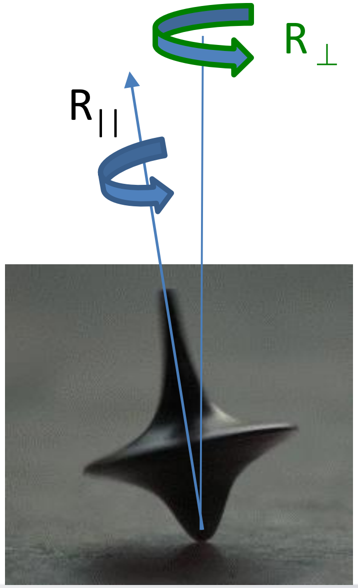
Tilted precession in triaxial nuclei calculated with the quasiparticle-rotor model

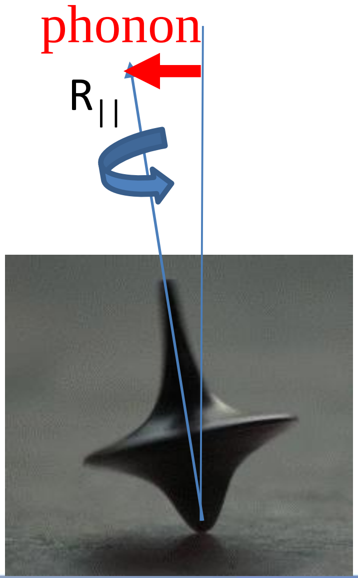
E.A. Lawrie, iThemba LABS, Cape Town, South Africa
O. Shirinda, Sol Plaatje University, Kimberley 8301, South Africa
C.M. Petrache, Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France

Tilted precession and wobbling?

Precession – only rotations



Wobbling – phonon excitation

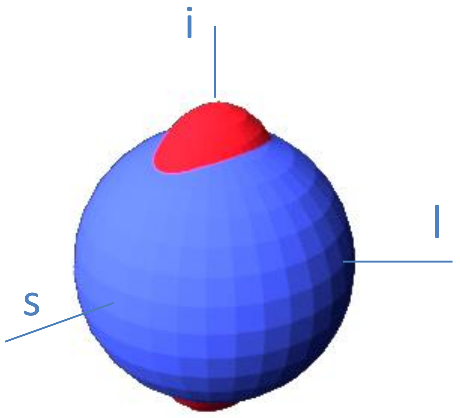


Rotation ??? Vibration (phonon excitation)

Rotation of even-even triaxial nucleus: tilted precession

spinning top

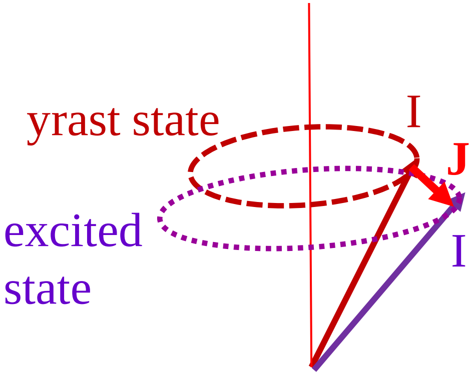
$$H_R = \frac{\hbar^2}{2\mathfrak{I}_1} R_1^{\,2} + \frac{\hbar^2}{2\mathfrak{I}_2} R_2^{\,2} + \frac{\hbar^2}{2\mathfrak{I}_3} R_3^{\,2}$$

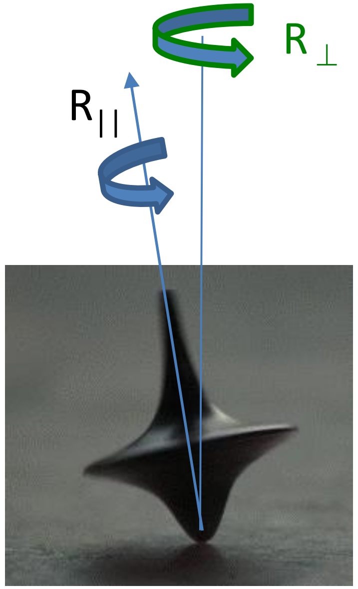


Precession of I around the axis with largest MoI

$I^2 = I_1^{\,2} + I_2^{\,2} + I_3^{\,2}$

$E = A_1 I_1^{\,2} + A_2 I_2^{\,2} + A_3 I_3^{\,2}$





yrast state, I → precession

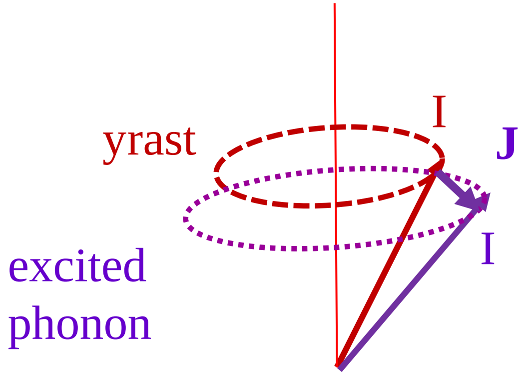
excited state, adding **J** → precession with a larger tilt angle

Wobbling phonon excitations

Coupling of rotation and vibration phonon excitation

$$H_V = \frac{\hbar^2}{2\mathfrak{I}_1} I^2 + \hbar\omega\,(\mathbf{n}+1/2)$$

where n is the number of the phonons,
ω is the vibrational frequency



J → wobbling phonon

Looking for the relationship between the two descriptions based on rotational and vibrational excitations…

Bohr and Mottelson, Nuclear Structure

190 文 ROTATIONAL SPECTRA *Ch. 4*

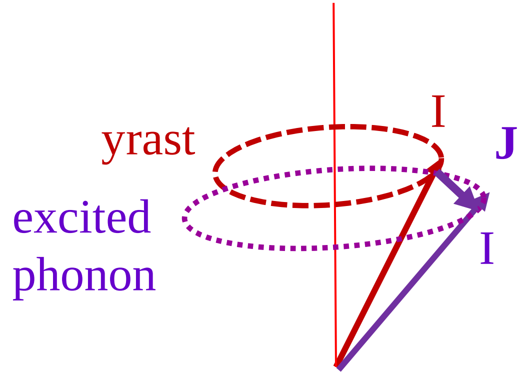
4-5e States with Large *I*

Simple and illuminating solutions for the asymmetric rotor can be obtained for the high angular momentum states in the yrast region. In the classical theory of the asymmetric rotor, the motion reduces to a simple rotation without precession of the axes, if the angular momentum is along the axis corresponding to the largest or smallest moment of inertia. Correspondingly, in the quantal theory, the states of smallest (or largest) energy for given *I* acquire a simple structure in the limit of large *I* (Golden and Bragg, 1949).

at **high spin** these two descriptions are **similar**!!!

What is “high spins”?

Bohr and Mottelson, Nuclear Structure



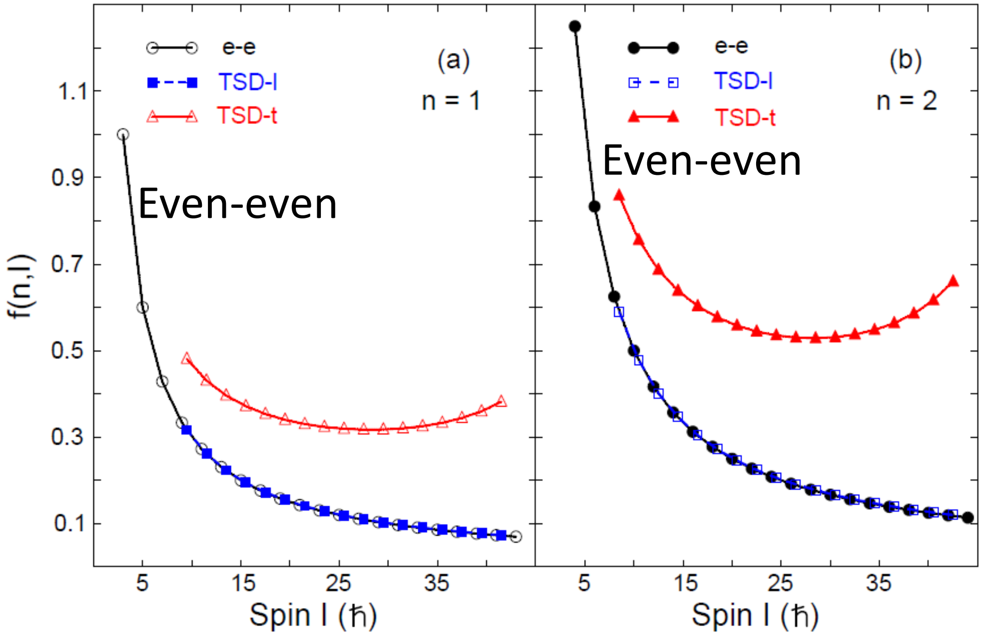
J $\rightarrow$ vibrational phonon

If the condition is satisfied

$$f(n, I) = (2n+1)\frac{(A_2 + A_3 - 2A_1)}{2I\sqrt{(A_2 - A_1)(A_3 - A_1)}} << 1$$

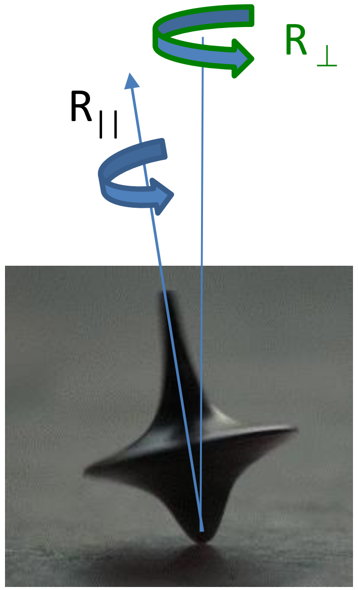
$$A_1 = \frac{\hbar^2}{2\mathfrak{J}_1}$$

f(n, I) < 0.15 for (n = 1 , I > 20)
(n = 2 , I > 34)

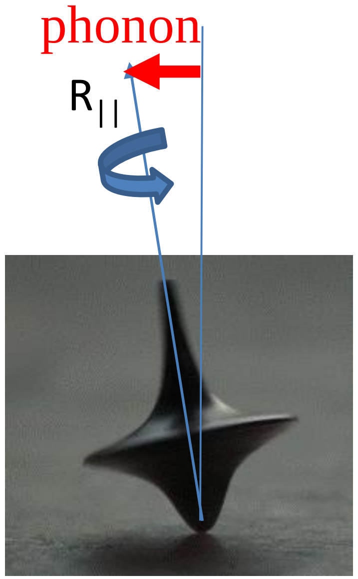


PRC, 101, 034306 (2020)

What does it mean that if the approximation condition is valid the two descriptions are **similar** ?



Rotational bands have **similar** features to harmonic wobbling phonon excitation



$$H_R = \frac{\hbar^2}{2\mathfrak{J}_1} R_1^2 + \frac{\hbar^2}{2\mathfrak{J}_2} R_2^2 + \frac{\hbar^2}{2\mathfrak{J}_3} R_3^2 \neq H_V = \frac{\hbar^2}{2\mathfrak{J}_1} I^2 + \hbar\omega\,(\mathbf{n}+1/2)$$

Rotational

phonon excitation

Precession is **similar to harmonic** phonon excitation

Precession is **equivalent to anharmonic** phonon excitation

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NRF National Research Foundation
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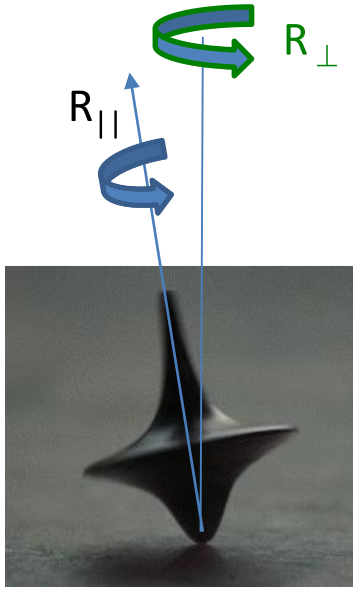
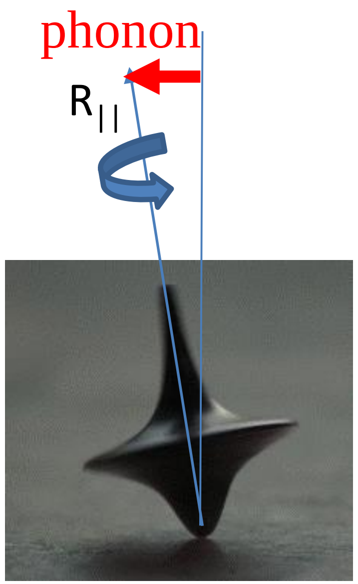
What at low spins where the approximation condition is not valid ?



Rotational bands **differ** substantially from harmonic wobbling phonon excitation



$$H_R = \frac{\hbar^2}{2\mathfrak{J}_1} R_1^{\,2} + \frac{\hbar^2}{2\mathfrak{J}_2} R_2^{\,2} + \frac{\hbar^2}{2\mathfrak{J}_3} R_3^{\,2} \quad \neq \quad H_V = \frac{\hbar^2}{2\mathfrak{J}_1} I^2 + \hbar\omega\,(\mathbf{n}+1/2)$$

Rotational

phonon excitation

The rotational and the vibrational descriptions are two **different** (competing) interpretations

Rotation of even-even triaxial nucleus: tilted precession

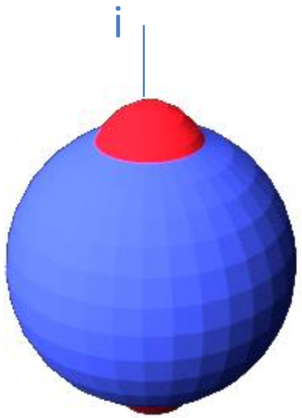
$$H_R = \frac{\hbar^2}{2\Im_1} R_1^{\,2} + \frac{\hbar^2}{2\Im_2} R_2^{\,2} + \frac{\hbar^2}{2\Im_3} R_3^{\,2}$$

With hydrodynamical-type MoI for **γ = 30º** there is a symmetry in H because $\Im_2$ (short) = $\Im_3$ (long) = ¼ $\Im_1$

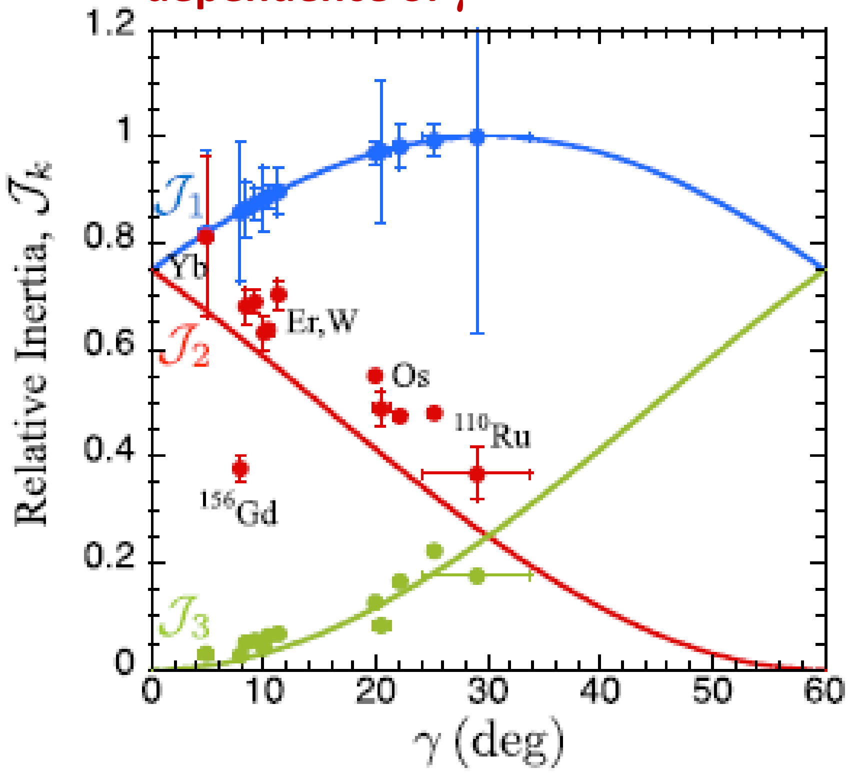
$$H = \frac{\hbar^2}{2\Im_1} R_1^{\,2} + \frac{4\hbar^2}{2\Im_1}(R_2^{\,2} + R_3^{\,2}) = \frac{\hbar^2}{2\Im_1}\{ R_1^{\,2} + 4[I(I+1) - R_1^{\,2}]\} =$$

$$= \frac{\hbar^2}{2\Im_1}\{4I(I+1) - 3R_1^{\,2}\}$$

- R_1 projection of I on the intermediate axis,
- R_1 is good q.n.
- $R_1 = I, I-1, I-2 \ldots.$
- each R_1 → a rotational band



Empirical evidence for MoI from measured energies and electric quadrupole matrix elements follow **hydrodynamical MoI dependence of γ**



J.M. Allmond, J.L.Wood,
Physics Letters B 767 (2017) 226–231

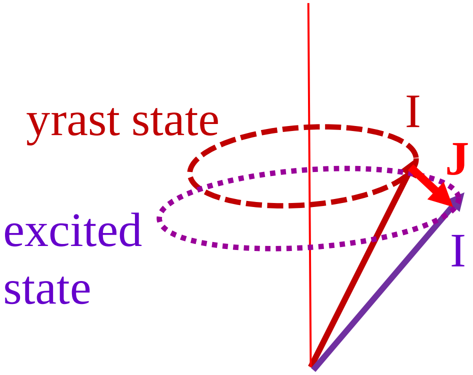
Rotation of even-even triaxial nucleus: tilted precession

$$H = \frac{\hbar^2}{2\mathfrak{J}_1}\{4I(I+1) - 3R_1^{\,2}\}$$

$R_1 = I$ → g.s. band

$R_1 = I - 1$ → γ band, odd spins

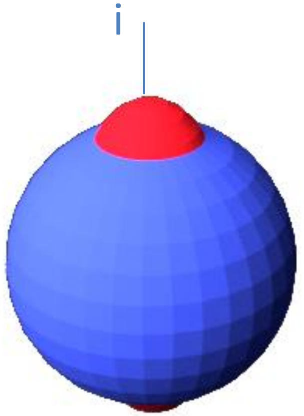
$R_1 = I - 2$ → γ band, even spins

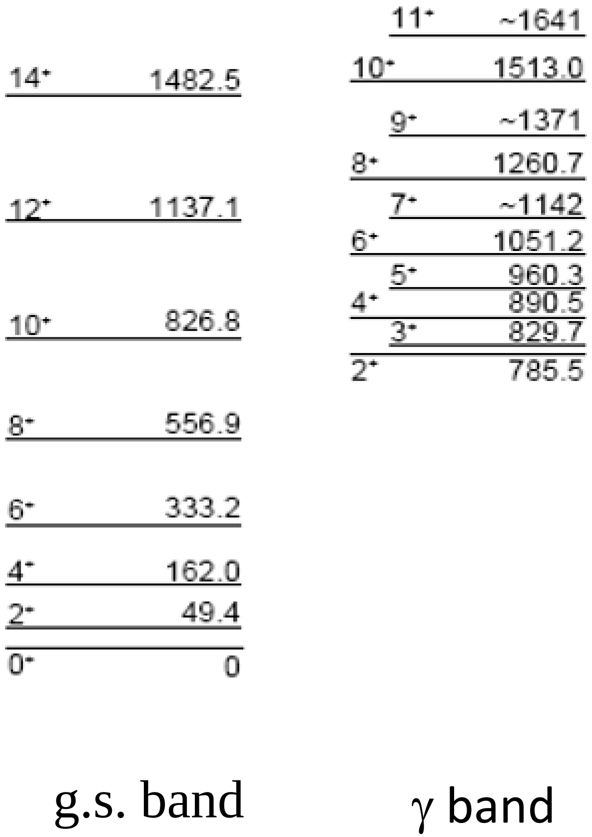


$R_1 = I, I - 1, I - 2 \ldots. = I - m$, where $m = 0,1,2,3\ldots$

$$E = \frac{\hbar^2}{2\mathfrak{J}_1}\{I(I+4) + 3m(2I - m)\}$$

Quadratic dependence on I
Quadratic dependence on m
} rotational nature





Wobbling due to phonon excitation

Precession in the γ band approximated at high spins with wobbling phonon

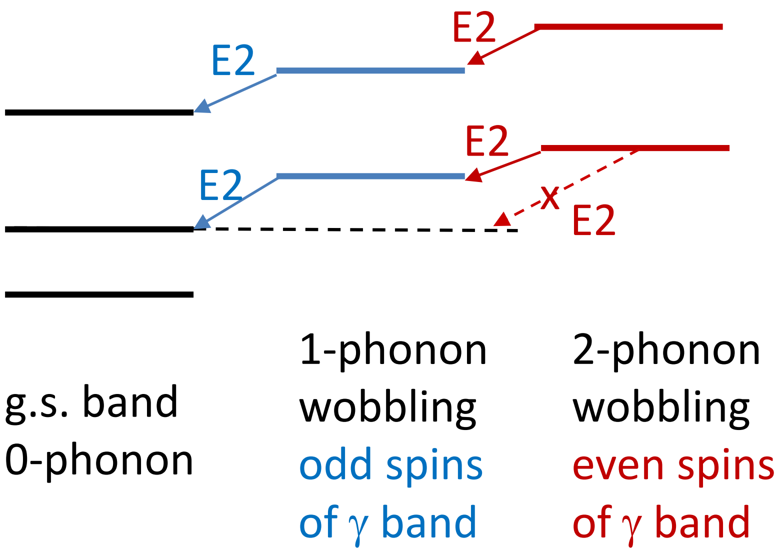
$$H = \frac{\hbar^2}{2\mathfrak{I}_1} I^2 + \hbar\omega\,(\mathbf{n}+1/2)$$

the quantization characteristics of phonon excitations:

- quantization in energy, $E(I, n) = \mathbf{n}\, E(I, 1)$, i.e. $E(I,n=2) = \mathbf{2}\, E(I,n=1)$
- quantization in $B(E2)_{out}$, $B(E2;\, n \rightarrow n\text{-}1) = \mathbf{n}\, B(E2;\, 1 \rightarrow 0)$,

  eg $B(E2;\, n=2 \rightarrow n=1) = \mathbf{2}\, B(E2;\, n=1 \rightarrow n=0)$,
- decays between the even-spin members of the γ band and the g.s. band are forbidden (simultaneous destruction of two phonons)

at high spins



Tilted precession due to rotation

Precession in the γ band at low spins (for $\gamma = 30^o$)

$$E = \frac{\hbar^2}{2\mathfrak{I}_1}\{I(I+4) + 3m(2I-m)\}$$

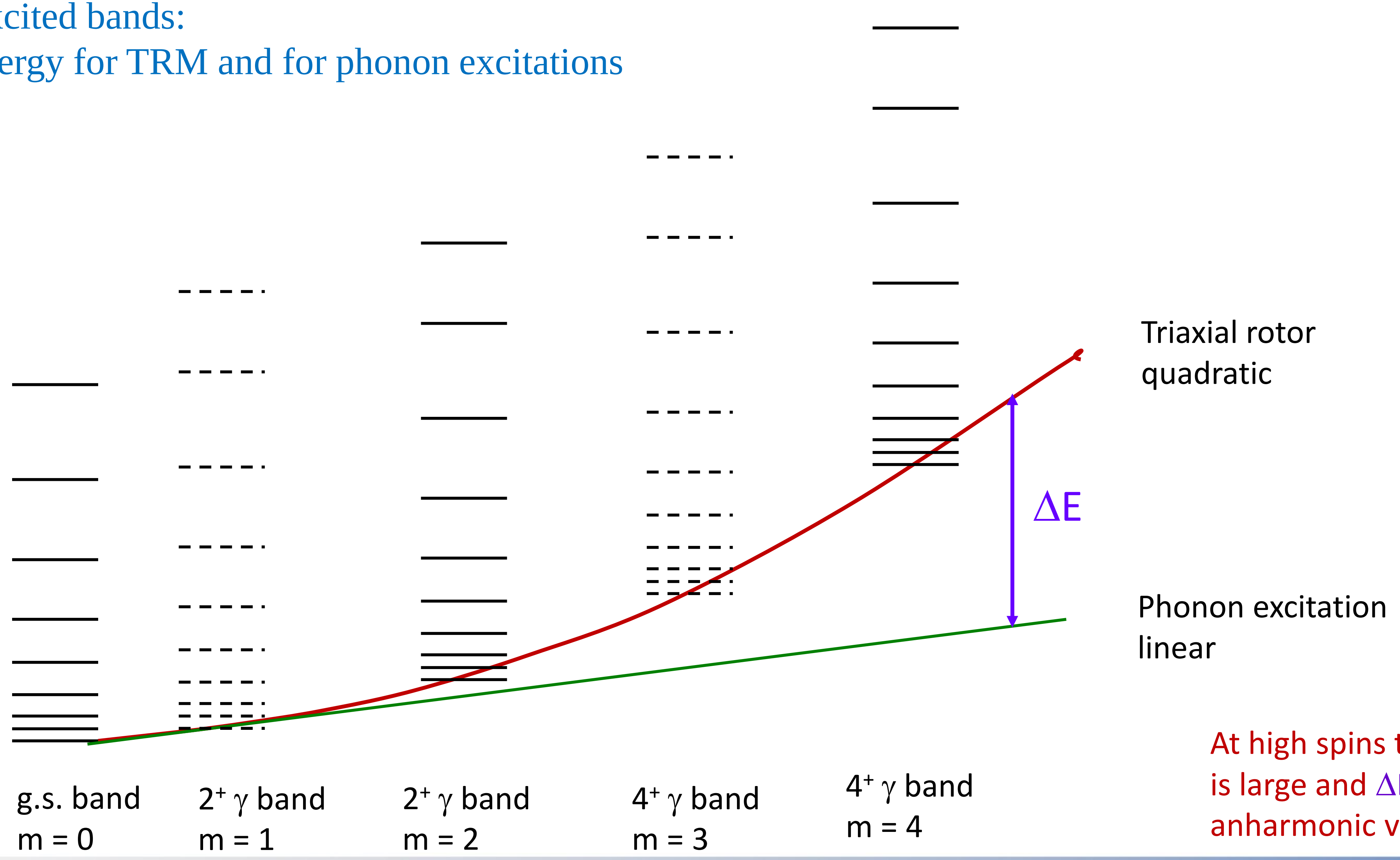
- The energy E(I, n) depends on m in quadrature, the quadratic term is small if $m \ll 2I$
- no quantization required in $B(E2)_{out}$,

  eg $B(E2;\, n=2 \rightarrow n=1) \neq \mathbf{2}\, B(E2;\, n=1 \rightarrow n=0)$,
- decays between the even-spin members of the γ band the g.s. band are allowed

γ band in triaxial-rotor model is understood as precession

It looks like anharmonic wobbling at high spins

Excited bands:
energy for TRM and for phonon excitations

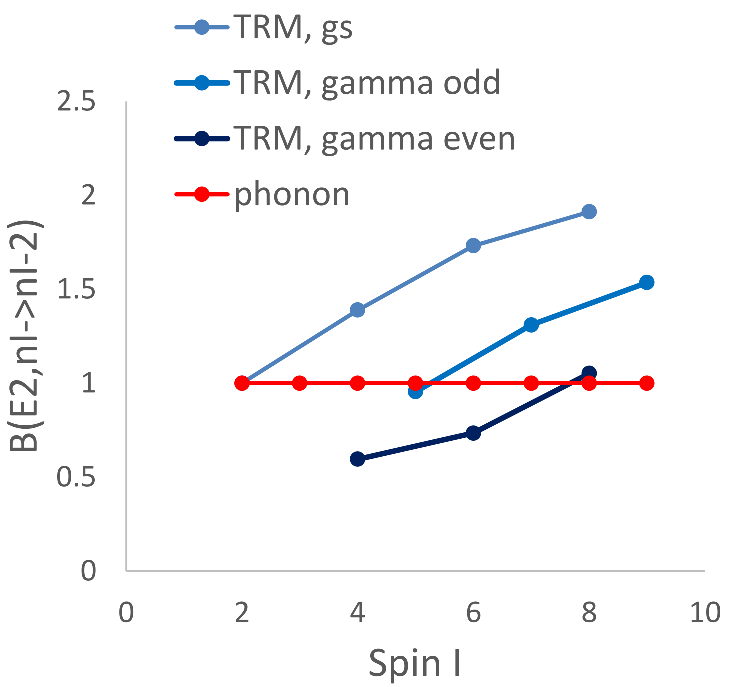


At high spins the rotational energy is large and ΔE can be small, anharmonic vibration

B(E2) transition probabilities:
TRM and phonon excitations

Intra-band

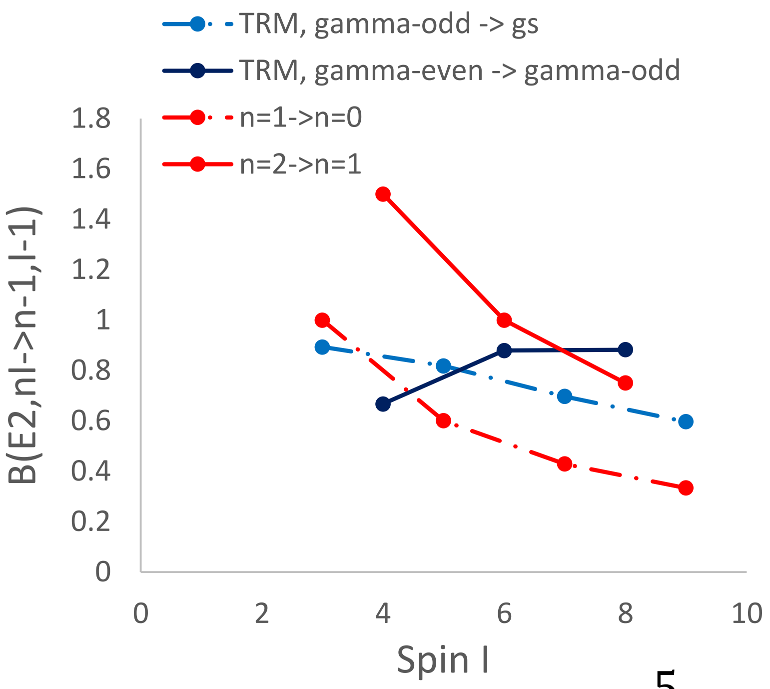
B(E2,n,I->n,I-2)



$$B(E2, n, I \rightarrow n, I \pm 2) = \frac{5}{16\pi} e^2 {Q_2}^2$$

Inter-band n → n-1

B(E2,n,I->n-1,I-1)



$$B(E2, n, I \rightarrow n-1, I-1) = \frac{5}{16\pi} e^2 \frac{n}{I} (\sqrt{3} Q_o x - \sqrt{2} Q_2 y)^2$$

Inter-band n → n-2

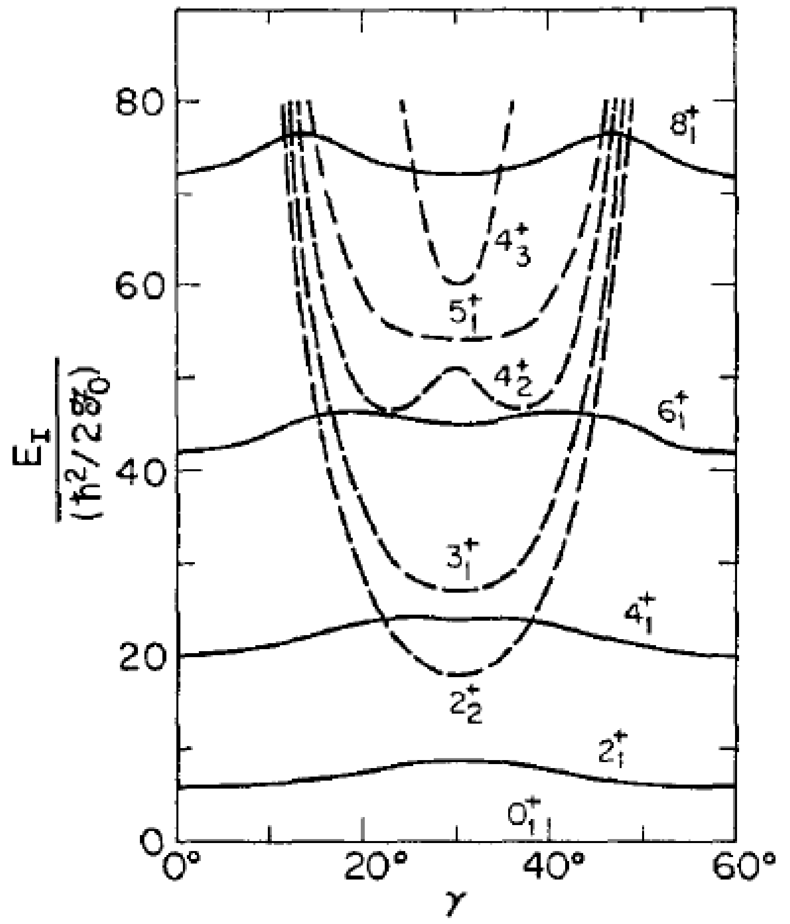
B(E2,n,I->n-2,I-2)
B(E2,n,I->n-2,I)

TRM – allowed,
$2^+_\gamma \rightarrow 2^+_{gs}$, $2^+_\gamma \rightarrow 0^+_{gs}$
$4^+_\gamma \rightarrow 4^+_{gs}$, $4^+_\gamma \rightarrow 2^+_{gs}$
$6^+_\gamma \rightarrow 6^+_{gs}$, $6^+_\gamma \rightarrow 4^+_{gs}$

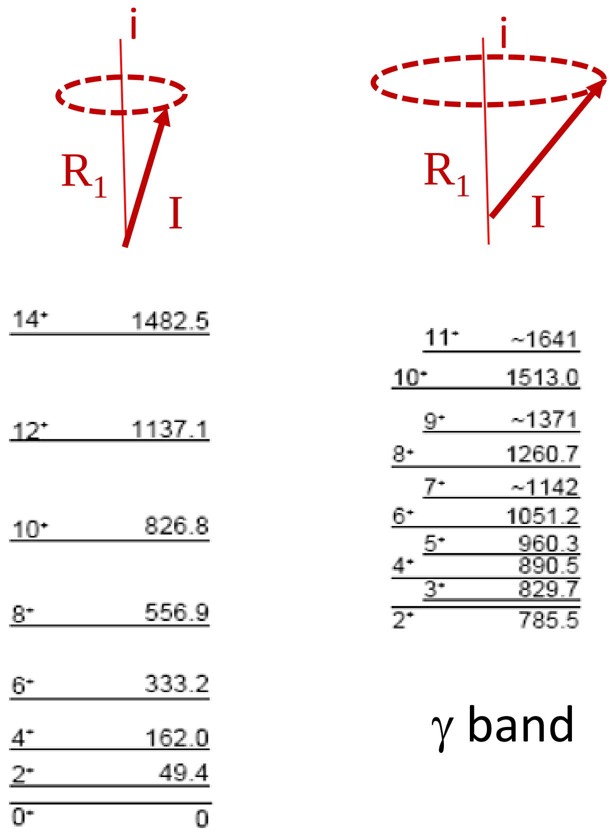
Phonon – forbidden destruction of 2 phonons

Considerable differences in the B(E2) probabilities for tilted precession and wobbling phonons

Rotation of triaxial nucleus: tilted precession



J. Meyer-ter-Vern, Nucl. Phys. A 249, 111 (1975)



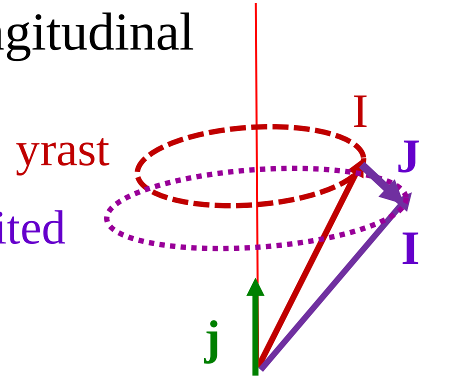
g.s. band

γ band is precession (rotational nature)

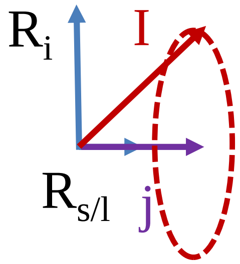
Since its introduction in the 1950s wobbling has been searched for in even-even nuclei for many decades.
Many γ bands were discovered at low spins, some of them have been interpreted using the triaxial-rotor model, but they were never considered as wobbling…

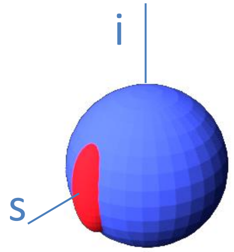
Precession in odd-mass nuclei

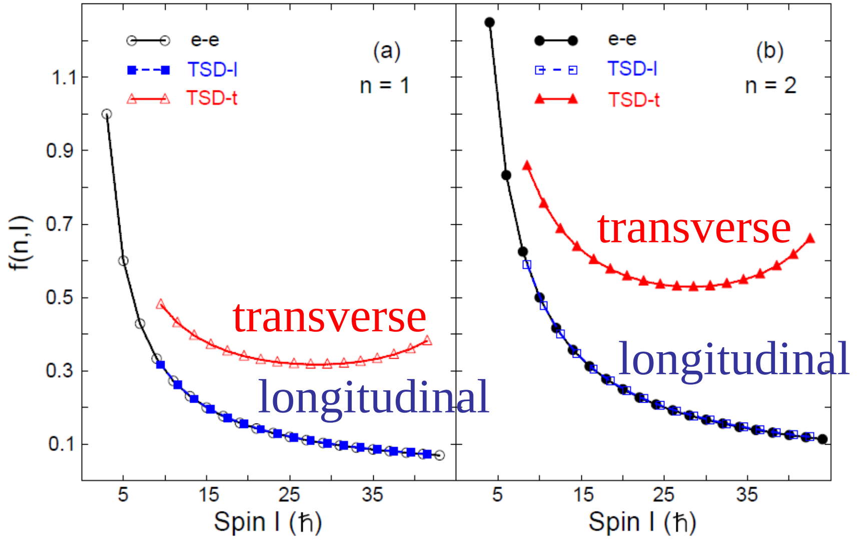
longitudinal

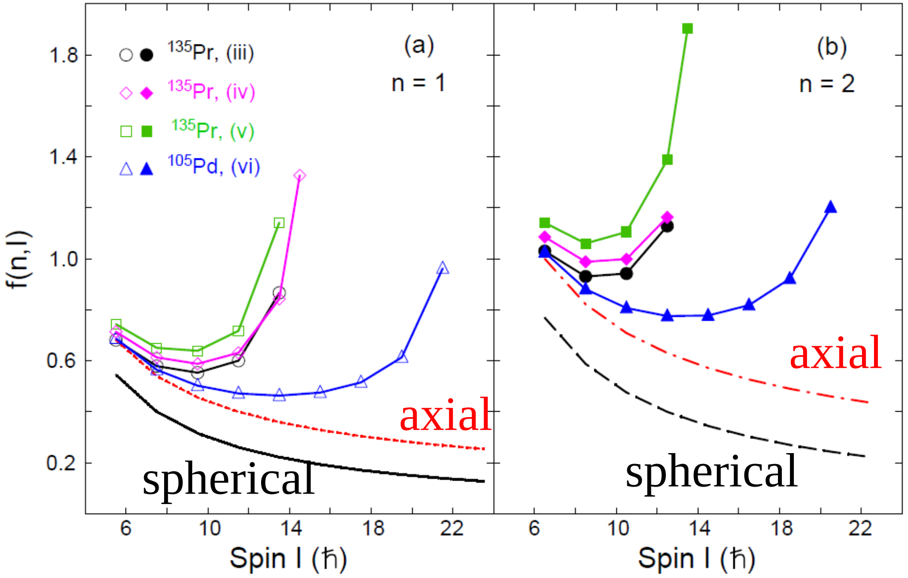


transverse









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Longitudinal coupling:

→ phonon approximation is applicable at high spins;

anharmonic phonons

Transverse coupling:

→ phonon approximation is not valid at any spin

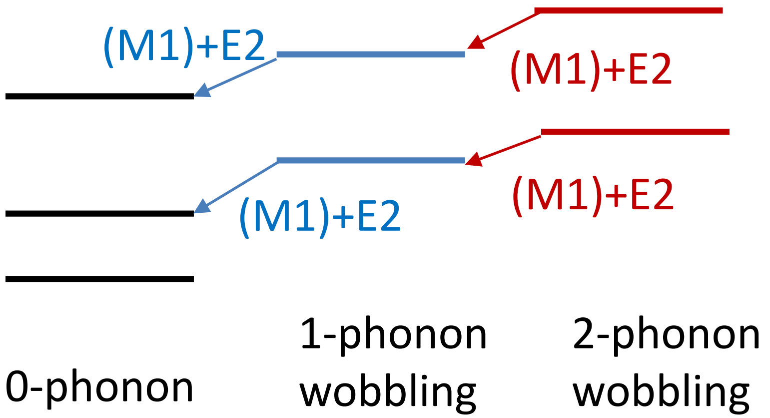
triaxial rotor model – odd mass nuclei	low spins	high spins
Longitudinal coupling	precession	precession equivalent to anharmonic wobbling
Transverse coupling	precession	precession

How to identify wobbling phonons?

- dominant E2 nature of linking transitions signifying collective nature of the excitation

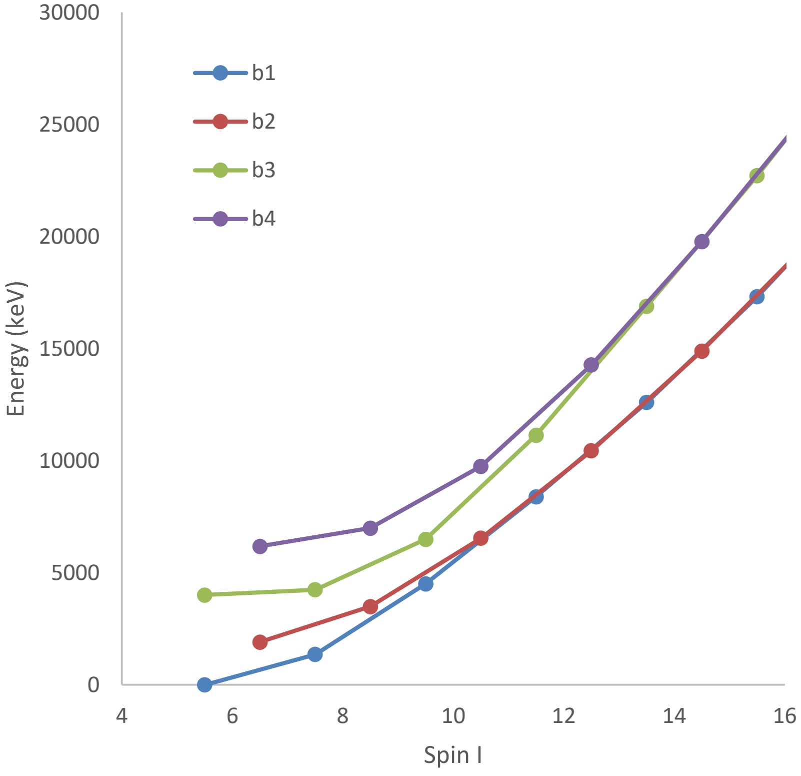
- there are other cases with dominant E2 nature, eg tilted precession, decay out of K = 2 γ vibrational band (small γ vibrations around an average axially symmetric shape), and others….

- To identify wobbling phonon excitation we should check for phonon quantization, eg quantization in excitation energy, in B(E2)s, in B(M1)s…



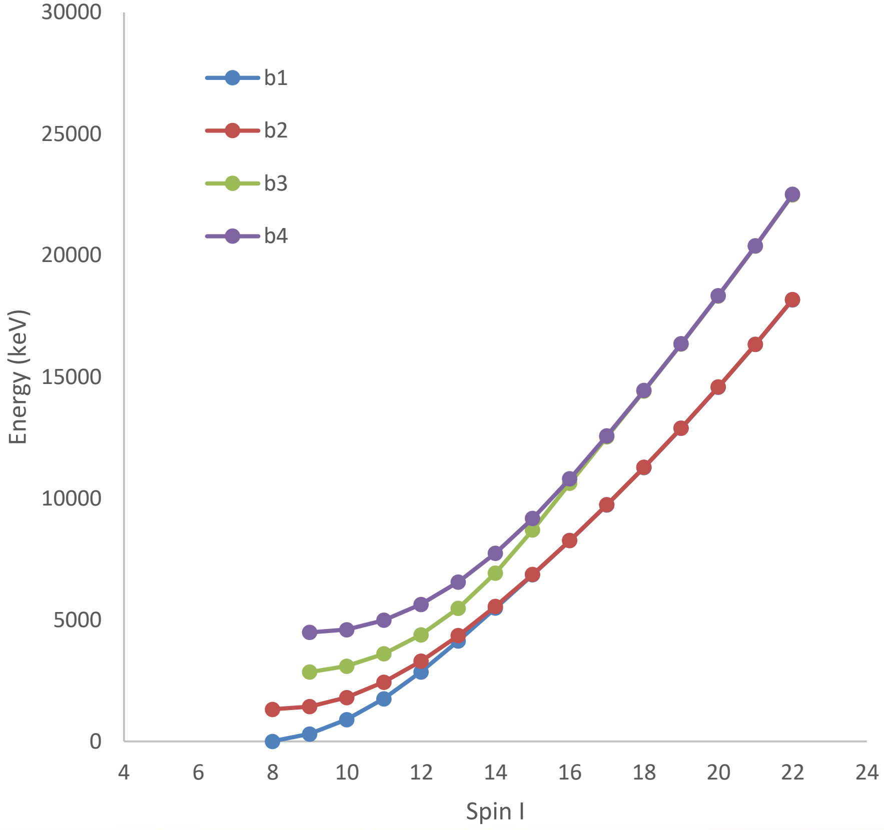
The precession of triaxial nuclei with configurations of **frozen $h_{11/2}$ and $\pi\, h_{11/2} \otimes \nu\, h_{11/2}^{-1}$ type**

Frozen $\pi\, h_{11/2}$,
hydrodynamic MoI, $\gamma = 30^o$



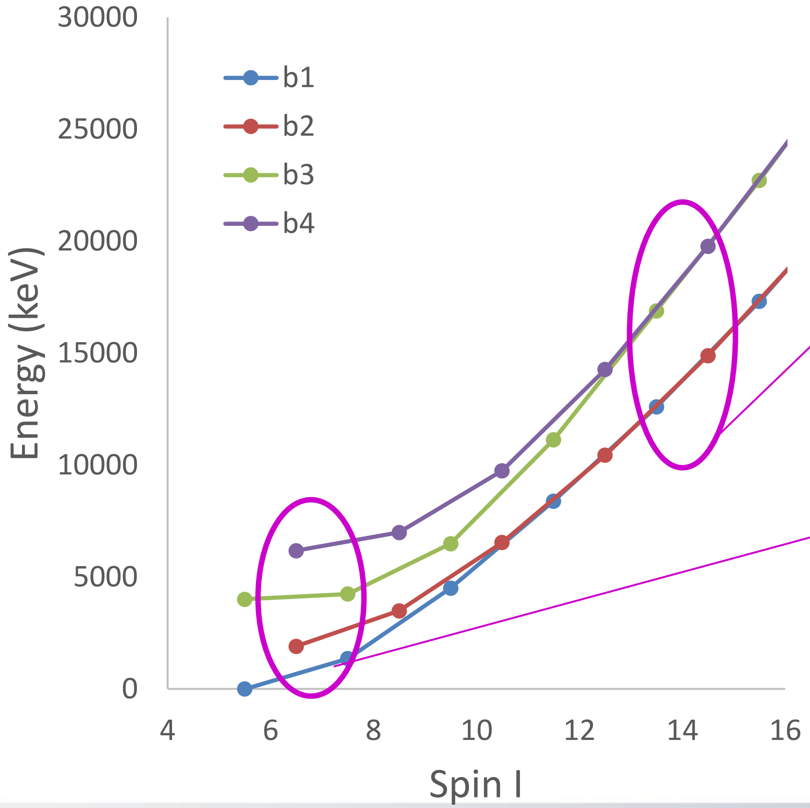
identical behaviour ??

Frozen $\pi\, h_{11/2} \otimes \nu\, h_{11/2}^{-1}$
hydrodynamic MoI, $\gamma = 30^o$



The precession of triaxial nuclei with configurations based on frozen $h_{11/2}$ and $\pi\, h_{11/2} \otimes \nu\, h_{11/2}^{-1}$ type

Frozen $\pi\, h_{11/2}$,
hydrodynamic MoI, $\gamma = 30^{o}$



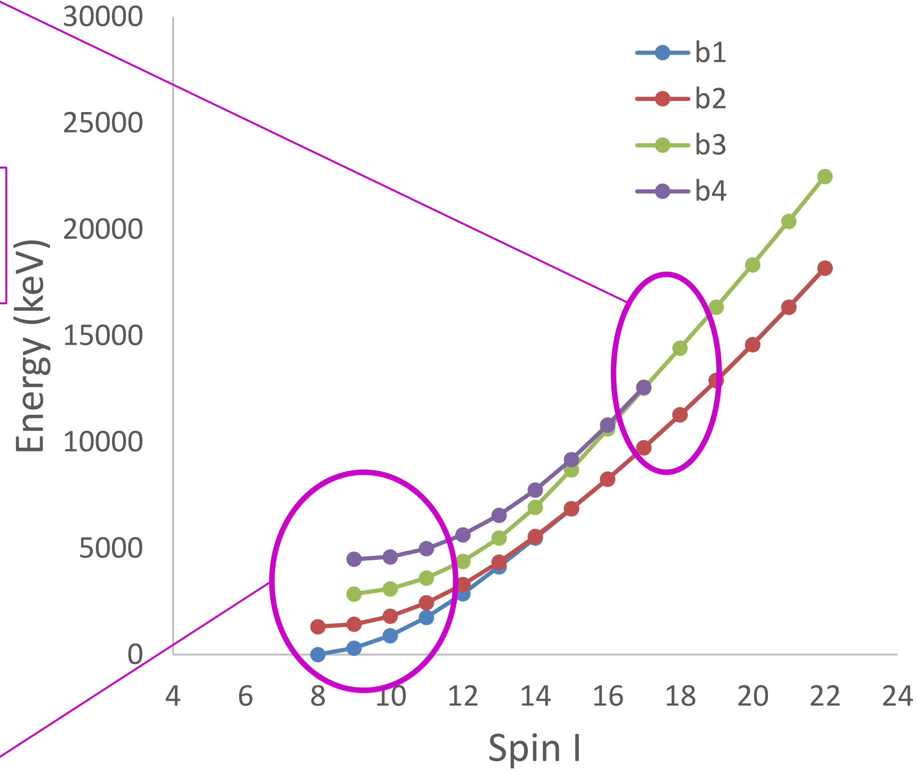
Chiral symmetry

Precession (wobbling) around the intermediate axis

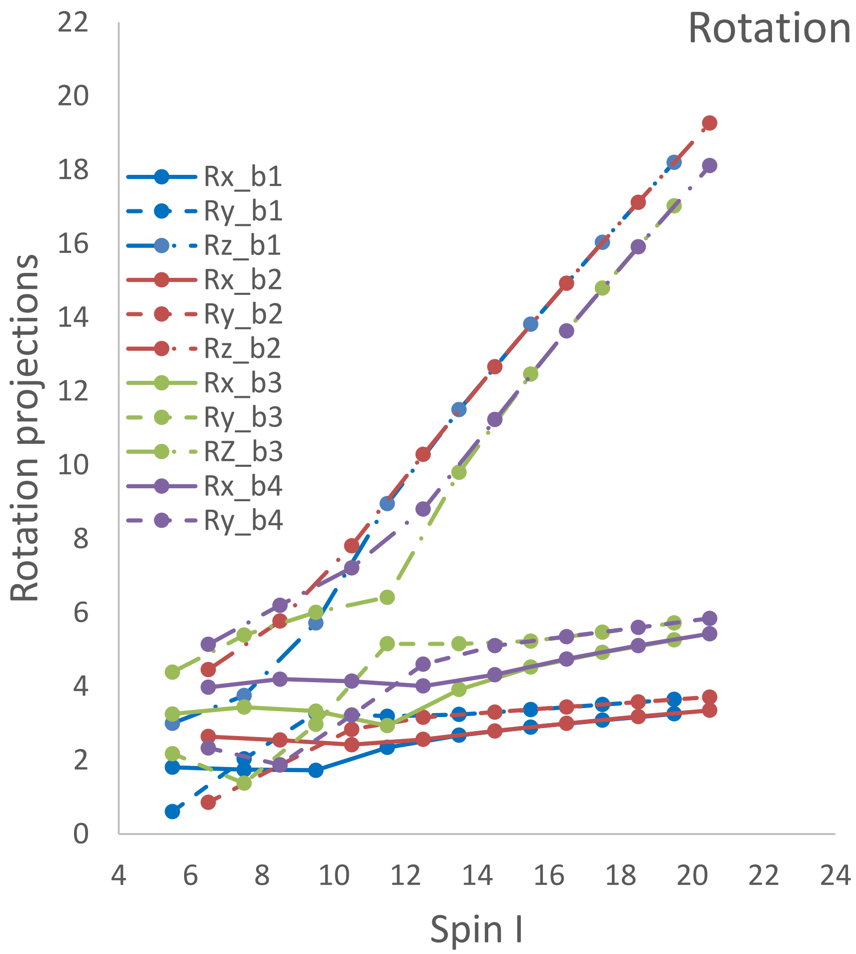
Precession (wobbling) around the short axis

Planar (non-chiral) components

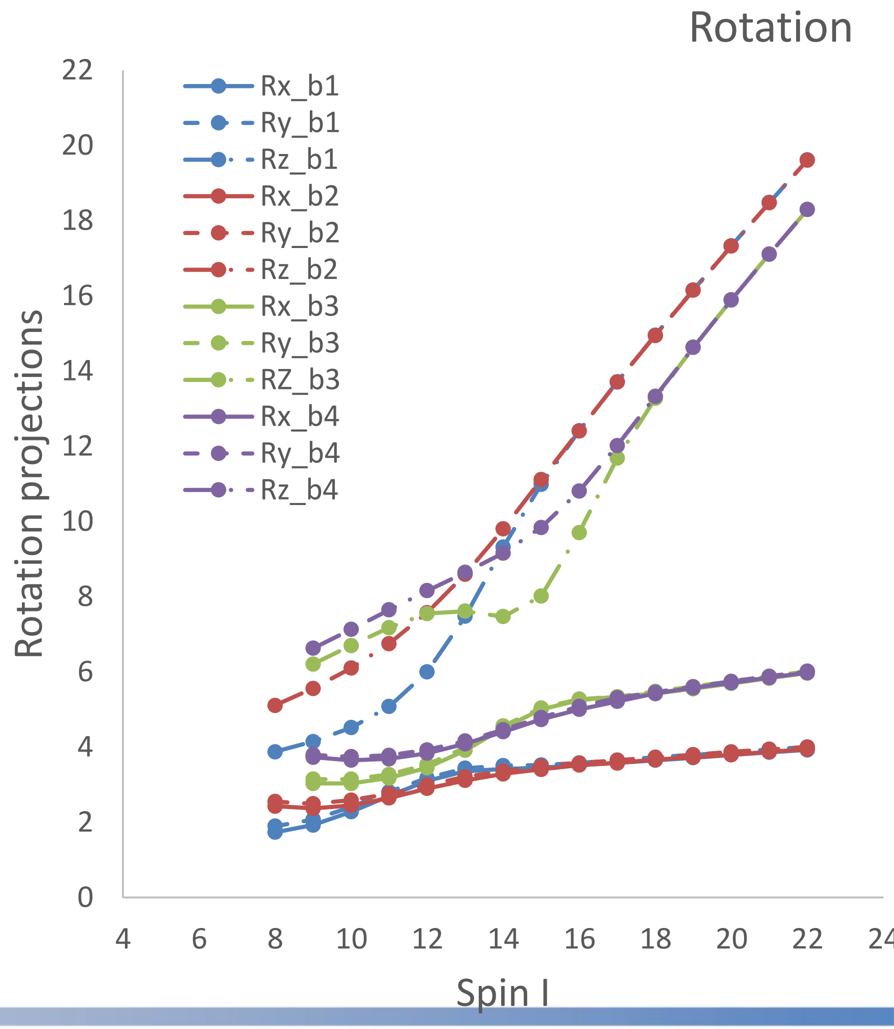
Frozen $\pi\, h_{11/2} \otimes \nu\, h_{11/2}^{-1}$
hydrodynamic MoI, $\gamma = 30^{o}$



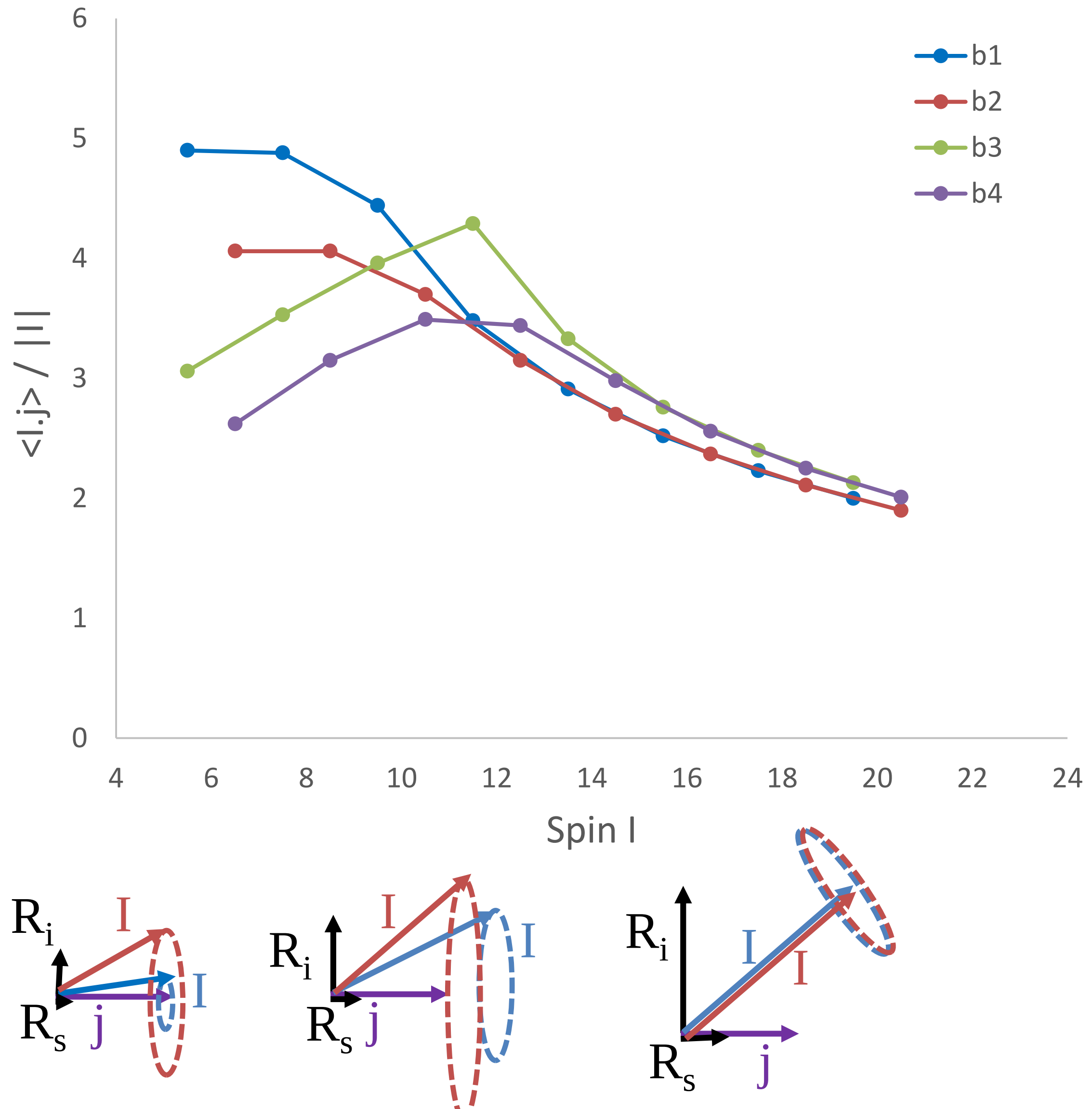
Frozen $\pi\ h_{11/2}$,
hydrodynamic MoI, $\gamma = 30^o$

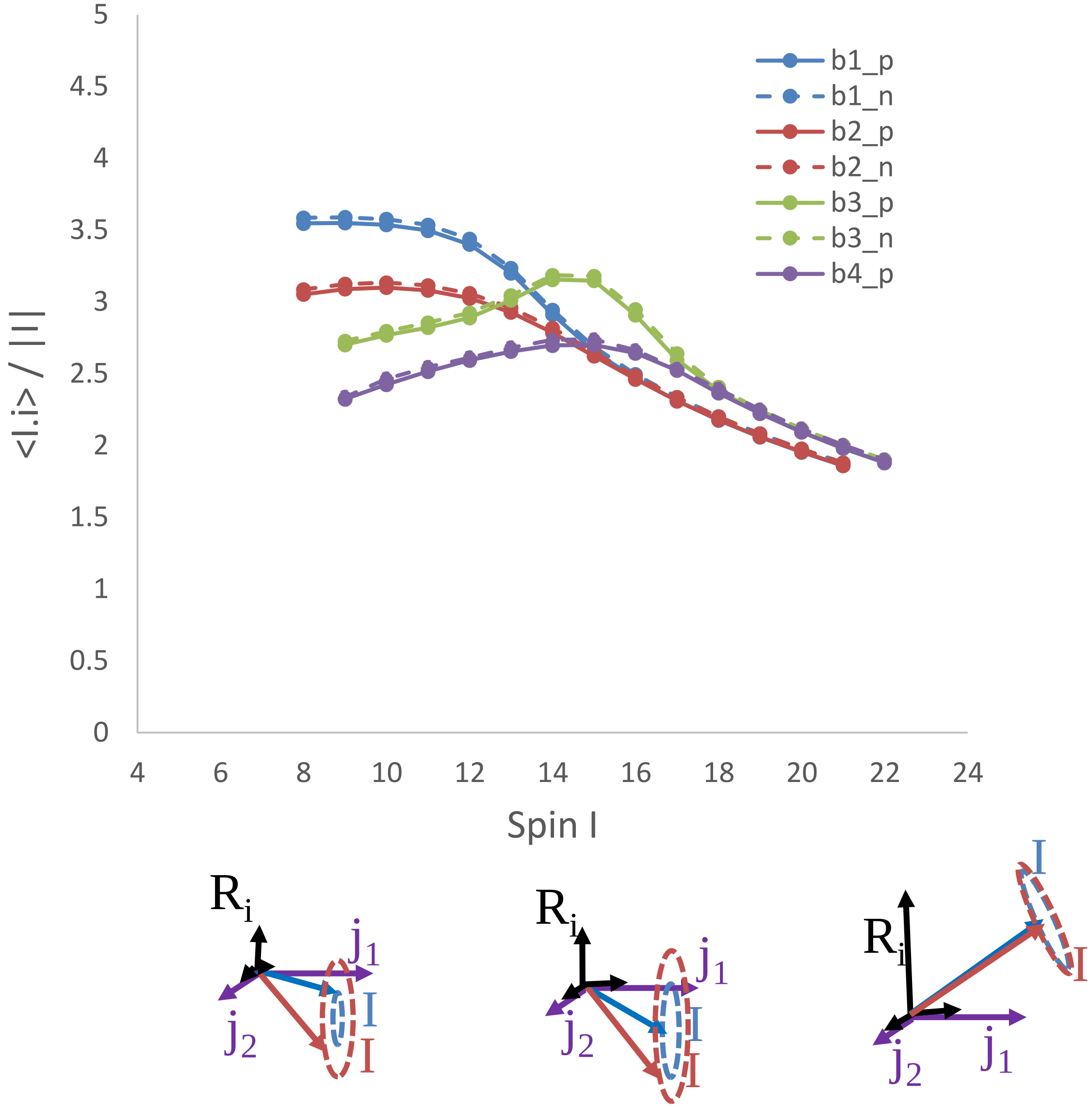


Frozen $\pi\ h_{11/2} \otimes \nu\ h_{11/2}^{-1}$
hydrodynamic MoI, $\gamma = 30^o$

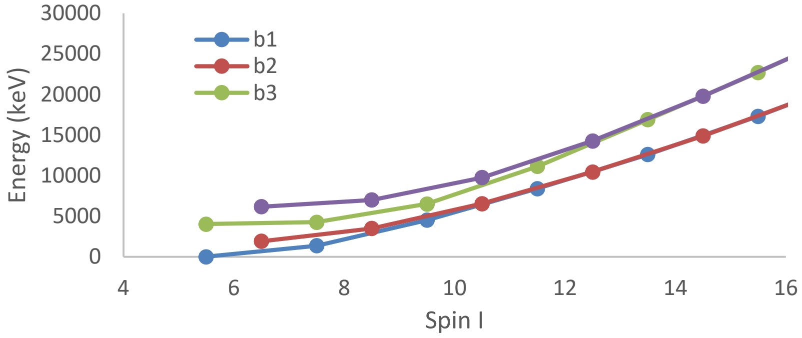


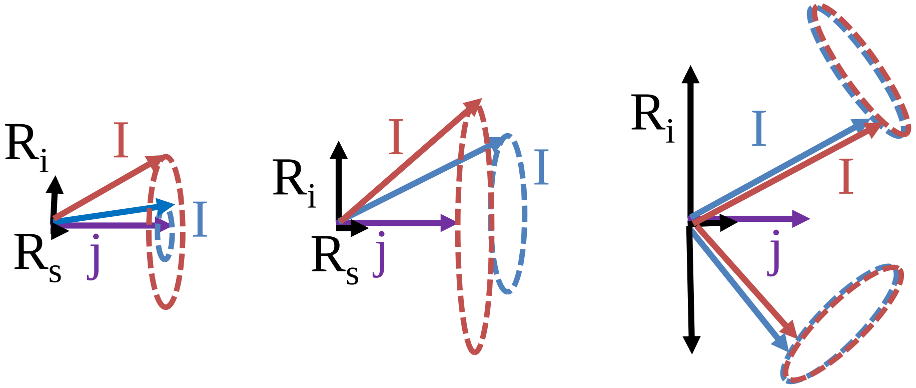
Projection of the single-particle angular momentum on the direction of the total angular momentum
< I . j> / |I|

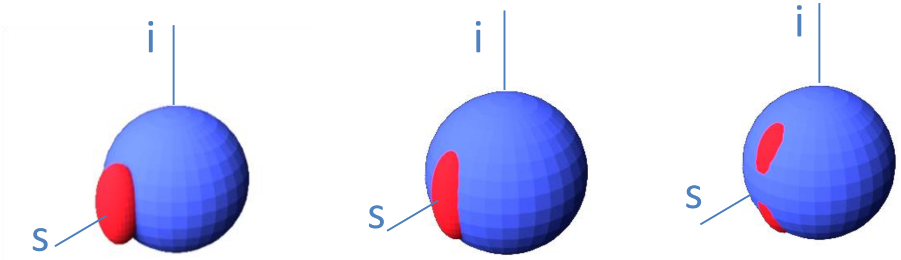




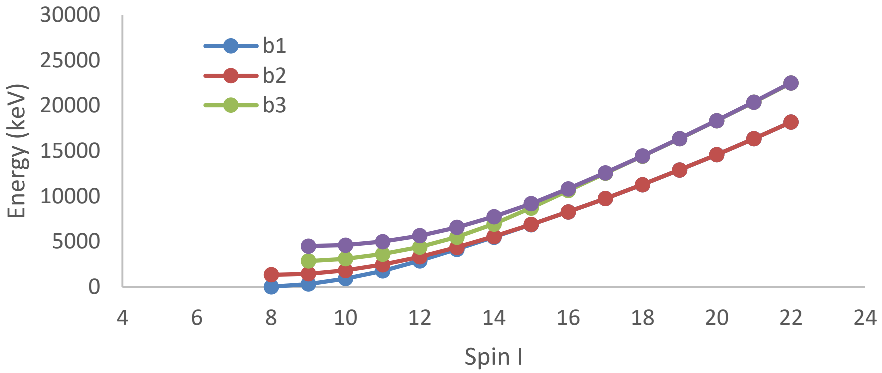
Frozen $\pi\, h_{11/2}$,
hydrodynamic MoI, $\gamma = 30^o$

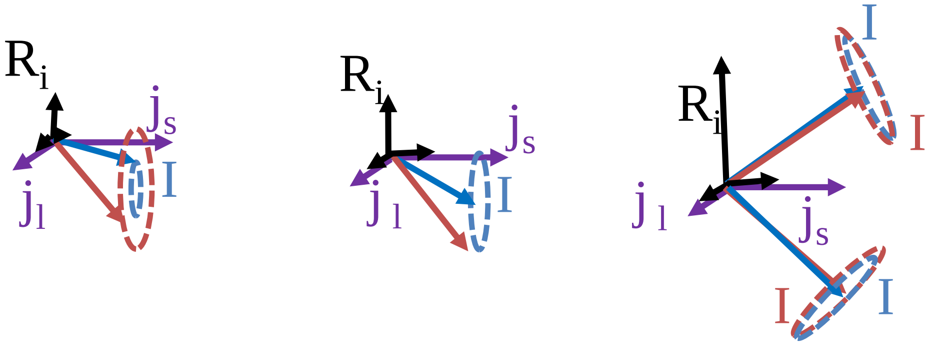


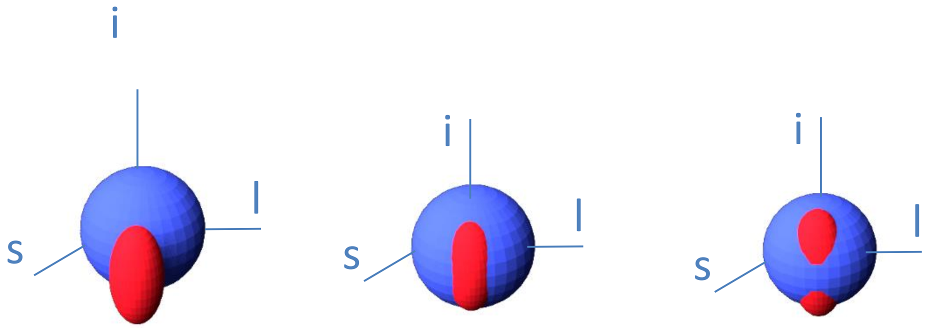




Frozen $\pi\, h_{11/2} \otimes \nu\, h_{11/2}^{-1}$
hydrodynamic MoI, $\gamma = 30^o$





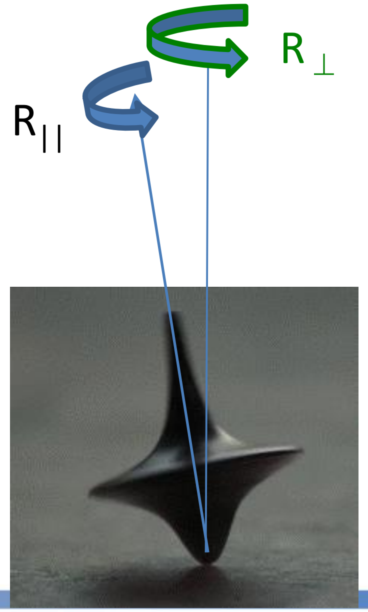


Summary

Triaxial rotor model → precession with rotational nature similar to the precession of a rotating top

Triaxial rotor model	**low spins**	**high spins**
Even-even	precession	precession is equivalent to anharmonic wobbling phonon excitation
Longitudinal coupling	precession	precession is equivalent to anharmonic wobbling phonon excitation
Transverse coupling	precession	precession

When approximation condition is not valid there are considerable differences between the calculated rotational and wobbling bands, eg in excitation energy, and the B(E2) probabilities, and these two descriptions should be considered as different interpretations.



Wobbling excitation should have the characteristic features of all phonon excitations – quantization in Eexc, B(E2)s, B(M1)s

The similarities in the behaviour of the calculated $\pi\, h_{11/2}$ and $\pi\, h_{11/2} \otimes \nu\, h_{11/2}^{-1}$ bands, including in excitation energies, rotational and total angular momenta are attributed to the same behaviour of the total angular momentum, which moves away from the short axis (short-long plane) for the former (latter) configuration.

Thank you for your attention

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