Recent Developments in Relativistic Embeddings of Modified Newtonian Dynamics

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MINISTRY OF EDUCATION,
YOUTH AND SPORTS

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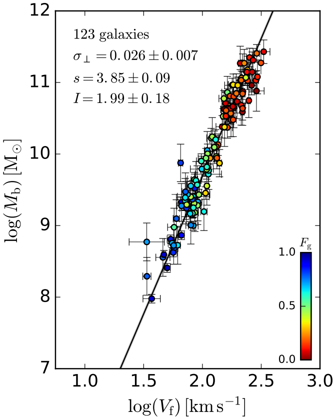

Figure: Power-law relation between v_∞ and M_b. Slope consistent with 4. (Lelli et al., 2019)

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- Squaring again find that $v_\infty^4 = (a_0 G)\, M_b$, the baryonic Tully-Fisher relation, with the constant now identified!

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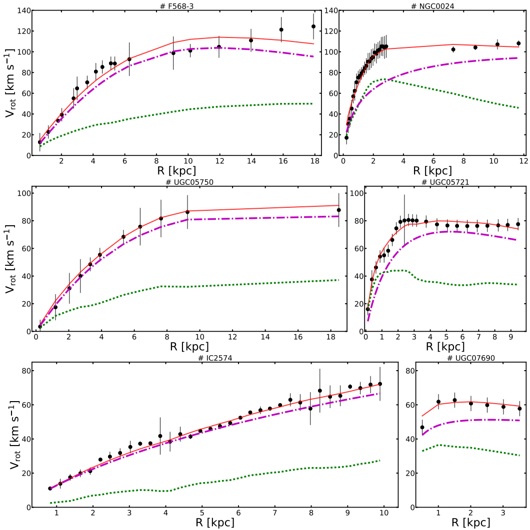
$v_\infty^4 = a_0 GM_b$ is not all. Diversity of rotation curves.



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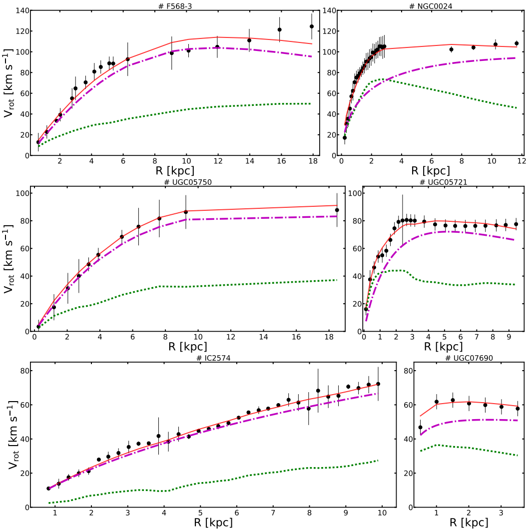

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- Same v_∞, slow/fast approach to v_∞.

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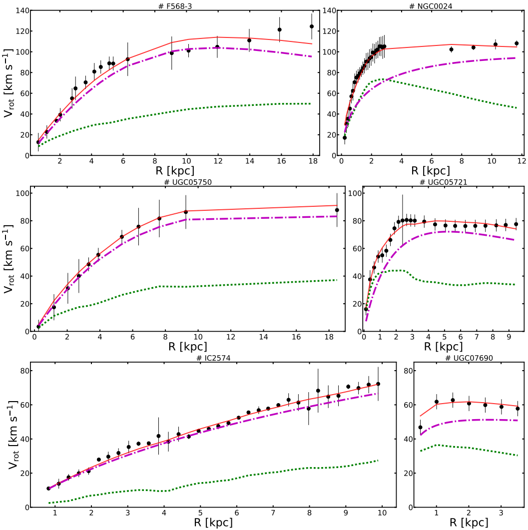

- Three v_∞ twins (Ghari, Famaey et al., 2019).
- Same v_∞, slow/fast approach to v_∞.
- Rotation curve tracks trend of baryonic contribution.

Old news.

Additional galactic regularity: the Radial Acceleration Relation (RAR)

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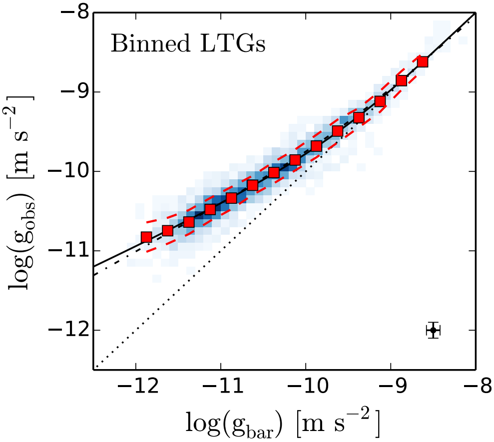
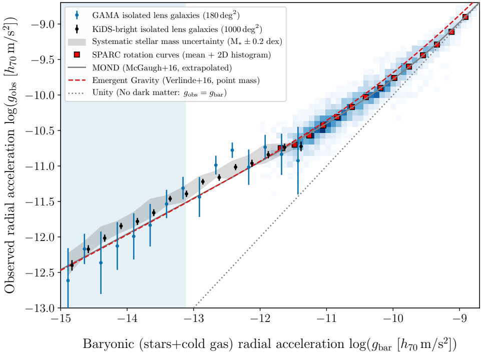


Figure: The baryons alone predict the dynamics (153 LTG) (Lelli et al., 2019).

Recent news.

Additional regularity: the radial acceleration relation (RAR) extended



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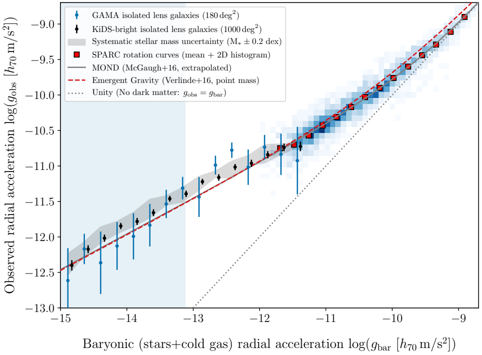

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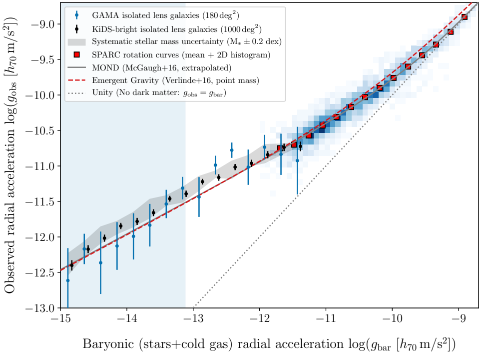

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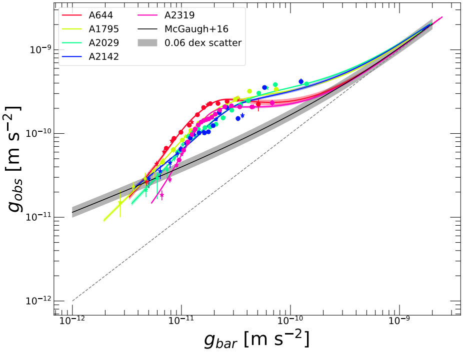

Figure: The RAR of galaxy clusters (tSZ and X-ray obs.) departs from the galactic RAR (Eckert et al., 2022).

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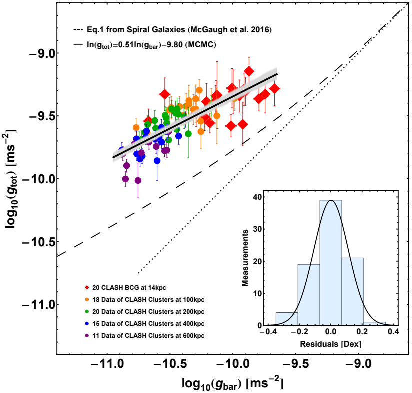

Figure: RAR of galaxy clusters inferred from lensing also departs from galactic RAR. (Tian et al., 2020)

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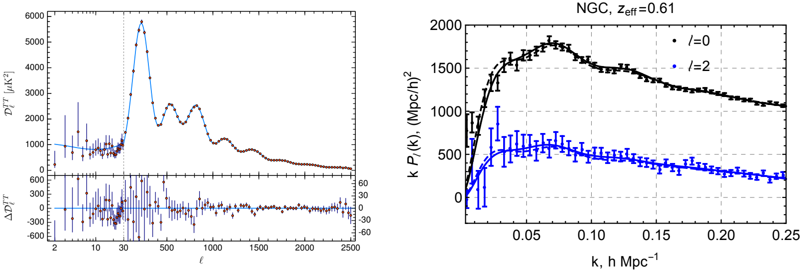

Figure: Left: Planck angular power spectrum. (Aghanim et al., 2018). Right: Matter power spectrum from BOSS (SDSS), monopole and quadrupole. (Ivanov et al., 2021)

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- In general relativity, the potential Φ is only the diagonal part of the larger metric tensor $g_{\mu\nu}$: $g_{00} = -1 + 2\Phi$, $g_{ii} = 1 + 2\Phi$.

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- Skordis and Złośnik (2020) proposed such a theory (called Aether-Scalar-Tensor theory) with a unit time-like vector field A_μ, a scalar field ϕ, and a metric/tensor $g_{\mu\nu}$.
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AeST vacuum solutions

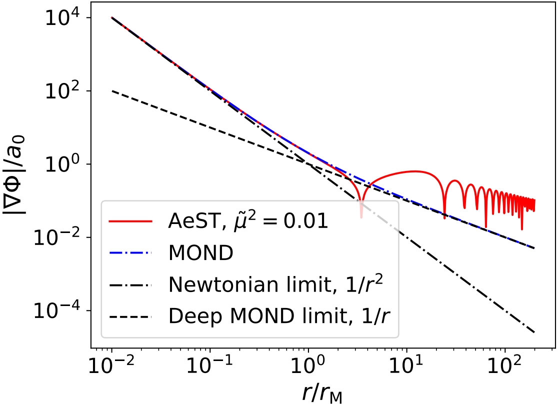

Figure: The magnitude of the force away from a point source in AeST (red). Newtonian regime $1/r^2$, MOND regime $1/r$ and oscillatory regime (with power-law enveloped). Distance in units of $r_{\rm M} = \sqrt{GM/a_0}$.

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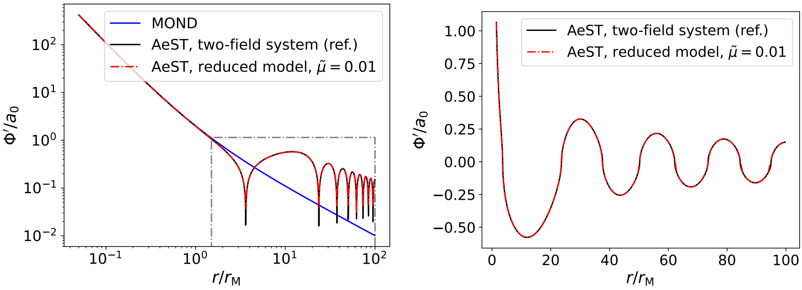

Figure: Oscillations in force. Left: Log-log plot of abs. value of force (AeST red, MOND blue). Right: Linear plot of force.

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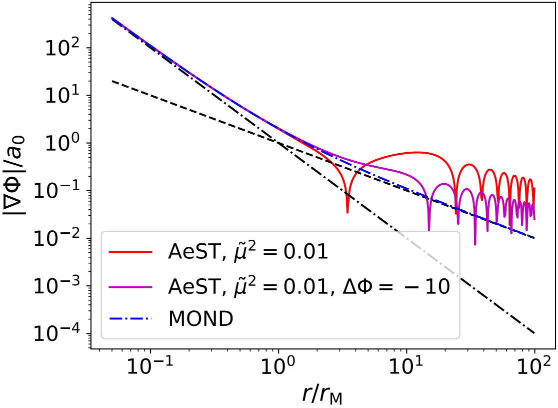

Figure: Onset of oscillatory regime depends on the boundary value of potential. Can be delayed by a lowered potential (magenta line).

Static spherically symmetric weak-field solutions of AeST: isothermal case

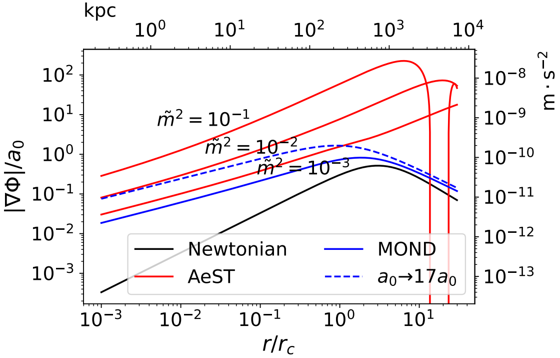

Figure: PRELIMINARY RESULT. Isothermal gas in hydrostatic equilibrium: $\rho = \exp(-\Phi)$. The force in AeST (red line) appears to be stronger than MOND (blue line) and also MOND with $a_0 \to 17a_0$ (dashed blue line).

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- Though further quantitative investigation is needed there are new weak-field effects in the theory that enhance the force compared to MOND.
- AeST has a rich phenomenology with other elements (non-static effects and vector field) which may address other issues, such as the apparent dark matter distribution around colliding clusters.

Role of vector fields

- Away from spherical symmetry the vector field cannot a priori be neglected. The weak field AeST system has been derived and reads

$$\nabla^2\tilde{\Phi} + \tilde{\mathcal{K}}_2\mathcal{Q}_0^2\left(\tilde{\Phi}+\chi\right) = 4\pi G\rho_b$$

$$\nabla^2\tilde{\Phi} = \nabla\cdot\left(\mathcal{J}'\left(\nabla\chi + \mathcal{Q}_0\boldsymbol{\beta}\right)\right)$$

$$-\frac{K_B}{2\left(2-K_B\right)}\nabla^2\boldsymbol{\beta} + \mathcal{Q}_0^2\boldsymbol{\beta} + \mathcal{J}'\left(\nabla\chi + \mathcal{Q}_0\boldsymbol{\beta}\right) + \nabla\Phi = 0$$

  where $\boldsymbol{\beta}$ is a divergenceless vector.

- Whether the new terms can be important for colliding galaxy clusters is currently under investigation.
- It appears that they have minor effect on the rotation curve of a MW-like galaxy (Mistele).

Addressing the displaced halo in galaxy clusters

- By construction the static equations have $\dot{\Phi}, \dot{\chi} = 0$.
- When expanding around Minkowski space it turns out that scalar, vector and tensor perturbations travel at different speeds.
- Could the scalar field lag behind baryons set in motion, only to join again when caught up? That would give a displacement of the baryonic distribution from the scalar field and potentially a displaced lensing signal.
- Another possibility: Though the galaxy clusters themselves are spherically symmetric the configuration that has both of them in collision is not, so there may be effects from the vector field.