Exotic heavy hadrons and Steiner-tree confinement

available at *http://www.ipnl.in2p3.fr/perso/richard/SemConf/Talks.html*

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Introduction

Multiquarks: Long and shaky history

- Z baryons with strangeness $S = +1$ in the late 60s,
- Baryonium in the late 70s and early 80s,
- Dibaryon resonances?
- H dibaryon with strangeness $S = -2$ predicted,
- Heavy pentaquark predicted in 1987,
- Light pentaquark predicted in 1997, base on earlier work,
- Light pentaquark candidate in 2003,
- Not confirmed in most other experiments
- Etc.
- Confusion added by theorists, jumping on a speculative idea, and producing tables and tables of multiquarks.

Theory

- state above the threshold: why is does not fall apart immediately into two ordinary hadrons?
  Cf, e.g., the discussion about baryonium
- state below the threshold? Why is such a clustering favoured?
  STRONG COMPETITION. e.g., in atomic physics
  $(\mu^+\mu^-e^+e^-)$ unbound, while $(\mu^+\mu^+, e^-e^-)$ bound.

Theory

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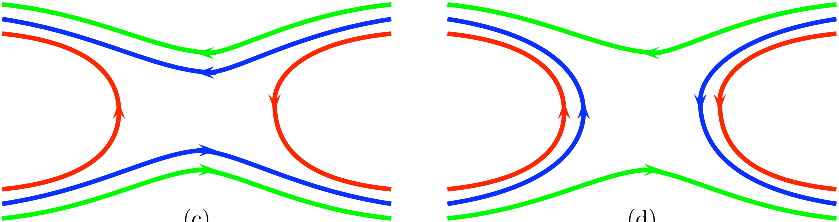
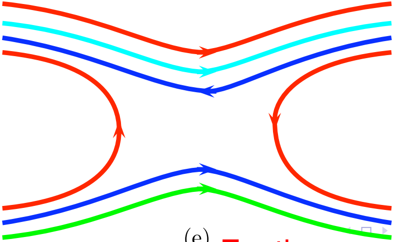
Duality

- *s*-channel exchanges vs. *t*-channel exchanges
- See Rosner, D.P. Roy, etc, baryonium partner of mesons in $\overline{N}N$,



(c) (d)

- Exotic baryon partner of mesons in baryonium–baryon, etc.



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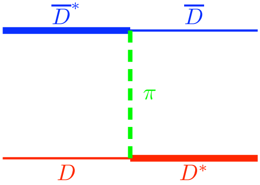
Nuclear forces: hadron molecules

- 

  Meson exchanges bind NN. Why not other hadrons containing light quarks? Non-local operator here (transfer of energy).
- In particular, the $X(3872)$ was predicted as a $D\bar{D}^*$ system,
- When the $X(3872)$ was found, greeted as a success of this approach,
- However, recent measurements also suggest a radial excitation of a $(c\bar{c})$ state,
- If the $X(3872)$ is eventually interpreted as (mainly) a molecule, other states predicted, but no proliferation (nuclear forces are spin and isospin dependent),
- In particular, the b-analogue predicted about 50 MeV below $B\bar{B}^*$,

Nuclear forces: hadron molecules

- Also baryon–antibaryon bound states possible with two or four heavy quarks or antiquarks (Riska).
- In the late 70s, a high-lying $(c\bar{c})$ with $J^P = 1^{--}$ state was claimed as a molecule, due to anomalous branching ratios into $D\bar{D}$, $D\bar{D}^* + \text{c.c.}$ and $D^*\bar{D}^*$ (Voloshin, DeRujula and Glashow, ...)
- In fact, the branching ratios are explained by the nodal structure of the state.
- Today, the quark model is almost abandoned in the light-quark sector. Either sophisticated Lattice QCD or QCD sum rules, or coupled-channel calculations, the so-called “dynamical generation of resonances”. Fifty years after Chew-Low!

Chromomagnetism

- In the 70s, the hyperfine splitting between hadrons ($J/\psi - \eta_c$, $\Delta - N$, etc.) explained à la Breit–Fermi, by a potential

$$V_{SS} = -A \sum_{i<j} \frac{\delta^{(3)}(\vec{r}_{ij})}{m_i\, m_j}\, \lambda_i^{(c)}.\lambda_j^{(c)}\, \vec{\sigma}_i.\vec{\sigma}_j \ ,$$

  a prototype being the magnetic part of one-gluon-exchange.
- Attractive coherences in the spin-colour part: $\langle \sum \lambda_i^{(c)}.\lambda_j^{(c)}\, \vec{\sigma}_i.\vec{\sigma}_j \rangle$ sometimes larger for multiquarks than for the threshold.
- In particular $\langle \ldots \rangle$ twice larger (and attractive) in the best (*uuddss*) as compared to $\Lambda + \Lambda$.
- But $\langle \delta^{(3)}(\vec{r}_{ij}) \rangle$ much weaker for multiquarks than for ordinary hadrons, and needs to be computed. Hence uncertainties in this approach.

Binding mechanisms: chromo-electricity-1

- What about a (confining) spin-independent confining interaction for $(q_1 q_2 \bar{q}_3 \bar{q}_4)$?
- Interesting properties if the interaction is flavour independent
- Mainly investigated with explicit constituent models,
- But probably more general.
- Early investigations based on the colour-additive model

$$V = -\frac{3}{16} \sum_{i<j} \tilde{\lambda}_i^{(c)}.\tilde{\lambda}_j^{(c)} \, v(r_{ij}) \ ,$$

(to be discussed shortly)

- No stable multiquark in this model, at least for equal masses

Binding mechanisms: chromo-electricity-2

Why multiquarks hardly bound with a pure colour-addive model?

- looks like $H = \sum_i \vec{p}_i^2/2m + \sum_{i<j} g_{ij}\, v(r_{ij})$
- with $\sum_{i<j} g_{ij}$ frozen (colour singlet)
- Easily shown (variational principle): symmetric case ($g_{ij} = 2G/N(N-1)\ \forall i,j$) has highest energy.
- Roughly: more asymmetric g_{ij} distribution, lower energy.
- In this respect, two mesons ($g_{12} = g_{34} = 1$, any other $g_{ij} = 0$), for instance, much favoured as compared to any tetraquark in which the g_{ij} are more clustered.

$(abcd)$	$v(r)$	g_{ij}	$\bar{g}$	Δg
(1,3)+(2,4)	$-1/r, r$	$\{0,0,1,0,1,0\}$	$1/3$	0.22
Ps_2	$-1/r$	$\{-1,-1,1,1,1,1\}$	$1/3$	0.89
$[(qq)_{\bar{3}}(\bar{q}\bar{q})_3]$	$-1/r, r$	$\{1/2,1/2,1/4,1/4,1/4,1/4\}$	$1/3$	0.01
$[(qq)_6(\bar{q}\bar{q})_{\bar{6}}]$	$-1/r, r$	$\{-1/4,-1/4,5/8,5/8,5/8,5/8\}$	$1/3$	0.17

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Binding mechanisms: chromo-electricity-3

Provisional conclusion:

- The chromoelectric model, at least the colour-additive version, does not bind multiquarks,
- We understand why: symmetry considerations on the colour coefficients
- Coupled-channel effects ($(qq\bar{q}\bar{q} = |\bar{3}3\rangle, |6\bar{6}\rangle)$ not enough
- Two ways out:
  1. find another symmetry that could overcome the effect (flavour symmetry, i.e., use unequal quark masses),
  2. modify more drastically the colour-additive ansatz → Steiner-tree model

Breaking flavour symmetry and/or charge conjugation

- Symmetry breaking: if $H = H_{\rm even} + \lambda\, H_{\rm odd}$, $E(\lambda) \le E(0)$
- But, the effect often benefits more to the threshold, and stability deteriorates,
- For instance, $\mathrm{Ps}_2 = (e^+, e^+, e^-, e^-)$ has many symmetries:
- Particle identity $\to$ (M^+, m^+, M^-, m^-) *unstable* for $M/m \gtrsim 2.2$.
- Charge conjugation $\to$ (M^+, M^+, m^-, m^-) *improves stability*
- Similarly, in pure chromoelectric models with flavour independence, $(QQ\bar{q}\bar{q})$ becomes stable if M/m large enough.
- Typically $(cc\bar{u}\bar{d})$ at the edge, $(bc\bar{q}\bar{q})$ or $(bb\bar{q}\bar{q})$ safer if one uses

$$V = -\frac{3}{16}\sum_{i<j} \tilde{\lambda}^{(c)}_i . \tilde{\lambda}^{(c)}_j \, v(r_{ij}) \ ,$$

- Question: what about a better model of confinement?

Linear confinement beyond quark–antiquark

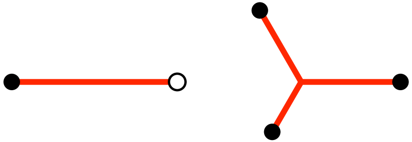
- First problem (Stanley and Robson, LIpkin, Martin & R., etc. link between meson and baryon spectroscopy.
-
$$V_{\text{baryon}} = \frac{1}{2}\left[v(r_{12}) + v(r_{23}) + v(r_{31})\right]$$
(including short-range and spin-dependent terms) works rather well.
- This is the result of the colour-additive model
$$V = -\frac{3}{16}\sum_{i<j}\tilde{\lambda}_i^{(c)}.\tilde{\lambda}_j^{(c)}\, v(r_{ij})\ ,$$
- Indeed, the most general two-body interaction reads
$$v(r) = v_1(\text{colour-singlet exch.}) + v_8(\text{octet exch.})$$
- But v_1 cannot dominates and cannot contribute to confinement,
- Hence pure v_8 seems the simplest solution.

Pairwise or multibody interaction?

Steiner tree: baryons-1

- For baryons, the linear confinement is described by a Y-shape interaction (Artru, Merkuriev, Dosch, Kuti et al., Kogut et al., etc.)

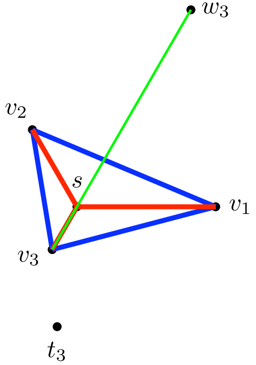
$$v = \sigma\, r_{12}\ , \qquad V_Y = \sigma \min_J \sum_{i=1}^{3} r_{iJ}\ .$$

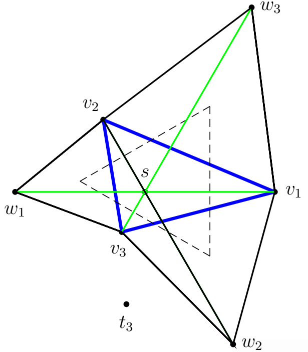


- No dramatic change for baryon spectroscopy, as compared to the $1/2$ rule.
- Except for solving the 3-body problem (Taxil et al., Semay et al., etc.)

Steiner tree: baryons-2

- This baryon potential is the solution of the famous Fermat-Torricelli problem of the minimal path linking three points, with an interesting symmetry restoration, intimately related to a theorem by Napoleon.





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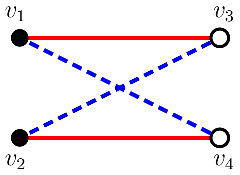
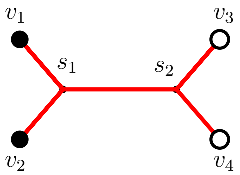
Steiner tree: tetraquarks-1

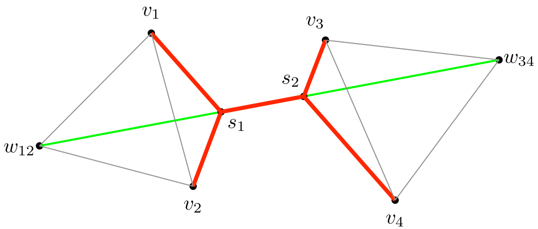
$$U = \min\{V_{\text{flip-flop}}, V_{\text{Steiner}}\}$$
$$V_{\text{flip-flop}} = \min\{d_{13} + d_{24}, d_{14} + d_{23}\} \ ,$$
$$V_{\text{Steiner}} = \min_{s_1, s_2}\left(\|v_1 s_1\| + \|v_2 s_1\| + \|s_1 s_2\| + \|s_2 v_3\| + \|s_2 v_4\| \right) \ ,$$

U dominated by the flip-flop term,





Steiner tree: tetraquarks-2



In the planar case, very simple construction of the connected term of the potential (this speeds up the computation).

$$V_4 = \sigma \left\| w_{12} w_{34} \right\| ,$$

maximal distance between the two Melznak points.

Steiner tree: tetraquarks-3

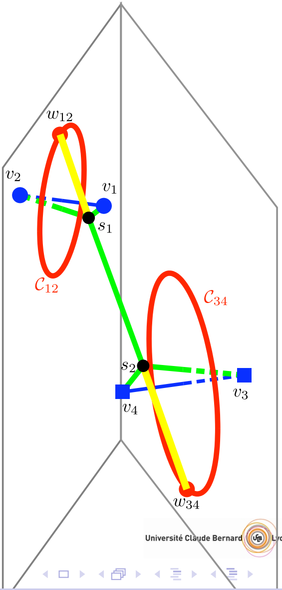
$$V_4 = \sigma \left\| w_{12} w_{34} \right\| \ ,$$

maximal distance between the two Melznak circles.

$$V_4 \le \sigma \left\{ \frac{\sqrt{3}}{2} \left[\|\vec{x}\| + \|\vec{y}\| \right] + \|\vec{z}\| \right\} \ ,$$

which is exactly solvable. The Jacobi var.

$$\vec{x} = v_1 v_2,$$
$$\vec{y} = v_3 v_4,$$
$$\vec{z} = (v_1 + v_2)/2 - (v_3 + v_4)/2 \ ,$$



Steiner tree: tetraquarks-4

- These crude, but rigorous, geometric considerations demonstrate stability at least for large M/m quark-to-antiquark mass ratio.
- What about an accurate numerical solution of this four-body problem?
- First estimate

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Absence of exotic hadrons in flux-tube quark models

- Second estimate (Vijande et al.)

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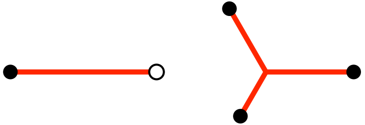
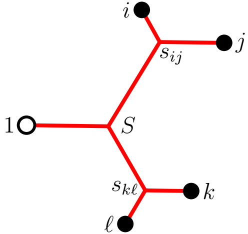
Stability of multiquarks in a simple string model

- However, the effect of antisymmetrisation and short-range forces not yet included.
- Tetraquarks with different flavours and large quark-to-antiquark mass ratio most likely, e.g., $(bc\bar{u}\bar{s})$.

Steiner tree: pentaquark

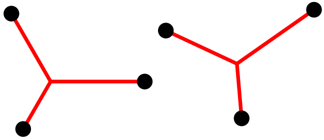


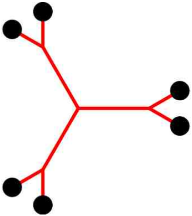
- $U = \min\{$flip-flop, Steiner$\}$,
- Flip-flop
- Connected Steiner tree



  - $(\bar{q}qqqq)$, as well as $(\overline{Q}qqqq)$, $(\bar{q}qqqQ)$ for $M \gg m$, and probably many other configurations bound vs. spontaneous dissociation. (hyperscalar approx. with flip-flop alone sufficient to prove binding)
  - But short-range forces and antisymmetrisation constraints not yet included.
  - $(\bar{c}uuds)$ should survive, as spin effects might help.

Steiner-tree: hexaquark

- Same scenario: flip-flop and connected diagrams,
- The latter, more interesting, but less important for the dynamics,
- Binding is obtained in most cases, where antisymmetrisation is neglected.
- 



Steiner-tree: baryon-antibaryon

- Again: flip-flop and connected diagrams,
- Binding obtained in some cases.
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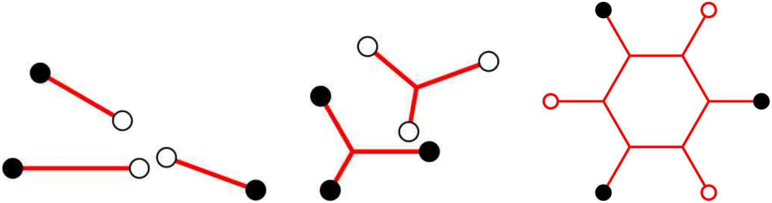

Conclusions. Multiquarks:

The stability of multiquarks remains a very important issue, with recent developments

- Lattice QCD, QCD sum rules, AdS/QCD entering the game very seriously,
- Support to the flip–flop – Steiner-tree model of confinement,
- more attractive than the empirical colour-additive model,
- Still needs antisymmetrisation, relativity, short-range forces, and non-adiabatic corrections
- Stable multiquarks likely in sectors with several flavours
- In particular: $(cc\bar{u}\bar{d})$, $(bc\bar{q}\bar{q})$
- Accessible with present detectors and accelerators, if some effort is devoted.
- Double charm baryons as a warm up.