The invisible dilaton

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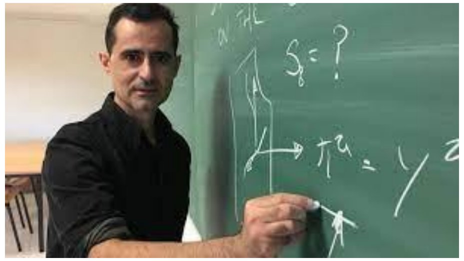

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2303.14469

In the standard model the quadratic μ term in the Higgs potential breaks scale invariance explicitly:

$$V(h) = -\frac{\mu^2}{2}h^2 + \frac{\lambda}{4}h^4$$

Poor man's Higgs: h neutral and not doublet of SU(2).

When fermions are coupled to the Higgs field, this gives rise to fermion masses $\mathcal{L} \supset \lambda_\psi h\bar{\psi}\psi$

$$\mu \neq 0 \;\Rightarrow\; m_\psi = \lambda_\psi \frac{\mu}{\sqrt{\lambda}}$$

Scale invariance could be broken spontaneously leaving a "Goldstone" field parameterising the breaking scale:

$$\mu \to \mu(\phi)$$

The physics is interesting when this field becomes massive with a small mass after electroweak breaking:

$$m_\phi \ll m_h$$

As the breaking scale is (almost) the only scale in the theory, the dilaton can acquire two types of potential terms:

$$V(\phi) = -g\Lambda_c^2\mu^2(\phi) + a\mu^4(\phi)$$

The cut-off scale introduce another scale which allows for this operator. This is automatically generated by closing the Higgs loop from:

$$\frac{\mu^2(\phi)}{2}h^2$$

The sign is chosen such that μ decreases in the cosmological evolution.

As the dilaton is light, the Higgs field can be integrated out:

$$h(\phi) = \frac{\mu(\phi)}{\sqrt{\lambda}}$$

In this Higgs phase, the potential picks up a term:

$$a = -\frac{1}{4\lambda}$$

In the Higgs phase, the Higgs and the fermions acquire a mass. This implies Coleman-Weinberg corrections to the dilaton potential:

$$V(\phi) \to V(\phi) + \frac{\mu^4(\phi)}{16\pi^2} \ln \frac{\Lambda_c^2}{2\mu^2(\phi)}$$

Here the Higgs contribution.

This can be reabsorbed in a redefinition of the Higgs self-coupling $\lambda(\phi) = \lambda(1 + \frac{\lambda}{16\pi^2} \ln \frac{\Lambda_c^2}{2\mu^2(\phi)})$

In the end, the only dynamical scale in the problem is the μ term and predominantly :

$$V(\phi) \simeq -g\Lambda_c^2\mu^2(\phi)$$

As a consequence, the minimum of the potential (before or after Higgs breaking) is always such that:

$$\partial_\phi \mu(\bar{\phi}) = 0$$

This has important consequences at low energy after integrating out the heavy Higgs field:

$$h(\phi) = \frac{\mu(\phi)}{\sqrt{\lambda}} = v + \frac{\partial_\phi^2 \mu(\bar{\phi})\varphi^2}{2\sqrt{\lambda}} \longrightarrow \delta\mathcal{L} = \frac{\varphi^2}{2\Lambda_f^2}\bar{\psi}\psi$$

The dilaton field $\varphi = \phi - \bar{\phi}$ couples **quadratically** to matter!

Recent into for quadratically coupled axions:

2307.10362

The suppression scales depends inversely on the mass of the dilaton :

$$\Lambda_f^2 = \frac{2g\mu^2(\bar{\phi})\Lambda_c^2}{m_\phi^2}$$

This coupling will be very much suppressed for light scalars, highly relevant to hide light scalars. Hence very light dilatons will be **invisible**.

In the presence of matter, this coupling has two effects:

- The mass of fermions becomes dilaton dependent:

$$m_\psi = \lambda_\psi v (1 - \frac{\varphi^2}{2\Lambda_f^2})$$

Very well-known quadratic dependence known as scalarisation /symmetron in the context of modified gravity.

- In the presence of macroscopic matter, the effective potential is modified and becomes:

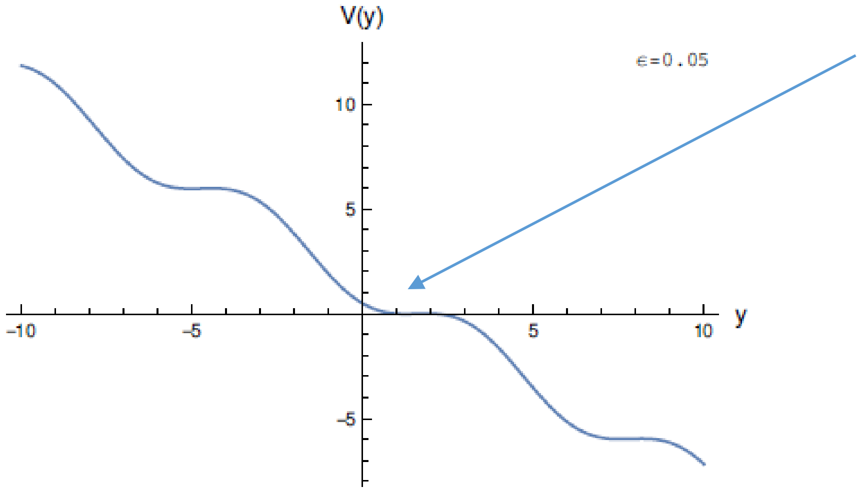
$$V_{\rm eff}(\phi) = V(\phi) + \lambda_\psi \frac{\mu(\phi)}{\sqrt{\lambda}} n_\psi$$

Number density

In the presence of matter, the field can be affected. Can lead to **screening.**

In practice, the minimum of the effective potential will be hardly sensitive to matter densities. So the electron mass will be the same in the Sun and in vacuum. On the other hand, a small ball of matter could affect the profile of the dilaton variation φ .

Typically we will consider potentials for the dilaton like this:

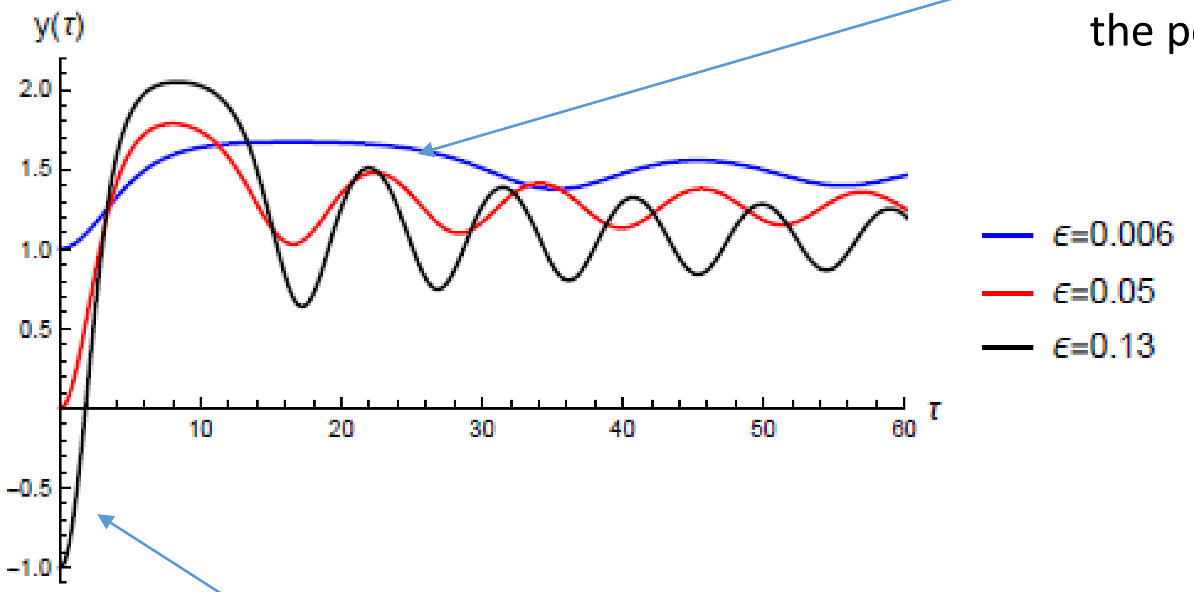


Minimum with a very flat potential around it. This guarantees that the dilaton is lighter than the Higgs.

What is the cosmology of such a model?

- Must be a spectator field during inflation.
- Must converge towards the “right” minimum at the right time (EW transition)

As the potential is not a tracking potential, the long time evolution of the field is dependent on the initial conditions. The convergence and oscillations around the "right" minimum specifies a basin of attraction for the initial condition.



Oscillations around the minimum of the potential.

Initial phase described by a slow roll evolution post-inflation.

This works provided the dilaton is at rest during inflation. This can be realised if, for instance, the dilaton couples conformally to the inflation field:

Jordan metric

$$S_{\rm inf} = \int d^4x\sqrt{-g_J}\mathcal{L}_{\rm inf}(\Phi, g^J_{\mu\nu} = A^2(\phi)g_{\mu\nu})$$

This is a “trick” which modifies that effective potential of the dilaton during inflation as:

$$V_{\rm eff}(\phi) = V(\phi) - T_{\rm infl}(A(\phi) - 1) \qquad T_{\rm inf} = -4V_{\rm inf}$$

The simplest choice is to pin the dilaton by the coupling to the inflaton energy density:

$$A(\phi) = 1 + \frac{(\phi - \phi_e)^2}{2m_{\rm Pl}^2}$$

For a recent review on stopping the relaxion:

1911.08473

This guarantees that the field is stuck at ϕ_e during inflation with a large mass:

$$m_{\phi,\rm inf}^2 = 12H_{\rm inf}^2$$

As a result no “isocurvature” fluctuations due to the dilaton during inflation. So at the background cosmology level, the dilaton only plays a role when it starts oscillating around its minimum → **misalignment mechanism**.

The quadratic coupling could imply that the dilaton is in thermal equilibrium with the fermion bath. This is only true at temperatures higher than:

$$T \geq T_{\rm de} = \frac{\Lambda_f^2}{m_\psi m_{\rm Pl}^2}$$

Large when the scalars are light.

This is always larger than the EW scale for light scalars of masses less than 1 eV, which is required to guarantee that the scalar field behaves like a classical condensate. So light scalars can be dark matter by the initial misalignment and not thermal equilibrium.

Let us give an example of interesting potential largely inspired by the relaxion models:

$$\mu^2(\phi) = \Lambda^2(1 - \frac{\phi}{f}) + M^2 \cos\frac{\pi}{2}\frac{\phi}{f}$$

$$\frac{\Lambda^2}{M^2} = \frac{\pi}{2}(1-\epsilon)$$

The minimum is at:

$$\frac{\bar\phi}{f} = 1 + \bar\delta \qquad \bar\delta = -\frac{2}{\pi}\sqrt{2\epsilon}$$

$$\epsilon = (\frac{3\lambda v^2}{2\sqrt{2}M^2})^{2/3} \ll 1$$

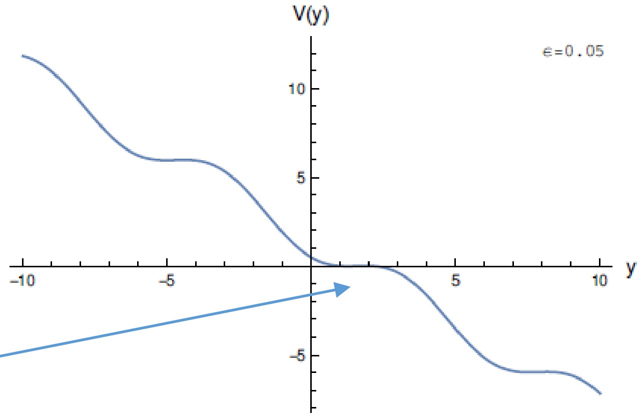
In these models the mass is typically hierarchically small:

See also 1810.01889

$$m_\phi = (2\epsilon)^{1/4} m_0$$

$$m_0^2 = \frac{\pi^2}{4} g \frac{M^2\Lambda_c^2}{f^2}$$

The field starts oscillating when: $H = m_0$

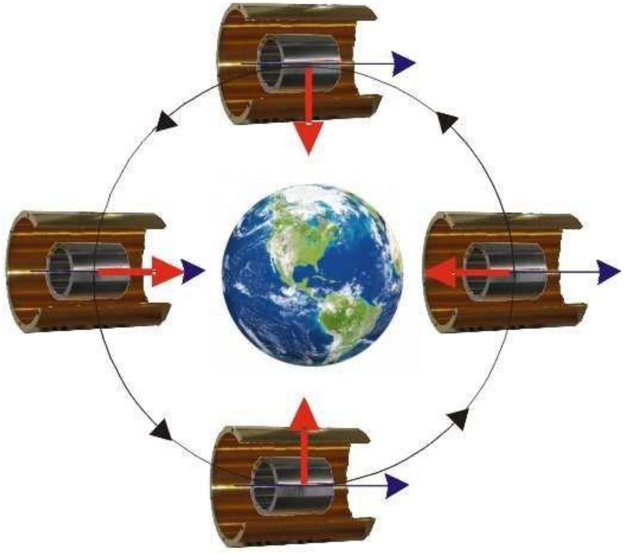


The initial dark matter density is suppressed:

$$\rho_{\rm ini} \simeq \frac{1}{2} m_\phi^2 \varphi_0^2 \simeq g\Lambda_c^2 M^2 \epsilon^{3/2}$$

This decays as the inverse of the scale factor cubed with time. This field could live its dark matter life without being bothered by matter if there was no quadratic couplings. Let us see what happens.

The dilaton couples to each constituents of the nuclei, i.e. the gluon condensate , the electromagnetic energy and the constituent quark masses. As atoms contain different numbers of nuclei, the effective coupling to different types of material will be different. This could lead to a violation of the equivalence principle.



$$\eta_{AB} = \frac{|\vec{a}_A - \vec{a}_B|}{|\vec{a}_A + \vec{a}_B|}$$

$$\eta_{AB} \leq 5.10^{-15}$$

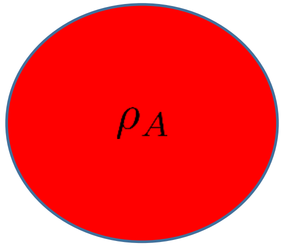
A=Platinum B=Titatium

2209.15487

The Klein-Gordon equation of the dilaton in the presence of matter is:

See also 1807.04512

$$\Box\varphi = m_\phi^2\varphi - Q_A\frac{\varphi}{\Lambda_f^2}\rho_A$$



The coupling depends on a phenomenological factor Q which is tabulated for different materials.

$$\varphi_\infty = \varphi_0 \cos m_\phi t$$

The solution reads:

$$\varphi(r) = \phi_\infty(t) f(\frac{r}{R_A})$$

$$Q_A < 0 \;:\; x < 1 \;\; f(x) = \frac{1}{\cosh u}\frac{\sinh ux}{ux}, \;\; x > 1 \;\; f(x) = 1 - \frac{1}{x}(1 - \frac{\tanh u}{u})$$

1/r behaviour which modifies gravity.

Gravity is modified depending on the screening parameter:

Similar to screening criterion for modified gravity.

$$u = \sqrt{|Q_A|\Phi_A \frac{6m_{\rm Pl}^2}{\Lambda_f^2}}$$

See for instance 1203.4812

As for screening of scalar dark energy, this happens for objects with a **large Newtonian potential**. The exterior field is then given by:

$$\varphi = \varphi_\infty(t)(1 - s_A \frac{G_N M_A}{r})$$

$$\begin{cases} u \gg 1: \; s_A = \dfrac{1}{\Phi_A} \\ u \ll 1: \; s_A = 2Q_A \dfrac{m_{\rm Pl}^2}{\Lambda_f^2} \end{cases}$$

The dilaton imparts an extra acceleration to bodies:

$$\vec{a}_A = -\frac{Q_A \varphi}{\Lambda_f^2}(\vec{\nabla}\varphi + \vec{v}_A \dot{\varphi})$$

This implies that a non-zero Eotvos parameter is generated:

$$\eta_{AB} = \frac{\varphi_0^2}{2\Lambda_f^2}|Q_A - Q_B| s_C (1 - s_C \frac{G_N M_C}{r})$$

No effect at the surface of the Earth C, only effect in satellites.

For a cut-off scale around 10 TeV, the screening criterion gives:

$$\begin{cases} m_\phi \geq 1\text{keV} \quad \text{Earth} \\ m_\phi \geq 10\text{eV} \ \ \text{Sun} \end{cases}$$

The dilaton is unscreened!

Despite being unscreened, the day is saved as the coupling is proportional to the square root of the dark matter density, so very small (unless we lived in a clump?)

$$\eta_{AB} \simeq \beta^2, \ \beta \simeq 10^{-24} \frac{m_\phi}{1\mathrm{eV}} (\frac{1\mathrm{TeV}}{\Lambda_c})^2$$

The small effective coupling β is what matters for other types of tests of this model:

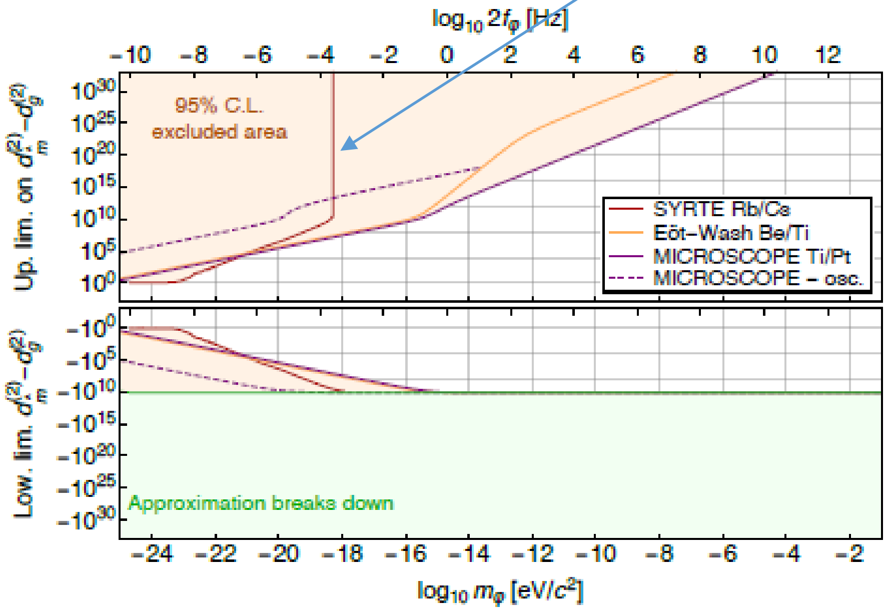
- Atomic clocks: the dilaton could induce a change of the energy level of atoms for very light dilaton. This requires

$$m_\phi \leq 10^{-18}\mathrm{eV} \ \Rightarrow \beta \leq 1$$

- The dilaton has a quadratically induced coupling to photons which leads to an effective coupling in the dark matter halo of order β. The decay rate is too small to be observable, its inverse is much larger than the age of the Universe.

$$\Gamma_{\varphi\to\gamma\gamma} \simeq \beta^2 \frac{m_\phi^3}{m_{\mathrm{Pl}}^2}$$

Atoms become screened.



1807.04512

The dilaton could also affect the growth of structure by modifying the coupling to gravity:

$$G_N \to (1 + 2\beta(\rho)^2) G_N \qquad \beta(\rho) = \frac{\beta}{a^{3/2}}$$

Scale factor.

This is valid on scales smaller than the Compton wavelength of the dilaton:

$$\frac{k}{a} \geq m_\phi$$

Even at the EW scale for a redshift,

$$z_{\mathrm{EW}} \sim \frac{v}{\Lambda_{\mathrm{DE}}} \simeq 10^{14}$$

This would not compensate the extreme smallness of the coupling.

CONCLUSIONS

- The dilaton coming from the breaking of scale invariance could lead to models of scalar dark matter quadratically coupling to matter and vector bosons.

- Quadratic effects lead to screening around compact objects.

- Light dilatons are not screened around the Earth and the Sun but evade all known tests by being extremely weakly coupled due to the very smallness of the local dark matter density.

- Maybe this type of dark matter could lead to fuzzy dark matter clumps of extremely small sizes where the density would be much higher as it varies like the inverse of the size to the fourth? Would this lead to observable effects locally if the Earth crossed a clump?

see 1710.04323