



北京大學

Chirality and Wobbling in Atomic Nuclei (CWAN'23)

July 10 - July 14, 2023, Huizhou, China

# Chiral dynamics in “rigid” and “soft” triaxial nuclei

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Peking University

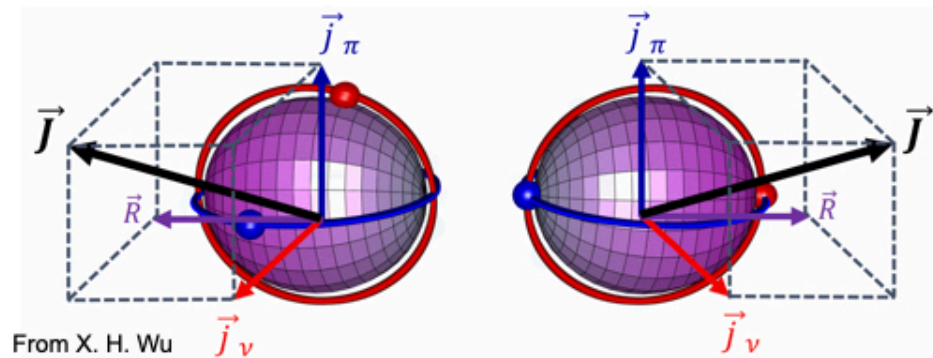
# Outline

- Nuclear chirality with covariant density functional theory
- Chiral dynamics in triaxial nuclei: chiral precession
- Chiral dynamics in soft triaxial nuclei
- Summary

# Nuclear spin-chirality

The **aplanar (3D-) rotation** of a **triaxial nucleus** could present **chiral geometry**.

Frauendorf and Meng, Nucl. Phys. A **617**, 131 (1997)



From X. H. Wu

Left-handed  $|\mathcal{L}\rangle$

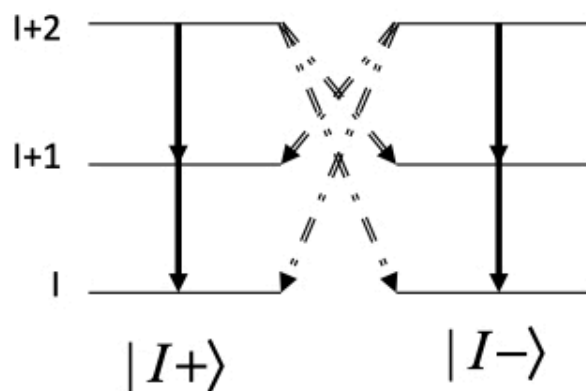
Right-handed  $|\mathcal{R}\rangle$

Intrinsic frame :

Chiral Symmetry breaking

$$\hat{\chi} = \hat{T} \hat{R}_y(\pi)$$

$$\hat{\chi} |\mathcal{L}\rangle = |\mathcal{R}\rangle \quad \hat{\chi} |\mathcal{R}\rangle = |\mathcal{L}\rangle$$



Lab. frame :

Chiral Symmetry restoration

$$|I+\rangle = \frac{1}{\sqrt{2}}(|\mathcal{L}\rangle + |\mathcal{R}\rangle)$$

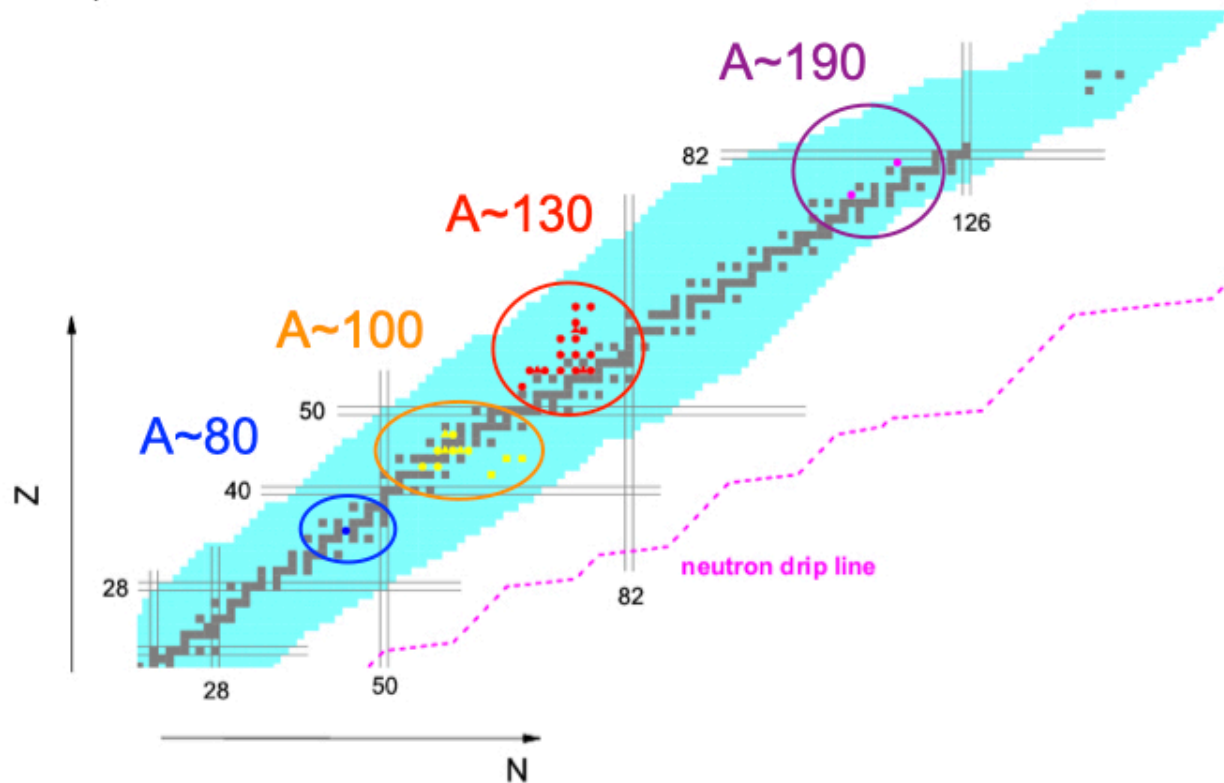
$$|I-\rangle = \frac{i}{\sqrt{2}}(|\mathcal{L}\rangle - |\mathcal{R}\rangle)$$

Exp. signal: **Two near degenerate  $\Delta I = 1$  bands**, called **chiral doublet bands**

# Observed chiral nuclei

More than 45 candidate chiral nuclei have been reported in the  $A \sim 80$ , 100, 130, and 190 mass regions, so far.

Xiong and Wang, *At. Data Nucl. Data Tables*, 2019, 125, 193-225



- ✓ **Triaxial deformation:** *less evident in the ground state*
- ✓ **Single-particle structure:** *isolate the role of specific orbitals*
- ✓ **Triaxial shape coexistence:** *a unique phenomenon of nuclear systems*



# Theoretical tools for nuclear chirality

## ➤ Particle Core Coupling

*(lab frame, phenomenological, quantal, with quantum tunneling)*

- ✓ Triaxial Particle Rotor Model      Frauendorf and Meng NPA(1997); Koike PRL(2004), Qi PLB(2009)
- ✓ Core-quasiparticle coupling model      Starosta PRC(2002); Koike PRC(2003)
- ✓ Interacting Boson Fermion Fermion Model      Brant PRC (2004), PRC (2008), Tonev PRL(2006)

## ➤ Tilted Axis Cranking Mean Field

*(intrinsic frame, microscopic, semi-classical, no quantum tunneling)*

- ✓ Single-j model      Frauendorf and Meng NPA(1997)
- ✓ Hybrid Woods-Saxon and Nilsson model      Dimitrov PRL(2000)
- ✓ Non-relativistic Skyrme DFT      Olbratowski PRL(2004), PRC(2006)
- ✓ **Covariant DFT**      PWZ PLB (2017)

## ➤ Shell Model

- ✓ Pair Truncated Shell Model      Higashiyama, PRC(2005)
- ✓ Projected Shell Model      Chen PRC (2017)

...

# Tilted axis cranking CDFT

The cranking mean-field model has been very successful for rotations

## Meson exchange version:

3-D Cranking: *Madokoro, Meng, Matsuzaki, Yamaji, PRC 62, 061301 (2000)*

2-D Cranking: *Peng, Meng, Ring, Zhang, PRC 78, 024313 (2008)*

## **Point-coupling version:**

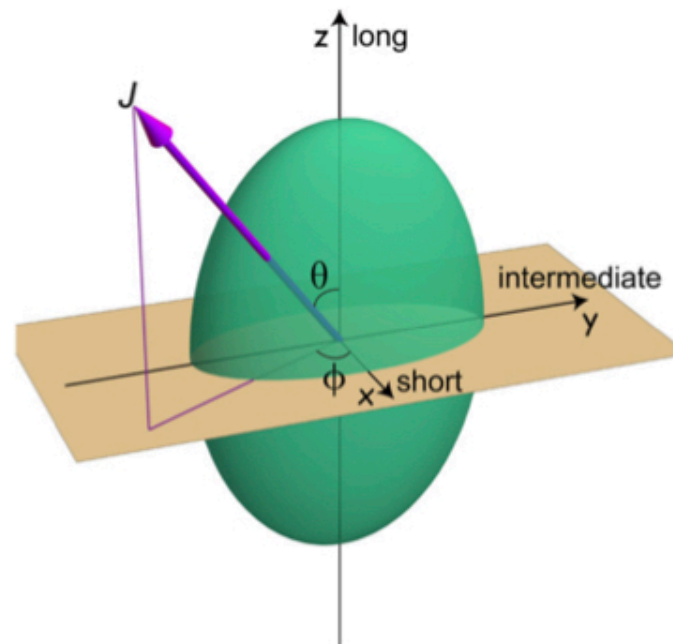
Simple and more suitable for systematic investigations

2-D Cranking: *PWZ, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)*

2-D Cranking + Pairing: *PWZ, Zhang, Meng, PRC 92, 034319 (2015)*

3-D Cranking: *PWZ, PLB 773, 1 (2017)*

3-D Cranking + Pairing: *in preparation*



## **Self-consistent and microscopic investigations**

- ✓ full account of polarization effects
- ✓ self-consistent treatment of the nuclear currents
- ✓ no additional parameter beyond a well-determined functional

# Density functional theory

The many-body problem is mapped onto a one-body problem

## Hohenberg-Kohn Theorem

The **exact ground-state energy** of a quantum mechanical many-body system is a **universal functional** of the **local density**.

$$E[\rho] = T[\rho] + U[\rho] + \int V(\mathbf{r})\rho(\mathbf{r}) d^3\mathbf{r}$$

## Kohn-Sham DFT

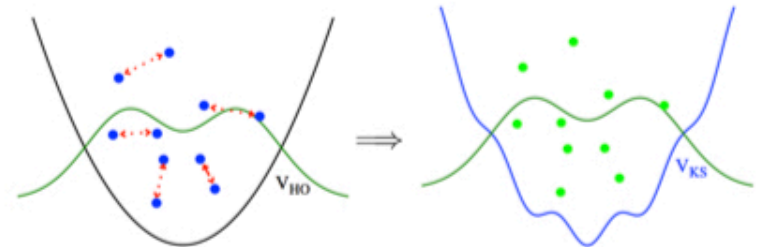


Figure from Drut PPNP 2010

$$T[\rho] \doteq \sum_{i=1}^N \left\langle \varphi_i \left| -\frac{\hbar^2}{2m} \nabla^2 \right| \varphi_i \right\rangle$$

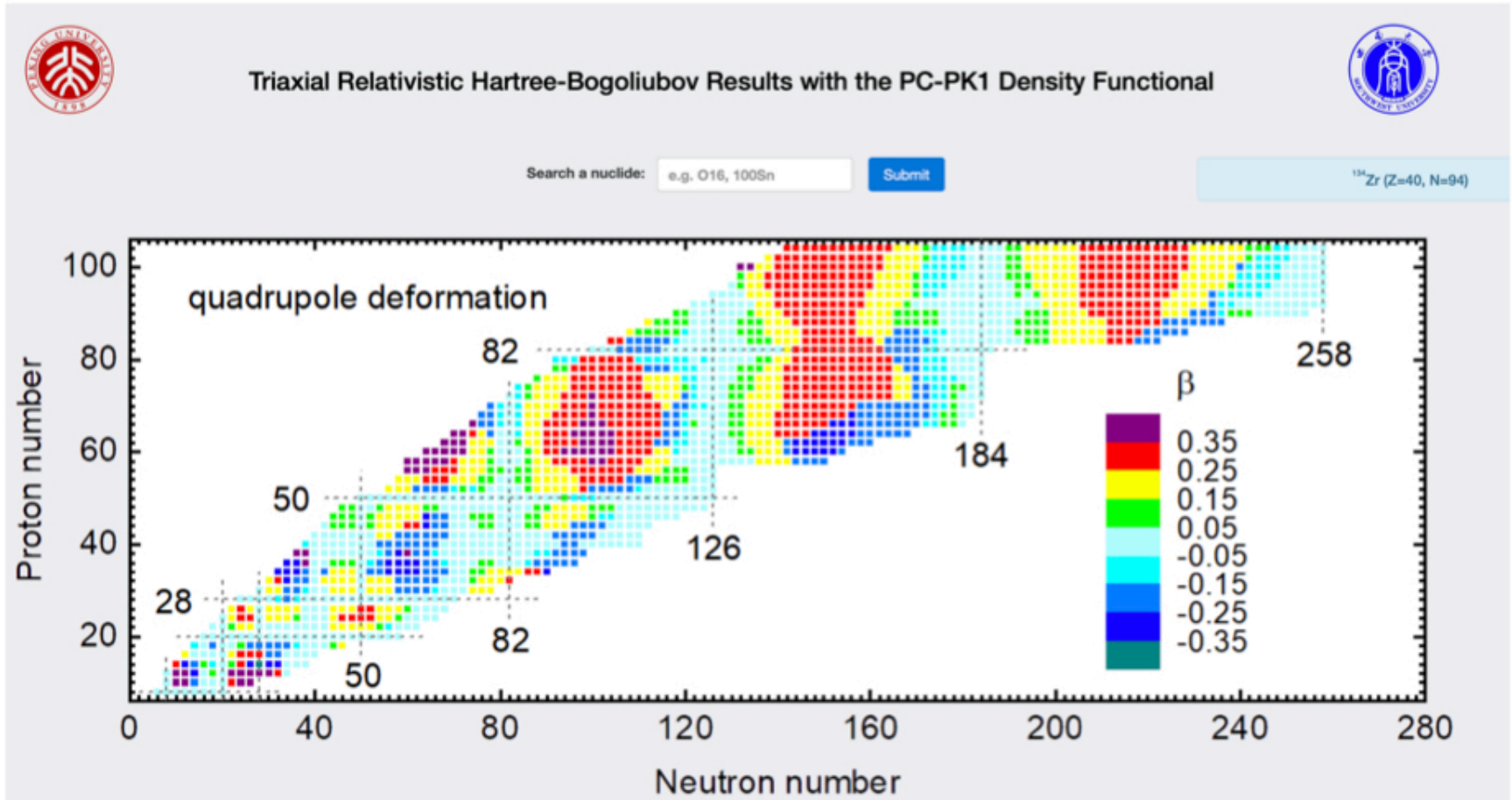
$$E[\rho] \Rightarrow \hat{h} = \frac{\delta E}{\delta \rho} \Rightarrow \hat{h}\varphi_i = \varepsilon_i\varphi_i \Rightarrow \rho = \sum_{i=1}^A |\varphi_i|^2$$

The practical usefulness of the Kohn-Sham theory depends entirely on whether an **Accurate Energy Density Functional** can be found!

# How many nuclei are bound?

<http://nuclearmap.jcnp.org/index.html> PC-PK1

Triaxial RHB



Yang, Wang, PWZ, Li, Phys. Rev. C 104, 054312 (2021)

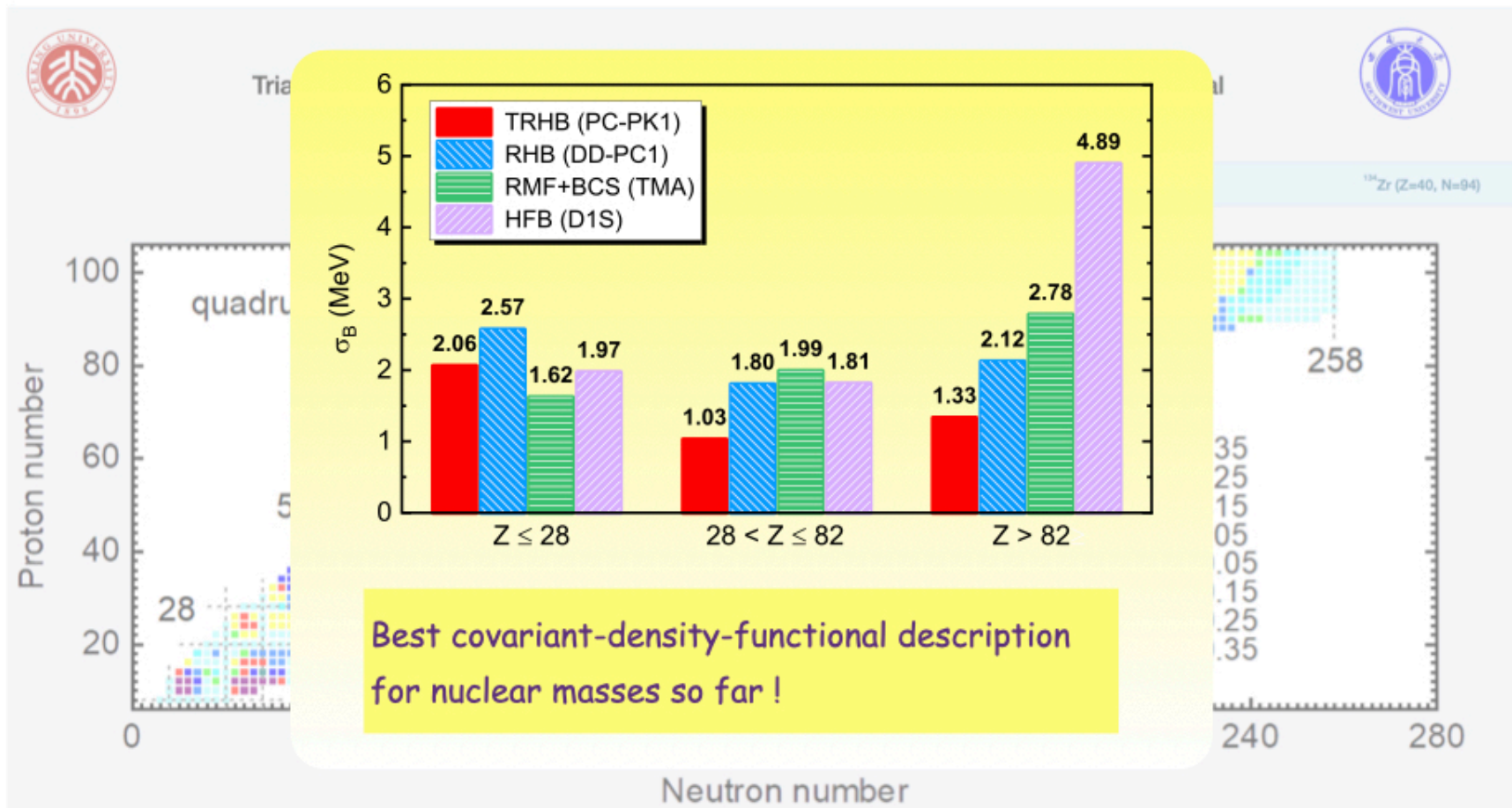
Yang, PWZ, Li, Phys. Rev. C 107, 024308 (2023)



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Triaxial RHB



Yang, Wang, PWZ, Li, Phys. Rev. C 104, 054312 (2021)

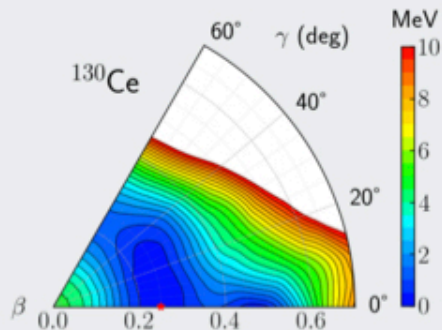
Yang, PWZ, Li, Phys. Rev. C 107, 024308 (2023)

# How many nuclei are bound?

<http://nuclearmap.jcnp.org/index.html> PC-PK1

Triaxial RHB

## Results for Cerium 130 (Z=58, N=72)



Potential energy surface calculated by RHB theory with PC-PK1 density functional. All energies are normalized with respect to the binding energy of the absolute minimum. The contours join points on the surface with the same energy, and the energy difference between adjacent contours is 0.5 MeV.

### Ground-state properties

Spectroscopy (coming soon)

$$E_{\text{RHB}} = -1079.46 \text{ MeV}$$

$$E_{5\text{DCH}} = -1083.50 \text{ MeV}$$

$$E_{\text{exp}} = -1083.32 \text{ MeV}$$

$$\beta = 0.25$$

$$\gamma = 0^\circ$$

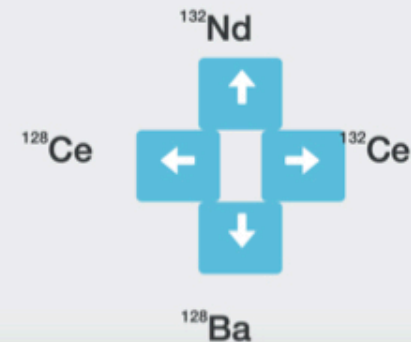
To know the meaning of a quantity, please hold the mouse still on it.

### Select a nuclide

Proton number

Neutron number

Submit



Yang, Wang, PWZ, Li, Phys. Rev. C 104, 054312 (2021)

Yang, PWZ, Li, Phys. Rev. C 107, 024308 (2023)

# Cranking Relativistic Kohn-Sham Equation

## Dirac Equation

$$\begin{pmatrix} m + V + S - \boldsymbol{\omega} \cdot \mathbf{J} & \boldsymbol{\sigma} \cdot \mathbf{p} - \boldsymbol{\sigma} \cdot \mathbf{V} \\ \boldsymbol{\sigma} \cdot \mathbf{p} - \boldsymbol{\sigma} \cdot \mathbf{V} & -m + V - S - \boldsymbol{\omega} \cdot \mathbf{J} \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = \varepsilon \begin{pmatrix} f \\ g \end{pmatrix}$$

$$S(\mathbf{r}) = \alpha_S \rho_S + \beta_V \rho_S^2 + \gamma_V \rho_S^3 + \delta_S \Delta \rho_S$$

$$V(\mathbf{r}) = \alpha_V \rho_V + \gamma_V \rho_V^3 + \delta_V \Delta \rho_V + \tau_3 \alpha_{TV} \rho_{TV} + \tau_3 \delta_{TV} \Delta \rho_{TV} + e \frac{1 - \tau_3}{2} A$$

$$\mathbf{V}(\mathbf{r}) = \alpha_V \mathbf{j}_V + \gamma_V \mathbf{j}_V^3 + \delta_V \Delta \mathbf{j}_V + \tau_3 \alpha_{TV} \mathbf{j}_{TV} + \tau_3 \delta_{TV} \Delta \mathbf{j}_{TV} + e \frac{1 - \tau_3}{2} \mathbf{A}$$

Consistent treatment for time-odd fields from nuclear currents

PWZ, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)

PWZ PLB 773, 1 (2017)



# Cranking Relativistic Kohn-Sham Equation

## Dirac Equation

Coriolis term

Time-odd mean fields

$$\begin{pmatrix} m + V + S - \omega \cdot J & \sigma \cdot p - \sigma \cdot V \\ \sigma \cdot p - \sigma \cdot V & -m + V - S - \omega \cdot J \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = \varepsilon \begin{pmatrix} f \\ g \end{pmatrix}$$

$$S(\mathbf{r}) = \alpha_S \rho_S + \beta_V \rho_S^2 + \gamma_V \rho_S^3 + \delta_S \Delta \rho_S$$

$$V(\mathbf{r}) = \alpha_V \rho_V + \gamma_V \rho_V^3 + \delta_V \Delta \rho_V + \tau_3 \alpha_{TV} \rho_{TV} + \tau_3 \delta_{TV} \Delta \rho_{TV} + e \frac{1 - \tau_3}{2} A$$

$$\mathbf{V}(\mathbf{r}) = \alpha_V \mathbf{j}_V + \gamma_V \mathbf{j}_V^3 + \delta_V \Delta \mathbf{j}_V + \tau_3 \alpha_{TV} \mathbf{j}_{TV} + \tau_3 \delta_{TV} \Delta \mathbf{j}_{TV} + e \frac{1 - \tau_3}{2} \mathbf{A}$$

Consistent treatment for time-odd fields from nuclear currents

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PWZ PLB 773, 1 (2017)

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## Dirac Equation

Coriolis term

Time-odd mean fields

$$\begin{pmatrix} m + V + S - \omega \cdot J & \sigma \cdot p - \sigma \cdot V \\ \sigma \cdot p - \sigma \cdot V & -m + V - S - \omega \cdot J \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = \varepsilon \begin{pmatrix} f \\ g \end{pmatrix}$$

$$S(\mathbf{r}) = \alpha_S \rho_S + \beta_V \rho_S^2 + \gamma_V \rho_S^3 + \delta_S \Delta \rho_S$$

$$V(\mathbf{r}) = \alpha_V \rho_V + \gamma_V \rho_V^3 + \delta_V \Delta \rho_V + \tau_3 \alpha_{TV} \rho_{TV} + \tau_3 \delta_{TV} \Delta \rho_{TV} + e \frac{1 - \tau_3}{2} A$$

$$\mathbf{V}(\mathbf{r}) = \alpha_V \mathbf{j}_V + \gamma_V \mathbf{j}_V^3 + \delta_V \Delta \mathbf{j}_V + \tau_3 \alpha_{TV} \mathbf{j}_{TV} + \tau_3 \delta_{TV} \Delta \mathbf{j}_{TV} + e \frac{1 - \tau_3}{2} \mathbf{A}$$

Consistent treatment for time-odd fields from nuclear currents

PWZ, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)

PWZ PLB 773, 1 (2017)

# Applications of TAC-CDFT for chiral nuclei

$102-107\text{Rh}$ ,  $106\text{Ag}$ ,  $^{135}\text{Nd}$ ,  $^{136}\text{Nd}$ ,  $120-134\text{Cs}$

PWZ PLB 773, 1-5 (2017)

PWZ, Wang, and Chen, PRC 99, 054319 (2019)

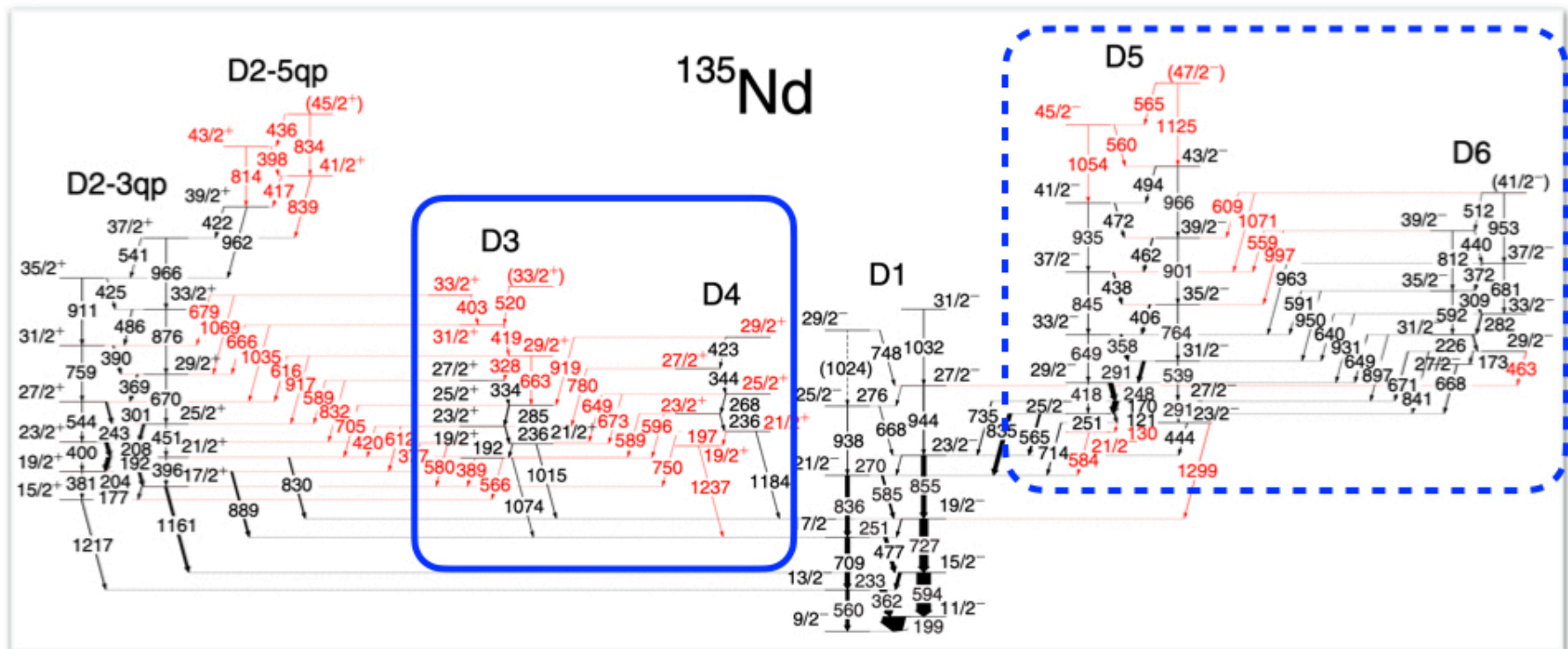
Peng and Chen, PLB 810, 135795 (2020)

Peng and Chen, PRC 105, 044318 (2022)

Chen, Li, and Guo, EPJA 59, 142 (2023)

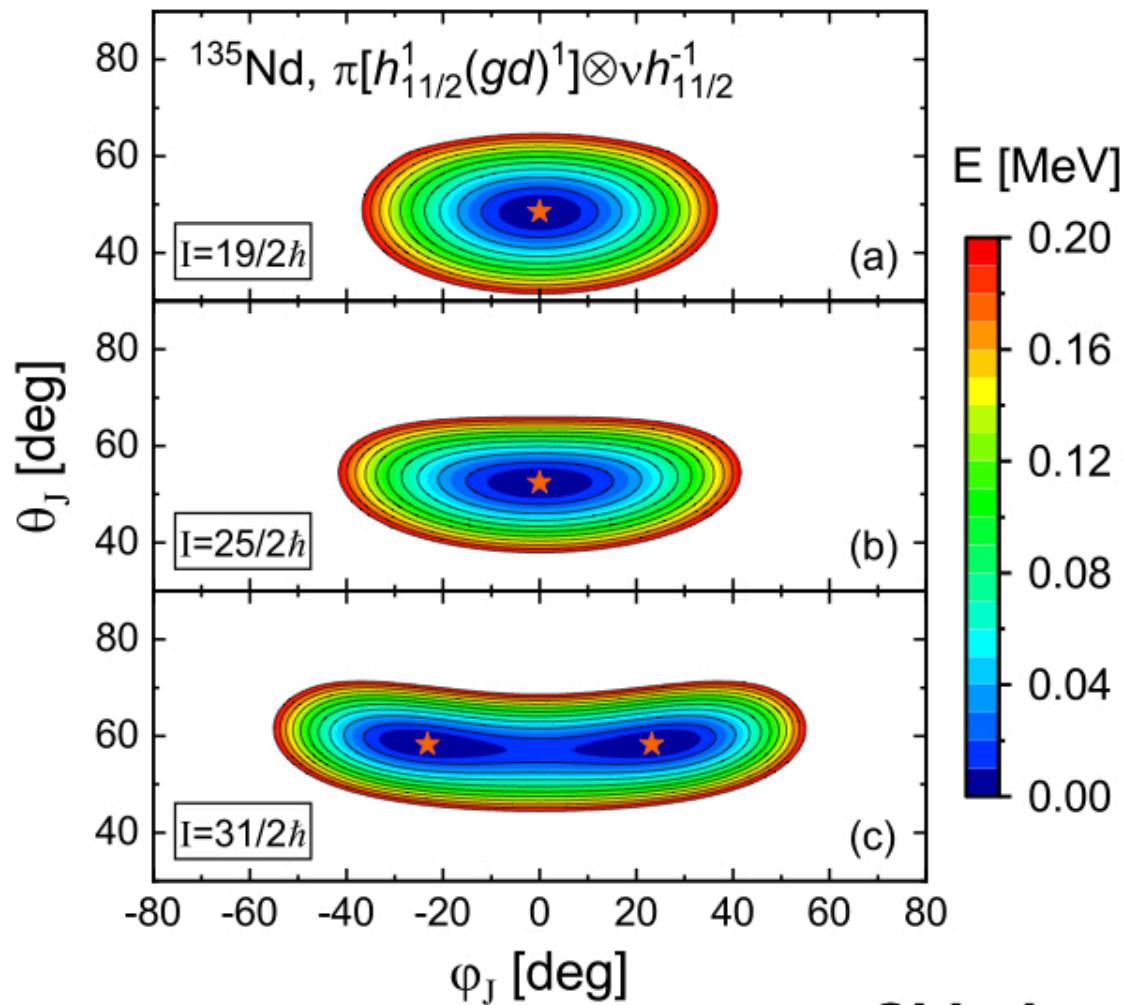
$$\text{D3-D4 } \pi[h_{11/2}^1(gd)^1] \otimes \nu h_{11/2}^{-1}$$

$$\text{D5-D6 } \pi h_{11/2}^2 \otimes \nu h_{11/2}^{-1}$$

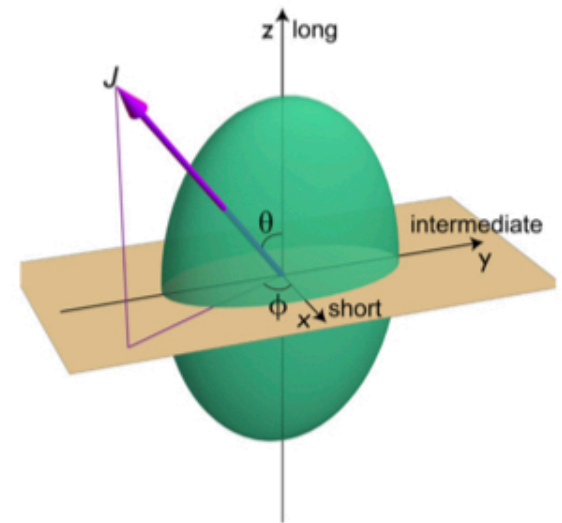


Experimental Data from Lv et al., Phys. Rev. C 100, 024314 (2019)

# Energy surface against tilted angles



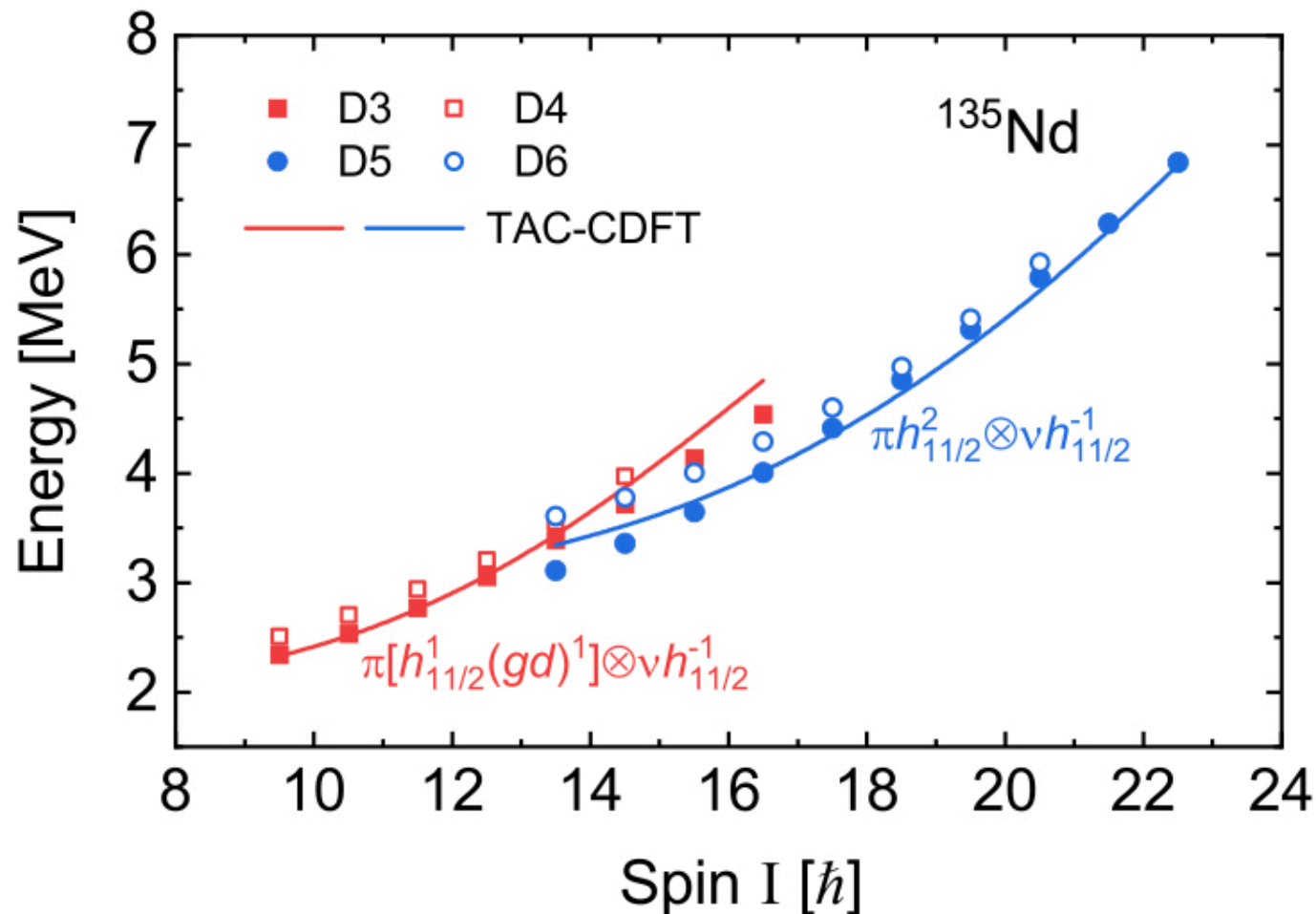
polar angle  $\theta$   
azimuth angle  $\varphi$



**Chiral geometry is clearly seen.**

# Energy spectrum

The **lower bands** are well reproduced.





# Time-dependent CDFT

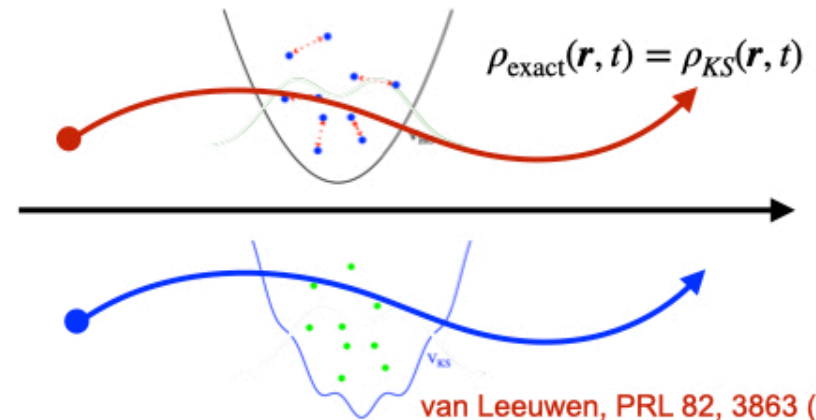
The many-body problem is mapped onto a one-body problem!

## Runge-Gross Theorem

There is a **unique mapping** between the **time dependent external potential** and the **density**, for many body systems evolving from a **given initial state**.

Runge and Gross, PRL 52, 997 (1984)

## Time-dependent Kohn-Sham DFT



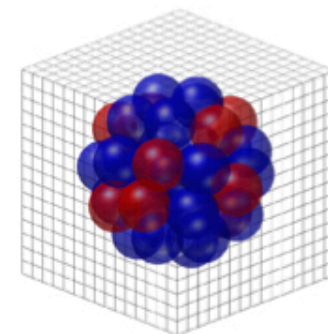
Ren, PWZ, Meng, PLB 801,135194 (2020)

$$i\partial_t \begin{pmatrix} f \\ g \end{pmatrix} = \begin{pmatrix} m + V + S & \boldsymbol{\sigma} \cdot \mathbf{p} - \boldsymbol{\sigma} \cdot \mathbf{V} \\ \boldsymbol{\sigma} \cdot \mathbf{p} - \boldsymbol{\sigma} \cdot \mathbf{V} & -m + V - S \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix}$$

$$\rho(\mathbf{r}, t) = \sum_i^N f_i^2 + g_i^2$$

# Applications of the TD-CDFT

## 3D Lattice: no spatial symmetry restriction



✓ Applications include:

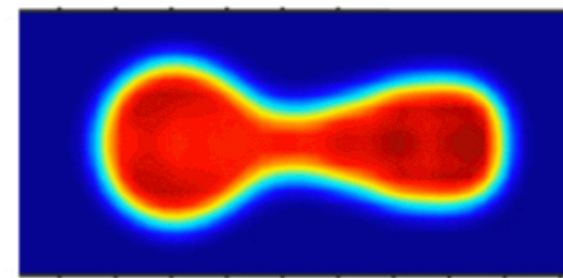
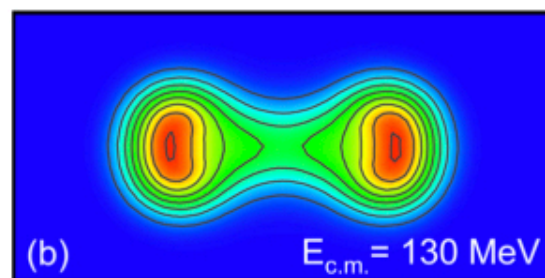
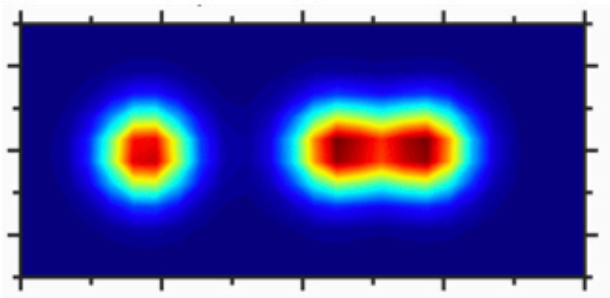
Linear alpha-chain PRL 115, 022501 (2015) PLB 801,135194 (2020)

Nuclear fission PRL 128, 172501 (2022) PRC 105, 044313 (2022) PRC 107, 014303 (2023)

Chiral dynamics PRC 105, L011301 (2022)

$^{16}\text{O} + ^{16}\text{O}$  reaction PRC 102, 044603 (2020)

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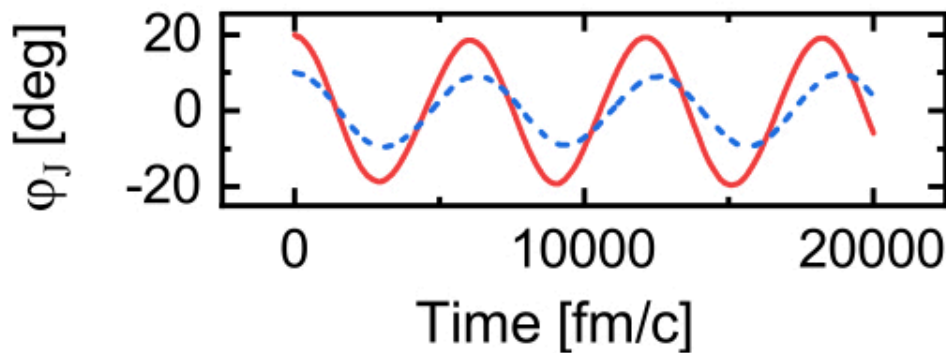
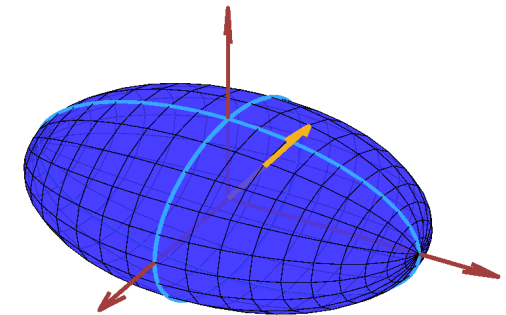
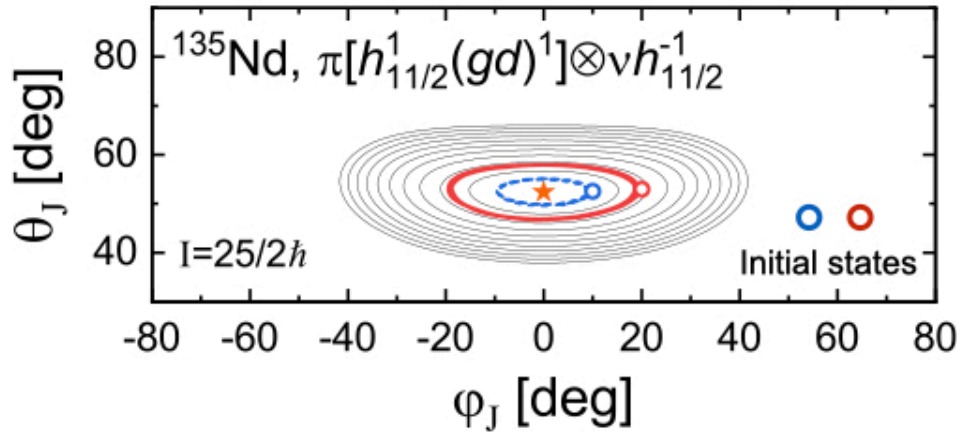




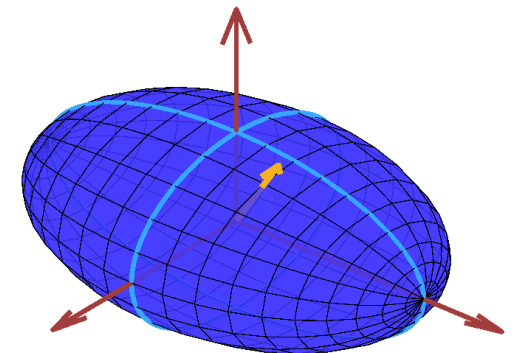
# Chiral dynamics in triaxial nuclei

**Chiral Precession** across left and right sectors

Space-fixed frame

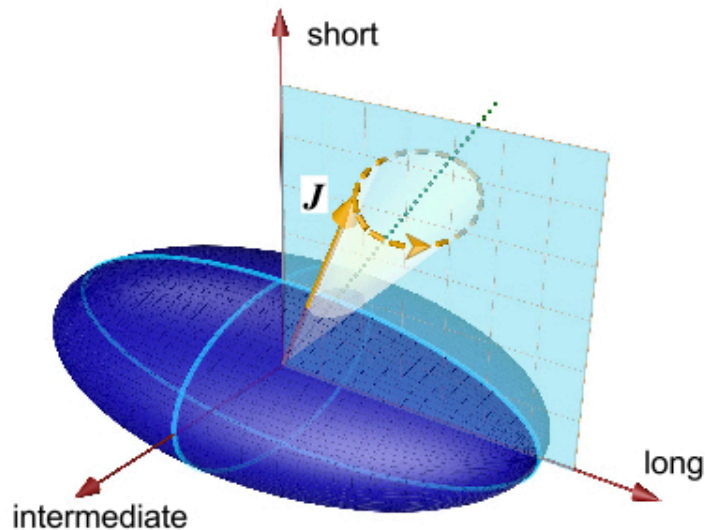


Body-fixed frame



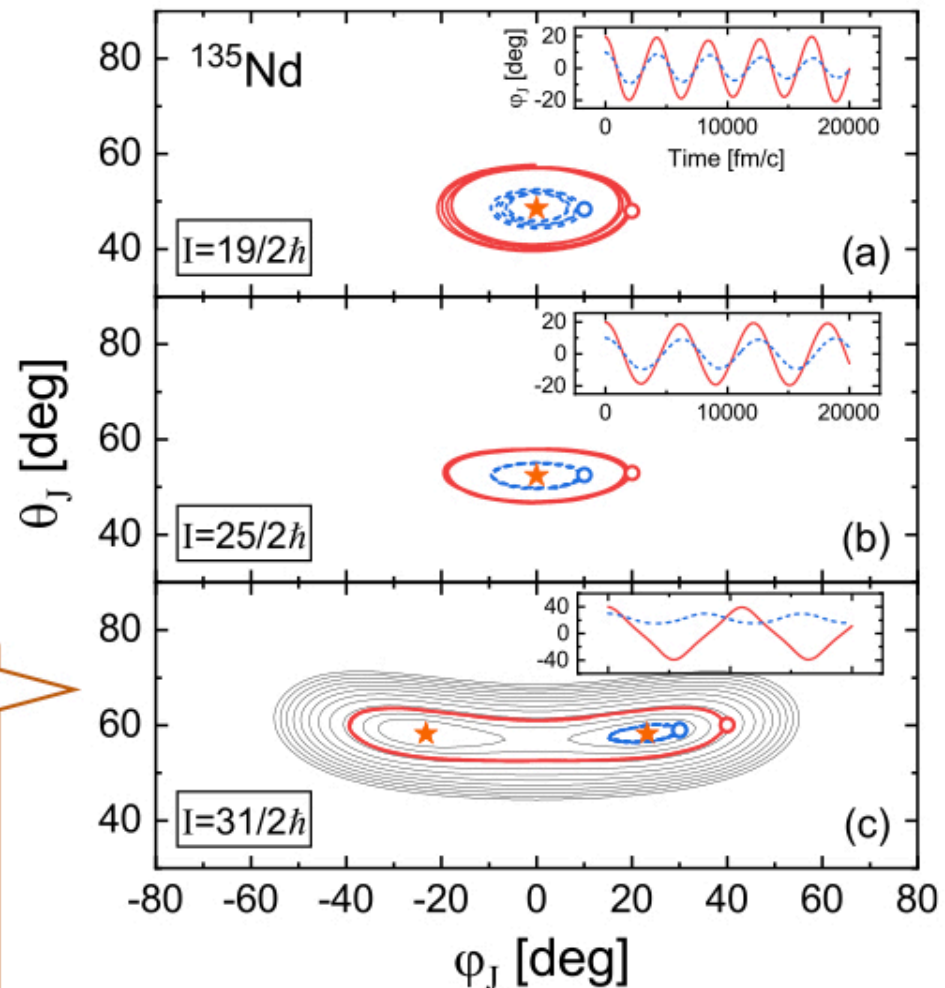
Precession periods are roughly **identical**.

# Chiral precession

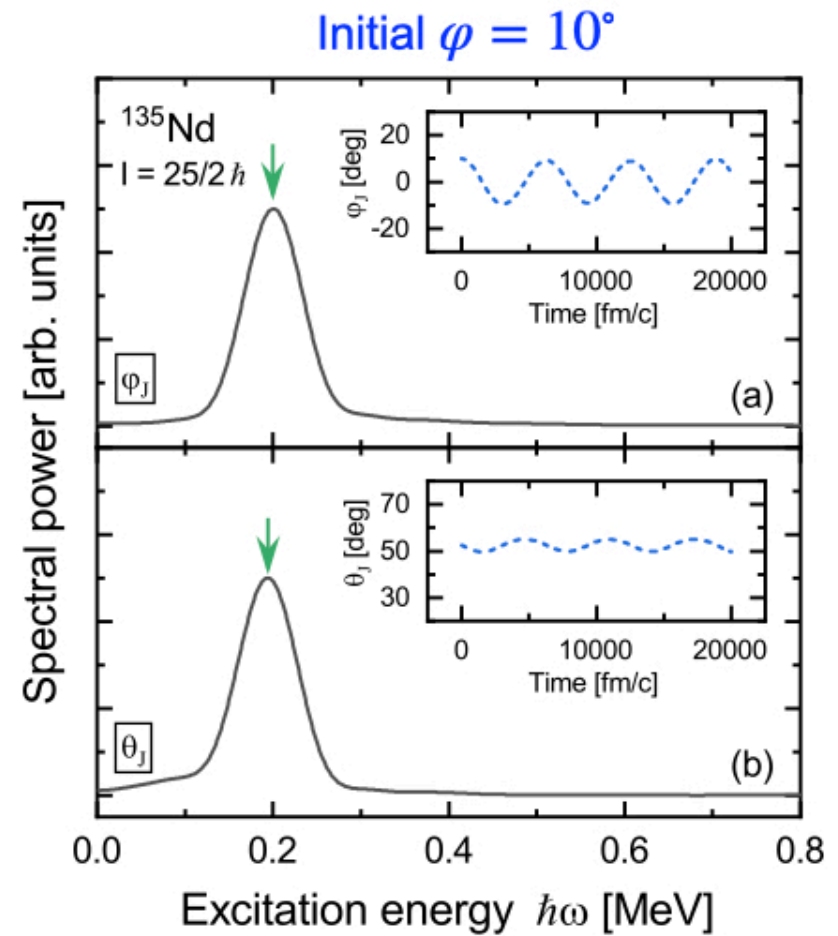
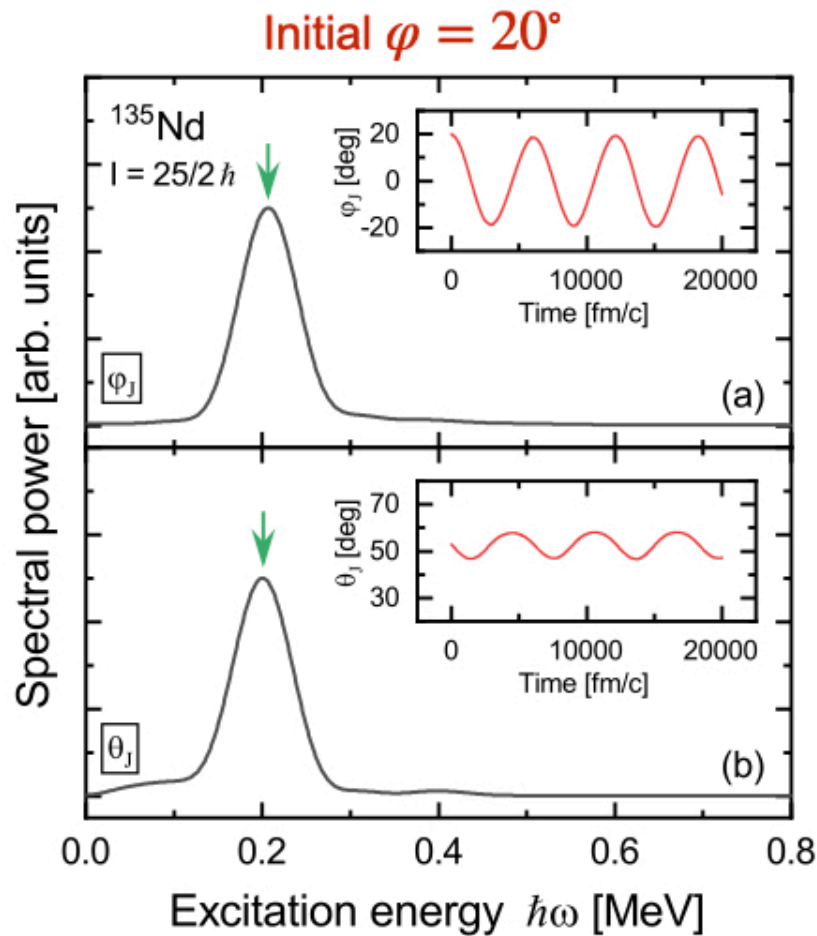


For  $31/2 \hbar$ , trajectories are disparate not only in amplitudes but also in periods.

anharmonic effects, quantum tunneling;  
**requantization of the TD-CDFT**

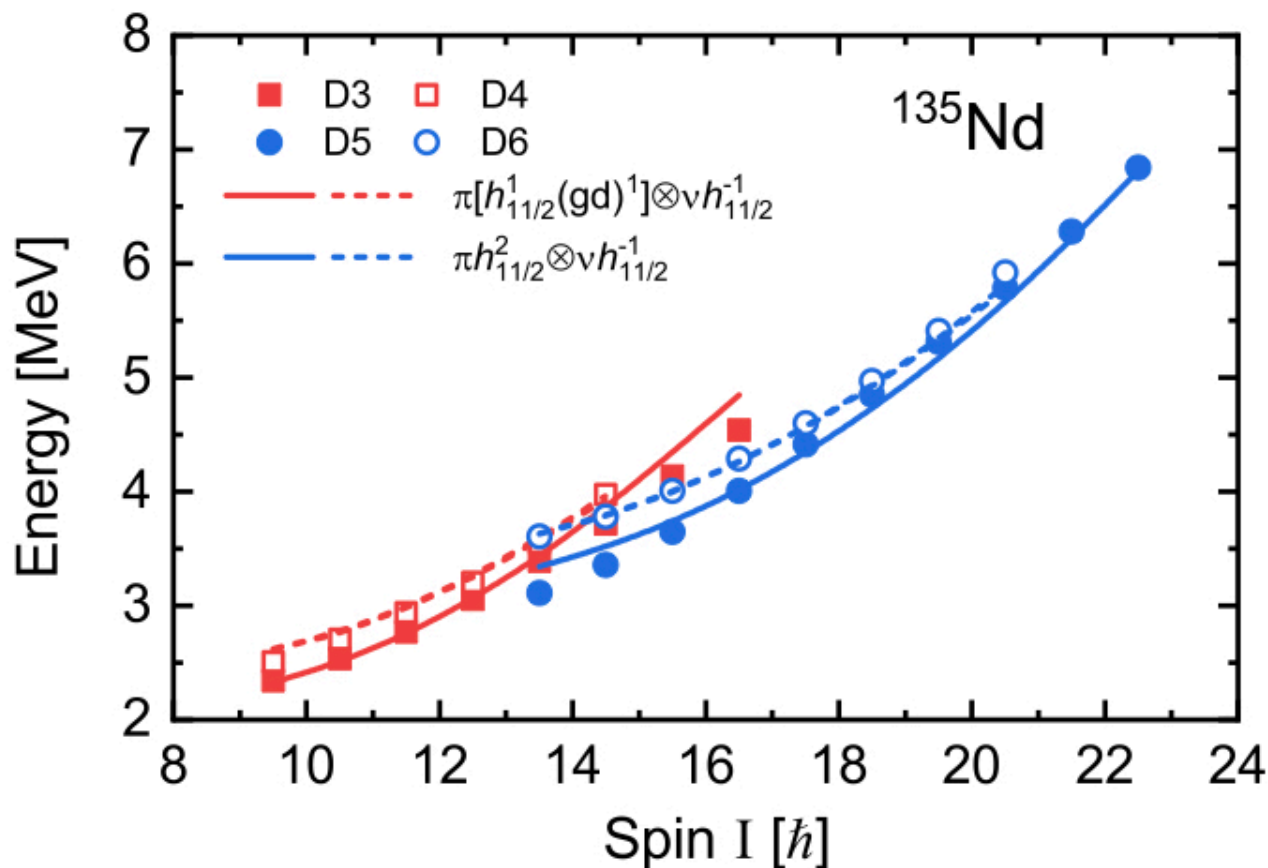


# Chiral excitation energies via Fourier analyses



# Energy spectrum

Ren, PWZ, Meng, PRC, **105**, L011301 (2022)



The first fully microscopic and self-consistent description for the **chiral twin bands** in the framework of DFTs.

# Chiral conundrum in $^{106}\text{Ag}$

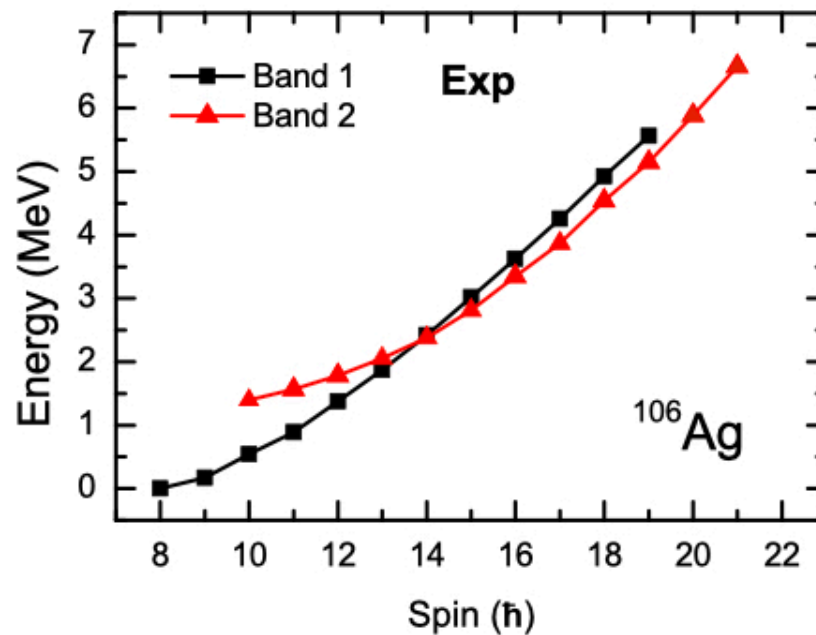
PRL **98**, 102501 (2007)

PHYSICAL REVIEW LETTERS

week ending  
9 MARCH 2007

## Effect of $\gamma$ Softness on the Stability of Chiral Geometry: Spectroscopy of $^{106}\text{Ag}$

P. Joshi,<sup>1</sup> M. P. Carpenter,<sup>2</sup> D. B. Fossan,<sup>3</sup> T. Koike,<sup>3,4</sup> E. S. Paul,<sup>5</sup> G. Rainovski,<sup>5,3,6</sup>  
K. Starosta,<sup>3,7</sup> C. Vaman,<sup>3,7</sup> and R. Wadsworth<sup>1</sup>





# Chiral conundrum in $^{106}\text{Ag}$

PRL **112**, 202503 (2014)

PHYSICAL REVIEW LETTERS

week ending  
23 MAY 2014

## Exploring the Origin of Nearly Degenerate Doublet Bands in $^{106}\text{Ag}$

N. Rather,<sup>1</sup> P. Datta,<sup>2,\*</sup> S. Chakrabarti,<sup>3</sup>  
R. Palit,<sup>4</sup> S. ...

PRL **112**, 202502 (2014)

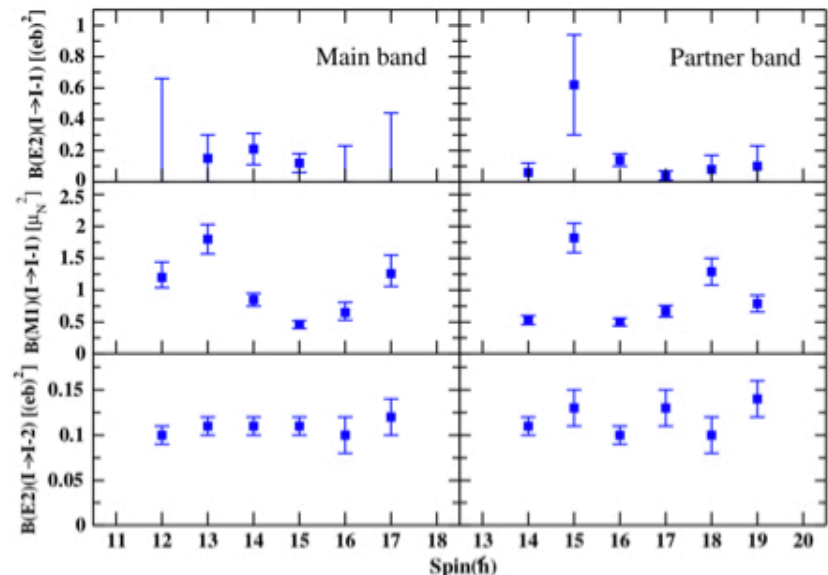
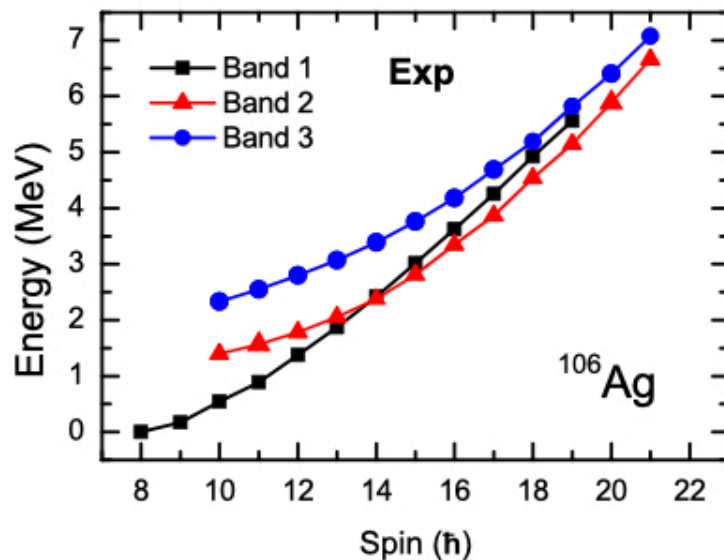
PHYSICAL REVIEW LETTERS

week ending  
23 MAY 2014

## Resolution of Chiral Conundrum in $^{106}\text{Ag}$ : Doppler-Shift Lifetime Investigation

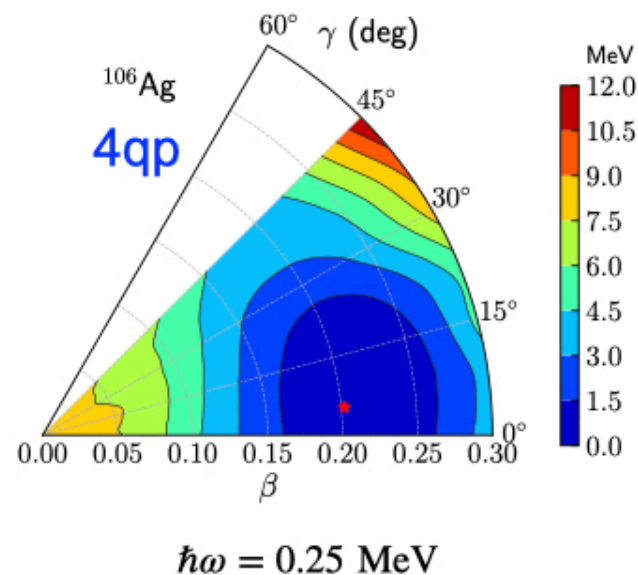
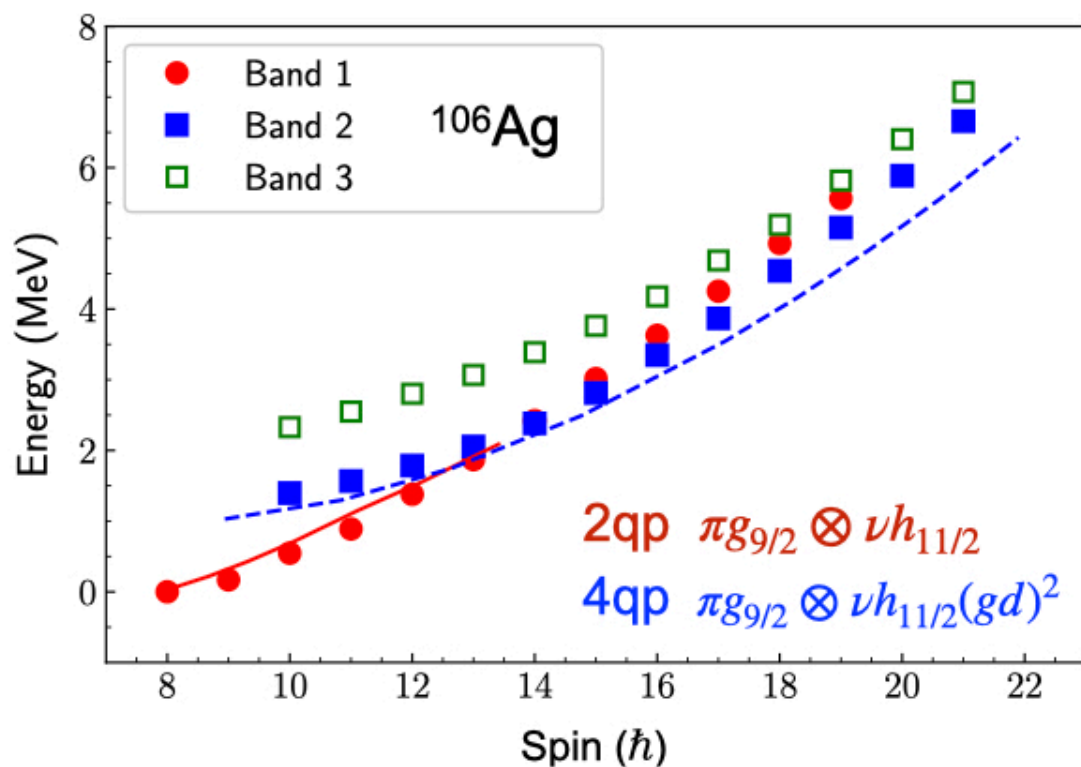
E. O. Lieder,<sup>1,2</sup> R. M. Lieder,<sup>1,\*</sup> R. A. Bark,<sup>1</sup> Q. B. Chen,<sup>3</sup> S. Q. Zhang,<sup>3</sup> J. Meng,<sup>3,4,5</sup> E. A. Lawrie,<sup>1</sup> J. J. Lawrie,<sup>1</sup>  
S. P. Bvumbi,<sup>1</sup> N. Y. Kheswa,<sup>1</sup> S. S. Ntshangase,<sup>1</sup> T. E. Madiba,<sup>1</sup> P. L. Masiteng,<sup>1</sup> S. M. Mullins,<sup>1</sup> S. Murray,<sup>1</sup>  
P. Papka,<sup>1</sup> D. G. Roux,<sup>6</sup> O. Shirinda,<sup>1</sup> Z. H. Zhang,<sup>3</sup> P. W. Zhao,<sup>3</sup> Z. P. Li,<sup>7</sup> J. Peng,<sup>8</sup> B. Qi,<sup>9</sup> S. Y. Wang,<sup>9</sup>  
Z. G. Xiao,<sup>10,11</sup> and C. Xu<sup>3</sup>

Consistent data but different interpretation



# TAC-CDFT for chiral conundrum in $^{106}\text{Ag}$

PWZ, Wang, Chen PRC 99, 54319 (2019)



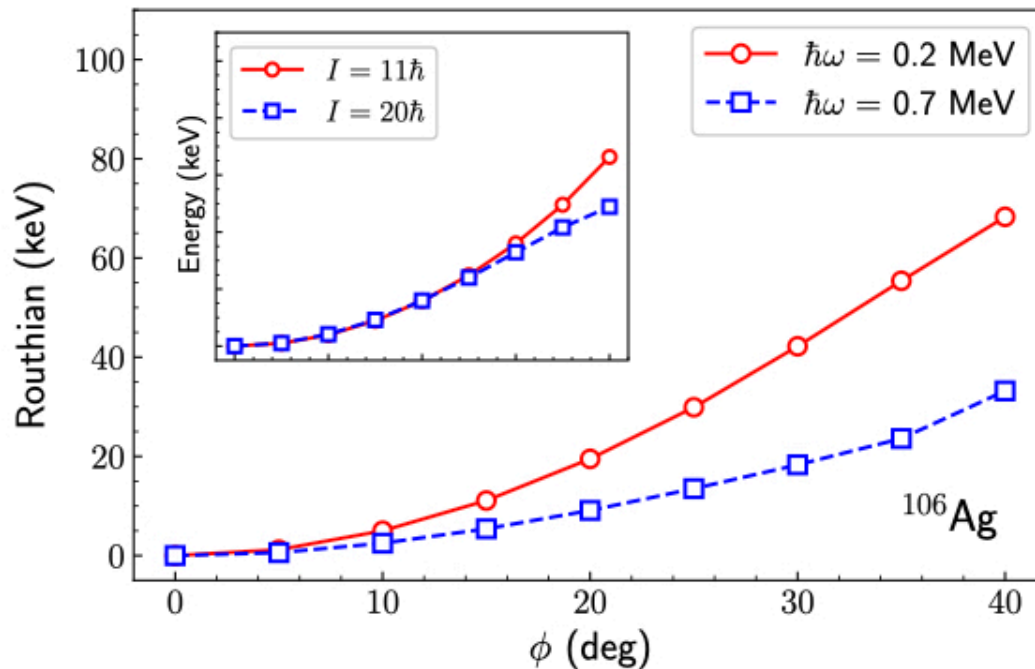
The energy for bands 1 and 2 are well reproduced with **two distinct configurations**.

For the 4qp configuration, the potential energy surface is **rather soft against triaxiality**.

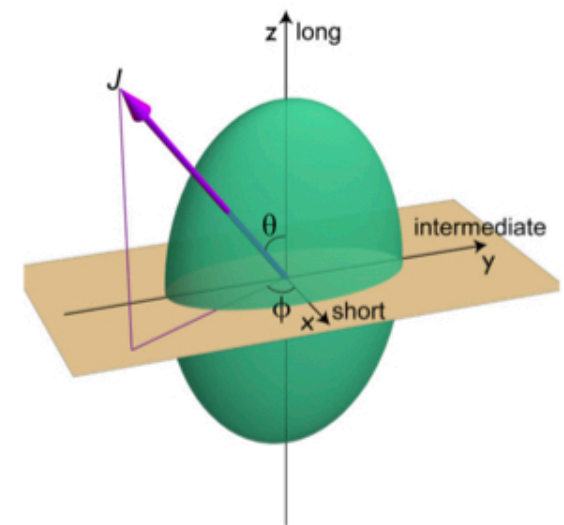


# Energy surface against tilted angles

PWZ, Wang, Chen PRC 99, 54319 (2019)



polar angle  $\theta$   
azimuth angle  $\varphi$

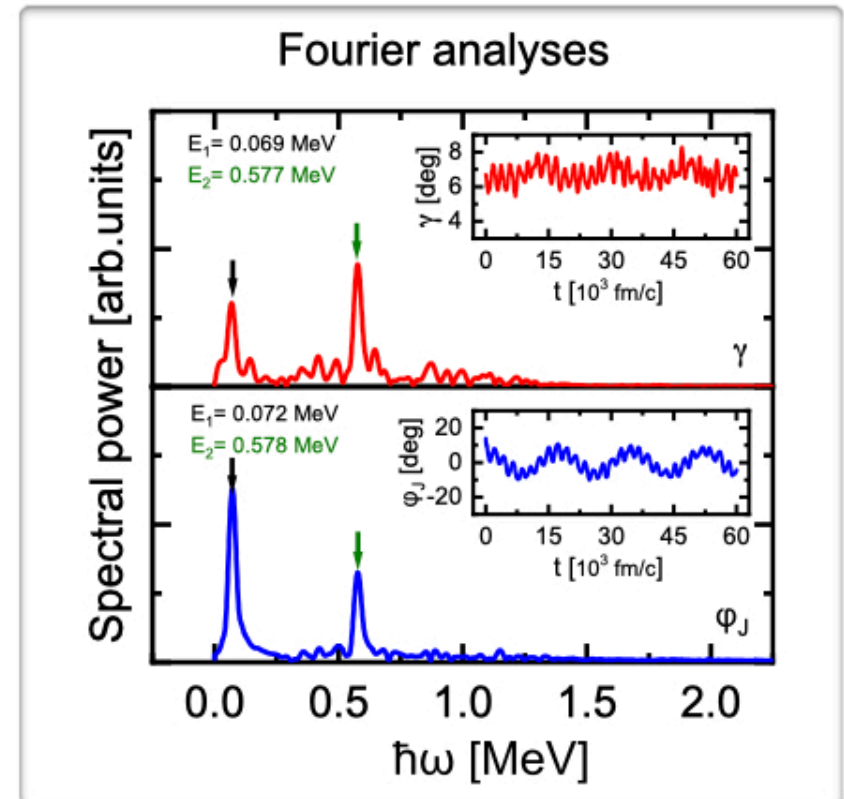
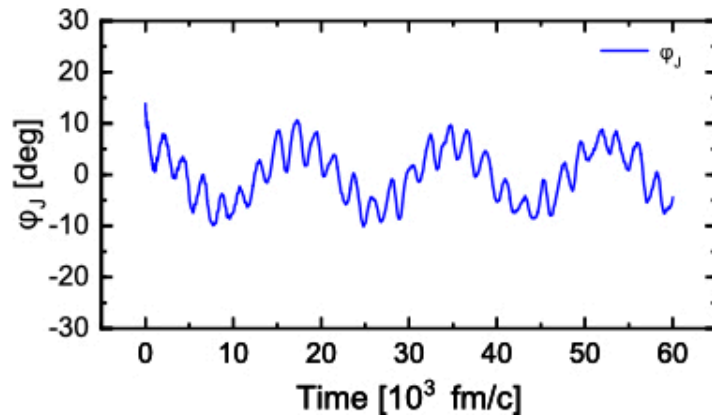
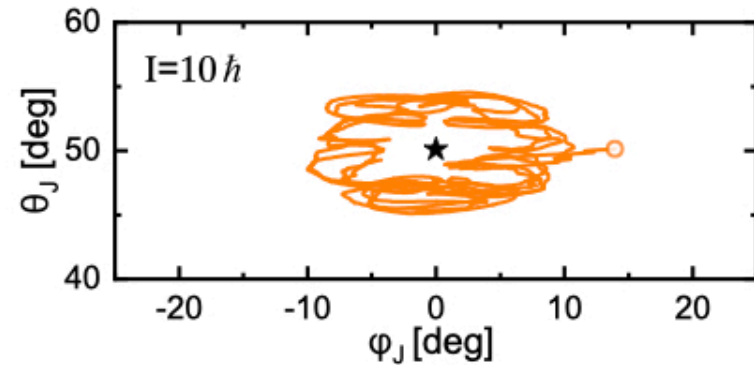


A chiral vibrational band could be expected due to the **soft Routhian curves**, which gives a possible explanation of band 3.

# Chiral dynamics in $^{106}\text{Ag}$

Li, et al., in preparation

**Chiral Precession** couples **Gamma Vibration**

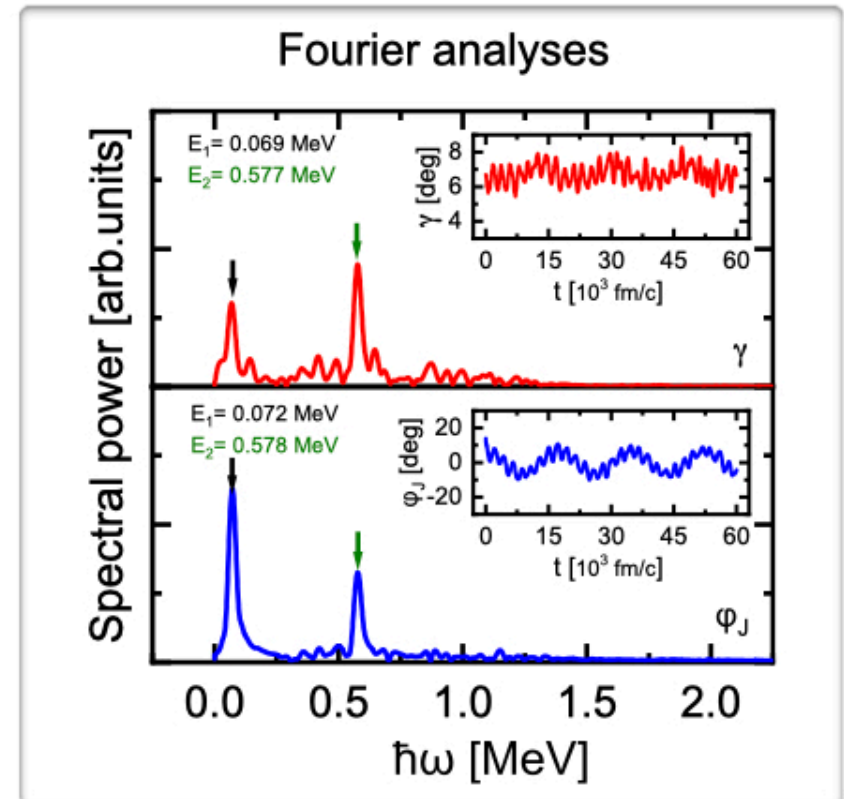
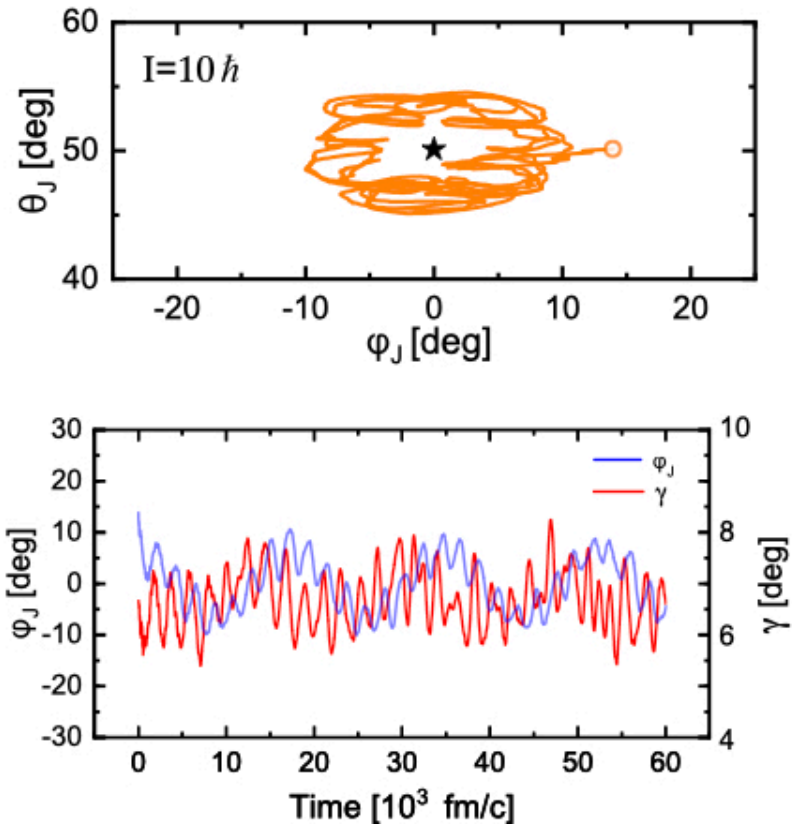


Two excitation modes can be extracted from the **Phi** and **Gamma** dynamics.

# Chiral dynamics in $^{106}\text{Ag}$

Li, et al., in preparation

**Chiral Precession** couples **Gamma Vibration**

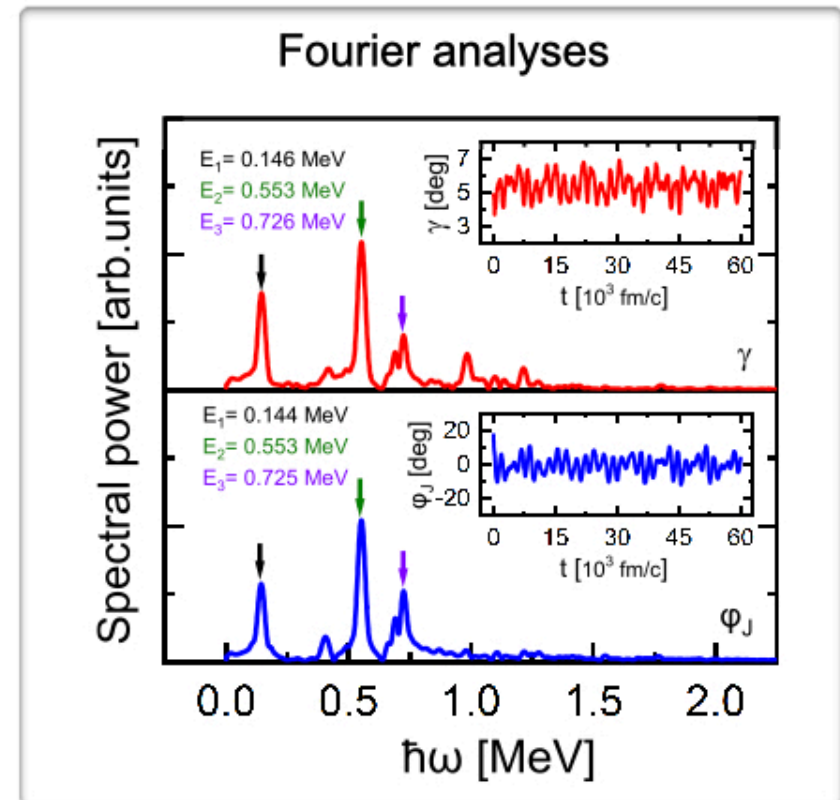
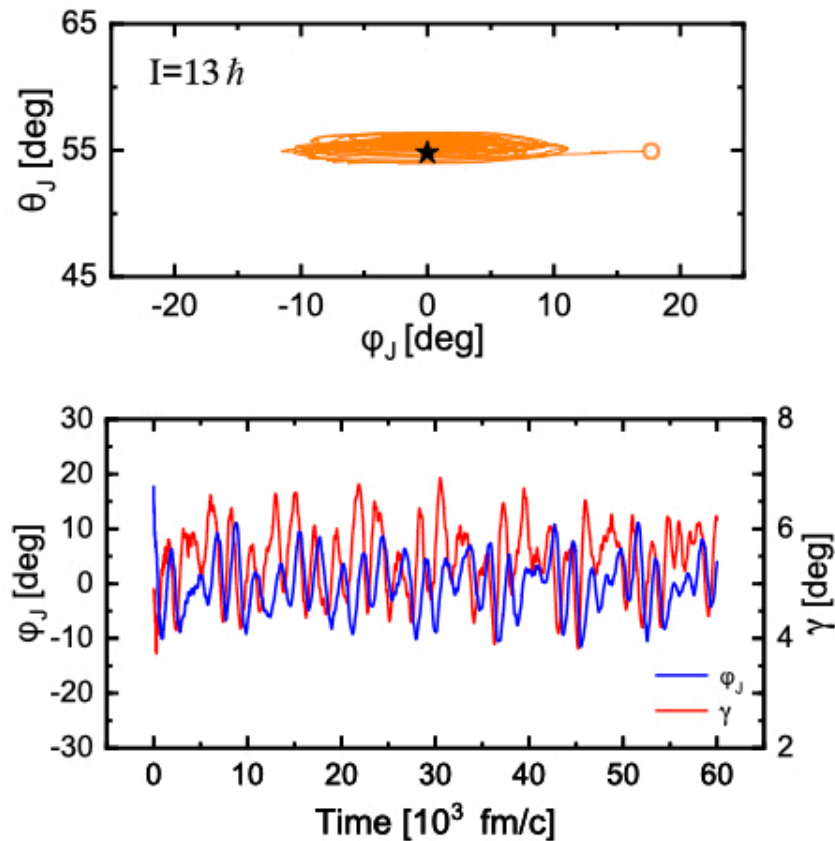


Two excitation modes can be extracted from the **Phi** and **Gamma** dynamics.

# Chiral dynamics in $^{106}\text{Ag}$

Li, et al., in preparation

**Chiral Precession** couples **Gamma Vibration**

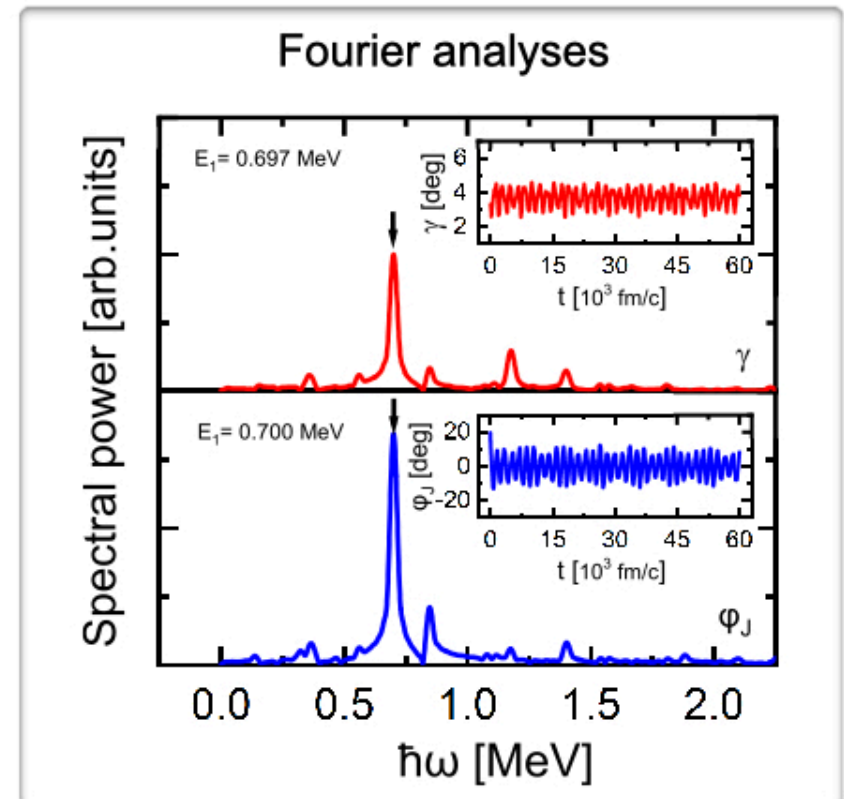
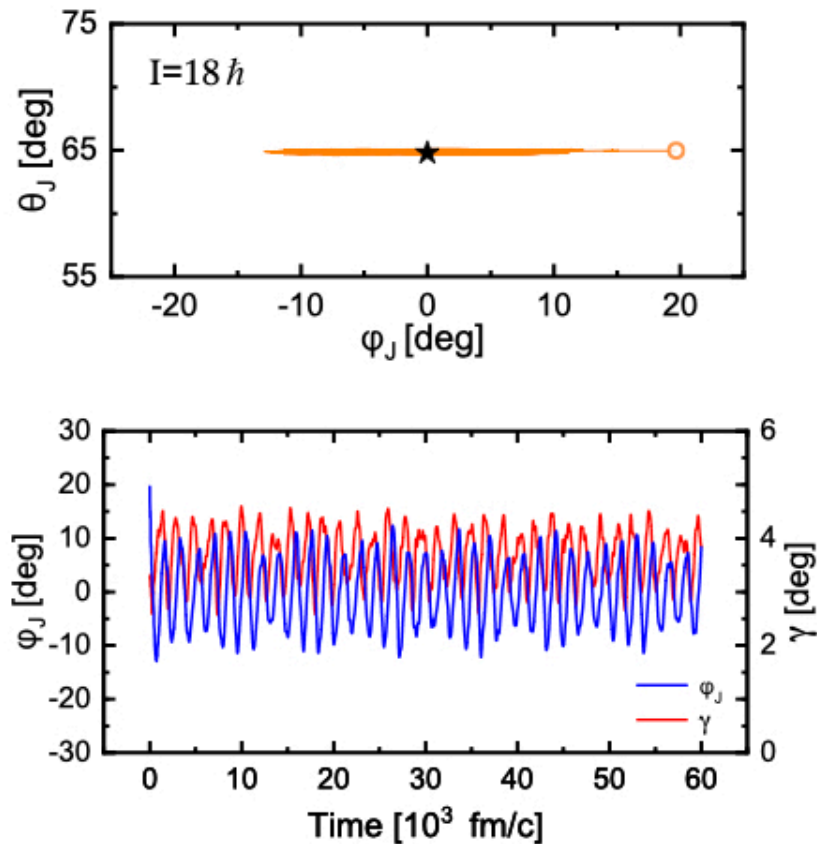


The two excitation modes are **strongly coupled** and the peak mainly associated with the gamma vibration are fragmented.

# Chiral dynamics in $^{106}\text{Ag}$

Li, et al., in preparation

**Chiral Precession** couples **Gamma Vibration**

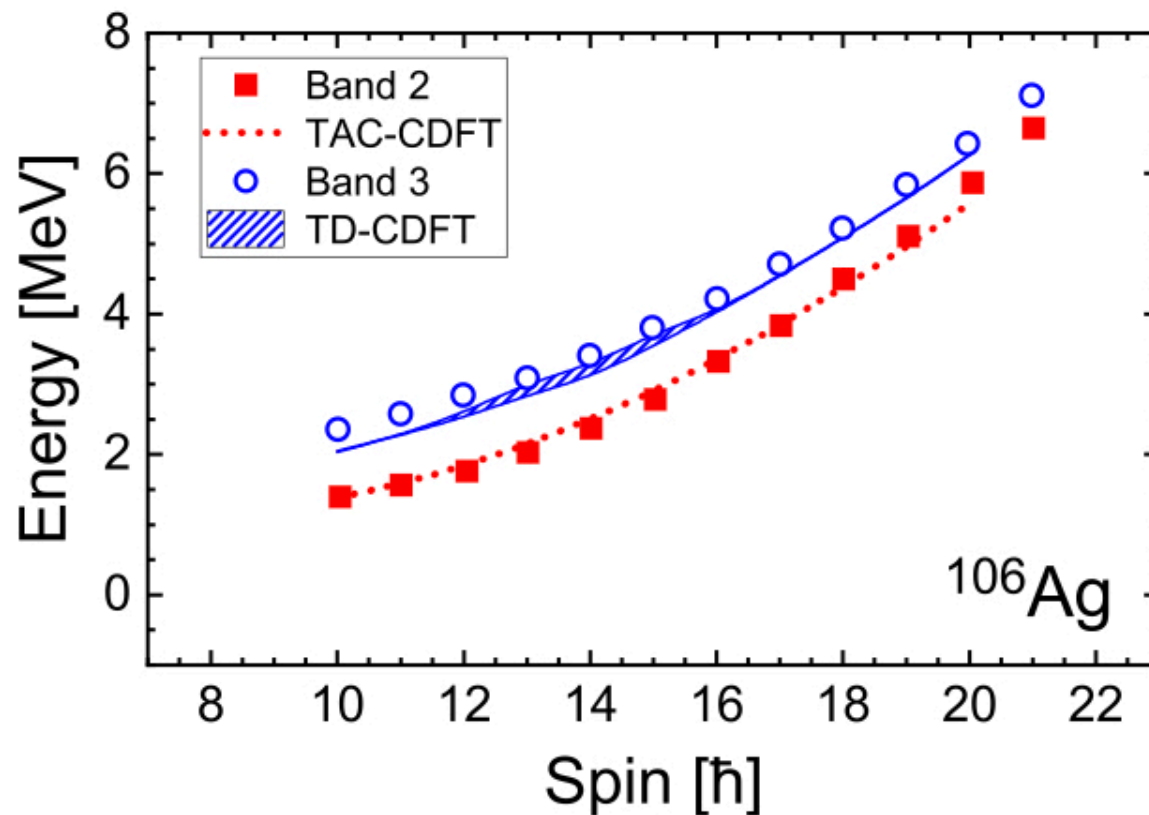


Gamma vibration is **dominated**, while chiral precession is **suppressed**.



# Energy spectrum

Li, et al., in preparation



The chiral excitation energy is given by the sum of the two modes, i.e., the chiral precession and gamma vibration.

# Summary

Covariant density functional theory has been improved and extended for a unified description of static and dynamic nuclear properties.

## ➤ Chiral dynamics in triaxial nuclei

### ✓ Static TAC-CDFT calculations

The **lower band** is well reproduced.

### ✓ Dynamic TD-CDFT calculations

The TAC-CDFT results are taken as the initial states.

The **upper band** is well reproduced.

### ✓ A novel mechanism, **chiral precession**, is revealed.

### ✓ The chiral dynamics and its interplay with the gamma vibration in **gamma soft nuclei** is explored.



# Collaborations

## Beijing

Jie Meng

Zhengxue Ren

Shuangquan Zhang

## Chongqing

Zhipan Li

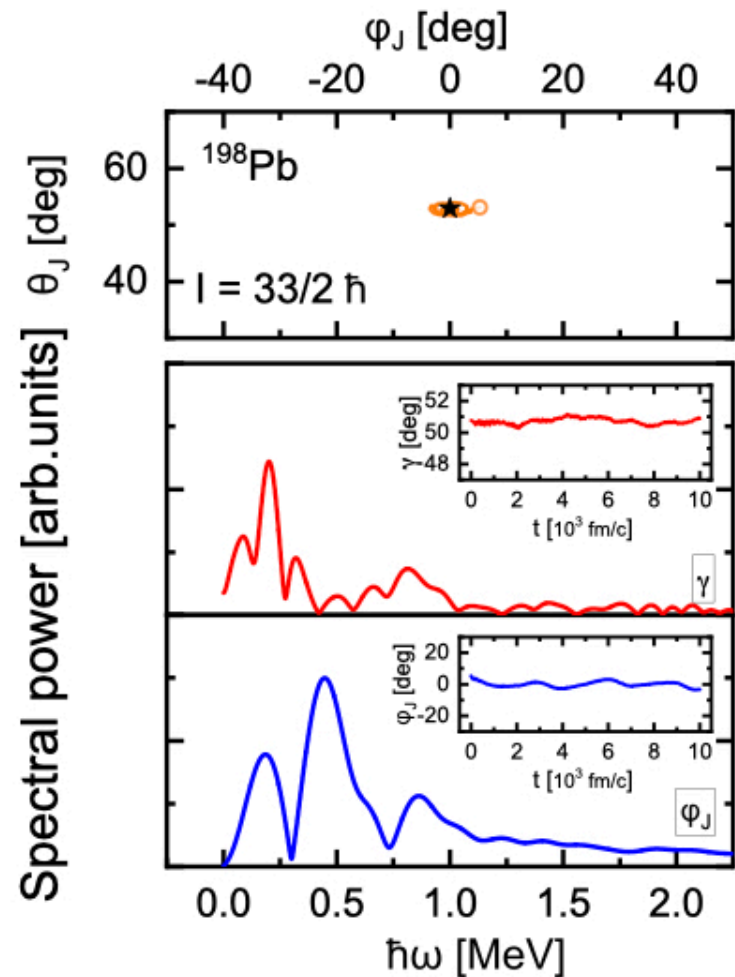
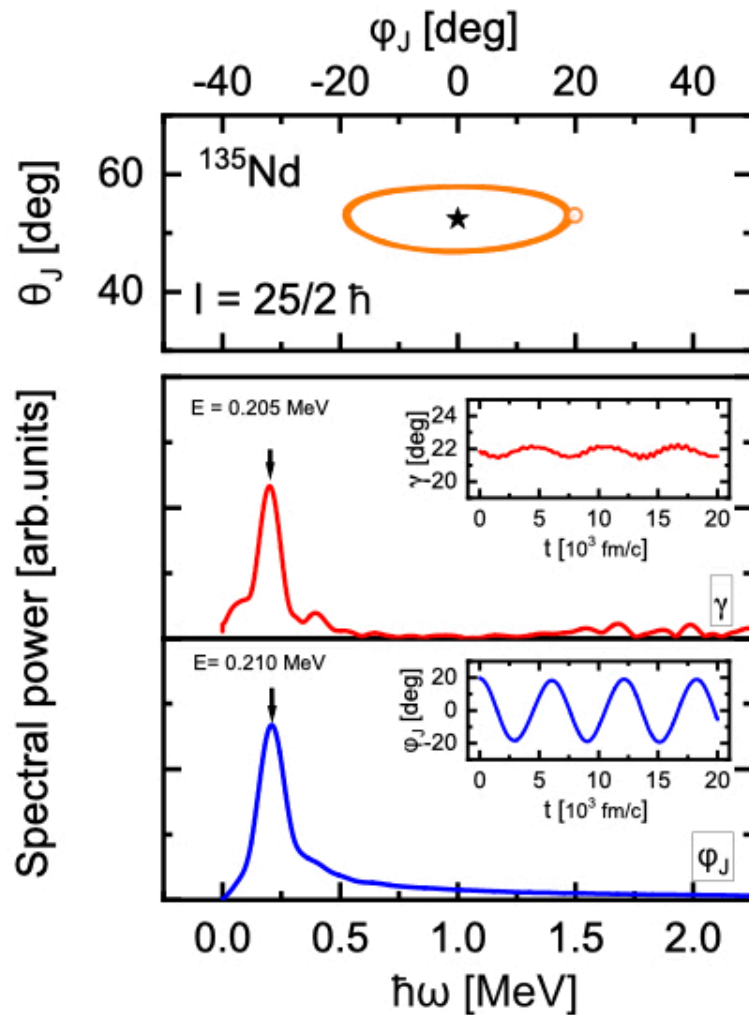
## Munich

Peter Ring

and many experimental colleagues ...

# Thank you for your attention!

# Chiral excitation energies from Fourier analyses



# TAC-CDFT for chiral conundrum in $^{106}\text{Ag}$

PWZ, Wang, Chen PRC 99, 54319 (2019)

