The Drell-Yan process with pions and polarized proton

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Outline

Introduction

- Transverse Momentum Dependent distribution factorization is valid only for $q_\perp \ll Q$ [1].

1. Collins(2011), TMD Handbook(2023)

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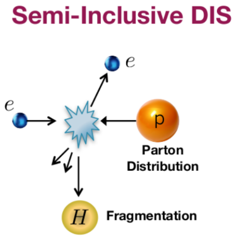

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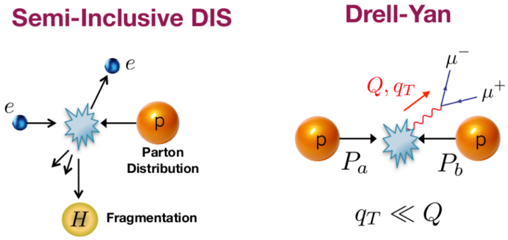

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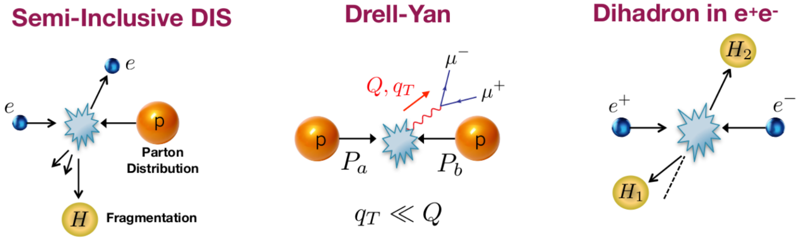

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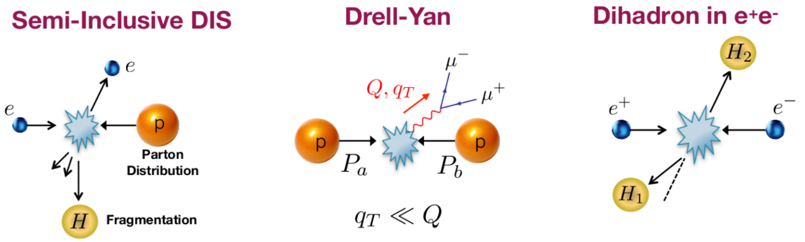

• The Drell-Yan process provides important information on the internal structure of hadrons including TMDs.

• The Drell-Yan technique poses significant experimental challenges due of its very low counting rates.

• However, theoretically, it is the cleanest hard hadron-hadron scattering process.

• The absence of a hadron in the final state simplifies factorization proof compared to hadron-hadron collisions with hadronic final states.

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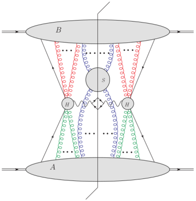
- Graphical structure corresponding to leading regions in Drell-Yan scattering process [1]



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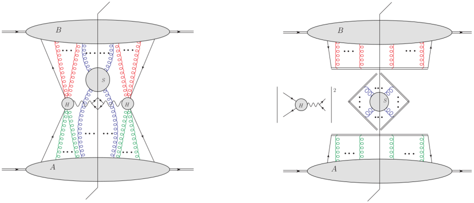

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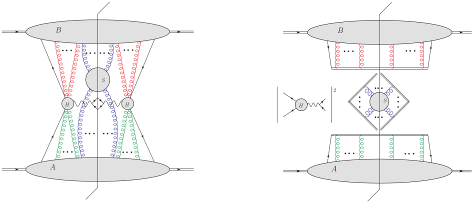

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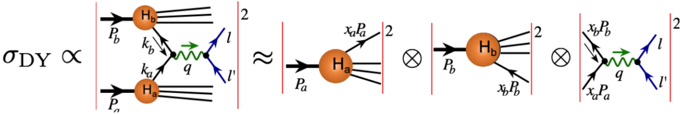

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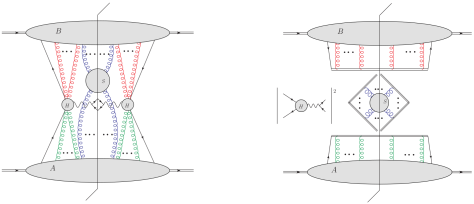

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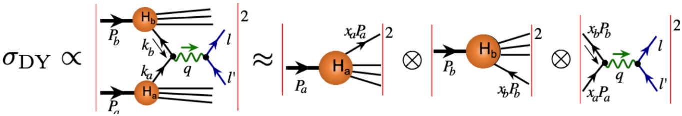

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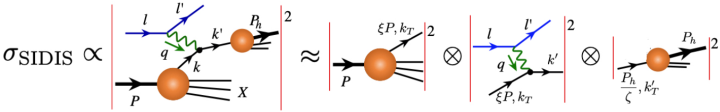

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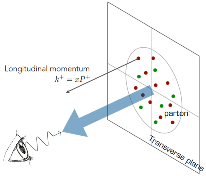

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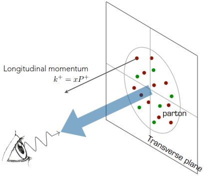

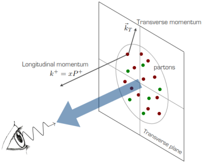

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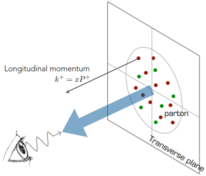

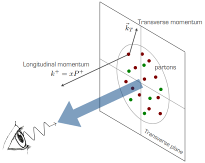

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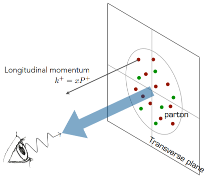

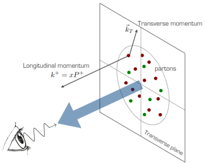

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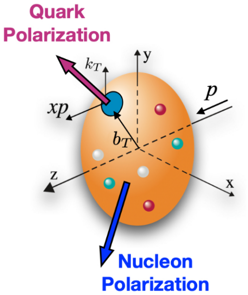

Leading Quark TMDPDFs (Nucleon Spin, Quark Spin)

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = Boer-Mulders
L		g_1 = Helicity	$h_{1L}^\perp$ = Worm-gear
T	$f_{1T}^\perp$ = Sivers	$g_{1T}^\perp$ = Worm-gear	h_1 = Transversity; $h_{1T}^\perp$ = Pretzelosity

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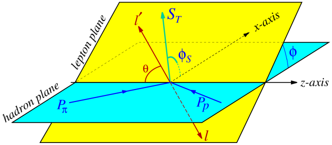
Pion-induced Drell-Yan process

• Within the TMD factorization $(q_\perp \ll Q)$ the differential cross section in the Drell-Yan process[1] :

$$\frac{d\sigma\left(\pi^- p \to l^+ l^- X\right)}{dx_1 dx_2 d^2\boldsymbol{q}_T d\Omega} = \frac{\alpha_{em}^2}{\mathcal{F}Q^2}\Big\{\Big[(1+\cos^2\theta)F_{UU}^1 + \sin^2\theta\cos(2\phi)F_{UU}^{\cos 2\phi}\Big] + S_L\sin^2\theta\sin(2\phi)F_{UL}^{\sin 2\phi} + S_T\sin^2\theta\Big[\sin\phi_s F_{UT}^{\sin\phi_s} + \sin(2\phi+\phi_s)F_{UT}^{\sin(2\phi+\phi_S)} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\Big]\Big\}$$



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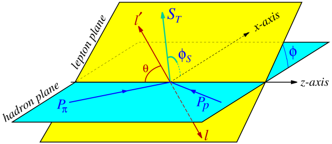

- Structure functions are the convolutions of TMDs :

$$\mathcal{C}[\omega f_\pi^{\bar{q}} f_p^q] = \frac{1}{N_c}\sum_q e_q^2 \int d^2\vec{k}_{\perp\pi} d^2\vec{k}_{\perp p}\delta^2(\vec{k}_{\perp\pi} + \vec{k}_{\perp p} - \vec{q}_\perp)\omega\Big[f_\pi^{\bar{q}}(x_\pi, k_{\perp\pi}^2) f_p^q(x_p, k_{\perp p}^2)\Big]$$

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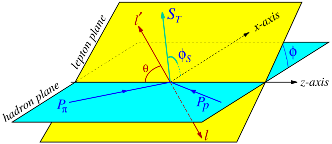

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- Transverse spin asymmetries :

$$A_{XY}^{\text{weight}}\left(x_\pi, x_p, q_T, Q^2\right) = \frac{F_{XY}^{\text{weight}}\left(x_\pi, x_p, q_T, Q^2\right)}{F_{UU}^1\left(x_\pi, x_p, q_T, Q^2\right)},$$

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- The TMD correlator for the pion (twist-2)[1] :

$$\Phi(x, k_\perp) = \frac{1}{2}\left\{ f_1 \not{n}_+ + i h_1^\perp \frac{[\not{k}_\perp, \not{n}_+]}{2M_\pi} \right\}$$

- The unpolarized pion TMD, $f_{1,\pi}$ describes the momentum distribution of unpolarized quarks within the pion.

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where, $\zeta = \sqrt{x(1-x)}b_\perp$ with $M^2 = M_\perp^2 + M_\parallel^2$.

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- The spin-improved holographic wave function :

$$\Psi_{h,\bar{h}}(x,\mathbf{k}_\perp) = \left[(M_\pi x\bar{x} + Bm_f)h\delta_{h,-\bar{h}} - Bk_\perp e^{-ih\theta_{k_\perp}}\delta_{h,\bar{h}}\right]\frac{\Psi(x,\mathbf{k}_\perp)}{x\bar{x}}\,.$$

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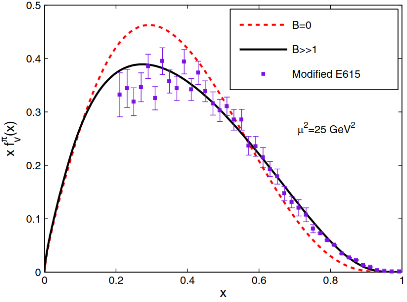

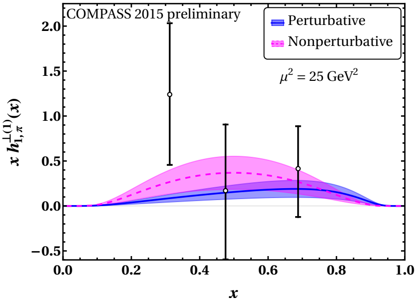

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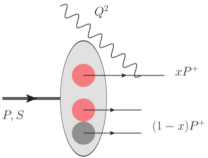
Proton TMDs : Light-Front Quark-Diquark model



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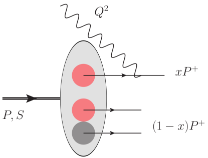

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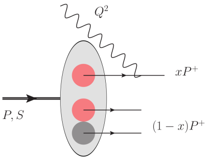

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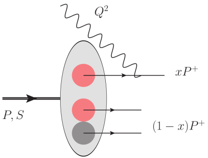

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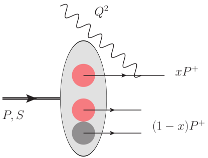

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• The two particle Fock-state expansion with $J_z = \pm\frac{1}{2}$ for scalar diquark :

$$|u\ S\rangle^{\pm} = \int \frac{dx d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \sum_{\lambda} \psi_{\lambda}^{\pm(u)}(x, \mathbf{p}_\perp)\, |\lambda, \Lambda_S; xP^+, \mathbf{p}_\perp\rangle\Big|_{\Lambda_S=0}$$

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- The light-front wavefunctions [1]

$$\psi^{\pm(\nu)}_{\lambda\Lambda}(x,\mathbf{p}_\perp) = N^{\nu} f(x,\mathbf{p}_\perp,\lambda,\Lambda)\varphi^{(\nu)}_i(x,\mathbf{p}_\perp)\Big|_{i=1,2}$$

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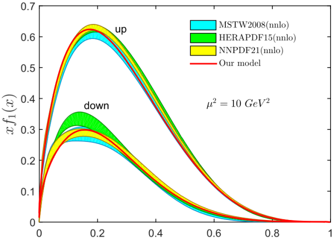
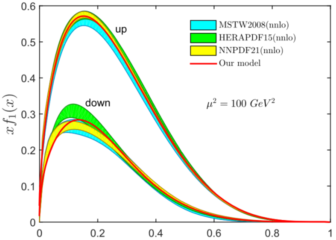
• Unpolarized parton densities are in good agreement with the available global analysis [1]





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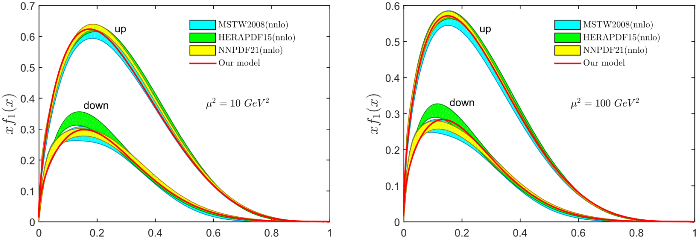

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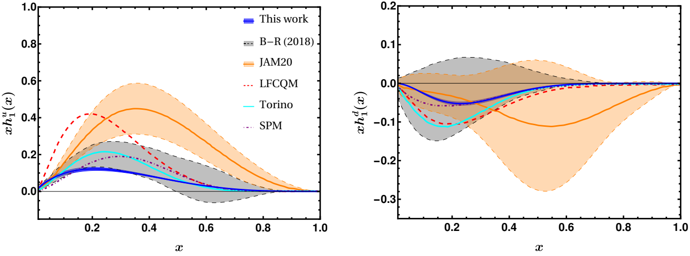

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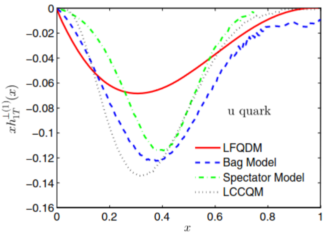

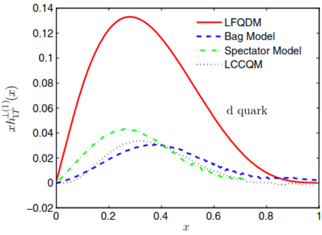

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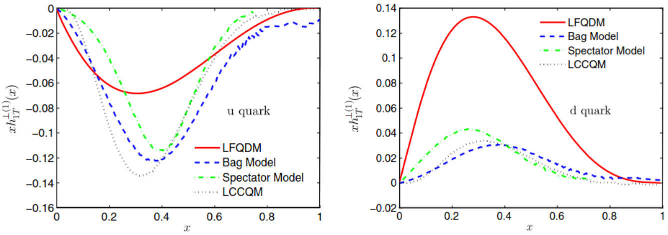

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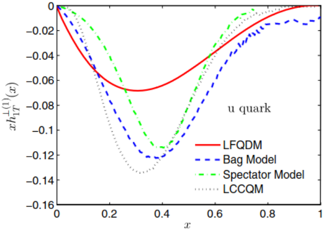

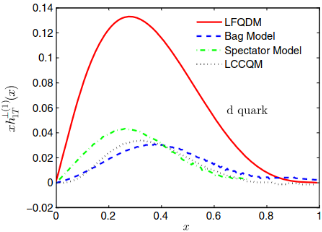

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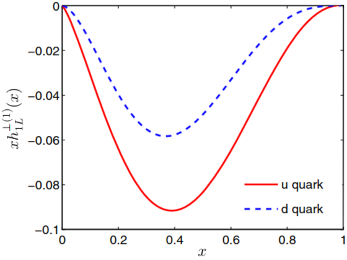

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Initial vs Final state interaction and T-odd TMDs

• A gluon exchange between the outgoing (incoming) quark (anti-quark) and the target spectator system is known as the FSI (ISI) [1].

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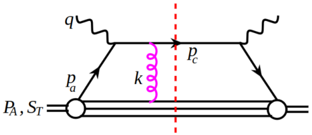

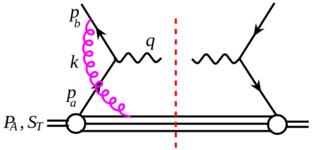

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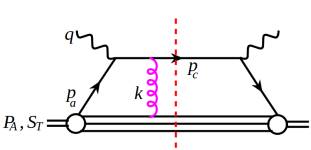
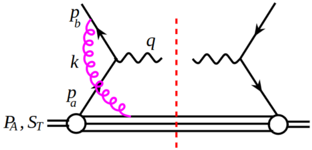
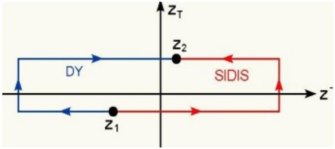




- Gauge link for T-even TMDs ⇒ Unity & T-odd TMDs ⇏ Unity

$$\mathcal{W}[z_1; z_2] = \mathcal{P}e^{-ig\int_{z_1}^{z_2} ds \cdot A(s)}$$



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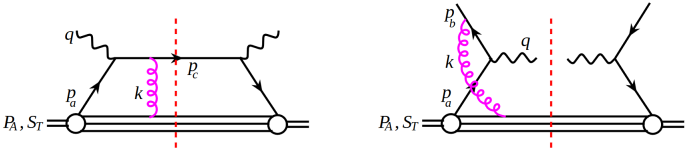

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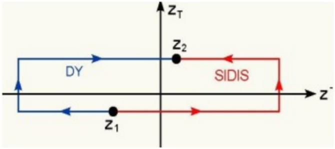


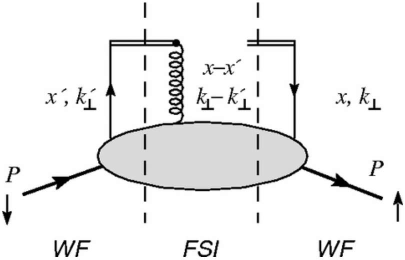
- Sivers and Boer-Mulders functions are T-odd and have opposite sign in SIDIS and DY processes. [☑ HERMES/COMPASS]

$$f_{1T}^{\perp q}|_{SIDIS} = -f_{1T}^{\perp q}|_{DY}, \qquad h_1^{\perp q}|_{SIDIS} = -h_1^{\perp q}|_{DY}$$

1. C. Pisano, L. Gamberg et al., (2011)

- The perturbative Abelian U(1) & the nonperturbative SU(3) kernel [1].

$$iG(x,q_\perp)\Big|_{U(1)} \propto \frac{C_F\alpha_S}{q_\perp^2}; \qquad iG(x,q_\perp)\Big|_{SU(3)} = -\frac{2}{(2\pi)^2}\frac{\bar{x}I(x,q_\perp)}{q_\perp}$$



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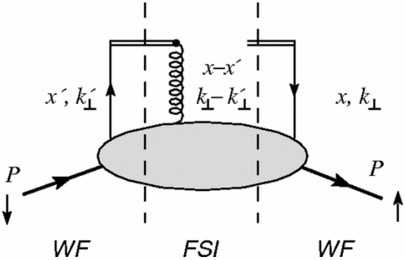

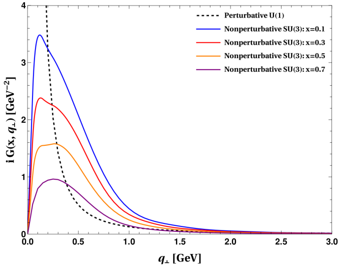

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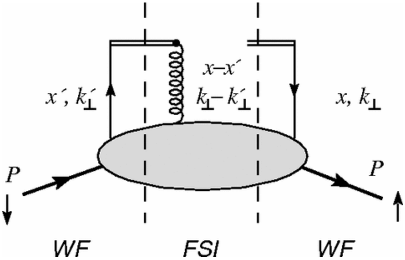

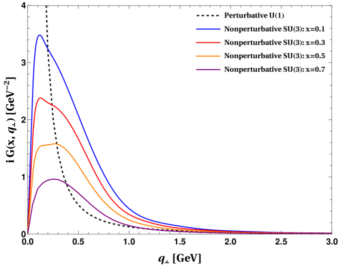

• The QCD lensing function is obtained from the eikonal amplitude of quark-antiquark scattering by exchanging non-Abelian soft gluons[2].

• The Lensing function, $I(x, q_\perp)$ connects $h_{1,\pi}^{\perp(1)}$ with chiral-odd pion GPD, $\mathcal{H}_1^\pi$ [2].

$$M_\pi^2 h_1^{\perp(1)}(x) = 2\pi \int_0^\infty db_\perp b_\perp^2 \mathcal{I}(x, b_\perp) \frac{\partial}{\partial b_\perp^2} \mathcal{H}_1^\pi(x, b_\perp^2)$$

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Sivers & Boer-Mulders Functions

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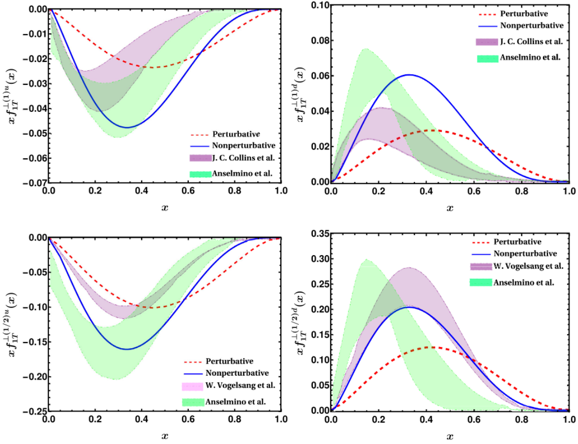

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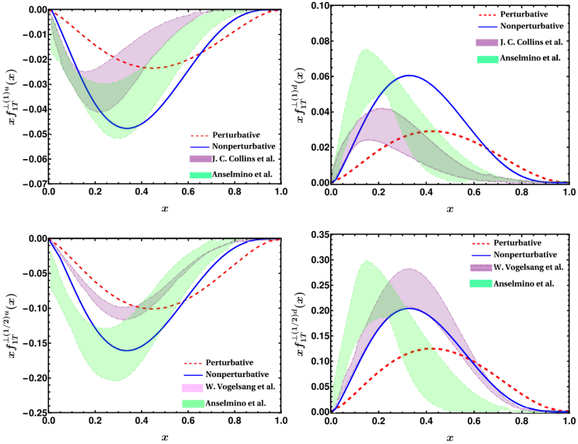

- Axial-vector diquark model [2] $f_{1T}^{\perp u}$ & $h_1^{\perp u} < 0$, and $f_{1T}^{\perp d} > 0$, $h_1^{\perp d} < 0$.

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TMD scale evolution

• TMD evolution is governed by two differential equations (RG-equation) in UV and rapidity renormalize scales.

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$$\tilde{F}(x, b_\perp; \mu) = \tilde{F}(x, b_\perp; \mu_0) e^{-[S_P(Q, b_*) + S_{NP}(Q, b_\perp)]}$$

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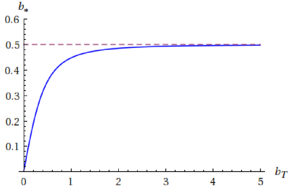
- b_* always remains in perturbative region $(0 \le b_* \le b_{\max})$ [2] :



1. M. G. Echevarria et al. (2014), S. M. Aybat et al., (2011)
2. Brock, Landry, Nadolsky, Yuan (2003)

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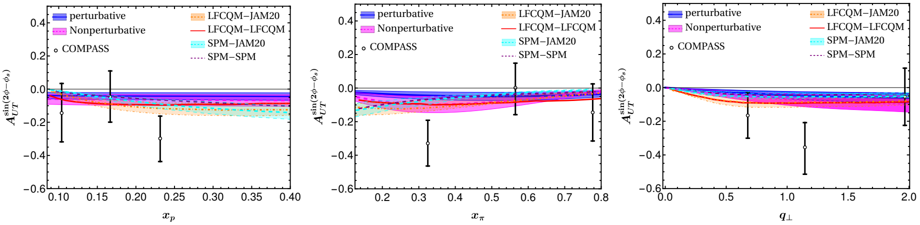

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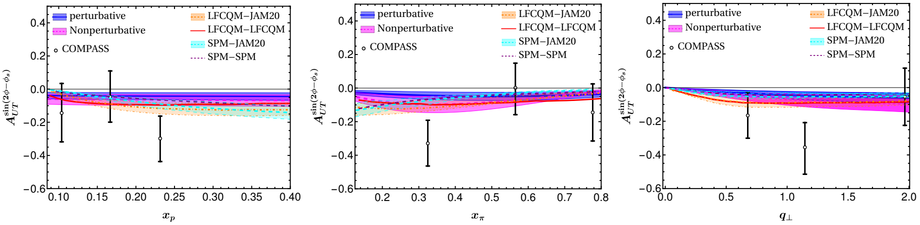

- $A_{UT}^{\sin(2\phi-\phi_s)} \propto -h^{\perp(1)\bar{u}}_{1,\pi^-}(x_\pi) h^{u}_{1,p}(x_p) < 0 \quad \Rightarrow$ COMPASS data indicate a positive sign for the pion Boer-Mulders TMD $h^{\perp(1)\bar{u}}_{1,\pi^-}$.

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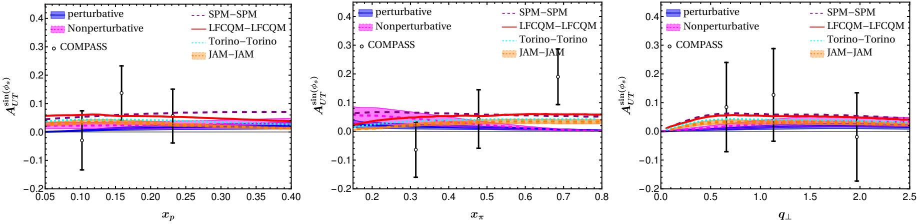

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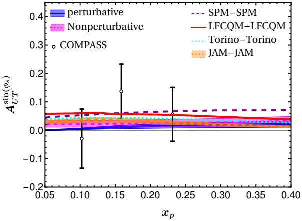

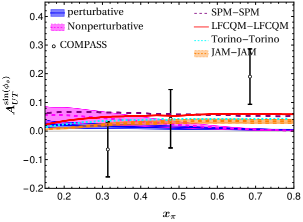

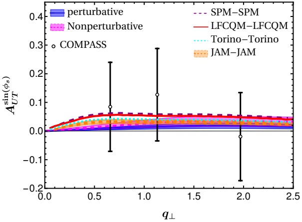

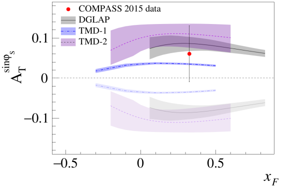

$\sin(2\phi + \phi_s)$ asymmetry

- In Drell-Yan process : $\sin(2\phi + \phi_s)$ asymmetry $\sim h_{1(\pi)}^{\perp} \otimes h_{1T(p)}^{\perp}$

$$A_{UT}^{\sin(2\phi+\phi_s)}(x_p, x_\pi, q_\perp) = \frac{F_{UT}^{\sin(2\phi+\phi_s)}(x_p, x_\pi, q_\perp)}{F_{UU}^{1}(x_p, x_\pi, q_\perp)}$$

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- The structure functions :

$$F_{UT}^{\sin(2\phi+\phi_S)} = -\mathcal{C}\left[\frac{2(\hat{h}\cdot\vec{k}_{\perp p})[2(\hat{h}\cdot\vec{k}_{\perp\pi})(\hat{h}\cdot\vec{k}_{\perp p}) - \vec{k}_{\perp\pi}\vec{k}_{\perp p}] - \vec{k}^2_{\perp p}(\hat{h}\cdot\vec{k}_{\perp\pi})}{2M_\pi M_p^2} h^{\perp}_{1,\bar{q}/\pi} h^{\perp}_{1T,q/p}\right]$$

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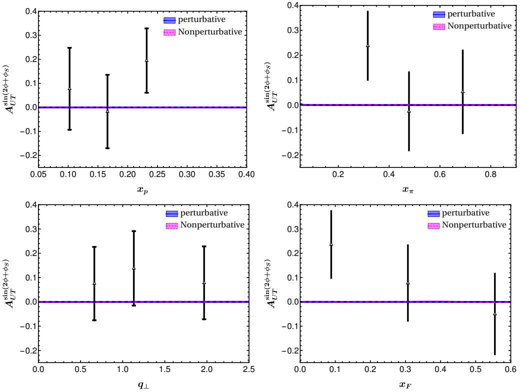

- This asymmetry is proportional to $q_\perp^3$ for $q_\perp \ll 1$ GeV.

$\sin(2\phi)$ asymmetry

- $\sin(2\phi)$ asymmetry $\sim h^{\perp}_{1,\bar{q}/\pi} \otimes h^{\perp}_{1L,q/p}$

$$A^{\sin(2\phi)}_{UL}(x_p, x_\pi, q_\perp) = \frac{F^{\sin(2\phi)}_{UL}(x_p, x_\pi, q_\perp)}{F^{1}_{UU}(x_p, x_\pi, q_\perp)}$$

1. S. Bastami, L. Gamberg, B. Pasquini et al. (2021)

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$$F^{1}_{UU} = \mathcal{C}\left[f_{1,\bar{q}/\pi} f_{1,q/p}\right], \qquad F^{\sin(2\phi)}_{UL} = -\mathcal{C}\left[\frac{2(\hat{h}\cdot\vec{k}_{\perp\pi})(\hat{h}\cdot\vec{k}_{\perp p}) - \vec{k}_{\perp\pi}\cdot\vec{k}_{\perp p}}{M_\pi M_p} h^{\perp}_{1,\bar{q}/\pi} h^{\perp}_{1L,q/p}\right]$$

1. S. Bastami, L. Gamberg, B. Pasquini et al. (2021)

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$$A_{UL}^{\sin(2\phi)}(x_p, x_\pi, q_\perp) = \frac{F_{UL}^{\sin(2\phi)}(x_p, x_\pi, q_\perp)}{F_{UU}^{1}(x_p, x_\pi, q_\perp)}$$

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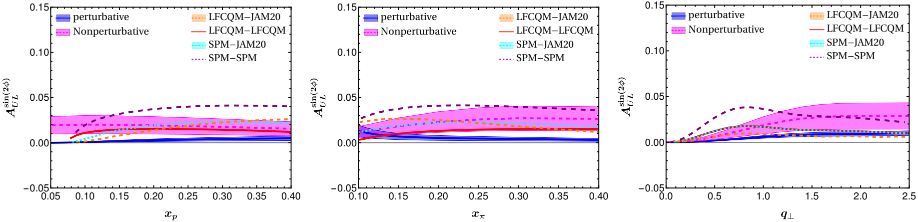
$$F_{UU}^{1} = \mathcal{C}\left[f_{1,\bar{q}/\pi} f_{1,q/p}\right], \qquad F_{UL}^{\sin(2\phi)} = -\mathcal{C}\left[\frac{2(\hat{h}\cdot\vec{k}_{\perp\pi})(\hat{h}\cdot\vec{k}_{\perp p}) - \vec{k}_{\perp\pi}\cdot\vec{k}_{\perp p}}{M_\pi M_p} h^{\perp}_{1,\bar{q}/\pi} h^{\perp}_{1L,q/p}\right]$$



- In DY $\sin(2\phi)$ asymmetry requires a longitudinal proton polarization and could be studied in DY with doubly polarized protons in a future NICA experiment. [1]

1. S. Bastami, L. Gamberg, B. Pasquini et al. (2021)

$\cos(2\phi)$ asymmetry

- $\cos(2\phi)$ asymmetry $\sim h^{\perp}_{1,\bar{q}/\pi} \otimes h^{\perp}_{1,q/p}$

$$^{1}A^{\cos(2\phi)}_{UU}(x_p, x_\pi, q_\perp) = \frac{F^{\cos(2\phi)}_{UU}(x_p, x_\pi, q_\perp)}{F^{1}_{UU}(x_p, x_\pi, q_\perp)}$$

1. S. Bastami, L. Gamberg, B. Pasquini et al. (2021)

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1. S. Bastami, L. Gamberg, B. Pasquini et al. (2021)

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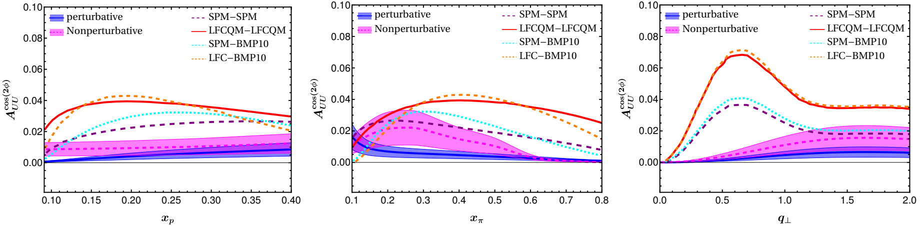

- The asymmetry, $A^{\cos(2\phi)}_{UU} \propto h^{\perp(1)\bar{u}}_{1,\pi^-} h^{\perp u}_{1,p} \Rightarrow h^{\perp u}_{1,p}$ is positive in DY, which is opposite to SIDIS analyses and hence in agreement with the prediction for the process dependence property of T-odd TMDs.

1. S. Bastami, L. Gamberg, B. Pasquini et al. (2021)

Conclusion

- We presented a complete description of polarized DY at leading twist using TMD evolution at NLL accuracy.
- The required TMDs include on the nucleon side $f_{1,p}$, $f_{1T,p}^{\perp}$, $h_{1,p}$, $h_{1p}^{\perp}$, $h_{1T}^{\perp q}$ and $h_{1L}^{\perp q}$; and on the pion side $f_{1,\pi}$, $h_{1,\pi}^{\perp}$.
- The T-odd TMDs requires gluon rescattering to obtain nonzero contribution. We explored the usage of a nonperturbative SU(3) gluon rescattering kernel, going beyond the typical approximation of perturbative U(1) gluons.
- Our analysis reveals that the above asymmetries at COMPASS can be qualitatively described (sign and magnitude) by analysing the pion TMDs in a holographic light-front QCD framework and the proton TMDs in a soft-wall AdS/QCD model.

Thanks for your attentation ! !