Gluon saturation:
toward precision using light front and covariant approaches

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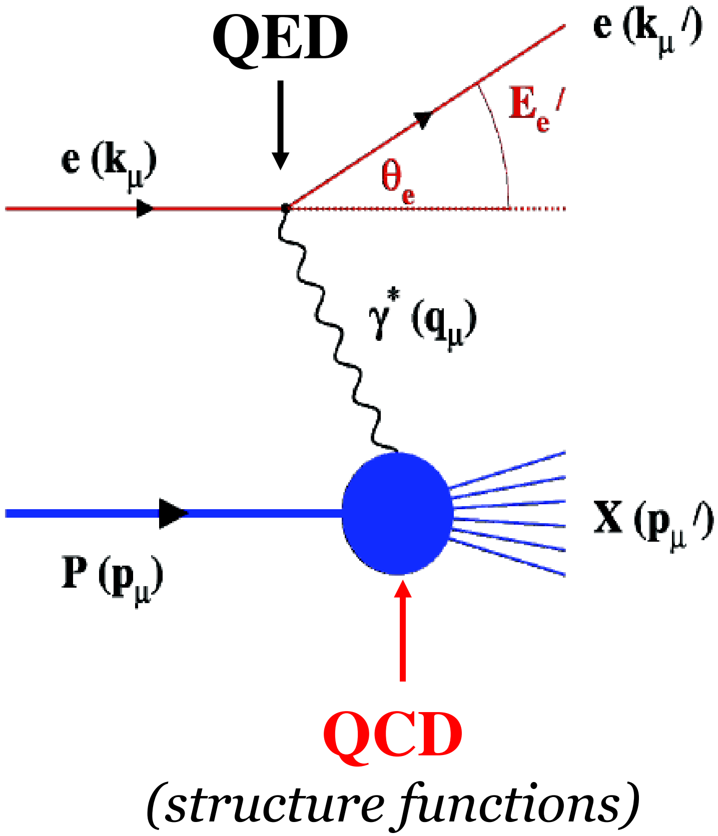
Light-Cone 2023
Centro Brasileiro de pesquisas Fisicas (CBPF)
Rio de Janeiro, Brazil
September 18-22, 2023

Deep Inelastic Scattering (DIS)

probing hadron structure



Kinematic Invariants

$$Q^2 = -q^2 = -(k_\mu - k'_\mu)^2$$

$$Q^2 = 4E_e E'_e \sin^2\left(\frac{\theta'_e}{2}\right)$$

Measure of resolution power

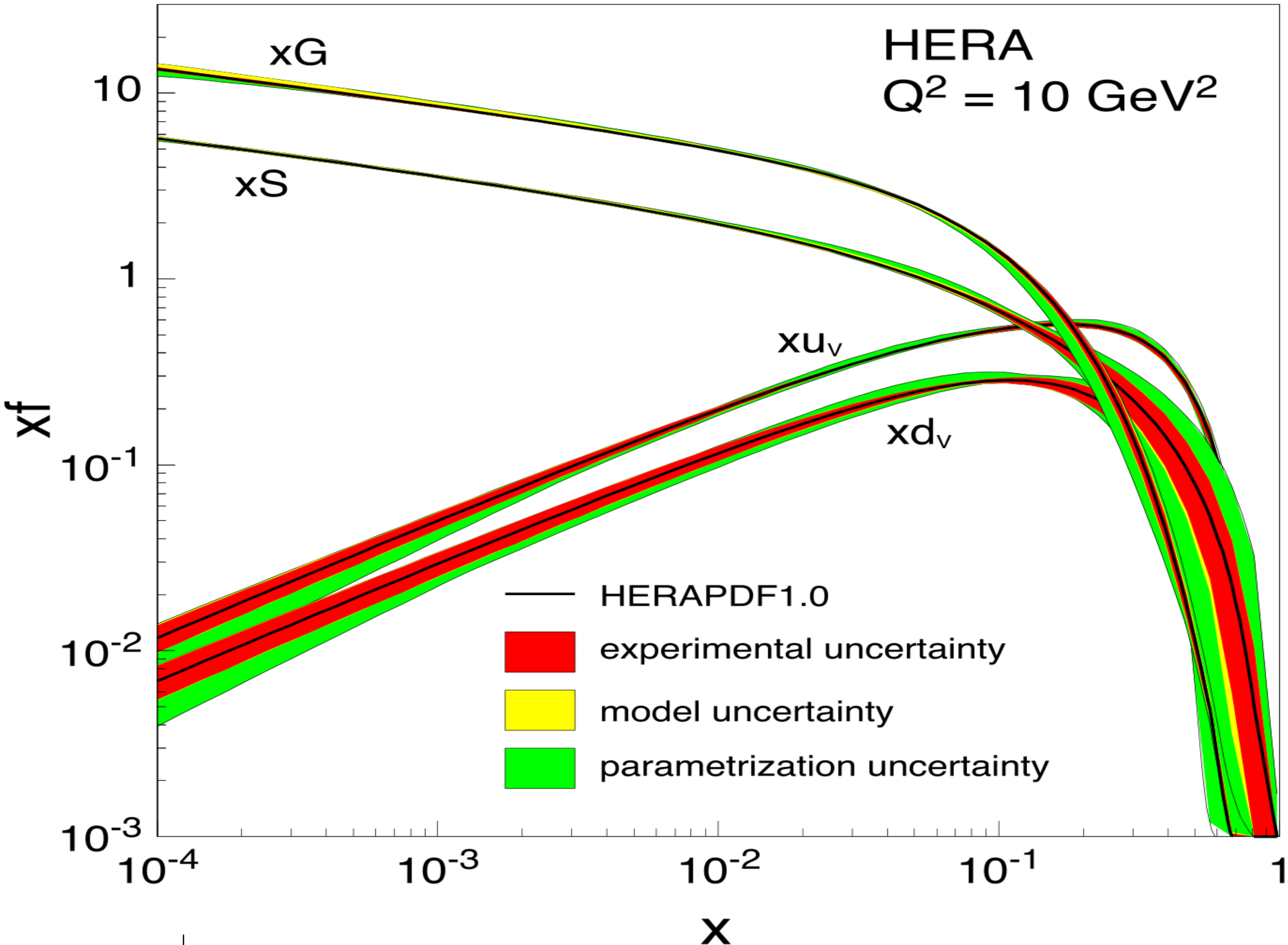
$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

Measure of momentum fraction of struck quark

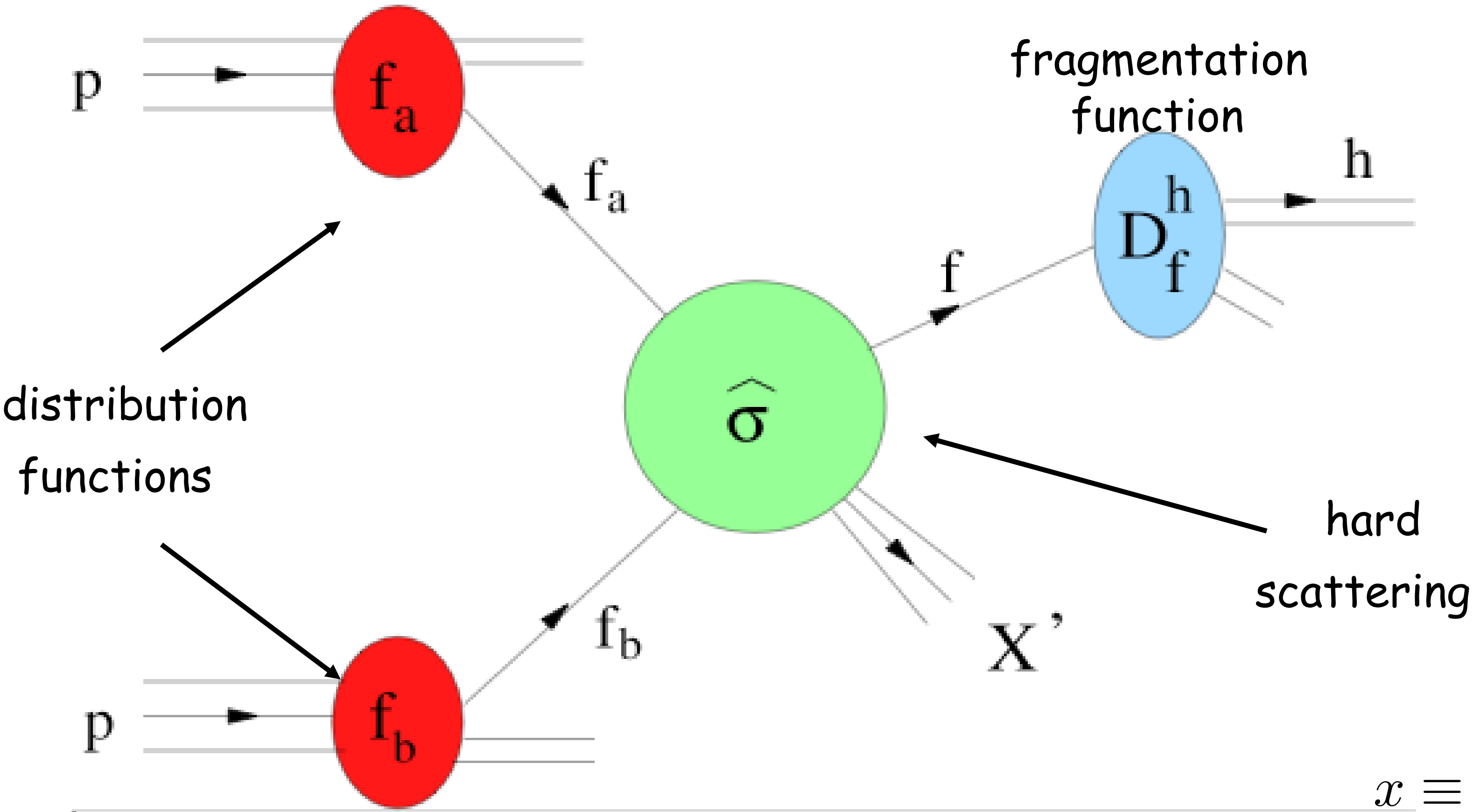
$$\mathbf{s} \equiv (\mathbf{p} + \mathbf{k})^2$$



$$x = \frac{p^+}{P^+}$$ ***x*** *is the fraction of hadron energy carried by a parton*

QCD in proton-proton collisions

collinear factorization: separation of soft (long distance) and hard (short distance)

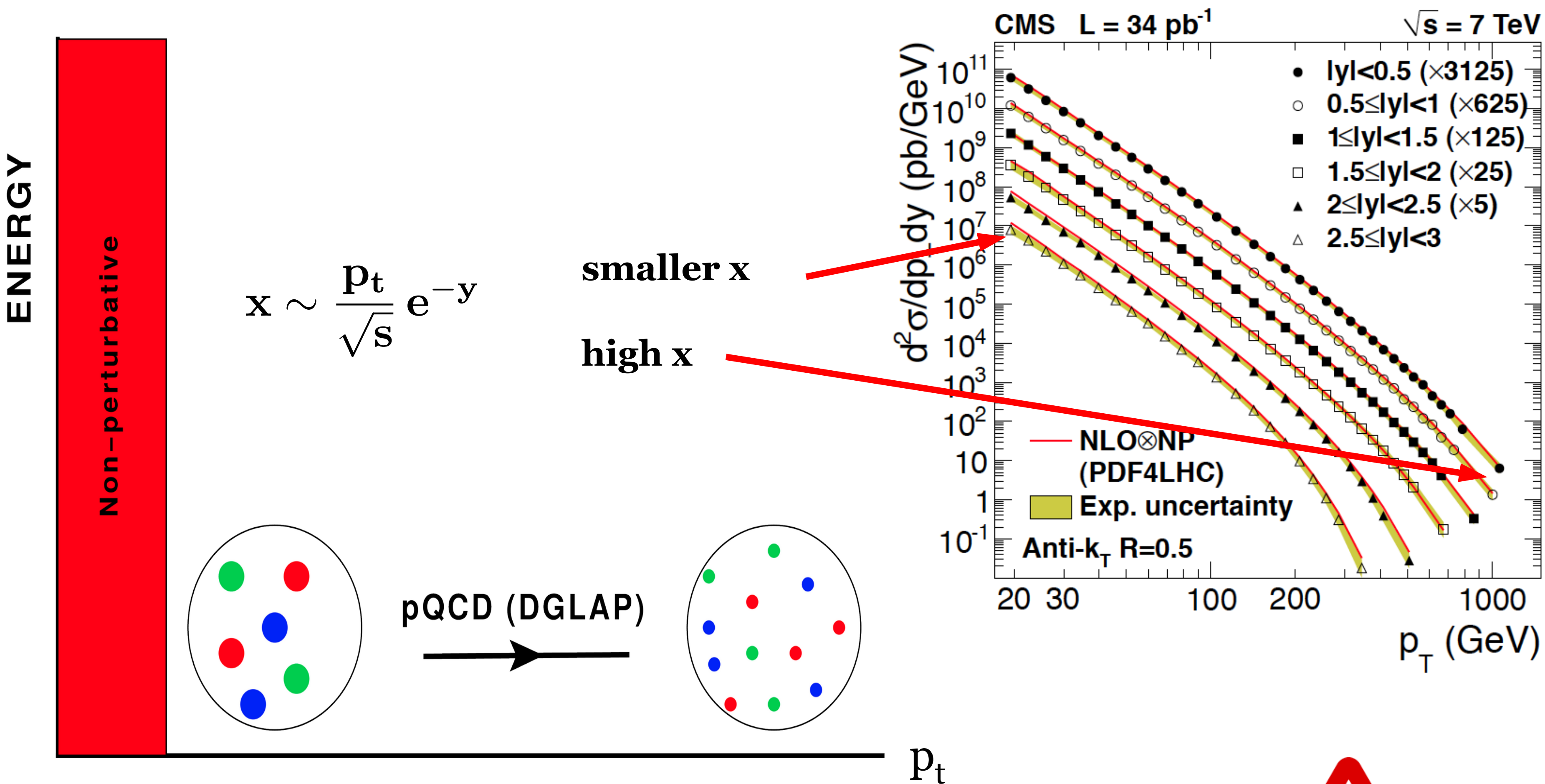


$$x \equiv \frac{p^+}{P^+}$$

$$\frac{d\sigma^{pp\to h\,X}}{d^2p_t\,dy} \sim f_a(x_1) \otimes f_b(x_2) \otimes \hat{\sigma} \otimes D_f^h(z) + \cdots\cdots$$

power corrections

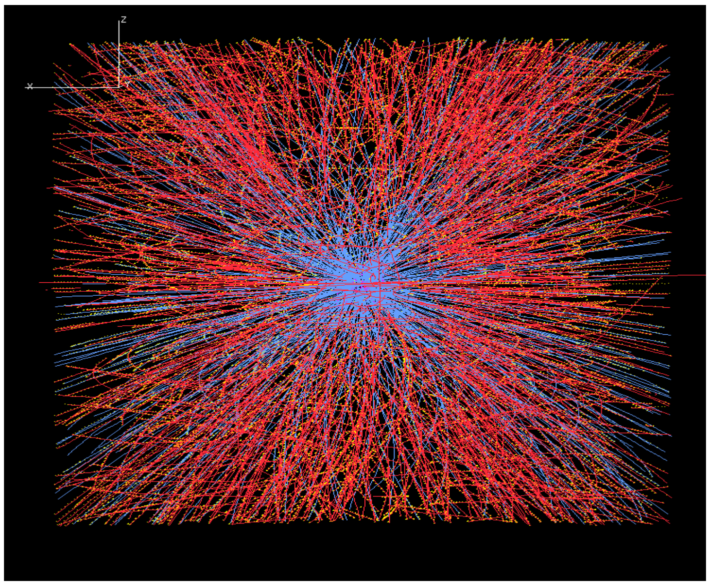
pQCD: the standard paradigm



bulk of QCD phenomena happens at low p_t (small x)

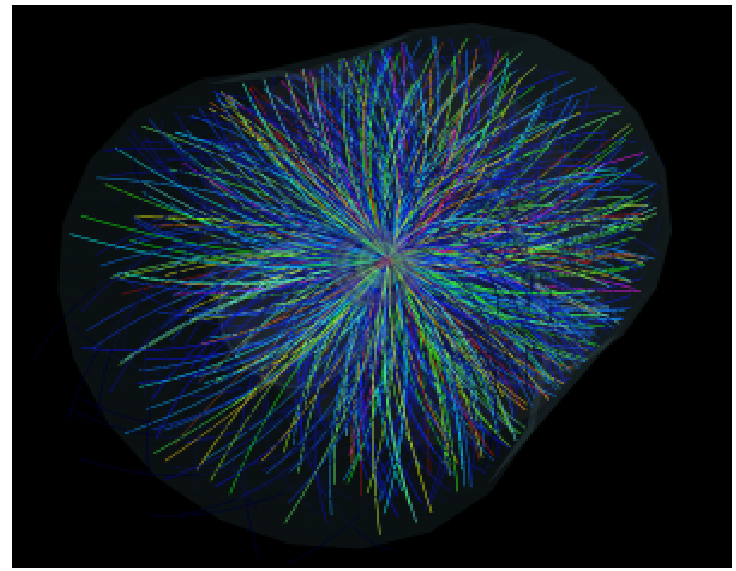
Nucleus-Nucleus (AA) Collisions:

Quark-Gluon Plasma



$$\sqrt{S} \sim 200\,\mathrm{GeV}$$

$$\mathrm{RHIC:}\ \frac{dN_{ch}}{d\eta} \sim 700$$

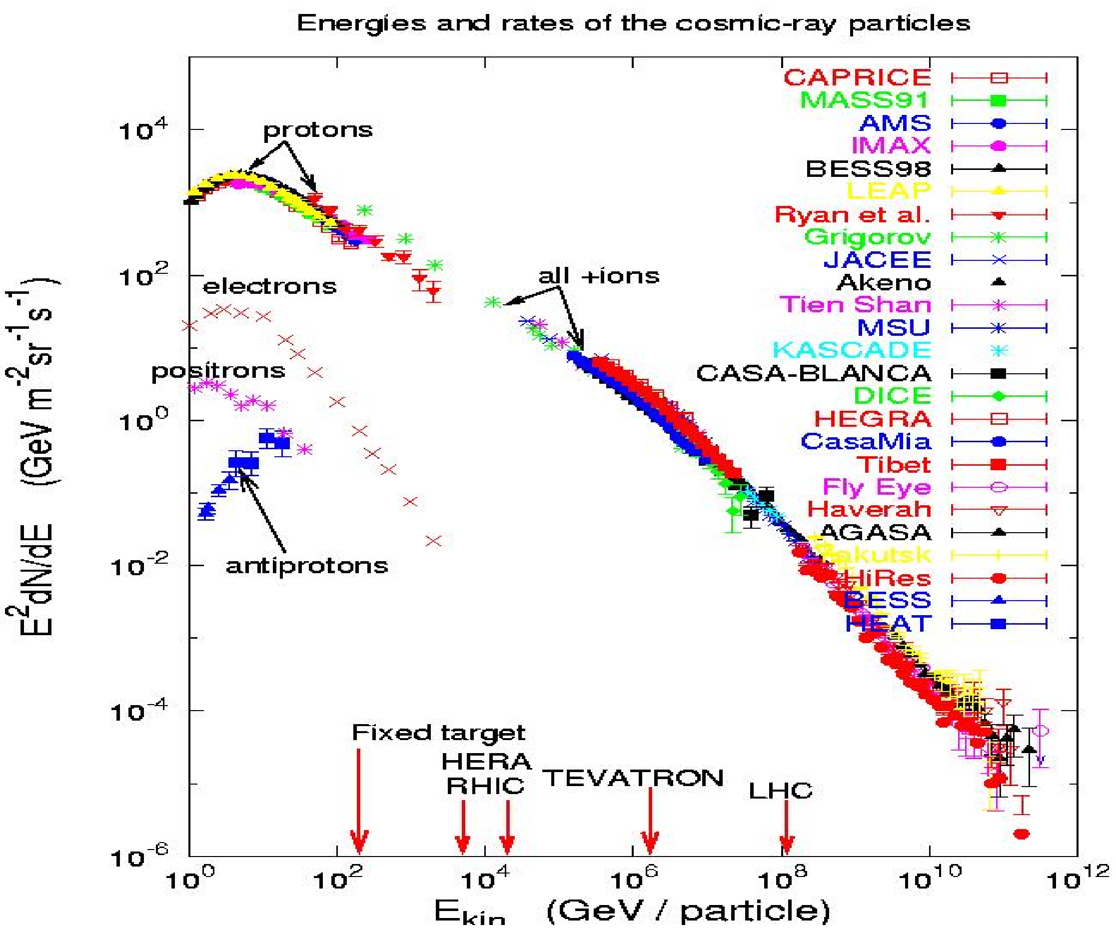


$$\sqrt{S} \sim 5\,\mathrm{TeV}$$

$$\mathrm{LHC:}\ \frac{dN_{ch}}{d\eta} \sim 1600$$

$$x \sim \frac{p_t}{\sqrt{S}} e^{-y} \rightarrow 0$$

High Energy Cosmic Rays



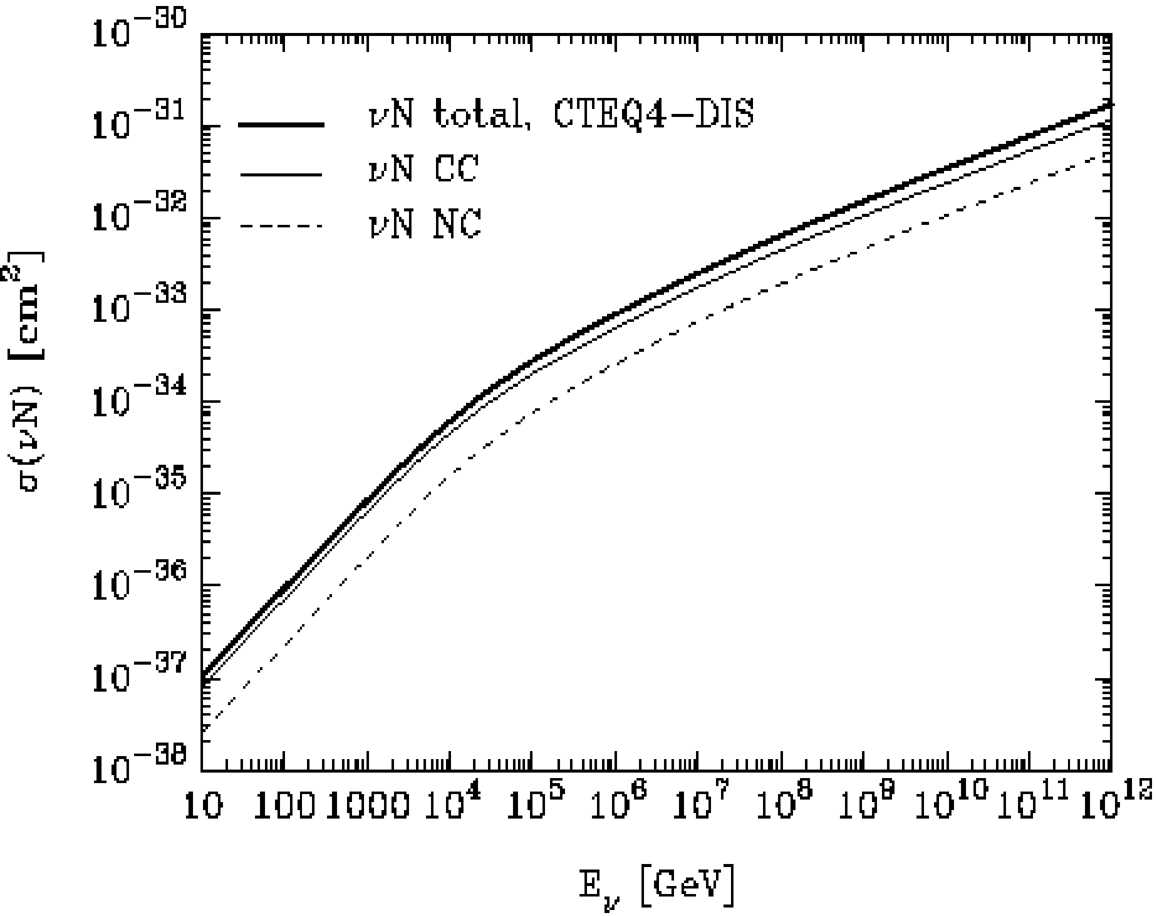
$$p\,A \rightarrow X$$

$$\sqrt{S} \sim 10^{2-3}\mathrm{TeV}$$

most particles/energy are in the forward rapidity region and have low p_t

$$x \sim \frac{p_t}{\sqrt{S}} e^{-y} \rightarrow 0$$

Ultra-High Energy Neutrinos



$$\nu\,\mathbf{N} \to \nu\,\mathbf{X}$$

$$\sqrt{\mathbf{S}} \sim \mathbf{10^{2-3}TeV}$$

$$\frac{\mathbf{M_z}}{\sqrt{\mathbf{S}}} \to \mathbf{0}$$

total cross section dominated by $Q \sim M_Z$

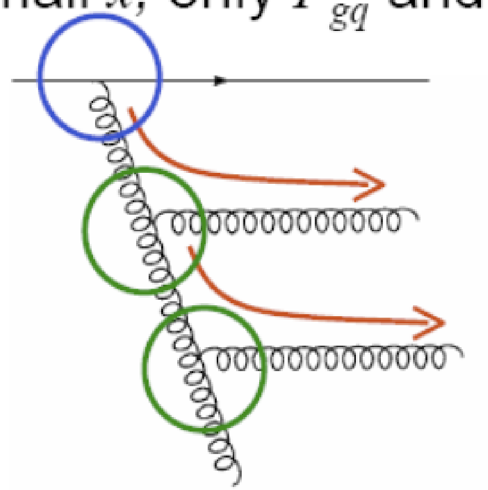
need to understand structure of hadrons at *very small x*

What drives the growth of parton distributions?

Splitting functions at leading order $O(\alpha_S{}^0)$ $\quad (x \neq 1)$

$$
\begin{aligned}
P_{qq}^{(0)}(x) &= C_F \frac{1+x^2}{1-x} \\
P_{qg}^{(0)}(x) &= \frac{1}{2}\left[x^2 + (1-x)^2\right] \\
P_{gq}^{(0)}(x) &= C_F \frac{1+(1-x)^2}{x} \\
P_{gg}^{(0)}(x) &= 2C_A\left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right]
\end{aligned}
$$

At small x, only P_{gq} and P_{gg} are relevant.



→ Gluon dominant at small *x*!

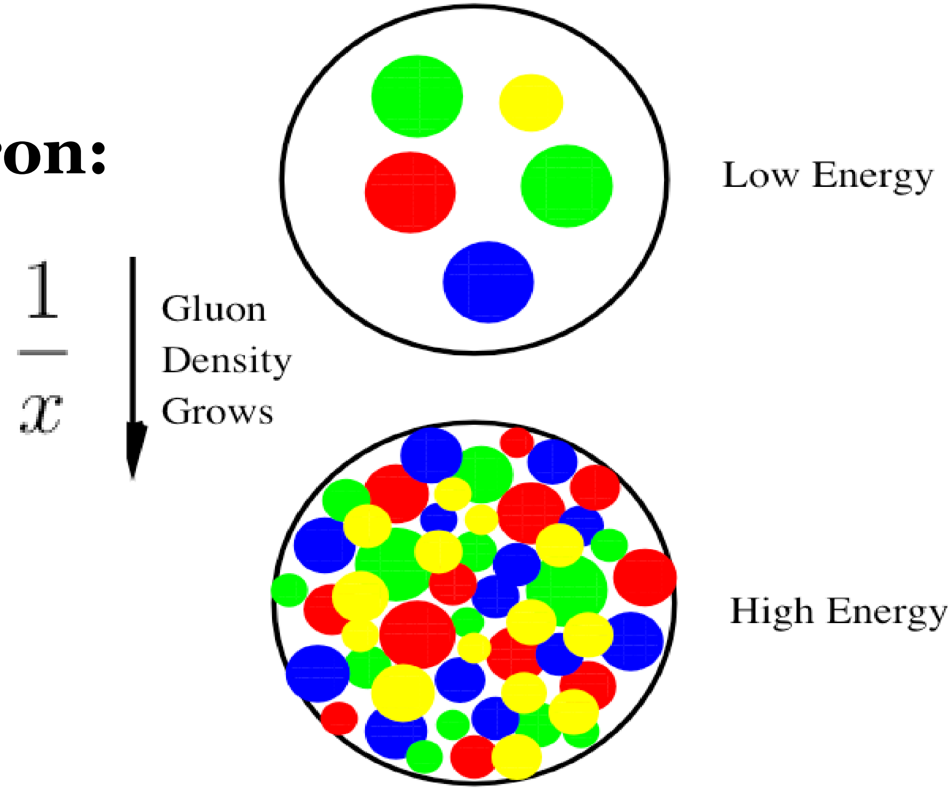
The double log approximation (DLA) of DGLAP is easily solved.

-- increase of gluon distribution at small x

$$\mathbf{xg(x, Q^2) \sim e^{\sqrt{\alpha_s (\log 1/x)(\log Q^2)}}}$$

Resolving the nucleus/hadron:

Regge-Gribov limit



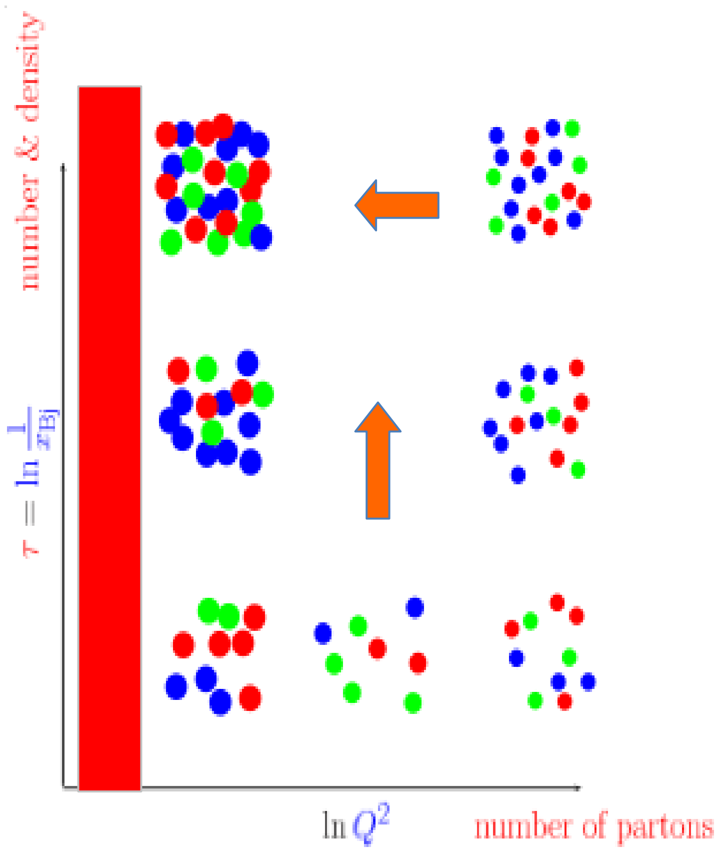
radiated gluons have the same size ($1/Q^2$) - the number of partons increase due to the increased longitudinal phase space

Saturation: hadron/nucleus becomes a dense system of gluons

Feynman's concept of a quasi-free parton is not useful

one can reach the same dense state in a nucleus at not so small x

Low x QCD:
many-body dynamics of universal gluonic matter (CGC)



How does this happen ?

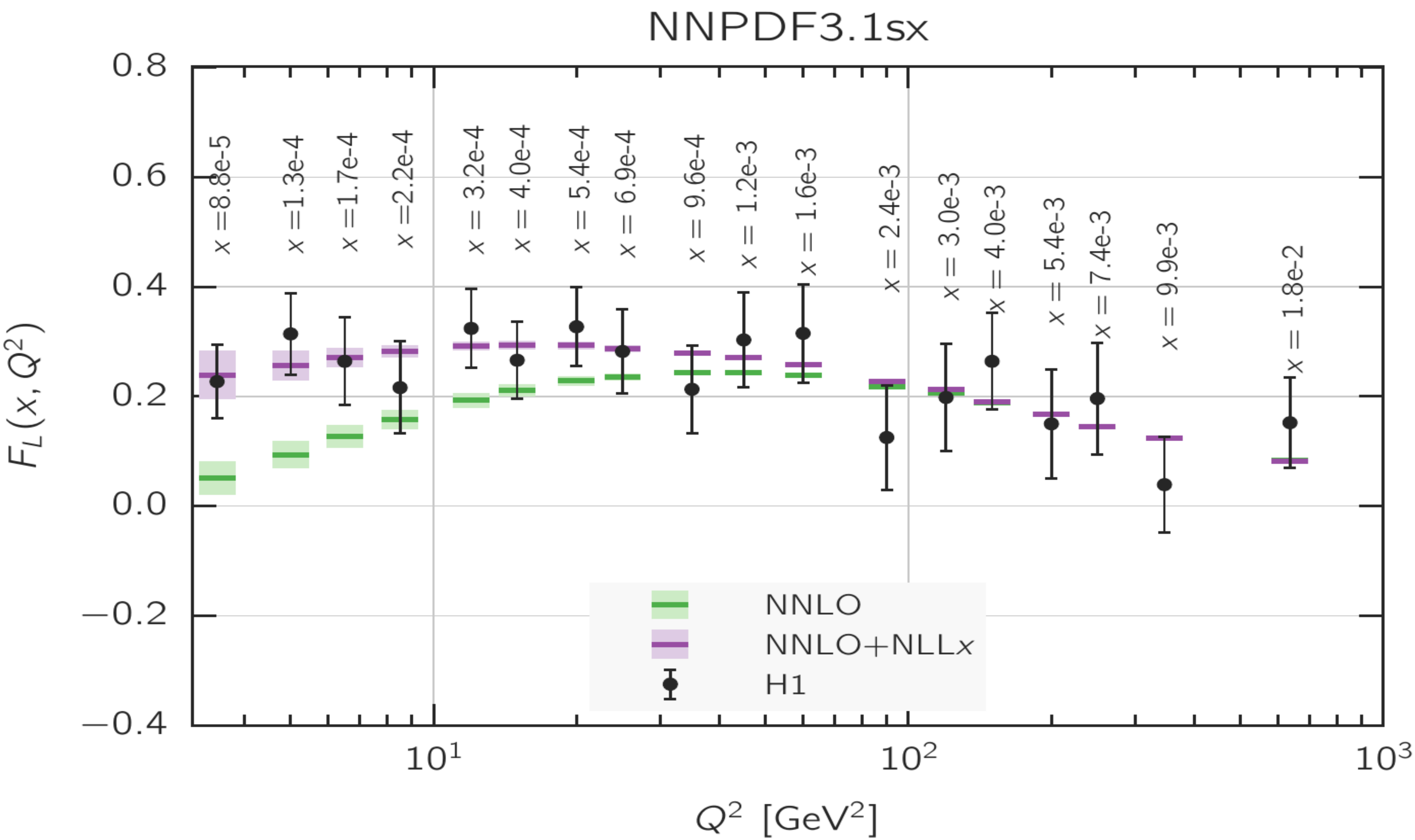
How do correlation functions of these evolve ?

Are there scaling laws ?

Can CGC explain aspects of HIC ?

Initial conditions for hydro?
Thermalization ?
Long range rapidity correlations ?
Azimuthal angular correlations ?
Nuclear modification factor ?

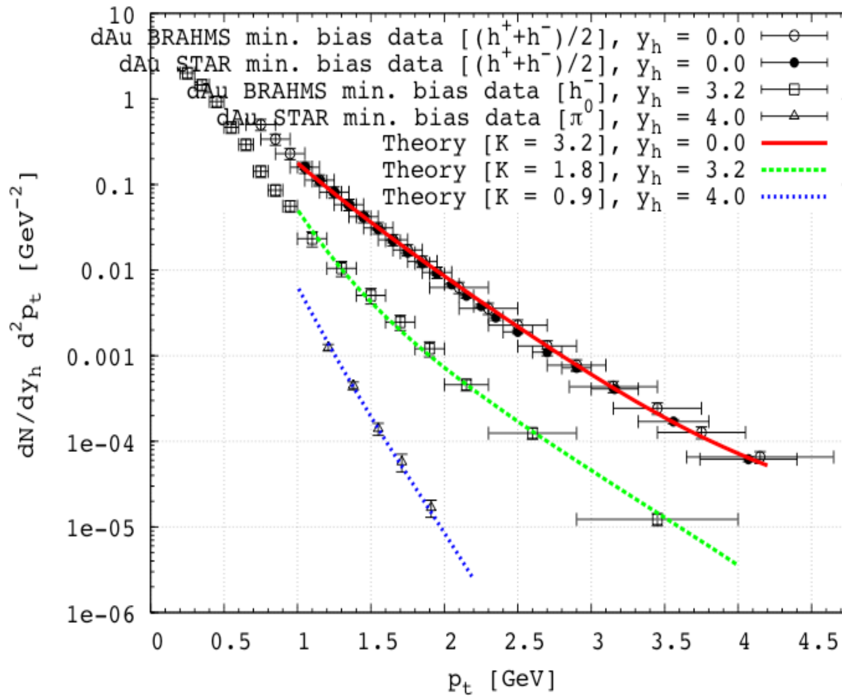
F_L at HERA



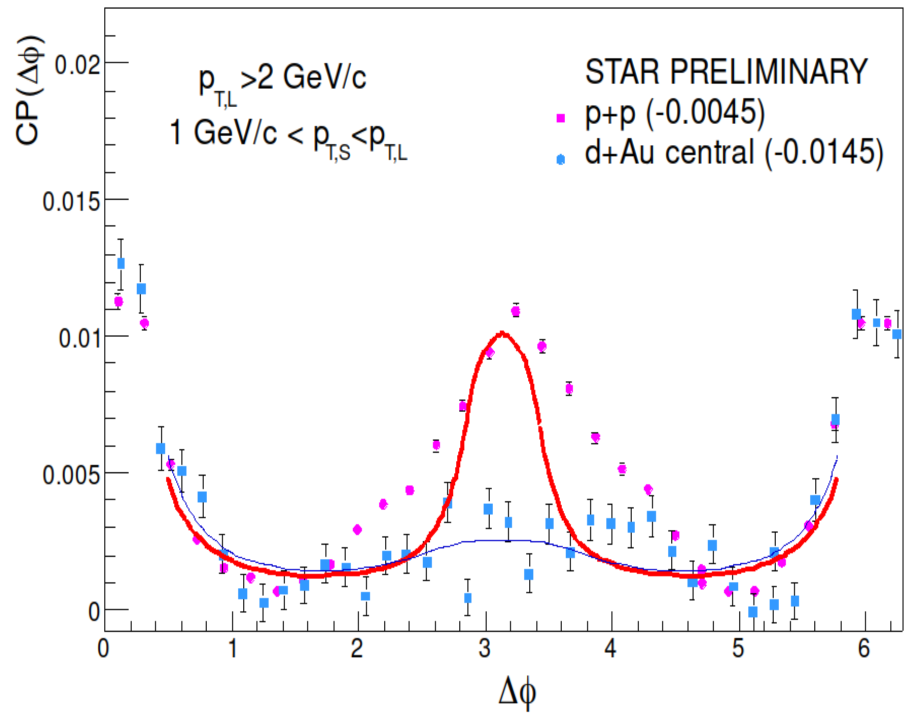
arXiv:1710.05935

CGC at RHIC

Single and double inclusive hadron production in dA collisions



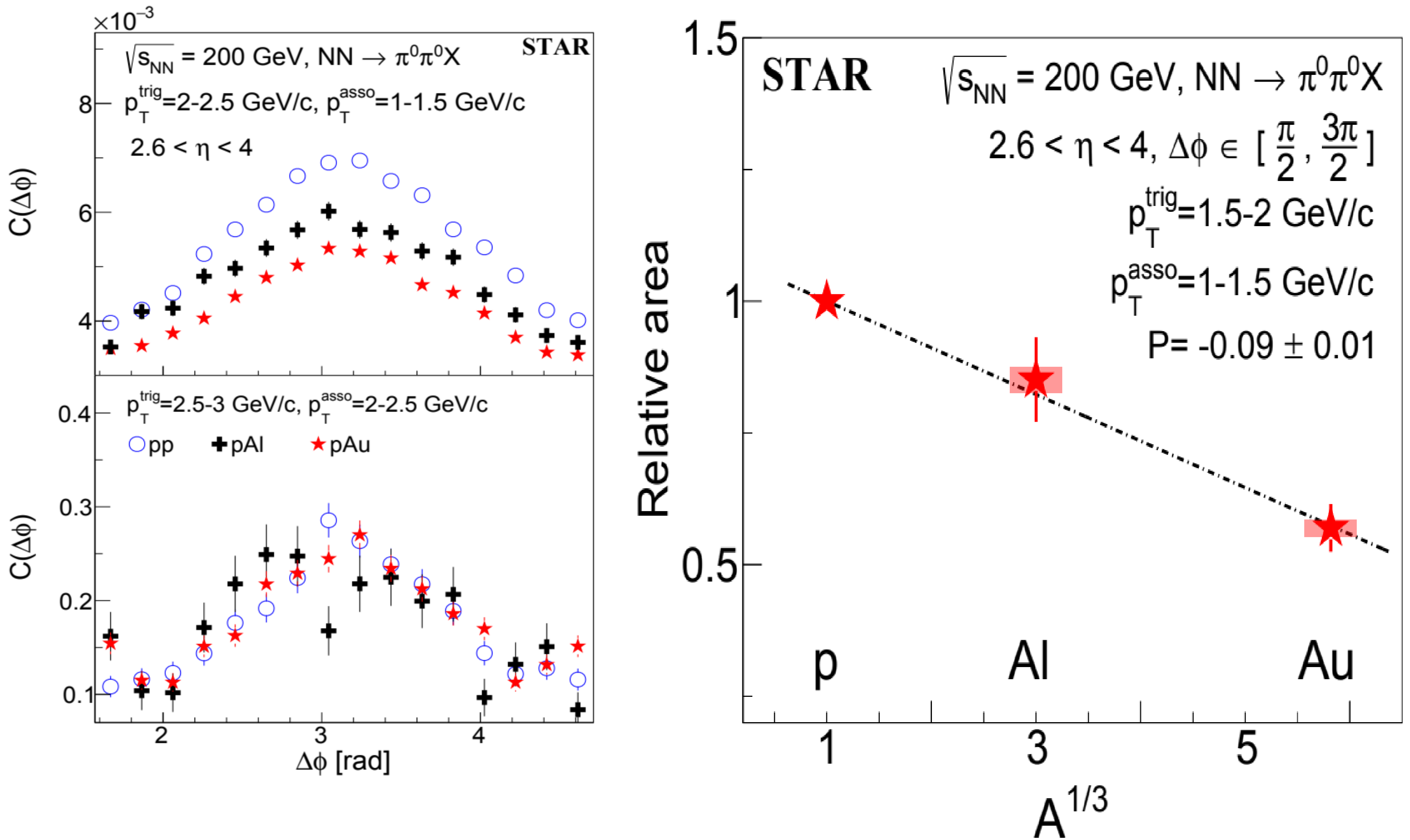
Dumitru,Hayashigaki, JJM, NPA770 (2006) 57



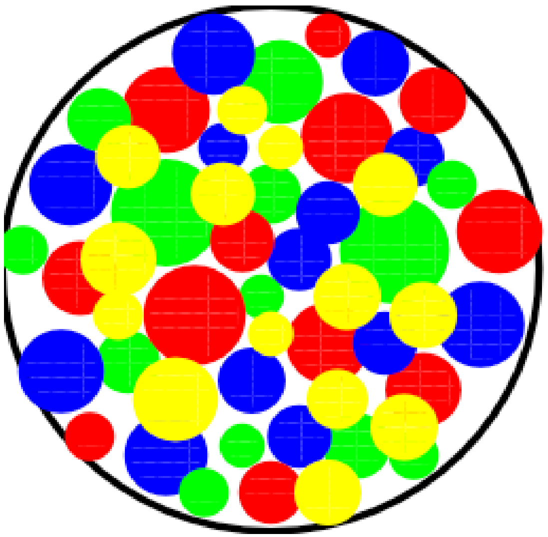
Albacete, Marquet, PRL105 (2010) 162301

Back to back hadron production in pA collisions: forward rapidity

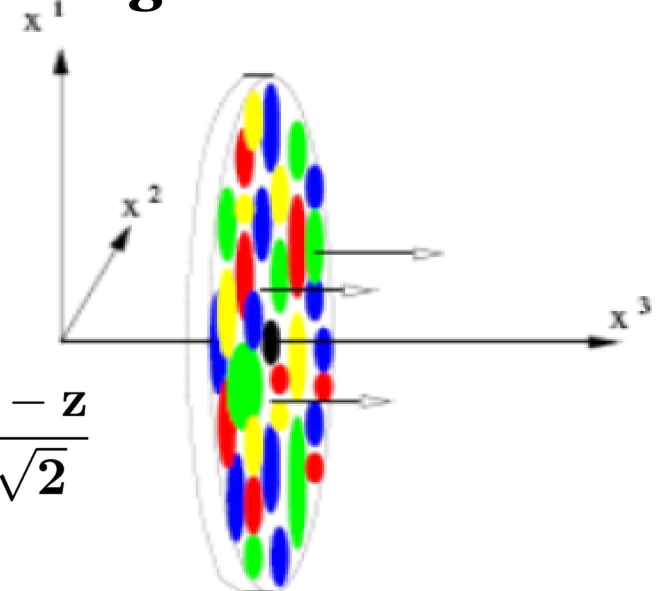
STAR collaboration(2021)
arXiv:2111.10396



dense target (proton/nucleus) as a background color field



boost



$$\mathbf{x^+ \equiv \frac{t+z}{\sqrt{2}} \qquad x^- \equiv \frac{t-z}{\sqrt{2}}}$$

sheet of color charge moving along x^+ and sitting at $x^- = 0$

$$\boxed{\mathbf{J^\mu_a(x) \equiv \delta^{\mu\,+}\,\delta(x^-)\,\rho_a(x_t)}}$$

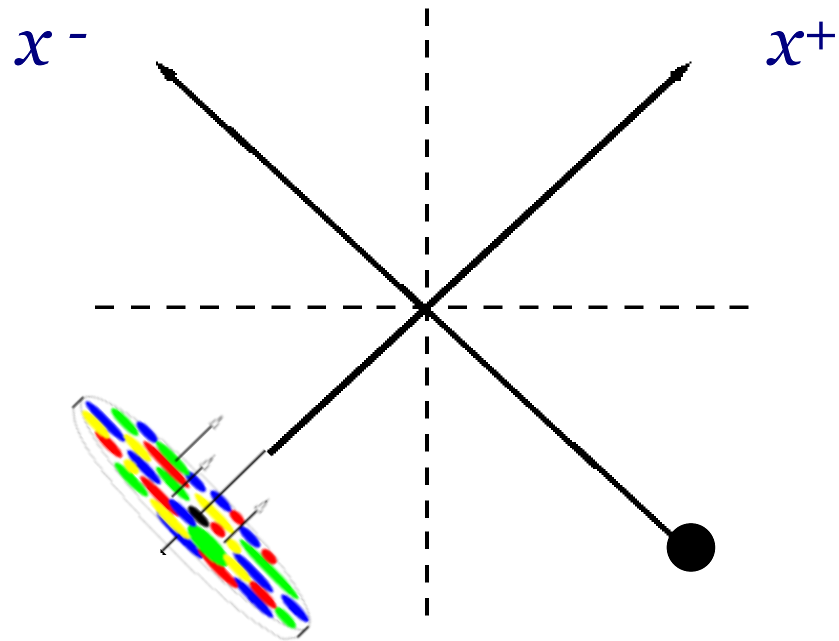
color current *color charge*

$$\mathbf{A^+_a(z^-, z_t) = \delta(z^-)\,\alpha_a(z_t)}$$

recall eikonal approximation

$$\bar{u}(q)\gamma^\mu u(p) \;\rightarrow\; \bar{u}(p)\gamma^\mu u(p) \sim p^\mu$$

$$\bar{u}(q)\not{A}\,u(p) \;\rightarrow\; p\cdot A \sim p^-\,A^+$$



Probing saturation in high energy collisions

nucleus-nucleus: “dense-dense”

proton-nucleus: “dilute-dense”

DIS (inclusive/diffractive)

- structure functions
- *particle production*
- angular correlations

need quite a bit of modeling

↓

much less modeling

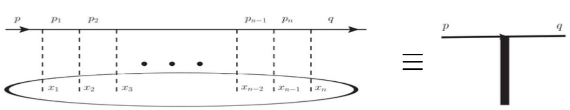
$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

for a proton target (quarks)

signatures in production spectra:

multiple scattering encoded in Wilson lines



$\equiv \mathbf{V}(\mathbf{x}_\perp)$

evolution with x (energy) via JIMWLK

$$\frac{d}{dy} < O >_y = \mathcal{H}_{JIMWLK} < O >_y$$

p_t broadening

suppression of spectra/away side peaks

Toward precision CGC at small x: inclusive DIS

NLO BK/JIMWLK evolution equations:

Balitsky, Chirilli (2007)

Kovner, Lublinsky, Mulian (2013)

....

NLO corrections to DIS structure functions:

Beuf (2017)

Beuf, Lappi, Paatelainen (2022)

....

NLO corrections to single inclusive hadron production in DIS:

Bergabo, JJM (2023)

NLO corrections to <u>inclusive dihadron</u>/dijet production in DIS:

Bergabo, JJM (2022, 2023)

Taels, Altinoluk, Beuf, Marquet (2022)

Caucal, Salazar, Schenke, Venugopalan (2022)

Caucal, Salazar, Venugopalan (2021)

....

DIS: sub-eikonal corrections at small x

Altinoluk, Armesto, Beuf (2023)

Altinoluk, Beuf, Czajka, Tymowska (2021, 2022)

....

Toward precision CGC: diffractive/exclusive DIS

NLO corrections to diffractive structure functions:

Beuf, Hanninen, Lappi, Mulian, Mantysaari (2022)

....

NLO corrections to diffractive dijet (+) production:

Boussarie, Grabovsky, Szymanowski, Wallon (2016)

Iancu, Mueller, Triantafyllopoulos (2021, 2022)

....

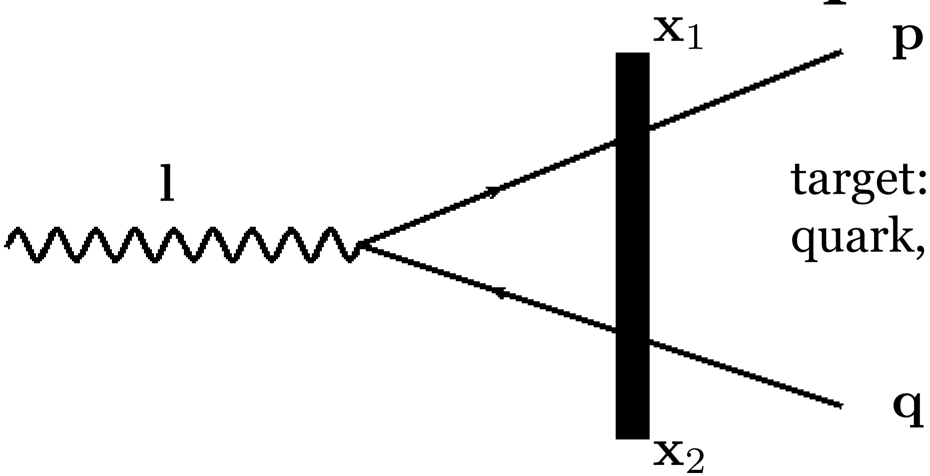
NLO corrections to exclusive light/heavy vector meson production:

Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2016)

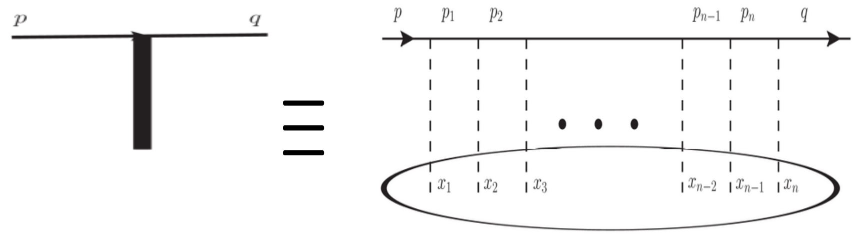
Mantysaari, Penttala (2021, 2022)

....

Inclusive dihadron production in DIS: LO



target: a classical color field
quark, antiquark multiply scatter on the target



$$\frac{d\sigma^{\gamma^* A \to q\bar{q} X}}{d^2p\, d^2q\, dy_1\, dy_2} = \frac{e^2 Q^2 (z_1 z_2)^2 N_c}{(2\pi)^7} \delta(1 - z_1 - z_2)$$

$$\int d^8 x_\perp e^{ip\cdot(x_1' - x_1)} e^{iq\cdot(x_2' - x_2)} \left[S_{122'1'} - S_{12} - S_{1'2'} + 1\right]$$

$$\Big\{ 4 z_1 z_2 K_0(|x_{12}|Q_1) K_0(|x_{1'2'}|Q_1) +$$

$$(z_1^2 + z_2^2) \frac{x_{12} \cdot x_{1'2'}}{|x_{12}||x_{1'2'}|} K_1(|x_{12}|Q_1) K_1(|x_{1'2'}|Q_1) \Big\}$$

with

dipole $\mathbf{S}_{12} \equiv \frac{1}{N_c} Tr\, V(x_1)\, V^\dagger(x_2)$

$$\mathbf{x}_{12} \equiv \mathbf{x}_1 - \mathbf{x}_2$$

quadrupole

$$\mathrm{S}_{122'1'} \equiv \frac{1}{N_c} Tr\, V(\mathbf{x}_1)\, V^\dagger(\mathbf{x}_2)\, V(\mathbf{x}_{2'})\, V^\dagger(\mathbf{x}_{1'})$$

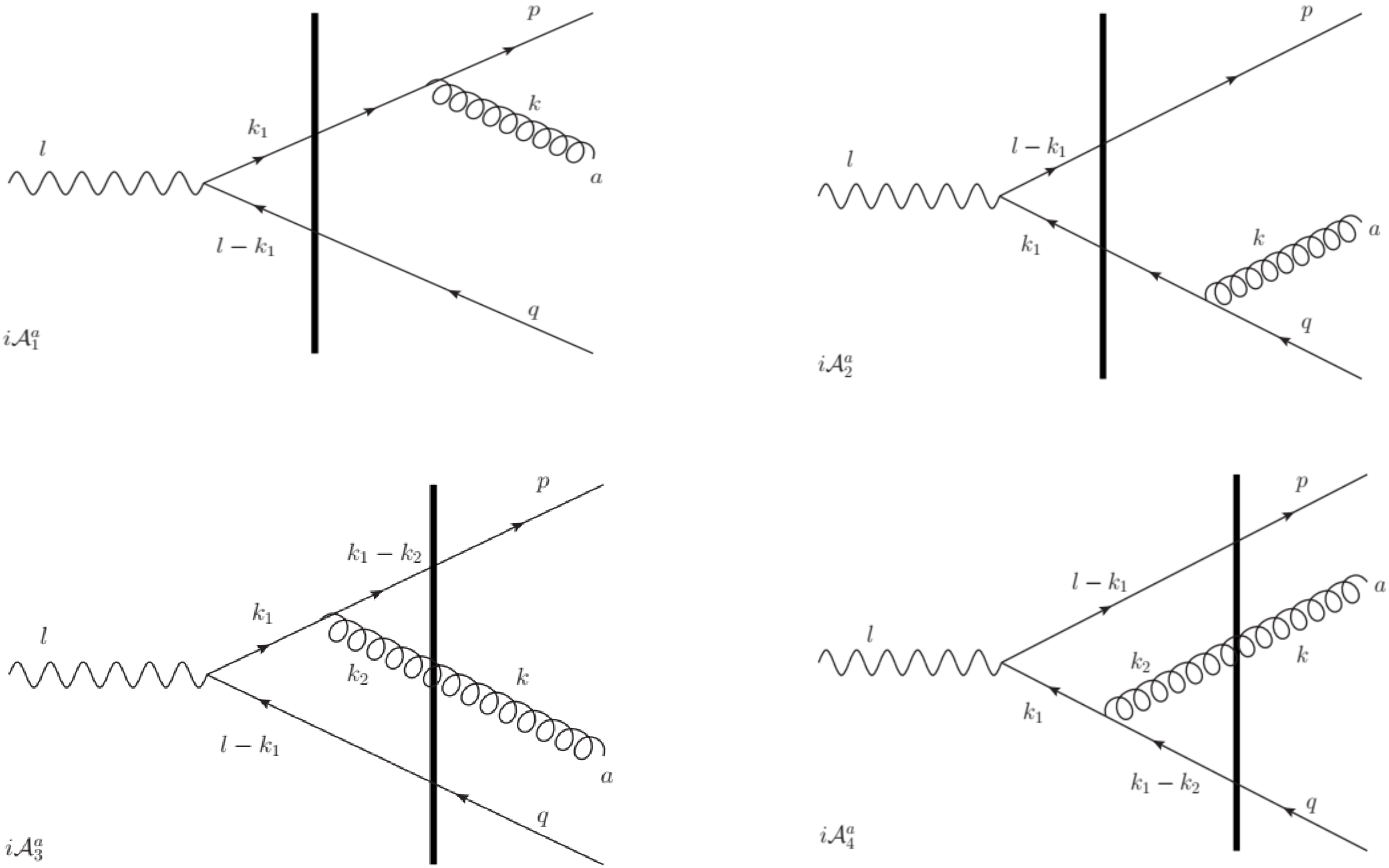
Inclusive dihadron production in forward rapidity: NLO

Based on F. Bergabo and JJM:

PRD 107 (2023) 5, 054036
NPA 1018 (2022) 122358
PRD 106 (2022) 5, 054035

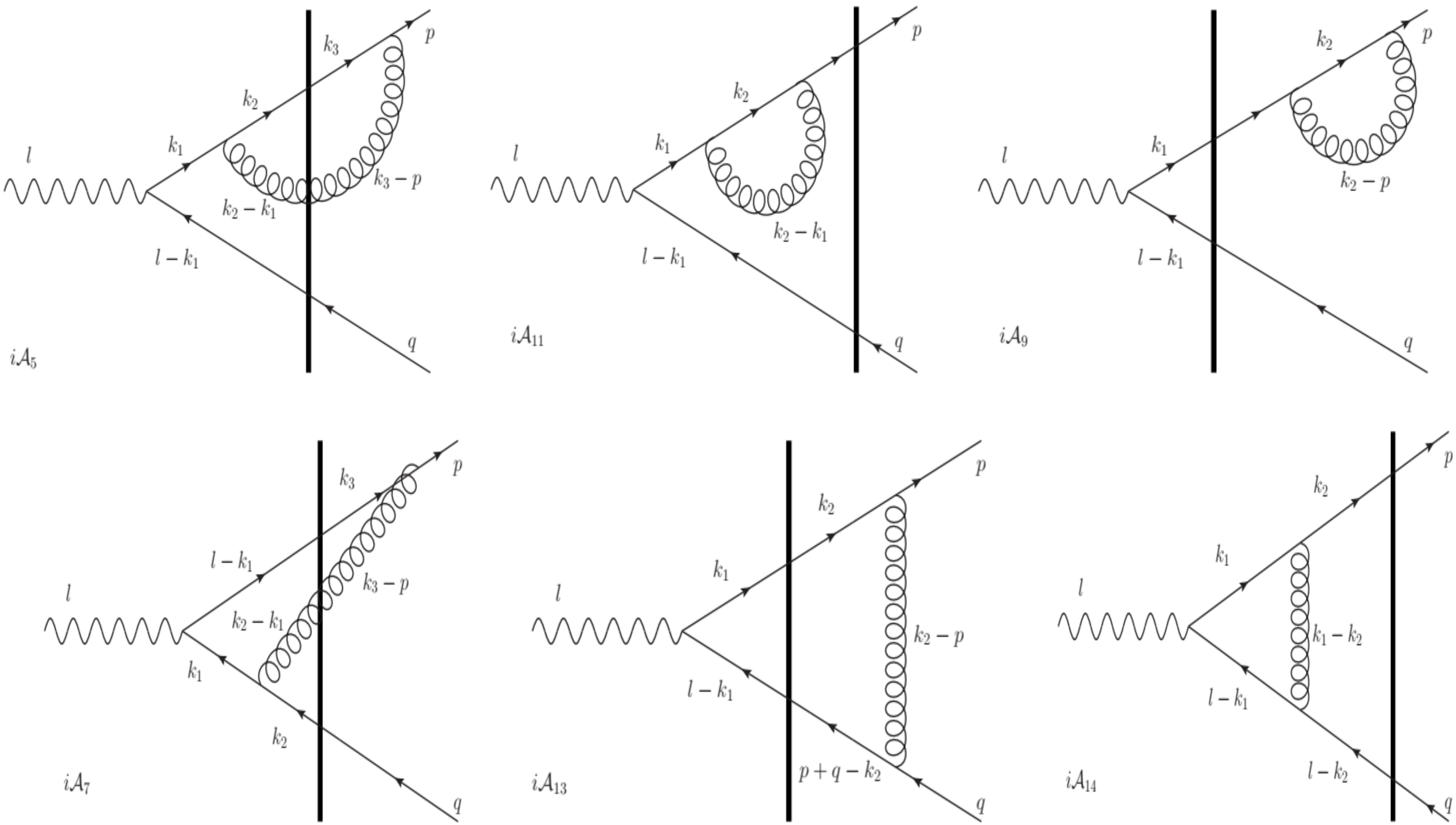
And work in progress (JJM)

NLO corrections - real diagrams (3-jet production)



3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans
PLB 761 (2016) 229 and NPB 920 (2017) 232

NLO corrections – virtual diagrams



F. Bergabo and JJM, dihadrons, 2207.03606, 2301.03117
P. Taels et al., dijets, 2204.11650
P. Caucal et al., dijets, 2108.06347,…….

$$\frac{\sigma^{\mathrm{Real},L}_{1-1}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_2^3(1-z_2)^2(z_1^2+(1-z_2)^2)}{(2\pi)^{10}z_1}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_2)K_0(|\mathbf{x}_{1'2'}|Q_2)\Delta^{(3)}_{11'}$$
$$[S_{122'1'}-S_{12}-S_{1'2'}+1]\,e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma^{\mathrm{Real},L}_{2-2}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1^3(1-z_1)^2(z_2^2+(1-z_1)^2)}{(2\pi)^{10}z_2}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{1'2'}|Q_1)\Delta^{(3)}_{22'}$$
$$[S_{122'1'}-S_{12}-S_{1'2'}+1]e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma^{\mathrm{Real},L}_{1-2}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{-2e^2g^2Q^2N_c^2z_1z_2(1-z_1)(1-z_2)(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_2)K_0(|\mathbf{x}_{1'2'}|Q_1)$$
$$\Delta^{(3)}_{12'}[S_{12}S_{1'2'}-S_{12}-S_{1'2'}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_2'-\mathbf{x}_3)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_1)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma^{\mathrm{Real},L}_{3-3}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1z_2^3(z_1^2+(1-z_2)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(QX)K_0(QX')\Delta^{(3)}_{11'}$$
$$[S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma^{\mathrm{Real},L}_{4-4}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1^3z_2(z_2^2+(1-z_1)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(QX)K_0(QX')\Delta^{(3)}_{22'}$$
$$[S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma^{\mathrm{Real},L}_{3-4}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{-2e^2g^2Q^2N_c^2z_1^2z_2^2(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(QX)K_0(QX')\Delta^{(3)}_{12'}$$
$$[S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma^{\mathrm{Real},L}_{1-3}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{-2e^2g^2Q^2N_c^2z_2^3(1-z_2)(z_1^2+(1-z_2)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_2)K_0(QX')\Delta^{(3)}_{11'}$$
$$[S_{122'3}S_{1'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_3-\mathbf{x}_1)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma^{\mathrm{Real},L}_{1-4}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1z_2^2(1-z_2)(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}K_0(|\mathbf{x}_{12}|Q_2)K_0(QX')\Delta^{(3)}_{12'}$$
$$[S_{122'3}S_{1'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_3-\mathbf{x}_1)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma^{\mathrm{Real},L}_{2-3}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1^2z_2(1-z_1)(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}K_0(|\mathbf{x}_{12}|Q_1)K_0(QX')\Delta^{(3)}_{21'}$$
$$[S_{1231'}S_{2'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$

$$\frac{\sigma^{\mathrm{Real},L}_{2-4}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{-2e^2g^2Q^2N_c^2z_1^3(1-z_1)(z_2^2+(1-z_1)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}K_0(|\mathbf{x}_{12}|Q_1)K_0(QX')\Delta^{(3)}_{22'}$$
$$[S_{1231'}S_{2'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}_2'-\mathbf{x}_2)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$

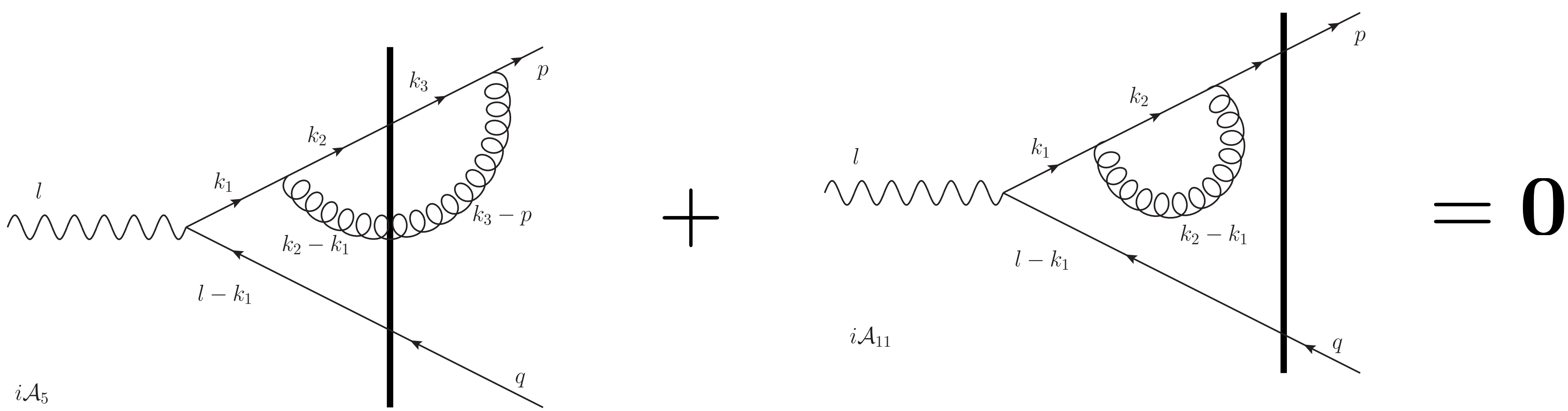
divergences

- ***Ultraviolet:***

Real corrections are UV finite

UV divergences cancel among virtual corrections $\mathbf{k \to \infty}$ **or** $\mathbf{x_3 \to x_i}$

$$\left(d\sigma_5 + d\sigma_{11}\right)_{UV} = 0$$
$$\left(d\sigma_6 + d\sigma_{12}\right)_{UV} = 0$$
$$\left(d\sigma_9 + d\sigma_{10} + d\sigma_{14(1)} + d\sigma_{14(2)}\right)_{UV} = 0$$



divergences

- ***Soft:*** $\mathbf{k^\mu \to 0 \;\; (x_3 \to \infty \;\; AND \;\; z \to 0)}$

Soft divergences cancel between real and virtual corrections

$$(d\sigma_{1-1} + d\sigma_9)_{soft} = 0,$$

$$\left(d\sigma_{1-2} + d\sigma_{13}^{(1)} + d\sigma_{13}^{(2)}\right)_{soft} = 0$$

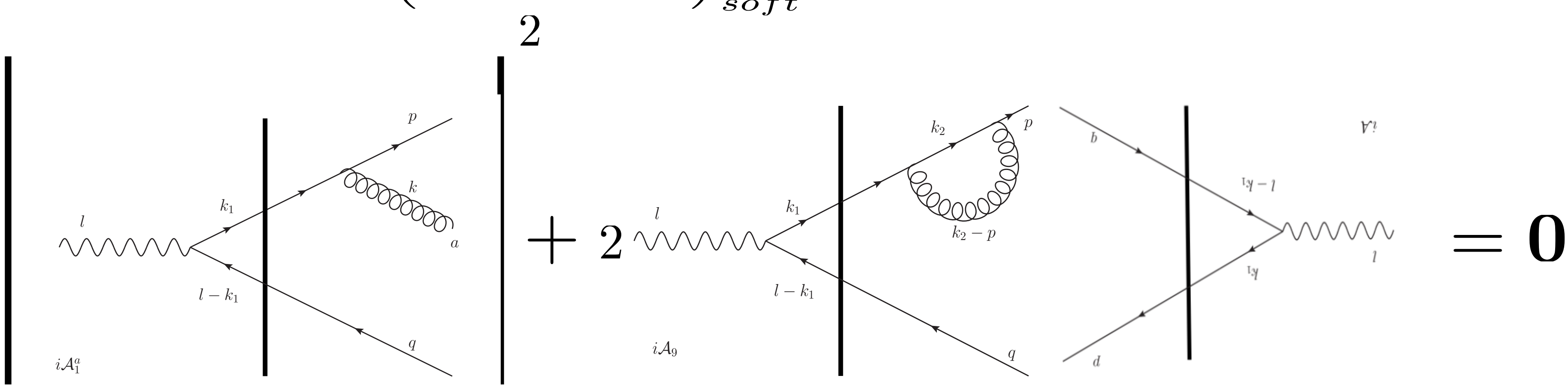
$$(d\sigma_{3-3} + d\sigma_{4-4} + d\sigma_{3-4})_{soft} = 0$$

$$(d\sigma_{1-3} + d\sigma_{1-4})_{soft} = 0$$

$$(d\sigma_{2-3} + d\sigma_{2-4})_{soft} = 0$$

$$(d\sigma_5 + d\sigma_7)_{soft} = 0$$

$$\left(d\sigma_{11} + d\sigma_{14}^{(1)}\right)_{soft} = 0$$



divergences

• ***Rapidity:*** $\mathbf{z} \to 0$ ***, but finite k_t***

$$\int_0^1 \frac{dz}{z} = \int_0^{z_f} \frac{dz}{z} + \int_{z_f}^1 \frac{dz}{z}$$

rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\frac{d\sigma^L_{\rm NLO}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, y_2} = \frac{2e^2 g^2 Q^2 N_c^2 (z_1 z_2)^3}{(2\pi)^{10}}\, \delta(1 - z_1 - z_2) \int_0^{z_f} \frac{dz}{z} \int d^{10}\mathbf{x}\, K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1)$$

$$e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \Big\{ \left(\tilde\Delta_{12} + \tilde\Delta_{22'} - \tilde\Delta_{12'}\right) S_{132'1'} S_{23} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{22'} - \tilde\Delta_{21'}\right) S_{1'321} S_{2'3}$$

$$+ \left(\tilde\Delta_{12} + \tilde\Delta_{11'} - \tilde\Delta_{21'}\right) S_{322'1'} S_{13} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{11'} - \tilde\Delta_{12'}\right) S_{32'21} S_{1'3}$$

$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} + \tilde\Delta_{12} + \tilde\Delta_{1'2'}\right) S_{122'1'} - \left(\tilde\Delta_{12} + \tilde\Delta_{1'2'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{12} S_{1'2'}$$

$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{11'} S_{22'} - 2\tilde\Delta_{12} \left(S_{13} S_{23} - S_{12}\right) - 2\tilde\Delta_{1'2'} \left(S_{1'3} S_{2'3} - S_{1'2'}\right) \Big\}$$

JIMWLK evolution of quadrupoles

JIMWLK evolution of dipoles

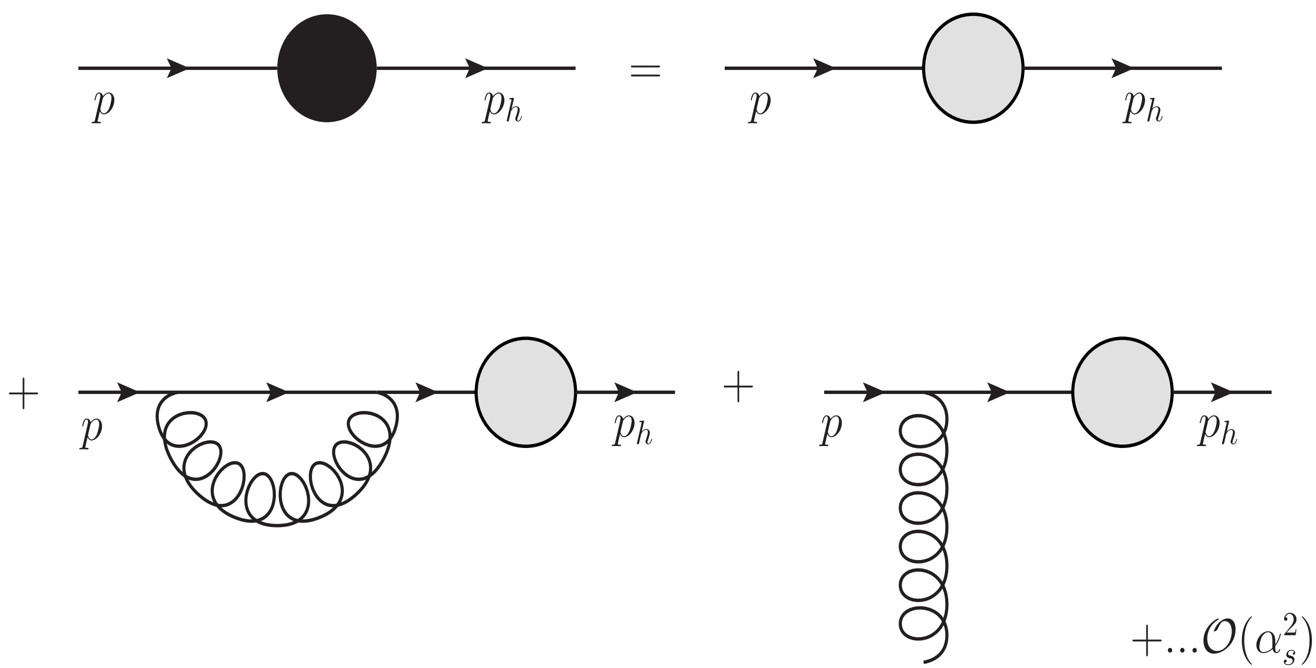
$$\tilde{\boldsymbol{\Delta}}_{\mathbf{12}} \equiv \frac{(\mathbf{x_1} - \mathbf{x_2})^2}{(\mathbf{x_1} - \mathbf{x_3})^2 (\mathbf{x_2} - \mathbf{x_3})^2}$$

divergences

- ***Collinear:***

$$\frac{1}{(p+k)^2} = \frac{1}{|\vec{p}||\vec{k}|(1-cos\,\theta)} \longrightarrow \infty \quad as \quad \theta \to 0$$

Collinear divergences are absorbed into evolution of parton-hadron fragmentation functions



collinear divergences

real corrections

$$\frac{d\sigma_{LO+1-1}^{\gamma^* A\to h_1 h_2 X}}{d^2\mathbf{p}_h\, d^2\mathbf{q}_h\, dy_1\, dy_2} = \int_0^1 dz_{h_1} \int_0^1 dz_{h_2} \frac{4e^2 Q^2 N_c (z_1 z_2)^3}{(2\pi)^7 (z_{h_1} z_{h_2})^2} H\left(\mathbf{p}, \mathbf{q}, z_2\right) D^0_{h_1/q}(z_{h_1}) D^0_{h_2/\bar{q}}(z_{h_2})$$

$$\int \frac{d\xi_1}{\xi_1^3} \delta(1 - z_2 - z_1/\xi_1) \left[\delta(1-\xi_1) + 2\alpha_s\, P_{qq}(\xi_1) \int d^2\mathbf{k} \frac{e^{i\mathbf{k}\cdot(\mathbf{x}_1' - \mathbf{x}_1)}}{\left(\xi_1 \mathbf{k} - (1-\xi_1)\mathbf{p}\right)^2}\right]$$

with $P_{qq}(\xi_1) = C_F \frac{(1+\xi_1^2)}{(1-\xi_1)}$

virtual corrections

$$\frac{d\sigma_9^{\gamma^* A\to h_1 h_2 X}}{d^2\mathbf{p}_h\, d^2\mathbf{q}_h\, dy_1\, dy_2} = -\int_0^1 dz_{h_1} \int_0^1 dz_{h_2} \frac{4e^2 Q^2 (z_1 z_2)^3 N_c}{(2\pi)^7 (z_{h_1} z_{h_2})^2} H(\mathbf{p}, \mathbf{q}, z_2) D^0_{h_1/q}(z_{h_1}) D^0_{h_2/\bar{q}}(z_{h_2})$$

$$\times\, \alpha_s \int_0^1 d\xi\, P_{qq}(\xi) \int d^2\mathbf{k} \frac{1}{\left(\mathbf{k} - (1-\xi)\mathbf{p}\right)^2} \delta(1 - z_1 - z_2)$$

these are combined into DGLAP evolution of fragmentation functions

$$D_{h_1/q}(z_{h1}, \mu^2) = \int_{z_{h1}}^1 \frac{d\xi}{\xi} D^0_{h_1/q}\left(\frac{z_{h1}}{\xi}\right) \left[\delta(1-\xi) + \frac{\alpha_s}{2\pi} P_{qq}(\xi)\, \log\left(\frac{\mu^2}{\Lambda^2}\right)\right]$$

Divergences

•*Ultraviolet*

Real corrections are UV finite
UV divergences cancel among virtual corrections

•*Soft*

Soft divergences cancel between real and virtual corrections

•*Collinear*

Collinear divergences are absorbed into hadron fragmentation functions

•*Rapidity*

rapidity divergences are absorbed into JIMWLK evolution of dipoles, quadrupoles

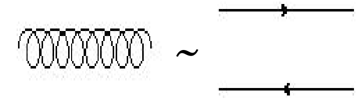
$$\sigma^{\gamma^* A\to h_1 h_2 X} = \sigma_{LO}\otimes \mathrm{JIMWLK} + \sigma_{LO}\otimes D_{h_1/q}(z_1,\mu^2) D_{h_2/\bar{q}}(z_2,\mu^2) + \sigma_{NLO}^{\mathrm{finite}}$$

phenomenology: EIC, UPC at the LHC,...

One-loop corrections: BK-JIMWLK eq.

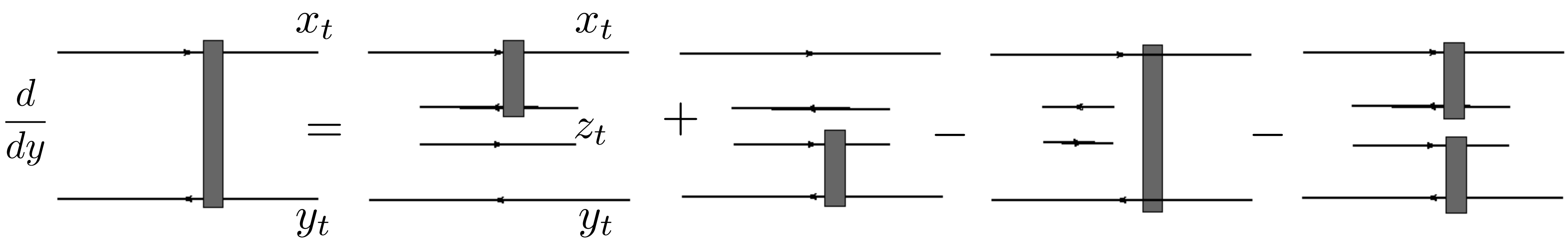
at large N_c

$3 \otimes \bar{3} = 8 \oplus 1 \simeq 8$



$$\frac{d}{dy}T(x_t, y_t) = \frac{N_c \alpha_s}{2\pi^2}\int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2}\left[T(x_t, z_t) + T(z_t, y_t) - T(x_t, y_t) - T(x_t, z_t)T(z_t, y_t)\right]$$

$$T \equiv 1 - S$$



$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \gg p_t^2$$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^\gamma \qquad Q_s^2 < p_t^2$$

nuclear modification factor

$$R_{pA} \equiv \frac{\frac{d\sigma^{pA}}{d^2 p_t dy}}{A^{1/3}\frac{d\sigma^{pp}}{d^2 p_t dy}}$$

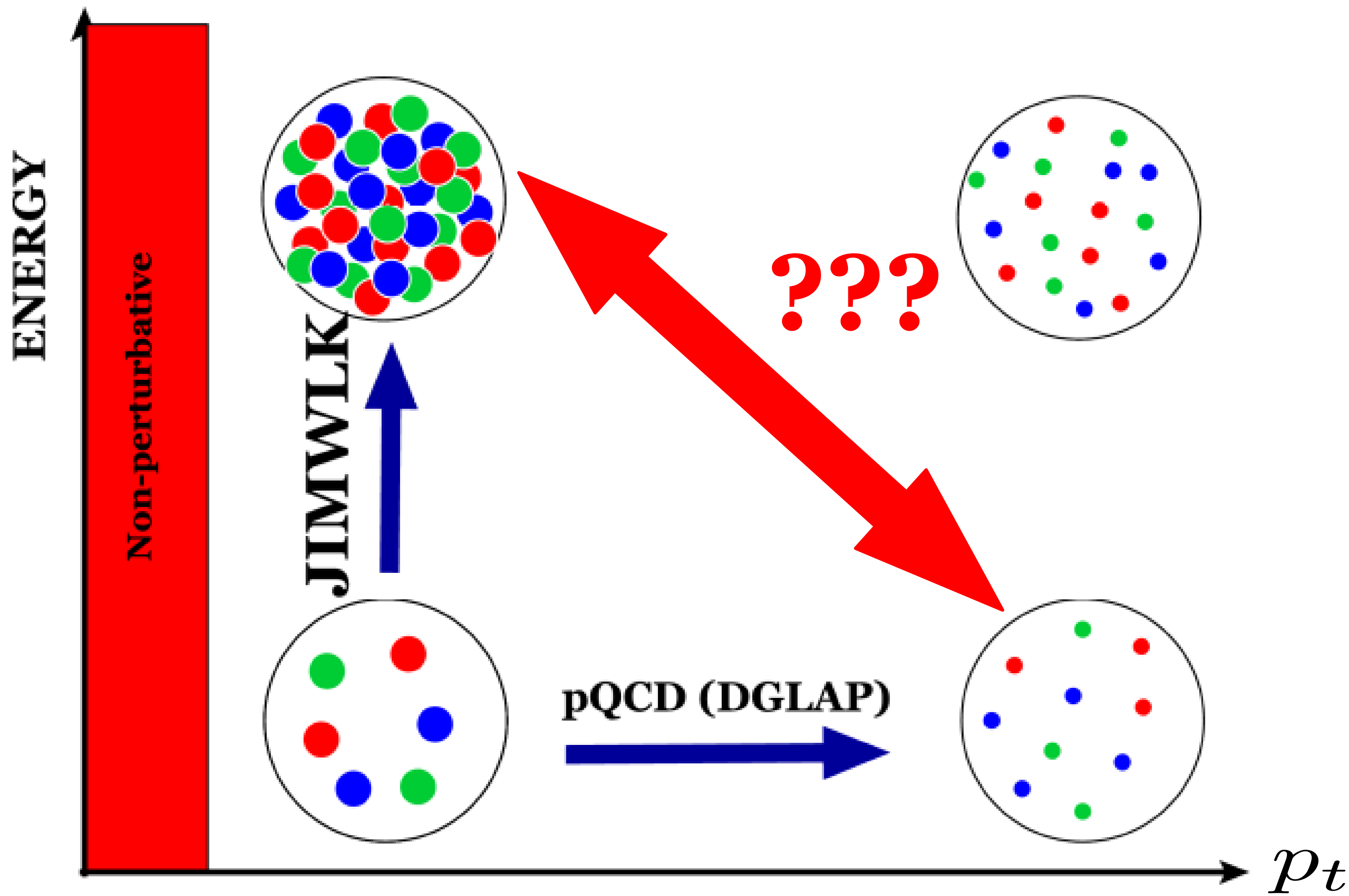
nuclear shadowing

suppression of p_t spectra

disappearance of back to back peaks

....

QCD kinematic phase space



SUMMARY

QCD at high p_t

dilute hadron: partons - collinear factorization

breaks down at small x/low p_t

QCD at high energy

dense hadron: gluon saturation, strong color fields - CGC

strong hints from RHIC, LHC,...

to be probed precisely at EIC

toward precision: NLO, sub-eikonal corrections, ...

CGC is limited to small x (low p_t)