

Dark matter from the centre of $SU(N)$

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In collaboration with **Michele Frigerio & Thomas Hambye**

Motivations

Observation today:

$$\tau \geq \tau_{\text{universe}} \approx 10^{10} \text{ yr}$$

Indirect detection, CMB, 21cm:

$$\tau \geq 10^{17-19} \text{ yr}$$

Stability of Dark Matter (DM)

Motivations

Origins of stability

Motivations

Origins of stability

Phase space

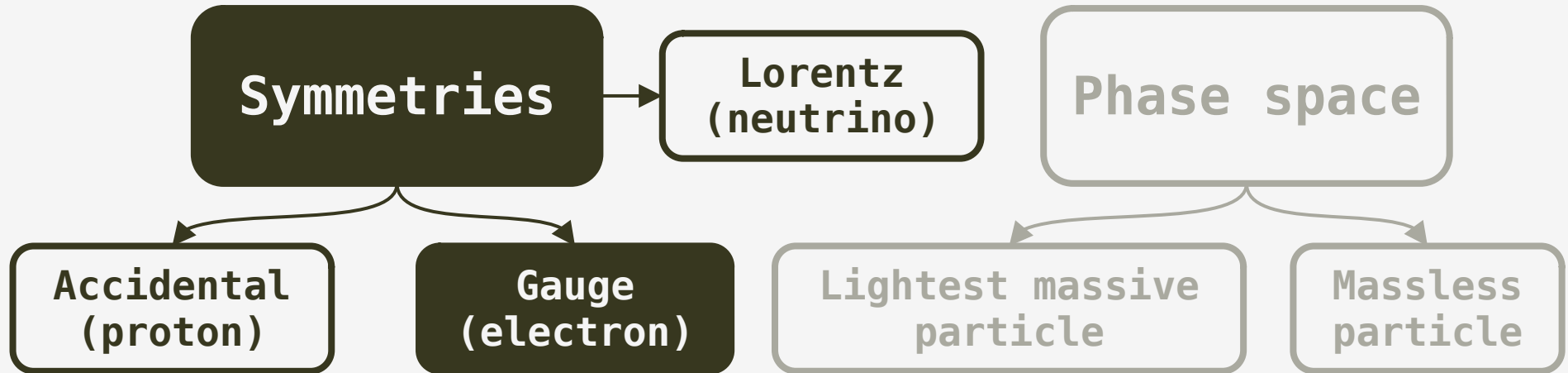
```
graph TD; A[Phase space] --> B[Lightest massive particle]; A --> C[Massless particle];
```

Lightest massive
particle

Massless
particle

Motivations

Origins of stability



Motivations

- DM **stability** from fundamental principles
- **Scalar** DM
- **Non-abelian** dark gauge symmetry
- Perturbative aspects (**SSB** to $U(1)_s$)
- **Minimal** particle content

**1. N-ality
property and
scalar DM
models**

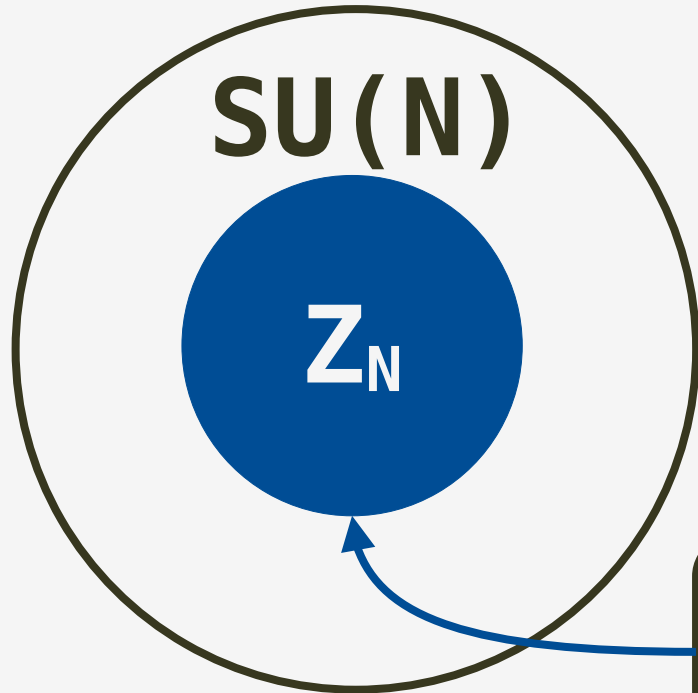
**2. Scalar
potential and
mass spectrum**

**3. DM
phenomenology**



1. N-ality property and scalar DM models

N-ality as a stabilisation mechanism:

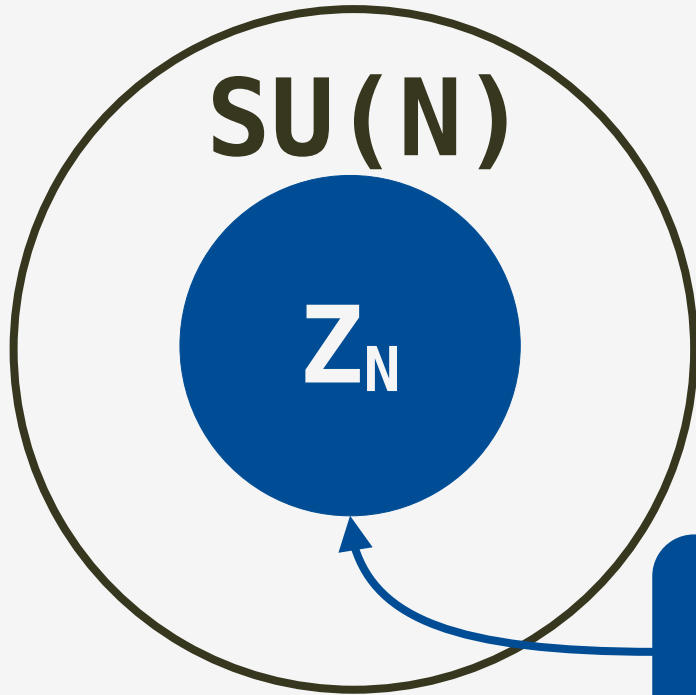


$$X_{j_1 j_2 j_3 \dots j_l}^{i_1 i_2 i_3 \dots i_k}$$

X has charge
 $Q_X = k - l \pmod N$
under Z_N

Centre of $SU(N)$ =
set of elements
that commute with
all $g \in SU(N)$

N-ality as a stabilisation mechanism:

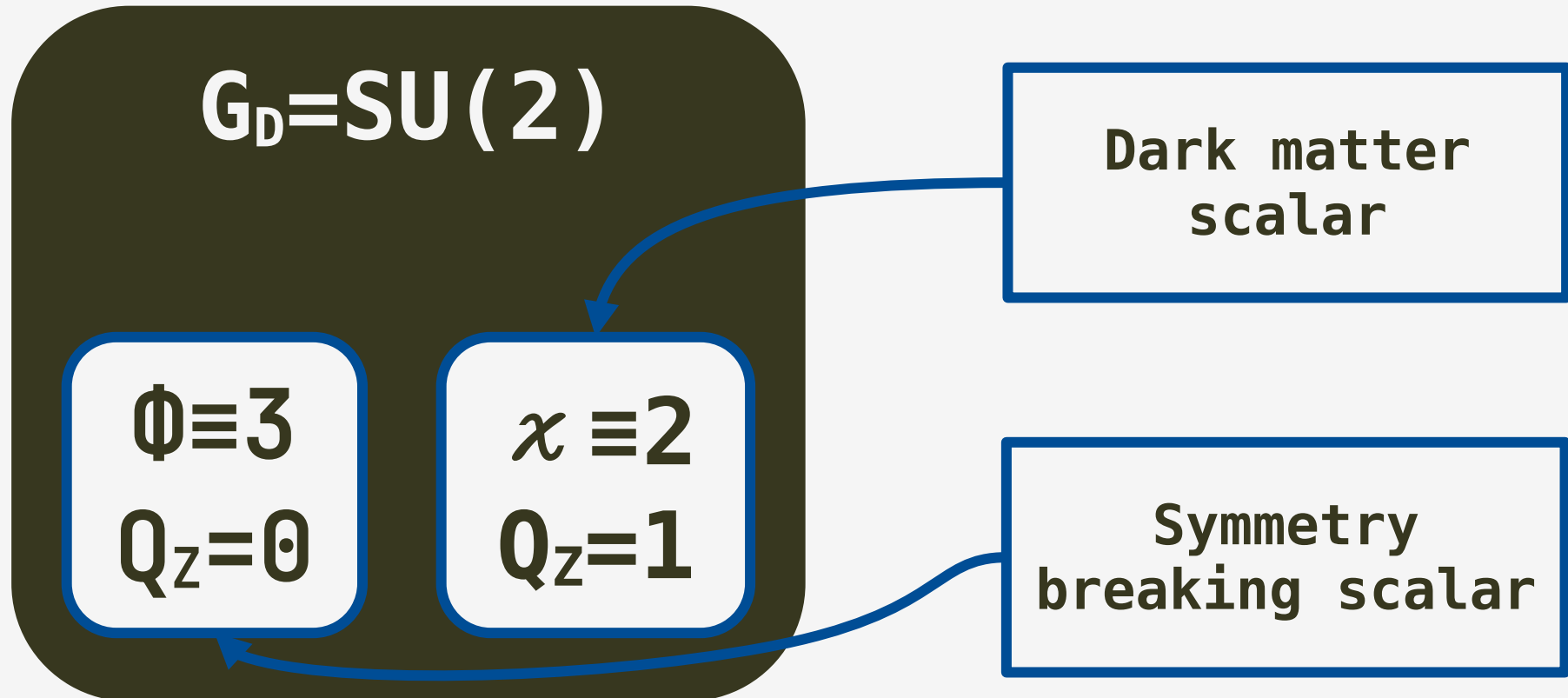


$$X \begin{matrix} i_1 i_2 i_3 \dots i_k \\ j_1 j_2 j_3 \dots j_l \end{matrix}$$

X has charge $Q_X = k - n \pmod N$ under Z_N

Z_N is preserved if $SU(N)$ broken by X with $Q_X = 0$

Particle content: $SU(2)$



Particle content: SU(2)

$$G_D = \text{SU}(2)$$

$$\Phi \equiv 3$$

$$Q_Z = 0$$

$$\chi \equiv 2$$

$$Q_Z = 1$$

Z_2 symmetry

$$V(\Phi, \chi) \supset \Phi_{ij} \chi^i \chi^j$$

Particle content: SU(3)

$G_D = \text{SU}(3)$

$\Phi \equiv 10$

$Q_Z = 0$

$\chi \equiv 3$

$Q_Z = 1$

Dark matter
scalar

Symmetry
breaking scalar

Particle content: SU(3)

$G_D = \text{SU}(3)$

$\Phi \equiv 10$

$Q_Z = 0$

$\chi \equiv 3$

$Q_Z = 1$

Z_3 symmetry

$$V(\Phi, \chi) \supset \Phi_{ijk}^* \chi^i \chi^j \chi^k$$



2. SSB: Scalar potential and mass spectrum

The SU(2) model

$$\mathcal{L}(\Phi) = \frac{1}{2} D_\mu \Phi^{ij} D^\mu \Phi_{ij} - V(\Phi)$$

$$V(\Phi) = -\frac{\mu^2}{2} \Phi^{ij} \Phi_{ji} + \frac{\lambda}{4} (\Phi^{ij} \Phi_{ji})^2$$

The ~~SU(2)~~ model

$$\mathcal{L}(\Phi) = \frac{1}{2} D_\mu \Phi^{ij} D^\mu \Phi_{ij} - V(\Phi)$$

U(1)

$$V(\Phi) = -\frac{\mu^2}{2} \Phi^{ij} \Phi_{ji} + \frac{\lambda}{4} (\Phi^{ij} \Phi_{ji})^2$$



$$\langle \Phi^{ij} \Phi_{ji} \rangle = \mu^2 / \lambda \equiv v_D^2$$

The SU(2) model

$$V(\chi) = \mu_\chi^2 \chi^i \tilde{\chi}_i + \lambda_\chi (\chi^i \tilde{\chi}_i)^2$$

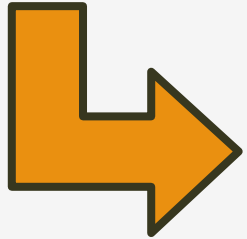
$\tilde{\chi}_i \equiv (\chi^i)^*$

$$V(\Phi, \chi) = -\kappa \chi^i \Phi_{ij} \tilde{\chi}^j + \frac{1}{2} \lambda_{\chi\Phi} (\Phi^{ij} \Phi_{ji}) (\chi^i \tilde{\chi}_i)$$

**Induces mass
splitting**

The SU(2) model

$$V_{\text{portal}} = \left(\lambda_{H\chi} \chi^i \tilde{\chi}_i + \frac{1}{2} \lambda_{\Phi H} \Phi^{ij} \Phi_{ji} \right) H^\dagger H$$



**Mixing between
dark and SM
higgs.**

$$\sin \theta_m \simeq \lambda_{\Phi H} v_D v / [2(\lambda_H v^2 - \lambda v_D^2)]$$

Mass spectrum

- Gauge bosons:

- 1 **massless** dark photon A_D

- 1 **complex charged boson** W_D^\pm : $m_{W_D}^2 = g_D^2 v_D^2$

Mass spectrum

- Gauge bosons:
 - 1 **massless** dark photon A_D
 - 1 complex charged boson W_D^\pm : $m_{W_D}^2 = g_D^2 v_D^2$
- **Real scalar triplet**:
 - 2 would-be NGBs
 - 1 **massive neutral mode** ρ : $m_\rho^2 \simeq 2\lambda v_D^2$

Mass spectrum

- Gauge bosons:
 - 1 **massless** dark photon A_D
 - 1 complex charged boson W_D^\pm : $m_{W_D}^2 = g_D^2 v_D^2$
- Real scalar triplet:
 - 2 would-be NGBs
 - 1 radial neutral mode ρ : $m_\rho^2 \simeq 2\lambda v_D^2$
- Complex scalar doublet (DM):
 - χ_\pm charged under A_D
 - $m_{\chi_\pm}^2 = \mu_\chi^2 + \frac{1}{2}\lambda_{\chi\Phi}v_D^2 + \frac{1}{2}\lambda_{\chi H}v^2 \pm \frac{1}{\sqrt{2}}\kappa v_D$

DM content

A) χ_- and χ_+ for $m_{\chi_+} + m_{\chi_-} < m_{W_D}$

B) χ_- , χ_+ and W_D for:

$$m_{\chi_+} - m_{\chi_-} \leq m_{W_D} \leq m_{\chi_+} + m_{\chi_-}$$

C) χ_- and W_D for $m_{W_D} < m_{\chi_+} - m_{\chi_-}$



3. DM
phenomenology

Annihilation processes

Case **A**: $DM \equiv \chi_{\pm}$

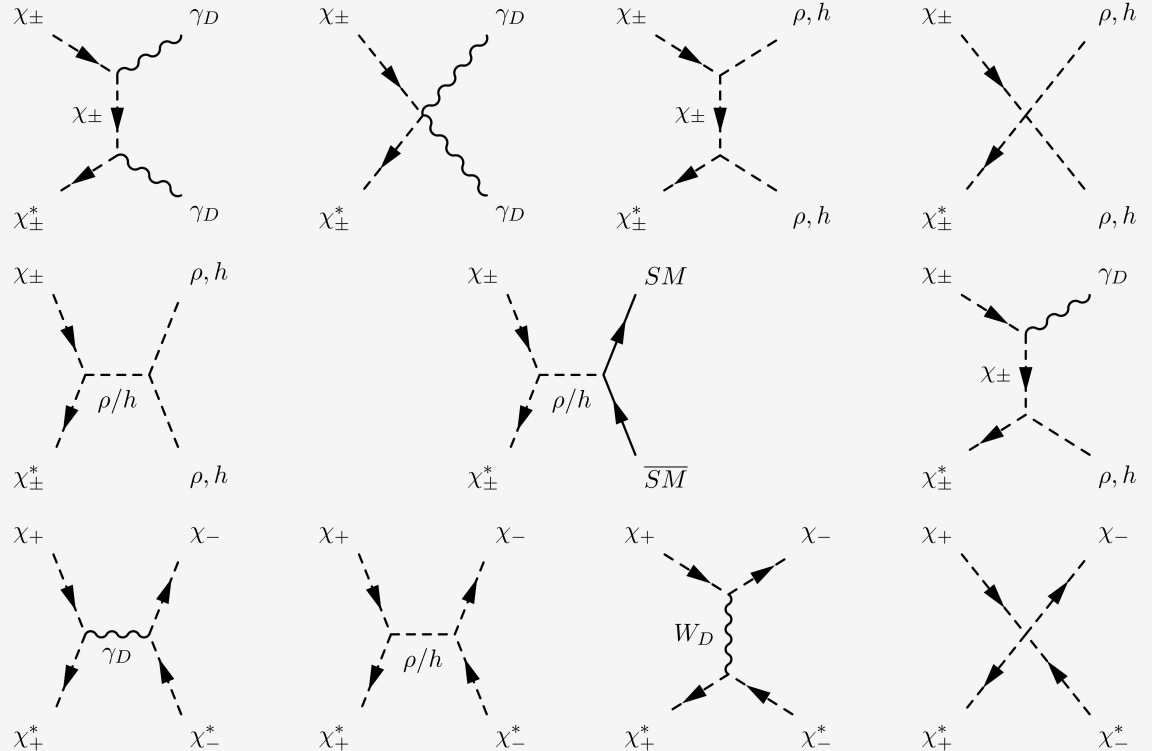
$$m_{\chi_+} + m_{\chi_-} < m_{W_D}$$

Focus on 3 limiting cases dominated by a single coupling:

$\alpha_D \rightarrow$ Into dark photons

$\lambda_{\chi\Phi} \rightarrow$ Into ρ scalars

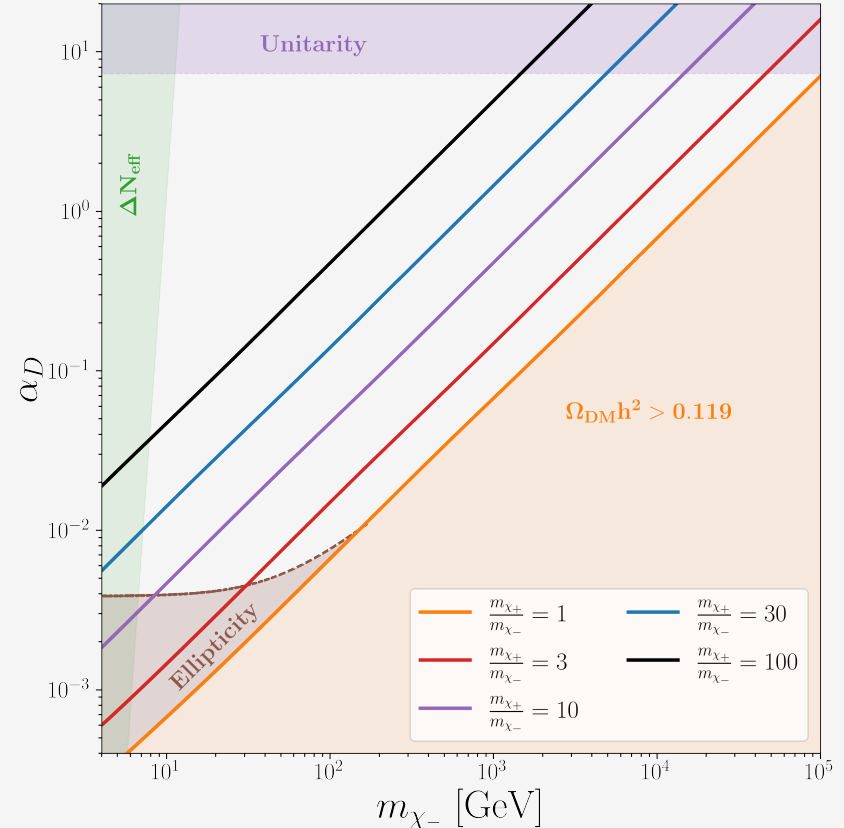
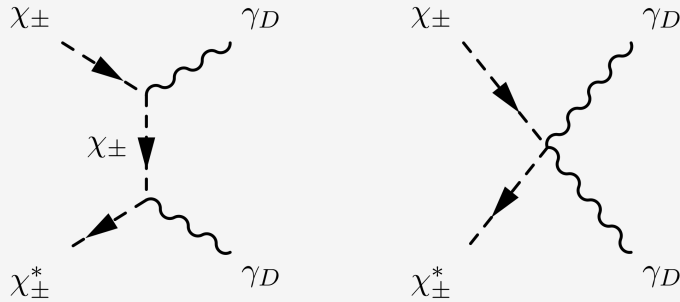
$\lambda_{H\chi} \rightarrow$ Into SM particles



Freeze-out to dark photons

$$\langle \sigma v \rangle_{\pm} \equiv \langle \sigma_{\chi_{\pm} \chi_{\pm}^* \rightarrow \gamma_D \gamma_D} v \rangle = \frac{\pi \alpha_D^2}{4 m_{\chi_{\pm}}^2}$$

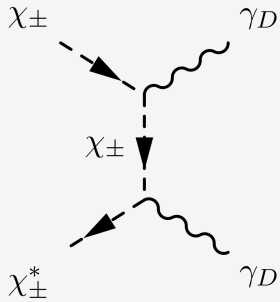
$$\Omega_{\chi_+} / \Omega_{\chi_-} \simeq m_{\chi_+}^2 / m_{\chi_-}^2$$



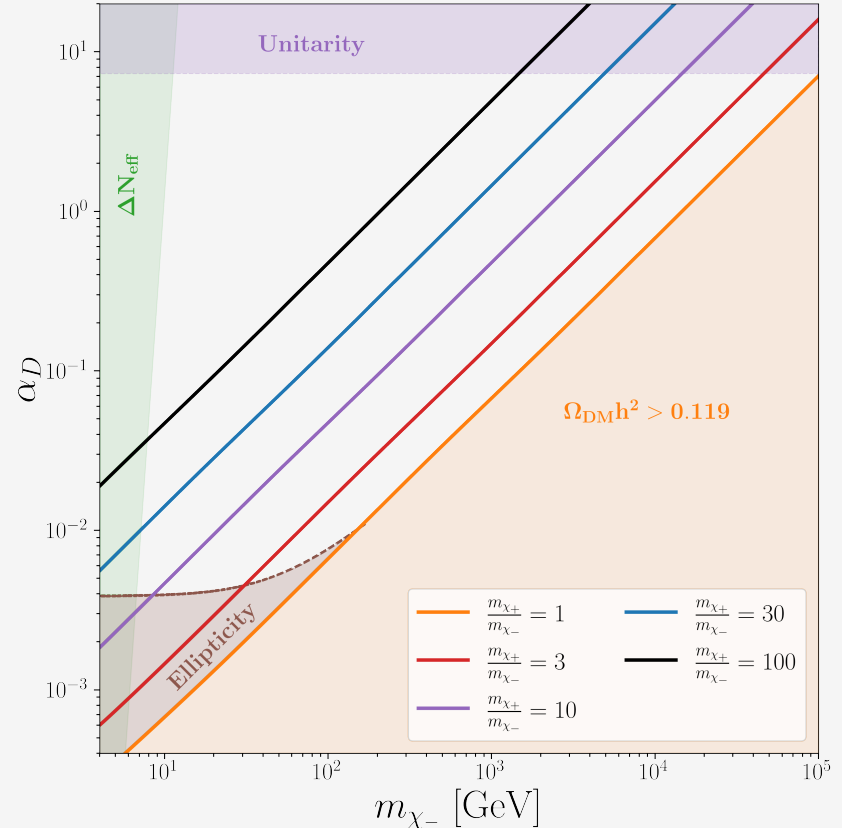
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χ_+
dominates
 Ω_{DM}



Freeze-out to dark photons

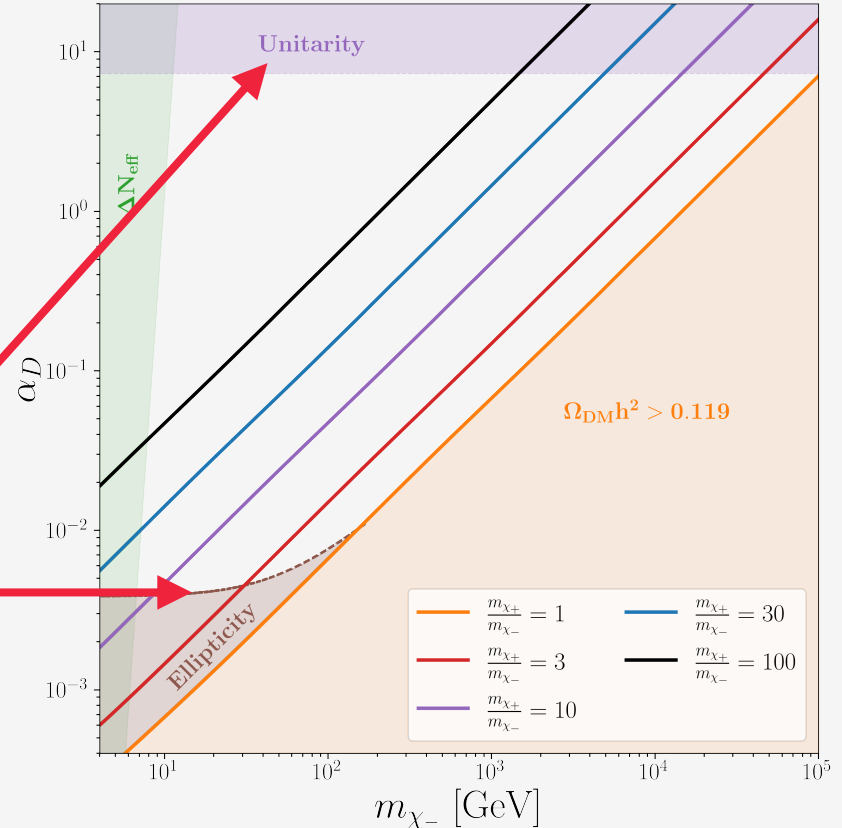
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$$\Omega_{\chi_+} / \Omega_{\chi_-} \simeq m_{\chi_+}^2 / m_{\chi_-}^2$$

$$\langle \sigma v \rangle_J \lesssim 4\pi(2J+1)/(m_{DM}^2 v)$$

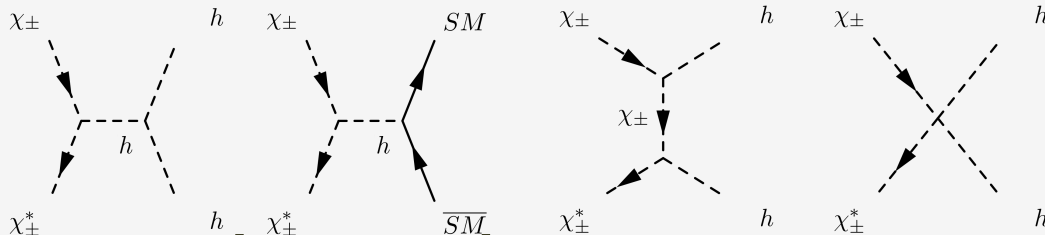
$$\alpha_D \lesssim 0.8 \sqrt{10^{-11} (m_{DM}/\text{GeV})^3}$$

See P. Agrawal, F. Cyr-Racine, L. Randall & J. Scholtz, 2016 (ArXiv :1610.04511)



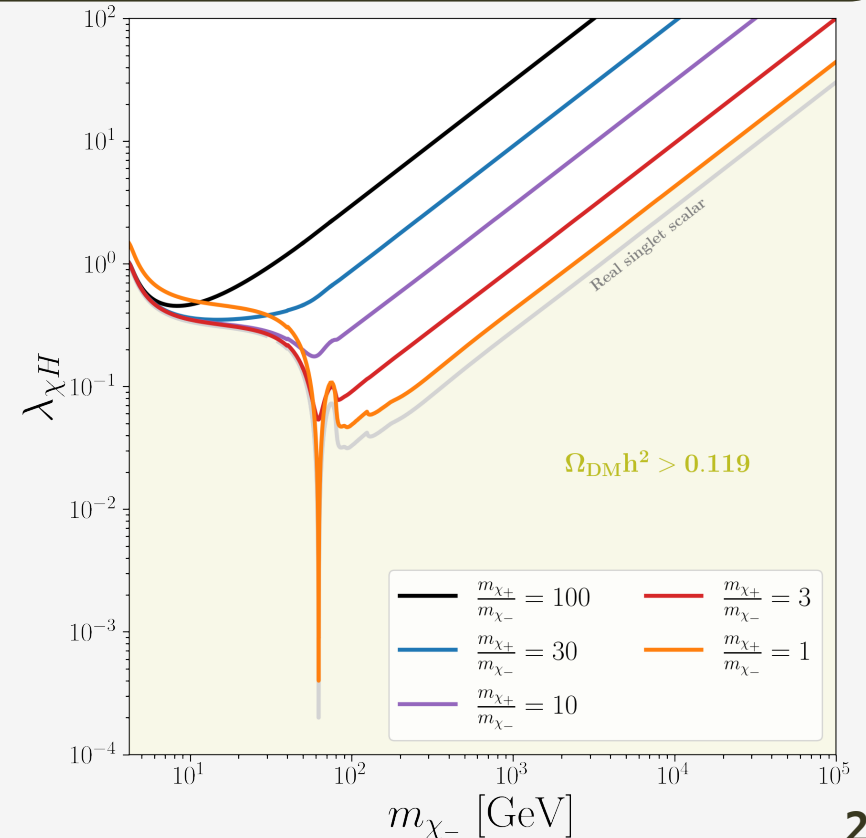
Freeze-out through Higgs portal

Usual Higgs portal cross sections



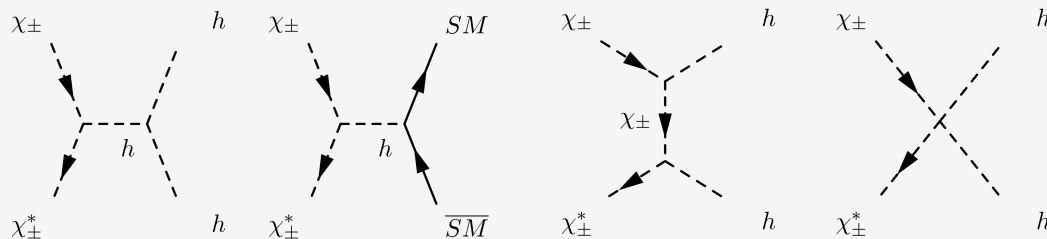
→ **Ordinary Higgs portal DM setup** (see G. Arcadi, A. Djouadi & M. Kado, ArXiv:2101.0257 for a review)

→ **But two DM components contributing to the relic density**



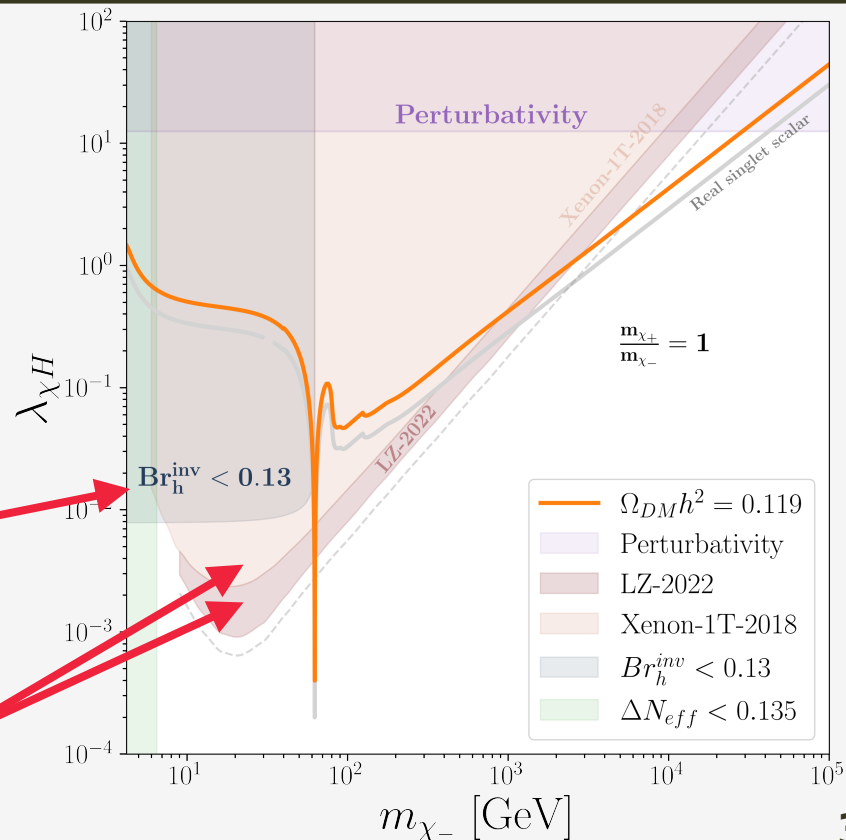
Freeze-out through Higgs portal

Usual Higgs portal cross sections (see e.g. 1204.2808)

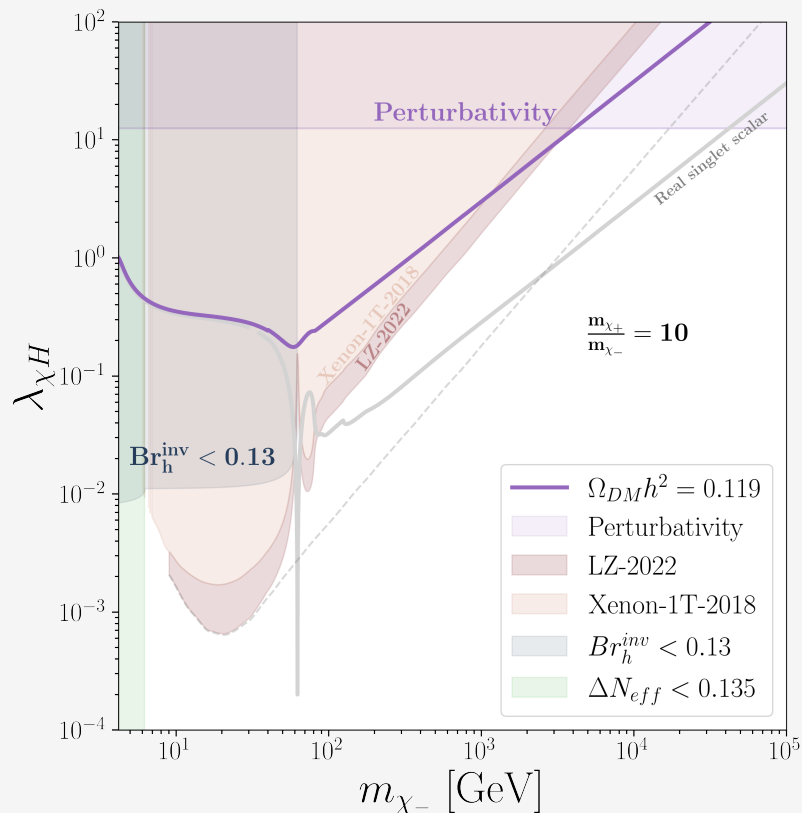


From Atlas latest analysis (2207.00092)

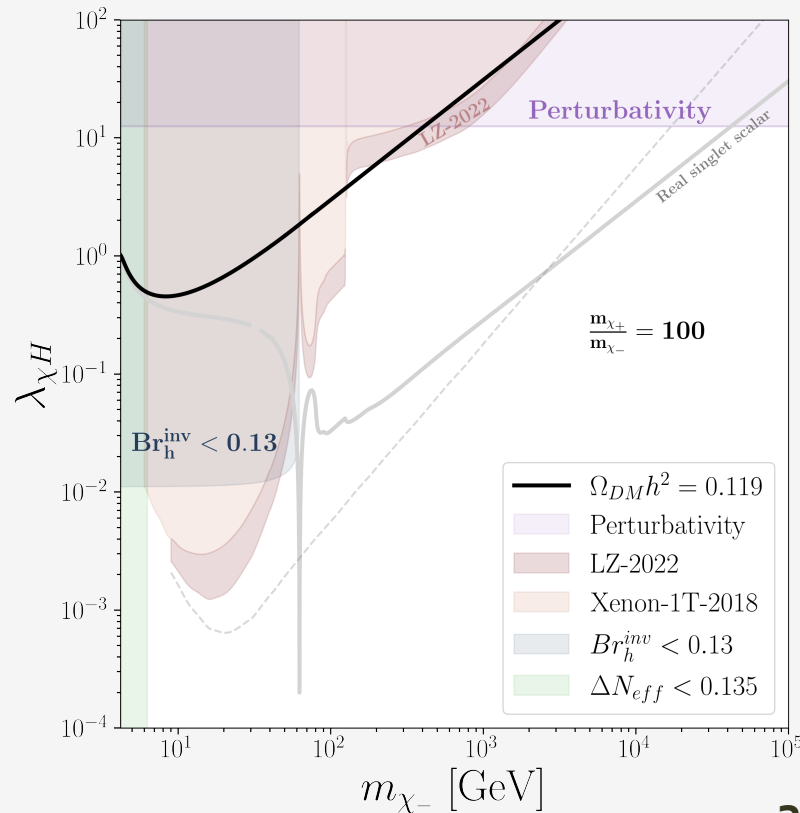
From Xenon1T and LZ (1805.12562 & 2207.03764)



Freeze-out through Higgs portal



DD
constraints,
driven by
subleading
 χ -component,
relax.



Summary

- N-ality provides a simple way to **automatically stabilise** scalar DM with non-abelian gauge symmetries
- **Scalar potentials** of the SU(2) and SU(3) models are **rich** and have intriguing properties
- **Mass splitting** leads to non-trivial interplay between DM components in the SU(2) model
- **Semi-annihilation** processes in the SU(3) model stem from the stabilisation mechanism

More details can be found here: [ArXiv:2212.11918](https://arxiv.org/abs/2212.11918)

**Thank you for your
attention**

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Back-up slides

A few words on the SU(3) model

- Accidental **flat** directions if additional U(1) symmetry
- SSB pattern : $SU(3) \rightarrow U(1) \times U(1)$
- Additional unbroken **discrete** symmetries (S_3)
- **Doublet** of massless DP/triplet of massive gauge bosons
- Scalar tenplet decomposition : **3-3-3-1**
- **Triplet** of scalar DM degenerate in mass

Symmetries of the SU(2) model

$$\text{SU}(2)_{D\Phi}$$
$$\Phi \rightarrow U_{D\Phi}\Phi$$

X

$$\text{SU}(2)_{D\chi} \times \text{SU}(2)_{\chi}$$
$$\chi \rightarrow U_{D\chi}\chi(U_{\chi})^{\dagger}$$

K-term

$$\text{SU}(2)_D \times \text{U}(1)_{\chi}$$

$$\chi \equiv (\tilde{\chi} \chi)$$

Symmetries of the SU(2) model

$$SU(2)_{D\Phi}$$

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X

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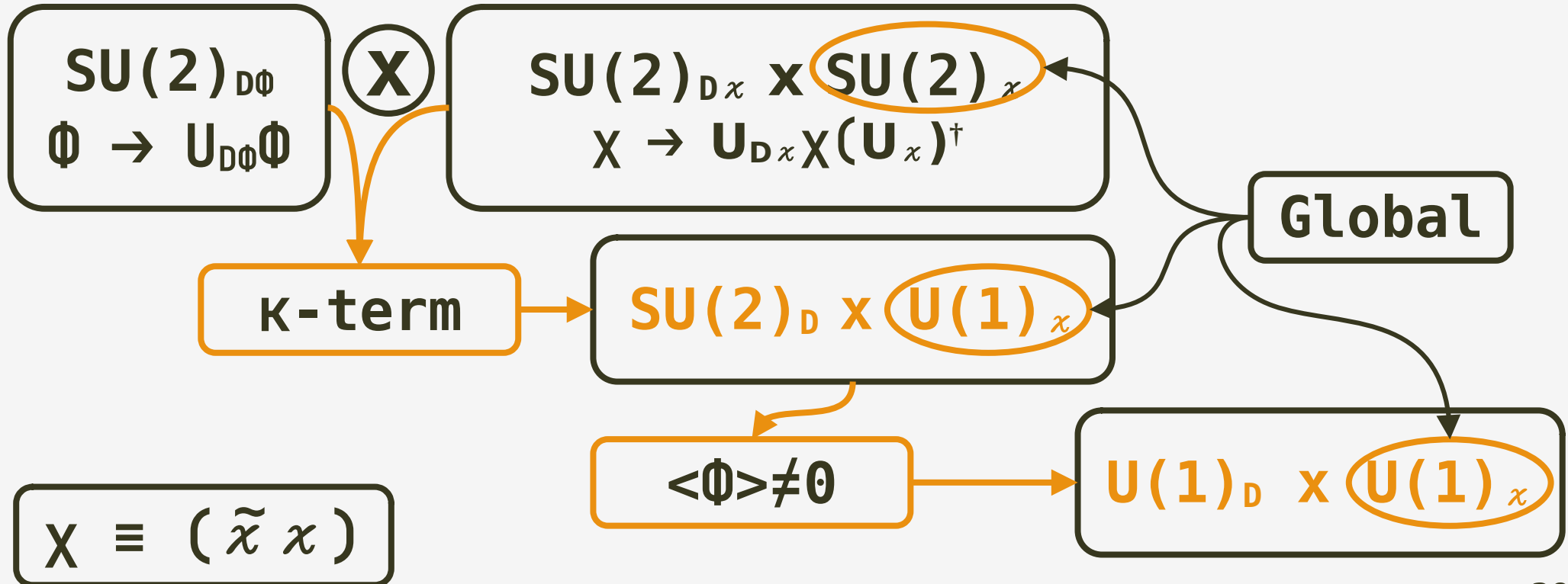
$$SU(2)_D \times U(1)_{\chi}$$

$$\langle \Phi \rangle \neq 0$$

$$U(1)_D \times U(1)_{\chi}$$

$$\chi \equiv (\tilde{\chi} \chi)$$

Symmetries of the SU(2) model



The SU(3) scalar potential

$$\begin{aligned}
 V(\Phi) = & -\mu^2 \Phi_{ijk}^* \Phi^{ijk} + \lambda \left(\Phi_{ijk}^* \Phi^{ijk} \right)^2 + \delta \Phi^{i_1 j_1 k_1} \Phi_{i_1 j_1 k_2}^* \Phi^{i_2 j_2 k_2} \Phi_{i_2 j_2 k_1}^* \\
 & + \left(\eta \epsilon_{i_1 i_2 i_3} \epsilon_{j_1 j_2 j_3} \Phi^{i_1 j_1 k_1} \Phi^{i_2 j_2 k_2} \Phi^{i_3 j_3 k_3} \Phi_{k_1 k_2 k_3}^* + h.c. \right) \\
 & + \left(\sigma \epsilon_{i_1 j_2 k_3} \epsilon_{i_4 j_1 k_2} \epsilon_{i_3 j_4 k_1} \epsilon_{i_2 j_3 k_4} \Phi^{i_1 j_1 k_1} \Phi^{i_2 j_2 k_2} \Phi^{i_3 j_3 k_3} \Phi^{i_4 j_4 k_4} + h.c. \right)
 \end{aligned}$$

The SU(3) scalar potential

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 & + \left(\eta \epsilon_{i_1 i_2 i_3} \epsilon_{j_1 j_2 j_3} \Phi^{i_1 j_1 k_1} \Phi_{j_2 k_2} \Phi^{i_3 j_3 k_3} \Phi_{k_1 k_2 k_3}^* + h.c. \right) \\
 & + \left(\sigma \epsilon_{i_1 j_2 k_3} \epsilon_{i_4 j_1 k_2} \epsilon_{i_3 j_4 k_1} \epsilon_{i_4 k_4} \Phi^{i_1 j_1 k_1} \Phi^{i_2 j_2 k_2} \Phi^{i_3 j_3 k_3} \Phi^{i_4 j_4 k_4} + h.c. \right)
 \end{aligned}$$

SU(3) x U(1)

1101.2417

The SU(3) scalar potential

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 & + \left(\eta \epsilon_{i_1 i_2 i_3} \epsilon_{j_1 j_2 j_3} \Phi^{i_1 j_1 k_1} \Phi^{i_2 j_2 k_2} \Phi^{i_3 j_3 k_3} \Phi_{k_1 k_2 k_3}^* + h.c. \right) \\
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 \end{aligned}$$

SU(3) x U(1)

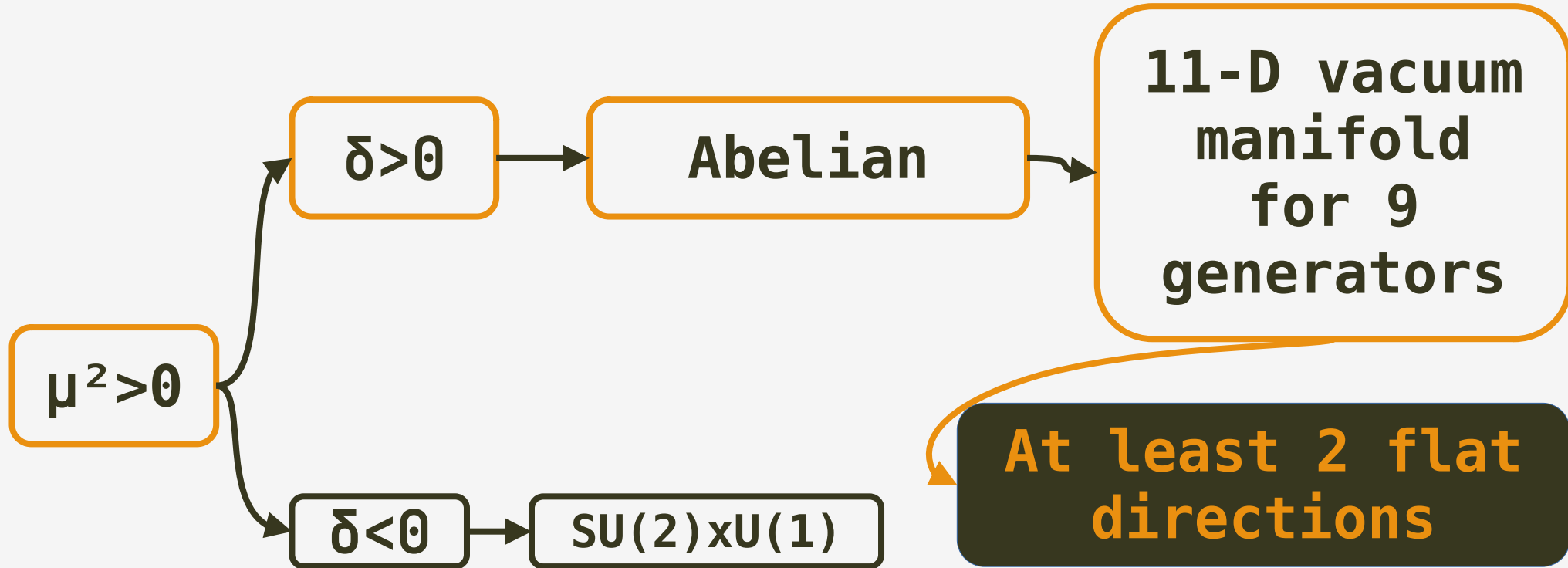


$$V(\Phi) = -\mu^2 A_i^i + \lambda (A_i^i)^2 + \delta A_j^i A_i^j$$

1101.2417

$$A_j^i \equiv \Phi^{ikl} \Phi_{jkl}^*$$

The $SU(3)$ scalar potential



Decoupling temperature

1 dark photon

$\Delta N_{\text{eff}} < 0.135$
at 2σ

2 dark photons

$$g_{\text{SM}}^*(T_{\text{dec}}) \geq 53$$

From
1912.01132

$$g_{\text{SM}}^*(T_{\text{dec}}) \geq 89$$

$$T_{\text{dec}} \geq 300 \text{ MeV}$$

$$T_{\text{dec}} \geq 30 \text{ GeV}$$

Freeze-out to dark photons

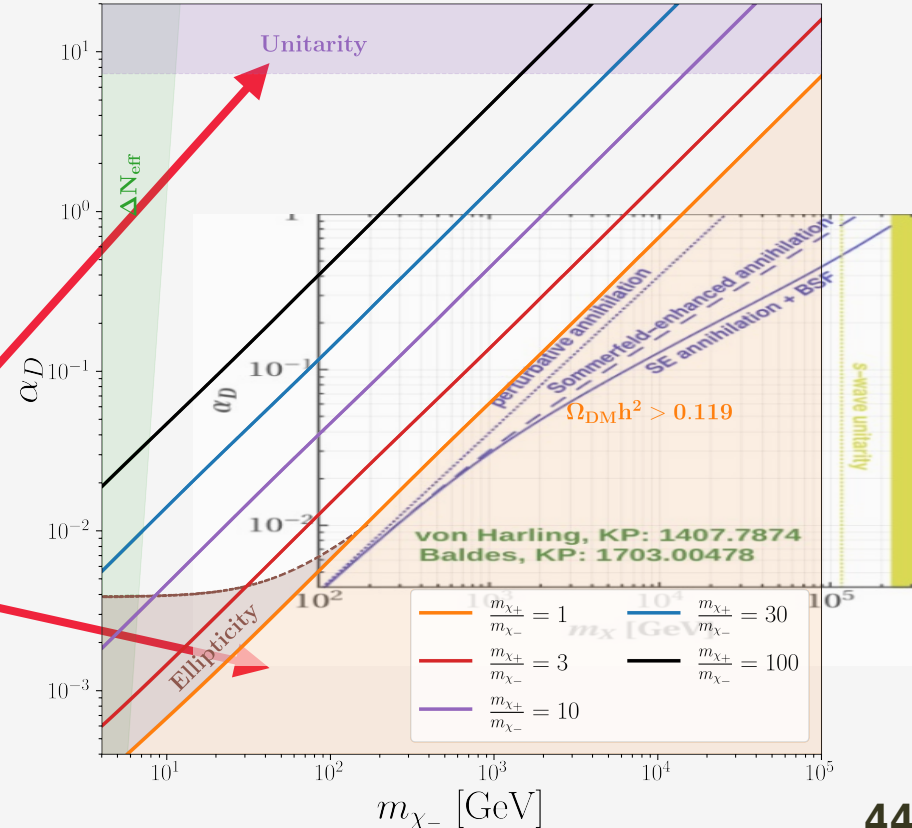
$$\langle \sigma v \rangle_{\pm} \equiv \langle \sigma_{\chi_{\pm} \chi_{\pm}^* \rightarrow \gamma_D \gamma_D} v \rangle = \frac{\pi \alpha_D^2}{4 m_{\chi_{\pm}}^2}$$

$$\Omega_{\chi_+} / \Omega_{\chi_-} \simeq m_{\chi_+}^2 / m_{\chi_-}^2$$

$$\langle \sigma v \rangle_J \lesssim 4\pi(2J + 1) / (m_{DM}^2 v)$$

$$\alpha_D \lesssim 0.8 \sqrt{10^{-11} (m_{DM} / \text{GeV})^3}$$

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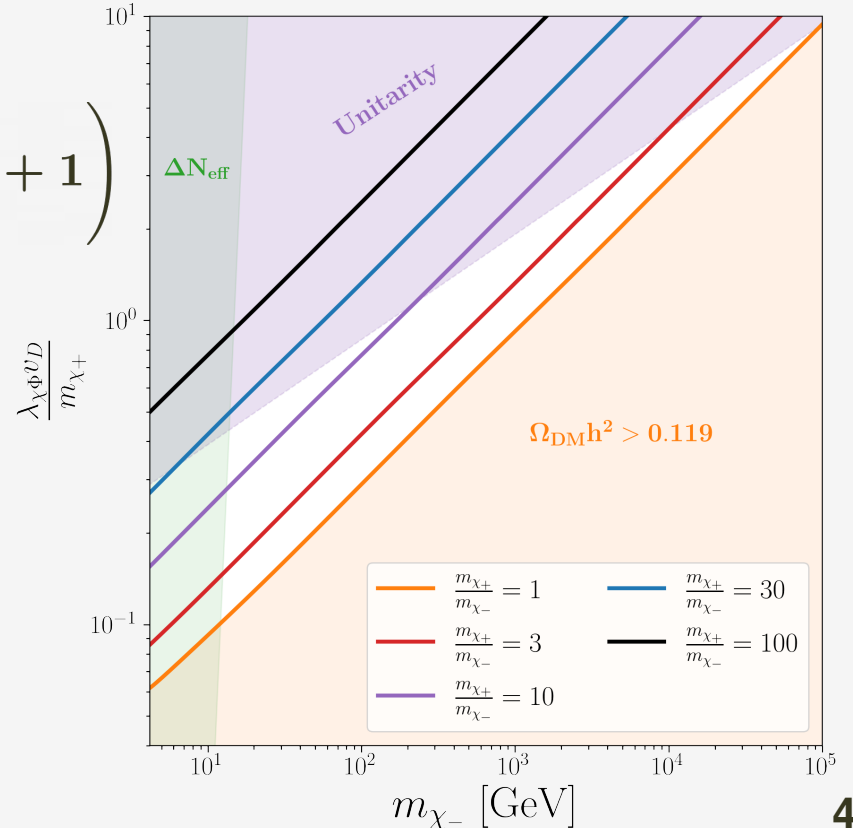
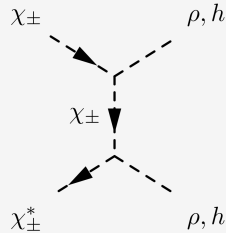


Freeze-out to dark scalars

$$\langle \sigma_{\chi_{\pm}\chi_{\pm}^* \rightarrow \rho\rho\nu} \rangle = \frac{\lambda_{\chi\Phi}^2}{64\pi m_{\chi_{\pm}}^2} \left(\lambda_{\chi\Phi}^2 \frac{v_D^4}{m_{\chi_{\pm}}^4} - 2\lambda_{\chi\Phi} \frac{v_D^2}{m_{\chi_{\pm}}^2} + 1 \right)$$

Assuming that the first term dominates:

$$\Omega_{\chi_+} / \Omega_{\chi_-} \simeq m_{\chi_+}^6 / m_{\chi_-}^6$$

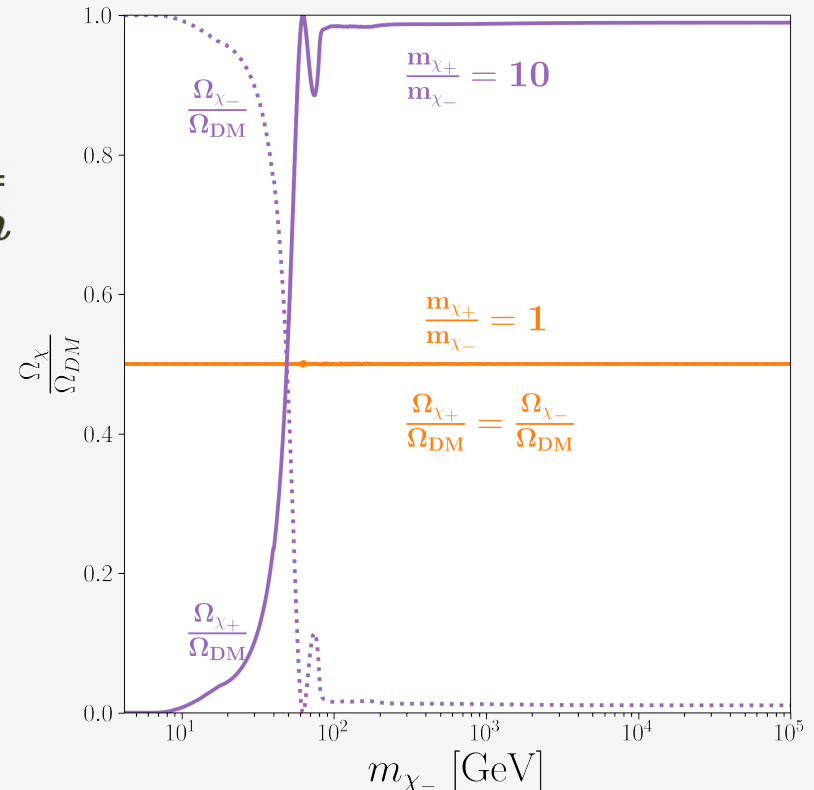


Higgs-portal DD constraints relaxation

$$m_{\chi_{\pm}} \gg m_h \longrightarrow \sigma \sim 1/m_{\chi_{\pm}}^2$$

$$m_{\chi_{\pm}} \ll m_h \longrightarrow \sigma \sim m_f^2/m_h^4$$

- For high masses and high mass ratios, the lightest component is suppressed
- DM-nuclei elastic cross-section scales as $1/m^2$
- DD upper limit relaxes at higher masses

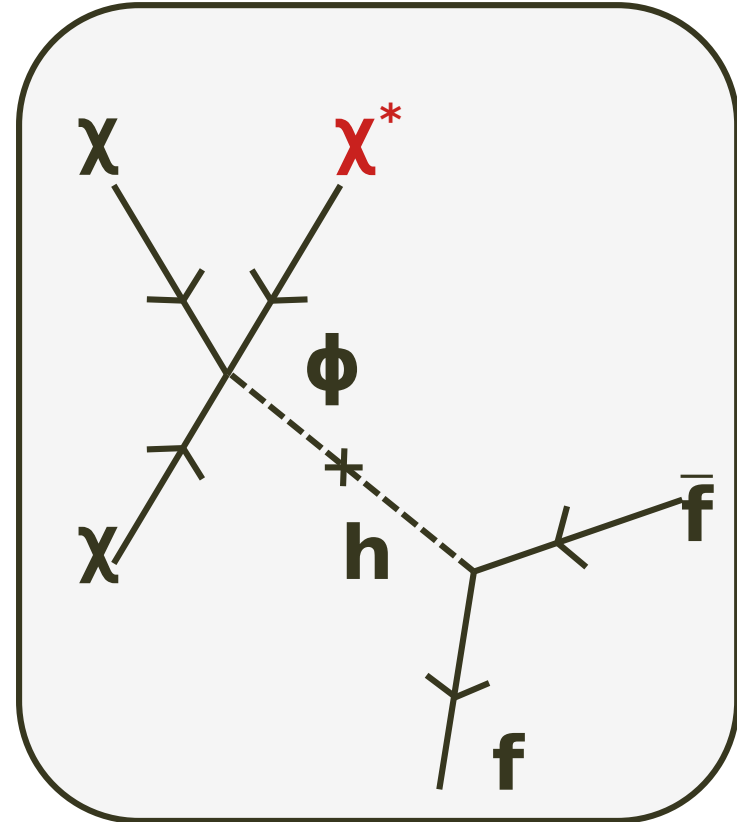


Semi-annihilations

$$LC \chi\chi\chi\phi^\dagger + \text{h.c.}$$

Direct detection of boosted DM in neutrino telescopes

(Berger & al., 2022 Snowmass)



$$\mathcal{L}_{eff} \supset \sqrt{3} \kappa \sin \theta_m (\chi^1 \chi^2 \chi^3 + \chi_1^* \chi_2^* \chi_3^*) h$$

Assuming **semi-annihilation to higgs boson** dominates the freeze-out process:

- All particles are heavier than DM triplet
- All coupling but trilinear and higgs-mixing coupling are small

Monochromatic flux of boosted DM particle:

$$E_\chi = (5m_\chi^2 - m_h^2)/(4m_\chi)$$

