Bridge between QCD and LFQM

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Light-Cone 2023: Hadrons and Symmetries

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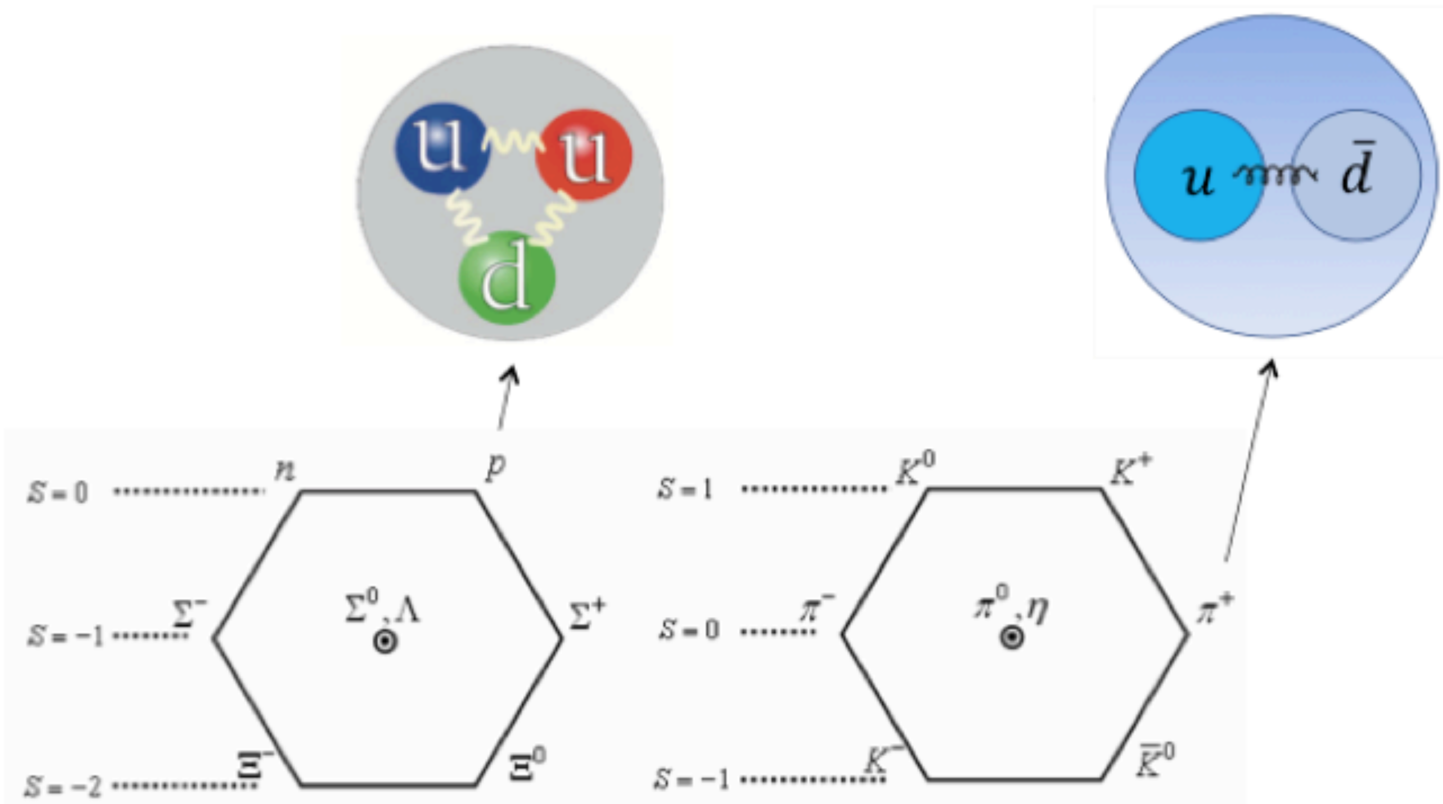
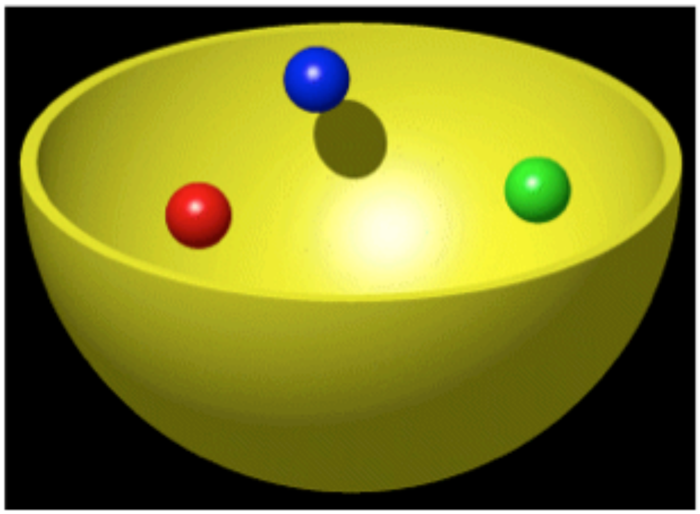
How do we understand the Quark Model in Quantum Chromodynamics?

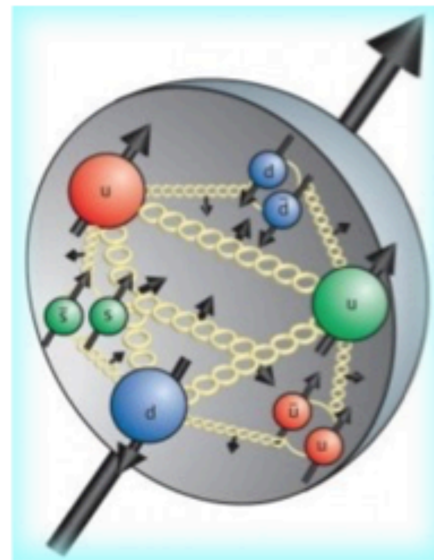


$$M_p = 938.272046 \pm 0.000021\ MeV$$

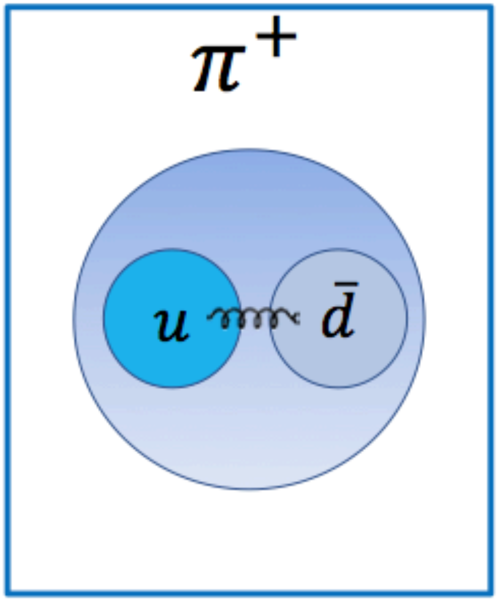
$$M_n = 939.565379 \pm 0.000021\ MeV$$



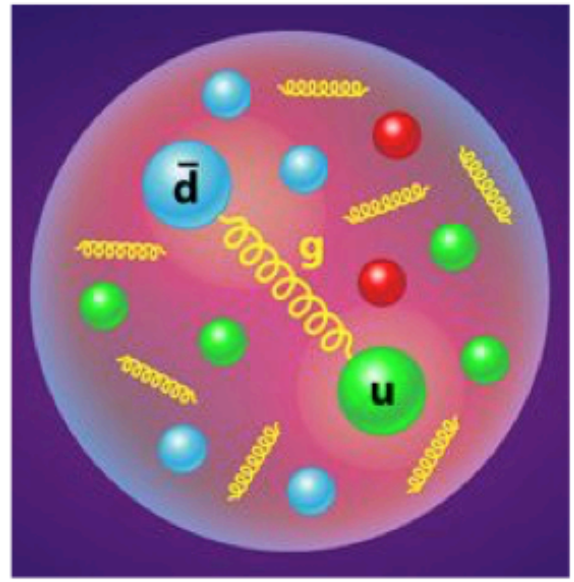
$\Longrightarrow$



$$m_u = 2.3^{+0.7}_{-0.5}\ MeV \quad ; \quad m_d = 4.8^{+0.7}_{-0.3}\ MeV$$



vs.



Constituent Quark Model

$$M = m_1 + m_2 + A\frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2}$$

$$m_u = m_d = 310 MeV/c^2$$

$$A = \left(\frac{2m_u}{\hbar}\right)^2 160 MeV/c^2$$

Quantum Chromodynamics

Isospin symmetry

Chiral symmetry

$$\mathrm{SU(2)_R \times SU(2)_L}$$

Spontaneous symmetry breakdown

Goldstone Bosons

$$F_\pi^2 M_\pi^2 = -(m_u + m_d)\langle 0|\bar{u}u|0\rangle$$

Effective field theory

Outline

- ‘tHooft model and short list of references
- Dirac’s proposition for relativistic dynamics
- Interpolation between IFD and LFD
- Mass gap solutions in ‘tHooft model
- Spontaneous breaking of chiral symmetry
- Meson spectroscopy and wavefunctions
- Application to hadron physics

Large N_c QCD in 1+1 dim. ('tHooft Model)

$$\mathcal{L} = -\frac{1}{4} F^a_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}\,a} + \bar{\psi}(i\gamma^{\hat{\mu}} D_{\hat{\mu}} - m)\psi$$

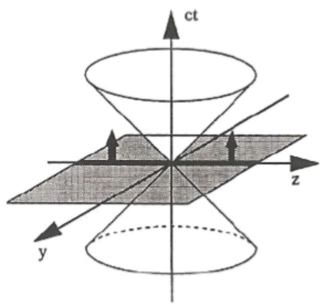
$$D_{\hat{\mu}} = \partial_{\hat{\mu}} - igA^a_{\hat{\mu}} t_a$$

$$F^a_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} A^a_{\hat{\nu}} - \partial_{\hat{\nu}} A^a_{\hat{\mu}} + g f^{abc} A^b_{\hat{\mu}} A^c_{\hat{\nu}}$$

'tHooft Coupling $\lambda = \frac{g^2 (N_c - 1/N_c)}{4\pi}$ and mass m

$$g \to 0, N_C \to \infty; \lambda \to finite$$

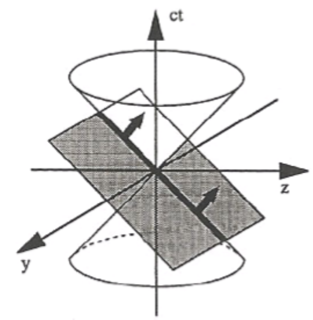
Dirac's Proposition for Relativistic Dynamics



The instant form

1949

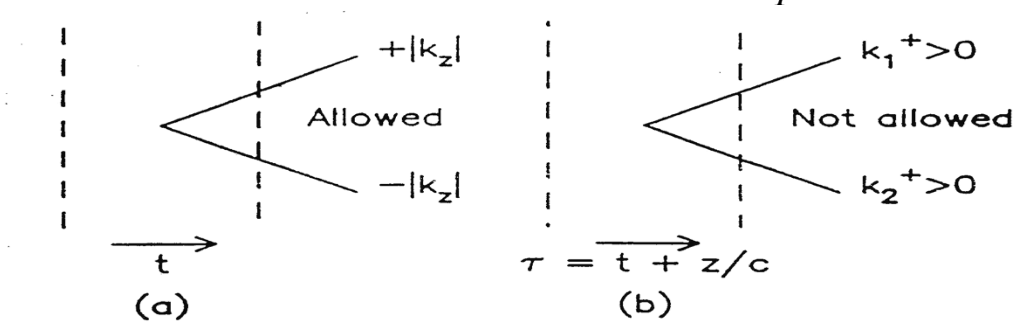
Equal t		Equal τ
p^0	$\leftrightarrow$	$p^- = p^0 - p^3$
(p^1, p^2)	$\leftrightarrow$	$\vec{p}_\perp$
p^3	$\leftrightarrow$	$p^+ = p^0 + p^3$



The front form

Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2} \qquad p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



Except zero-modes

$k_1^+ = k_2^+ = 0$

IFD

Instant Form Dynamics

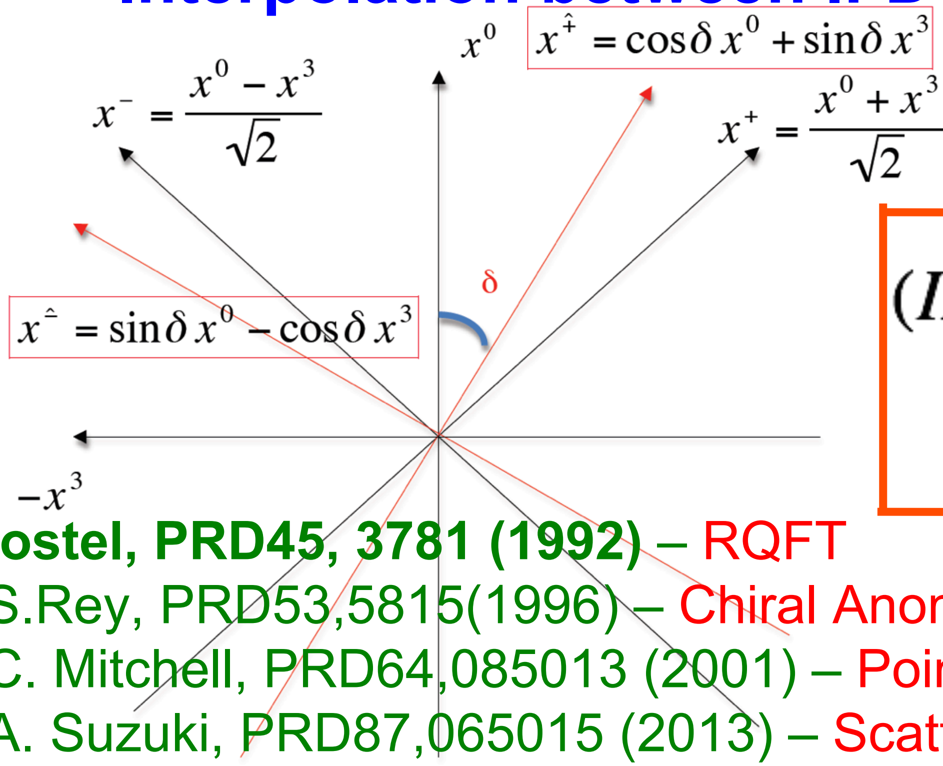
LFD

Light-Front Dynamics

Short List of References

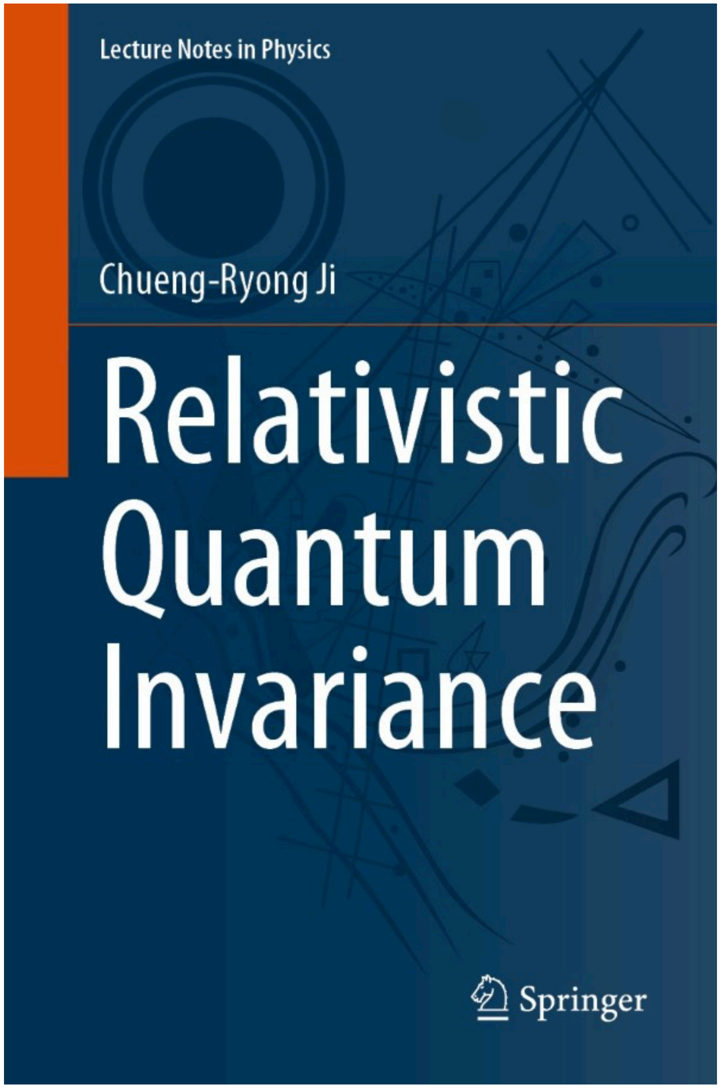
- G.‘tHooft, NPB75,461(74) - LFD
- Y.Frishman, et al., PRD15(75) - Interpol Gauges IFD&LFD
- I.Bars&M.Green, PRD17,537(78) - IFD(formulation)
- A.Zhitnitsky, PLB165,405(85) - LFD(chiral sym breaking)
- M.Li, et al., JPG13, 915(87) - IFD(rest frame)
- K.Hornbostel, Ph.D. Dissertation(88) - LFD(DLCQ)
- M.Burkardt, PRD53,933(96) - LFD(vacuum condensates)
- Y.Kalashnikov&A.Nefed’ev,Phys.-Usp.45,346(‘02) - IFD(rev)
- Y. Jia, et al., JHEP11, 151(‘17) - IFD(moving frame)
- Y. Jia, et al., PRD98, 054011(‘18) - IFD(quasi-PDFs)
- B.Ma&C.Ji, PRD104,036004(‘21) - Link IFD&LFD

Interpolation between IFD and LFD

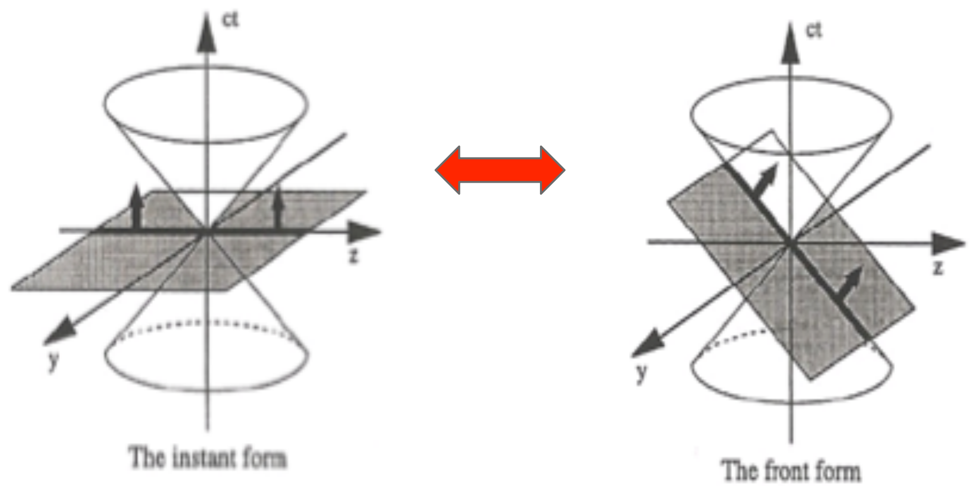


$$\boxed{\begin{array}{ccc} (IFD) & 0 \le \delta \le \dfrac{\pi}{4} & (LFD) \\ 1 \ge & C \equiv \cos(2\delta) & \ge 0 \end{array}}$$

K. Hornbostel, PRD45, 3781 (1992) – RQFT
C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly
C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra
C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps
C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges
Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors
C.Ji, Z.Li, B.Ma and A.Suzuki, PRD98, 036017(2018) – QED
B.Ma and C.Ji, PRD194,036004(2021) – QCD_{1+1}



Interpolating instant form dynamics and light-front dynamics



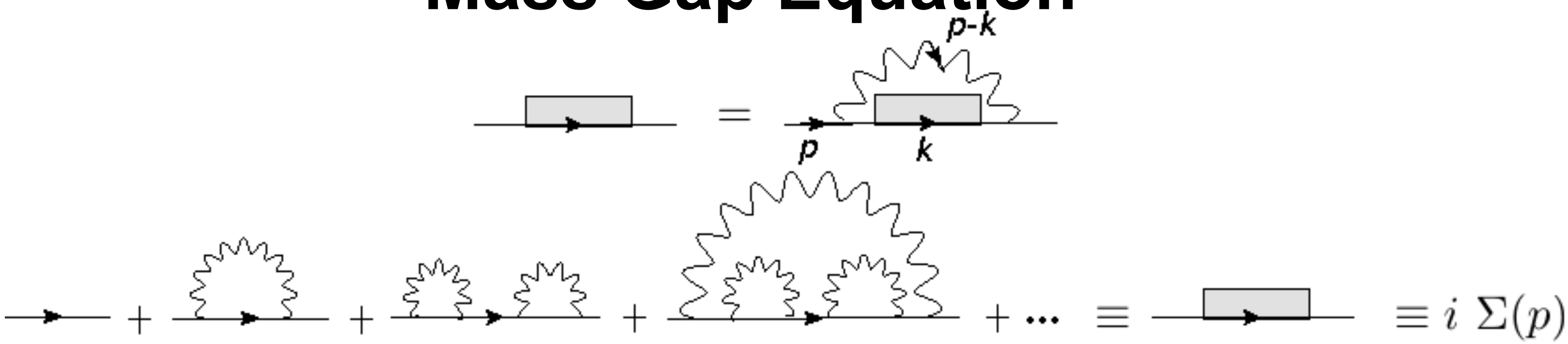
Interpolating Axial Gauge

$$A^a_{\hat{-}} = 0$$

$$A^a_1 = 0 \qquad\qquad A^+_a = 0$$

$$\mathcal{L} = \frac{1}{2}(\partial_{\hat{-}} A^a_{\hat{+}})^2 + \bar{\psi}(i\gamma^{\hat{+}} D_{\hat{+}} + i\gamma^{\hat{-}}\partial_{\hat{-}} - m)\psi$$

Mass Gap Equation



$$\Sigma(p_{\hat{-}}) = i\frac{\lambda}{2\pi}\int \frac{dk_{\hat{-}}\, dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\gamma^{\hat{+}} \frac{1}{\not{k} - m - \Sigma(k_{\hat{-}}) + i\epsilon}\gamma^{\hat{+}}$$

Fermion Propagator

Free Propagator → **Interacting Propagator**

$$S_f(p) = \frac{1}{\not{p} - m + i\varepsilon}$$

$$S(p) = \frac{1}{\not{p} - m - \sum(p) + i\varepsilon}$$

$$= \frac{F(p)}{\not{p} - M(p) + i\varepsilon}$$

$$\boxed{\Sigma(p) = \Sigma_s(p) + \Sigma_v(p)\not{p}}$$

$$F(p) = \left(1 - \Sigma_v(p)\right)^{-1}$$ "Wave function renormalization factor"

$$M(p) = \frac{m + \Sigma_s(p)}{1 - \Sigma_v(p)}$$ "Renormalized fermion mass function"

Energy-Momentum Dispersion Relation

Free particle | **Interacting particle**

$$E = \sqrt{p_z^2 + m^2}$$

$$\frac{F(p'_\perp)E(p'_\perp)}{\sqrt{C}} = \sqrt{p'^2_\perp + M(p'_\perp)^2} \equiv \tilde{E}(p'_\perp)$$

$$\theta_f = \tan^{-1}(p_z / m)$$

$$\theta(p'_\perp) = \theta_f(p'_\perp) + 2\zeta(p'_\perp)$$

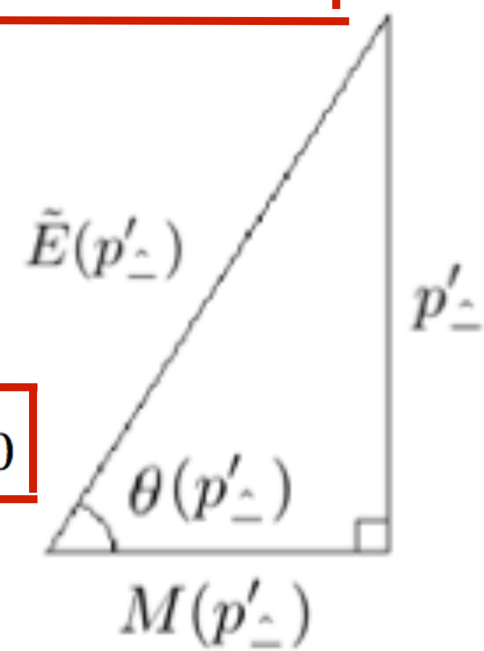
$$\beta = p_z / E$$

$$\begin{pmatrix} b^i(p'_\perp) \\ d^{+i}(p'_\perp) \end{pmatrix} = \begin{pmatrix} \cos\zeta(p'_\perp) & -\sin\zeta(p'_\perp) \\ \sin\zeta(p'_\perp) & \cos\zeta(p'_\perp) \end{pmatrix} \begin{pmatrix} b^i_f(p'_\perp) \\ d^{+i}_f(p'_\perp) \end{pmatrix}$$

$$= \sin\theta_f$$

$$= \tanh\eta$$

$$b^i_f|0>=0, d^i_f|0>=0 \quad \text{vs.} \quad b^i|\Omega>=0, d^i|\Omega>=0$$



Interpolation

$$(E, p_z) \Rightarrow (p^+ / \sqrt{C}, p_\perp / \sqrt{C} \equiv p'_\perp)$$

Mass Gap Equation in Scaled Variables

$$\bar{p}'_{\underline{\wedge}} = \frac{\bar{p}_{\underline{\wedge}}}{\sqrt{\mathbb{C}}},\ \ \bar{E}' = \frac{\bar{E}}{\sqrt{\mathbb{C}}},\ \bar{p}_{\underline{\wedge}} = \frac{p_{\underline{\wedge}}}{\sqrt{2\lambda}},\ \ \bar{E} = \frac{E}{\sqrt{2\lambda}},\ \bar{m} = \frac{m}{\sqrt{2\lambda}}$$

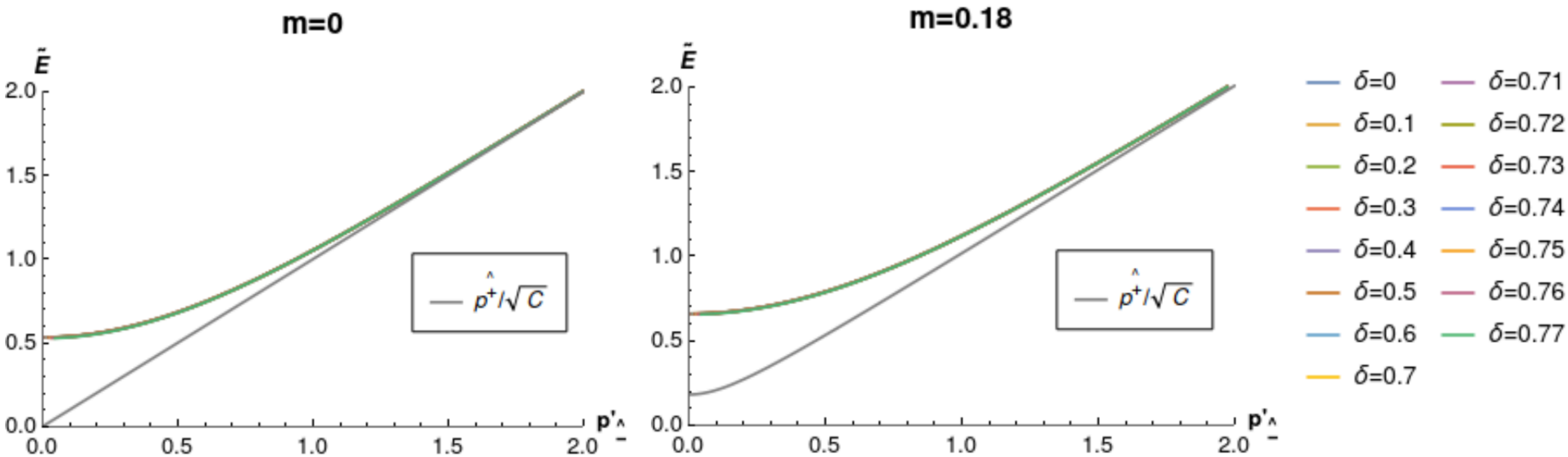
$$\bar{p}'_{\underline{\wedge}} \cos\theta(\bar{p}'_{\underline{\wedge}}) - \bar{m}\sin\theta(\bar{p}'_{\underline{\wedge}}) = \frac{1}{4}\int \frac{d\bar{k}'_{\underline{\wedge}}}{(\bar{p}'_{\underline{\wedge}} - \bar{k}'_{\underline{\wedge}})^2} \sin\left(\theta(\bar{p}'_{\underline{\wedge}}) - \theta(\bar{k}'_{\underline{\wedge}})\right)$$

$$\bar{E}'(\bar{p}'_{\underline{\wedge}}) = \bar{p}'_{\underline{\wedge}} \sin\theta(\bar{p}'_{\underline{\wedge}}) + \bar{m}\cos\theta(\bar{p}'_{\underline{\wedge}}) + \frac{1}{4}\int \frac{d\bar{k}'_{\underline{\wedge}}}{(\bar{p}'_{\underline{\wedge}} - \bar{k}'_{\underline{\wedge}})^2} \cos\left(\theta(\bar{p}'_{\underline{\wedge}}) - \theta(\bar{k}'_{\underline{\wedge}})\right)$$

$$\frac{p_{\underline{\wedge}}}{\mathbb{C}} \cos\theta(p_{\underline{\wedge}}) - \frac{m}{\sqrt{\mathbb{C}}}\sin\theta(p_{\underline{\wedge}}) = \frac{\lambda}{2}\int \frac{dk_{\underline{\wedge}}}{(p_{\underline{\wedge}} - k_{\underline{\wedge}})^2} \sin\left(\theta(p_{\underline{\wedge}}) - \theta(k_{\underline{\wedge}})\right)$$

$$E(p_{\underline{\wedge}}) = p_{\underline{\wedge}} \sin\theta(p_{\underline{\wedge}}) + \sqrt{\mathbb{C}}m\cos\theta(p_{\underline{\wedge}}) + \frac{\mathbb{C}\lambda}{2}\int \frac{dk_{\underline{\wedge}}}{(p_{\underline{\wedge}} - k_{\underline{\wedge}})^2} \cos\left(\theta(p_{\underline{\wedge}}) - \theta(k_{\underline{\wedge}})\right)$$

Mass Gap Solutions



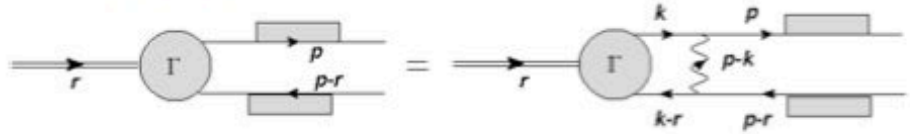
$$\tilde{E}(0) = \frac{F(0)E(0)}{\sqrt{\mathbb{C}}} = M(0)$$

m	0	0.045	0.18	0.749	1.00	2.11	4.23
$M(0)$	0.532778	0.563644	0.659112	1.10105	1.31167	2.30969	4.34358
$F(0)$	−0.495173	−0.584175	−0.987673	4.11079	2.17976	1.22134	1.05526

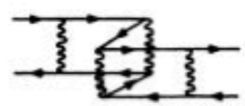
$$m \lesssim 0.56$$

BOUND-STATE EQUATION

$$\Gamma(r,p) = \frac{i\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2} S(p)\gamma^{\hat{+}}\Gamma(r,k)\gamma^{\hat{+}}S(p-r)$$

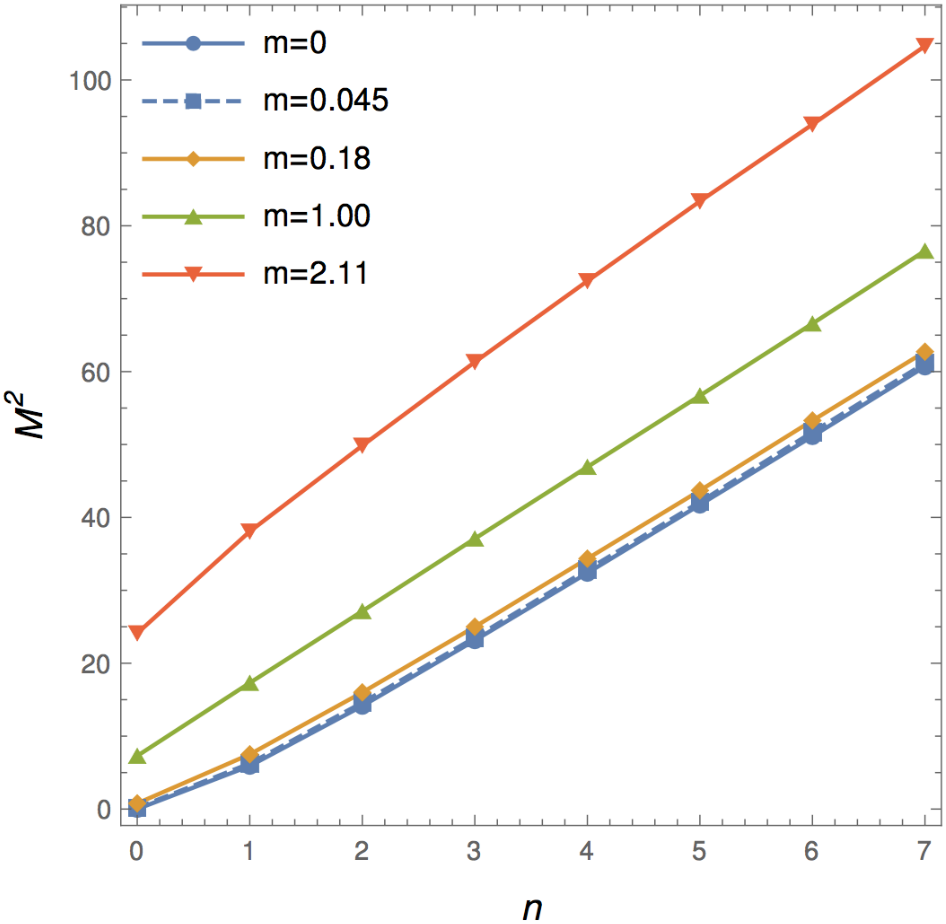


$$\left[-r_{\hat{+}} + \frac{-\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_+(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda \int \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}})\right],$$
$$\left[r_{\hat{+}} + \frac{-\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_-(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda \int \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}})\right].$$



LFD $\left[\mathcal{M}^2 - \frac{m^2 - 2\lambda}{x} - \frac{m^2 - 2\lambda}{1-x}\right]\phi(x) = -2\lambda \int_0^1 \frac{dy}{(x-y)^2}\phi(y)$

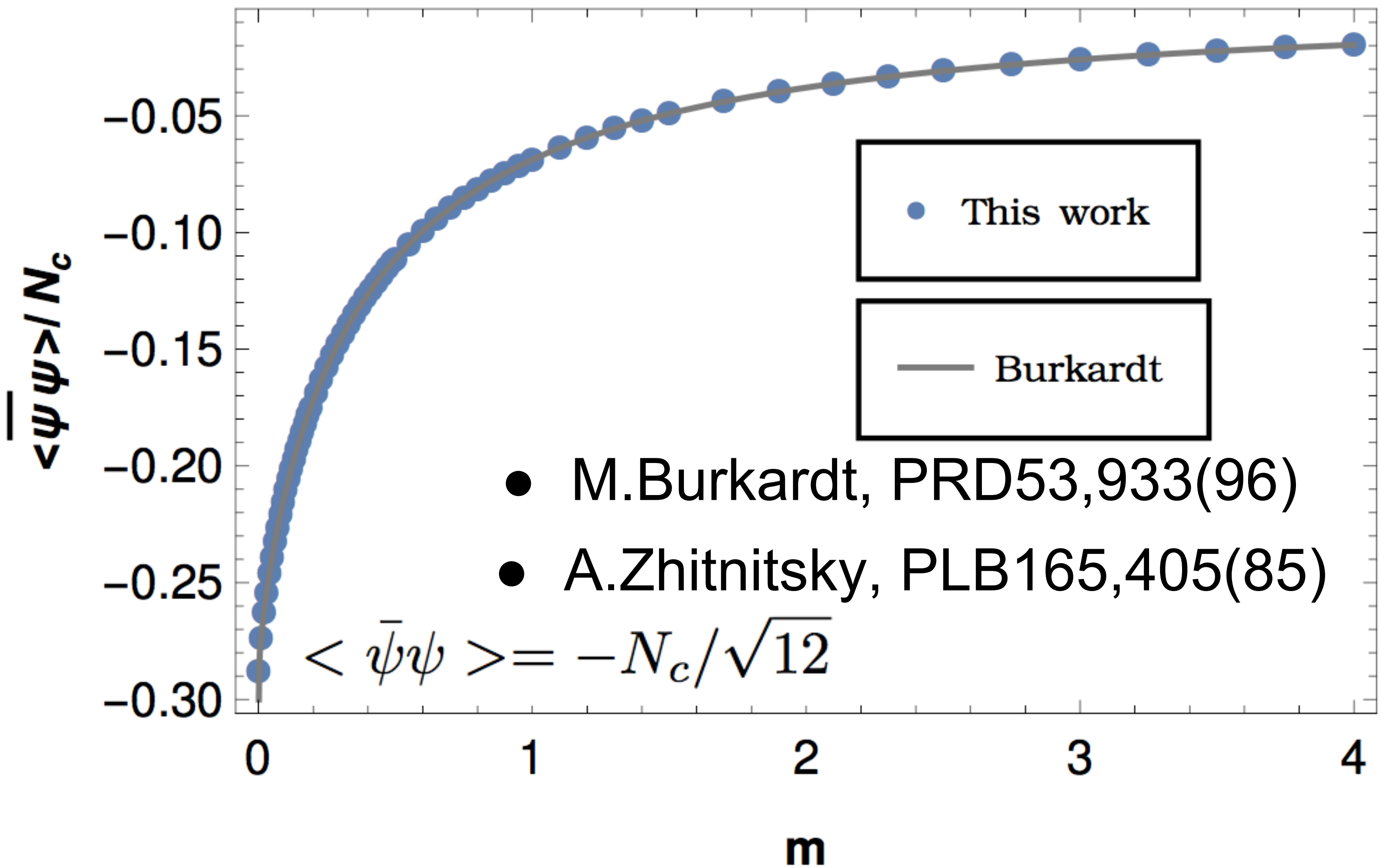
Meson Spectroscopy



- G.'tHooft, NPB75, 461(74) - LFD

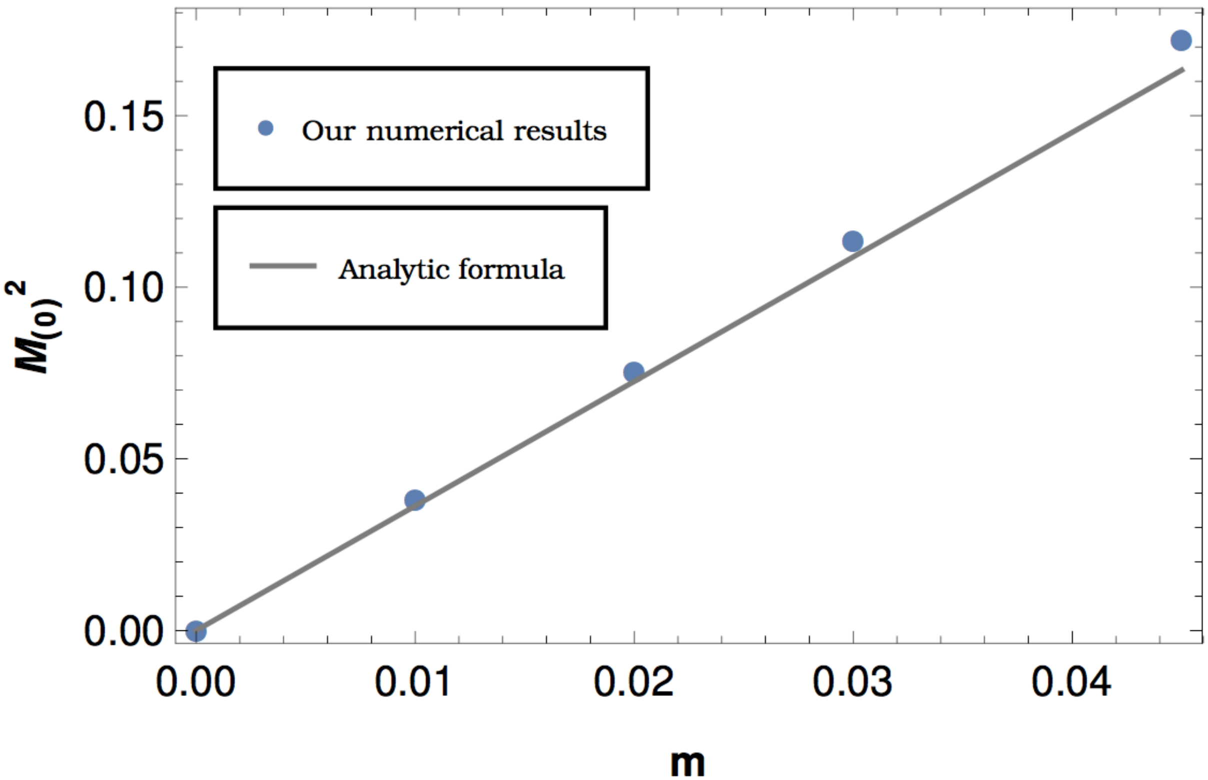
- M.Li, et al., JPG13, 915(87) - IFD (rest frame)

- Y. Jia, et al., JHEP11, 151('17) - IFD (moving frame)



$\langle\bar{\psi}\psi\rangle/N_c$
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
0
1
2
3
4
m
This work
Burkardt
• M.Burkardt, PRD53,933(96)
• A.Zhitnitsky, PLB165,405(85)
$<\bar{\psi}\psi>=-N_c/\sqrt{12}$

Gell-Mann - Oaks - Renner Relation

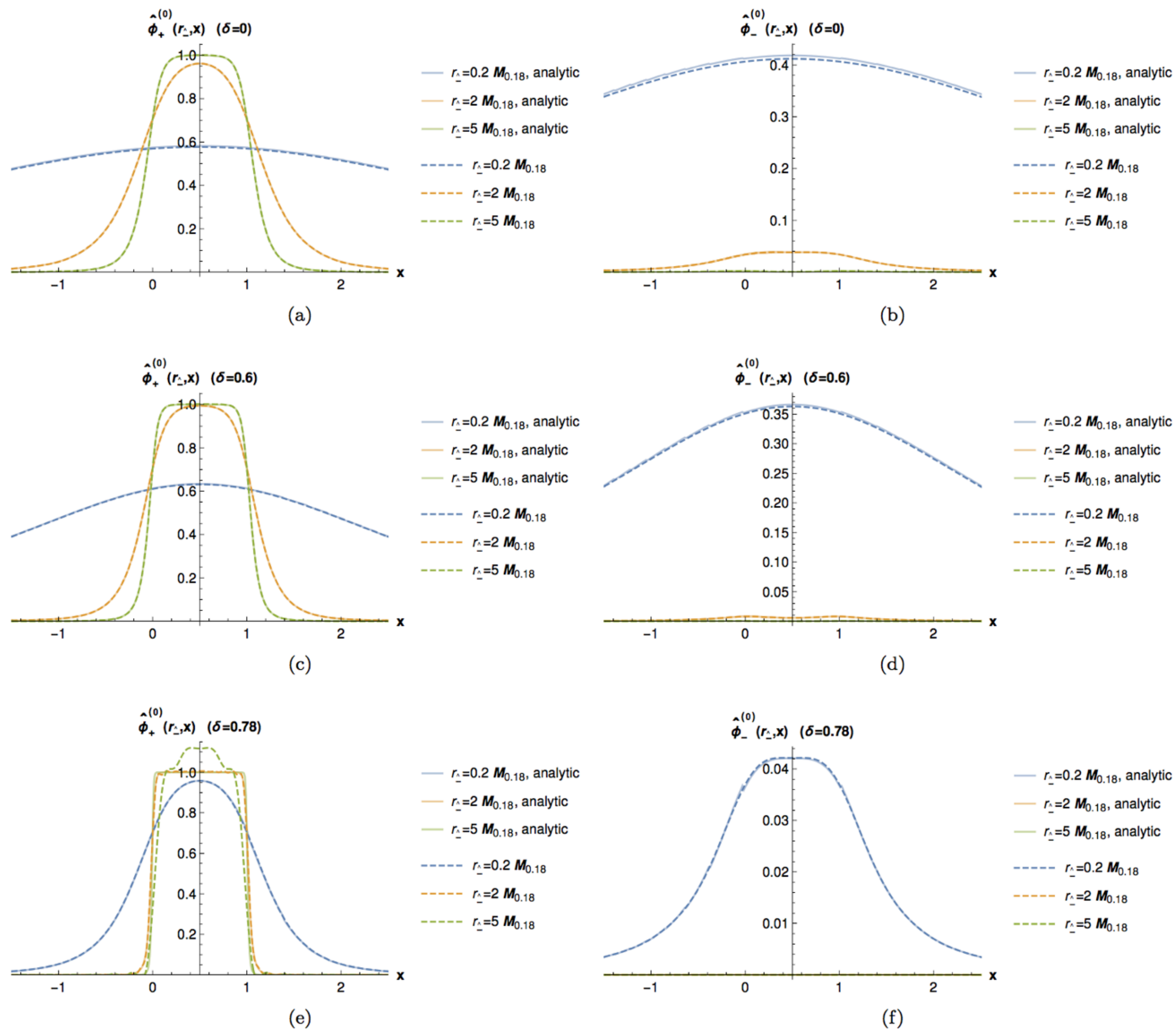


$$\mathcal{M}_\pi^2 = -\frac{4m < \bar{\psi}\psi >}{f_\pi^2} = \sqrt{\frac{8\pi^2 m^2 \lambda}{3}}$$

$$f_\pi = \sqrt{N_c/\pi}$$

Meson Ground-state Wave-function for m=0 case

$$\boxed{\hat{\phi}^{(0)}_{\pm}(r_{\stackrel{\wedge}{-}}, p_{\stackrel{\wedge}{-}}) = \frac{1}{2}\left(\cos\frac{\theta(r_{\stackrel{\wedge}{-}} - p_{\stackrel{\wedge}{-}}) - \theta(p_{\stackrel{\wedge}{-}})}{2} \pm \sin\frac{\theta(r_{\stackrel{\wedge}{-}} - p_{\stackrel{\wedge}{-}}) + \theta(p_{\stackrel{\wedge}{-}})}{2}\right)}$$



Parton Distribution Functions (PDFs)

$$q_n(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-}$$

$$\times \langle P_n^-, P^+ | \bar{\psi}(\xi^-)\gamma^+ \mathcal{W}[\xi^-,0]\psi(0) | P_n^-, P^+ \rangle_C,$$

$$\mathcal{W}[\xi^-,0] = \mathcal{P}\left[\exp\left(-ig_s \int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\right]$$

$A^+=0$ Gauge in LFD

Quasi-PDFs

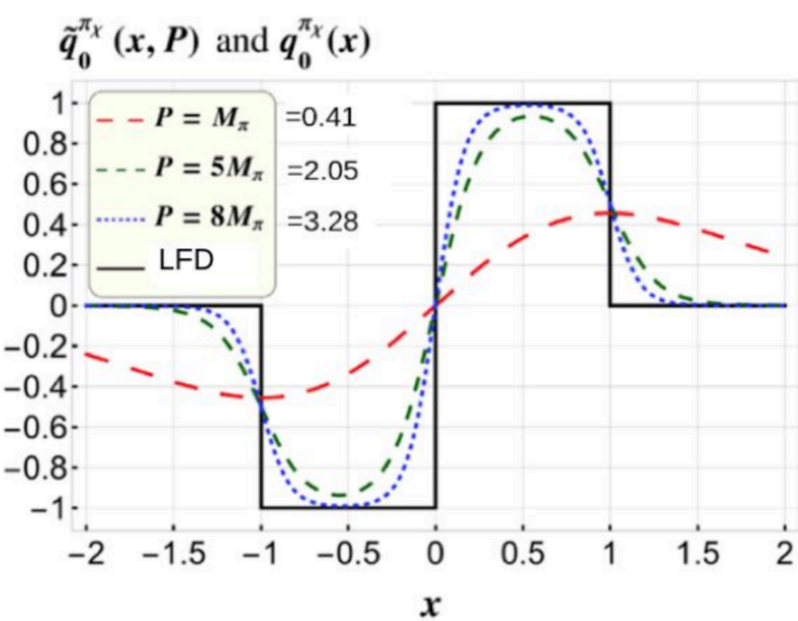
$$\tilde{q}_{(n)}(r_{\hat{-}}, x) = \int_{-\infty}^{+\infty} \frac{dx^{\hat{-}}}{4\pi} \, e^{ix^{\hat{-}} r_{\hat{-}}}$$

$$\times < r^{\hat{+}}_{(n)}, r_{\hat{-}} \mid \bar{\psi}(x^{\hat{-}}) \, \gamma_{\hat{-}} \, \mathcal{W}[x^{\hat{-}}, 0] \, \psi(0) \mid r^{\hat{+}}_{(n)}, r_{\hat{-}} >_C,$$

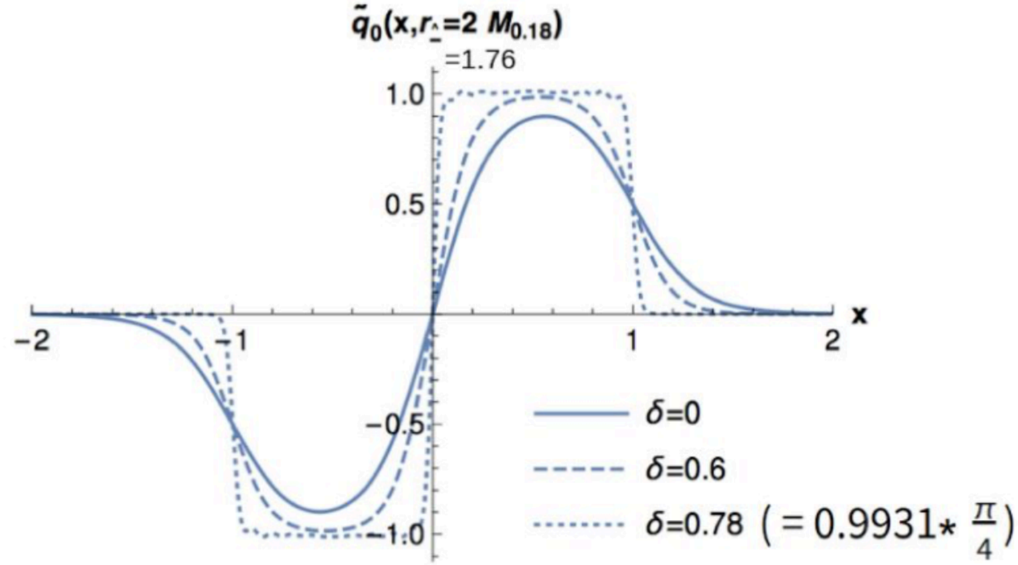
$$\mathcal{W}[x^{\hat{-}},0] = \mathcal{P}\left[\exp\left(-ig \int_0^{x^{\hat{-}}} dx'^{\hat{-}} A_{\hat{-}}(x'^{\hat{-}})\right)\right]$$

Interpolating dynamics

Quasi-PDF



Quark quasi-PDFs and light-front PDF for the chiral pion.



Interpolating "quasi-PDFs" for the chiral pion.

All quantities are in proper units of $\sqrt{2\lambda}$.

Jia, Y., Liang, S., Xiong, X., and Yu, R. (2018). Phys. Rev. D, 98:054011.

Ma, B. and Ji, C.-R. (2021). Phys. Rev. D, 104:036004.

Extended Wick Rotation

$$p^0 \to \tilde{P}^0 = ip^0 \quad (\delta = 0)$$

$$For \quad 0 < \delta < \pi / 4 \, ,$$

$$p^{\hat{+}} / \sqrt{C} \to \tilde{P}^{\hat{+}} / \sqrt{C} = ip^{\hat{+}} / \sqrt{C} \, .$$

$$Correspondence\ to\ Euclidean\ Space$$

$$p_{\underline{\wedge}}'^{2} = p_{\underline{\wedge}}^{2} / C \leftrightarrow -\tilde{P}^2$$

Conclusion and Outlook

- QCD(1+1) in large Nc “tHooft model’ is interpolated between IFD and LFD and solved for its mass gap to find interpolation angle independent energy function including the wavefunction renormalization.
- Chiral condensate, meson mass spectra bearing the feature of Regge trajectories and GOR relation are found independent of interpolation angle indicating the persistence of nontrivial vacuum even in LFD.

- **Applying to quasi-PDFs in the interpolating formulation, we note a possibility of utilizing not only the reference frame dependence but also the interpolation angle dependence to get an alternative effective approach to the LFD's PDFs.**
- **Interpolation of QCD(3+1) in Nc=3 between IFD and LFD needs to be explored, in particular, in the timelike region to study the color confinement.**