) = m_X^2\, X^2 + \lambda\, X^4$$

The tachyonic field X maps to the quark mass operator $< \bar{q}\, q >$

Fixing the conformal dimension to $\Delta = 3$ we obtain $m_X^2 = -3$

Assuming isospin symmetry we take the ansatz $X(z) = \frac{1}{2} v(z) I_{2\times 2}$

and obtain the non-linear differential equation

$$\left[\partial_z^2 + (3A_s' - \Phi')\partial_z\right]v - e^{2A_s} f(\Phi)\left(m_X^2 v - \frac{\lambda}{2} v^3\right) = 0$$

We introduce the non-minimal dilaton coupling $$f(\Phi) = \frac{1}{1 + b_0\Phi + a_0\Phi^2}$$

The role of this coupling is to turn off the tachyon potential in the region where confinement should be dominant.

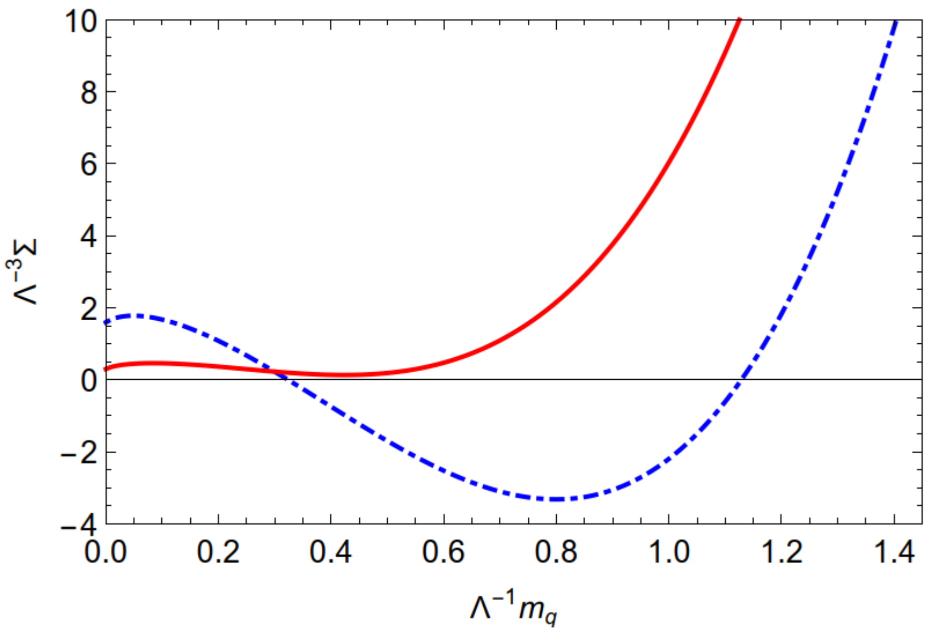
The tachyonic field behaves at small z as $$v(z) = c_1 z + c_3 z^3 + d_3 z^3 \ln z + \cdots$$

From the UV coefficient we obtain the chiral condensate $\Sigma = \langle \bar{q}\, q \rangle$

In the figure we display the chiral condensate as a function of the quark mass for

i) $a_0 = 1$ and $b_0 = 0$ (blue dashed)

ii) $a_0 = 0.02,\ b_0 = 1.7$ (red solid)



We obtain the masses and decay constants of scalar, pseudo-scalar, vector and axial-vector mesons and find good agreement with experimental data.

For more details see arXiv 2308.07503

Conclusions

- We have built a minimal holographic QCD model that describes vector mesons and nucleons in a single fashion
- The model contains only one parameter associated with hadron mass generation and confinement. In this way the model improves previous bottom-up approaches
- The comparison between our results for the spectrum of vector mesons and nucleons and experimental data is better for the higher excited states than the first states
- Incorporating effects of spontaneous chiral symmetry breaking on vector mesons and nucleons should improve the results for the first states

Next steps

- Calculate couplings between vector mesons and nucleons
- Obtain the electromagnetic and gravitational form factors of vector mesons and nucleons
- Transition to the regime of heavy quarks
- Turn on the temperature and investigate the transition to deconfinement, chiral symmetry restoration and hadron melting