

# Constraints from disk heating

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*News from the dark 8*

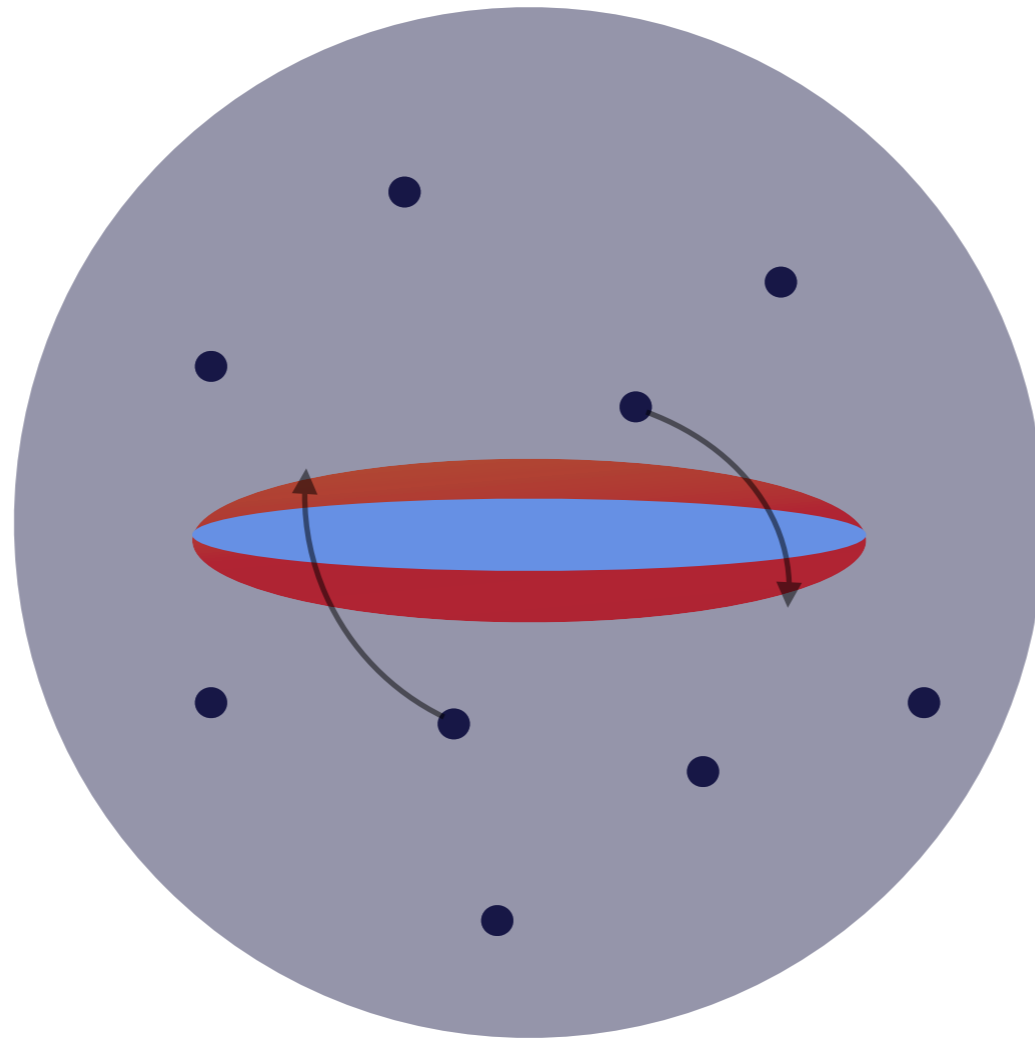
In collaboration with Julien Lavalle



Francesca Scarcella



# *Heating of the galactic disk*



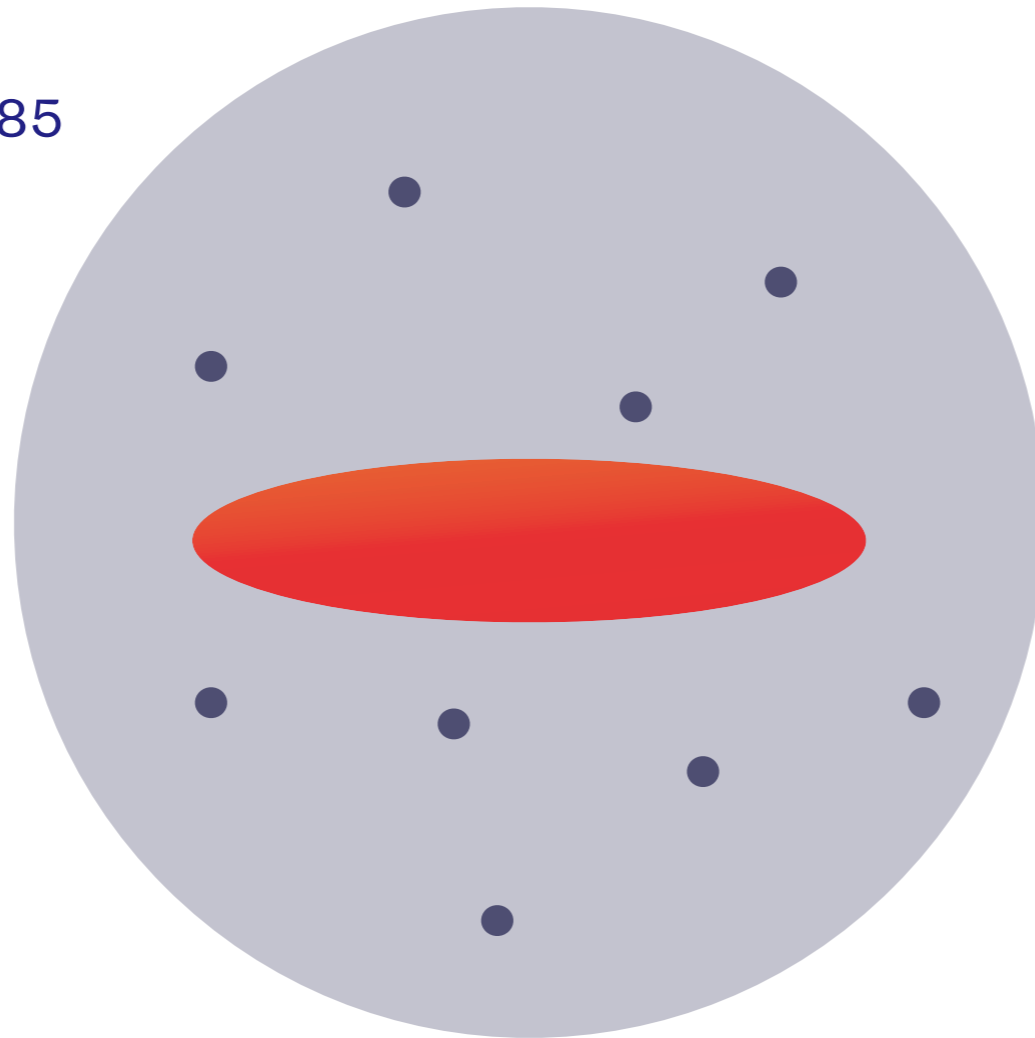
# Heating of the galactic disk

- ▶ MACHOs / PBHs

Lacey and Ostriker 1985

$$M \sim 10^6 M_{\odot}$$

Carr and Lacey 1986



- ▶ DM galactic substructures:

- ▶ PBH clusters

- ▶ Ultracompact minihalos

- ▶ ...

- ▶ Fuzzy DM

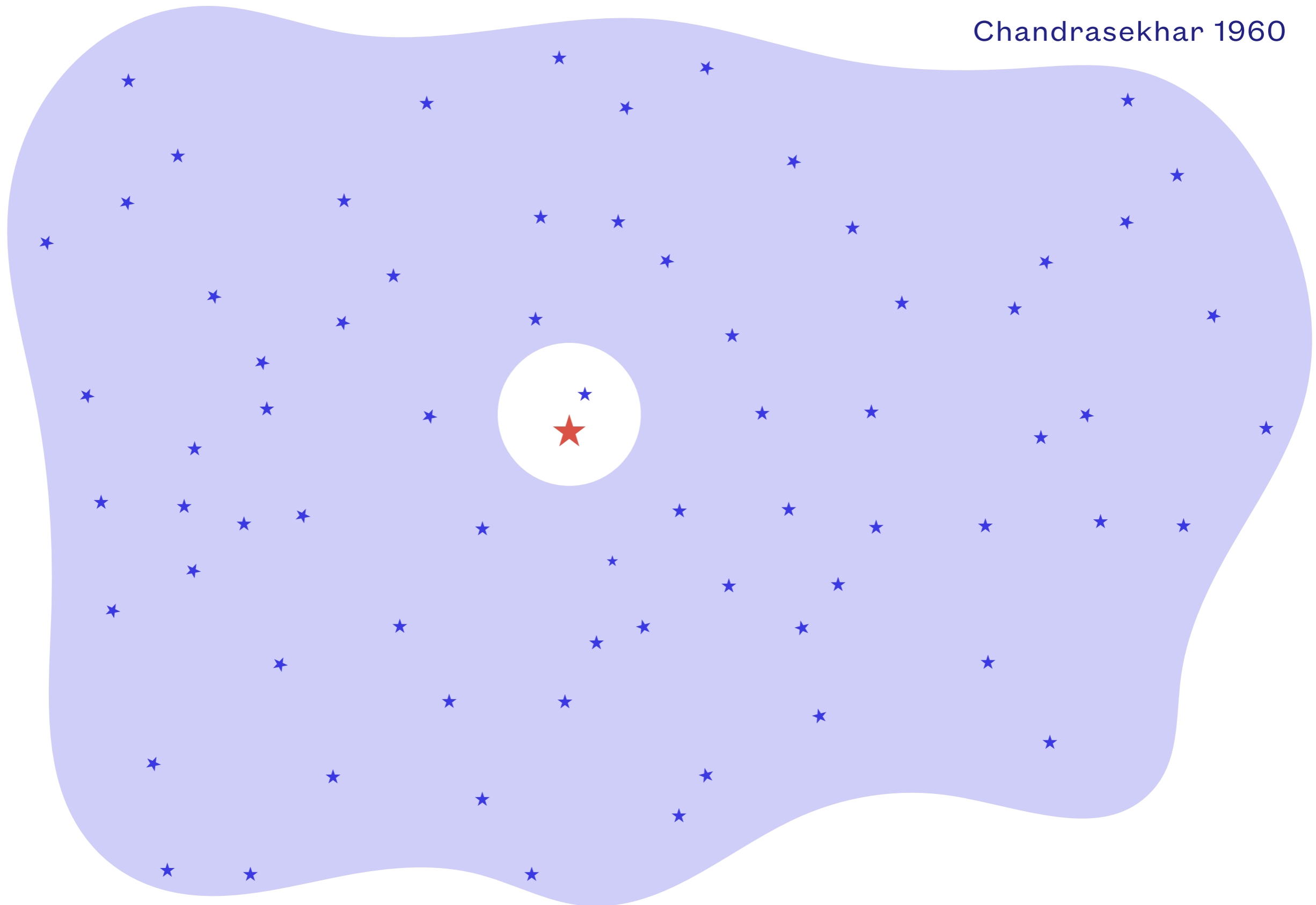
2211.07452

- ▶ Very precise observations of the velocity dispersion of MW stars

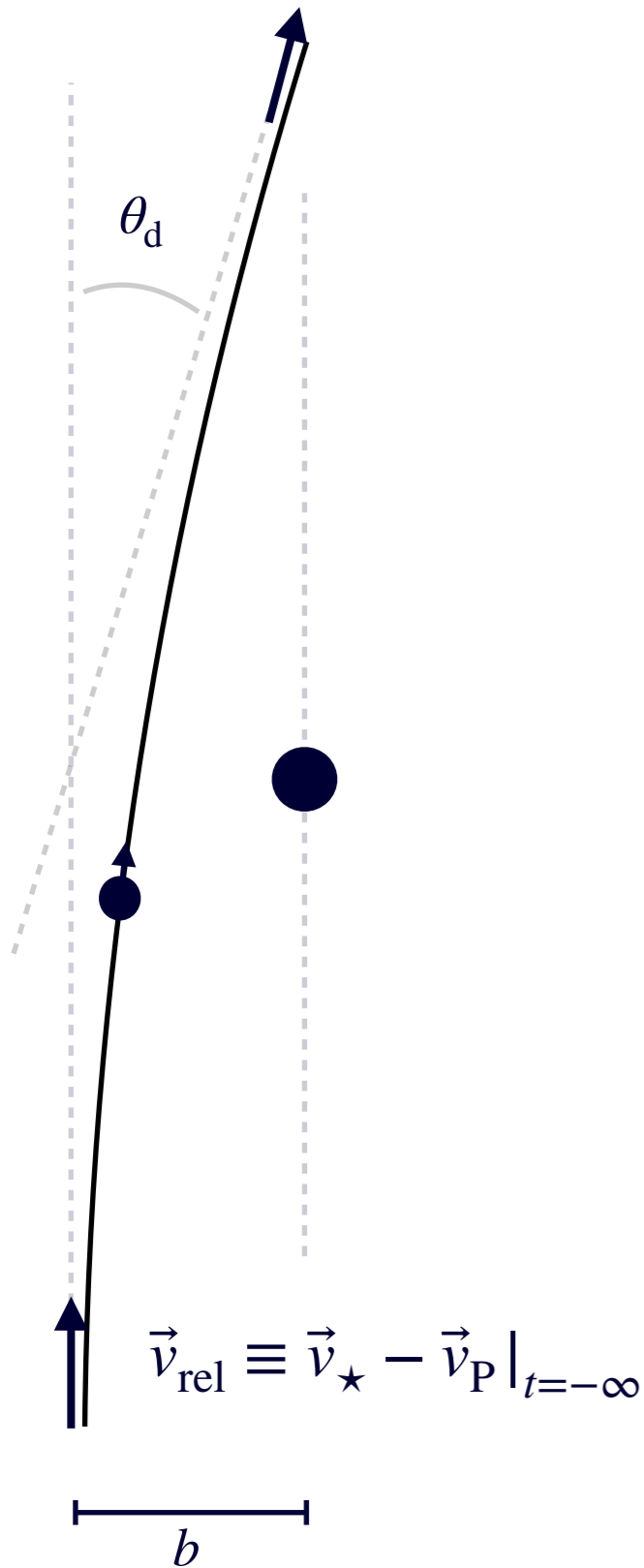
- ▶ Sharma et al. 2004.06556 (Gaia DR2)

# *The classical calculation: setup*

Chandrasekhar 1960



# Variation of the star velocity



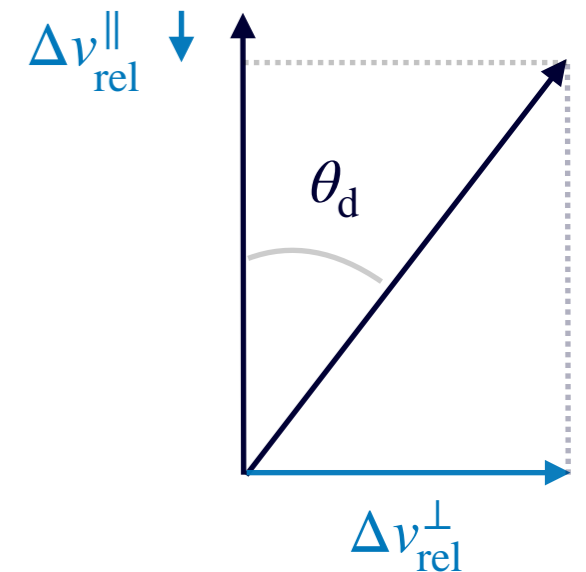
► Deflection angle

$$\theta_d = 2 \tan^{-1} \left( \frac{b_{90}}{b} \right)$$

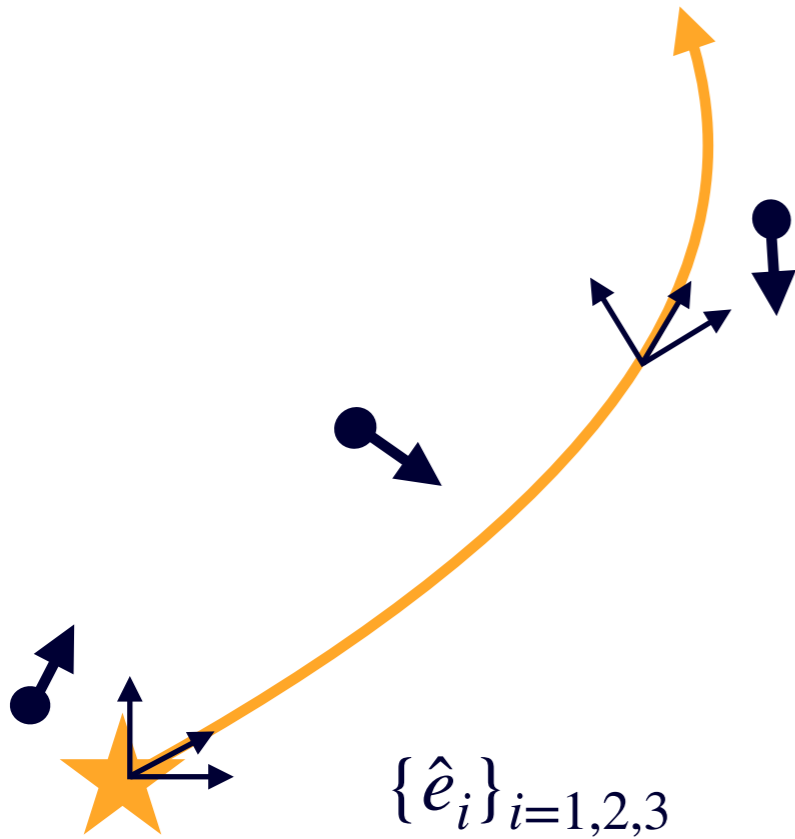
$$b_{90} \equiv \frac{G (m_{\text{P}} + m_{\star})}{v_{\text{rel}}^2}$$

► For  $b \gg b_{90}$

$$\left\{ \begin{array}{l} \Delta v_{\star}^{\perp} \simeq \frac{2v_{\text{rel}}m_{\text{P}}}{m_{\text{P}} + m_{\star}} \frac{b_{90}}{b} = \frac{2Gm_{\text{P}}}{bv_{\text{rel}}} \\ \Delta v_{\star}^{\parallel} \simeq -\frac{2v_{\text{rel}}m_{\text{P}}}{m_{\text{P}} + m_{\star}} \left( \frac{b_{90}}{b} \right)^2 \end{array} \right.$$



# Isotropy and projection on star coordinate system



- ▶ Assume isotropy  $\langle \Delta v_{\star}^{\perp} \rangle = 0$
- ▶ Project along star trajectory

$$\Delta v_i \doteq - |\Delta v_{\star}^{\parallel}| \langle \hat{e}_i | \hat{v}_{\text{rel}} \rangle$$

$$\propto \left( \frac{b_{90}}{b} \right)^2$$

$$\Delta v_i \Delta v_j \doteq \frac{1}{2} |\Delta v_{\star}^{\perp}|^2 \left( \delta_{ij} - \langle \hat{e}_i | \hat{v}_{\text{rel}} \rangle \langle \hat{e}_j | \hat{v}_{\text{rel}} \rangle \right)$$

$$\propto \left( \frac{b_{90}}{b} \right)^2$$

- ▶ The star velocity varies by

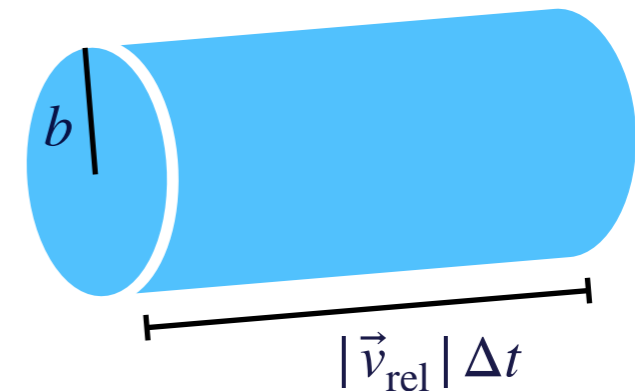
$$\Delta v_i \simeq - \frac{2G^2(m_{\text{P}} + m_{\star})m_{\text{P}}}{|v_{\text{rel}}| b^2} \frac{\partial}{\partial v_i} \left( \frac{1}{|v_{\text{rel}}|} \right)$$

$$\Delta v_i \Delta v_j \simeq \frac{2G^2 m_{\text{P}}^2}{|v_{\text{rel}}| b^2} \frac{\partial^2}{\partial v_i \partial v_j} |v_{\text{rel}}|$$

# Summing over many encounters

- ▶ Number of encounters with impact parameter  $[b, b+db]$ , relative velocity  $[\vec{v}_{\text{rel}}, \vec{v}_{\text{rel}} + d^3\vec{v}_{\text{rel}}]$ , in time interval  $\Delta t$

$$dN(b, \vec{v}_{\text{rel}}) = n_{\text{P}} 2\pi b db |\vec{v}_{\text{rel}}| \Delta t f(\vec{v}_{\text{rel}}) d^3\vec{v}_{\text{rel}}$$



- ▶ Density of perturbers  $n_{\text{P}}$  approximately constant in neighbourhood of star
- ▶ Velocity distribution  $f(\vec{v}_{\text{rel}})$  independent of  $b$

- ▶ Integrate over  $\vec{v}_{\text{rel}} = \vec{v}_{\star} - \vec{v}_{\text{P}}$ 
  - ▶  $\vec{v}_{\star} = \text{const}$  ( $\Delta v_{\star} \ll v_{\star}$ )
  - ▶  $\vec{v}_{\text{P}} \sim MB(\sigma_{\text{P}})$

$$h(\vec{v}_{\star}) = \int d^3v_{\text{P}} \frac{f(\vec{v}_{\text{P}})}{|\vec{v}_{\star} - \vec{v}_{\text{P}}|}$$

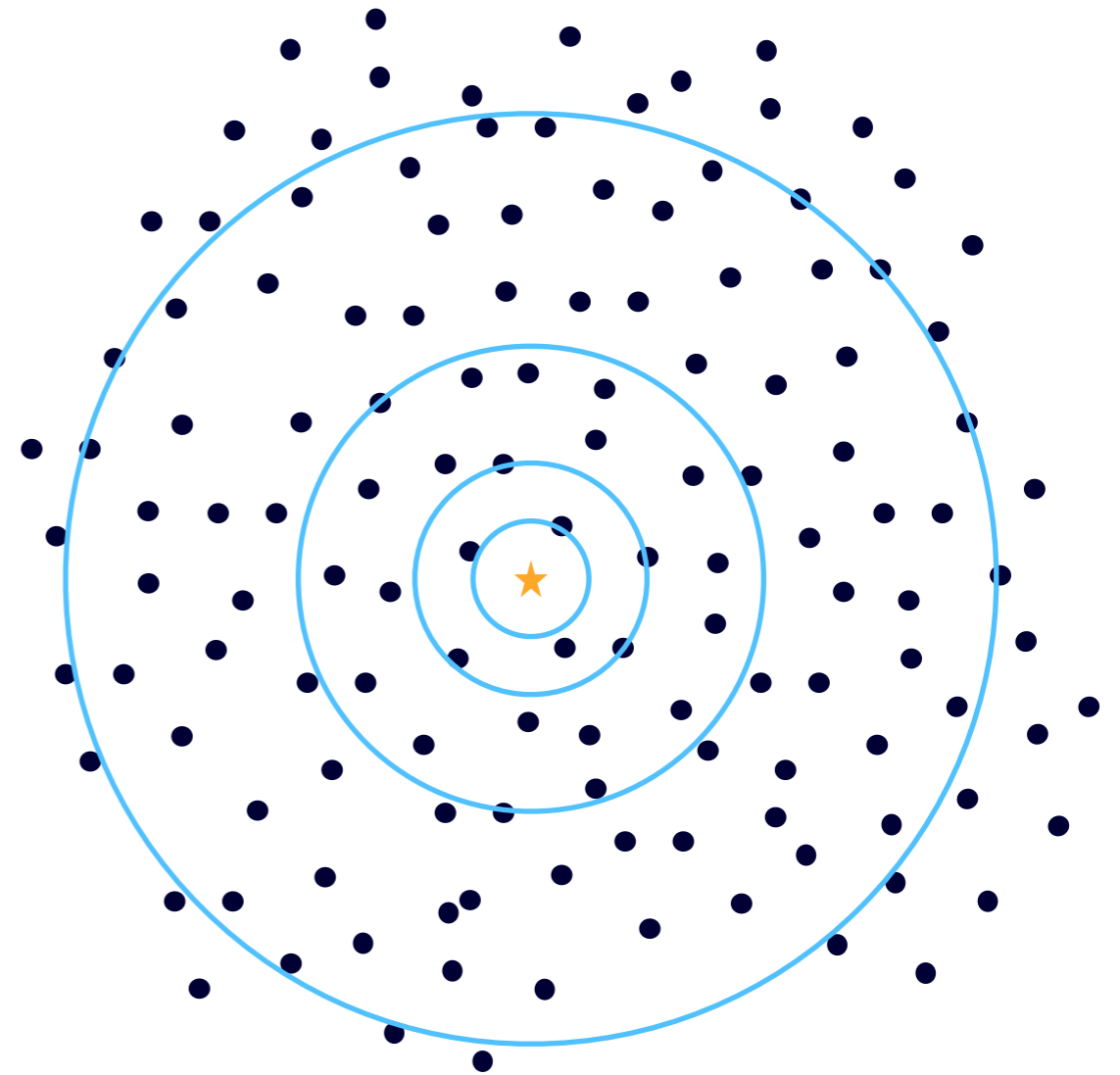
$$g(\vec{v}_{\star}) = \int d^3v_{\text{P}} f(\vec{v}_{\text{P}}) |\vec{v}_{\star} - \vec{v}_{\text{P}}|$$

# *Integral over the impact parameter*

- ▶ Integrate over the impact parameter

$$\int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \log \frac{b_{\max}}{b_{\min}} \equiv \log \Lambda$$

- ▶ Logarithmic divergence with  $b_{\min}$ ,  $b_{\max}$
- ▶ Minimum impact parameter  $b_{\min} > b_{90}$
- ▶  $b_{\max}$  to be set to reasonable value





# *Velocity change of the star due to many encounters*

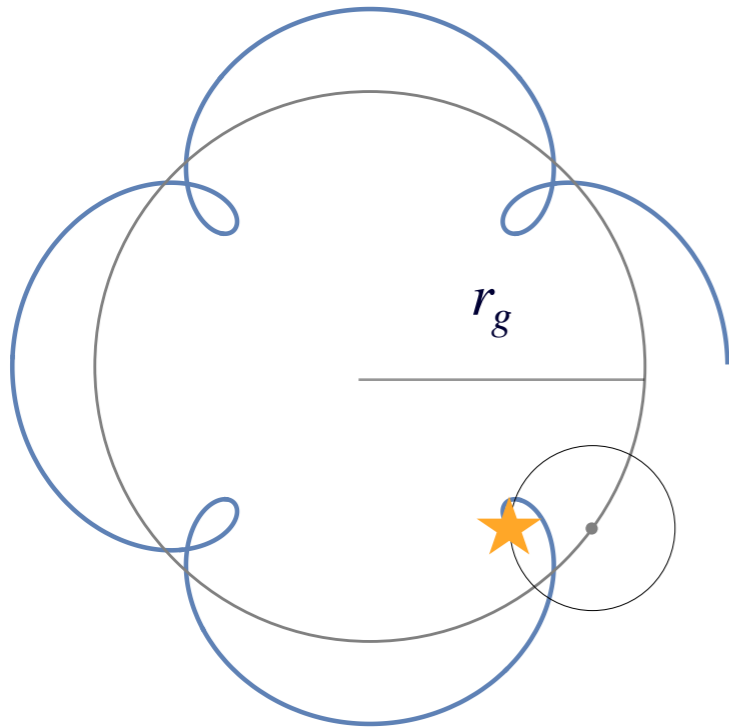
- ▶ Components parallel and perpendicular to the star's velocity

$$\frac{\langle \Delta v_{\parallel} \rangle}{\Delta t} = - \frac{4\pi G^2 n_p m_p (m_p + m_{\star}) \ln \Lambda}{\sigma_p^2} G(X)$$

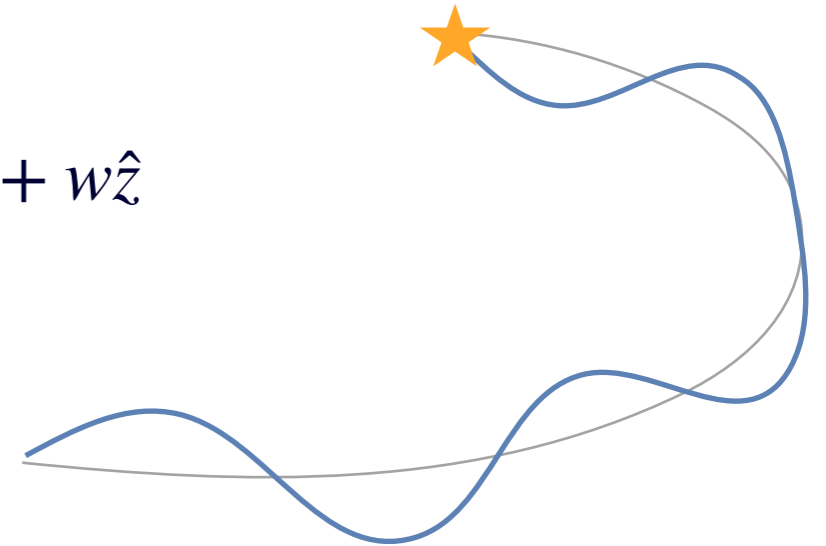
$$\frac{\langle \Delta v_{\parallel}^2 \rangle}{\Delta t} = \frac{4\sqrt{2}\pi G^2 n_p m_p^2 \ln \Lambda}{\sigma_p} \frac{G(X)}{X} \quad X \equiv \frac{v_{\star}}{\sqrt{2}\sigma_p}$$

$$\frac{\langle \Delta v_{\perp}^2 \rangle}{\Delta t} = \frac{4\sqrt{2}\pi G^2 n_p m_p^2 \ln \Lambda}{\sigma_p} \frac{\text{Erf}(X) - G(X)}{X}$$

# Application to epicyclic orbits



$$\vec{v}_\star = u\hat{r} + (v_c(r) + v)\hat{\theta} + w\hat{z}$$



$$\frac{d\sigma_r^2}{dt} = \beta^2 \frac{d\sigma_\theta^2}{dt} = D_e$$

$$\frac{d\sigma_z^2}{dt} = D_z$$

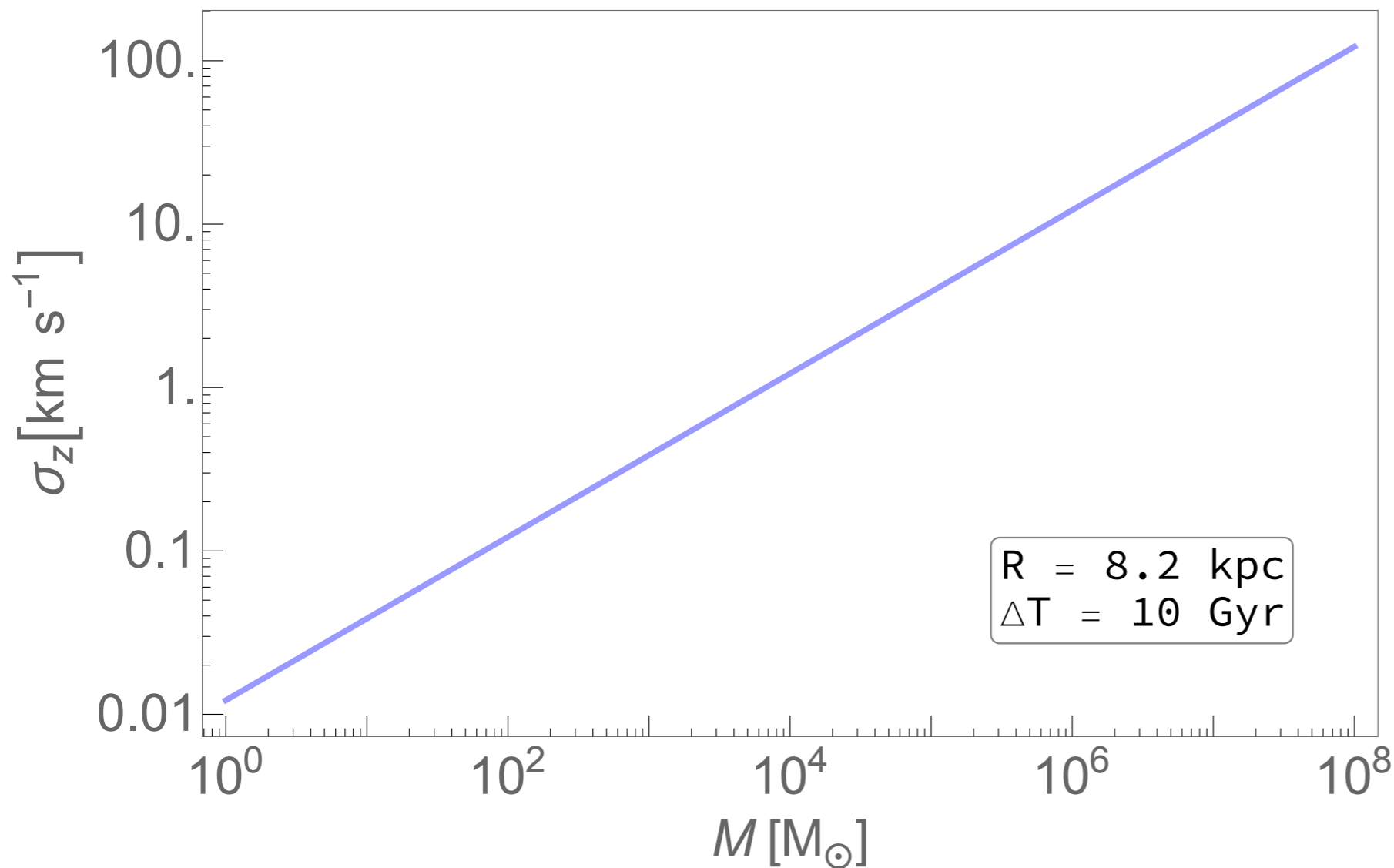
$$D_e \equiv \frac{1}{4} \frac{\langle \Delta v_\perp^2 \rangle_{v_c}}{\Delta t} + \frac{\beta^2}{2} \frac{\langle \Delta v_\parallel^2 \rangle_{v_c}}{\Delta t}$$

$$D_z \equiv \frac{1}{4} \frac{\langle \Delta v_\perp^2 \rangle_{v_c}}{\Delta t}$$

$$\beta = \frac{2\Omega_g}{\kappa}$$

# Vertical heating

$$\frac{d\sigma_z^2}{dt} = \frac{\sqrt{2}\pi G^2 n_p m_p^2 \ln \Lambda}{\sigma_p} \frac{\text{Erf}(X) - G(X)}{X}$$



- ▶ Bkg potential from McMillan 2016 + NFW halo
- ▶ Jeans equation for  $\sigma_p$
- ▶  $\ln \Lambda = 3$

# *Prospect: extended objects*

- ▶ Validity of the pointlike approximation

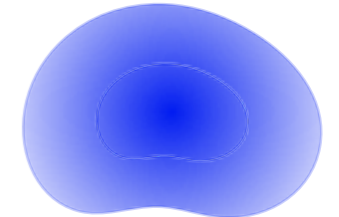
- ▶ Interstellar distance  $\sim 1$  pc



- ▶ Tidal stripping  $\rightarrow$  mass loss

- ▶ R dependent

- ▶ Mass concentration relation



# *Conclusions*

- ▶ Disk heating can allow us to constrain the dark matter distribution on small scale through observations of stellar motion
- ▶ Timely: stellar dynamics data
- ▶ Ongoing work on theoretical modelling: penetrative encounters, tidal stripping, ....
- ▶ Stay tuned !