Quantum Stress on the Light Front

Xianghui Cao

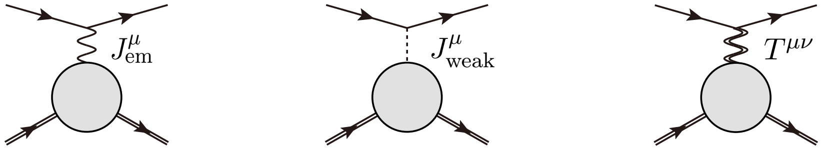
University of Science and Technology of China, Hefei, China

Light-Cone 2023
September 19, 2023, Rio de Janeiro, Brazil

Based on: XC, Yang Li, James Vary, arXiv:2308.06812

The last global unknown

- We can probe a hadron by other three fundamental forces



- Nucleon energy-momentum tensor (EMT):

$$\langle p', s'|\hat{T}^{\mu\nu}(0)|p, s\rangle = \frac{1}{2M}\bar{u}_{s'}(p')\left[2P^\mu P^\nu A(q^2) + iP^{\{\mu}\sigma^{\nu\}\rho}q_\rho J(q^2) + \frac{1}{2}(q^\mu q^\nu - q^2 g^{\mu\nu})D(q^2)\right]u_s(p).$$

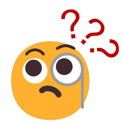
where $P = (p' + p)/2$, $q = p' - p$.

- Global properties:

em: $\partial_\mu J^\mu_{\rm em} = 0$	$\langle N'\|J^\mu_{\bf em}\|N\rangle$	$\longrightarrow$	Q	$= 1.602176487(40) \times 10^{-19}$C
			μ	$= 2.792847356(23)\mu_N$
weak: PCAC	$\langle N'\|J^\mu_{\bf weak}\|N\rangle$	$\longrightarrow$	g_A	$= 1.2694(28)$
			g_p	$= 8.06(55)$
gravity: $\partial_\mu T^{\mu\nu}_{\bf grav} = 0$	$\langle N'\|T^{\mu\nu}_{\bf grav}\|N\rangle$	$\longrightarrow$	m	$= 938.272013(23)$ MeV$/c^2$
			J	$= \frac{1}{2}$
			$\boldsymbol{D}$	$= ?$

[Polyakov:2018zvc]

What does $D = D(0)$ stand for?



Mechanical properties of hadrons

[Perevalova:2016dln]

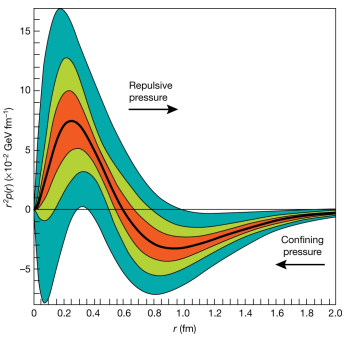
$$T^{ij}(\boldsymbol{r}) = [\hat{\boldsymbol{r}}^i\hat{\boldsymbol{r}}^j - \frac{1}{3}\delta^{ij}]\underbrace{s(r)}_{\text{shear}} + \delta^{ij}\underbrace{p(r)}_{\text{pressure}}$$

- Force balance inside the nucleon:

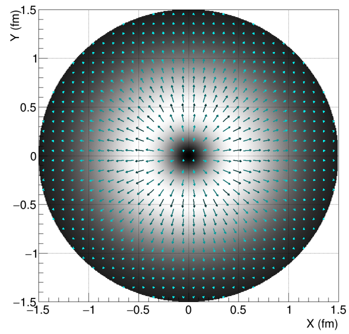
von Laue condition: $\partial_\mu T^{\mu\nu} = 0 \Rightarrow \int d^3r p(r) = 0$

Normal (radial) forces: $dF_r/dS_r = \frac{2}{3}s(r) + p(r) > 0$ conjecture

$$\Rightarrow D = -\frac{2m}{3}\int d^3r r^2\left[\frac{2}{3}s(r) + p(r)\right] < 0$$



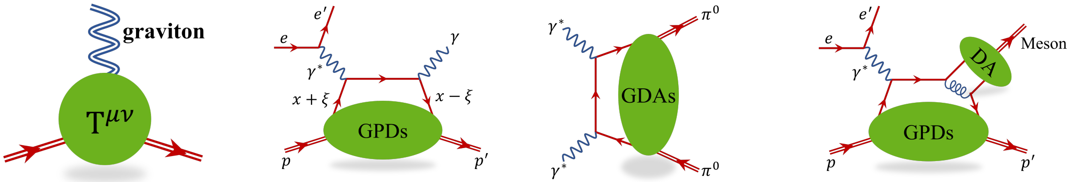
Pressure distribution in the proton
[Burkert:2018bqq]



Normal force distribution in the proton
[Burkert:2021ith]

How to measure the GFFs

- Experimental access to gravitational form factors (GFFs): [Burkert:2023wzr]



  - Deeply virtual Compton scattering (DVCS) to probe quark GFFs
  - Deeply virtual meson production to probe gluon GFFs
  - $\gamma\gamma^* \to \pi^0\pi^0$ to probe pion GFFs
  - ...

- Ji's sum rule:

$$\int_{-1}^{1} dx x H^{q,g}(x,\xi,t) = A^{q,g}(t) + \xi^2 D^{q,g}(t), \quad \int_{-1}^{1} dx x E^{q,g}(x,\xi,t) = B^{q,g}(t) - \xi^2 D^{q,g}(t).$$

Here, $H^{q,g}$ and $E^{q,g}$ are generalized parton distributions (GPDs) [Ji:1996nm]

- GPD formalism is convenient to obtain individual contributions of GFFs

Theoretical understanding

- Lattice QCD: proton and pion GFFs [Shanahan:2018pib]
- Perturbative QCD: asymptotic behaviors of nucleon [Tong:2022zax]

$$A(q^2) \sim 1/(q^2)^2 \quad J(q^2),\ D(q^2) \sim 1/(q^2)^3$$

- Phenomenological models:
  - Bag model, chiral quark soliton model: fermion D-term [Hudson:2017oul]
  - Liquid drop model for nuclei: $D \propto A^{7/3}$ [Polyakov:2002yz]
  - . . .
- Light-front quantization
  - A natural framework of non-perturbative hadron physics [Brodsky:2000ii]
  - Light-front wave functions (LFWFs) provide a physical interpretation, e.g. $B(0) = 0$
  - constituent quark models, AdS/QCD models, NJL models, $\cdots$

[Sun:2020wfo,Chakrabarti:2015lba,Freese:2019bhb]

D-term contains interaction, needs proper renormalization

Scalar Yukawa model

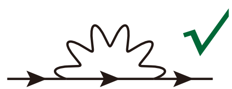
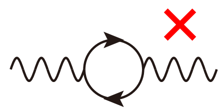
$$\mathcal{L} = \partial_\mu \chi^\dagger \partial^\mu \chi - m^2 \chi^\dagger \chi + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2}\mu^2 \varphi^2 + g_0 \chi^\dagger \chi \varphi + \delta m^2 \chi^\dagger \chi$$

$$\Downarrow$$

$$\hat{T}^{\mu\nu} = \partial^{\{\mu} \chi^\dagger \partial^{\nu\}} \chi - g^{\mu\nu}\left[\partial_\sigma \chi^\dagger \partial^\sigma \chi - (m^2 - \delta m^2)\chi^\dagger \chi\right] - g^{\mu\nu} g_0 \chi^\dagger \chi \varphi + \partial^\mu \varphi \partial^\nu \varphi - \frac{1}{2} g^{\mu\nu}\left(\partial^\rho \varphi \partial_\rho \varphi - \mu_0^2 \varphi^2\right)$$

where $m = 0.94\text{GeV}$, $\mu = 0.14\text{GeV}$. g_0 and δm^2 are bare parameters.

- χ : mock nucleon, φ : mock pion
- Quenched approximation: to avoid vacuum instability [Gross:2001ha]

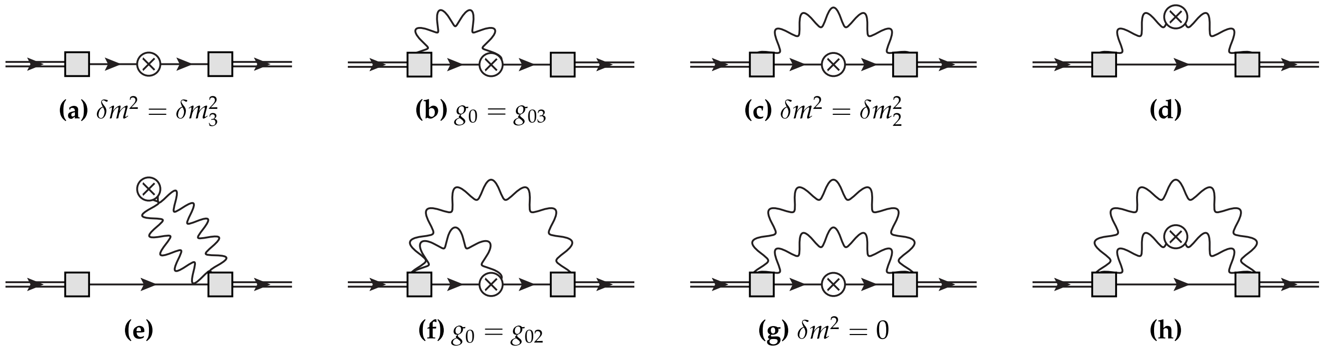



- Fock sector expansion:

$$|p\rangle = |\chi\rangle + |\chi\varphi\rangle + |\chi\varphi\varphi\rangle + |\chi\varphi\varphi\varphi\rangle + \cdots$$

- Solved up to $|\chi\varphi\varphi\varphi\rangle$ sector at non-perturbative couplings [Li, Karmanov & Vary:2014kfa]
  - Fock sector dependent renormalization [Karmanov:2008br]
  - Fock sector expansion converged up to $|\chi\varphi\varphi\rangle$ sector

Diagrammatic representation of EMT



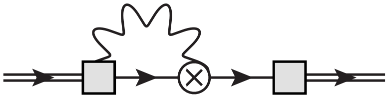
- Fock space truncation up to 3-body
- LFWFs from [Li, Karmanov & Vary:2015iaw,2016yzu]
- Light-front graphical rules extended to non-perturbative regime using LFWFs

[Carbonell:1998rj]

EMT renormalization

- Fock sector dependent renormalization [Karmanov:2008br]
- Sector dependent counterterms from [Li, Karmanov & Vary:2015iaw,2016yzu]
- All divergences cancel out with sector dependent counterterms, e.g. $(a) + (b)$

(a) $\delta m^2 = \delta m_3^2$



(b) $g_0 = g_{03}$

$$t_a^{\alpha\beta} = Z[(\frac{1}{2}q^2 - \delta m_3^2)g^{\alpha\beta} + p^{\{\alpha}p'^{\beta\}}]$$
$$= Z[2P^\alpha P^\beta + (\frac{1}{2}q^2 - \delta m_3^2)g^{\alpha\beta} - \frac{1}{2}q^\alpha q^\beta]$$

$$t_b^{\alpha\beta} = -\sqrt{Z}g^{\alpha\beta}\int\frac{dx}{2x(1-x)}\int\frac{d^2k_\perp}{(2\pi)^3}g_{03}\psi_2(x,k_\perp)$$
$$= g^{\alpha\beta}Z\delta m_3^2$$

GFFs on the light front

[Carbonell:1998rj, Karmanov:2002qu]

- Covariant light-front dynamics analysis (Drell-Yan frame $q^+ = 0$)

$$\langle p'|\hat{T}^{\alpha\beta}(0)|p\rangle = 2P^\alpha P^\beta A(q^2) + \frac{1}{2}(q^\alpha q^\beta - q^2 g^{\alpha\beta})D(q^2) + \frac{(q^2)^2\omega^\alpha\omega^\beta}{(P^+)^2}S_1(q^2) + \frac{1}{(P^+)^2}\epsilon^{\alpha\mu\nu\gamma}P_\mu q_\nu \omega_\gamma \epsilon^{\beta\rho\sigma\lambda}P_\rho q_\sigma \omega_\lambda S_2(q^2).$$

$S_{1,2}(q^2)$ are two spurious GFFs due to the violation of the Lorentz symmetry.

- t^{++} and t^{+-} are free of the spurious contributions. In Breit frame ($\boldsymbol{P}_\perp = 0$):

$$t^{\alpha\beta} = \langle p'|\hat{T}^{\alpha\beta}(0)|p\rangle / (2P^+)$$

$$t^{++} = P^+ A(-\boldsymbol{q}_\perp^2),$$

$$t^{+-} = \frac{m^2 + \frac{1}{4}\boldsymbol{q}_\perp^2}{P^+}A(-\boldsymbol{q}_\perp^2) + \frac{1}{2P^+}\boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2),$$

$$\mathrm{tr}t^{ij} = -\frac{1}{4P^+}\boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2) + \frac{1}{2P^+}\boldsymbol{q}_\perp^2 S_2(-\boldsymbol{q}_\perp^2).$$

$$\Rightarrow A(-\boldsymbol{q}_\perp^2) = \frac{t^{++}}{P^+}, \quad \boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2) = 2P^+ t^{+-} - 2\frac{m^2 + \frac{1}{4}\boldsymbol{q}_\perp^2}{P^+}t^{++}$$

Conservation laws

- Light-front Schrödinger equation,

$$\hat{P}^\mu |p\rangle = p^\mu |p\rangle ,$$

$$\Rightarrow p^\mu 2p^+ \delta^{(3)}(p-p') = \langle p'|\hat{P}^\mu|p\rangle = \langle p'| \int d^3x \hat{T}^{+\mu}(x)|p\rangle$$

$$= \int d^3x e^{iq\cdot x} \langle p'|\hat{T}^{+\mu}(0)|p\rangle = \delta^{(3)}(p-p') \langle p'|\hat{T}^{+\mu}(0)|p\rangle .$$

- Forward limit ($q^2 = 0$):

$$\hat{P}^+ = \int d^3x \hat{T}^{++}(x), \Rightarrow \langle p|\hat{T}^{++}(0)|p\rangle = 2p^+p^+, \Rightarrow A(0) = 1,$$

$$\hat{P}^i = \int d^3x \hat{T}^{+i}(x), \Rightarrow \langle p|\hat{T}^{+i}(0)|p\rangle = 2p^+p^i, \Rightarrow A(0) = 1,$$

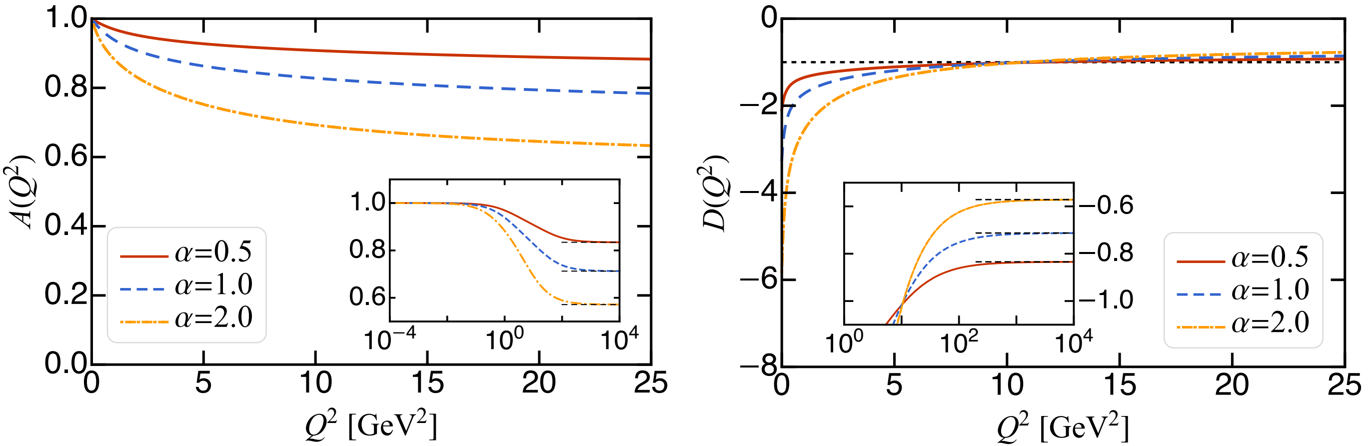
$$\hat{P}^- = \int d^3x \hat{T}^{+-}(x), \Rightarrow \langle p|\hat{T}^{+-}(0)|p\rangle = 2p^+p^-, \Rightarrow \underbrace{\lim_{q_\perp \to 0} \boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2) = 0}_{\text{von Laue condition}} .$$

Here, $d^3x = \frac{1}{2}dx^- d^2x_\perp$.

- Indeed, D is finite in our model

[Varma:2020crx]

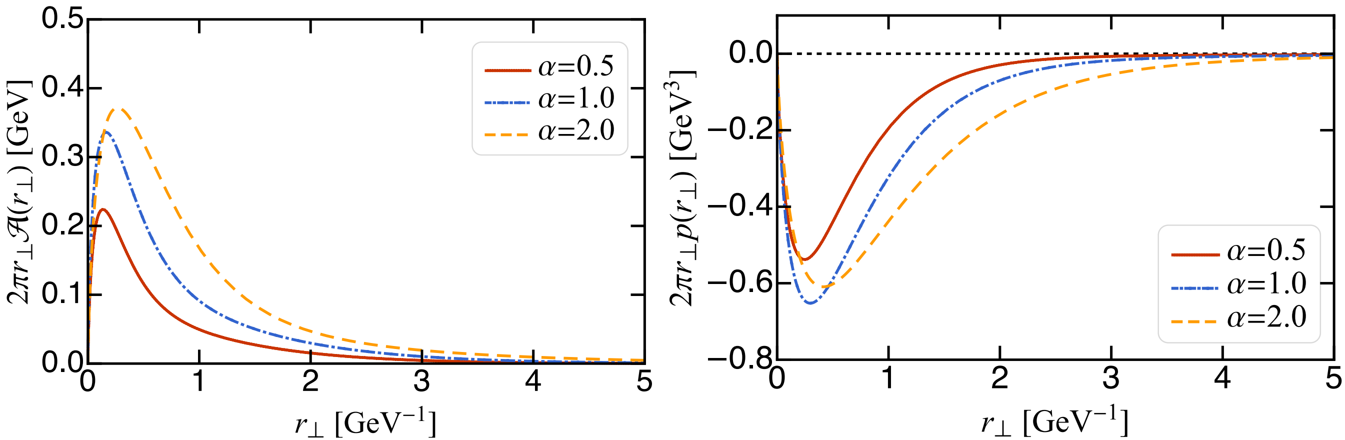
Numerical results: $A(Q^2)$ and $D(Q^2)$ up to strong coupling



$Q^2 = -q^2 = \boldsymbol{q}_\perp^2$, $\alpha = g^2/(16\pi m^2)$

- For small α, $D(Q^2)$ is close to -1, the free scalar particle's result. As α increases, $D(0)$ becomes more negative
- Forward limit: $A(0) = 1$, D is finite and negative
- For large Q^2, $A(Q^2 \to \infty) = Z$, $D(Q^2 \to \infty) = -Z$, revealing a pointlike core

Matter density and pressure



- Light-front distribution:

fitting functions: $f(Q^2) = f(\infty) + \frac{a_1}{1+Q^2/\Lambda_1^2} + \frac{a_2}{1+Q^2/\Lambda_2^2}$

$$\mathcal{A}(r_\perp) = \int \frac{d^2q_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_\perp} A(-\boldsymbol{q}_\perp^2), \quad p(r_\perp) = -\frac{1}{6M}\int \frac{d^2q_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_\perp} \boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2)$$

- A point-like repulsive core at $r_\perp = 0$

$$\int \frac{d^2q_\perp}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_\perp} \boldsymbol{q}_\perp^2 \frac{1}{1+\boldsymbol{q}_\perp^2/\Lambda_1^2} = \frac{\Lambda_1^2}{2\pi}\delta^{(2)}(\boldsymbol{r}_\perp) - \frac{\Lambda_1^4}{2\pi}K_0(\Lambda_1 r_\perp)$$

Light-front wave function representation

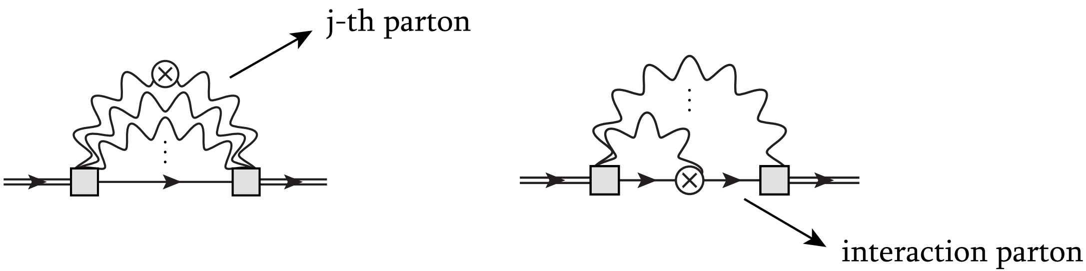
[Brodsky:2000ii]

$$t^{++}(-\boldsymbol{q}_\perp^2) = \sum_n \int \left[dx_i d^2 r_{i\perp}\right]_n \left|\widetilde{\psi}_n(\{x_i, \boldsymbol{r}_{i\perp}\})\right|^2 \sum_j x_j P^+ e^{i\boldsymbol{r}_{j\perp}\cdot\boldsymbol{q}_\perp}$$

A new LFWF representation for t^{+-} and $D(q^2)$:

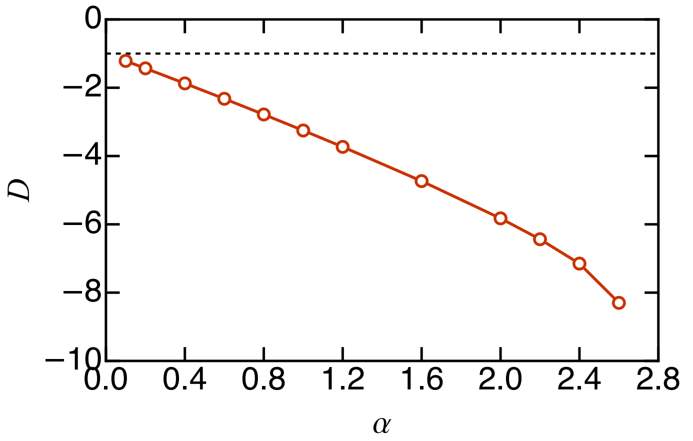
$$t^{+-} = \sum_n \int \left[dx_i d^2 r_{i\perp}\right]_n \widetilde{\psi}_n^*(\{x_i, \boldsymbol{r}_{i\perp}\}) \sum_j e^{i\boldsymbol{r}_{j\perp}\cdot\boldsymbol{q}_\perp} \frac{-\boldsymbol{\nabla}_{j\perp}^2 + m_j^2 - \frac{1}{4}\boldsymbol{q}_\perp^2}{x_j P^+} \widetilde{\psi}_n(\{x_i, \boldsymbol{r}_{i\perp}\})$$
$$+ \sum_n \int \left[dx_i d^2 r_{i\perp}\right]_n \widetilde{\psi}_n^*(\{x_i, \boldsymbol{r}_{i\perp}\}) e^{i\boldsymbol{r}_n\cdot\boldsymbol{q}_\perp} \left[\frac{M^2}{P^+} - \sum_j \frac{-\boldsymbol{\nabla}_{j\perp}^2 + m_j^2}{x_j P^+}\right] \widetilde{\psi}_n(\{x_i, \boldsymbol{r}_{i\perp}\})$$

$$\boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2) = 2P^+[T(-\boldsymbol{q}_\perp^2) + V(-\boldsymbol{q}_\perp^2)] - (2M^2 + \frac{1}{2}\boldsymbol{q}_\perp^2) A(-\boldsymbol{q}_\perp^2)$$



- D is finite:

$$D = -1 + 2\sum_n \int \left[dx_i d^2 r_{i\perp}\right]_n \widetilde{\psi}_n^*(\{x_i, \boldsymbol{r}_{i\perp}\})$$
$$\times \sum_j \frac{1}{x_j}\left\{(r_n^2 - r_{j\perp}^2)(-\boldsymbol{\nabla}_{j\perp}^2 + m_j^2 - x_j^2 M^2) + \frac{1}{4}(x_j^2 - 1)\right\}\widetilde{\psi}_n(\{x_i, \boldsymbol{r}_{i\perp}\}).$$



Summary

XC, Yang Li, James Vary, arXiv:2308.06812

- Form factor $D(q^2)$ of hadrons remain to be "the last global unknown"
- We calculate the gravitational form factors of a strongly-coupled scalar nucleon using light-front Hamiltonian formalism and extract matter distrubutions and pressure
- We obtain a non-perturbative light-front wave function representation of the D-term
- This representation can be adapted in phenomenological QCD models

$$V(q^2) = \sum_n \int \left[dx_i d^2 r_{i\perp}\right]_n \widetilde{\psi}_n^*(\{x_i, \boldsymbol{r}_{i\perp}\}) e^{i\boldsymbol{R}_\perp \cdot \boldsymbol{q}_\perp} \mathcal{V}_{\text{eff}} \widetilde{\psi}_n(\{x_i, \boldsymbol{k}_{i,n\perp}\})$$

$\boldsymbol{R}_\perp$: means where the interaction acts on.

Thank You

All hadron matrix elements

$$t^{++} = P^+ A(-\boldsymbol{q}_\perp^2)$$

$$t^{+i} = 0$$

$$t^{ij} = \frac{1}{4P^+}(q^i q^j - \delta^{ij}\boldsymbol{q}_\perp^2) D(-\boldsymbol{q}_\perp^2) + \frac{1}{2P^+}\epsilon^{in}\epsilon^{jm} q_\perp^n q_\perp^m S_2(-\boldsymbol{q}_\perp^2)$$

$$\mathbf{tr} t^{ij} = -\frac{1}{4P^+}\boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2) + \frac{1}{2P^+}\boldsymbol{q}_\perp^2 S_2(-\boldsymbol{q}_\perp^2)$$

$$t^{+-} = \frac{m^2 + \frac{1}{4}\boldsymbol{q}_\perp^2}{P^+} A(-\boldsymbol{q}_\perp^2) + \frac{1}{2P^+}\boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2)$$

$$t^{--} = \frac{4(m^2 + \frac{1}{4}\boldsymbol{q}_\perp^2)^2}{(P^+)^3} A(-\boldsymbol{q}_\perp^2) + \frac{2\boldsymbol{q}_\perp^4}{(P^+)^3} S_1(-\boldsymbol{q}_\perp^2)$$

$$t^{-i} = 0$$

t^{++}

$$x_i = p_i^+/p^+,\ \boldsymbol{k}_{i\perp} = \boldsymbol{p}_{i\perp} - x_i \boldsymbol{p}_\perp,\ Z = \psi_1^2$$

$$\begin{aligned}
t^{++} =& 2(P^+)^2 Z \\
&+ 2(P^+)^2 \int \frac{dx}{2x(1-x)} \int \frac{d^2k_\perp}{(2\pi)^3} \psi_2(x, \boldsymbol{k}_\perp) \psi_2^*(x, \boldsymbol{k}_\perp - x\boldsymbol{q}_\perp)(1-x) \\
&+ 2(P^+)^2 \int \frac{dx}{2x(1-x)} \int \frac{d^2k_\perp}{(2\pi)^3} \psi_2(x, \boldsymbol{k}_\perp) \psi_2^*(x, \boldsymbol{k}_\perp + (1-x)\boldsymbol{q}_\perp) x \\
&+ 2(P^+)^2 \frac{1}{2!} \int \frac{dx}{2x} \int \frac{d^2k_\perp}{(2\pi)^3} \int \frac{dx'}{2x'(1-x-x')} \int \frac{d^2k'_\perp}{(2\pi)^3} \\
&\times \psi_3(x, \boldsymbol{k}_\perp, x', \boldsymbol{k}'_\perp) \psi_3^*(x, \boldsymbol{k}_\perp - x\boldsymbol{q}_\perp, x', \boldsymbol{k}'_\perp - x'\boldsymbol{q}_\perp)(1-x-x') \\
&+ 2(P^+)^2 \frac{1}{2!} \int \frac{dx}{2x} \int \frac{d^2k_\perp}{(2\pi)^3} \int \frac{dx'}{2x'(1-x-x')} \int \frac{d^2k'_\perp}{(2\pi)^3} \\
&\times \psi_3(x, \boldsymbol{k}_\perp, x', \boldsymbol{k}'_\perp) \psi_3^*(x, \boldsymbol{k}_\perp + (1-x)\boldsymbol{q}_\perp, x', \boldsymbol{k}'_\perp - x'\boldsymbol{q}_\perp) x \\
&+ 2(P^+)^2 \frac{1}{2!} \int \frac{dx}{2x} \int \frac{d^2k_\perp}{(2\pi)^3} \int \frac{dx'}{2x'(1-x-x')} \int \frac{d^2k'_\perp}{(2\pi)^3} \\
&\times \psi_3(x, \boldsymbol{k}_\perp, x', \boldsymbol{k}'_\perp) \psi_3^*(x, \boldsymbol{k}_\perp - x\boldsymbol{q}_\perp, x', \boldsymbol{k}'_\perp + (1-x')\boldsymbol{q}_\perp) x'.
\end{aligned}$$

t^{+-}

One-body and two-body: $\boldsymbol{\ell}_\perp = \boldsymbol{k}_\perp - \frac{1}{2}x\boldsymbol{q}_\perp$

$$
\begin{aligned}
t^{+-}_{1+2} &= Z[2(m^2 + \boldsymbol{P}_\perp^2) - \frac{1}{2}\boldsymbol{q}_\perp^2] \\
&+ \int \frac{dx}{2x(1-x)} \int \frac{d^2\ell_\perp}{(2\pi)^3} \psi_2(x, \boldsymbol{\ell}_\perp + \frac{1}{2}x\boldsymbol{q}_\perp)\psi_2^*(x, \boldsymbol{\ell}_\perp - \frac{1}{2}x\boldsymbol{q}_\perp) \\
&\times \frac{2(\boldsymbol{\ell}_\perp - (1-x)\boldsymbol{P}_\perp)^2 + 2m^2 - \frac{1}{2}\boldsymbol{q}_\perp^2}{1-x} \\
&+ \int \frac{dx}{2x(1-x)} \int \frac{d^2\ell_\perp}{(2\pi)^3} \psi_2(x, \boldsymbol{\ell} - \frac{1}{2}(1-x)\boldsymbol{q}_\perp)\psi_2^*(x, \boldsymbol{\ell}_\perp + \frac{1}{2}(1-x)\boldsymbol{q}_\perp) \\
&\times \frac{2(\boldsymbol{\ell}_\perp + x\boldsymbol{P}_\perp)^2 + 2\mu^2 - \frac{1}{2}\boldsymbol{q}_\perp^2}{x} \\
&- \int \frac{dx}{2x(1-x)} \int \frac{d^2\ell_\perp}{(2\pi)^3} \psi_2(x, \boldsymbol{\ell}_\perp)\psi_2^*(x, \boldsymbol{\ell}_\perp - x\boldsymbol{q}_\perp) \\
&\times 2\left(\frac{\boldsymbol{\ell}_\perp^2 + \mu^2}{x} + \frac{\boldsymbol{\ell}_\perp^2 + m^2}{1-x} - M^2\right)
\end{aligned}
$$

t^{+-}

Three-body:

$$t_3^{+-} = \frac{1}{2!}\int\frac{dx}{2x}\int\frac{d^2\ell_\perp}{(2\pi)^3}\int\frac{dx'}{2x'(1-x-x')}\int\frac{d^2\ell'_\perp}{(2\pi)^3}\Bigg\{$$

$$\psi_3(x,\boldsymbol{\ell}_\perp+\frac{1}{2}x\boldsymbol{q}_\perp,x',\boldsymbol{\ell}'_\perp+\frac{1}{2}x'\boldsymbol{q}_\perp)\psi_3(x,\boldsymbol{\ell}_\perp-\frac{1}{2}x\boldsymbol{q}_\perp,x',\boldsymbol{\ell}'_\perp-\frac{1}{2}x'\boldsymbol{q}_\perp)$$

$$\times\frac{2(\boldsymbol{\ell}_\perp+\boldsymbol{\ell}'_\perp-(1-x-x')\boldsymbol{P}_\perp)^2+2m^2-\frac{1}{2}\boldsymbol{q}_\perp^2}{1-x-x'}$$

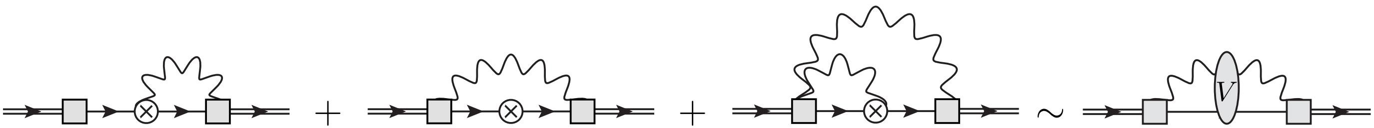
$$+\psi_3(x,\boldsymbol{\ell}_\perp-\frac{1}{2}(1-x)\boldsymbol{q}_\perp,x',\boldsymbol{\ell}'_\perp+\frac{1}{2}x'\boldsymbol{q}_\perp)\psi_3(x,\boldsymbol{\ell}_\perp+\frac{1}{2}(1-x)\boldsymbol{q}_\perp,x',\boldsymbol{\ell}'_\perp-\frac{1}{2}x'\boldsymbol{q}_\perp)$$

$$\times\frac{2(\boldsymbol{\ell}_\perp+x\boldsymbol{P}_\perp)^2+2\mu^2-\frac{1}{2}\boldsymbol{q}_\perp^2}{x}$$

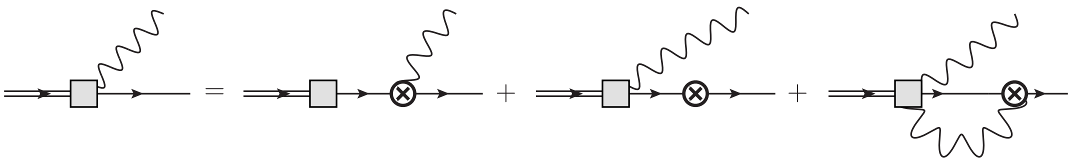
$$-\psi_3(x,\boldsymbol{\ell}_\perp,x',\boldsymbol{\ell}'_\perp)\psi_3^*(x,\boldsymbol{\ell}_\perp-x\boldsymbol{q}_\perp,x',\boldsymbol{\ell}'_\perp-x'\boldsymbol{q}_\perp)$$

$$\times 2\left[\frac{(\boldsymbol{\ell}_\perp+\boldsymbol{\ell}'_\perp)^2+m^2}{1-x-x'}+\frac{\boldsymbol{\ell}_\perp^2+\mu^2}{x}+\frac{\boldsymbol{\ell}'^2_\perp+\mu^2}{x'}-M^2\right]\Bigg\}$$

Two-body diagonalization



Equation of motion:



EMT decomposition

Drell-Yan frame ($q^+ = 0$):

$$\langle p' | \hat{T}^{\mu\nu} | p \rangle = 2P^\mu P^\nu A(q^2) + \frac{1}{2}(q^\mu q^\nu - q^2 g^{\mu\nu}) D(q^2) \\ + \frac{(q^2)^2}{(\omega \cdot P)^2} \omega^\mu \omega^\nu S_1(q^2) + \frac{q^2}{\omega \cdot P}(P^\mu \omega^\nu + P^\nu \omega^\mu) S_2(q^2)$$

Breit frame ($\boldsymbol{P}_\perp = 0$):

$$t^{++} = (P^+) A(-\boldsymbol{q}_\perp^2)$$

$$t^{+-} = \frac{m^2 + \frac{1}{4}\boldsymbol{q}_\perp^2}{P^+} A(-\boldsymbol{q}_\perp^2) + \frac{1}{2P^+}\boldsymbol{q}_\perp^2 [D(-\boldsymbol{q}_\perp^2) - 2S_2(-\boldsymbol{q}_\perp^2)]$$

$$\mathrm{tr} t^{ij} = -\frac{1}{4P^+}\boldsymbol{q}_\perp^2 D(-\boldsymbol{q}_\perp^2)$$