The hidden gauge symmetry of relativistic dissipative hydrodynamics

or... Hydrodynamics with 50 particles. What does it mean and how to think about it?

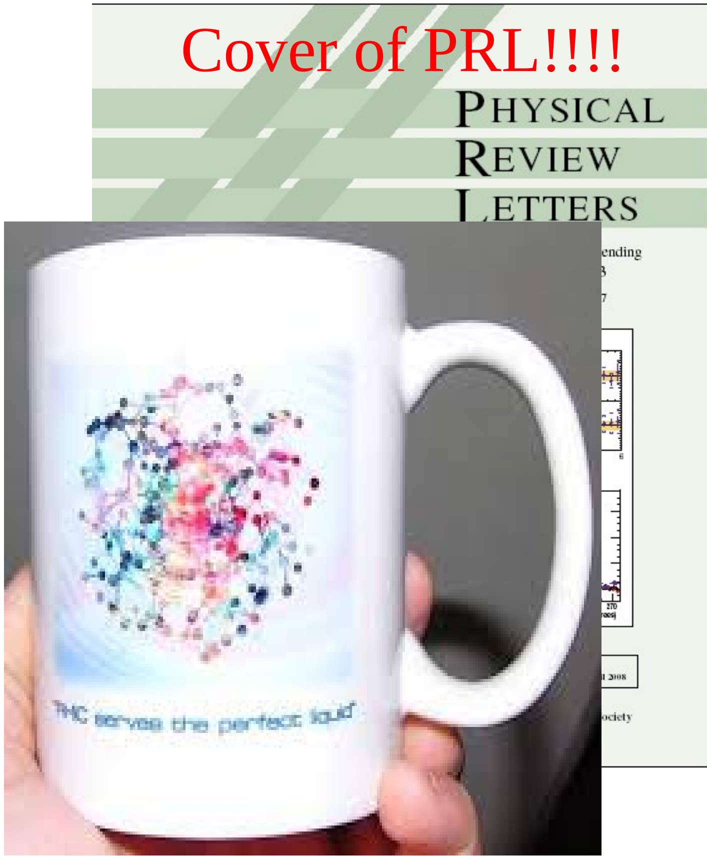
G.Torrieri

UNICAMP

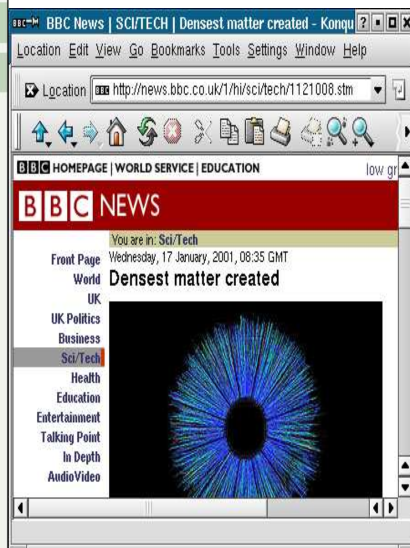
FAPESP
FUNDAÇÃO DE AMPARO À PESQUISA
DO ESTADO DE SÃO PAULO

2307.07021 (Submitted,PRL), 2309.05154 2007.09224 (JHEP),
2109.06389 (Annals of Physics, With T.Dore,M.Shokri,L.Gavassino,D.Montenegro)
Answers somewhat speculative... but I think I am asking good questions!

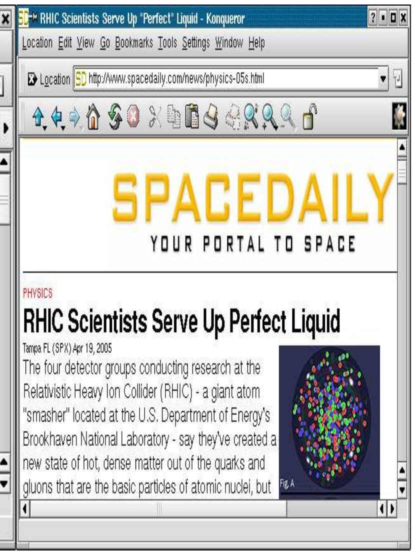
Cover of PRL!!!!



BBC!

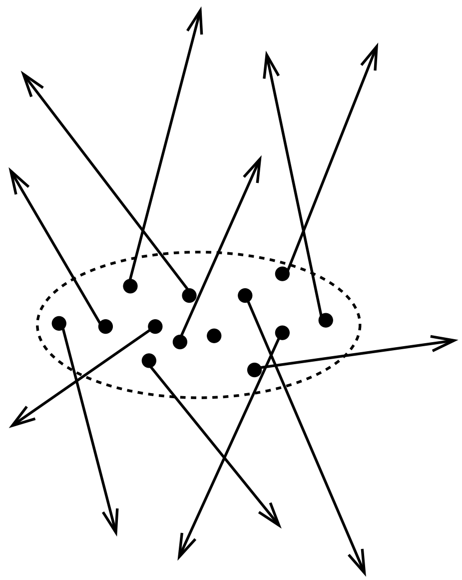


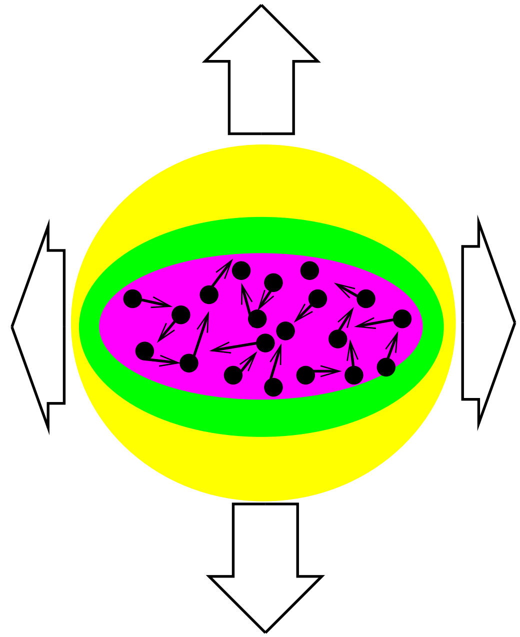
SPACE DAILY!



Heavy ion physicists found the perfect liquid! our field largely redefined to this

A "dust"
Particles ignore each other, their path is independent of initial shape.



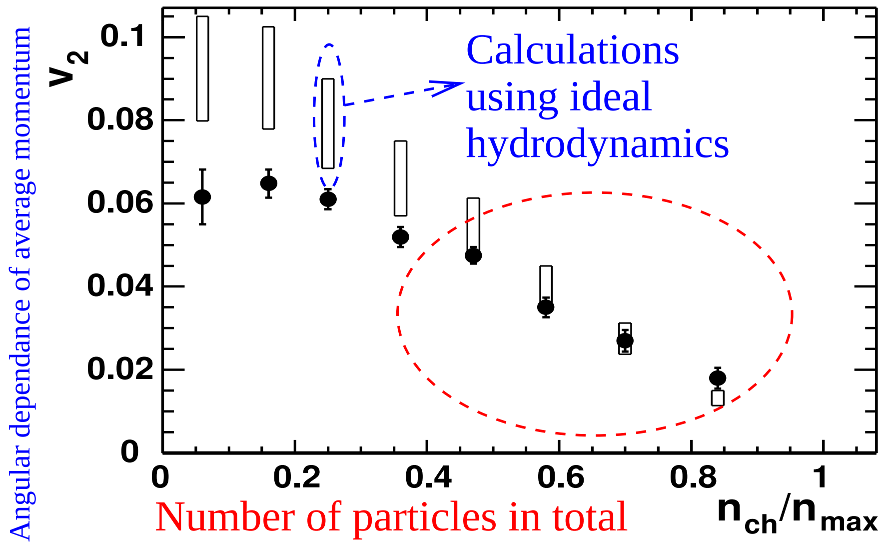


A "fluid"
Particles continuously interact. Expansion determined by density gradient (shape).

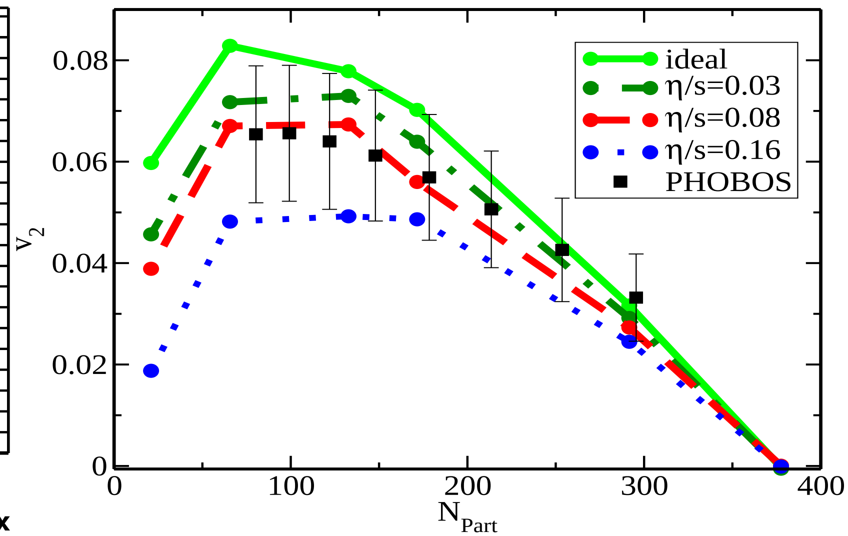
Observable: $\frac{dN}{p_T dp_T dy d\phi} = \frac{dN}{p_T dp_T dy} \left[1 + 2 v_n(p_T, y) \cos\left(n\left(\phi - \phi_0\left(n, p_T, y\right)\right)\right)\right]$

"Collectivity" Same v_n <u>appears</u> in $\forall$ n-particle correlations , $\left\langle \frac{dN}{d\phi_1}\frac{dN}{d\phi_2}... \right\rangle$

P.Kolb and U.Heinz,Nucl.Phys.A702:269,2002.



P.Romatschke,PRL99:172301,2007



Fits ideal hydro , fitted upper limit on viscosity low Spurned a lot of theoretical and numerical/phenomenological development of relativistic hydrodynamics. Restarted the controversy over viscous relativistic hydrodynamics of the 70s

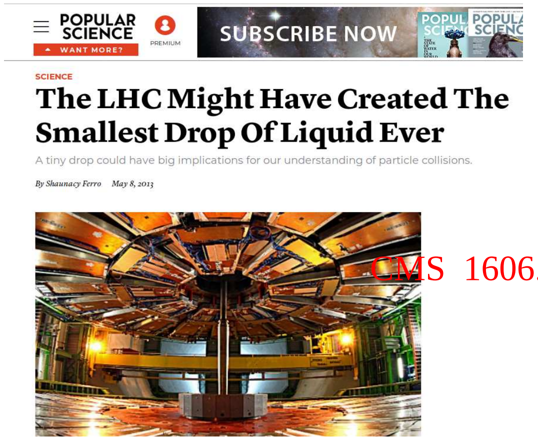
Conventional widsom: hydro EFT of gradients of conserved currents

$$\partial_\mu T^{\mu\nu} = 0; T^{\mu\nu} = \underbrace{T^{\mu\nu}_{eq}}_{Thermal} + \underbrace{\Pi^{\mu\nu}}_{Relax} \equiv T^{\mu\nu} = T^{\mu\nu}_0(e,u) + \eta\mathcal{O}\left(\partial u\right) + \tau\mathcal{O}\left(\partial^2 u\right) + ...$$

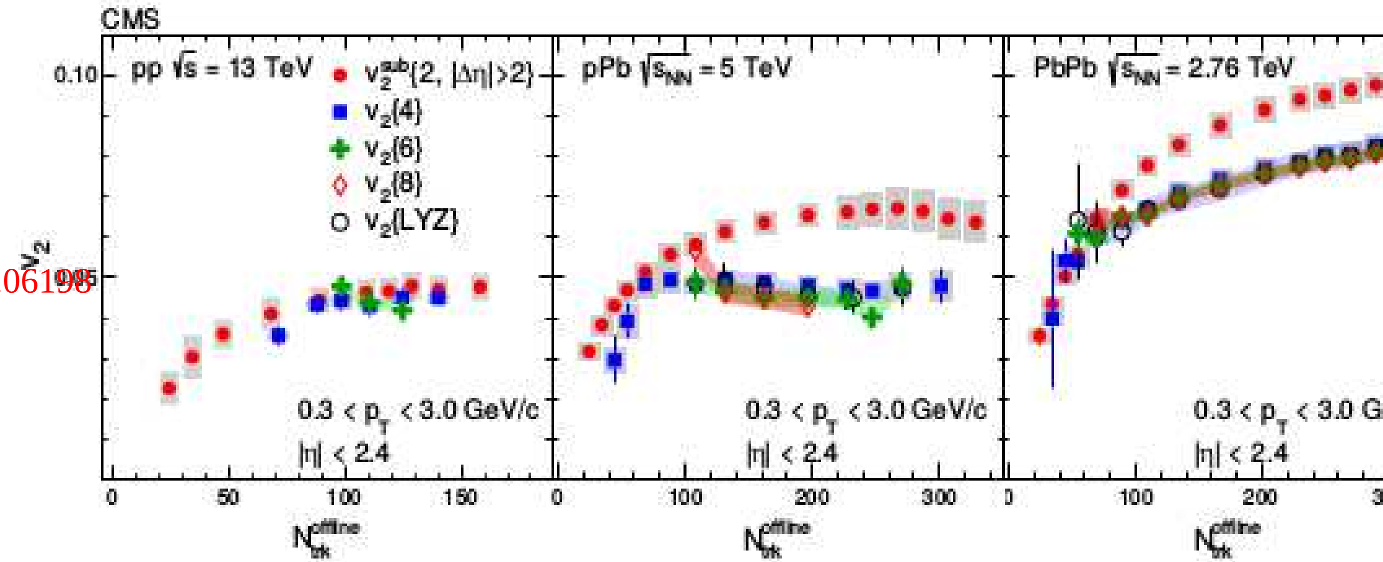
$$\eta = \lim_{k\to 0}\frac{1}{k}\mathrm{Im}\int dx \left\langle \hat{T}_{xy}(x)\hat{T}_{xy}(y)\right\rangle \exp\left[ik(x-y)\right] \quad , \quad \tau \sim \frac{\partial^2}{\partial k^2}\int e^{ikx}\left\langle TT\right\rangle ,$$

This is a <u>classical</u> theory , $\hat{T}_{\mu\nu} \to \langle T_{\mu\nu}\rangle$ Correlators $\langle T_{\mu\nu}(x)...T_{\mu\nu}\rangle$ play role in coefficients, <u>not</u> in EoM (if you know initial conditions, you know the whole evolution!) Kubo formula $w \to 0$ cuts out thermal fluctuations. <u>Implicitly assumed$\ll$</u> mean free path

Both top-down ultimately derived from "microscopic" theories (Boltzmann equation,AdS/CFT), not "bottom up" statistical mechanics ("universality", independent from microscopic physics)!



CMS 1606.06198



1606.06198 (CMS) : When you consider geometry differences and multi-particle cumulants (remove momentum conservation), hydro with $\mathcal{O}(20)$ particles "just as collective" as for 1000. Also cold atom fluids with 10,000 particles $\sim 1mm^3$. Controversial but AFAIK no evidence collectivity goes down with A, N! Little understanding of this in "conventional widsom". What si the smallest possible fluid?

What is hydrodynamics if $N \sim 50$?

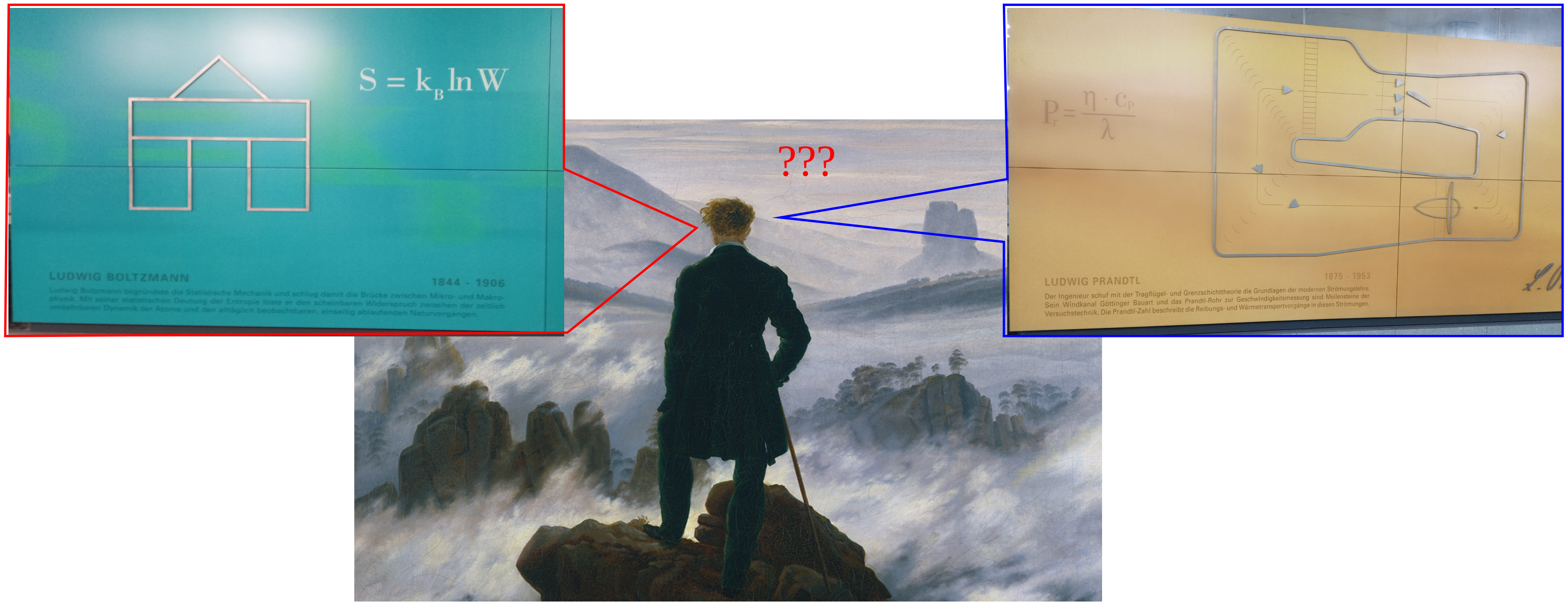
- Ensemble averaging , $\langle F\left(\{x_i\},t\right)\rangle \neq F\left(\{\langle x_i\rangle\},t\right)$
suspect for any non-linear theory. molecular chaos in Boltzmann, Large N_c in AdS/CFT, all assumed . But for $\mathcal{O}\left(50\right)$ particles?!?!

- For water, a cube of length $\eta/(sT)$ has $\mathcal{O}\left(10^9\right)$ molecules,

$$P(N \neq \langle N\rangle) \sim \exp\left[-\langle N\rangle^{-1}\left(N - \langle N\rangle\right)^2\right] \ll 1$$

.

How do microscopic, macroscopic and quantum corrections talk to eac other? EoS is given by $p = T\ln Z$ but $\partial^2 \ln Z/\partial T^2, dP/dV$??
NB: nothing to do with equilibration timescale . Kubo Formulae hide this issue by taking asymptotic limits



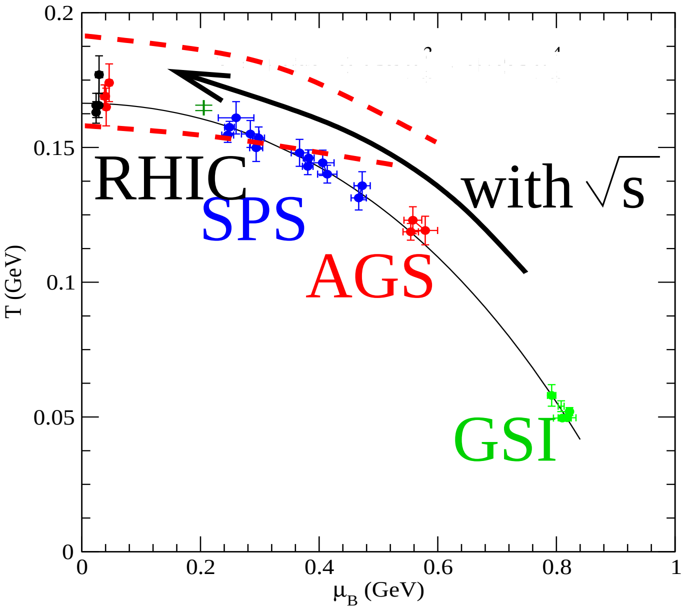
Bottom line: Either hydrodynamics is not the right explanation for these observables (possible! But small/big systems similar!) or we are not understanding something basic about what's behind the hydrodynamics! What do fluctuations do? Just a lower limit to dissipation?
more fundamentally , the relationship between hydrodynamics and statistical mechanics is not as understood as one might think!

Perhaps even related to everyday physics?

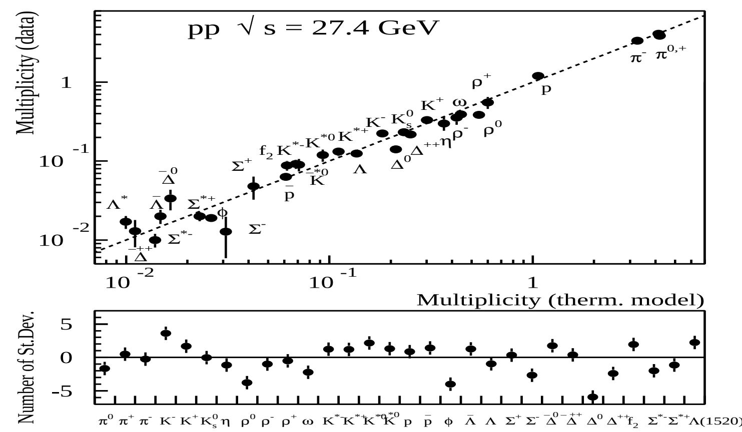
The
Brazil
nut effect

Statistical mechanics in small systems

Smaller
energies
Cleymans
Oeschler
Redlich

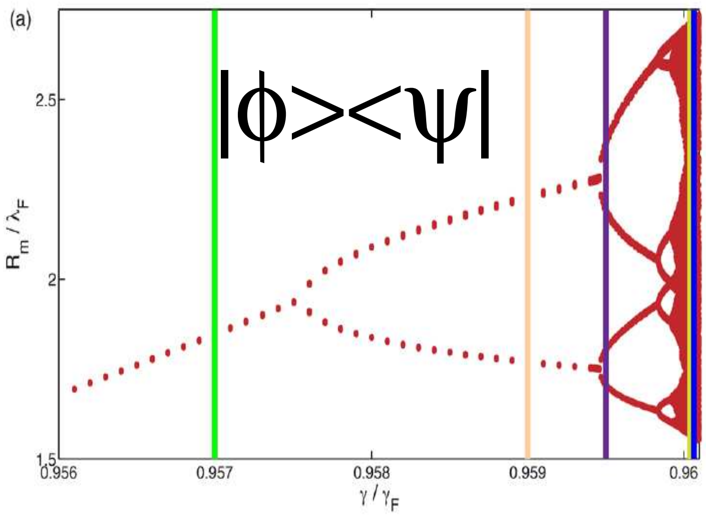


Smaller sizes (p−p,e−e)
Becattini



If you consider chemistry (particle ratios), statistical mechanics does seem to work reasonably well to smallest systems! What is the relationship between this,hydrodynamics and microscopic theory?

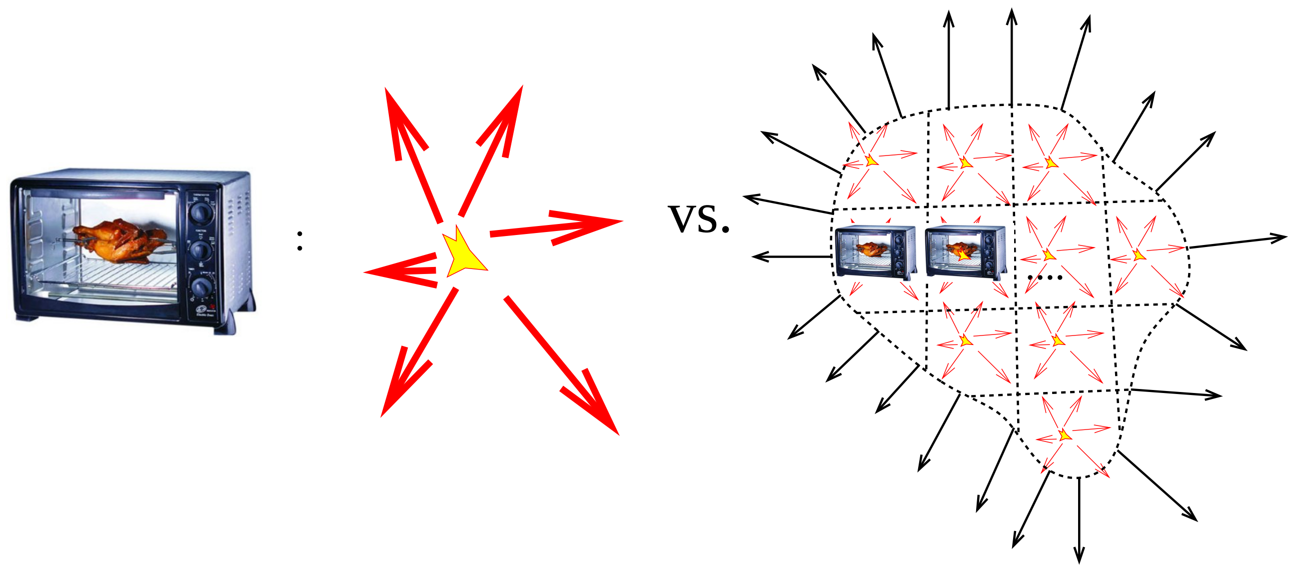
Statistical behavior is actually not surprising



=

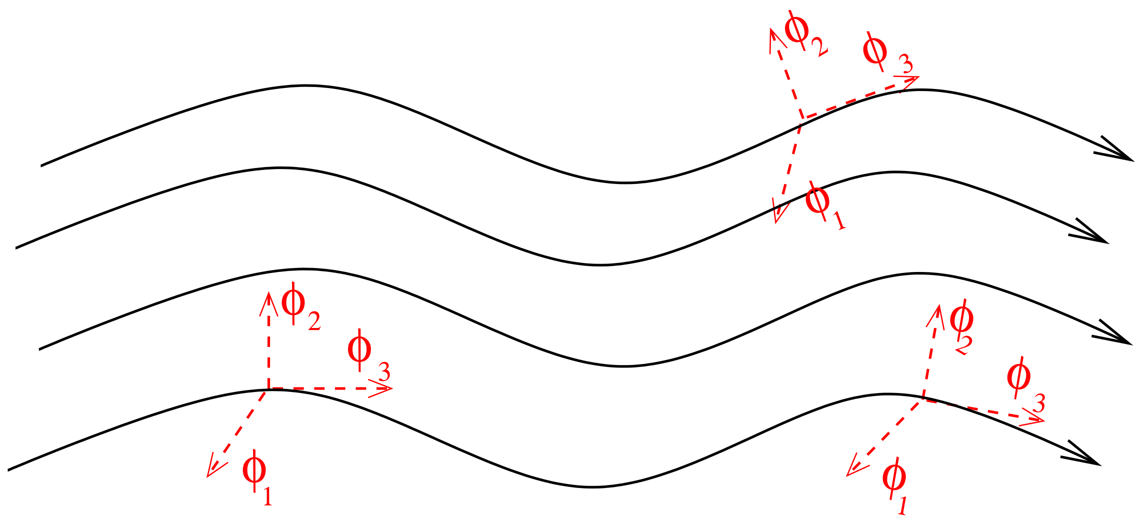
Berry/Bohigas/Eigenstate thermalization hypothesys: $E_{n>>1}$ of quantum systems whose classical correspondent is chaotic have density matrices that look like pseudo-random. If off-diagonal elements oscillate fast or observables simple, indistinguishable from Micro-canonical ensemble!

What we lack...



We need to build a hydrodynamics from such a picture away from the many particle limit So fluctuations are included. Boltzmann,AdS/CFT both assume $s^{-1/3} \ll \eta/(sT)$ Can intuition that fluctuations "only add dissipation" away from thermalization be wrong? actually in Gauge theory the opposite happens! Fluctuations "add to equilibrium"!

Lets set-up EFT around local equilibrium (Nicolis et al,1011.6396 (JHEP))
Continuus mechanics (fluids, solids, jellies,...) is written in terms of 3-coordinates $\phi_I(x^\mu), I = 1...3$ of the position of a fluid cell originally at $\phi_I(t=0, x^i), I = 1...3$. (Lagrangian hydro . NB: no conserved charges)



Translation invariance at Lagrangian level $\leftrightarrow$ Lagrangian can only be a function of $B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J$ Now we have a "continuus material"!

Homogeneity/Isotropy means the Lagrangian can only be a function of $B = \det B^{IJ}, \mathrm{diag} B^{IJ}$

The comoving fluid cell must not see a "preferred" direction $\Leftarrow SO(3)$ invariance

Invariance under Volume-preserving diffeomorphisms means the Lagrangian can only be a function of B (actually $b=\sqrt{B}$)

In all fluids a cell can be infinitesimally deformed

(with this, we have a fluid. If this last requirement is not met, Nicolis et all call this a "Jelly")

Simplified space-like GR 4D local Lorentz invariance becomes local SO(3) invariance Vierbein $g_{\mu\nu} = \eta^{\alpha\beta} e_\mu^\alpha e_\nu^\beta$ is $\frac{\partial x_I^{comoving}}{\partial x_\mu} = \partial_\mu \phi_I$ Killing vector becomes u_μ $\mathcal{L} \sim \sqrt{-g}\,(\Lambda + R + ...)$ becomes $\mathcal{L} \sim F(B) \equiv f(\sqrt{-g})$

Just cosmological constant, expanding fluid $\equiv$ dS space

A few exercises for the bored public Check that L = -F(B) leads to

$$T_{\mu\nu} = (P+\rho)u_\mu u_\nu - P g_{\mu\nu}$$

provided that

$$\rho = F(B)\ , \qquad p = F(B) - 2F'(B)B\ , \qquad u^\mu = \frac{1}{6\sqrt{B}}\epsilon^{\mu\alpha\beta\gamma}\epsilon_{IJK}\,\partial_\alpha\phi^I\partial_\beta\phi^J\partial_\gamma\phi^K$$

(A useful formula is $\frac{db}{d\partial_\mu\phi_I}\partial_\nu\phi_I = u^\mu u^\nu - g^{\mu\nu}$)
Equation of state chosen by specifying $F(b)$. "Ideal": $\Leftrightarrow F(B) \propto b^{2/3}$
b is identified with the entropy and $b\frac{dF(B)}{dB}$ with the microscopic temperature.
u^μ fixed by $u^\mu\partial_\mu\phi^{\forall I} = 0$. Vortices become Noether currents of diffeomorphisms!
This is all really smart, but why?

And chemical potential? Dubovsky et al, 1107.0731

Within Lagrangian field theory a <u>scalar</u> chemical potential is added by adding a $U(1)$ symmetry to system.

$$\phi_I \to \phi_I e^{i\alpha} \quad , \quad L(\phi_I, \alpha) = L(\phi_I, \alpha + y) \quad , \quad J^\mu = \frac{dL}{d\partial_\mu \alpha}$$

generally flow of b and of J not in same direction. Can impose a well-defined u^μ by adding <u>chemical shift symmetry</u>

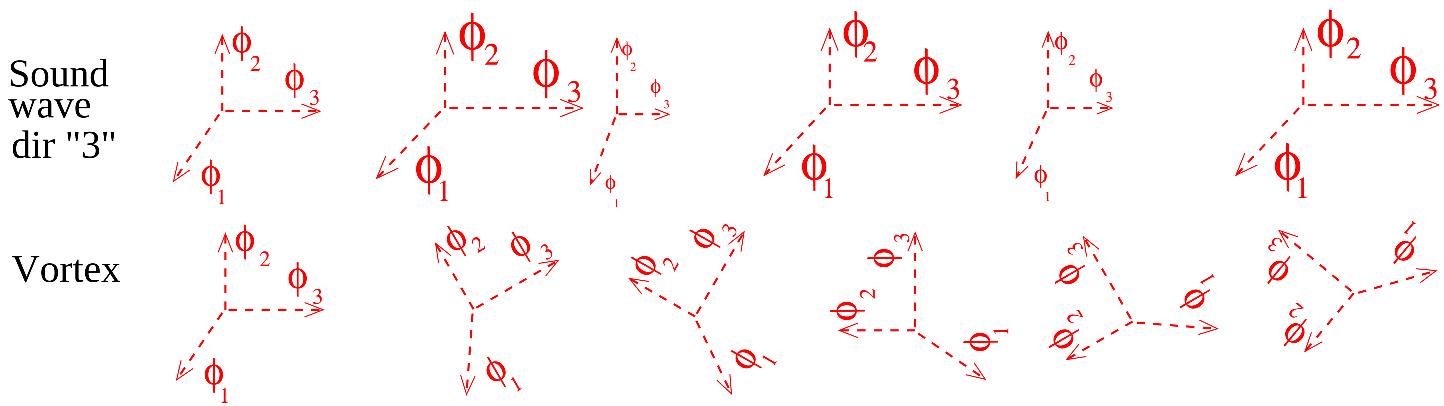
$$L(\phi_I, \alpha) = L(\phi_I, \alpha + y(\phi_I)) \to L = L\left(b, y = u_\mu \partial^\mu \alpha\right)$$

A comparison with the usual thermodynamics gives us

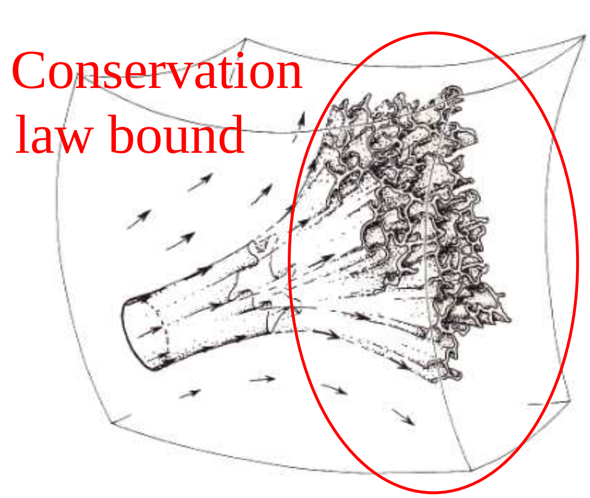
$$\mu = y \quad , \quad n = dF/dy$$

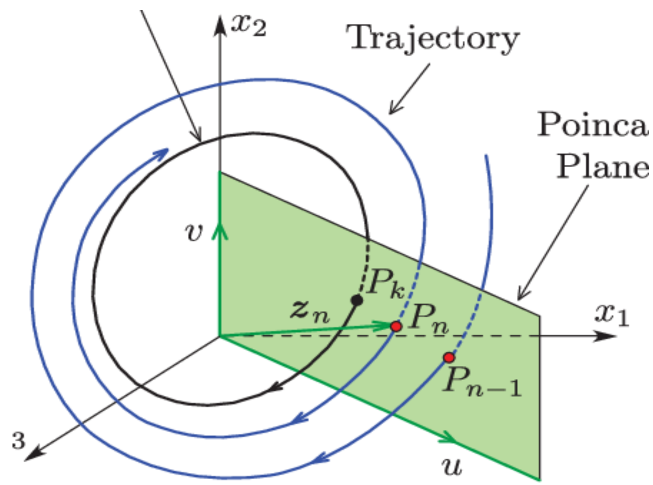
For analytical calculations fluid can be perturbed around a <u>hydrostatic</u> ($\phi_I = \vec{x}$) background

$$\phi_I = \vec{x} + \underbrace{(\vec{\pi}_L)}_{sound} + \underbrace{(\vec{\pi}_T)}_{vortex}$$



Where does statistical mechanics come from? Ergodicity





Classical evolution via Hamilton's equations

$$\dot{x} = \frac{\partial H}{\partial p} \quad , \quad \dot{p} = -\frac{\partial H}{\partial x} \quad , \quad \dot{O} = \{O, H\}$$

"Chaos",conservation laws$\rightarrow$ phase space more "fractal", recurring

"After some time", for <u>any</u> observable <u>ergodic</u> limit applies

$$\int_0^{(large)\ \ T} \dot{O}(p,q)dt = \int P(O(p,q))dqdp$$

where $P(...)$ probability independent of time. This probability can only be given by conservation laws

$$P(O) = \frac{\left(\sum_i O_i\right)\delta^4\left(\sum_i P_i^\mu - P^\mu\right)\delta\left(\sum_i Q_i - Q\right)}{N} \quad , \quad N = \int P(O)dO = 1$$

this is the <u>microcanonicanal ensemble</u>. In <u>thermodynamic limit</u>

$$P(O) \to \delta(O - \langle O \rangle)$$

Hydrodynamics is "thermodynamics in every cell

$$\int_0^{(large)\quad T} \dot{O}(p,q)dt \to \frac{\Delta\phi}{\Delta t}$$

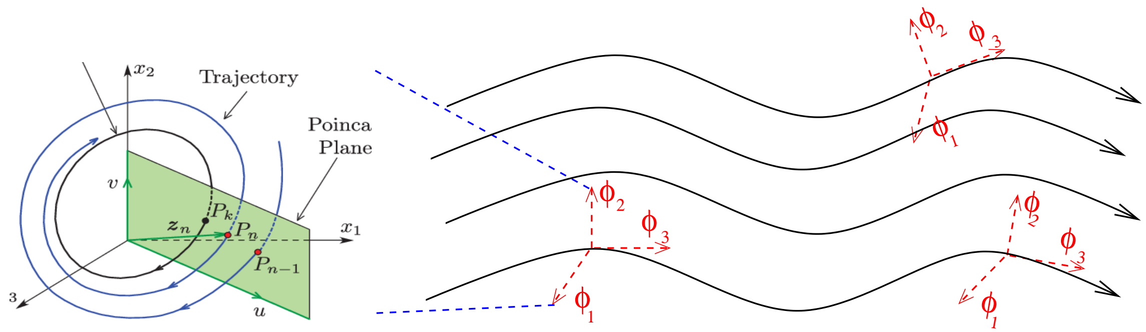
where ϕ is some local observable.

$$\left.\frac{\Delta\phi}{\Delta t}\right|_{t-t'=\Delta} \simeq \frac{1}{d\Omega(Q,E)}\times$$

$$\times \sum \delta^4_{P^\mu, P^\mu_{macro}(t)} \delta_{Q,Q_{macro}(t)} \delta\left(\sum_j^\infty p_j^\mu - P^\mu\right) \delta\left(\sum_j^\infty Q_j - Q\right)$$

Problem: This is not relativistically covariant!

Solution: Foliation!



$$t \to \Sigma_0 \quad , \quad x_\mu \to \Sigma_\mu \quad , \quad \Delta \to \text{“}smooth\text{''} \; \frac{\partial \Sigma_\mu}{\partial \Sigma_\nu}$$

Smooth: $R_{curvature}$ of metric change smaller than “cell size” (New f_{mfp})

$$\frac{\Delta\phi}{\Delta\Sigma_0} = \int P(\phi, \Sigma_\mu) d\Sigma_i \quad , \quad \Sigma_\mu \to \Sigma'_\mu \quad , \quad \frac{\Delta\phi}{\Delta\Sigma'_0} = \frac{\Delta\phi}{\Delta\Sigma_0}$$

What kind of effective lagrangian would enforce

$$\frac{\Delta\phi}{\Delta\Sigma_0} = \int P(\phi, \Sigma_\mu) d\Sigma_i \quad , \quad \frac{\Delta\phi}{\Delta\Sigma_0'} = \frac{\Delta\phi}{\Delta\Sigma_0}$$

with

$$P(...) \sim \delta(\sum_i P_i^\mu - P)\delta(\sum_i Q_i - Q)$$

Now Remember Noether's theorem!

$$p_\mu = \int d^3\Sigma^\nu T_{\mu\nu} \quad , \quad T_{\mu\nu} = \frac{\partial L}{\partial \partial^\mu \phi}\Delta_\nu\phi - g_{\mu\nu}L \quad , \quad \Delta_\nu\phi(x_\mu) = \phi(x_\mu + dx_\nu)$$

$$Q = \int d^3\Sigma^\nu j_\nu \quad , \quad j_\nu = \frac{\partial L}{\partial \partial^\mu \phi}\Delta_\psi\phi \quad , \quad \Delta_\psi\phi = |\phi(x)| e^{i(\psi(x) + \delta\psi(x))}$$

momentum generates spatial translations, conserved charges generate complex rotations!

Space-like foliations decompose

$$d\Sigma_\mu = \epsilon_{\mu\nu\alpha\beta}\frac{\partial\Sigma^\nu}{\partial\Phi_1}\frac{\partial\Sigma^\alpha}{\partial\Phi_2}\frac{\partial\Sigma^\beta}{\partial\Phi_3}d\Phi_1 d\Phi_2 d\Phi_3$$

where the determinant (needed for integrating out $\delta - functions$ is only in the volume part

$$\frac{\partial\Sigma'_\mu}{\partial\Sigma_\nu} = \Lambda_\mu^\nu \det\frac{d\Phi'_I}{d\Phi_J} \quad , \qquad \det\Lambda_\mu^\nu = 1$$

Physically, Λ_μ^ν moves between the frame $d\Sigma^\mu_{rest} = d\Phi_1 d\Phi_2 d\Phi_3 (1, \vec{0})$

so lets try

$$\underbrace{L(\phi)}_{microscopic\ \ DoFs} \simeq L_{eff}(\Phi_{1,2,3})$$

with

$$\frac{\Delta\phi}{\Delta\Sigma_0} = \int P(\phi, \Sigma_\mu) d\Sigma_i \quad , \quad P(\ldots) = \delta(\ldots)\delta(\ldots)$$

the general covariance requirement of $\frac{\Delta\phi}{\Delta\Sigma_0} = \frac{\Delta\phi}{\Delta\Sigma_0'}$ means the invariance of the RHS

$$\frac{d\Omega(dP_\mu', dQ', \Sigma_0')}{d\Omega(dP_\mu, dQ, \Sigma_0)} =$$

$$= \frac{d\Sigma_0'}{d\Sigma_0} \frac{\int da_\mu d\psi \delta^4 \left(d\Sigma^\nu a_\alpha \partial^\alpha \left(\delta_\nu^\mu L\right) - dP^\mu(\Sigma_0)\right) \delta\left(d\Sigma^\mu \psi \partial_\mu L - dQ(\Sigma_0)\right)}{\int da_\mu' d\psi' \delta^4 \left(d\Sigma_\nu' a_\alpha' \partial^\alpha \left(\delta_\nu^\mu L\right) - dP_\mu'(\Sigma_0')\right) \delta\left(d\Sigma_\mu' \psi' \partial^\mu L - dQ'(\Sigma_0')\right)}$$

It is then easy to see,via

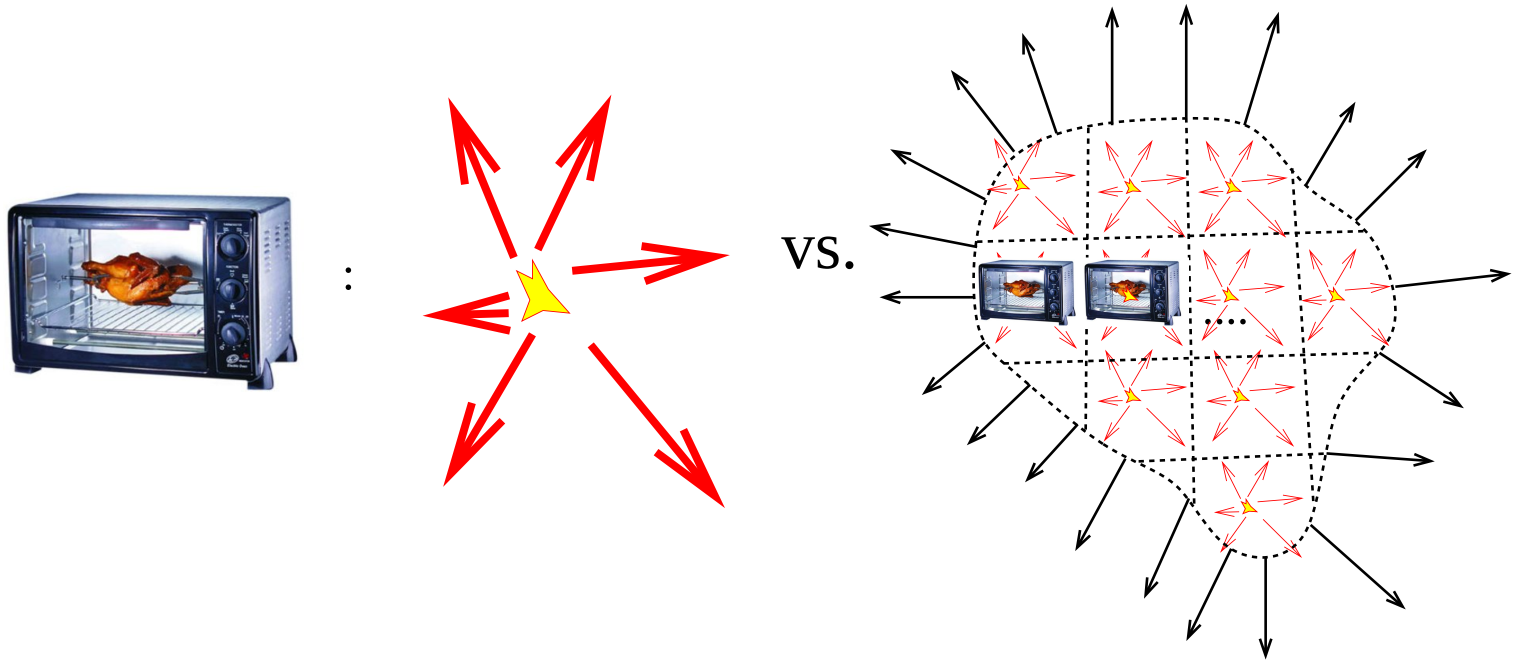
$$\delta((f(x_i))) = \sum_i \frac{\delta(x_i - a_i)}{\underbrace{f'(x_i = a_i)}_{f(a_i)=0}} \quad , \quad \phi'_I = \frac{\partial_\alpha \Sigma'_I}{\partial^\alpha \Sigma^J} \Phi_J \quad , \quad \delta^4(\Sigma_\mu) = \det \left| \frac{\partial \Sigma^\mu}{\partial \Sigma^\nu} \right| \delta^4$$

that for general covariance to hold

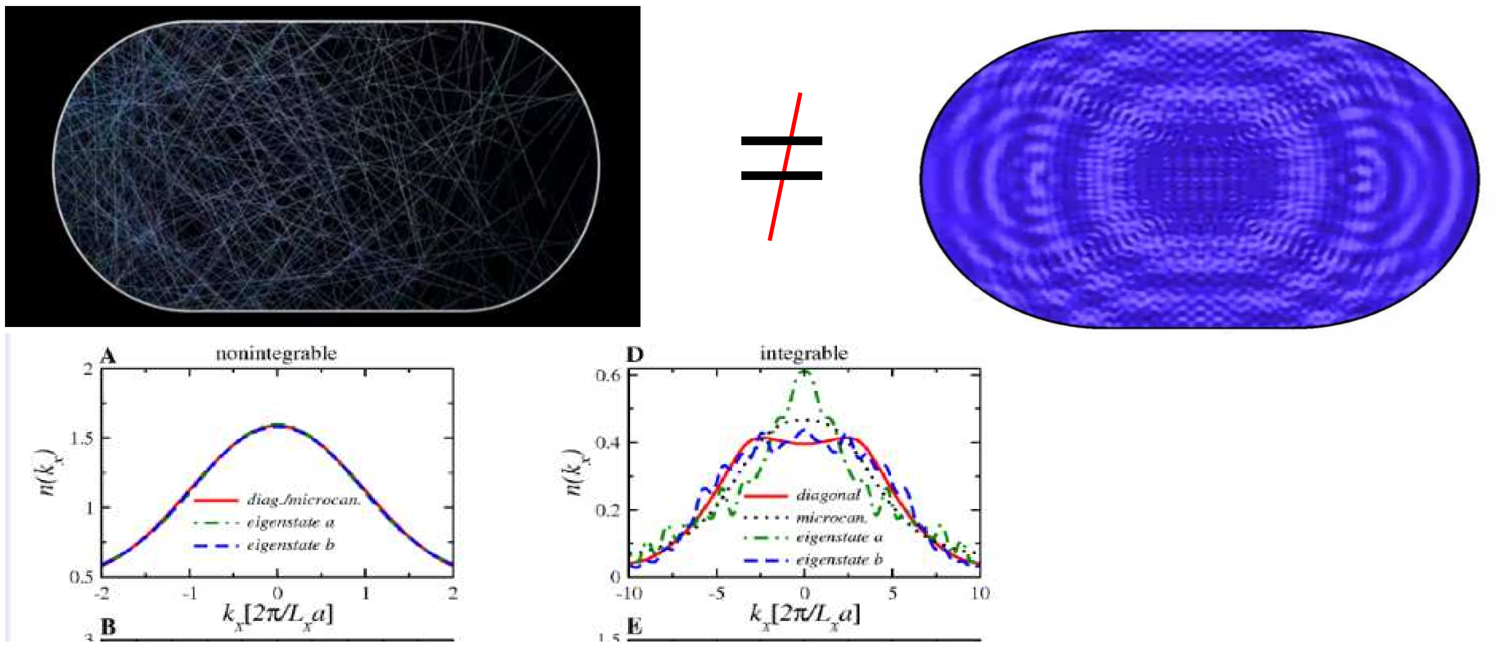
$$L(\Phi_I, \psi) = L(\Phi'_I, \psi') \quad , \quad \det \frac{\partial \phi_I}{\partial \phi_J} = 1 \quad , \quad \psi' = \psi + f(\phi_I)$$

the symmetries of perfect fluid dynamics are equivalent to requiring the ergodic hypothesys to hold for generally covariant causal spacetime foliations!!!!

Classical to quantum F.Becattini, 0901.3643



Berry's conjecture: quantum systems with Chaotic classical counterparts and Above ground state $E_{n \gg 1}$
Density matrix pseudorandom , indistinguishable from microcanonical ensemble. born in equilibrium



M. Rigol, V. Dunjko, and M. Olshanii,Nature 452, 854 (2008)

Quantum billiard balls very different from classical and semi-classical ones! Any "non-integrability" modifies "initial state" which already "looks thermal". All evolution does is randomize phase . Related to divergences in finite temperature QFT? "loop" corrections to transport hard!

Applying the Eigenstate thermalization hypothesis to every cell in every foliation is equivalent to promoting $J_{\mu\nu}, \theta, P, Q$ to functions of x_μ and imposing foliation independence on the "pseudo-randomness" of $\hat{\rho}$.

$$\left.\frac{d\hat{\rho}}{d\Sigma_0}\right|_{\Sigma_0-\Sigma_0'\simeq\Delta} = 0 \quad , \quad \hat{\rho} \simeq \frac{1}{d\Sigma}\hat{\delta}_{E,E'}\hat{\delta}_{Q,Q'}$$

$$\hat{U}^{-1}(x)\hat{\rho}\hat{U}(x) \simeq \hat{\rho} \quad , \quad \hat{U}(x) = \exp\left[i\hat{T}^{\mu\nu}d^3\Sigma_\mu \delta x_\nu\right]\exp\left[i\partial_\alpha\theta d^3\Sigma^\alpha\delta\hat{Q}\right]$$

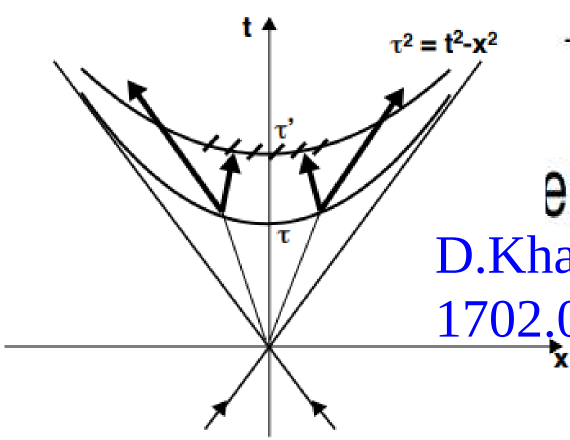
for arbitrary $d^3\Sigma_\mu$. Above derivation follows.

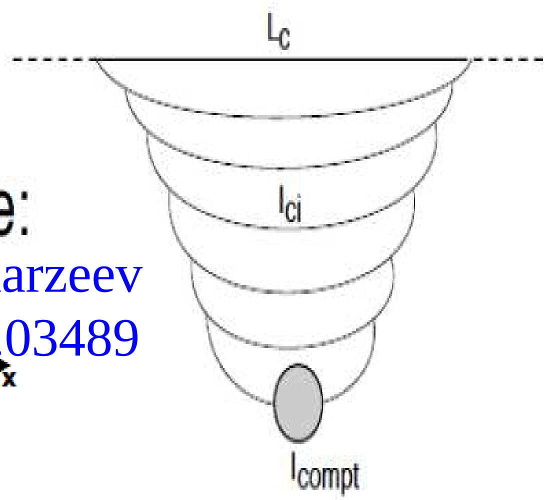
So one expects hydro together with statistical hadronization!

Prospects and why am I here

Lightcone picture Diffeomorphisms in $d\Sigma \to (d\Sigma_+, \phi_1, \phi_2, d\Sigma_-)$

Parton entanglement entropy parton decorrelation in rapidity in the strongly coupled regime as hydro?





e:

D.Kharzeev
1702.03489

Conclusions

A strongly coupled system could be "close the ergodic limit" at every cell even if the number of dofs in each cell is small. Hydro with Gibbsian equilibrium (no transport, Khinchin conditions violated)
Since ergodicity and frame Independence are different concepts, in such a limit, foliation independence should still be valid.

Foliation-independent ergodicity generates ideal hydro "on average" with small numbers of particles. Could be applied for "realistic" level densities on light-cone?

Connection to parton model via entanglement entropy?

What is a gauge theory,exactly?

$$\mathcal{Z} = \int \mathcal{D}A^\mu \exp\left[S[F_{\mu\nu}]\right] \equiv \int \mathcal{D}A_1^\mu \mathcal{D}A_2^\mu \exp\left[S[A_1^\mu]\right]$$

$A^\mu_{1,2}$ can be separated since physics sensitive to derivatives of $\ln \mathcal{Z}$

$$\ln \mathcal{Z} = \Lambda + \ln \mathcal{Z}_G \quad , \quad Z_G = \int \mathcal{D}\mathcal{A}^\mu \delta\left(G(A^\mu)\right) \exp\left[S(A_\mu)\right]$$

Ghosts come from expanding $\delta(...)$ term. In KMS condition/Zubarev

$$Z = \int \mathcal{D}\phi \quad , \quad "S" \rightarrow d\Sigma_\nu \beta_\mu T^{\mu\nu}$$

Multiple $T_{\mu\nu}(\phi) \rightarrow$ Gauge-like configuration . Related to Phase space fluctuations of ϕ

A proposal for a different point of view: Inverse ("Bayesian") attractor

Close to local equilibrium is not on gradient expansion but the approximate applicability of fluctuation-dissipation
These are not automatically the same!

For smaller fluctuating systems many equivalent definitions of $T_0^{\mu\nu}, \Pi^{\mu\nu}$

Different Boltzmannian entropy but all counted as Gibbsian entropy

If many equivalent choices of $\Pi_{\mu\nu}$ likely in one its "small"! Ideal hydro behavior.

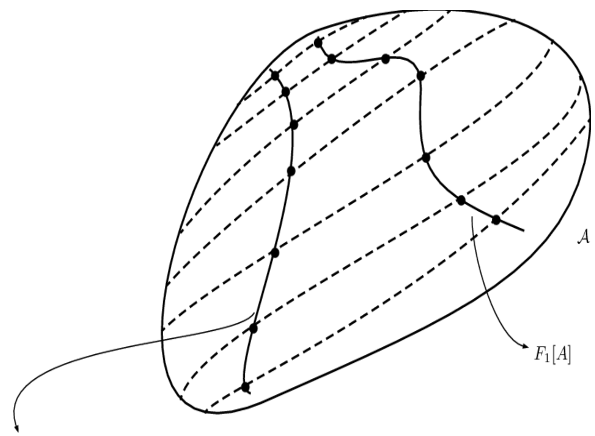
So indeed Ambiguity from fluctuations makes system look like a fluid.

So could fluctuations help thermalize? A key insight is redundances

Some qualitative developments: $T_0^{\mu\nu}, \Pi^{\mu\nu}, u^\mu$ are not actually experimental observables! Only total energy momentum tensor

$$\hat{T}^{\mu\nu} = \hat{T}_0^{\mu\nu} + \hat{\Pi}^{\mu\nu}$$

and its correlators are! Changing $d\Sigma_\mu$ in Zubarev $\equiv$ changing $\Pi^{\mu\nu}, T_0^{\mu\nu}$!



Analogy to choosing a gauge in gauge theory?

This is relevant for current hydrodynamic research
Causal relativistic hydrodynamics still contentious, with many definitions

Israel-Stewart Relaxing $\Pi_{\mu\nu}$.
Causal, but up to 9 additional DoFs (not counting conserved charges), blow-up possible (M.Disconzi, 2008.03841). $\Pi_{\mu\nu}$ "evolving" microstates!

BDNK,earlier Hiscock,Lindblom,Geroch,... $\Pi_{\mu\nu} \sim \partial u$ At a price

- Arbitrary (up to causality constaints) u_μ .
- Entropy "temporarily decreases" with perturbations (Gavassino et al, arXiv:2006.09843). Kovtun in 2112.14042 derives BDNK from a truncation of the Boltzmann equation generally violating the H-theorem

For phenomenology because of conservation laws "any" $\partial_\mu T^{\mu\nu}$ "can be integrated" but lack of link with equilibration and multiple definitions of "near-equilibrium" problematic.

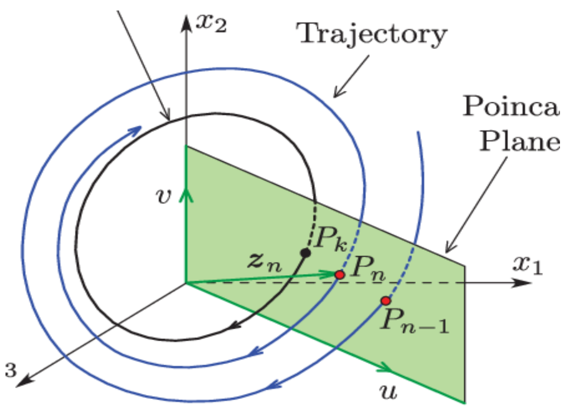
If you care about statistical mechanics, price is steep!
"special" time foliation from ergodic hypothesis/Poncaire cycles!

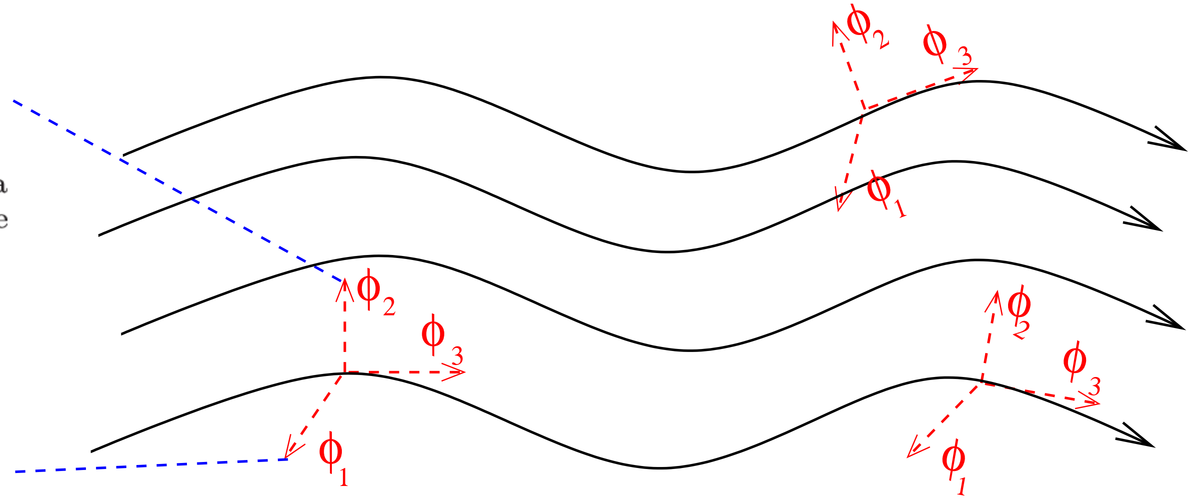
But entropy decrease physically reasonable from Zubarev definition. But not from H-theorem!

Fluctuations come with redundances in $T_0^{\mu\nu}, \Pi^{\mu\nu}$

Could these definitions of u_μ be just "Gauge" choices?

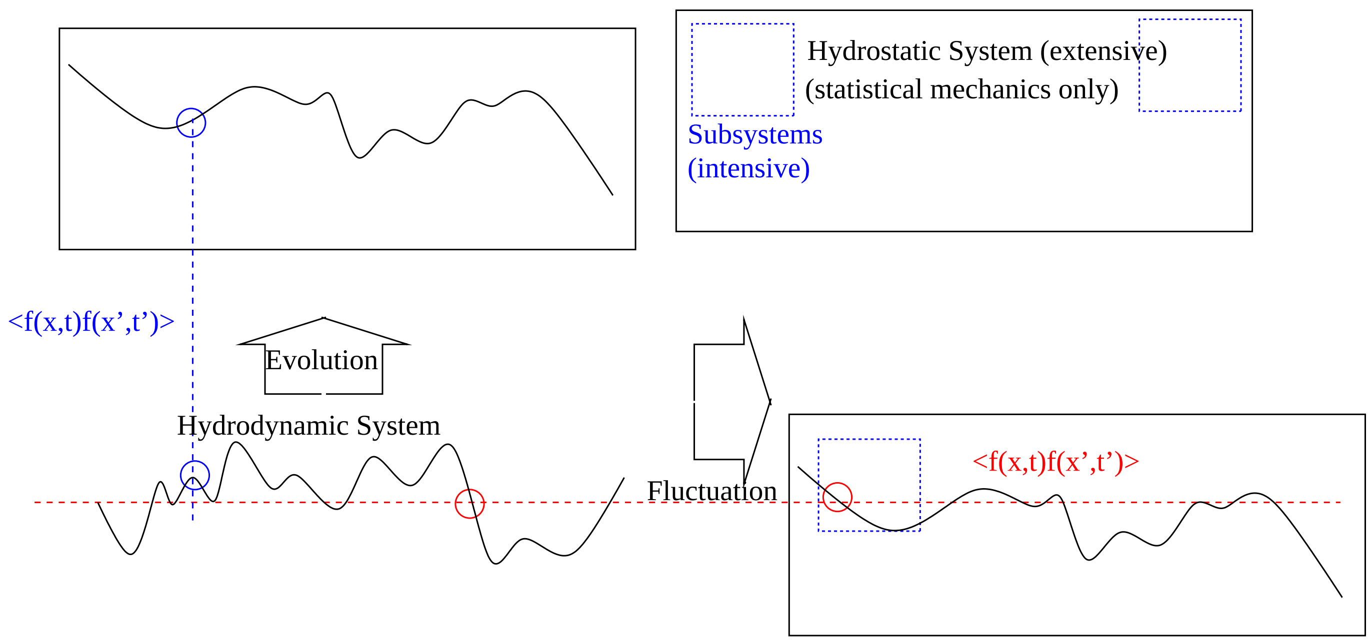
How to make physics fully "gauge"-invariant? Ergodicity/Poncaire cycles meet relativity slightly away from equilibrium!



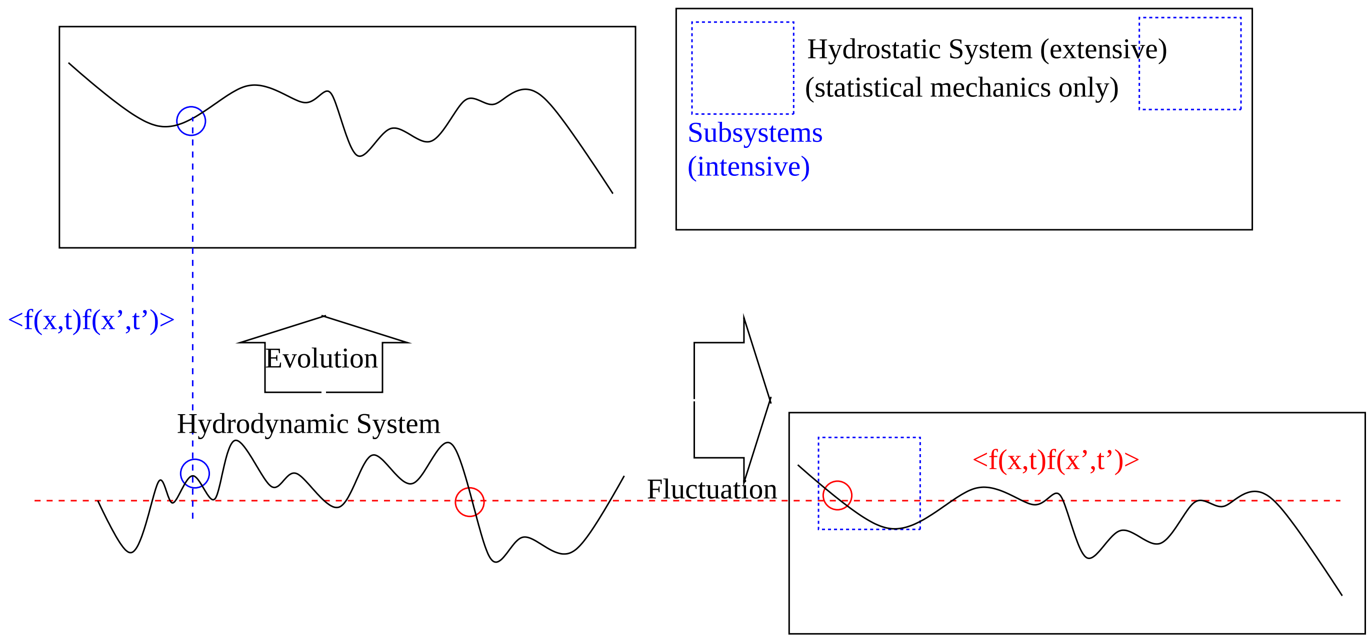


Gibbs entropy level+relativity : Lack of equilibrium is equivalent to "loss of phase" of Poncaire cycles. one can see a slightly out of equilibrium cell <u>either</u> as a "mismatched u_μ" (fluctuation) or as lack of genuine equilibrium (dissipation)

How to make physics fully "gauge"-invariant?



Fluctuation-dissipation at the cell level could do it! We don't know if a "step" is fluctuation ($T_0^{\mu\nu}$ or evolution ($\Pi_{\mu\nu}$)-driven!



But in hydro $T_0^{\mu\nu}, \Pi_{\mu\nu}$ treated very differently! "Sound-wave"

$u \sim \exp[ik_\mu x^\mu]$ or "non-hydrodynamic Israel-Stewart mode?"

$D\Pi_{\mu\nu} + \Pi_{\mu\nu} = \partial u$

Only in EFT $1/T \ll l_{mfp}$ they are truly different!

Infinitesimal transformation $dM_{\mu\nu}$ such that $dM_{\mu\nu}(x)\frac{\delta \ln \mathcal{Z}_E[\beta_\mu]}{dg^{\alpha\mu}(x)} = 0$

Change in microscopic fluctuation $\ln \mathcal{Z} \rightarrow \ln \mathcal{Z} + d\ln \mathcal{Z}$

$$d\ln \mathcal{Z} = \sum_{N=0}^{\infty} \int \prod_{j=1}^{N} d^4 p_j \delta\left(E_N(p_1, ...p_j) - \sum_j p_j^0 \right) \sqrt{|dM|} \exp\left(-\frac{dM_{0\mu} p^\mu}{T} \right)$$

Change in macroscopic dissipative term

$$\Pi_{\mu\nu} \rightarrow \Pi_{\alpha\gamma} \left(g_\mu^\alpha g_\nu^\gamma - g_\mu^\alpha dM_\nu^\gamma - g_\nu^\gamma dM_\mu^\alpha \right) \quad , \quad u_\mu \rightarrow u_\alpha \left(g_\mu^\alpha - dM_\mu^\alpha \right)$$

For $1/T \ll l_{mfp}$ probability$\rightarrow 0$, $1/T \sim l_{mfp}$ many "similar" probabilities!

The “gauge-symmetry” in practice

Generally $dM_{\mu\nu} = \Lambda^{-1}_{\alpha\mu} dU^{\alpha\beta} \Lambda_{\beta\mu}$

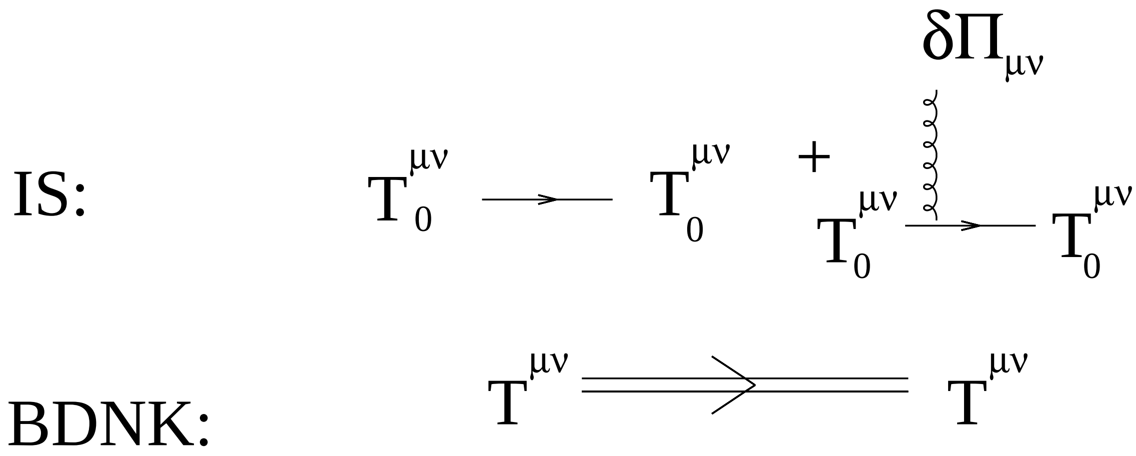
$$d\left[\ln \Pi_{\alpha\beta}\right] \Lambda^{\alpha\mu} \left(\Lambda^{\beta\nu}\right)^{-1} = \eta^{\mu\nu} d\mathcal{A} + \sum_{I=1,3} \left(d\alpha_I \hat{J}_I^{\mu\nu} + d\beta_I \hat{K}_I^{\mu\nu}\right)$$

$$K_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, K_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, K_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

which move components from $\Pi_{\mu\nu}$ to Q_μ as well as $K_{1,2,3}$

$$J_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, J_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, J_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

An example... bulk viscosity



$$e_{IS} \to e + (e+p)\tau \frac{\dot{e}}{e+p} + \left((e+p)\tau + \frac{c_V}{s}\zeta \right) \partial_\mu u^\mu \quad , \quad p_{IS} \to p + \Pi$$

Considering c_V controls energy fluctuations, shift from IS to BDNK equivalent to relabeling Π dynamics as interaction with a fluctuation-generated sound wave.

Characterizing these gauge redundancies

Grossi,Floerchinger, 2102.11098 (PRD) Let us define a J co-moving with u_μ and use the "exact" (before coarse-graining) partition function to build

$$\Gamma(\phi) = \mathrm{Sup_J}\left(\int J(x)\phi(x) - i \ln \mathcal{Z}[J] \right)$$

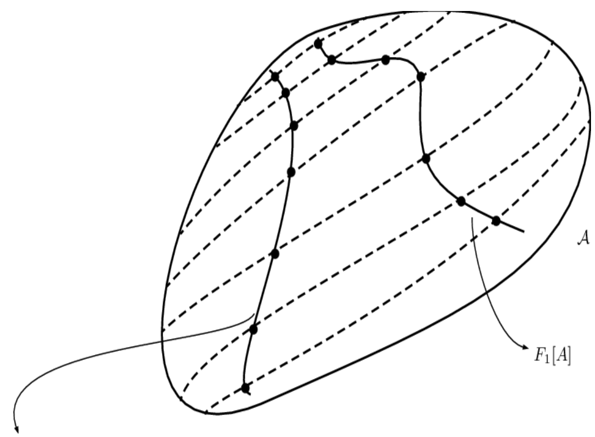
$u_\mu \to u'_\mu$ non-inertial and does not change $\langle T_{\mu\nu} \rangle$, so one can define

$$J_{\mu\nu\gamma} = \frac{1}{\sqrt{g}} \frac{\delta \ln \mathcal{Z}[J']}{\delta \Gamma^{\alpha\nu\gamma}} \quad , \quad D_\mu J^{\mu\nu\gamma} = 0$$

Setting the gauge at the level of the microscopic approximately thermalized partition function equivalent adding auxiliary field $D_\mu M_{\alpha\beta} = 0$ to

$$\mathcal{Z}[J_{\alpha\beta\gamma}] = \int \mathcal{D}\phi \mathcal{D} M_{\alpha\beta} \exp\left[\int det[M] d^4x \mathcal{L}\left(\phi, \partial_\mu + \Gamma ...\right) + \int d\Sigma^\gamma M^{\alpha\beta} J_{\alpha\beta\gamma} \right]$$

Cool but what about thermalization in small systems?
Initial and final state described by many equivalent trajectories



One of them could be close to an ideal-looking one. “reverse” attractor Few particles with strong interaction (Eigenstate thermalization?) correspond to many hydro like-configurations $\{u_\mu, \Pi_{\mu\nu}\}$ with fluctuations , within same Gibbs entropy class. some closer to ideal? No symmetries necessary!

Irrelevant in everyday liquids since $l_{mfp} \gg 1/T$ or AdS/CFT since $N_c \ll \infty$ but perhaps not for QGP!

Every statistical theory needs a "state space" and an "evolution dynamics"
The ingredients

State space:Zubarev hydrodynamics Mixes micro and macro DoFs

Dynamics: Crooks fluctuation theorem provides the dynamics via a definition of $\Pi_{\mu\nu}$ from fluctuations

$\hat{T}^{\mu\nu}$ is an operator, so any decomposition, such as $\hat{T}_0^{\mu\nu} + \hat{\Pi}^{\mu\nu}$ must be too!

Zubarev partition function for local equilibrium: think of Eigenstate thermalization...

Let us generalize the GC ensemble to a co-moving frame $E/T \to \beta_\mu T^\mu_\nu$

$$\hat{\rho}(T_0^{\mu\nu}(x), \Sigma_\mu, \beta_\mu) = \frac{1}{Z(\Sigma_\mu, \beta_\mu)} \exp\left[- \int_{\Sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}_0^{\mu\nu} \right]$$

Z is a partition function with a field of Lagrange multiplies β_μ , with microscopic and quantum fluctuations included.

Effective action from $\ln[Z]$. Correction to Lagrangian picture?

All normalizations diverge but hey, it's QFT! (Later we resolve this!)

This is perfect global equilibrium. What about imperfect local? Two vectors, $d\Sigma_\mu u_\mu T_0^{\mu\nu}$ $d\Sigma_\mu$ foliation. We can coarse-grain and gradient expand, but Kubo already proven ,can we do better?

An operator formulation $\hat{T}^{\mu\nu} = \hat{T}_0^{\mu\nu} + \hat{\Pi}_{\mu\nu}$
and $\hat{T}_0^{\mu\nu}$ truly in equilibrium! Each microscopic particle "does not know" if it "belongs" to $\hat{T}_0^{\mu\nu}, \hat{\Pi}_{\mu\nu}$

$$\hat{\rho}(T_0^{\mu\nu}(x), \Sigma_\mu, \beta_\mu) = \frac{1}{Z(\Sigma_\mu, \beta_\mu)} \exp\left[- \int_{\Sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}_0^{\mu\nu} \right]$$

describes all cumulants and probabilities

$$\langle T_0^{\mu\nu}(x_1) T_0^{\mu\nu}(x_2) ... T_0^{\mu\nu}(x_n) \rangle = \prod_i \frac{\delta^n}{\delta \beta_\mu(x_i)} \ln Z$$

Equilibrium at "probabilistic" level and KMS Condition obeyed by "part of density matrix" in equilibrium, "expand" around that! An operator constrained by KMS condition is still an operator! $\equiv$ time dependence in interaction picture

Entropy/Deviations from equilibrium

$$n^\nu \partial_\nu \left(s u^\mu\right) = n^\mu \frac{\Pi^{\alpha\beta}}{T} \partial_\alpha \beta_\beta \quad , \quad \geq 0$$

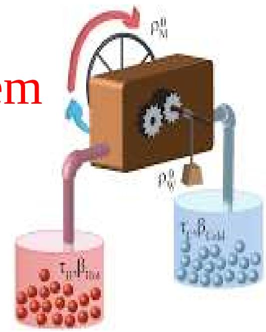
- If n_μ arbitrary cannot be true for "any" choice
- 2nd law is true for "averages" anyways, <u>sometimes</u> entropy can decrease

We need a fluctuating formulation!

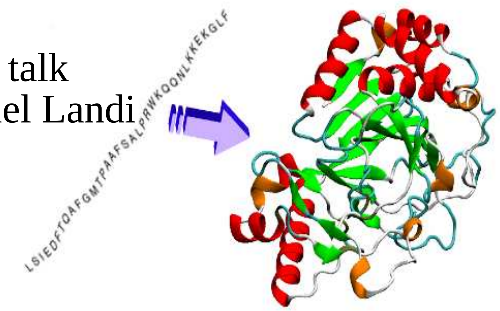
- "Statistical" (probability depends on "local microstates")
- Dynamics with fluctuations, time evolution of β_μ distribution

Crooks fluctuation theorem

$P(W)/P(-W)=e^{\Delta s}$



From talk
Gabriel Landi



Relates fluctuations, entropy in small fluctuating systems (Nano,proteins)

P(W) Probability system doing work in its usual thermal evolution

P(-W) Probability of the same system "running in reverse" and decreasing entropy due to a thermal fluctuation

ΔS Entropy produced by $P(W)$

Valid far from equilibrium,proven for non-Boltzmannian processes

How is Crooks theorem useful for what we did? Guarnieri et al, arXiv:1901.10428 (PRX) derive Thermodynamic uncertainity relations from

$$\hat{\rho}_{ness} \simeq \hat{\rho}_{les}(\lambda) e^{\hat{\Sigma}} \frac{Z_{les}}{Z_{ness}} \quad , \quad \hat{\rho}_{les} = \frac{1}{Z_{les}} \exp\left[-\frac{\hat{H}}{T} \right]$$

$\hat{\rho}_{les}$ is Zubarev operator while Σ is calculated with a Kubo-like formula

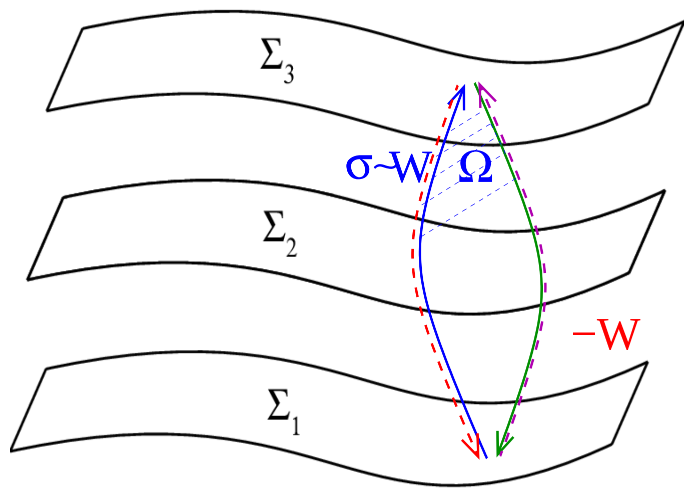
$$\hat{\Sigma} = \delta_\beta \Delta \hat{H}_+ \quad , \quad \hat{H}_+ = \lim_{\epsilon \to 0^+} \epsilon \int dt e^{\epsilon t} e^{-\hat{H} t} \Delta \hat{H} e^{\hat{H} t}$$

Relies on

$$\lim_{w \to 0} \left\langle \left[\hat{\Sigma}, \hat{H} \right] \right\rangle \to 0 \equiv \lim_{t \to \infty} \left\langle \left[\hat{\Sigma(t)}, \hat{H}(0) \right] \right\rangle \to 0$$

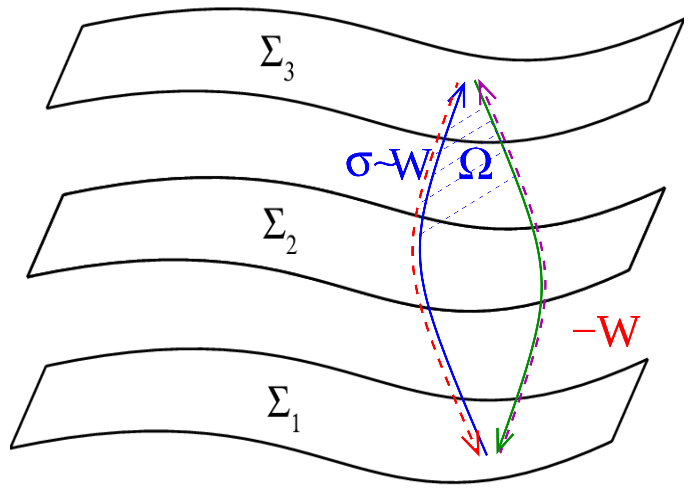
This "infinite" is "small" w.r.t. hydro gradients. $\equiv$ Markovian as in Hydro with $l_{mfp} \to \partial$ but with operators$\to$ carries all fluctuations with it!

Applying Crooks theorem to Zubarev hydrodynamics: Stokes theorem



$$-\int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu} \beta_\nu \right) = -\int_{\Sigma(\tau')} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu} \beta_\nu \right) + \int_\Omega \mathrm{d}\Omega \left(\widehat{T}^{\mu\nu} \nabla_\mu \beta_\nu \right),$$

true for “any” fluctuating configuration.

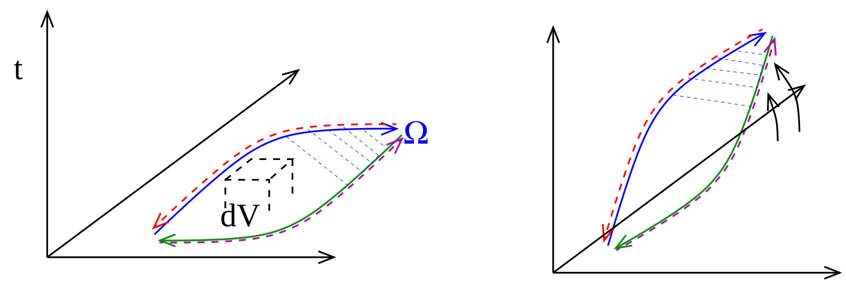


Let us now invert one foliation so it goes "backwards in time" assuming Crooks theorem means

$$\frac{\exp\left[-\int_{\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu}\right]}{\exp\left[-\int_{-\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu}\right]} = \exp\left[\frac{1}{2}\int_\Omega d\Omega_\mu^\mu \left[\frac{\hat{\Pi}^{\alpha\beta}}{T}\right]\partial_\beta \beta_\alpha\right]$$

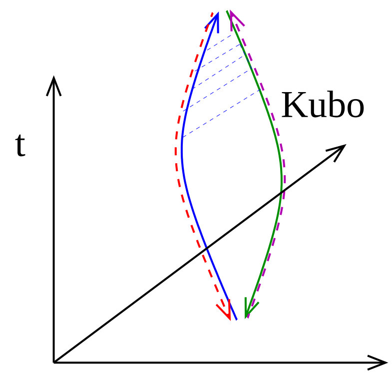
Small loop limit $\left\langle \exp\left[\oint d\Sigma_\mu \omega^{\mu\nu}\beta^\alpha \hat{T}_{\alpha\nu}\right]\right\rangle = \left\langle \exp\left[\int \frac{1}{2} d\Sigma_\mu \beta^\mu \hat{\Pi}^{\alpha\beta}\partial_\alpha \beta_\beta\right]\right\rangle$
A non-perturbative operator equation,divergences cancel out...

$$\left.\frac{\hat{\Pi}^{\mu\nu}}{T}\right|_\sigma = \left(\frac{1}{\partial_\mu \beta_\nu}\right)\frac{\delta}{\delta\sigma}\left[\int_{\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu} - \int_{-\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu}\right]$$



A sanity check: For a an equilibrium spacelike $d\Sigma_\mu = (dV, \vec{0})$ (left-panel) we recover Boltzmann's $\Pi^{\mu\nu} \Rightarrow \Delta S = \frac{dQ}{T} = \ln\left(\frac{N_1}{N_2}\right)$, for an analytically continued "tilted" panel, Kubo's formula

A sanity check



When $\eta \to 0$ *and* $s^{-1/3} \to 0$ (the first two terms in the hierarchy), Crooks fluctuation theorem gives $P(W) \to 1$ $P(-W) \to 0$ $\Delta S \to \infty$ so Crooks theorem reduces to δ-functions of the entropy current

$$\delta\left(d\Sigma_\mu\left(s u^\mu\right)\right) \Rightarrow n^\mu \partial_\mu\left(s u^\mu\right) = 0$$

We therefore recover conservation equations for the entropy current, a.k.a. ideal hydro

A numerical formulation

Define a field β_μ field and n_μ

Generate an ensemble of

$$\ln Z|_{t+dt} = \int \mathcal{D}g_{\mu\nu}(x) T^{\mu\nu}|_{t+dt} \quad , \quad \beta_\mu|_{t+dt} = \frac{\delta \ln Z|_{t+dt}}{\delta T_{\mu\nu}} n_\nu$$

According to a Metropolis algorithm ran via Crooks theorem

Reconstruct the new β and $\Pi_{\mu\nu}$. The Ward identity will make sure $\beta_\mu \beta^\mu = -1/T^2$

Computationally intensive (an ensemble at every timestep), but who knows?

Conclusions

- Linking hydrodynamics to statistical mechanics is still an open problem
  Only top-down models (Boltzmann,AdS/CFT) rather than bottom-up theory
  Is hydro universal? what are its limits of applicability? still open question

  The observation of hydro-like behavior in small systems liable to fluctuations makes this explicit!

- Crooks fluctuation theorem could provide such a link!

- redundances play crucial role in fluctuations, could mean small systems achieve "thermalization" quicker! inverse attractor!

SPARE SLIDES

PS: transfer of micro to macro DoFs experimentally proven!

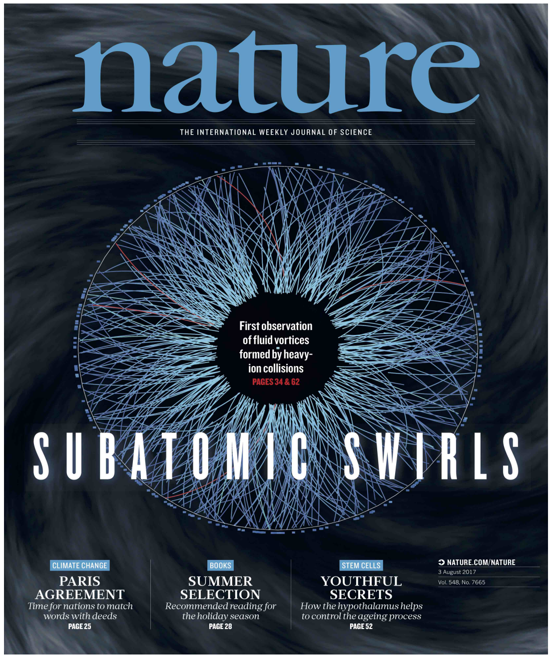
STAR collaboration

1701.06657

NATURE

August 2017

Polarization by vorticity in heavy ion collisions

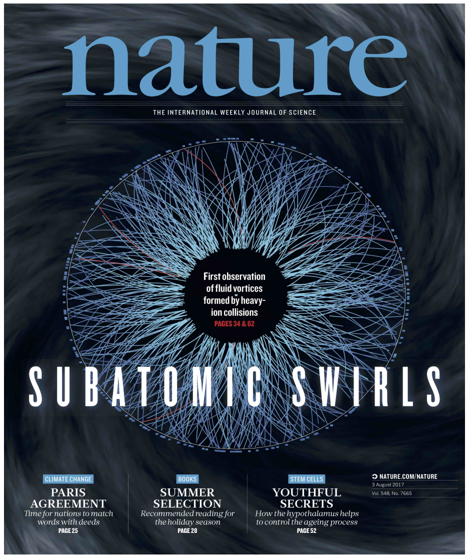


Could give new talk about this, but will mention hydro with spin not developed and a lot of conceptual debates Pseudo-gauge dependence if both spin and angular momentum present in fluid? Gauge symmetry "ghosts"? GT,1810.12468 (EPJA) . redundances?

STAR
collaboration
1701.06657

NATURE
August 2017

Polarization by vorticity
in heavy ion collisions



Pseudo-gauge symmetries physical interpretation: T.Brauner, 1910.12224

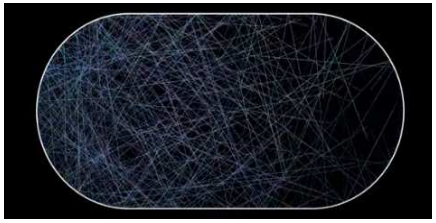
$$x^\mu \to x^\mu + \epsilon \zeta^\mu(x) \quad , \quad \psi_a \to \psi_a + \epsilon \psi'_a \to \mathcal{L} \to \mathcal{L}$$

$\ln \mathcal{Z}$ Invariant, but $\langle O \rangle$ generally is not. Spin $\leftrightarrow$ fluctuation, need equivalent of DSE equations! $D \langle O \rangle = 0 \to D \langle O \rangle = \langle O_I O_J \rangle$

Statistical mechanics: This is a system in global equilibrium, described by a partition function $Z(T, V, \mu)$, whose derivatives give expectation values $\langle E \rangle$,fluctuations $\langle (\Delta E)^2 \rangle$ etc. in terms of conserved charges. All microstates equally likely, which leads to preferred macrostates!

Fluid dynamics: This is the state of a field in local equilibrium which can be perturbed in an infinity of ways. The perturbations will then interact and dissipate according to the Euler/N-S equations. many issues connecting to Stat.Mech. Wild weak solutions, millenium problem!

The problem with general "transport thinking"

 $=$  ?

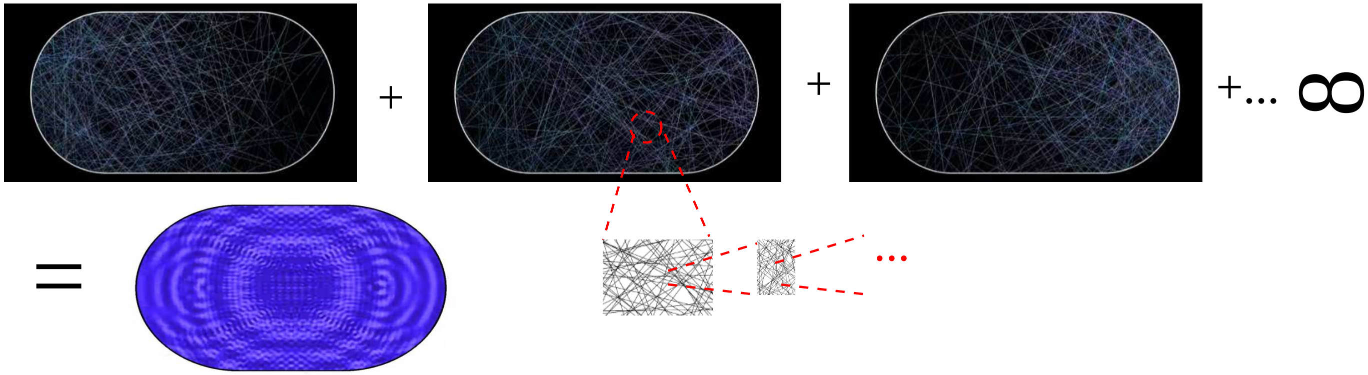
Let's solve the simplest transport equation possible: Free particles

$$\frac{p^\mu}{m}\partial_\mu f(x,p) = 0 \rightarrow f(x,p) = f\left(x_0 + \frac{p}{m}t, p\right)$$

<u>obvious</u> solution is just to propagate
What is <u>weird</u> is that "hydro-like" solution possible too (eg vortices)!

$$f(x,p) \sim \exp\left[-\beta_\mu p^\mu\right] \quad , \quad \partial_\mu\beta_\nu + \partial_\nu\beta_\mu = 0$$

But obviously unphysical, no force! What's up?



This paradox is resolved by remembering that $f(x,p)$ is defined in an ensemble average limit where the number of particles is not just "large" but uncountable . curvature from continuity!

BUt this suggests Boltzmann equation disconnected from any finite number of particles!

What if $e^{-\beta_\mu p^\mu}$ used to sample strongly coupled particles in "many finite events"? Thermal fluctuations,Vlasov correlations and Boltzmann scattering "mix these words". Many ways to mix,some wrong! What is appropriate?

How "different events" correlated is crucial
Villani , `https://www.youtube.com/watch?v=ZRPT1Hzze44`

Vlasov equation contains all <u>classical</u> correlations. Relativistically numer of particles varies in each event but "evolves" deterministically. but instability-ridden, "filaments", cascade in scales.
$N_{DOF} \to \infty$ invalidates KAM theorem stability

Boltzmann equation "Semi-Classical UV-completion" ov Vlasov equation, first term in BBGK hyerarchy, written in terms of Wigner functions.

Infinitely unstable jerks on infinitely small scales Random scattering Statistical behavior emerges from <u>both</u> instabilities (chaos, Poncaire cycles) <u>and</u> scattering (H-theorem) but interplay non-trivial. Strong coupling away from molecular chaos not understood!

There is more to hydro than the Knudsen number

Power counting:

B.Betz,D.Henkel,D.Risc

0812.1440

3 length scales: 2 ~~microscopic,~~ 1 macroscopic

- thermal wavelength $\lambda_{\rm th} \sim \beta \equiv 1/T$
- mean free path $\ell_{\rm mfp} \sim (\langle\sigma\rangle n)^{-1}$

What if these are ~?

$\langle\sigma\rangle$ averaged cross section, $n \sim T^3 = \beta^{-3} \sim \lambda_{\rm th}^{-3}$

- length scale over which macroscopic fluid fields vary $L_{\rm hydro}$, $\partial_\mu \sim L_{\rm hydro}^{-1}$

Note: since $\eta \sim (\langle\sigma\rangle \lambda_{\rm th})^{-1} \Longrightarrow$ $\boxed{\frac{\ell_{\rm mfp}}{\lambda_{\rm th}} \sim \frac{1}{\langle\sigma\rangle n}\frac{1}{\lambda_{\rm th}} \sim \frac{\lambda_{\rm th}^3}{\langle\sigma\rangle\lambda_{\rm th}} \sim \frac{\lambda_{\rm th}^3}{\langle\sigma\rangle\lambda_{\rm th}} \sim \frac{\eta}{s}}$

s entropy density, $s \sim n \sim T^3 = \beta^{-3} \sim \lambda_{\rm th}^{-3}$

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

Second inequality was developed so far, but first is suspect! experimentally

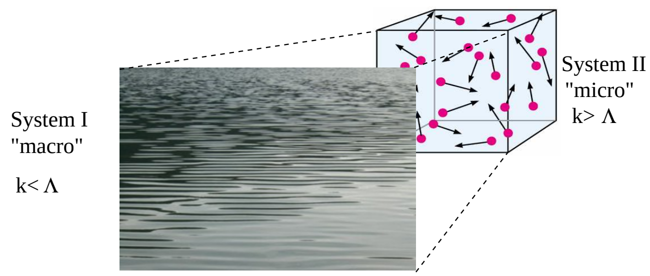
$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

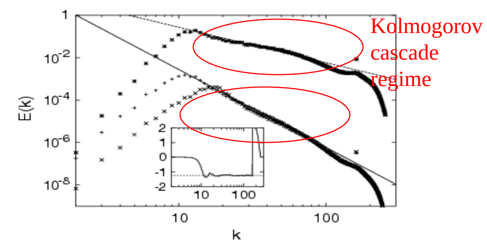
Weakly coupled: Ensemble averaging in Boltzmann equation good up to $\mathcal{O}\left((1/\rho)^{1/3}\partial_\mu f(...)\right)$
Strongly coupled: classical supergravity requires $\lambda \gg 1$ but $\lambda N_c^{-1} = g_{YM} \ll 1$ so

$$\frac{1}{T N_c^{2/3}} \ll \frac{\eta}{sT} \quad \left(\quad or \quad \frac{1}{\sqrt{\lambda} T} \right) \ll L_{macro}$$

QGP: $N_c = 3 \ll \infty$,so $l_{micro} \sim \frac{\eta}{sT}$. Cold atoms: $l_{micro} \sim n^{-1/3} > \frac{\eta}{sT}$?

Why is $l_{micro} \ll l_{mfp}$ necessary? microscopic fluctuations (which have nothing to do with viscosity) will drive fluid evolution. $\Delta\rho/\rho \sim C_V^{-1} \sim N_c^{-2}$



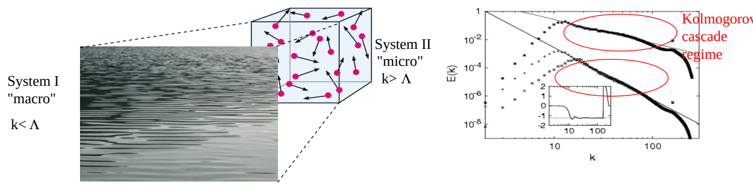


A classical low-viscosity fluid is turbulent. Typically, low-k modes cascade into higher and higher k modes In a non-relativistic incompressible fluid

$$\eta/(sT) \ll L_{eddy} \ll L_{boundary} \quad , \quad E(k) \sim \left(\frac{dE}{dt}\right)^{2/3} k^{-5/3}$$

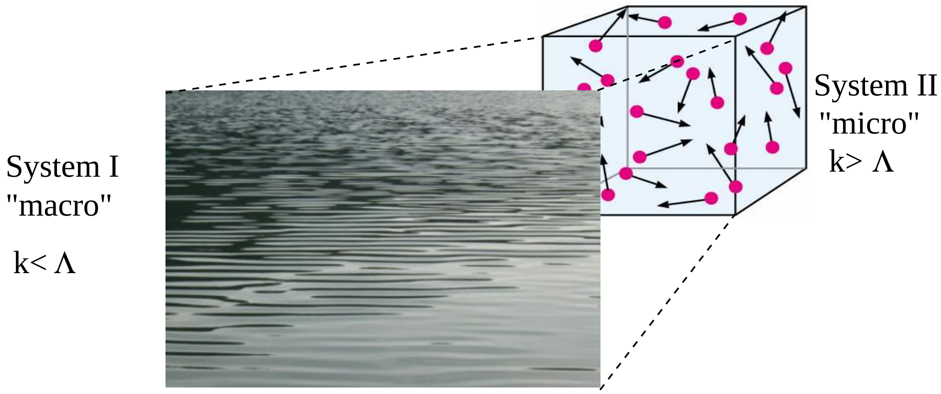
For a classical ideal fluid, no limit! since $\lim_{\delta\rho\to 0, k\to\infty} \delta E(k) \sim \delta\rho k c_s \to 0$ but quantum $E \geq k$ so energy conservation has to cap cascade.

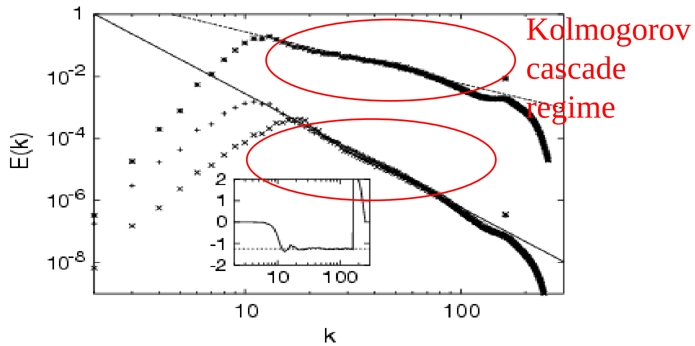
More fundamentally: take stationary slab of fluid at local equilibrium.



Statistical mechanics: This is a system in global equilibrium, described by a partition function $Z(T, V, \mu)$, whose derivatives give expectation values $\langle E \rangle$,fluctuations $\langle (\Delta E)^2 \rangle$ etc. in terms of conserved charges. All microstates equally likely, which leads to preferred macrostates!

Fluid dynamics: This is the state of a field in local equilibrium which can be perturbed in an infinity of ways. The perturbations will then interact and dissipate according to the Euler/N-S equations. Smaller η/s , the closer to local equilibrium (SM applies to cell) but the longer the timescale to global equilibrium (SM applies to system).





- Provided state is localized, local equilibrium is "global equilibrium in every cell", global equilibrium with spin, forces "non-local" A.Palermo et al,2007.08249,2106.08340 "global" equilibrium not necessarily stable against hydro perturbations I think "real" global equilibrium built up from local equilibria

- Dissipation scale in local equilibrium $\eta/(Ts)$, global equilibration timescale $(Ts)/\eta$ .turbulence drastically changes this ,but "when does a small perturbation become a microstate?"

Some insight from maths

Millenium problem: existence and smoothness of the Navier-Stokes equations

Important tool are "weak solutions" , similar to what we call "coarse-graining".

$$F\left(\frac{d}{dx}, f(x)\right) = 0 \Rightarrow F\left(\int \frac{d}{dx}\phi(x)..., f(x)\right) = 0$$

$\phi(x)$ "test function", similar to coarse-graining!

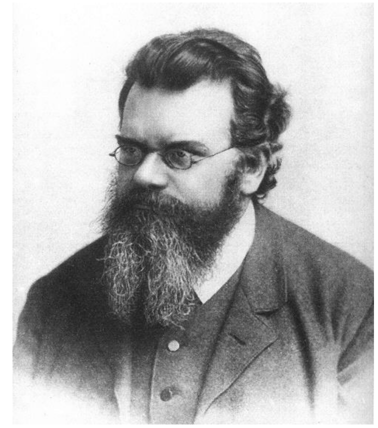
Existance of Wild/Nightmare solutions and non-uniqueness of weak solutions shows this tension is non-trivial, coarse-graining “dangerous”

I am a physicist so I care little about the ”existence of ethernal solutions” to an approximate equation, Turbulent regime and microscopic local equilibria need to be consistent

Thermal fluctuations could both ”stabilize” hydrodynamics and ”accellerate” local thermalization
But where do microstates,”local” microstates fit here?

the battle

of the entropies



Boltzmann entropy is usually a property of the "DoF", and is "kinetic" subject to the H-theorem which is really a consequence of the not-so-justified molecular chaos assumption. Gibbsian entropy is the log of the area of phase space, and is justified from coarse-graining and ergodicity , but hard to define it in non-equilibrium . The two are different even in equilibrium, with interactions! Note, Von Neumann $\langle ln\hat{\rho}\rangle$ Gibbsian

Gauge theory and local thermalization

The formalism we introduced earlier is ok for quark polarization but problematic for gluon polarization: Gauge symmetry means one can exchange locally angular momentum states for transversely polarized spin states. So vorticity vs polarization is ambiguus

Using the energy-momentum tensor for dynamics is even more problematic for spin $T_{\mu\nu}$ aquires a "pseudo-gauge" transformation

$$T_{\mu\nu} \rightarrow T_{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda,\mu\nu} + \Phi^{\mu,\nu\lambda} + \Phi^{\nu,\mu\lambda} \right)$$

where Φ is fully antisymmetric. $\delta S/\delta g_{\mu\nu}$ and canonical tensors are lmits of choice of Φ . But vorticity global (and gauge invariant), $y_{\mu\nu}$ local (and gauge dependent). Affects EFTs based on $T_{\mu\nu}$ (Hong Liu,Florkowski and collaborators)

Generalization from $U(1)$ to generic group easy

$$\alpha \to \{\alpha_i\} \qquad , \qquad \exp(i\alpha) \to \exp\left(i\sum_i \alpha_i \hat{T}_i\right)$$

One subtlety: Currents stay parallel to u_μ but chemical potentials become adjoint, since rotations in current space still conserved

$$y = J^\mu \partial_\mu \alpha_i \to y_{ab} = J_a^\mu \partial_\mu \alpha_b$$

Lagrangian still a function of $dF(b, \{\mu\})/dy_{ab}$, "flavor chemical potentials"

From global to gauge invariance! Lagrangian invariant under

$$\{y_{ab}\} \to y'_{ab} = U_{ac}^{-1}(x) y_{cd} U_{db}(x) \quad , \quad U_{ab}(x) = \exp\left(i \sum_i \alpha_i(x) \hat{T}_i \right)$$

However, gradients of x obviously change y .

$$y_{ab} \to U_{ac}^{-1}(x) y_{cd} U_{bd}(x) = U^{-1}(x)_{ac} J_f^{\mu} U_{cf} U_{fg}^{-1} \partial_\mu \alpha_g U_{bg} =$$

$$= U^{-1}(x)_{ac} J_f^{\mu} U_{cf} \partial_\mu \left(U_{fg}^{-1} \alpha_d U_{bd}(x) \right) - J_a^{\mu} \left(U \partial_\mu U \right)_{fb} \alpha_f$$

Only way to make lagrangian gauge invariant is

$$F\left(b, J_j^{\mu} \partial_\mu \alpha_i\right) \to F\left(b, J_j^{\mu}\left(\partial_\mu - U(x) \partial_\mu U(x)\right) \alpha_i\right)$$

Which is totally unexpected, profound and <u>crazy</u>

The swimming ghost!

$$F\left(b, J_j^\mu \partial_\mu \alpha_i\right) \to F\left(b, J_j^\mu \left(\partial_\mu - U(x)\partial_\mu U(x)\right)\alpha_i\right)$$

Means the ideal fluid lagrangian depends on velocity!. no real ideal fluid limit possibl the system "knows it is flowing" at local equilibrium! NB: For U(1)

$$\hat{T}_i \to 1 \quad , \quad y_{ab} \to \mu_Q \quad , \quad u_\mu \partial^\mu \alpha_i \to A_\tau$$

So second term can be gauged to a redefinition of the chemical potential (the electrodynamic potentials effect on the chemical potential).

Cannot do it for Non-Abelian gauge theory, "twisting direction" in color space It turns out this has an old analogue...

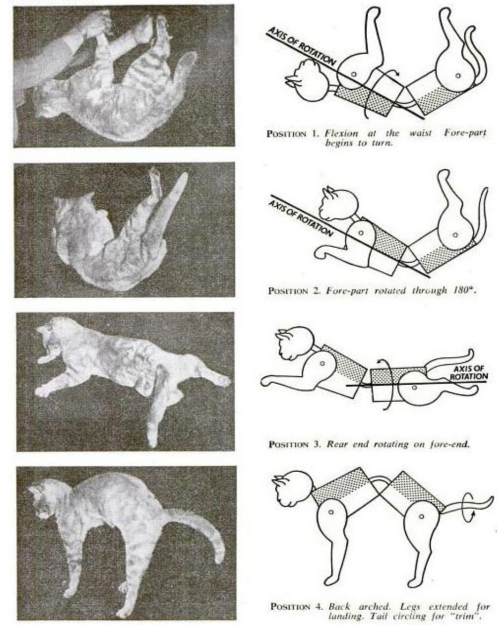
The swirling ghost

Since $u^\mu \partial_\mu$ is in the Lagrangian,let us compare vorticity and Wilson loops!

$$\mathrm{Vorticity}: \oint J_\mu dx^\mu \neq 0 \quad , \quad \mathrm{Wilsonloop}: \oint dx_\mu \partial^\mu U_{ab} \equiv \int_\Sigma d\Sigma_{\mu\nu} F_{ab}^{\mu\nu}$$

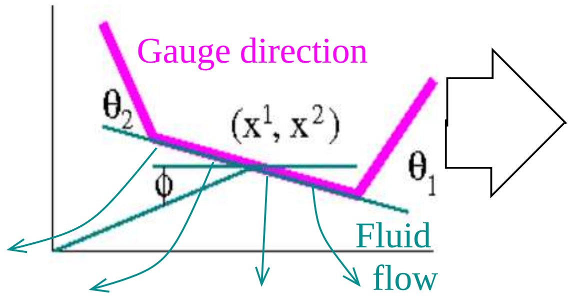
Lagrangian will in general have gauge-invariant terms proportional to $Tr_a \omega_{\mu\nu a} F_a^{\mu\nu}$ Unlike in Jackiw et al , $F_{\mu\nu}$ is <u>not</u> field strength but just a polarization tensor, whose value is set by entropy maximization.

But circular modes correlating angle around vortex of u_μ and direction a of $F_{\mu\nu}^a$ non-dissipative (unlike in polarization hydro described earlier)



S. Montgomery (2003): How does a cat always fall on its feet without anything to push themsevles against? The shape of spaces a cat can deform themselves into defines a "set of gauges" a cat can choose without change of angular momentum.

Purcell,Shapere+Wilczek,Avron+Raz : A similar process enables swimmers to move through viscous liquids with no applied force



Now imagine each fluid cell filled with a "swimmer", with arms and legs outstretched in "gauge" directions...

Hydrostatic vacuum unstable against purcell swimmers in Gauge space!

A statistical mechanics/Gauge explanation

Hydrodynamic limit: $\partial^\mu s_\mu \equiv \partial^\mu \left(u_\mu \ln N_{microstates} \right) = 0$

In thermal Gauge theory microstates contain gauge redundancies,

$N_{microstates} \to N_{microstates} - N_{gauge}$ But s_μ^{real} not parallel to s_μ^{gauge} so no local equilibrium!. recall hydrostatic limit perturbation

$$\phi_I = X_I + \vec{\pi}_I^{sound} + \vec{\pi}_I^{vortex} \quad , \quad \nabla . \vec{\pi}_I^{vortex} = \nabla \times \vec{\pi}_I^{sound} = 0$$

Since the derivative of the free energy w.r.t. b is positive, sound waves and vortices do "work". Let us now assume the system has a "color chemical potential". Let us vary the color chemical potential in space according to

$$\Delta\mu(x) = \sum_i \left(\mu_i(x)^{swim} + \mu_i(x)^{swirl} \right) \hat{T}_i \quad , \quad \nabla_i . \mu_i^{swim} = \nabla_i \times \mu_i^{swirl} = 0$$

"color susceptibility" typically negative. So the two can balance!!!!

But this breaks the "hyerarchy" of statistical mechanics

It mixes micro and macro perturbations!

In statistical mechanics, what normally distinguishes "work" from "heat" is coarse-graining, the separation between micro and macro states. Quantitatively, probability of thermal fluctuations is normalized by $1/(c_V T)$ and microscopic correlations due to viscosity are $\sim \eta/(Ts)$. Since for a usual fluid, there is a hyerarchy between microscopic scale, Knudsen number and gradient

$$\frac{1}{c_V T} \ll \frac{\eta}{(Ts)} \ll \partial u_\mu$$

Gauge symmetry breaks it, since it equalizes perturbations at both ends of this!

Is there a Gauge-independent way of seeing this? Perhaps!

One can write the effective Lagrangian in a Gauge-invariant way using Wilson-Loops . But the effective Lagrangian written this way will have an infinite number of terms, in a series weighted by the characteristic Wilson loop size. For a locally equilibrated system, this series does not commute with the gradient. Just like with Polymers, the system should have multiple anisotropic non-local minima which mess up any Knuden number expansion. Some materials are inhomogeneus and anisotropic at equilibrium, YM could be like this!

Lattice would not see it , as there are no gradients there. There is an entropy maximum, and it is the one the lattice sees. The problems arise if you "coarse-grain" this maximum into each microscopic cell and try to do a gradient expansion around this equilibrium, unless you have color neutrality.

Development of EoMs, linearization, etc. of this theory in progress!

A crazy guess, speculation Remember that all flow dependence through μ_{ab} color chemical potentials. What if local equilibrium happens when they go to zero, i.e. color density is neutral.

Could colored-swimming ghosts quickly be produced, and then locally thermalize and color-neutralize the QGP?

Similar to Positivity violation picture of confinement (Alkofer)

What about gauge-gravity duality?

Large N non-hydrodynamic modes go away in the planar limit
There are N ghost modes and N^2 degrees of freedom

Conformal fixed point most likely means ghosts non-dynamical
Not yet sure of this, but conformal invariance reduces pseudo-Gauge transformations to

$$\Phi_{\lambda,\mu\nu} \underbrace{\rightarrow}_{conformal} g_{\sigma\mu}\partial_\nu\phi - g_{\sigma\mu}\partial_\mu\phi$$

where ϕ is a scalar function. Irrelevant for dynamics.
As shown in Capri et al (1404.7163) Gribov copies for a Yang-Mills theory non-dynamical there. It would be a huge job to do this for hydrodynamics.