Dark matter in the exponential growth scenarios
arXiv:2308.09801 [hep-ph]

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Null results of dark matter

Rethink:

Production mechanisms

Detection prospects

Various production mechanisms of dark matter

Thermal Freeze-out

Freeze-out to SM states

$\chi\chi \to SM\,SM$

via SM particles: Higgs, Z

$(m_\chi > O(10\,\text{GeV}))$

(Lee-Weinberg)

via BSM particles

DM can be lighter

Freeze-out to new states

$\chi\chi \to \phi\phi$

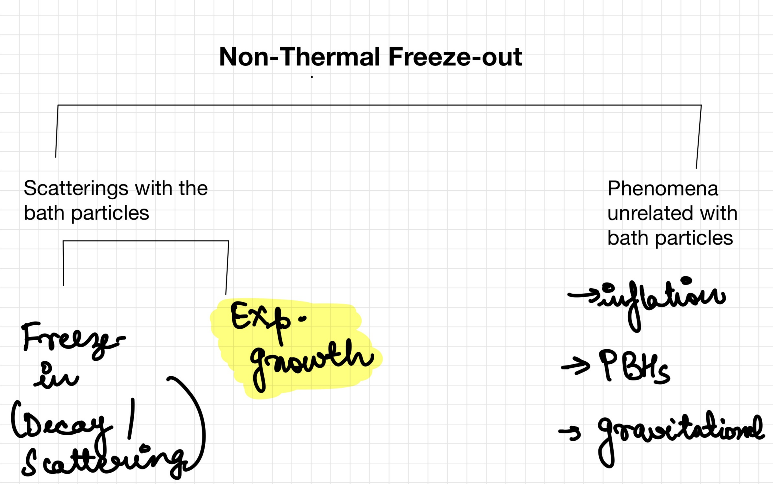
$\chi\chi \to \chi\phi$

Cannibalism

$n\chi \to 2\chi$

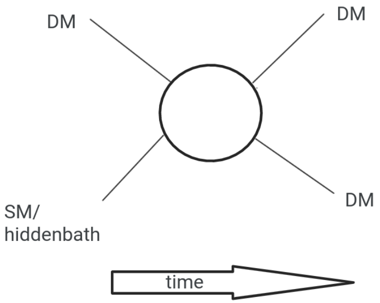
$(n > 2)$

Various production mechanisms of dark matter



Exponential growth mechanism

T. Bringmann, J.T. Rudermann et al, PRL 21, A. Hryczuk, M. Laletin, JHEP, 21



- Non-thermal process
- Initial dark matter density must be non-zero.

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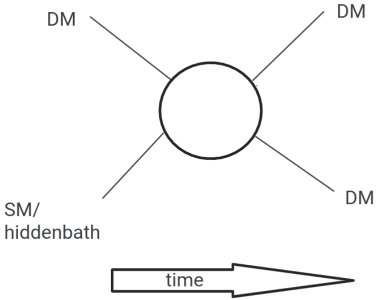

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Boltzmann equation for dark matter abundance

- For scattering: $\chi(p) + \phi(p') \to \chi(k) + \chi(k')$
- Abundance through Boltzmann equation i.e. $\mathcal{L}[f_\chi] = \mathcal{C}[f_\chi]$

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\begin{aligned}
\mathcal{L}[f_\chi(p,t)] &= \left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right) f_\chi(p,t)\,, \\
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Dark matter abundance continued...

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i.e. dark matter is in (*kinetic)* equilibrium
(Gondolo and Gelmini, 91)

- With this, the dark matter density can be estimated by considering the zeroth moment of the Boltzmann equation:

$$\frac{1}{a^3}\frac{d}{dt}(n_\chi a^3) \;=\; \langle \sigma v\rangle n_\phi^{\rm eq} n_\chi \left(1 - \frac{n_\chi}{n_\chi^{\rm eq}}\right)$$

Standard case for semi-annihilations

- Or coupled Boltzmann equation in n_χ and T'

(Binder, Bringmann, PRD 2017)

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Exponential production

- Solved using f_χ tracing equilibrium pattern

- Initial DM density small

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Since the production is non-thermal, does the equilibrium assumption hold true?

DB, 2308.09801[hep-ph]

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- Model dependence $\Rightarrow$ S and $|M|^2$ and in the initial condition.

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Which can be solved in the comoving frame

$$(p, t) \rightarrow (q, t)$$

$$\left[q = p\,a = \frac{p}{s^{1/3}} \right]$$

where $\frac{dq}{dt} = 0$

hence $\frac{d}{dt} f_\chi(q, t) = \frac{\partial}{\partial t} f_\chi(q, t)$

Boltzmann equation reduces to infinitely many ordinary differential equations

Generalised solutions to exponential growth scenarios

$$
\begin{aligned}
f_\chi(q,x) &= f_\chi(q,x_{\rm init}) \exp\left(\int_{x_{\rm init}}^{x} dx\, P'(q,x)\right) \quad \text{where ,} \\
P'(q,x) &= \frac{h_{\rm eff}(x) g_\chi g_\phi}{S\; H(x)\; x} \int \frac{d^3p'}{(2\pi)^3} \left(\sigma v_{\rm mol}(q,p')\right)\; f_\phi^{\rm eq}(p',x)\;.
\end{aligned}
$$

The above equation is similar to any growth/decay equation
$\Rightarrow P'(q,x)$ is a growth function.

$P'(q, x) > 0$ For DM production

$\to 0$ To avoid DM overproduction

$\downarrow$

$m_\phi > m_\chi \to f_\phi^{eq} \to$ Boltzmann supp.

$m_\chi > m_\phi \to (\sigma v)$ is Boltzmann suppressed.

Growth function

- Since the growth function depends on q, not all momentum modes grow similarly with the expansion of the universe.
- DM growth in the universe is purely exponential if $P'(q, x)$ is constant in x.
- In general the growth function can be complicated function in x and it is hard to parameterise the growth as simple exponential.
- The growth of the distribution in general can scale as a factor of exponential $\exp[A(x)]$.
- We still call growth as exponential to avoid new names.

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Comparison between the generalised approach and the equilibrium assumption:

Generalised approach for distribution function:

$$Y_\chi(x) = g_\chi \int \frac{d^3q}{(2\pi)^3} \left[f_\chi(q, x_{\rm init}) \exp\left(\int_{x_{\rm init}}^{x} dx \; P'(q,x) \right) \right] \quad \text{where} \; ,$$

$$P'(q,x) = \frac{h_{\rm eff}(x) g_\chi g_\phi}{S \; H(x) \; x} \int \frac{d^3p'}{(2\pi)^3} \left(\sigma v_{\rm mol}(q,p')\right) \; f_\phi^{\rm eq}(p',x) \; .$$

Simplified approach for distribution function:

$$Y_\chi(x) = Y_\chi(x = x_{\rm init}) \exp\left(\int_{x_{\rm init}}^{x} dx \; P(x) \right) \; , \quad \text{where}$$

$$P(x) = \frac{h_{\rm eff}(x) \; n_\phi^{\rm eq} \langle \sigma v \rangle}{\mathrm{S} \; x \; H(x)}$$

Comparison between the generalized approach and the equilibrium assumption:

- Simplified approach:
  - The dependence on initial conditions is through appearance of $Y_\chi(x = x_{\rm init})$.
  - Do not need to specify the process of generation of the initial density.
- Generalised approach:
  - The dependence on initial conditions is through appearance of $f_\chi(q, x = x_{\rm init})$
  - The final relic intrinsically depends on the initial process populating dark matter and the momentum modes.

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What kind of models?

- Basically all models explaining semi-annhililations in the low coupling regions.
- A Z_3 symmetric dark matter interaction with a singlet scalar $\Rightarrow \chi^3\phi+$ h.c.
- A mass-mixed fermion having self interactions via scalar/gauge boson $\Rightarrow \Delta m\overline{\chi_1}\chi_2$ with $\overline{\chi_1}\chi_1\phi$ or $\overline{\chi_1}\gamma^\mu\chi_1 Z'_\mu$
- As the process requires some initial density of dark matter, some mechanism beyond the above must be present to generate the initial abundance.

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First: Toy model for exponential growth mechanism

- The models with fermionic mass mixings/ or Z_3 symmetric dark matter can give rise to scatterings of the form: $\chi + \phi \to \chi + \chi$.
- We consider a scalar DM coupled with a bath particle ϕ.
- For simplicity, we assume ϕ to be coupled with the SM bath.
- We consider the case where $m_\phi > m_\chi$ but $m_\phi < 3m_\chi$ to avoid $\phi \to \chi\chi\chi$.
- The growth function in this case:

$$P'(q,x) = \frac{|\lambda_{\rm tr}|^2 h_{\rm eff}(x)}{32\pi E_q \, x \, H(x)} \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} \frac{\sqrt{s - 4m_\chi^2}}{\sqrt{s}} f_\phi^{\rm eq}(p', x)$$

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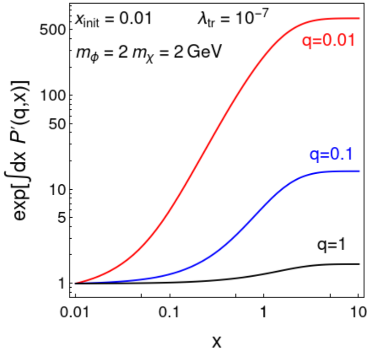
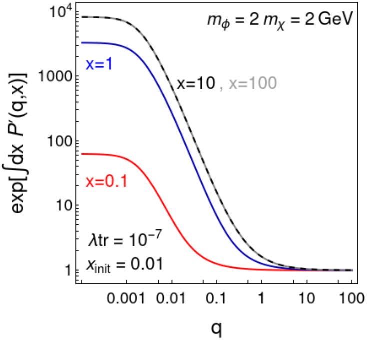
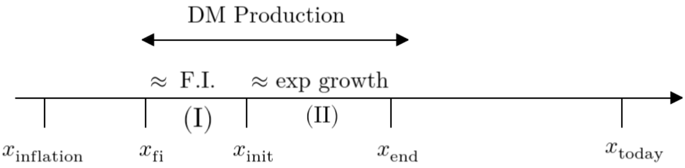




$P'(q,x) \propto 1/E_q \Rightarrow$ low momentum modes populate more than high momentum modes.

Initial conditions

Freeze-in



- The collision operator in region (I) can be approximated as:

$$\mathcal{C}^{(\mathrm{I})}[f_\chi(q,x)] \approx \frac{1}{2E_q\, S}\int \frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{d^3k}{(2\pi)^3 2E_k}\frac{d^3k'}{(2\pi)^3 2E_{k'}}(2\pi)^4\delta^4(P_i - P_f) \times f_\chi^{\rm eq}(q,x) f_\chi^{\rm eq}(p',x)\,|M|^2_{\phi\phi\to\chi\chi}$$

- Here also we neglect the backward scattering, and can easily compute $f_\chi(q, x_{\rm init})$

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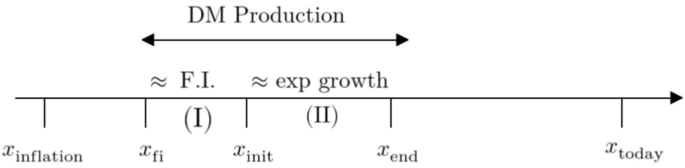

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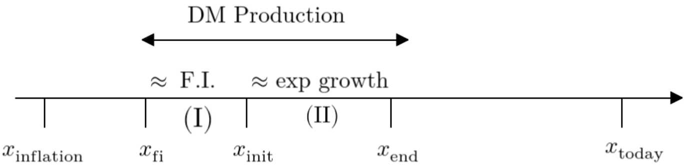

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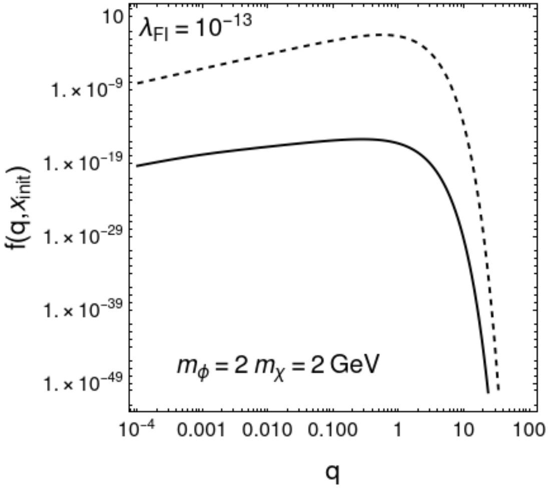

$\lambda_{FI} = 10^{-13}$
$m_\phi = 2\, m_\chi = 2\,\mathrm{GeV}$
f(q, x_{init})
q
10
1. × 10^-9
1. × 10^-19
1. × 10^-29
1. × 10^-39
1. × 10^-49
10^-4
0.001
0.010
0.100
1
10
100

Out-of-equilibrium decay of a heavy non-relativistic particle

- The initial distribution function can be solved by considering: $\Phi \to \chi\chi$
- Ideally it depends on the f_Φ.
- Simplification when $m_\Phi \gg m_\chi \Rightarrow p_\chi \approx m_\Phi/2$
- The initial distribution function in this case (heuristically):

$$f_\chi(q,x) = A\,\delta(|q| - q_{\rm init})\,\Theta\left(x - x_{\rm init}\right) \ .$$

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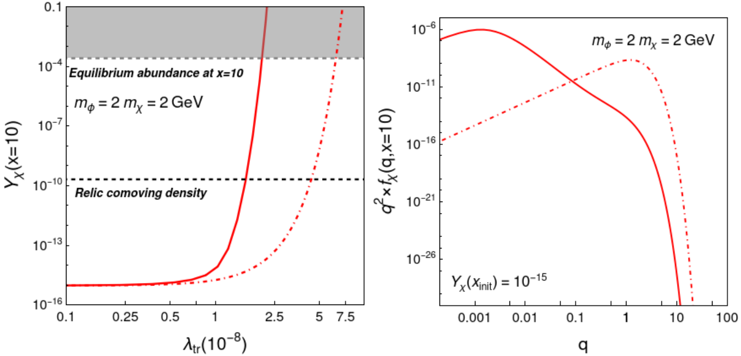
Freeze-in results:



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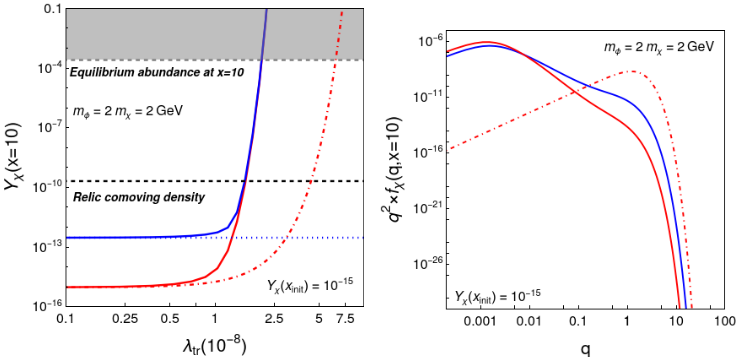
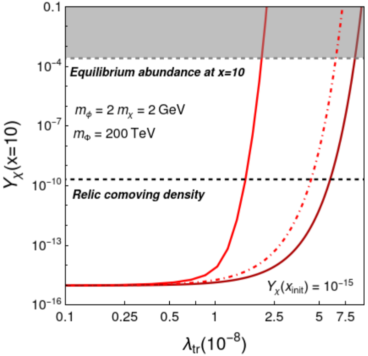


Out-of-equilibrium decay of a heavy non-relativistic particle continued...



Summary and concluding remarks

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- Large number of possibilities to explore.
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