Oak Ridge National Laboratory

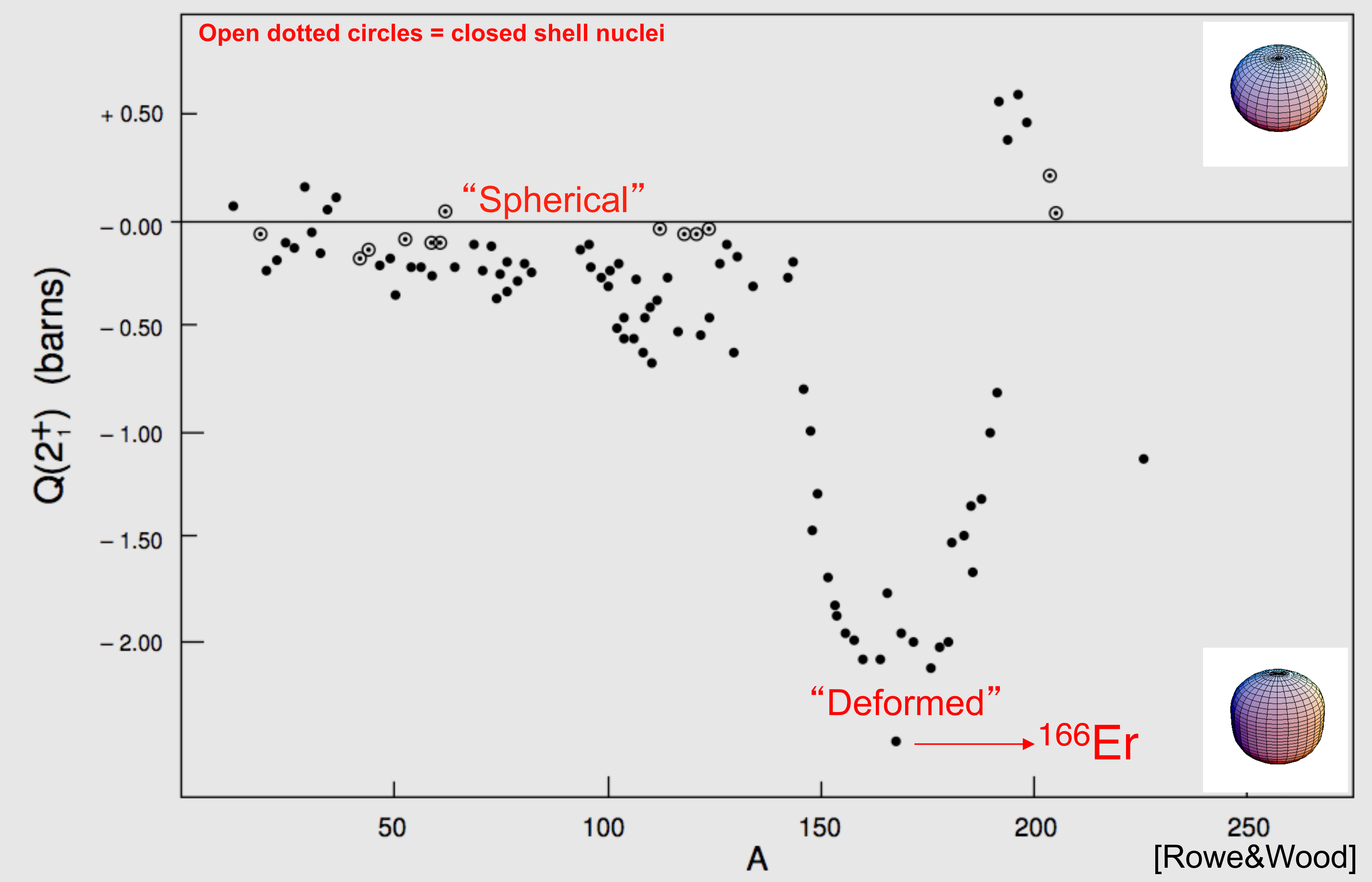
Updated View on Empirical Moments of Inertia of Axially Asymmetric Nuclei

J.M. Allmond – ORNL

U.S. DEPARTMENT OF ENERGY

First 2^+ Quadrupole Moment

Nuclei are predominantly between spherical and prolate deformed



[Rowe&Wood]

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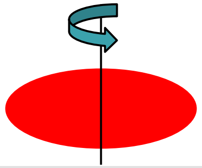
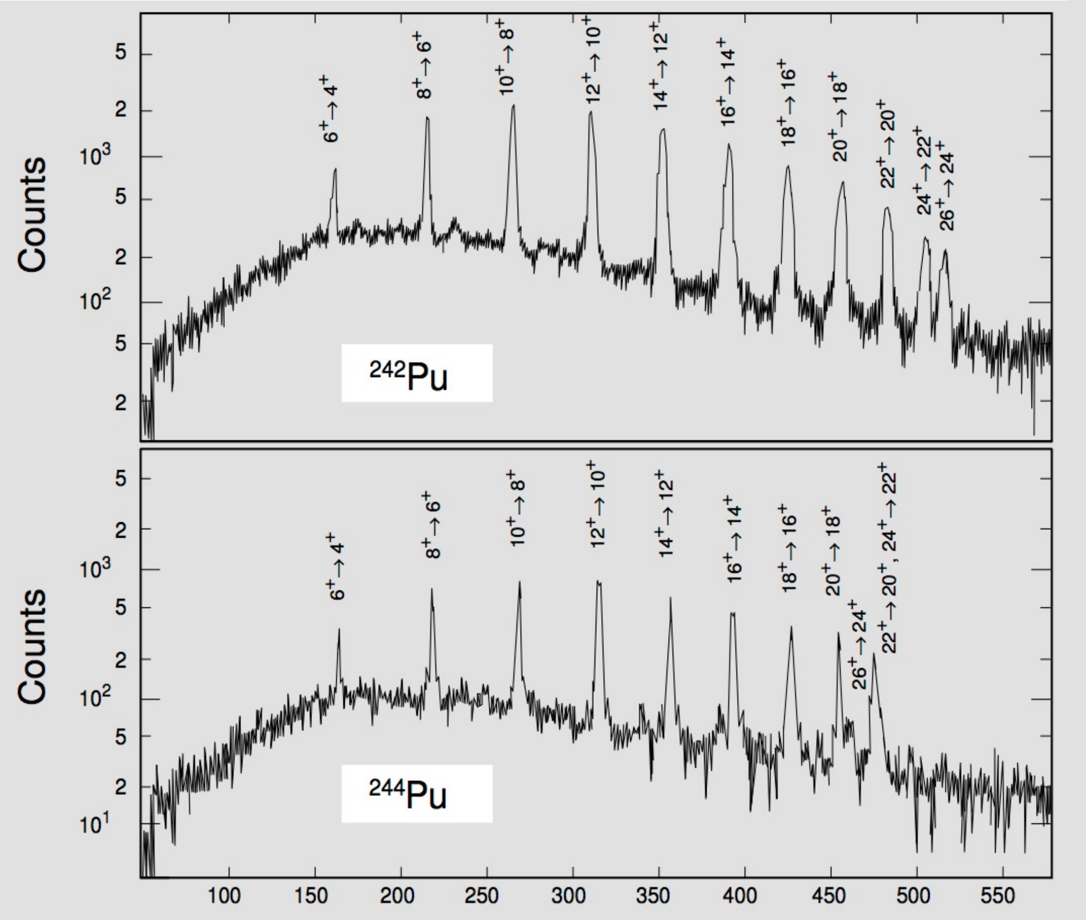
Rotor Example

Sequence of γ-rays following Coulomb excitation of Pu isotopes reveals rotor-like pattern

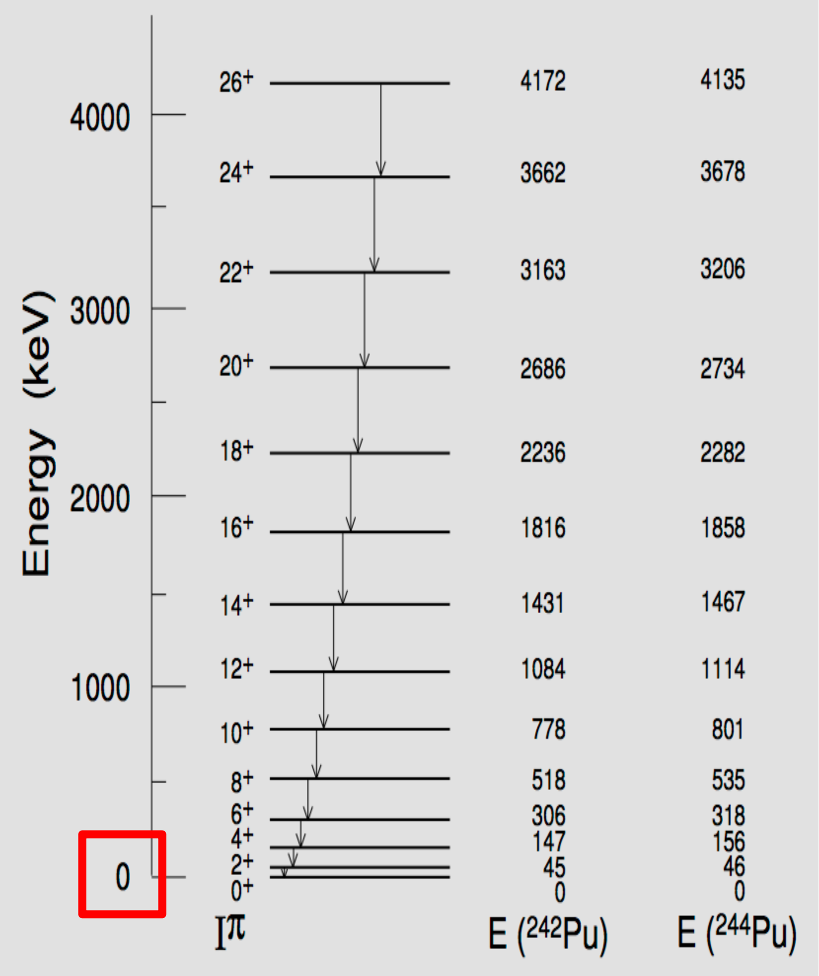


$E = A\,I(I+1)$

condition=γ-ray singles



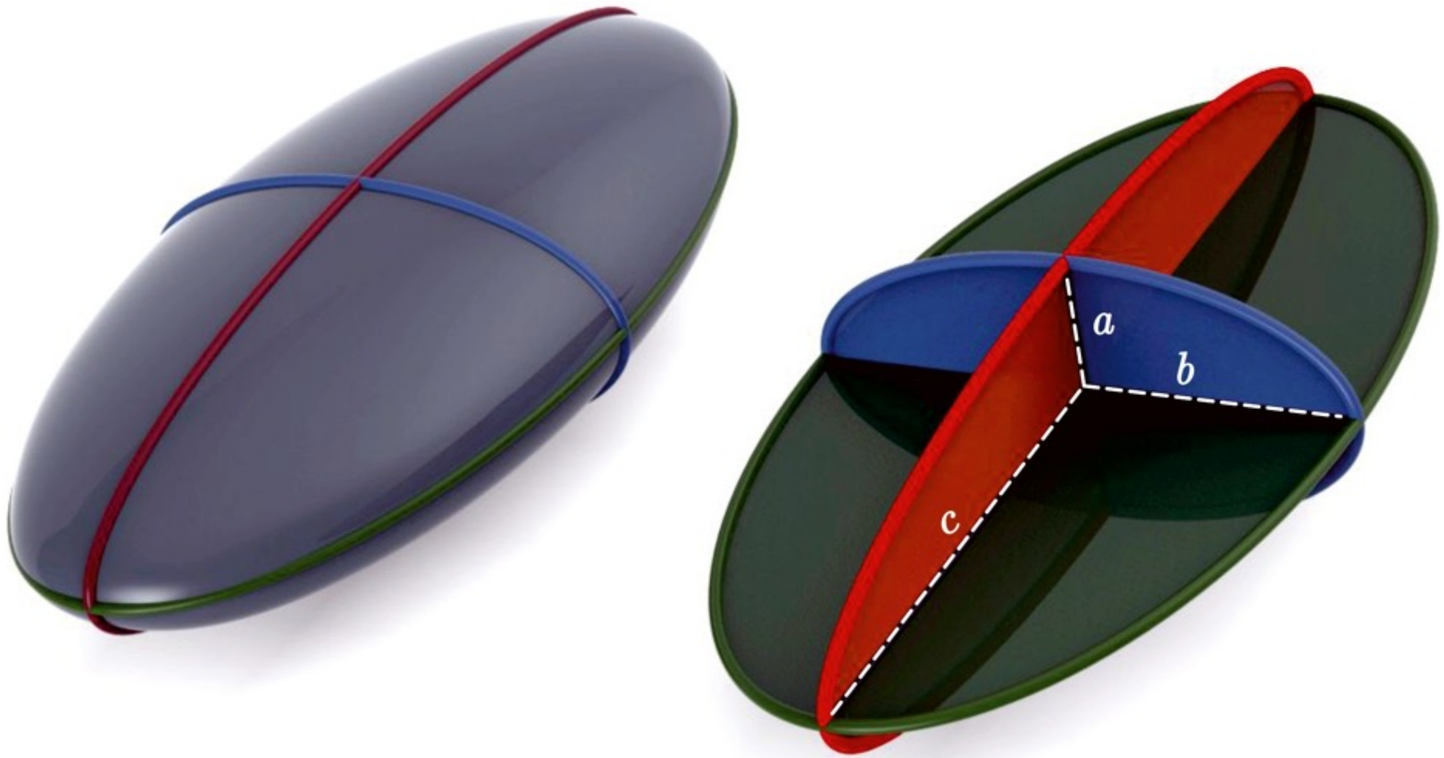
[Rowe&Wood]



[Rowe&Wood]

Triaxial (ellipsoid) Shapes?

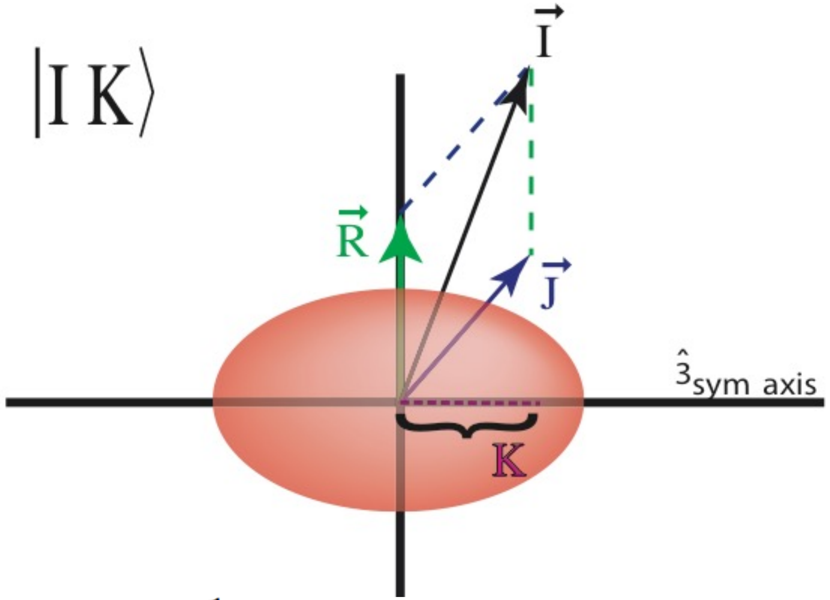
Three unique axes



c > b > a

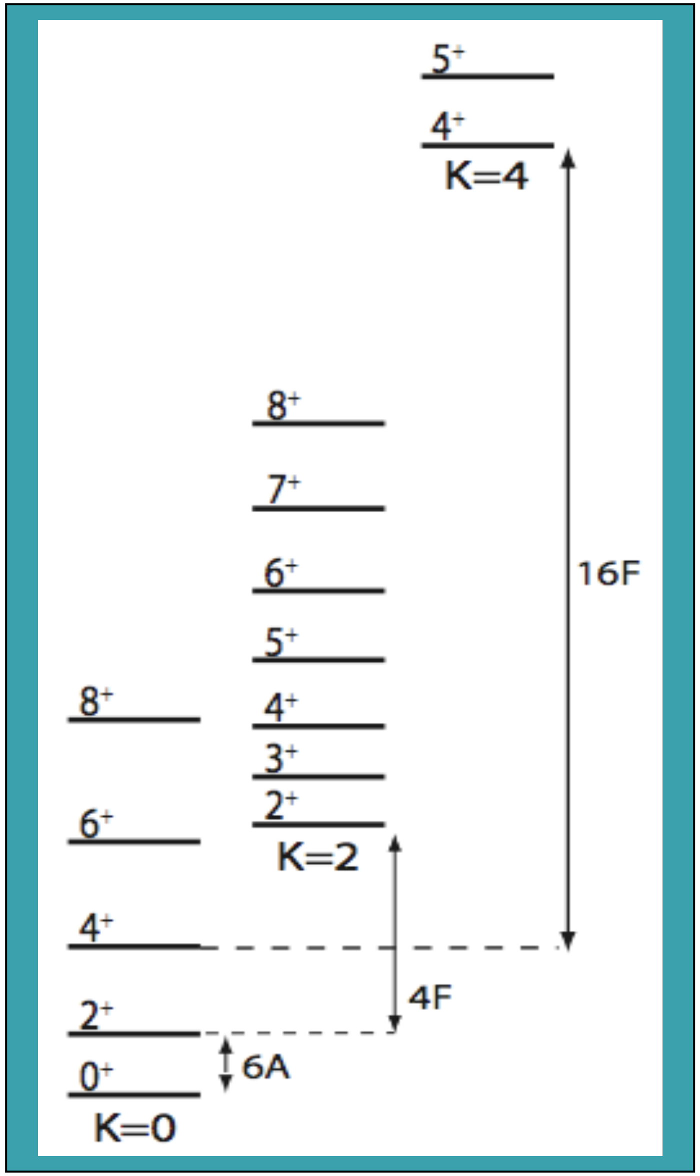
Rotor Basis

Body-frame projection (K) is used instead of lab-frame projection (M)



$$|IK\rangle_\diamond = \frac{1}{\sqrt{2}}\left(|IK\rangle + (-1)^{I+K}|I,-K\rangle\right)$$

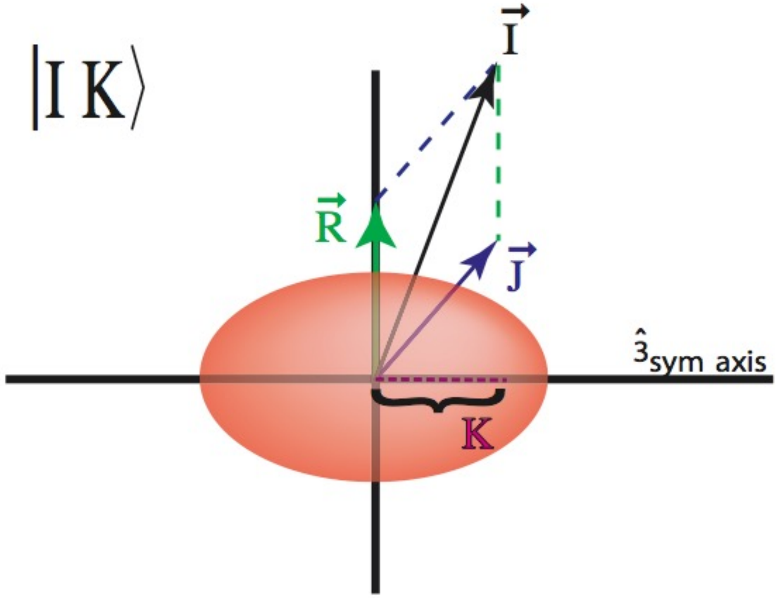
*Because nucleus has a plane of reflection symmetry



$$_\diamond\langle IK|\hat{H}|IK\rangle_\diamond = AI(I+1) + FK^2$$

Generalized Triaxial Rotor Model

$\boxed{\Gamma}$ Consequence of making no assumption on rigid or irrotational flow MOI in Hamiltonian. Induces ΔK=2 mixing from differences in 1- and 2-axis MOI.



E2 deformation

E2 asym.

inertia asym.

$$\langle||\hat{T}(E2)||\rangle = f(Q_0, \gamma, \Gamma_I)$$

Hamiltonian

$$H = A_1\hat{I}_1^2 + A_2\hat{I}_2^2 + A_3\hat{I}_3^2$$
$$= A\hat{I}^2 + F\hat{I}_3^2 + G(\hat{I}_+^2 + \hat{I}_-^2),$$

where

$$A = \frac{1}{2}(A_1 + A_2),\ F = A_3 - A,\ G = \frac{1}{4}(A_1 - A_2),$$

$$H(2^+) = \begin{pmatrix} 6A & 4\sqrt{3}G \\ 4\sqrt{3}G & 6A + 4F \end{pmatrix}$$

$$|I_g\rangle = \cos\Gamma_I |I0\rangle_\diamond - \sin\Gamma_I |I2\rangle_\diamond$$
$$|I_\gamma\rangle = \sin\Gamma_I |I0\rangle_\diamond + \cos\Gamma_I |I2\rangle_\diamond$$
$$\tan(2\Gamma_2) = 2\sqrt{3}\frac{G}{F}$$

E2 Operator

$$\hat{T}(E2) = \cos\gamma \hat{T}_0^{(2)} + \frac{\sin\gamma}{\sqrt{2}}(\hat{T}_{+2}^{(2)} + \hat{T}_{-2}^{(2)})$$

Rigid and Irrotational Couplings of Inertia and E2 Tensors

Generic model with use of Γ can recapture the traditional assumptions, e.g., of Davydov-Filippov Irrotational Model

$$\mathcal{J}_{rigid,\,k} = B_{rigid}\left[1 - \sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma - k\frac{2\pi}{3}\right)\right]$$

and

$$\mathcal{J}_{irrot.,\,k} = 4B_{irrot.}\beta^2\sin^2\left(\gamma - k\frac{2\pi}{3}\right),$$

$$\Gamma = \frac{1}{2}\tan^{-1}\left(\sqrt{3}\frac{\mathcal{J}_2 - \mathcal{J}_1}{\frac{2\mathcal{J}_1\mathcal{J}_2}{\mathcal{J}_3} - \mathcal{J}_2 - \mathcal{J}_1}\right)$$

Apply Irrotational MOI Assumption →

$$\Gamma_{irrot.} = -\frac{1}{2}\cos^{-1}\left(\frac{\cos 4\gamma + 2\cos 2\gamma}{\sqrt{9 - 8\sin^2 3\gamma}}\right)$$

Becomes equivalent to Davydov-Filippov model

Γ Consequence of making no assumption on rigid or irrotational flow MOI in Hamiltonian. Induces ΔK=2 mixing from differences in 1- and 2-axis MOI.

$\gamma = 0 \rightarrow \Gamma = 0$
$\gamma = 30 \rightarrow \Gamma = -30$
$\gamma = 60 \rightarrow \Gamma = -60$

Independent of β

Triaxial I=0,2 Model Space and Shape Parameters

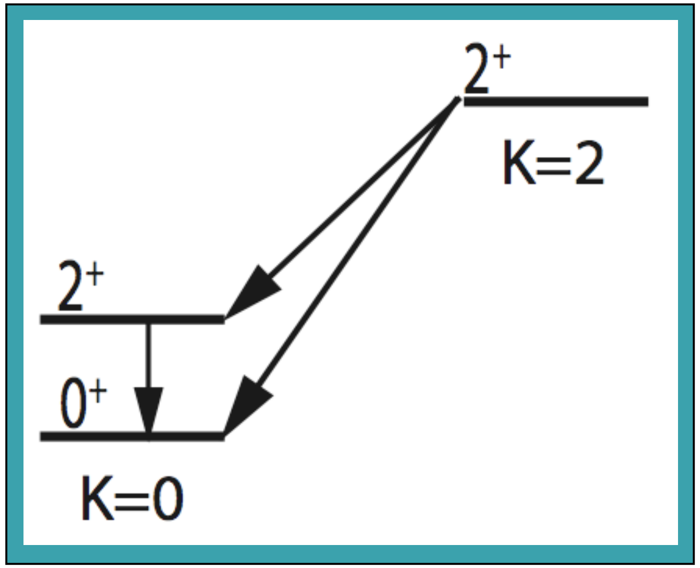
*Only need 3 of 5 M.E.s to determine Q_o, γ, Γ; Typically use the 3 transition M.E.s due to precision

$$*\langle 0_1||\hat{T}^{(2)}||2_1\rangle = \sqrt{\frac{5}{16\pi}}Q_0\cos(\gamma+\Gamma_2)$$

$$*\langle 0_1||\hat{T}^{(2)}||2_2\rangle = \sqrt{\frac{5}{16\pi}}Q_0\sin(\gamma+\Gamma_2)$$

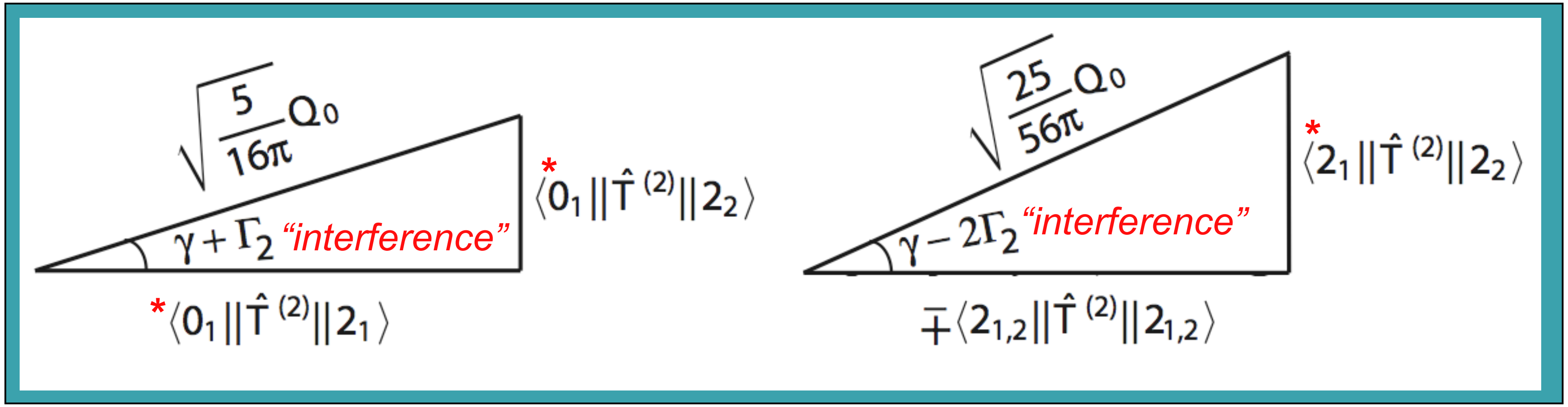
$$*\langle 2_1||\hat{T}^{(2)}||2_2\rangle = \sqrt{\frac{25}{56\pi}}Q_0\sin(\gamma-2\Gamma_2)$$

$$\langle 2_1||\hat{T}^{(2)}||2_1\rangle = -\sqrt{\frac{25}{56\pi}}Q_0\cos(\gamma-2\Gamma_2) = -\langle 2_2||\hat{T}^{(2)}||2_2\rangle$$



2^+ mixing, Γ_2 (deg)

*generated from difference in 1- and 2-axis MOI



Parametrization reveals a destructive interference effect between E2 and inertia asymmetry

Empirical Moments of Inertia

Summary of Procedure:

- β, γ, Γ from E2 Matrix Elements
- A, F, G from 2+ energies and Γ
- MOI from A, F, G

$$\mathcal{J}_1 = \frac{1}{2}\frac{\hbar^2}{A+2G},$$

$$\mathcal{J}_2 = \frac{1}{2}\frac{\hbar^2}{A-2G},$$

$$\mathcal{J}_3 = \frac{1}{2}\frac{\hbar^2}{A+F}.$$

where,

$$F = \frac{E(2_\gamma^+) - E(2_g^+)}{4\sqrt{1+\tan^2(2\Gamma)}},$$

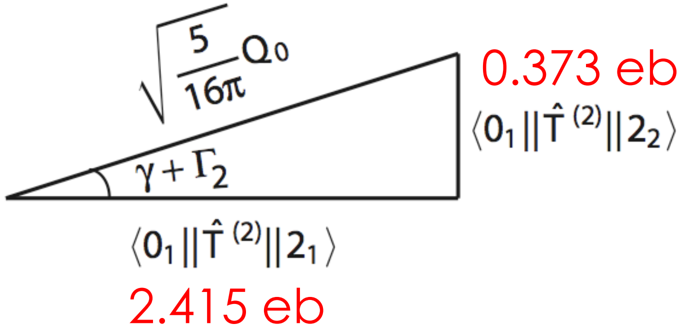
$$A = \frac{E(2_g^+) + E(2_\gamma^+) - 4F}{12},$$

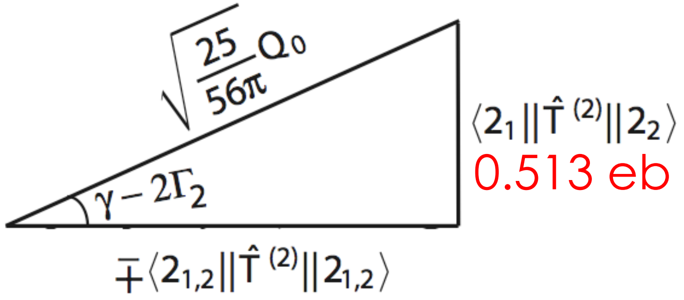
$$G = \frac{F}{2\sqrt{3}}\tan 2\Gamma,$$

Experimental 2^+ energies

Mixing angle (inertia asymmetry) from experimental E2 Matrix Elements *(see previous slide)*

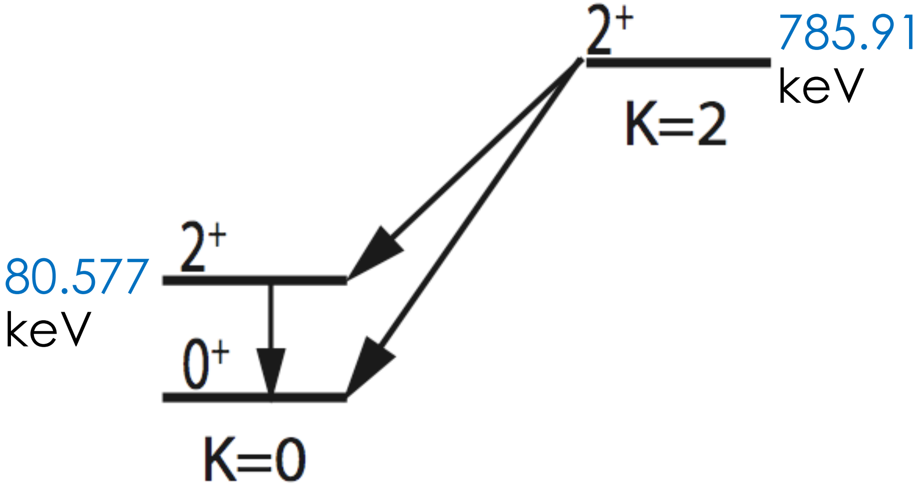
Example Fit to ^{166}Er





-2.3 eb
+2.9 eb
exp

-/+ 2.8 eb
Prediction



Q_o = 7.75(3) eb
γ = 9.23(17)°
Γ_2 = -0.45(12)°

$$F = \frac{E(2^+_\gamma) - E(2^+_g)}{4\sqrt{1+\tan^2(2\Gamma)}},$$ F = 176.31 keV

$$A = \frac{E(2^+_g) + E(2^+_\gamma) - 4F}{12},$$ A = 13.44 keV

$$G = \frac{F}{2\sqrt{3}}\tan 2\Gamma,$$ G = -0.79 keV

MOI from combining E2 and Energy values

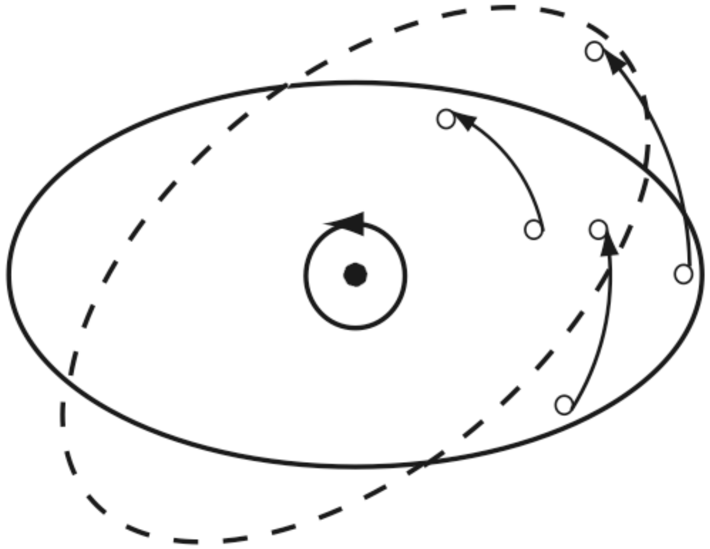
$$\mathcal{J}_1 = \frac{1}{2}\frac{\hbar^2}{A+2G},$$ = 42.2 h-bar²/MeV

$$\mathcal{J}_2 = \frac{1}{2}\frac{\hbar^2}{A-2G},$$ = 33.3 h-bar²/MeV

$$\mathcal{J}_3 = \frac{1}{2}\frac{\hbar^2}{A+F}.$$ = 2.6 h-bar²/MeV

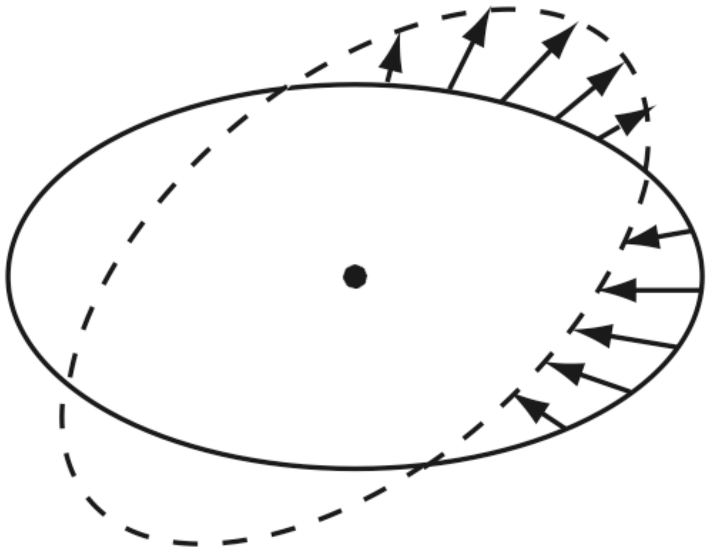
How Do Nuclei Rotate: Rigid or Irrotational Flow?

Rigid Flow



e.g., like a spinning football

Irrotational Flow



e.g., like a surface wave

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View in 2017 Study

E(4)/E(2) > 2.7 cases

Physics Letters B 767 (2017) 226–231

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PHYSICS LETTERS B

Empirical moments of inertia of axially asymmetric nuclei

CrossMark

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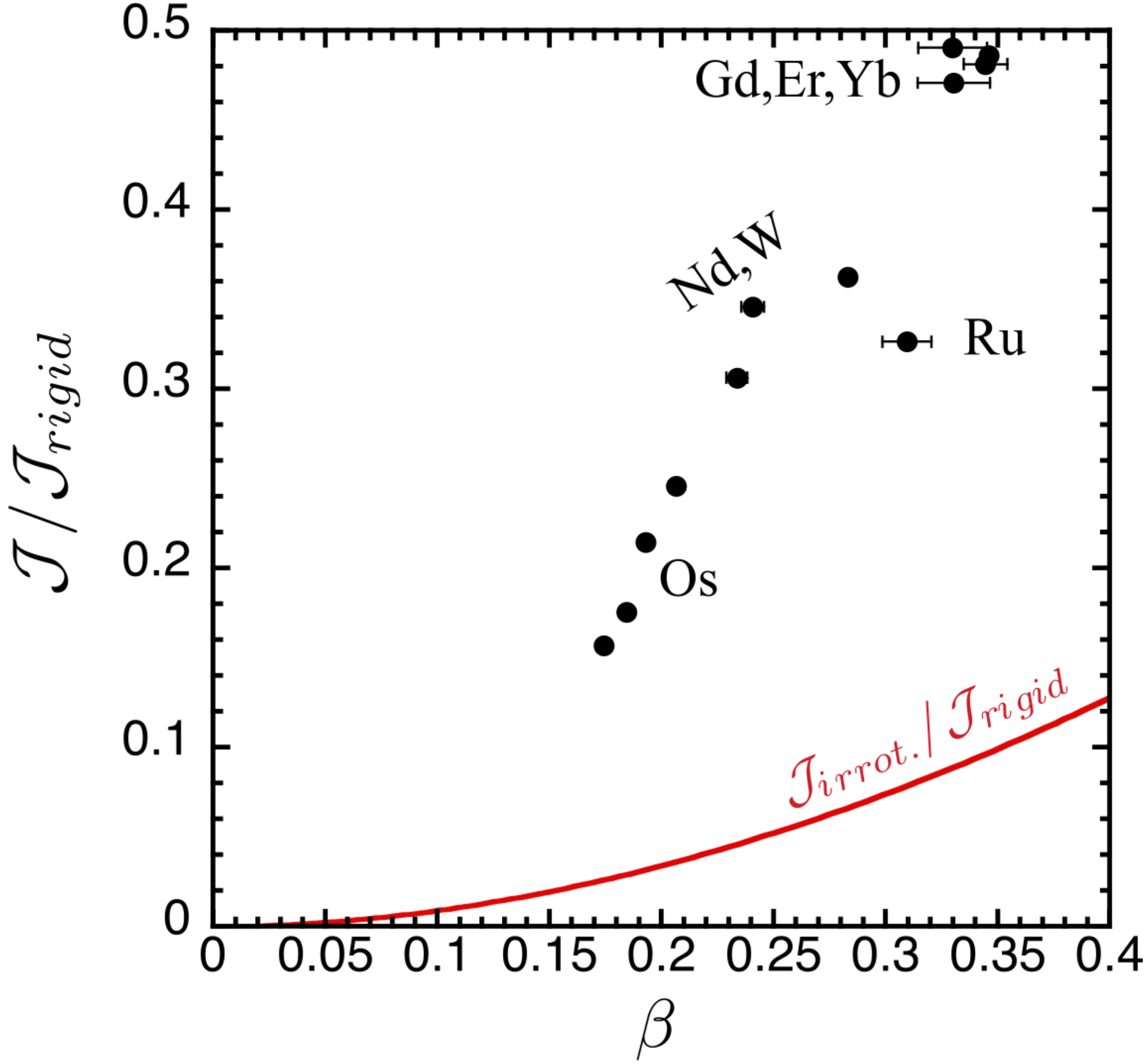
Empirical 1D View

E(4)/E(2) > 2.7

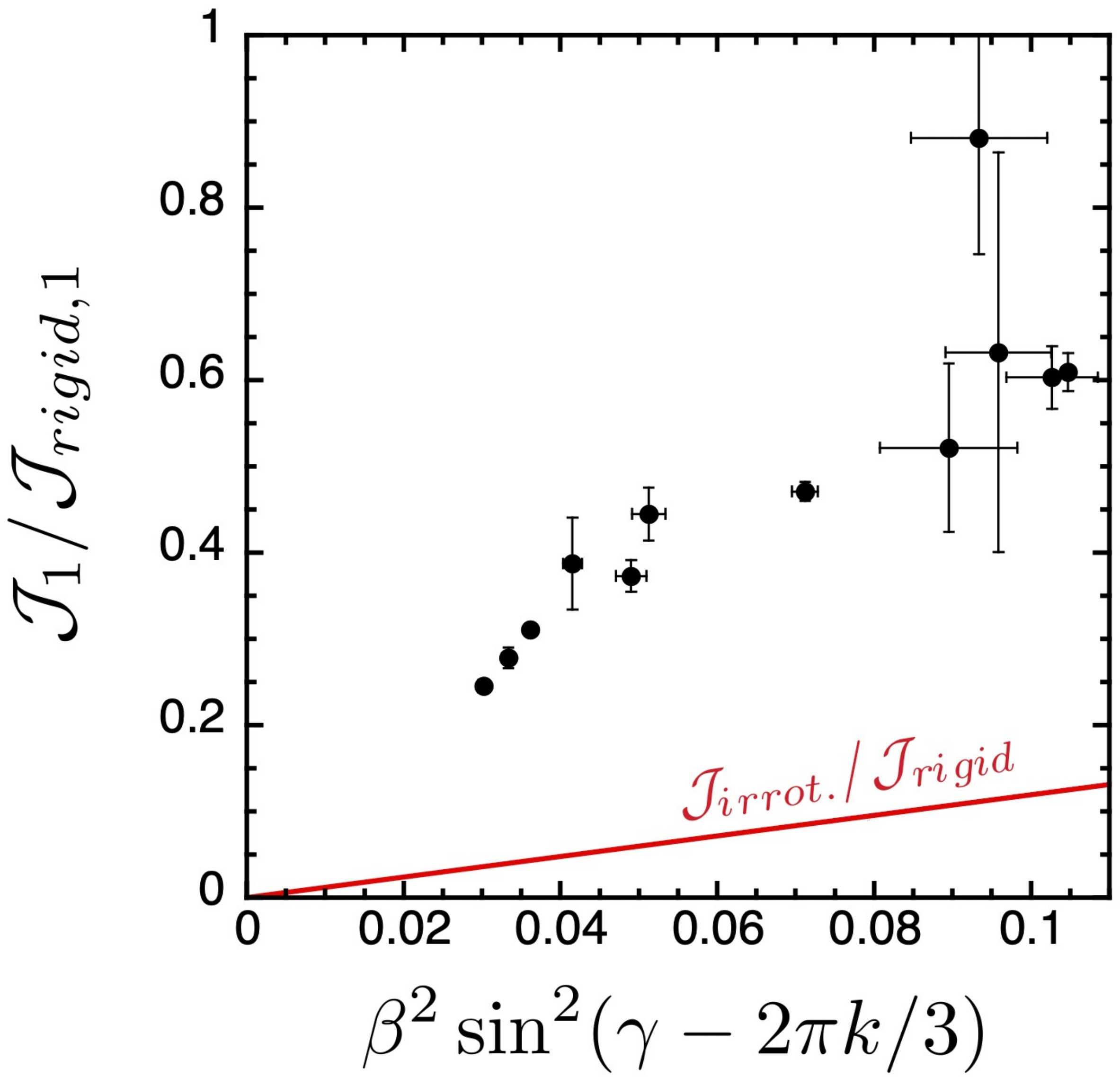


Presumes Axial Symmetry, γ=0

1-axis = 2-axis MOI = 3/E(2_1)
3-axis = 0

Empirical 3D View: the 1-Axis

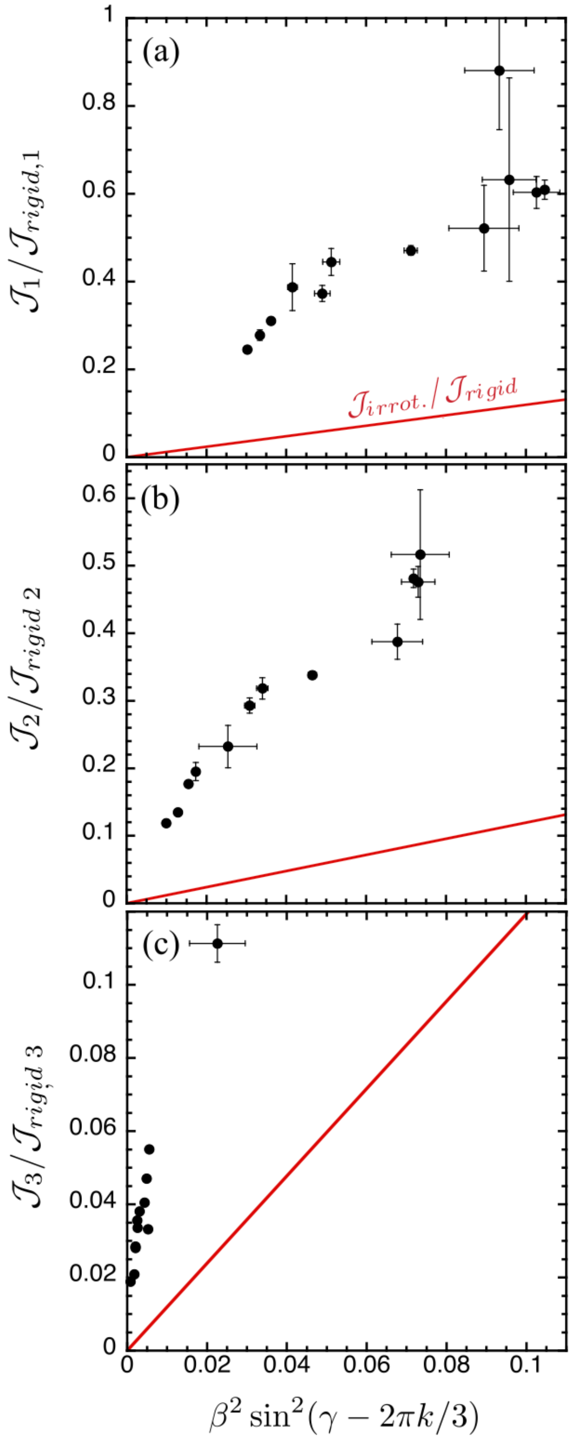
E(4)/E(2) > 2.7



Presumes Axial Asymmetry
1-axis ≠ 2-axis ≠ 3-axis

Empirical 3D View

E(4)/E(2) > 2.7



1-axis

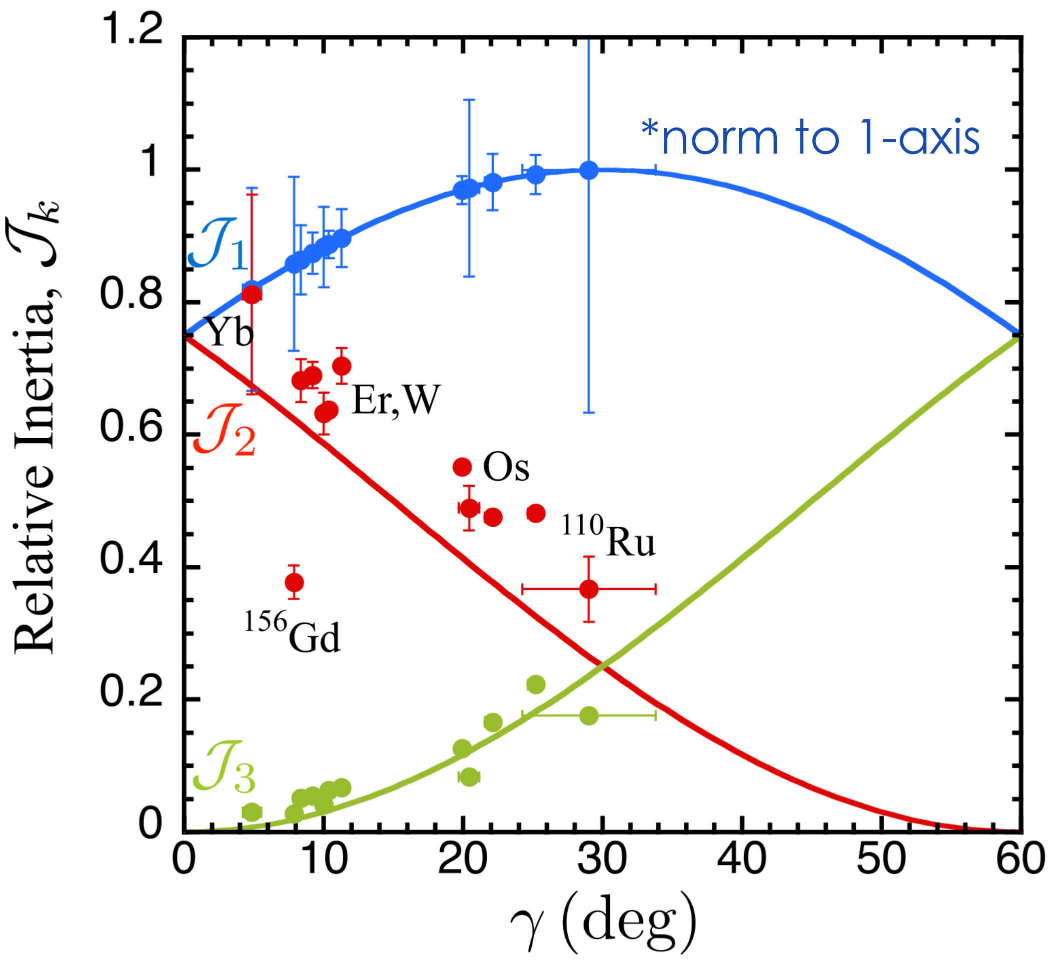
2-axis

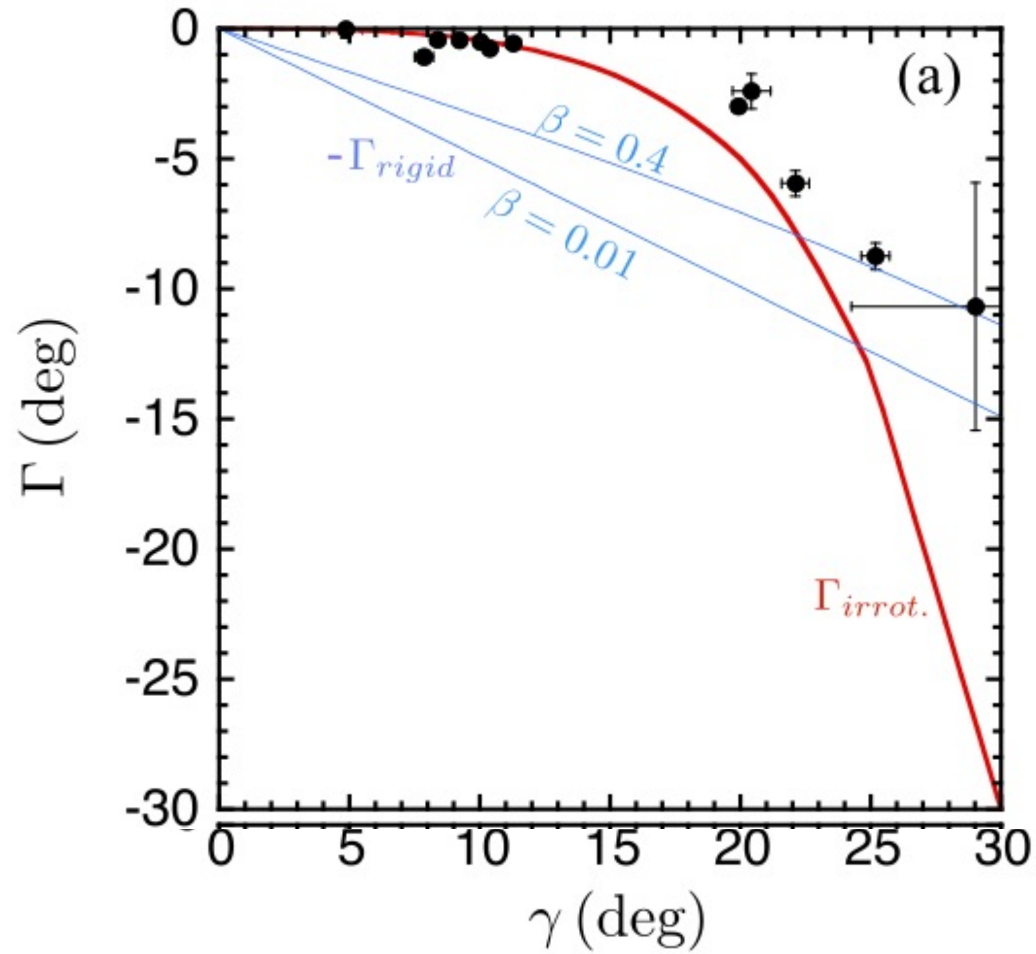
3-axis

Presumes Axial Asymmetry
1-axis ≠ 2-axis ≠ 3-axis

Empirical 3D View

E(4)/E(2) > 2.7

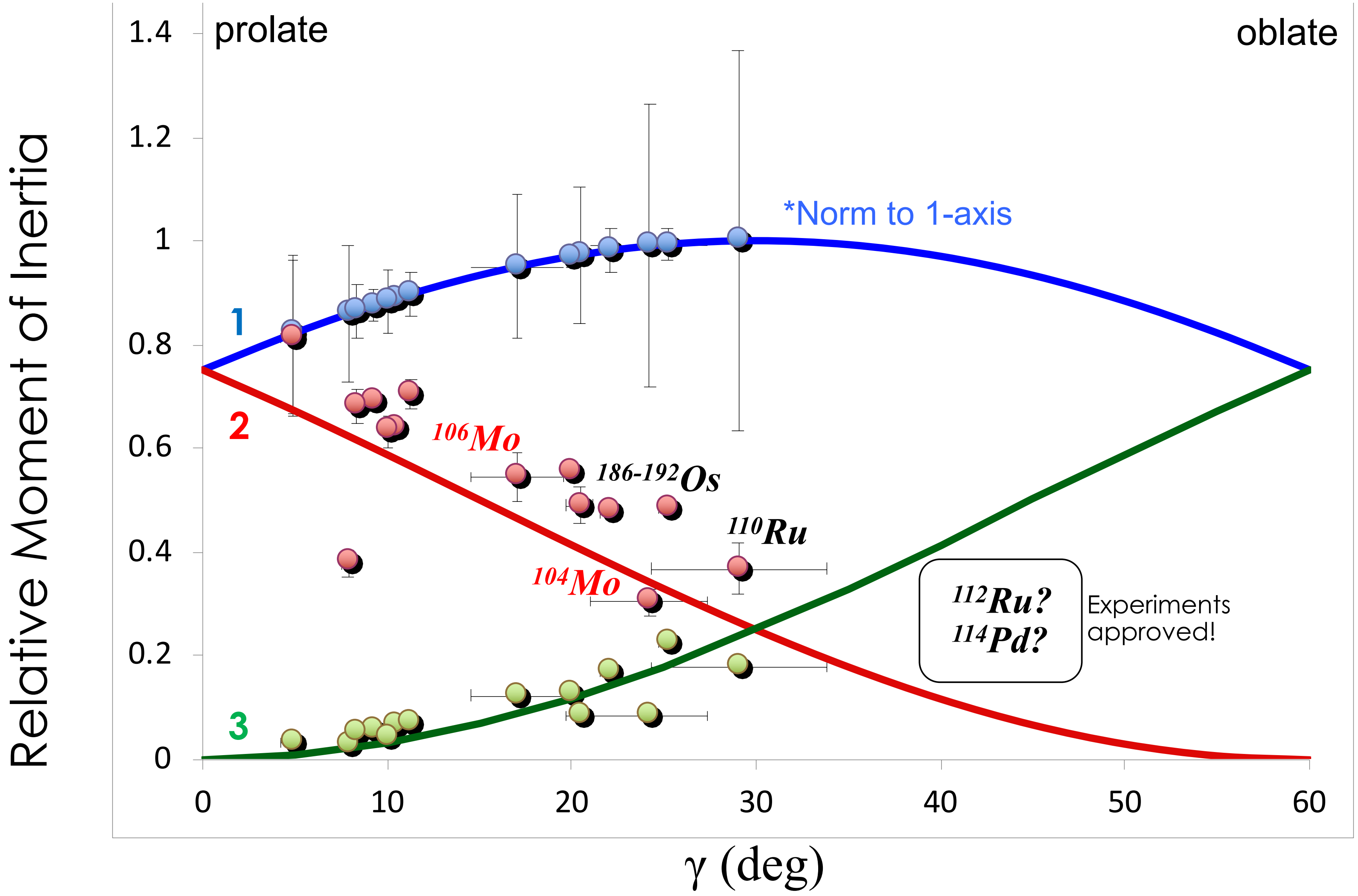




Absolute values: *Between Rigid and Irrotational Flow*
Relative values: *Qualitatively consistent with SO(5) symmetry of Bohr Hamiltonian*
***irrotational flow happens to be SO(5) symmetric*
***E2 matrix elements only dependent on relative MOI*

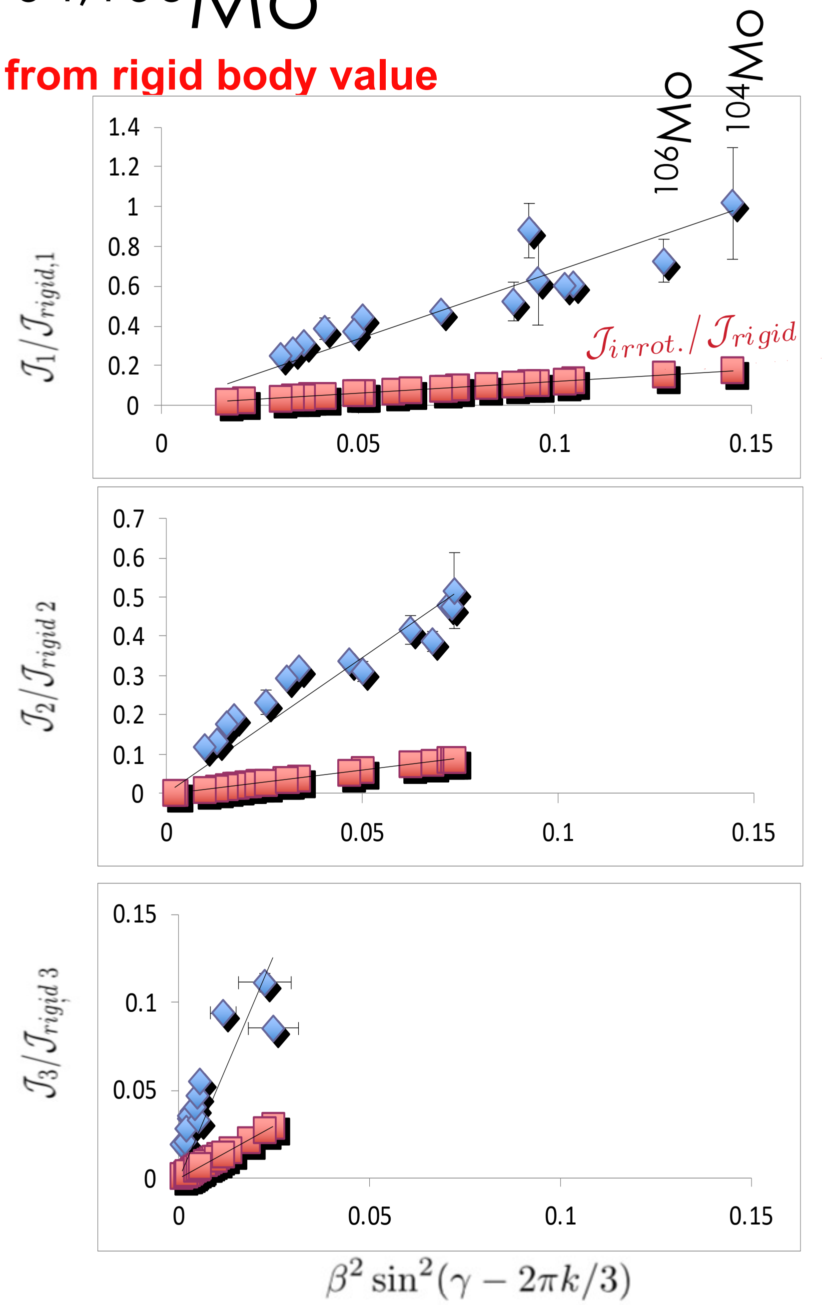
Update to Include ^{104,106}Mo

Add E(4)/E(2) = 2.92 and 3.03 cases; ^{104,106}Mo are similar to the stable Os isotopes



Update to Include $^{104,106}Mo$

1-axis MOI of ^{104}Mo nearly unquenched from rigid body value



1-axis

2-axis

3-axis

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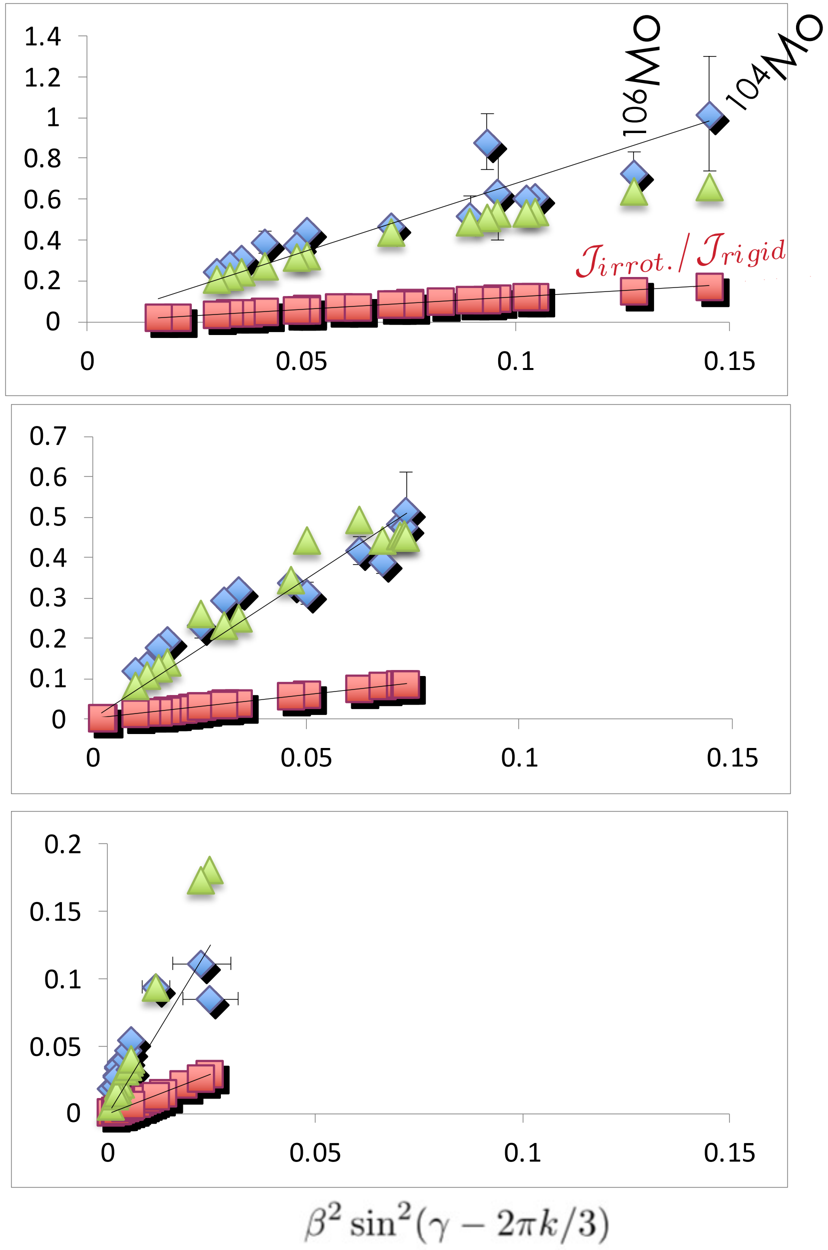
3D MOI Values ~ Similar to a Trapped Cold Fermi Gas



PHYSICAL REVIEW A **67**, 033611 (2003)

Slow rotation of a superfluid trapped Fermi gas

Michael Urban[1] and Peter Schuck[1,2]

Green Points (Cold Fermi Gas MOI)

- 41-MeV "trapping potential"
- 1.5-MeV "interaction strength"
- β and γ from Coulex data

PHYSICAL REVIEW C **100**, 031301(R) (2019)

Rapid Communications

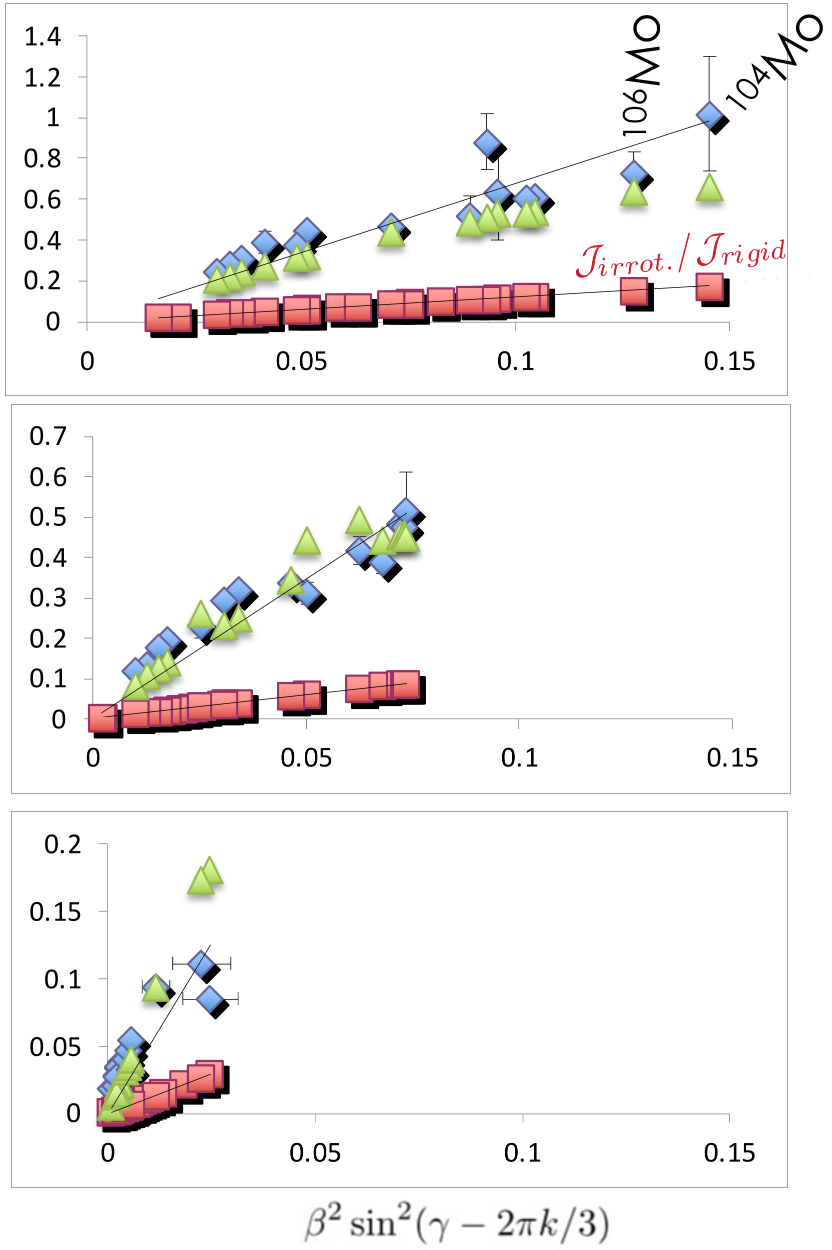
Macroscopic manifestations of rotating triaxial superfluid nuclei

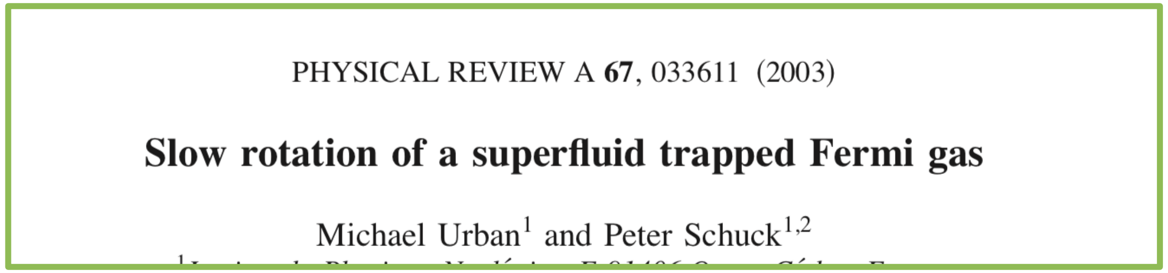
P. Schuck[1,2,*] and M. Urban[1,†]

[1] *Institut de Physique Nucléaire, CNRS-IN2P3, Université Paris-Sud, Université Paris-Saclay, F-91406 Orsay Cedex, France*

[2] *Université Grenoble Alpes, CNRS, LPMMC, F-38000 Grenoble, France*

3D MOI Values ~ Similar to a Trapped Cold Fermi Gas




PHYSICAL REVIEW A **67**, 033611 (2003)

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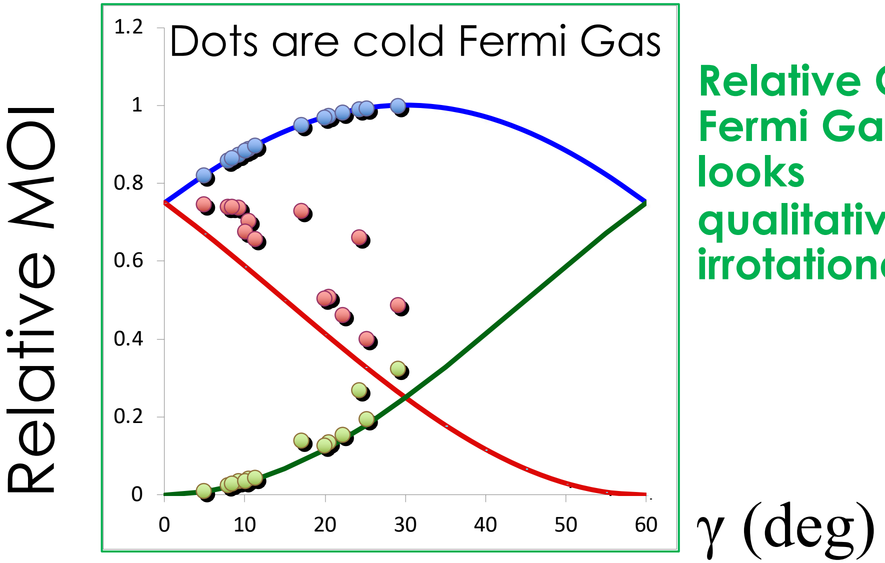


Relative Cold Fermi Gas MOI looks qualitatively like irrotational flow

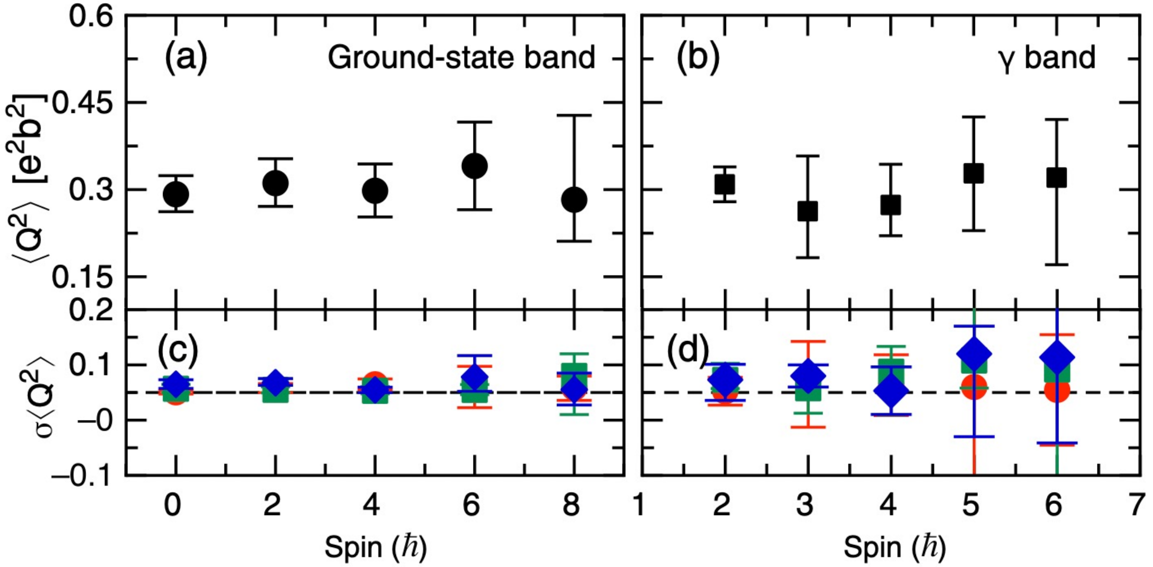
Recent Surprises: ^{76}Ge is β and γ Rigid

Energies imply shape softness, E(4)/E(2) = 2.5, but E2 matrix elements imply shape rigidity

PHYSICAL REVIEW LETTERS **123**, 102501 (2019)

Evidence for Rigid Triaxial Deformation in ^{76}Ge from a Model-Independent Analysis

A. D. Ayangeakaa,[1,*] R. V. F. Janssens,[2,3,†] S. Zhu,[4,‡] D. Little,[2,3] J. Henderson,[5] C. Y. Wu,[5] D. J. Hartley,[1] M. Albers,[4] K. Auranen,[4] B. Bucher,[5,§] M. P. Carpenter,[4] P. Chowdhury,[6] D. Cline,[7] H. L. Crawford,[8] P. Fallon,[8] A. M. Forney,[9] A. Gade,[10,11] A. B. Hayes,[7] F. G. Kondev,[4] Krishichayan,[3,12] T. Lauritsen,[4] J. Li,[4] A. O. Macchiavelli,[8] D. Rhodes,[10,11] D. Seweryniak,[4] S. M. Stolze,[4] W. B. Walters,[9] and J. Wu[4]

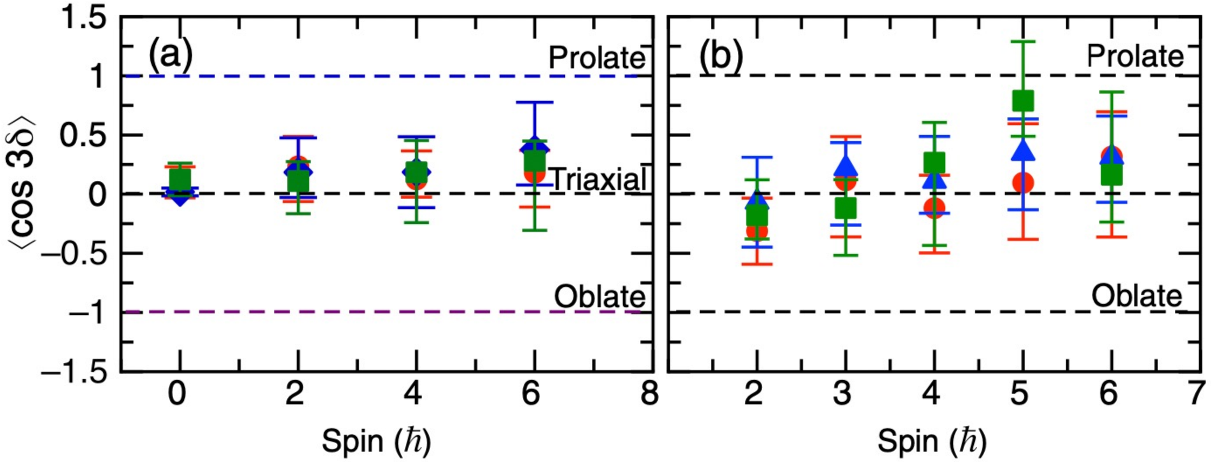


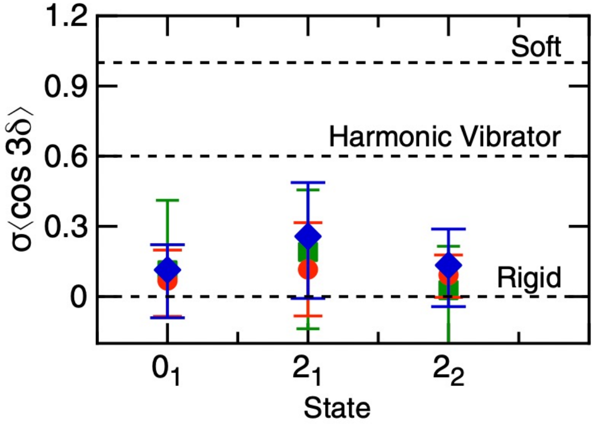
red (circle), blue (diamond), and green (square) correspond to J=0, 2, 4 E2 vector coupling solutions, giving three independent measures of variance

PHYSICAL REVIEW C **107**, 044314 (2023)

Triaxiality and the nature of low-energy excitations in ^{76}Ge

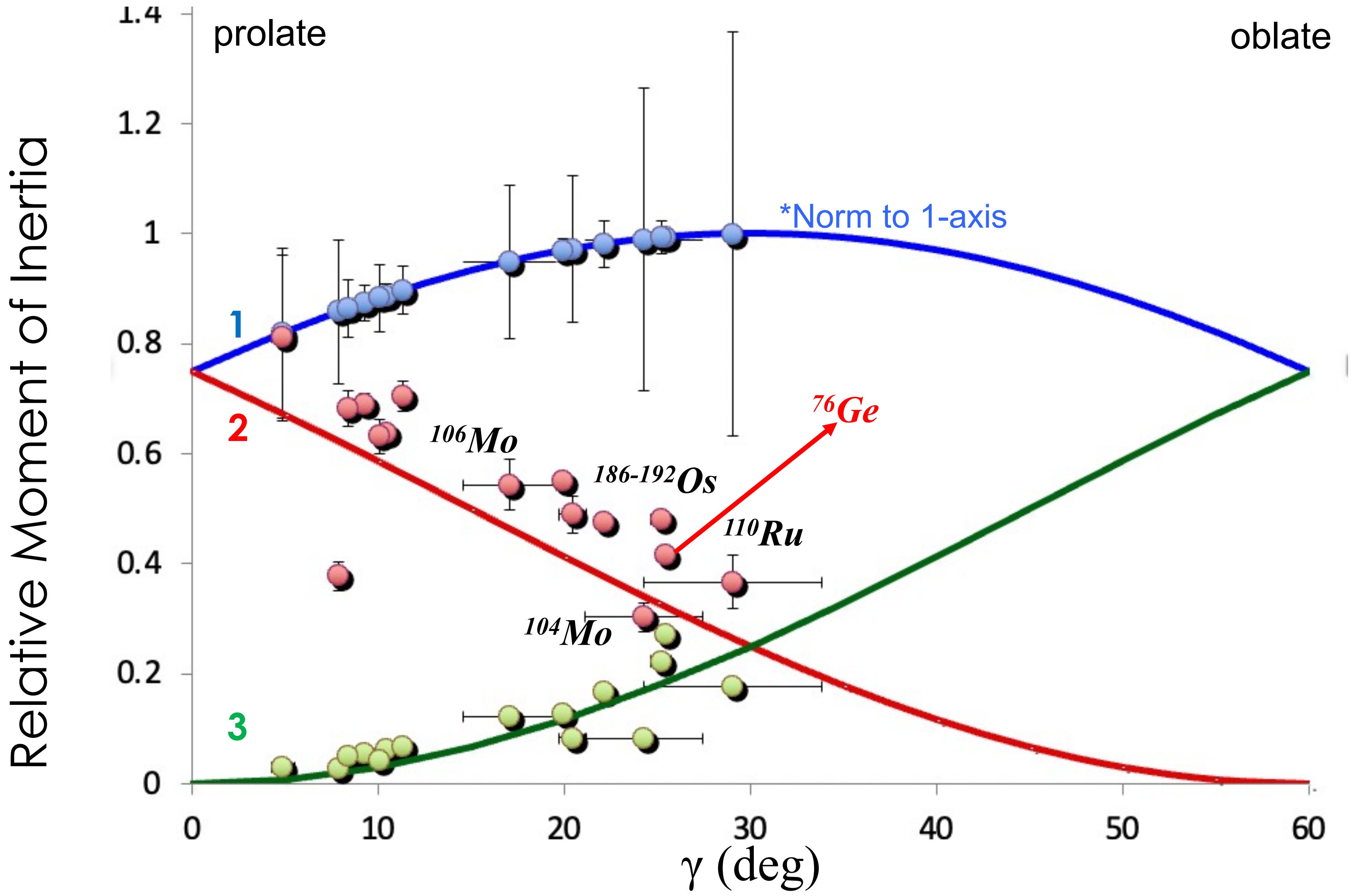
A. D. Ayangeakaa,[1,2,3,*] R. V. F. Janssens,[1,2,†] S. Zhu,[4,‡] J. M. Allmond,[5] B. A. Brown,[6,7] C. Y. Wu,[8] M. Albers,[9,§] K. Auranen,[9] B. Bucher,[8,‖] M. P. Carpenter,[9] P. Chowdhury,[10] D. Cline,[11] H. L. Crawford,[12] P. Fallon,[12] A. M. Forney,[13] A. Gade,[6,7] D. J. Hartley,[3] A. B. Hayes,[11] J. Henderson,[8] F. G. Kondev,[9] Krishichayan,[2,14] T. Lauritsen,[9] J. Li,[9] D. Little,[1,2] A. O. Macchiavelli,[12,¶] D. Rhodes,[6,7,#] D. Seweryniak,[9] S. M. Stolze,[9] W. B. Walters,[13] and J. Wu[9,**]





Recent Surprises: ^{76}Ge is β and γ Rigid

Energies imply shape softness, E(4)/E(2) = 2.5, but E2 matrix elements imply shape rigidity



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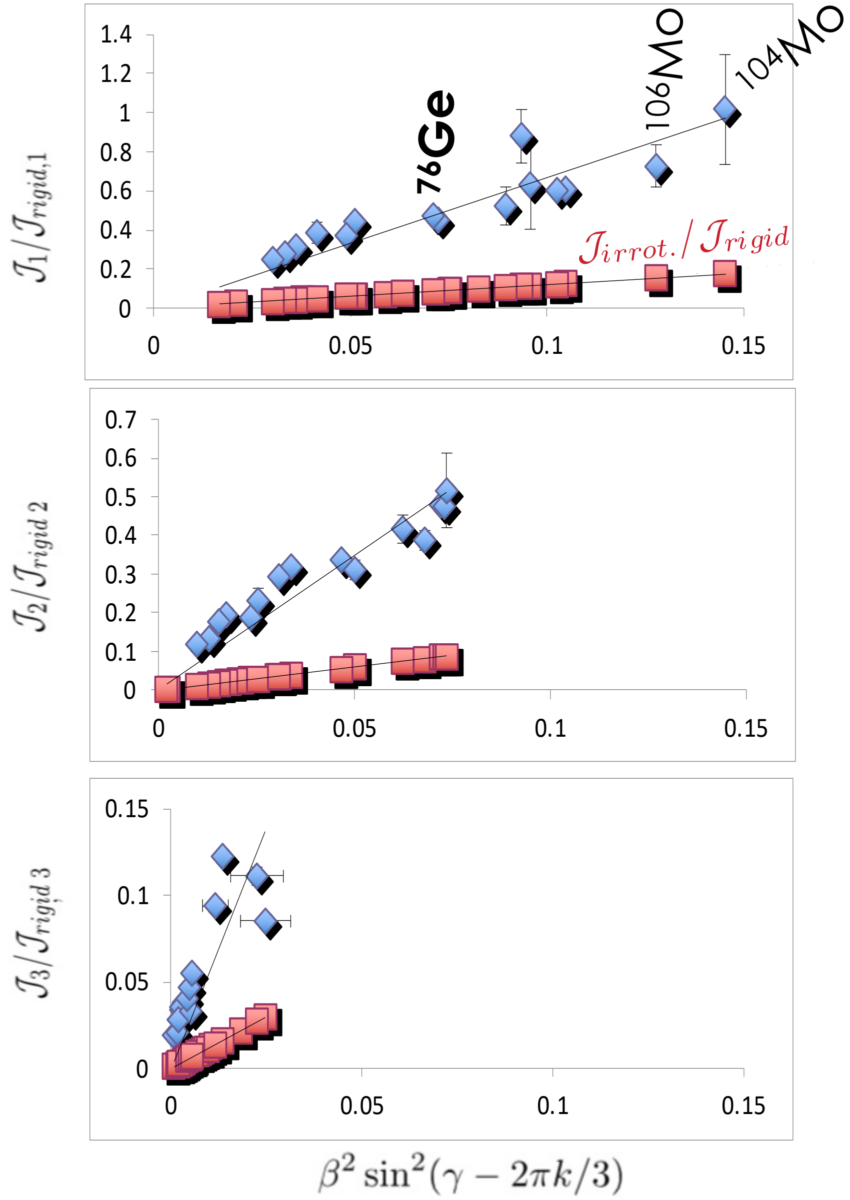


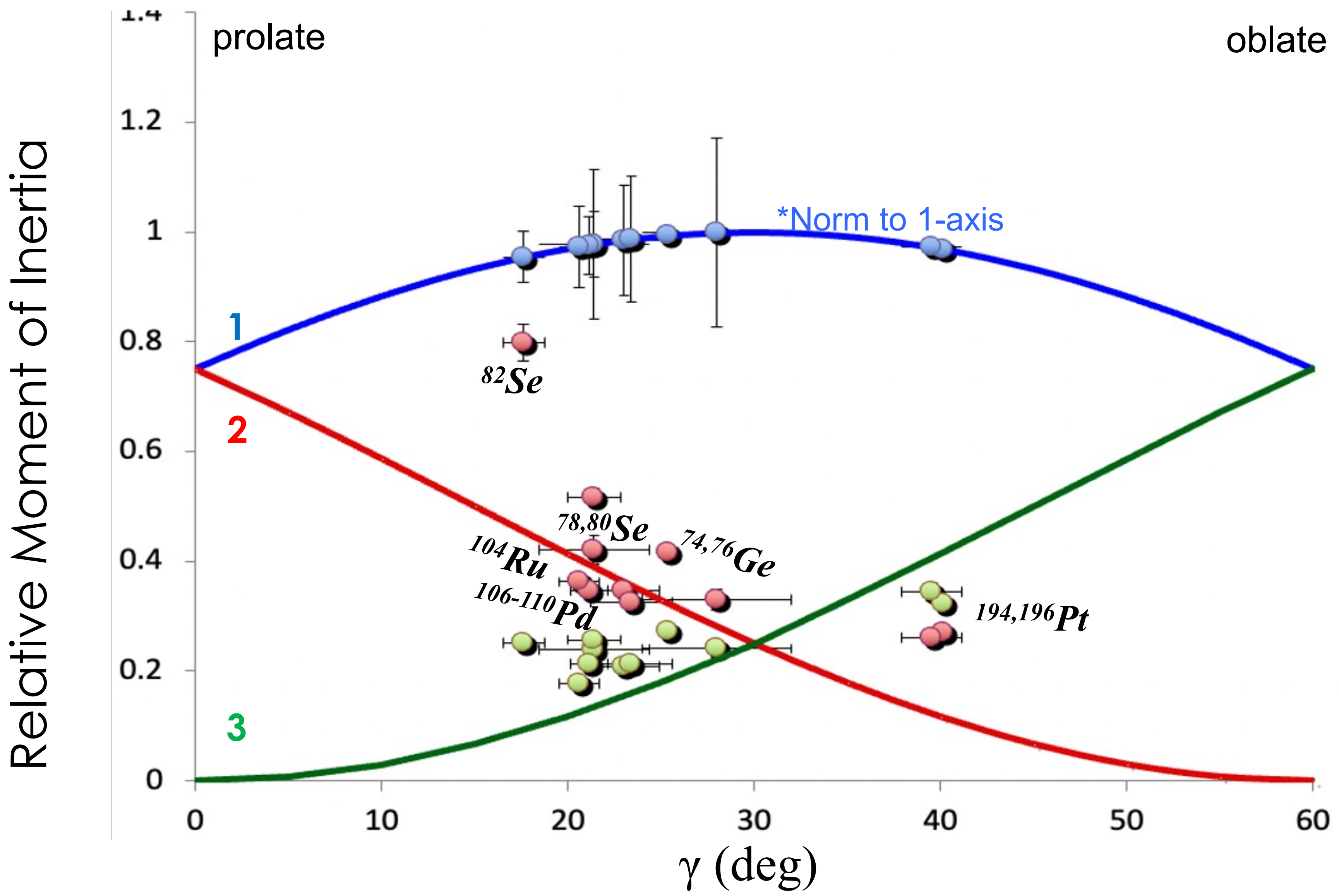
1-axis

2-axis

3-axis

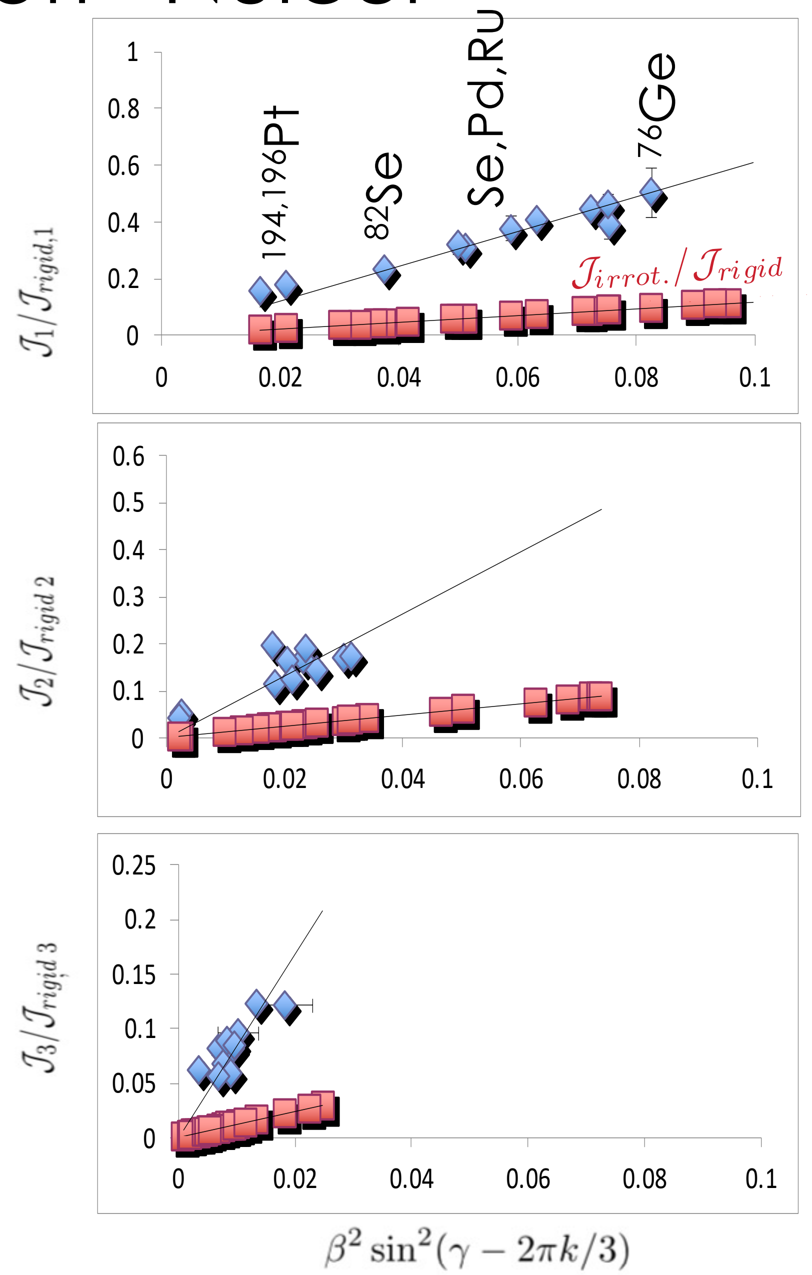
Empirical MOI of "Soft" Nulcei

Cases with E(4)/E(2) = 2.4 – 2.7



Empirical MOI of "Soft" Nulcei

Cases with E(4)/E(2) = 2.4 – 2.7



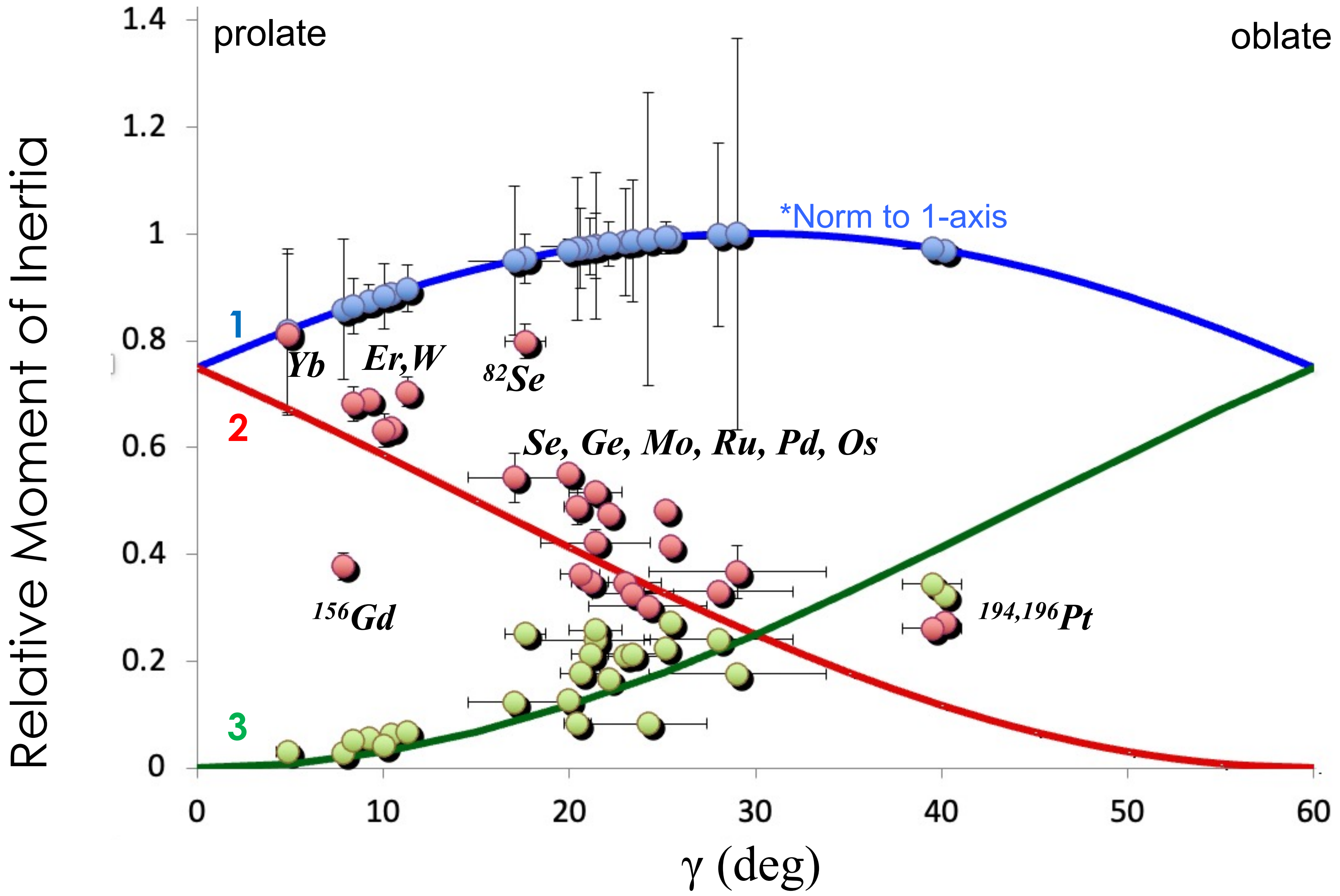
1-axis

2-axis

3-axis

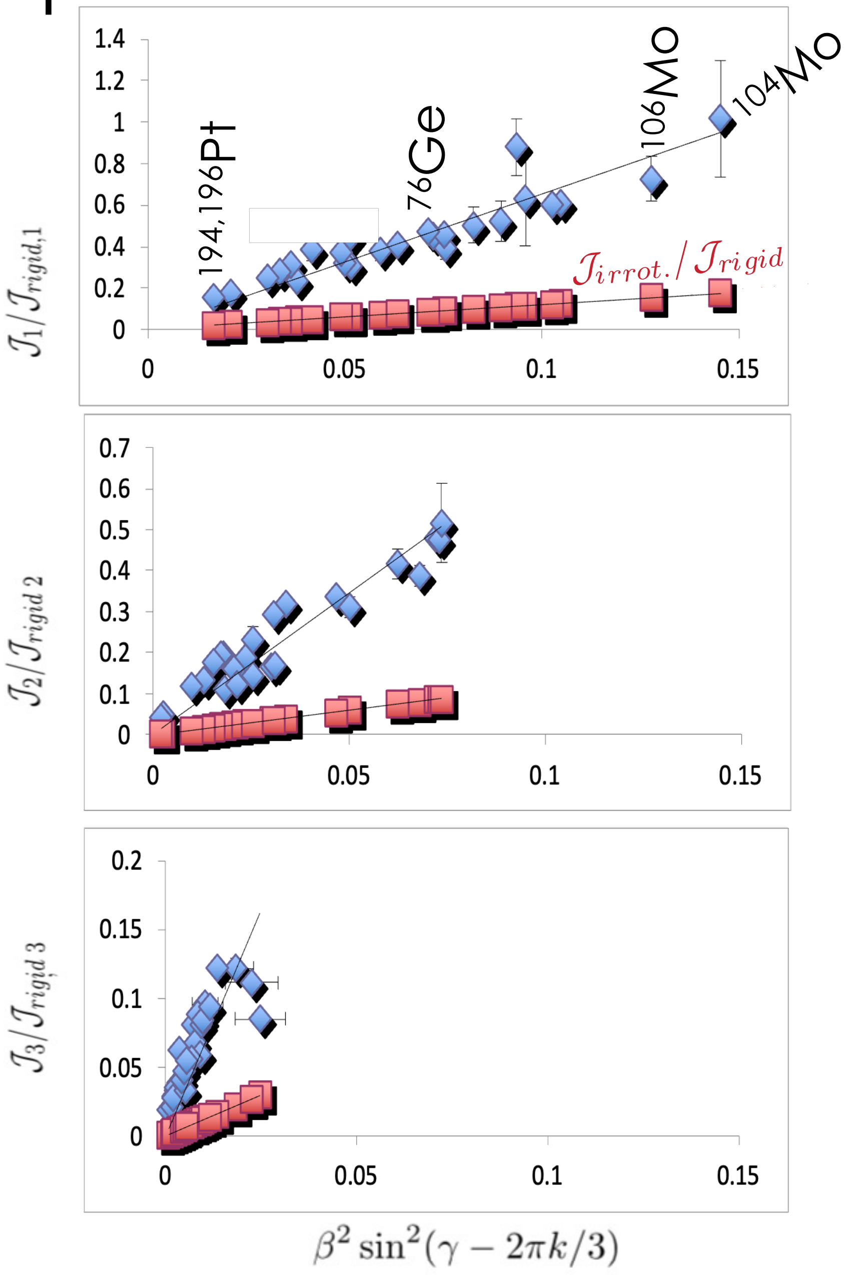
Updated View of Empirical MOI

Cases with E(4)/E(2) = 2.4 – 3.3



Updated View of Empirical MOI

Cases with E(4)/E(2) = 2.4 – 3.3



1-axis

2-axis

3-axis

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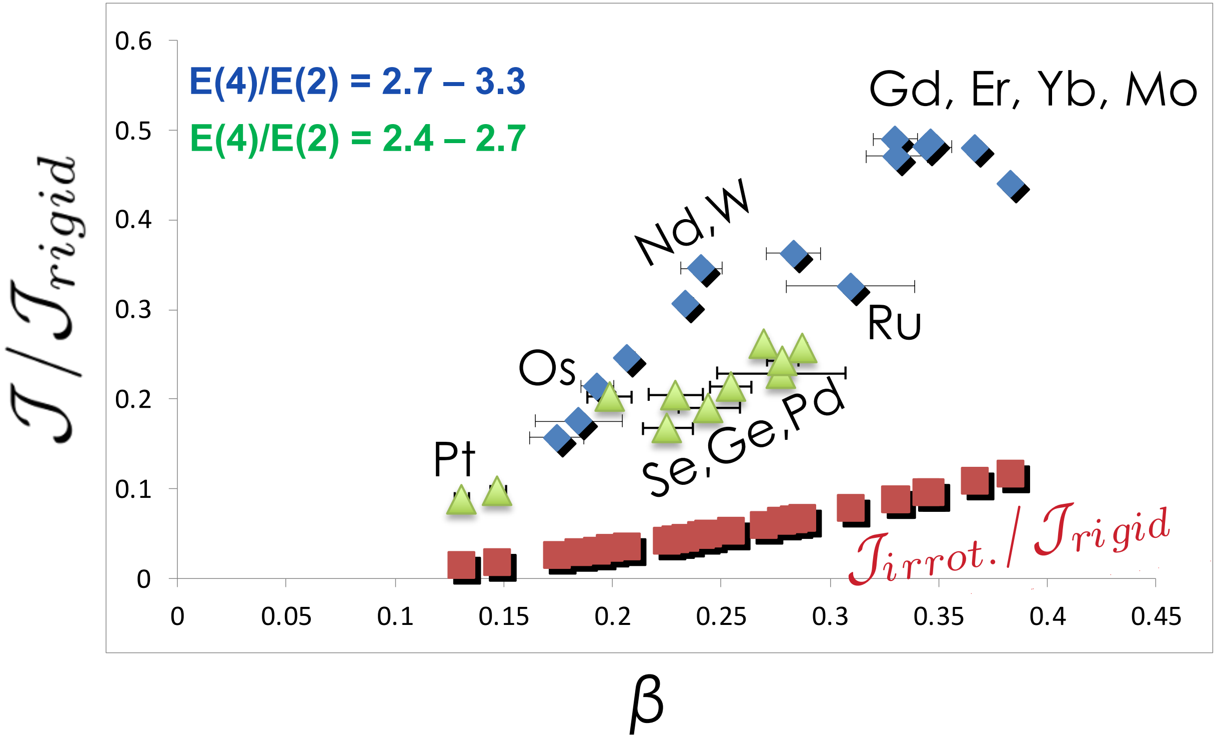
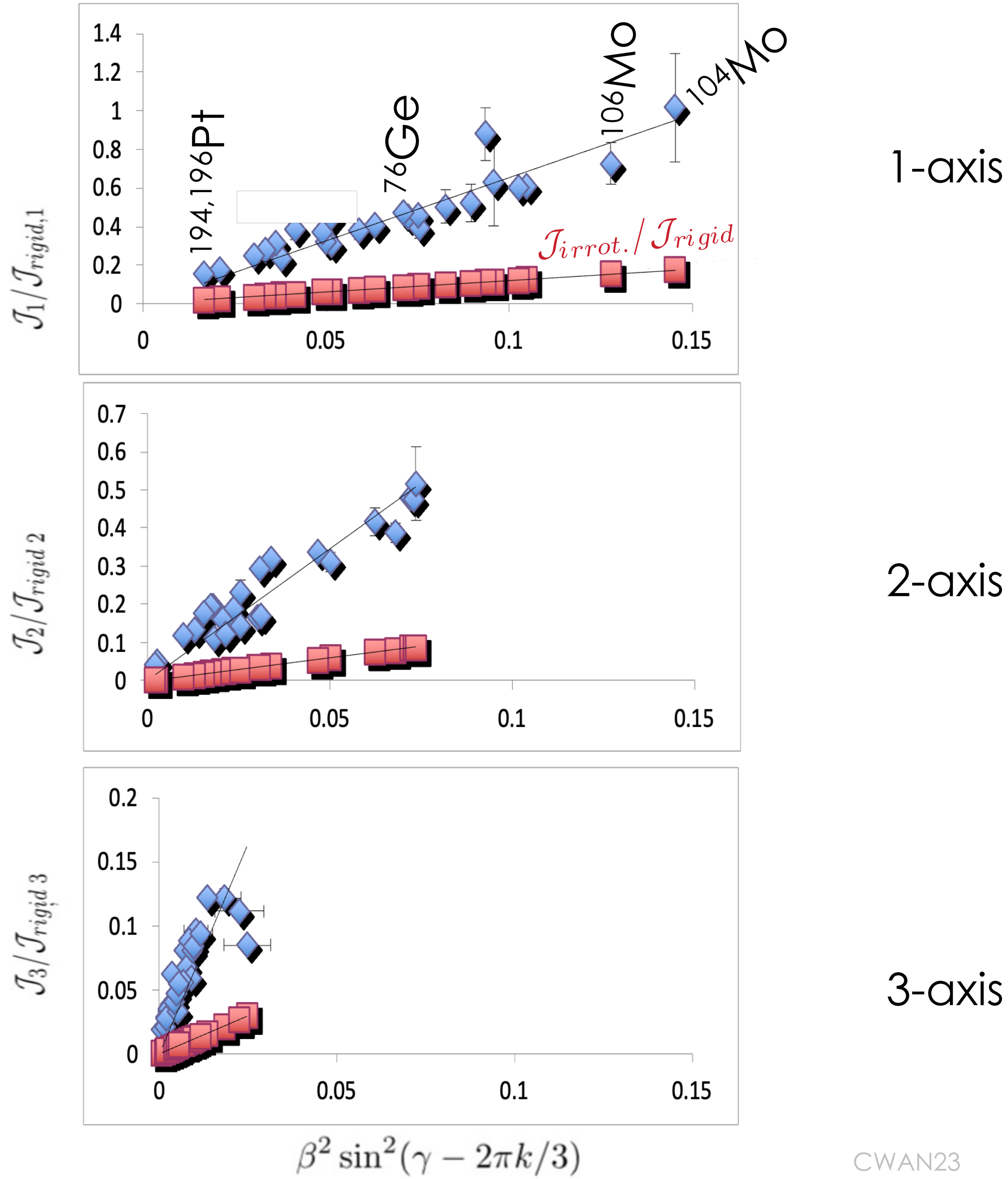
Updated View of Empirical MOI

Cases with E(4)/E(2) = 2.4 – 3.3

1D View shows bifurcation in MOI from "soft" nuclei



3D view removes strong bifurcation



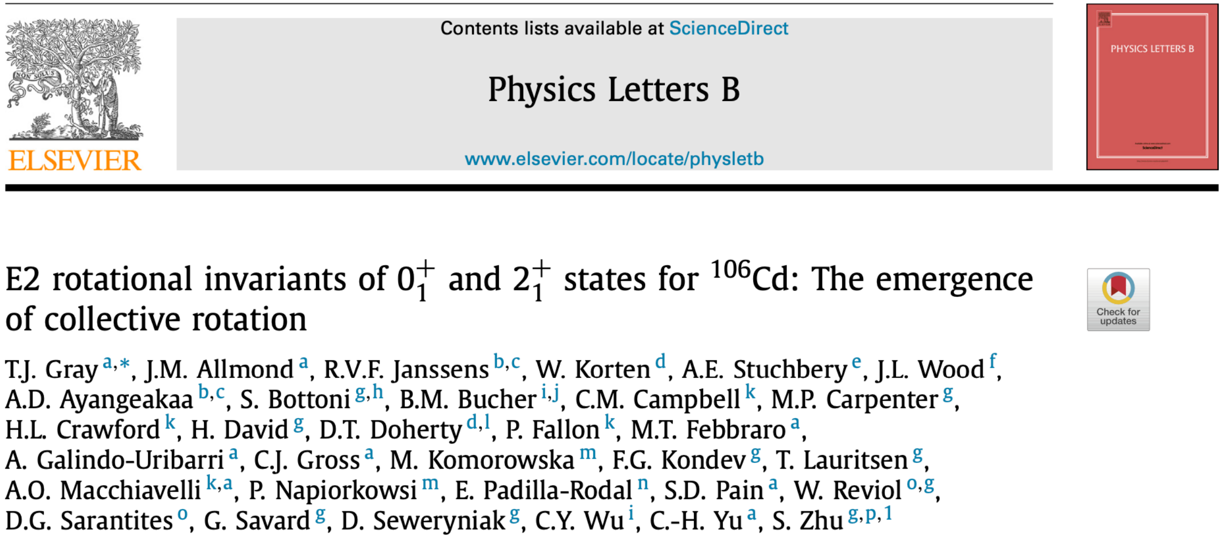
Summary

- Absolute MOI are between rigid and irrotational inertial flow values
- Relative MOI are qualitatively (not quantitatively) consistent with irrotational flow
- Bohr Hamiltonian is SO(5) symmetric and Irrotational flow is also fortuitously SO(5) symmetric, explaining limited success of Davydov-Filippov Model
- Abs. and Rel. MOI are qualitatively consistent with the slow rotation of a trapped cold Fermi Gas
- Similar MOI behavior for both E(4)/E(2) > 2.7 and E(4)/E(2) = 2.4 - 2.7
- 2- and 3-axis crossing happens after γ = 30 degrees
- Inertial dynamics may not be understood or constrained for more oblate nuclei, γ > 40 degrees
- Independent E2 and Inertia tensor required empirically and must begin thinking in terms of inertia or energy "softness" versus E2 "softness"; E2 character could be significantly more "rigid" than expected from energies.

Recent Case in Point for E(4)/E(2) < 2.4 and Future Teaser

- **E2 character could be significantly more "rigid" than expected from energies, cf. results on ^{106}Cd**

- Analysis of other Cd isotopes underway
- Preliminary results hint at constant <Q^2> values like ^{76}Ge
- Do low-energy quadrupole "vibrations" even exist?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E2 rotational invariants of 0_1^+ and 2_1^+ states for ^{106}Cd: The emergence of collective rotation

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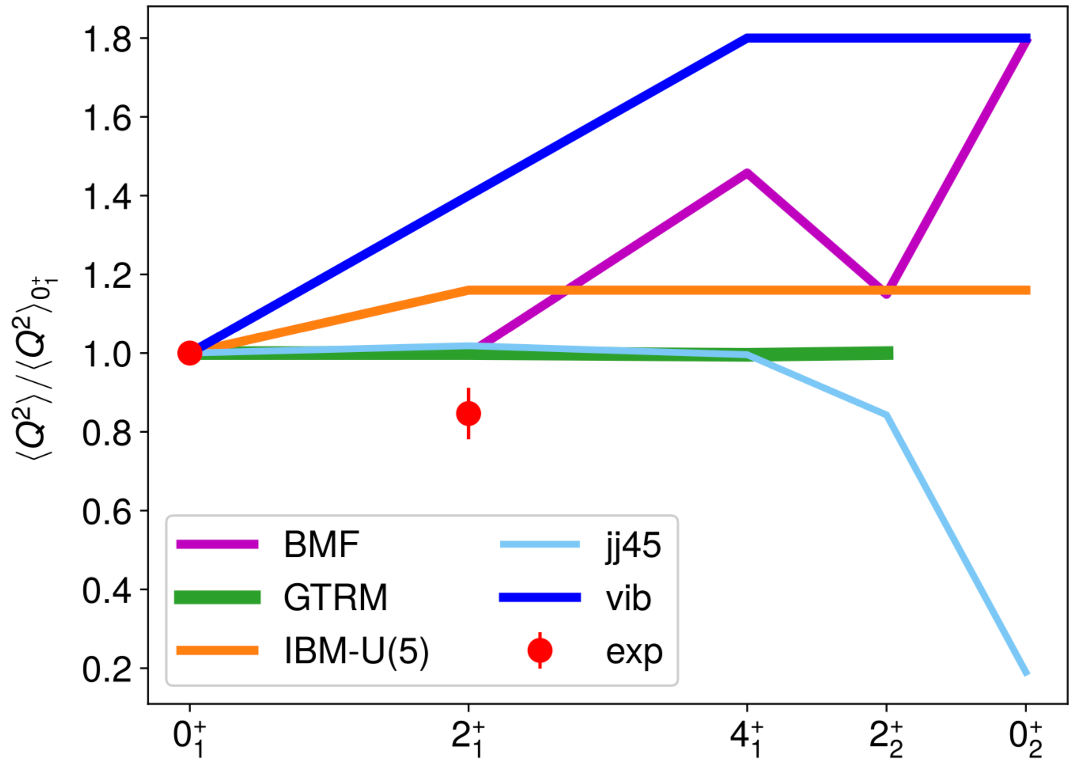
^{106}Cd



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