Exploring the covariant form factors for spin-1 particles

Light-Cone 2023: Hadrons and Symmetries

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Outline

Light-Front Motivations

- **Ligh-Front is the Ideal Framework to Describe Hadronic Bound States**
- **Constituent Picture and Unanbiguous Partons Content of the Hadronic System**
- **Light-Front Wavefunctions: Representation of Composite Systems in QFT**
- **Invariant Under Boosts**
- **Light-Front Vacuum is Trivial**
- **After Integrate in k^-: Bethe-Salpeter Amplitude (Wave Function)**
- **LF Lorentz Invariant Hamiltonian:** $P^2 = P^+P^- - P_\perp^2$

Light-Front Coordinates and Elect. Current

Four-Vector $\Longrightarrow \quad x^\mu = (x^0, x^1, x^2, x^3) \; = \; (x^+, x^-, x_\perp)$

$x^+ = t + z \qquad x^+ = x^0 + x^3 \quad \Longrightarrow$ **Time**

$x^- = t - z \qquad x^- = x^0 - x^3 \quad \Longrightarrow$ **Position**

Metric Tensor and Scalar product

$$x \cdot y = x^\mu y_\mu = x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 = \frac{x^+ y^- + x^- y^+}{2} - \vec{x}_\perp \vec{y}_\perp$$

$$p^+ = p^0 + p^3 \; , \; p^- = p^0 - p^3 \; , \; p^\perp = (p^1, p^2)$$

$p^\mu x_\mu = \frac{p^+ x^- + p^- x^+}{2} - \vec{p}_\perp \vec{x}_\perp$, $\quad x^+, x^-, \vec{x_\perp} \quad \Longrightarrow p^+, p^-, \vec{p}_\perp$

- **Dirac Matrix and Electromagnetic Current**

$\gamma^+ = \gamma^0 + \gamma^3 \; \Longrightarrow$ **Electr. Current** $J^+ = J^0 + J^3$

$\gamma^- = \gamma^0 - \gamma^3 \; \Longrightarrow$ **Electr. Current** $J^- = J^0 - J^3$

$\gamma^\perp = (\gamma^1, \gamma^2) \; \Longrightarrow$ **Electr. Current** $J^\perp = (J^1, J^2)$

$p^- \quad \Longrightarrow$ **Light-Front Energy**

$$p^2 = p^+ p^- - (\vec{p}_\perp)^2 \Longrightarrow \quad p^- = \frac{(\vec{p}_\perp)^2 + m^2}{p^+}$$

On-shell

Bosons $\quad \Longrightarrow \quad S_F(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$

Fermions $\Longrightarrow \quad S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + \imath\epsilon} + \frac{\gamma^+}{2p^+}$

Review Papers:

- **Phys. Rept. 301, (1998) 299-486, Brodsky, Pauli and Pinsky**
- **A. Harindranath, Pramana, Journal of Indian Academy of Sciences Physics Vol.55, Nos 1 & 2, (2000) 241.**
- **An Introduction to Light-Front Dynamics for Pedestrians Avaroth Harindranath**

Light-Front book organizers: James Vary and Frank Wolz,(1997)

General expression for the spin-1 electromagnetic current

Plus and minus components

$$\begin{aligned} J^{\pm}_{\lambda'\lambda} &= (p'^{\pm} + p^{\pm})[F_1(q^2)(\epsilon_{\lambda'}.\epsilon_{\lambda}) - \frac{F_2(q^2)}{2m_v^2}(q.\epsilon_{\lambda'})(q.\epsilon_{\lambda})] \\ &- F_3(q^2)((q.\epsilon_{\lambda'})\epsilon^{\pm}_{\lambda} - (q.\epsilon_{\lambda})\epsilon^{\pm}_{\lambda'}) \ . \end{aligned}$$

- $\Longrightarrow$ F_1, F_2 **and** F_3 **: Covariant electromagnetic form factors**
- m_v, **Vector bound state mass**
- **Ref. Gilman, Ronald A. and Gross, Franz;**

J. Phys. G 28 (2002) R37–R116.

Frame

- Breit frame
- $\Longrightarrow \quad p^{\mu} = (p_0, -q/2, 0, 0)$, **Initial state**
- $\Longrightarrow \quad p'^{\mu} = (p_0, q/2, 0, 0)$ **Final state**

Transfer momentum $===>> \quad q^{\mu} = (0, q, 0, 0)$

- Polarization in the cartesian basis

$\epsilon_x^{\mu} = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0), \epsilon_y^{\mu} = (0, 0, 1, 0), \epsilon_z^{\mu} = (0, 0, 0, 1)$

for the vector meson in the initial state

- **And in the final state**

$\epsilon'^{\mu} = (\sqrt{\eta}, \ \sqrt{1+\eta}, 0, 0), \ \epsilon_y'^{\mu} = (0, 0, 1, 0), \ \epsilon_z'^{\mu} = (0, 0, 0, 1)$

- **with,** $\eta = \frac{q^2}{4m_{\rho}^2}$

Relation between the electromagnetic current, J_{ji}^{+}, and the covariant form factors

- **Plus component of the electromagnetic current**

$$
\begin{aligned}
J_{xx}^{+} &= 2p^{+}\left(-2F_1(1+2\eta) - \frac{F_2}{2m_v^2}q^2(1+\eta)\right) - F_3 2q\sqrt{\eta}\sqrt{1+\eta},\\
J_{yy}^{+} &= -2p^{+}F_1,\\
J_{zz}^{+} &= -2p^{+}F_1,\\
J_{zx}^{+} &= -F_3 q\sqrt{1+\eta},\\
J_{xz}^{+} &= F_3 q\sqrt{1+\eta},\\
J_{yx}^{+} &= J_{xy}^{+} = J_{zy}^{+} = J_{yz}^{+} \;=\; 0.
\end{aligned}
$$

- **Electromagentic form factors in terms of matrix elements of the current**

$$
\begin{aligned}
F_1^+ &= -\frac{J_{yy}^+}{2p^+} = -\frac{J_{zz}^+}{2p^+}, \\
F_2^+ &= \frac{m_v^2}{p^+ q^2 (1+\eta)} \left[J_{yy}^+ (1+2\eta) - J_{xx}^+ + J_{zx}^+ 2\sqrt{\eta} \right], \\
F_3^+ &= -\frac{J_{zx}^+}{q\sqrt{1+\eta}} = \frac{J_{xz}^+}{q\sqrt{1+\eta}} .
\end{aligned}
$$

Relation between the electromagnetic current, J^-_{ji}, and the covariant form factors

- **Minus component of the electromagnetic current**

$$
\begin{aligned}
J^-_{xx} &= 2p^- \left(-F_1(1+2\eta) - \frac{F_2}{2m_v^2} q^2 (1+\eta) \right) - F_3 2q\sqrt{\eta}\sqrt{1+\eta}, \\
J^-_{yy} &= -2p^- F_1, \\
J^-_{zz} &= -2p^- F_1, \\
J^-_{zx} &= F_3 q \sqrt{1+\eta}, \\
J^-_{xz} &= -F_3 q \sqrt{1+\eta}, \\
J^-_{yx} &= J^-_{xy} = J^-_{zy} = J^-_{yz} = 0.
\end{aligned}
$$

- **Electromagnetic form factors in terms of matrix elements of the current**

$$
\begin{aligned}
F_1^- &= -\frac{J_{yy}^-}{2p^-} = -\frac{J_{zz}^-}{2p^-}, \\
F_2^- &= \frac{m_v^2}{p^- q^2(1+\eta)}\left[J_{yy}^-(1+2\eta) - J_{xx}^- + J_{zx}^- 2\sqrt{\eta}\right], \\
F_3^- &= -\frac{J_{zx}^-}{q\sqrt{1+\eta}} = \frac{J_{xz}^-}{q\sqrt{1+\eta}}\,.
\end{aligned}
$$

- **Basis**
- **In spherical basis**

$$
\begin{aligned}
\epsilon^{\mu}_{+-} &= -(+)\frac{\epsilon^{\mu}_{x} + (-)\imath\epsilon^{\mu}_{y}}{\sqrt{2}} \\
\epsilon^{\mu}_{0} &= \epsilon^{\mu}_{z} \; .
\end{aligned}
$$

- **Matrix elements electromagnetic current (plus)**

$$J^{+}_{ji} = \frac{1}{2}\begin{pmatrix} J^{+}_{xx} + J^{+}_{yy} & \sqrt{2}J^{+}_{zx} & J^{+}_{yy} - J^{+}_{xx} \\ -\sqrt{2}J^{+}_{zx} & 2J^{+}_{zz} & \sqrt{2}J^{+}_{zx} \\ J^{+}_{yy} - J^{+}_{xx} & -\sqrt{2}J^{+}_{zx} & J^{+}_{xx} + J^{+}_{yy} \end{pmatrix}$$

- **Matrix elements electromagnetic current (minus)**

$$J^{-}_{ji} = \frac{1}{2}\begin{pmatrix} J^{-}_{xx} + J^{-}_{yy} & \sqrt{2}J^{-}_{zx} & J^{-}_{yy} - J^{-}_{xx} \\ -\sqrt{2}J^{-}_{zx} & 2J^{-}_{zz} & \sqrt{2}J^{-}_{zx} \\ J^{-}_{yy} - J^{-}_{xx} & -\sqrt{2}J^{-}_{zx} & J^{-}_{xx} + J^{-}_{yy} \end{pmatrix}$$

- **Light-front matriz electromagnetic current:**

$$I^{\pm}_{m'm} = \begin{pmatrix} I^{\pm}_{11} & I^{\pm}_{10} & I^{\pm}_{1-1} \\ -I^{\pm}_{10} & I^{\pm}_{00} & I^{\pm}_{10} \\ I^{\pm}_{1-1} & -I^{\pm}_{10} & I^{\pm}_{11} \end{pmatrix}$$

- **Relation (by Melosh matriz)**

$$R_M.J^{\pm}.R_M = I^{\pm} \quad (VIP!!)$$

$$R_M = \begin{pmatrix} \frac{1+\cos\theta}{2} & -\frac{\sin\theta}{2} & \frac{1-\cos\theta}{2} \\ \frac{\sin\theta}{\sqrt{2}} & \cos\theta & -\frac{\sin\theta}{2} \\ \frac{1-\cos\theta}{2} & \frac{\sin\theta}{\sqrt{2}} & \frac{1+\cos\theta}{2} \end{pmatrix} .$$

- **With**

$\cos\theta = \frac{1}{\sqrt{1+\eta}}, \sin\theta = \frac{\sqrt{\eta}}{\sqrt{1+\eta}}$ **and** $\eta = \frac{\vec{p}^2}{m^2}$, **if** $p_x < 0$, $\theta < 0$

- **Relations between the matrix elements of the current in the Cartesian spin basis, J^+**

$$I_{11}^{+} = \frac{J_{xx}^{+} + (1+\eta)J_{yy}^{+} - \eta J_{zz}^{+} - 2\sqrt{\eta}J_{zx}^{+}}{2(1+\eta)},$$

$$I_{10}^{+} = \frac{\sqrt{2\eta}J_{xx}^{+} + \sqrt{2\eta}J_{zz}^{+} - \sqrt{2}(\eta-1)J_{zx}^{+}}{2(1+\eta)},$$

$$I_{1-1}^{+} = \frac{-J_{xx}^{+} + (1+\eta)J_{yy}^{+} + \eta J_{zz}^{+} + 2\sqrt{\eta}J_{zx}^{+}}{2(1+\eta)},$$

$$I_{00}^{+} = \frac{-\eta J_{xx}^{+} + J_{zz}^{+} - 2\sqrt{\eta}J_{zx}^{+}}{(1+\eta)}$$

• **Relations between the matrix elements of the current in the Cartesian spin basis, J^-**

$$I_{11}^- = \frac{J_{xx}^- + (1+\eta)J_{yy}^- - \eta J_{zz}^- - 2\sqrt{\eta}J_{zx}^-}{2(1+\eta)},$$

$$I_{10}^- = \frac{\sqrt{2\eta}J_{xx}^- + \sqrt{2\eta}J_{zz}^- - \sqrt{2}(\eta-1)J_{zx}^-}{2(1+\eta)},$$

$$I_{1-1}^- = \frac{-J_{xx}^- + (1+\eta)J_{yy}^- + \eta J_{zz}^- + 2\sqrt{\eta}J_{zx}^-}{2(1+\eta)},$$

$$I_{00}^- = \frac{-\eta J_{xx}^- + J_{zz}^- - 2\sqrt{\eta}J_{zx}^-}{(1+\eta)}$$

Model

- $\Gamma(k, p) = \gamma^{\mu}$

$$J_{ji}^{\pm} = \imath \int \frac{d^4k}{(2\pi)^4} \frac{Tr\,[\Gamma\Gamma]_{ji}^{\pm}\;\Lambda(k, p_f)\,\Lambda(k, p_i)}{((k-p_i)^2 - m^2 + \imath\epsilon)(k^2 - m^2 + \imath\epsilon)((k-p_f)^2 - m^2 + \imath\epsilon)}$$

- Dirac trace:

$$Tr\,[\Gamma\Gamma]_{ji}^{\pm} = Tr\left[\epsilon_j \cdot \Gamma(k, p_f)(\not{k} - \not{p}_f + m)\gamma^{\pm}(\not{k} - \not{p}_i + m)\epsilon_i \cdot \Gamma(k, p_i)(\not{k} + m)\right]$$

- Regularization function

$\Lambda(k, p) = N/[(k-p)^2 - m_R^2 + \imath\epsilon]^2$ which is chosen to turn the loop integration finite.

Angular condition

- **Light-front we have the angular condition equation for $I^{+}_{m'm}$**
- **Plus component of the electromagnetic current**

$$\begin{aligned}\Delta^{(+)}(Q^2) &= (1+2\eta)I^{+}_{11} + I^{+}_{1-1} - \sqrt{8\eta}I^{+}_{10} - I^{+}_{00} \\ &= (1+\eta)(J^{+}_{yy} - J^{+}_{zz}) = 0;\end{aligned}$$

- **Minus component of the electromagnetic current**

$$\begin{aligned}\Delta^{(-)}(Q^2) &= (1+2\eta)I^{-}_{11} + I^{-}_{1-1} - \sqrt{8\eta}I^{-}_{-} - I^{-}_{00} \\ &= (1+\eta)(J^{-}_{yy} - J^{-}_{zz}) = 0;\end{aligned}$$

Ref. I.L. Grach, L.A. Kondratyuk, Sov.J.Nucl.Phys.38 (1984) 198
I.L. Grach, L.A. Kondratyuk, M. Strikman, Phys.Rev.Lett.62 (1989) 387

- **Angular Condition: Violation!!**

$$q_x \Longrightarrow J^{+}_{yy} = J^{+}_{zz} \begin{cases} Parity \\ + \\ Rotations \end{cases}$$

- **Equal time:** $\Delta^{\pm}(Q^2) = 0$, **is true!!**

- **Light-front:** $\Delta^{\pm}(Q^2) = 0$, **is not true!!**

- **With the covariant form factors**

$$
\begin{aligned}
\mathrm{J}^{\pm}_{\mathrm{yy}} &= -2p^{\pm}F^{\pm}_{1}, \\
J^{\pm}_{zz} &= -2p^{\pm}F^{\pm}_{1}.
\end{aligned}
$$

$$
\begin{aligned}
\Delta^{\pm}(\mathrm{Q}^2) &= (1+2\eta)(J^{\pm}_{yy} - J^{\pm}_{zz}) \\
&= (1+2\eta)(-2p^{\pm}F^{\pm}_{1} + 2p^{\pm}F^{\pm}_{1}) = 0.
\end{aligned}
$$

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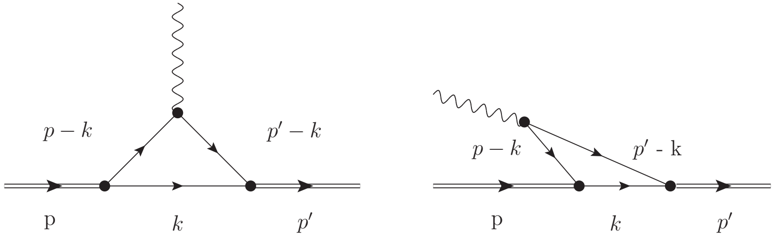


Fig. 1. **Feynman diagrams for the valence contribution (left panel) and the non-valence contribuition (right panel) for the electromagnetic current.**

- **See talks by (LC2023)**

$$\begin{cases} \textbf{Wayne Polyzou} \\ \textbf{Lubomir Martinovic} \\ \textbf{James Vary} \\ \textbf{Chandan Mondal} \end{cases}$$

Pole Dislocation Method

$p^+ \quad \Longrightarrow \quad p'^+ = p^+ + \delta$

Boson Eletromagnetic Current

Breit Frame $\Longrightarrow \; q^- = 0, \; q^+ \Longrightarrow 0_+, \; \vec{q}_\perp \neq 0$

$J^+ = J^- +$ **restoration covariance term**

$J_\perp \propto q^+ \Rightarrow 0$

J. de Melo, Sales and T.Frederico Nucl. Phys. B631, (1998) 574.

Ward-Takahashi Identity $\Longrightarrow$ **Pair Contribuition**

Naus, de Melo and Frederico

Few-Body Syst. 24, 1998, 99-107

- **Chang e Yan, Phys. Rev. D7 (73) 1147, Phys. Rev. D7 (73) 1780.**
- **Sawicki, Phys. Rev. D44 (91) 433, Phys. Rev. D46 (92) 474.**

Prescriptions

$$\begin{cases} FFS\ (Frederico, Frankfurt, Strikman) \\ GK\ (Grach, Kondratyku) \\ CCKP\ (Coester, Chung, Keister, Polyzou) \\ BH\ (Brodsky, Hiller) \\ KA\ (Karmanov) \end{cases}$$ **vs** **COVARIANT**

- **Breit Frame** $\Longrightarrow P^{+} = P'^{+}$, $P^{-} = P'^{-}$, $\vec{P'}_{\perp} = -\vec{P}_{\perp} = \vec{q}/2$
- $J^{+}_{\rho} = \begin{cases} 4\ \ Current\ \ \ Elements \\ 3\ \ Form\ \ Factors\ \ \ G_0\ ,\ \ G_1\ \ and\ \ G_2 \end{cases}$

• **Ref. J.P.B.C. de Melo, T. Frederico, Phys. Rev.C55 (1997) 2043**

Electromagnetic form factors: G_0, G_1 *and* G_2

- **I.L. Grach, L.A. Kondratyuk prescrition**

====>> **Eliminate the $I^{\pm}_{00}$ component of the electromagnetic current**

$$
\begin{aligned}
G_0^{GK} &= \frac{1}{3}[(3-2\eta)I^{\pm}_{11} + 2\sqrt{2\eta}I^{\pm}_{10} + I^{\pm}_{1-1}] \\
&= \frac{1}{3}[J^{\pm}_{xx} + (2-\eta)J^{\pm}_{yy} + \eta J^{\pm}_{zz}], \\
G_1^{GK} &= 2[I^{\pm}_{11} - \frac{1}{\sqrt{2\eta}}I^{\pm}_{10}] = J^{\pm}_{yy} - J^{\pm}_{zz} - \frac{J^{\pm}_{zx}}{\sqrt{\eta}}, \\
G_2^{GK} &= \frac{2\sqrt{2}}{3}[\sqrt{2\eta}I^{\pm}_{10} - \eta I^{\pm}_{11} - I^{\pm}_{1-1}] = \frac{\sqrt{2}}{3}[J^{\pm}_{xx} - (1+\eta)J^{\pm}_{yy} + \eta J^{\pm}_{zz}] .
\end{aligned}
$$

Ref. (GK) I.L. Grach, L.A. Kondratyuk, Sov. J. Nucl. Phys., 38 (1984) 198

CCKP

$$
\begin{aligned}
G_0^{CCKP} &= \frac{1}{3(1+\eta)}[(\frac{3}{2}-\eta)(I^+_{11}+I^+_{00})+5\sqrt{2\eta}I^+_{10}+(2\eta-\frac{1}{2})I^+_{1-1}] \\
&= \frac{1}{6}[2J^+_{xx}+J^+_{yy}+3J^+_{zz}] \\
G_1^{CCKP} &= \frac{1}{(1+\eta)}[I^+_{11}+I^+_{00}-I^+_{1-1}-\frac{2(1-\eta)}{\sqrt{2\eta}}I^+_{10}] = -\frac{J^+_{zx}}{\sqrt{\eta}} \\
G_2^{CCKP} &= \frac{\sqrt{2}}{3(1+\eta)}[-\eta I^+_{11}-\eta I^+_{00}+2\sqrt{2\eta}I^+_{10}-(\eta+2)I^+_{1-1}] = \\
&\quad \frac{\sqrt{2}}{3}[J^+_{xx}-J^+_{yy}]
\end{aligned}
$$

Ref. **Chung, Polyzou, Coester, Keister, Phys. Rev. C37 (1988) 2000**

Brodsky-Hiller - (BH) - I_{11}^{+}

$$G_0^{BH} = \frac{1}{3(1+2\eta)}[(3-2\eta)I_{00}^{+} + 8\sqrt{2\eta}I_{10}^{+} + 2(2\eta-1)I_{1-1}^{+}]$$
$$= \frac{1}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) + J_{yy}^{+}(2\eta-1) + J_{zz}^{+}(3+2\eta)]$$
$$G_1^{BH} = \frac{2}{(1+2\eta)}[I_{00}^{+} - I_{1-1}^{+} + \frac{(2\eta-1)}{\sqrt{2\eta}}I_{10}^{+}]$$
$$= \frac{1}{(1+2\eta)}[\frac{J_{zx}^{+}}{\sqrt{\eta}}(1+2\eta) - J_{yy}^{+} + J_{zz}^{+}]$$
$$G_2^{BH} = \frac{2\sqrt{2}}{3(1+2\eta)}[\sqrt{2\eta}I_{10}^{+} - \eta I_{00}^{+} - (\eta+1)I_{1-1}^{+}]$$
$$= \frac{\sqrt{2}}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) - J_{yy}^{+}(1+\eta) - \eta J_{zz}^{+}]$$

Ref. **Brodsky, Hiller, Phys. Rev. D46 (1992) 2141**

FFS

$$
\begin{aligned}
G_0^{FFS} &= \frac{1}{3(1+\eta)}[(\frac{3}{2}-\eta)(I_{11}^{+}+I_{00}^{+})+5\sqrt{2\eta}I_{10}^{+}+(2\eta-\frac{1}{2})I_{1-1}^{+}] \\
&= \frac{1}{6}[2J_{xx}^{+}+J_{yy}^{+}+3J_{zz}^{+}] \\
G_1^{FFS} &= G_1^{CCKP}\ ,\ G_2^{FFS}=G_2^{CCKP}
\end{aligned}
$$

Ref. **Frankfurt, Frederico, Strikman, Phy. Rev. C48 (1993) 2182**

Karmanov

$$
\begin{aligned}
G_0^{KA} &= \frac{1}{3}\left[2(1-\eta)I_{11}^{+} + 4\sqrt{2\eta}I_{10}^{+} + I_{00}^{+}\right] \\
&= \frac{1}{3}[J_{xx}^{+} + J_{yy}^{+}(1-2\eta) + (2\eta+1)J_{zz}^{+}] \\
G_1^{KA} &= \left[2I_{11}^{+} - \sqrt{\frac{2}{\eta}}I_{10}^{+}\right] = \left[J_{yy}^{+} - \frac{J_{zx}^{+}}{\sqrt{\eta}} - J_{zz}^{+}\right] \\
G_2^{KA} &= \frac{2\sqrt{2}}{3}\left[(1+\eta)I_{11}^{+} - \sqrt{2\eta}I_{10}^{+} - I_{00}^{+}\right] \\
&= \frac{\sqrt{2}}{3}\left[J_{xx}^{+} + (1+\eta)J_{yy}^{+} - (2+\eta)J_{zz}^{+}\right]
\end{aligned}
$$

Ref.: **V. Karmanov, Nucl. Physics A608 (1996) 316**

- **Zero-mode contributions to the matrix elements of the current**

$$J_{yy}^{+Z} = 0, J_{xx}^{+Z} = -\eta J_{zz}^{+Z} \text{ and } J_{zx}^{+Z} = -\sqrt{\eta} J_{zz}^{+Z},$$

- **Only from valence contribuitions as**

$$J_{zz}^{+Z} = J_{yy}^{+V} - J_{zz}^{+V},$$

$===>>$ **Is a consequence of the angular condition**

- **Final relations for the matrix elements of the plus component of the current, computed solely in terms of valence matrix elements**

$$\begin{aligned} J_{xx}^{+} &= J_{xx}^{+V} - \eta \left(J_{yy}^{+V} - J_{zz}^{+V} \right) \\ J_{zx}^{+} &= J_{zx}^{+V} - \sqrt{\eta} \left(J_{yy}^{+V} - J_{zz}^{+V} \right) \end{aligned}$$

The elimination of zero-modes for the matrix elements of the current $I^{+}_{m' m}$, leads to the following

$$I^{+Z}_{11} = 0, \quad I^{+Z}_{10} = 0, \quad I^{+Z}_{1-1} = 0,$$

and

$$I^{+Z}_{00} = (1+\eta) J^{+Z}_{zz} = (1+\eta)\left(J^{+V}_{yy} - J^{+V}_{zz}\right)$$

Ref.

J.P.B.C. de Melo, T. Frederico, Phys. Rev.C55 (1997) 2043
J.P.B.C. de Melo, T. Frederico, Phys. Lett. B 708 (2012) 87
J.P.B.C. de Melo, Phys. Lett. B788 (2019) 152
J.P.B.C. de Melo, 2309.07890 (2023) [hep-ph]

- Similar Results are found by Ji, Bakker and Choi
- Phy.Rev.D65 (2002) 116001
- Phy.Rev.D70 (2004) 053015

- **With the relations above, we have**

$$G_0^{GK(+Z)} = \frac{1}{3}\left[J_{xx}^{(+Z)} + \eta J_{zz}^{+Z}\right] = \frac{1}{3}\left[-\eta J_{zz}^{+Z} + \eta J_{zz}^{+Z}\right] = 0,$$

$$G_1^{GK(+Z)} = \left[-J_{zz}^{+Z}[gg] - \frac{J_{zX}^{+Z}}{\sqrt{\eta}}\right] = -J_{zz}^{+Z} + \sqrt{\eta}\frac{J_{zz}^{+Z}}{\sqrt{\eta}} = 0,$$

$$G_2^{GK(+Z)} = \frac{\sqrt{2}}{3}\left(J_{xx}^{+Z} + \eta J_{zz}^{+Z}\right) = \frac{\sqrt{2}}{3}\left[-\eta J_{zz}^{+} + \eta J_{zz}^{+Z}\right] = 0,$$

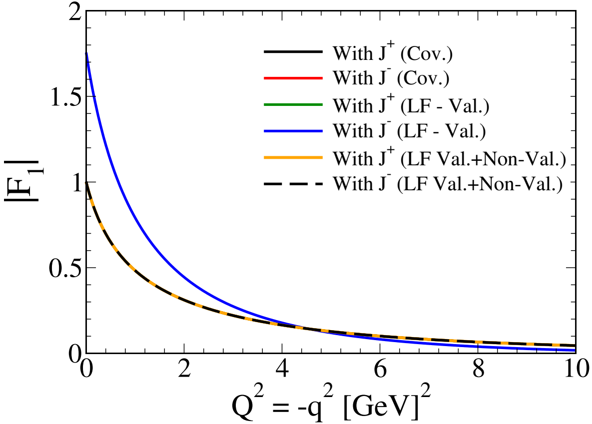
- **Prove** GK **Prescription is free of the zero modes contribuitions!!**

Ref.

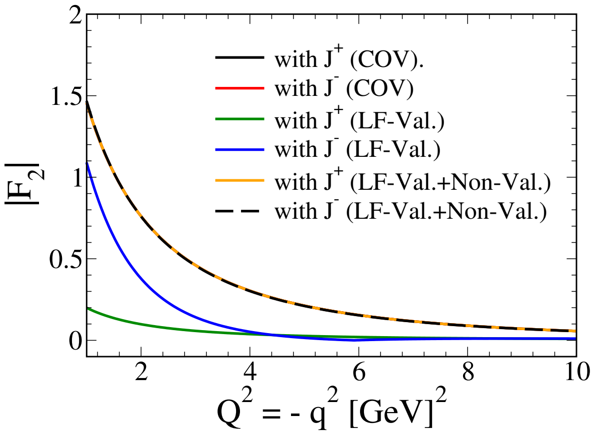
J.P.B.C. de Melo, T. Frederico, Phys. Lett. B 708 (2012) 87

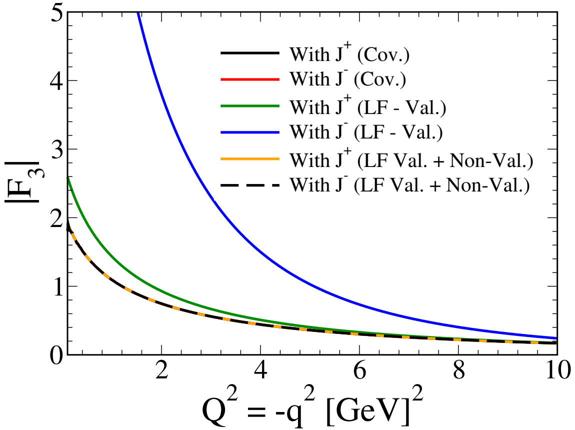
J.P.B.C. de Melo, Phys. Lett. B788 (2019) 152

Covariant Electromagnetic Form Factors



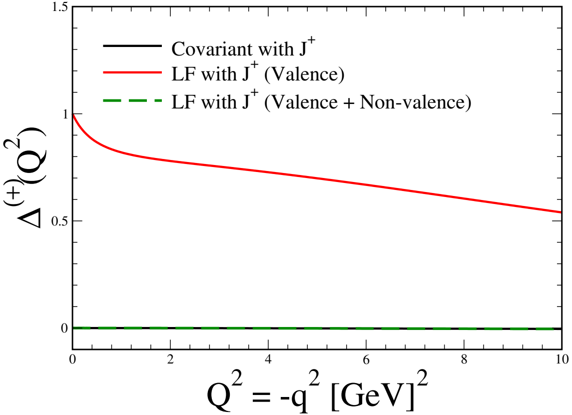
- $m_q = 0.430 \ GeV, \ m_\rho = 0.775 \ GeV, \ m_R = 3.0 \ GeV$
- **Fixed by the exp.** $f_\rho = 153 \pm 8 \ MeV$ **(PDG)**


with J+ (COV).
with J- (COV)
with J+ (LF-Val.)
with J- (LF-Val.)
with J+ (LF-Val.+Non-Val.)
with J- (LF-Val.+Non-Val.)
|F2|
Q2 = - q2 [GeV]2
0
0.5
1
1.5
2
2
4
6
8
10

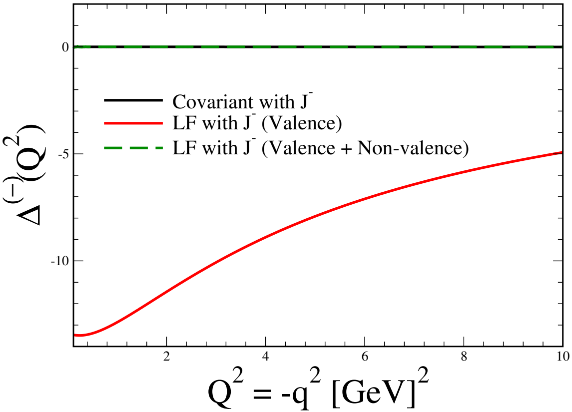


With J+ (Cov.)
With J- (Cov.)
With J+ (LF - Val.)
With J- (LF - Val.)
With J+ (LF Val. + Non-Val.)
With J- (LF Val. + Non-Val.)
|F3|
Q2 = -q2 [GeV]2
0
1
2
3
4
5
2
4
6
8
10

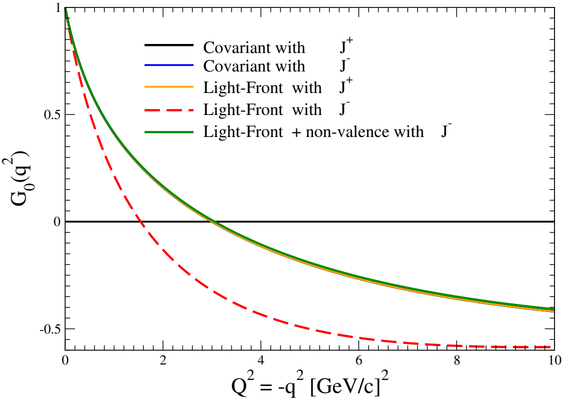
Angular Condition (with the plus component of e.m)



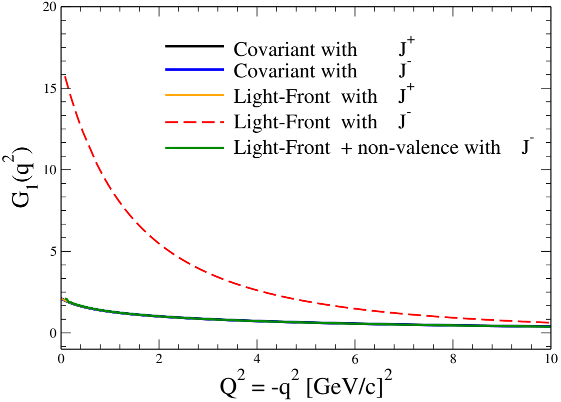
Angular Condition (with the minus component of e.m)



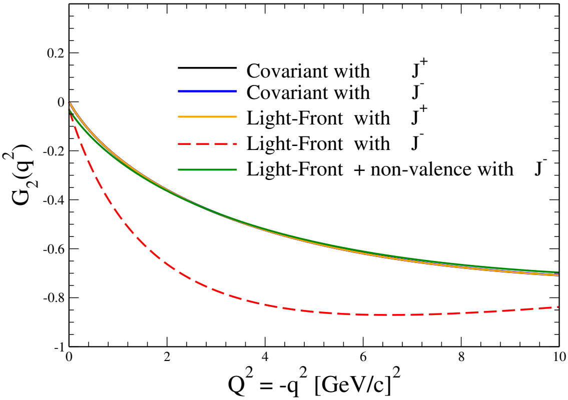
Charge Electromagnetic form factor: $G_0(q^2)$



Magnetic Electromagnetic form factor: $G_1(q^2)$



Quadrupole Electromagnetic form factor: $G_2(q^2)$



Remarks

- **Light-front approach correctly describes hadronic bound states**
- **Take New Informations about Bound States**
- **Breaking of the rotational invariance has to be evaluated**
- **The inclusion of zero modes or pair terms is crucial**
- **The break of the rotational symmetry for J^-_{ji} case is very pronounced**
- **We can see that with the inclusion of pair terms, we have the covariance restored**
- **Next: Full Spin-1 vertex and others meson with S=1**

Thanks!! Obrigado!!

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