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Microscopic study of nuclear shapes and excitations

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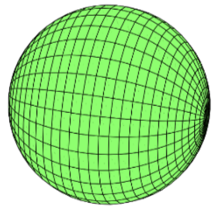
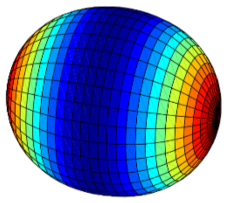
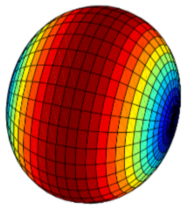
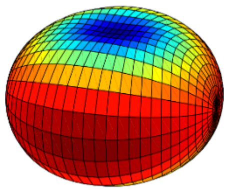
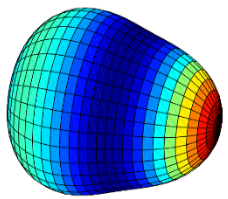
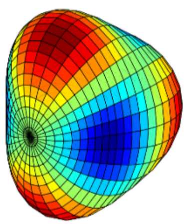
Outline

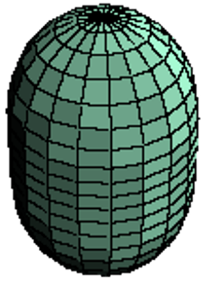
1. Introduction
2. Microscopic collective Hamiltonian
3. Results and discussion
   a. Quadrupole shape coexistence
   b. Octupole shapes and parity doublets
4. Summary and outlook

Nuclear shapes and excitations

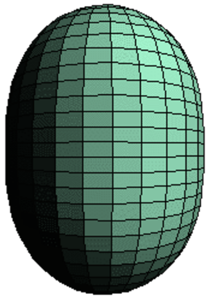
$$R = R_0[1+\beta_{00}+\sum_{\lambda=1}^{\infty}\sum_{\mu=-\lambda}^{\lambda}\beta_{\lambda\mu}^{*}Y_{\lambda\mu}(\theta,\varphi)]$$

$\beta_{\lambda\mu}=0$	$\beta_{20}>0$	$\beta_{20}<0$
		
$\beta_{22}\neq 0$	$\beta_{30}\neq 0$	$\beta_{32}\neq 0$
		

B.N. Lu, Ph.D. thesis, Academy of Sciences (2012)

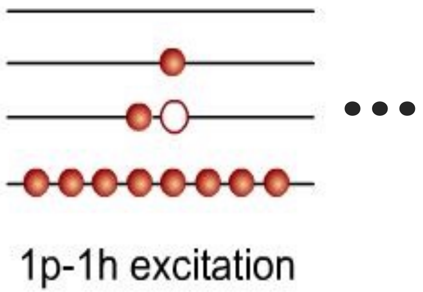


vibration



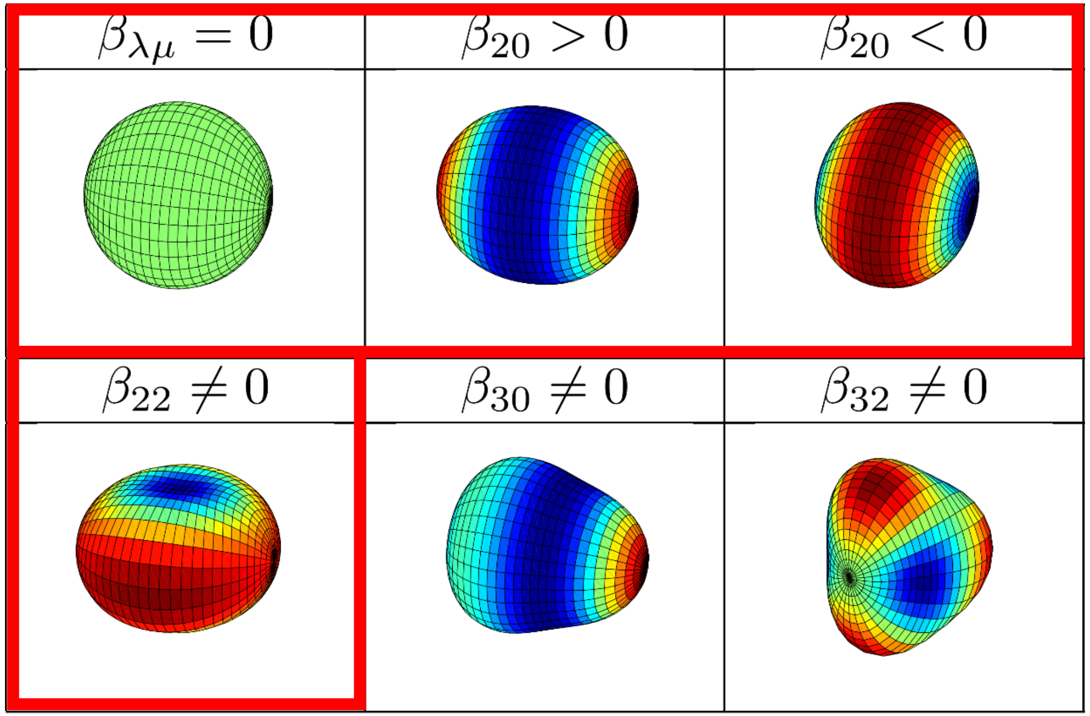
rotation

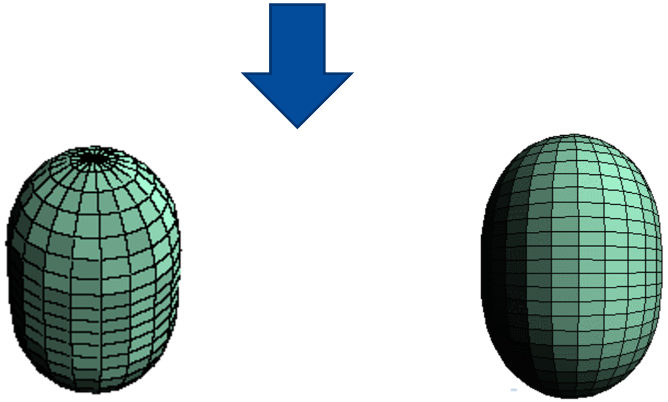
fission



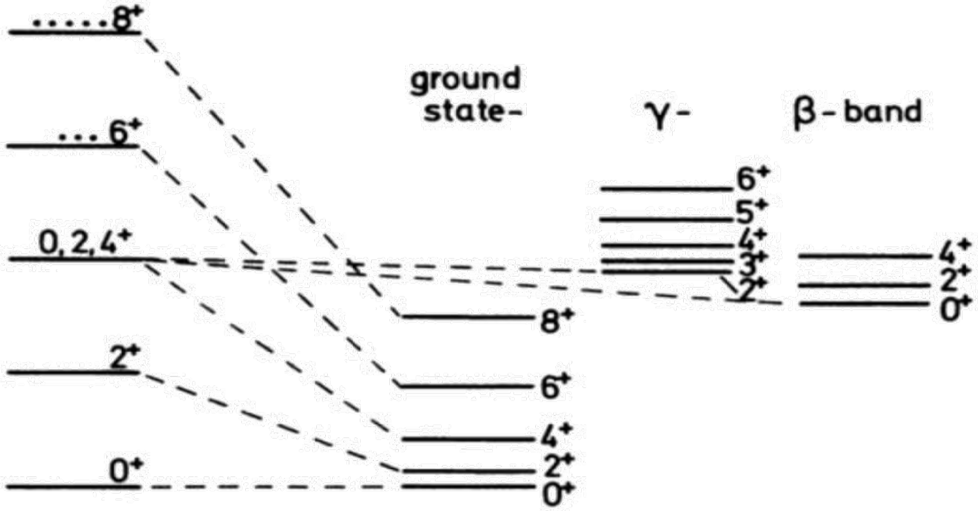
s.p. excitation

Nuclear quadrupole shapes





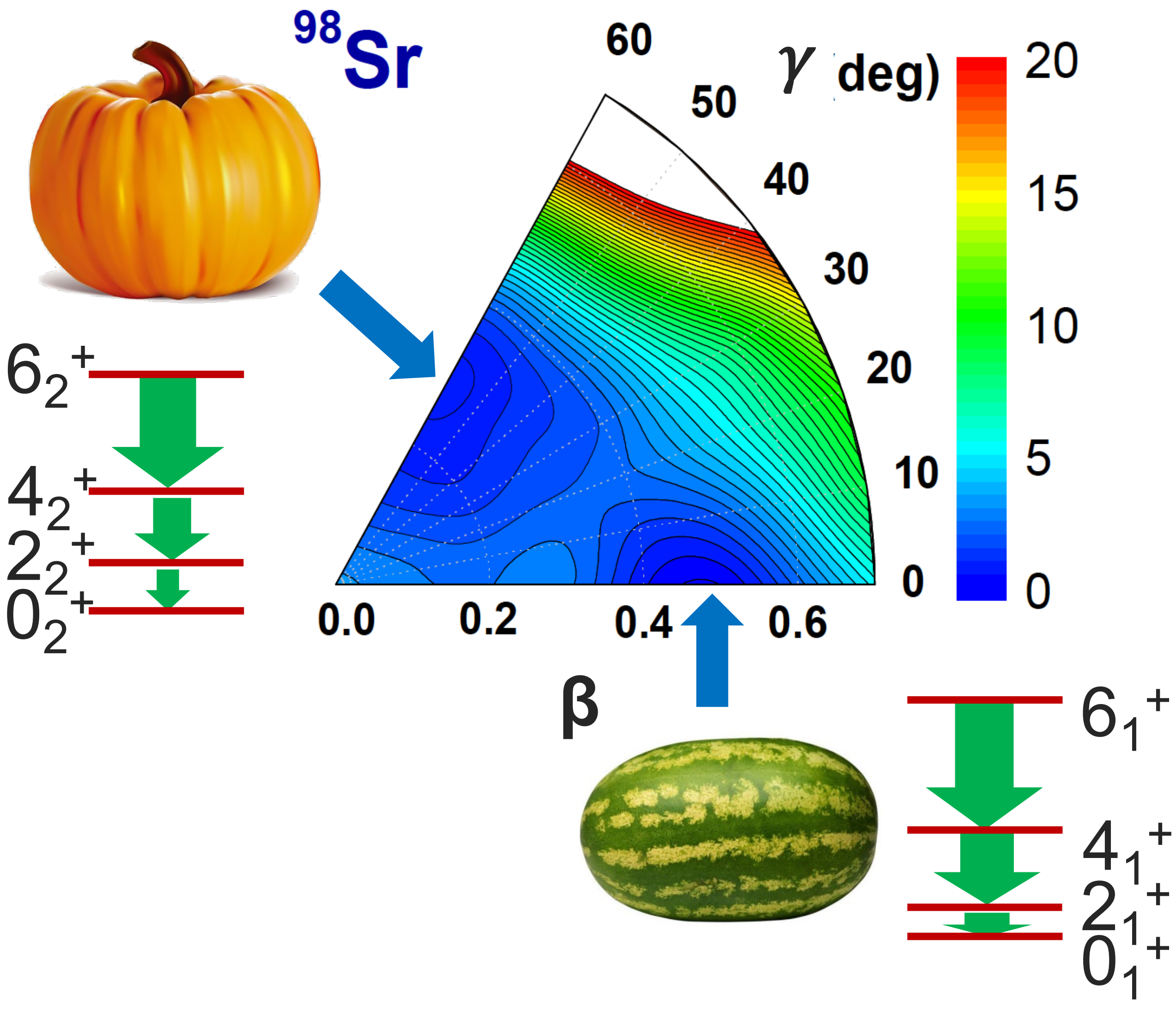
$$\hat{H} = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2}\frac{1}{\sin(3\gamma)}\frac{\partial}{\partial\gamma}\sin(3\gamma)\frac{\partial}{\partial\gamma}\right] + \sum_{k=1}^{3}\frac{\hat{J}_k'^2}{2\mathcal{J}_k} + V(\beta,\gamma) \quad .$$



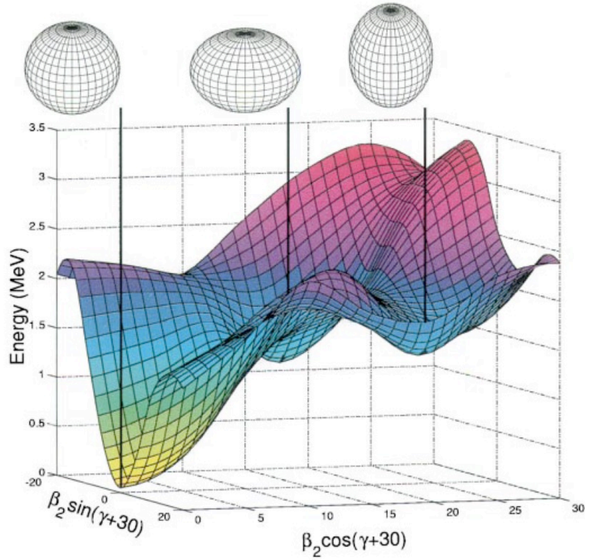
A. Bohr, Mat. Fys. Medd. K. Dan. Vidensk Selsk. 26, 14 (1952).

A. Bohr & Mottelson, Mat. Fys. Medd. K. Dan. Vidensk Selsk. 27, 16 (1953).

Nuclear shape coexistence

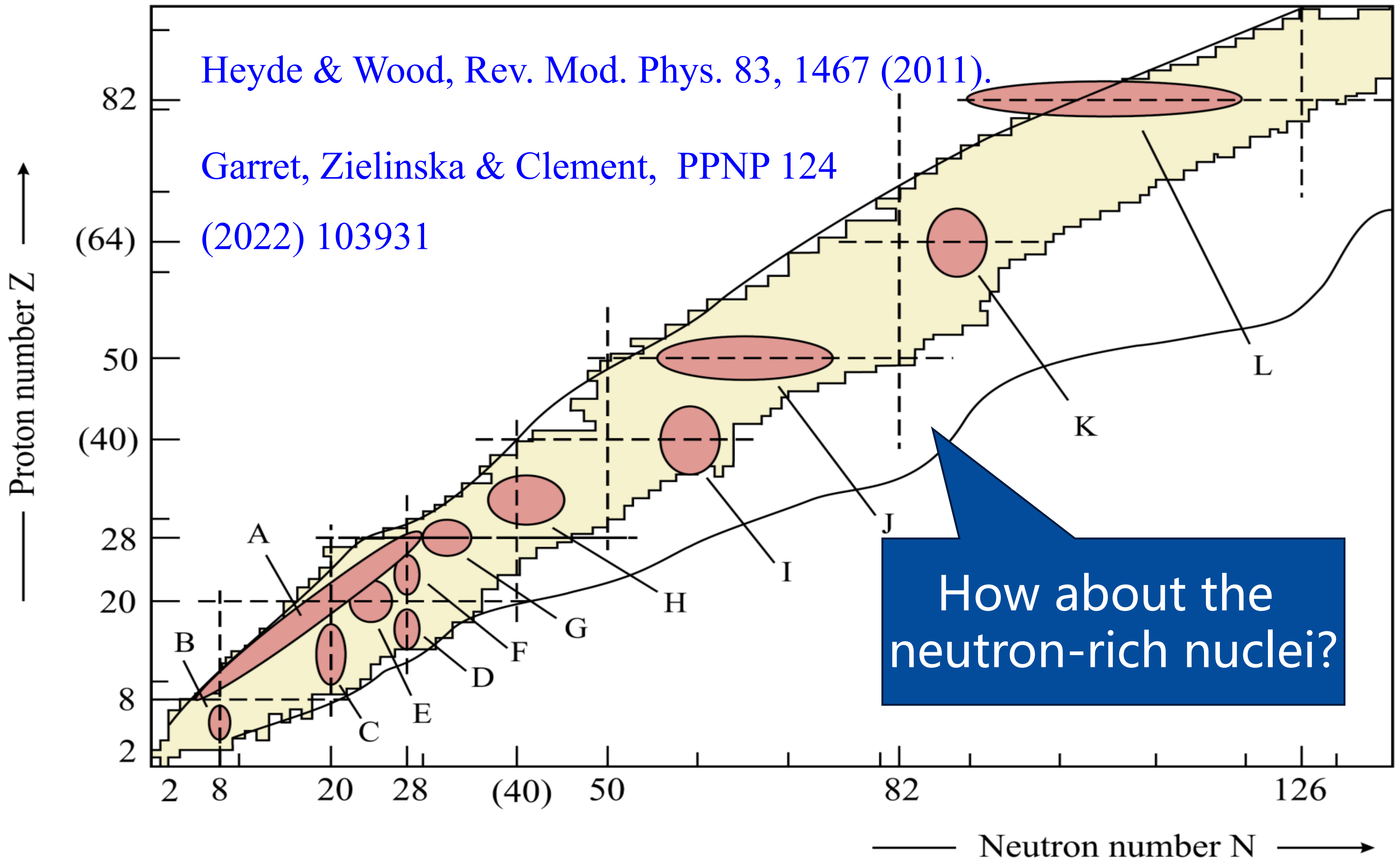


- There may be deformation in the 0^+ excited state of ^{16}O.
  H. Morinaga, Phys. Rev. 101, 254 (1956).
- Late 1960s, near Z = 50,82
- In 2000, three shapes coexisted in ^{186}Pb.

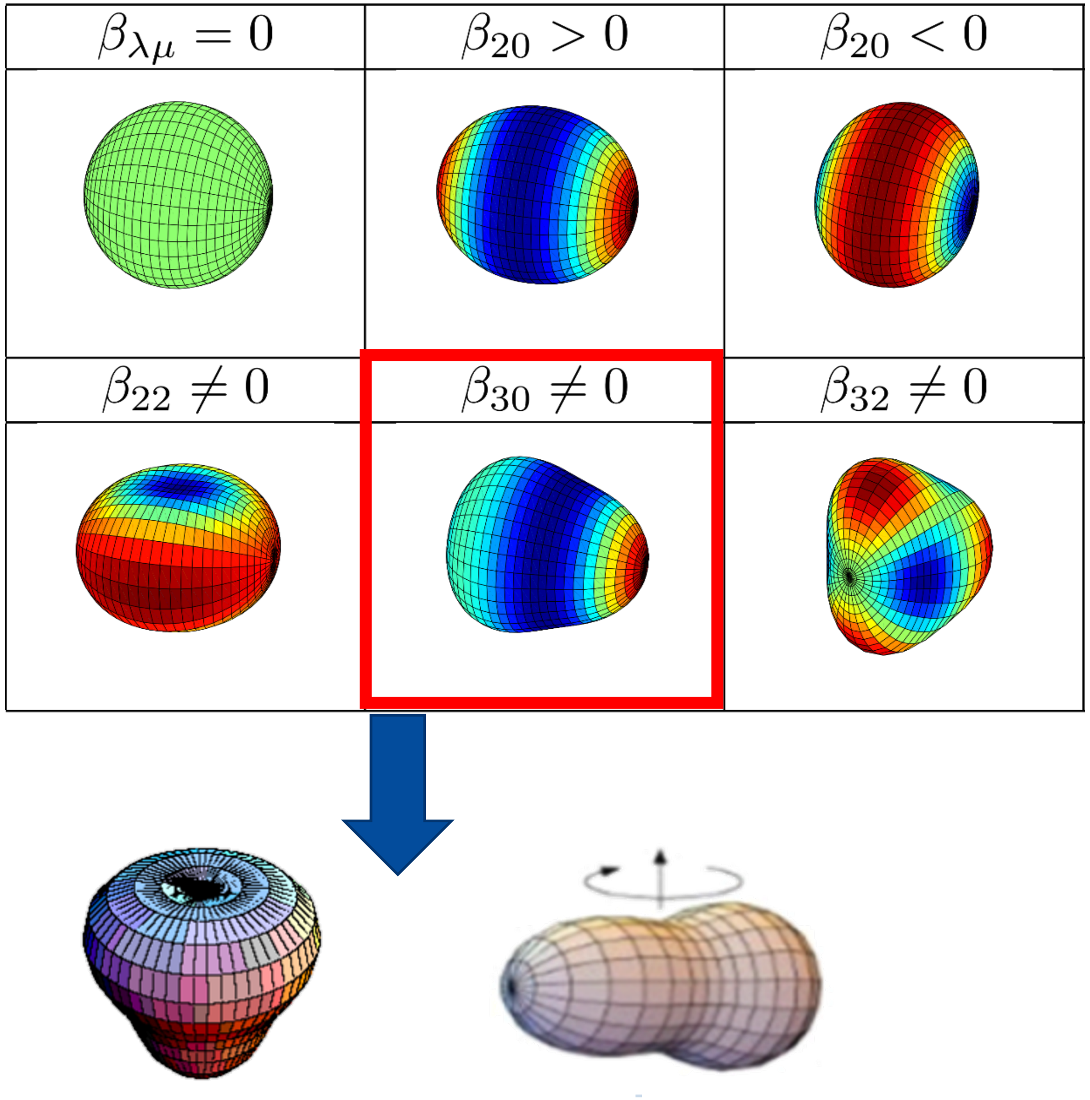


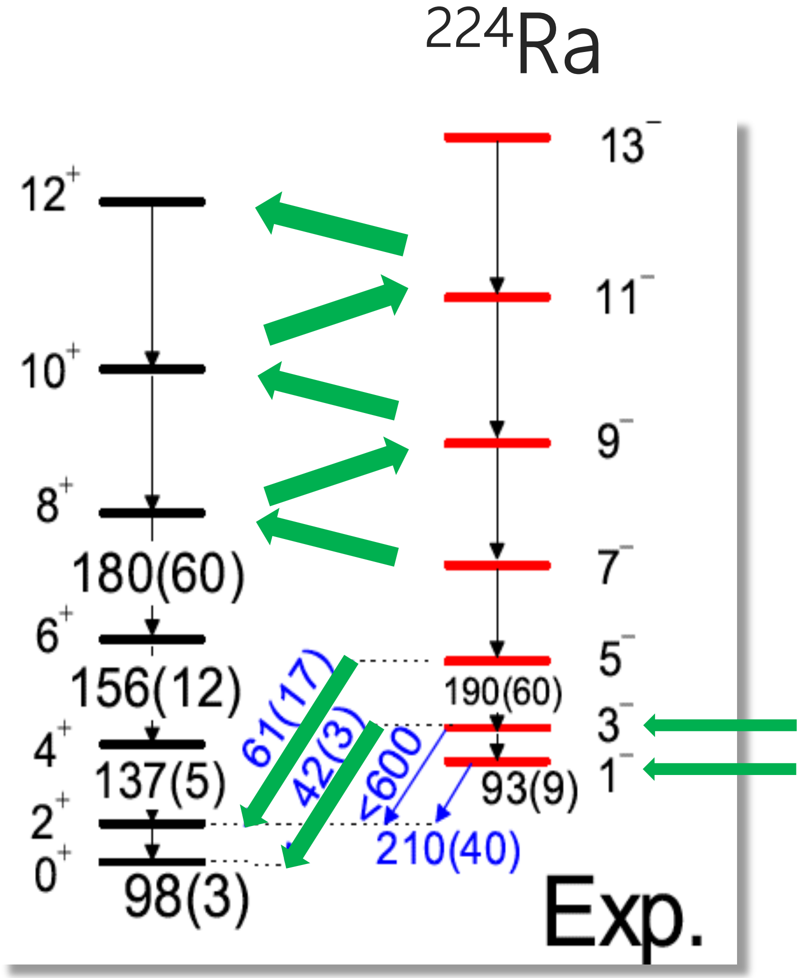
Andreyev et al., Nature 405, 430 (2000).

Nuclear shape coexistence



Axially symmetric octupole shapes





Butler & Nazarewicz, RMP68, 349 (1996).

Axially symmetric octupole shapes

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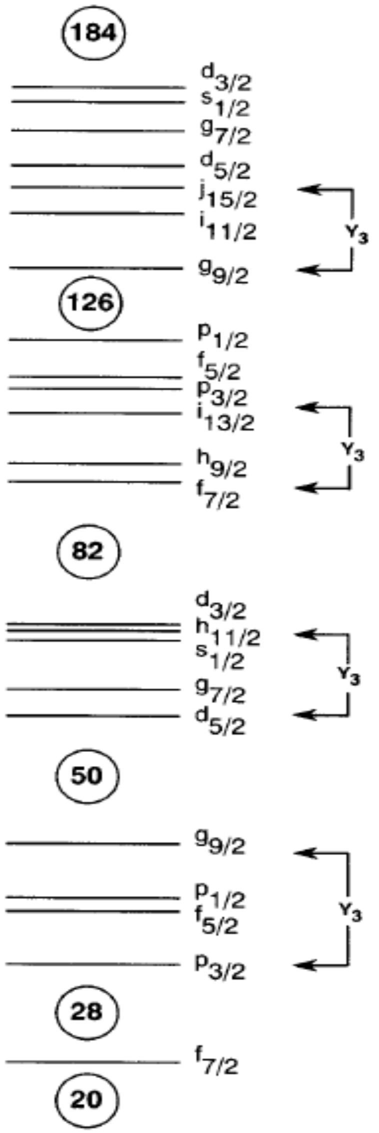


$\langle j \| Q_3 \| j' \rangle$

~134 $j_{15/2} \leftrightarrow g_{9/2}$

~88 $i_{13/2} \leftrightarrow f_{7/2}$

~56 $h_{11/2} \leftrightarrow d_{5/2}$

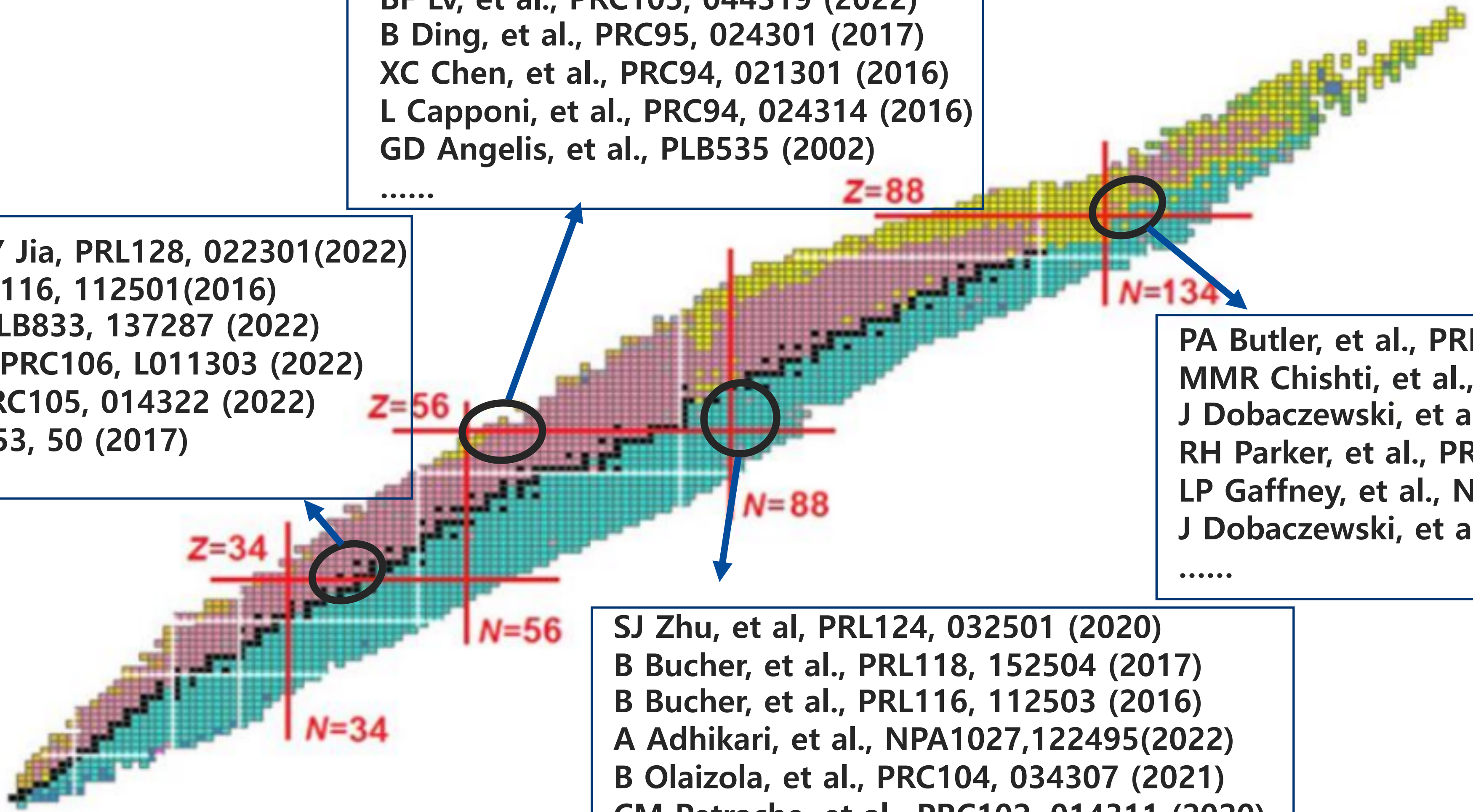
~34 $g_{9/2} \leftrightarrow p_{3/2}$

Near Fermi surface :

Strong octupole coupling of levels with $(\Delta l = \Delta j = 3)$

Butler & Nazarewicz, RMP68, 349 (1996).

Experimental progress for octupole correlations



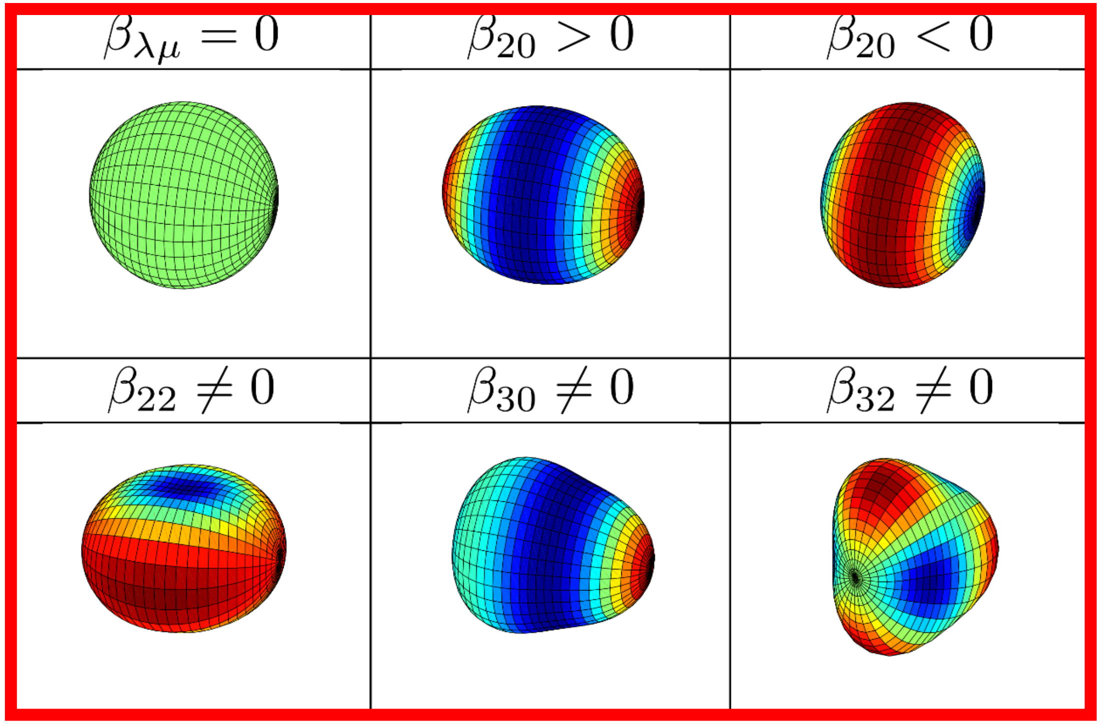
A Yagi, et al., PRC105, 044325 (2022)
BF Lv, et al., PRC105, 044319 (2022)
B Ding, et al., PRC95, 024301 (2017)
XC Chen, et al., PRC94, 021301 (2016)
L Capponi, et al., PRC94, 024314 (2016)
GD Angelis, et al., PLB535 (2002)
……

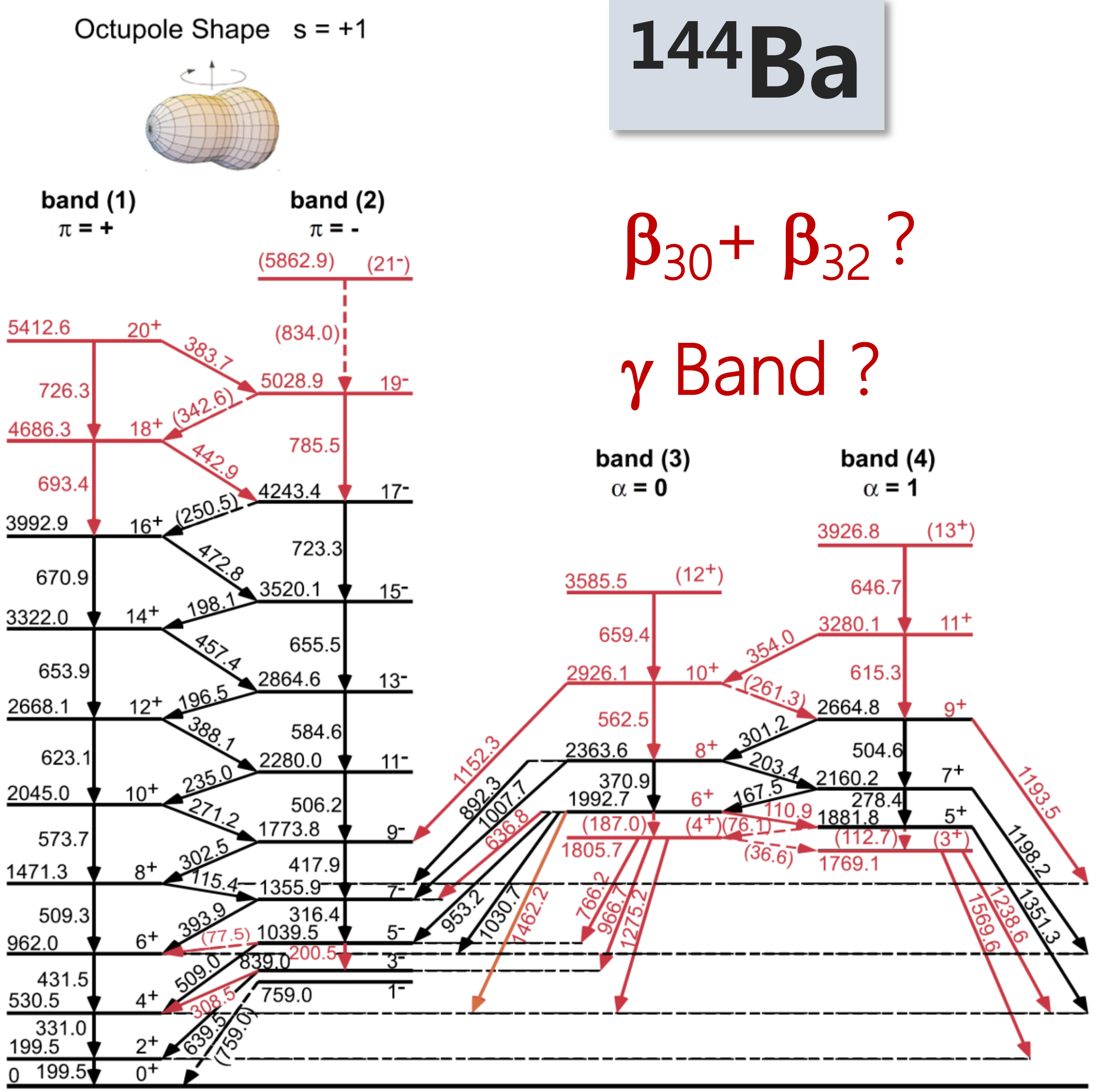
CJ Zhang and JY Jia, PRL128, 022301(2022)
C. Liu et al., PRL116, 112501(2016)
W.Z. Xu et al., PLB833, 137287 (2022)
CG Wang et al., PRC106, L011303 (2022)
A Mukherjee, PRC105, 014322 (2022)
ET Gregor, EPJA53, 50 (2017)
……

PA Butler, et al., PRL124, 042503 (2020)
MMR Chishti, et al., Nat. Phys. (2020)
J Dobaczewski, et al., PRL121, 232501 (2018)
RH Parker, et al., PRL114, 233002 (2015)
LP Gaffney, et al., Nature, 497:199 (2013)
J Dobaczewski, et al., PRL94, 232502 (2005)
……

SJ Zhu, et al, PRL124, 032501 (2020)
B Bucher, et al., PRL118, 152504 (2017)
B Bucher, et al., PRL116, 112503 (2016)
A Adhikari, et al., NPA1027,122495(2022)
B Olaizola, et al., PRC104, 034307 (2021)
CM Petrache, et al., PRC102, 014311 (2020)
HJ Li, et al., PRC90, 047303 (2014)
……

Triaxial octupole deformation ?





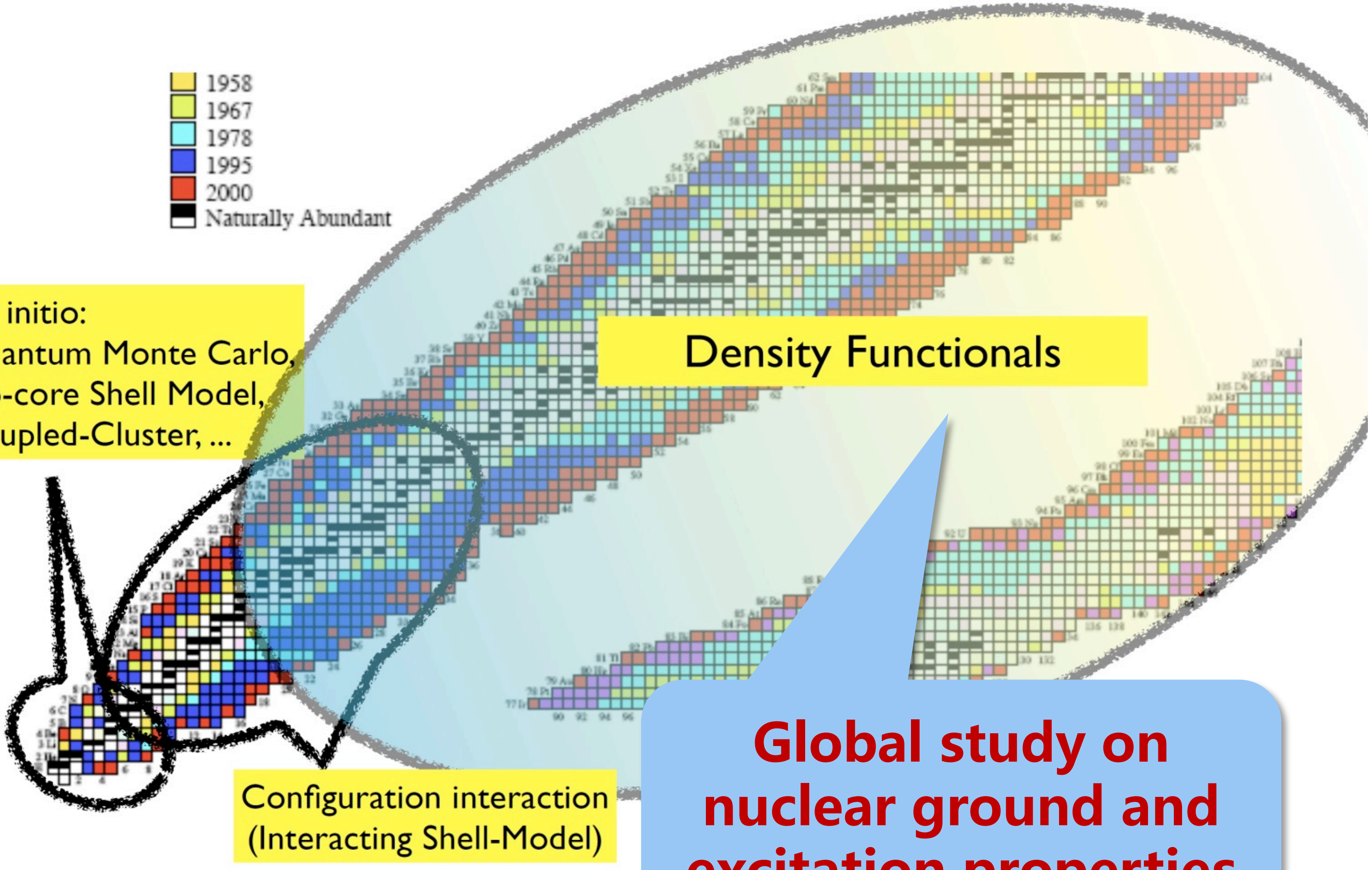
^{144}Ba

$\beta_{30} + \beta_{32}$?

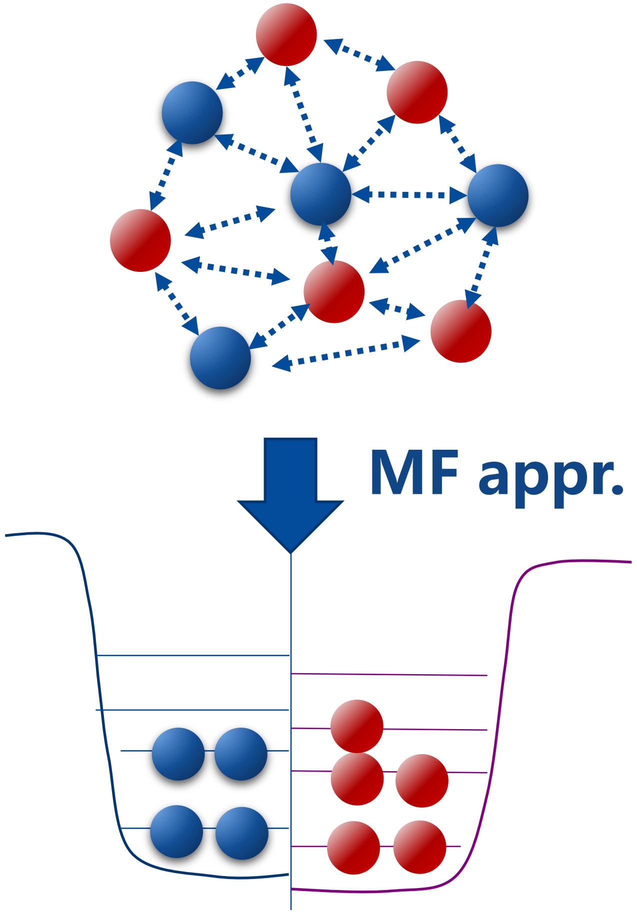
γ Band ?

S. J. Zhu et. al., PRL124, 032501 (2020).

Microscopic models for nuclear excitation



Global study on nuclear ground and excitation properties



- Simpler wave function
- More correlations: ph, pp

《The Nuclear Many-body Problem》

Beyond-mean-field methods

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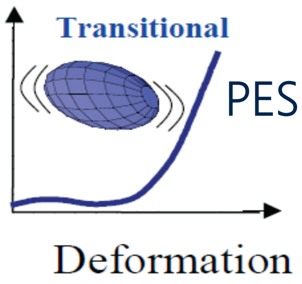
Limitation of MF

- intrinsic ground state properties: mass, radius shape…; PES

"Real" ground state (good quantum numbers) and excitations:

beyond the MF approximation

- Restoration of broken symmetry, e.g. rotational, parity
- Mixing of different shapes
- Inclusion of quasiparticles



Collective Hamiltonian (DFT)

Outline

1. Introduction
2. **Microscopic collective Hamiltonian**
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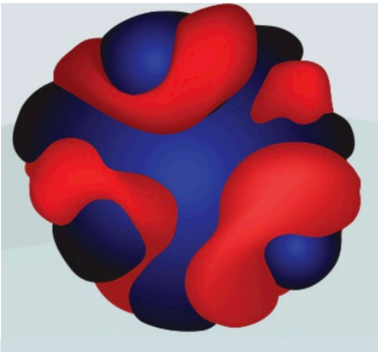
Collective Hamiltonian based on CDFT



CDFT

Construct

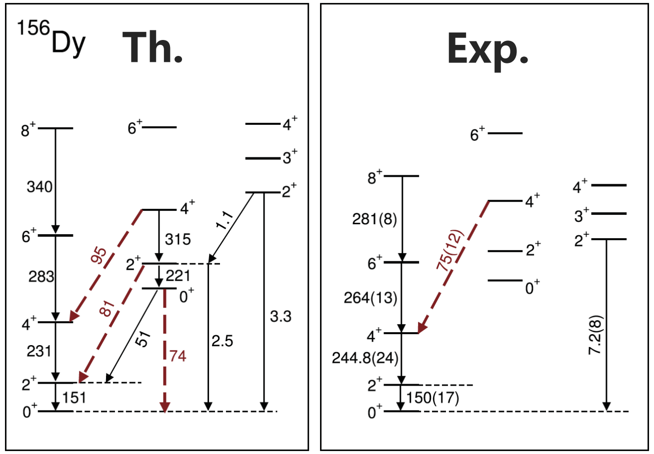
Collective parameters

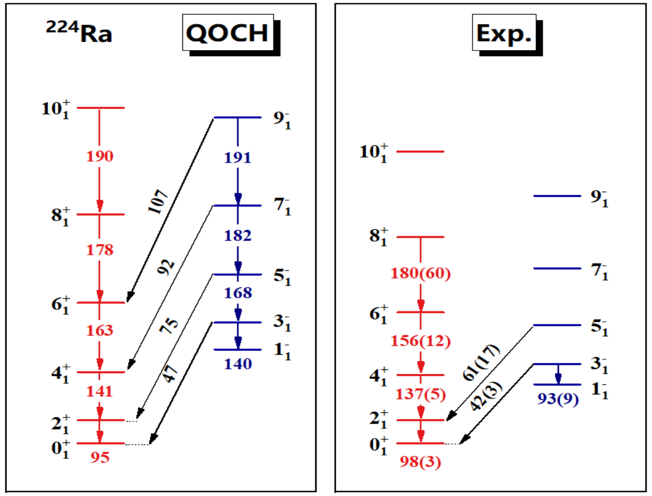
Collective Hamiltonian

$$\hat{H}_{\text{coll}} = \hat{T}_{\text{vib}} + \hat{T}_{\text{rot}} + V_{\text{coll}}$$

$$(\beta_{20}, \beta_{22}, \beta_{30}, \beta_{32}, \Omega \ldots)$$

Solve





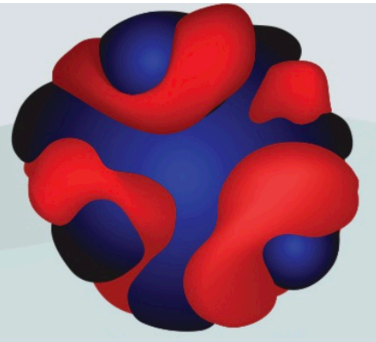
ZPLi, Niksic, Vretenar, Meng, Lalazissis, & Ring, PRC (2009)

ZPLi, Song, Yao, Vretenar & Meng. PLB (2013)

Xiang, ZPLi, Niksic, Vretenar & Long, PRC (2020)

ZPLi & Vretenar, *Model for Collective Motion. In: Handbook of Nuclear Physics*. Springer, Singapore.

Collective Hamiltonian based on CDFT



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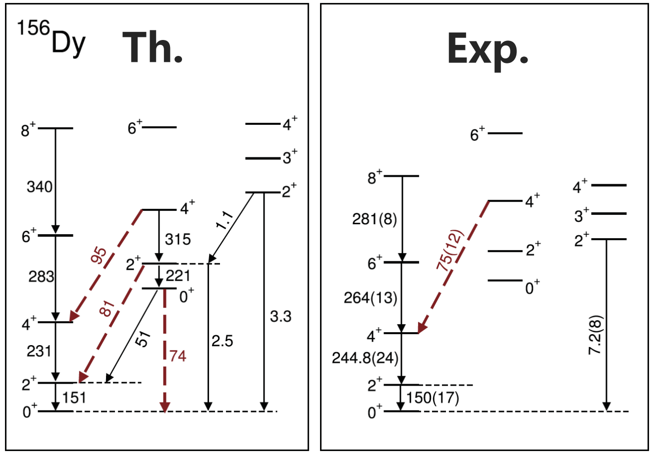
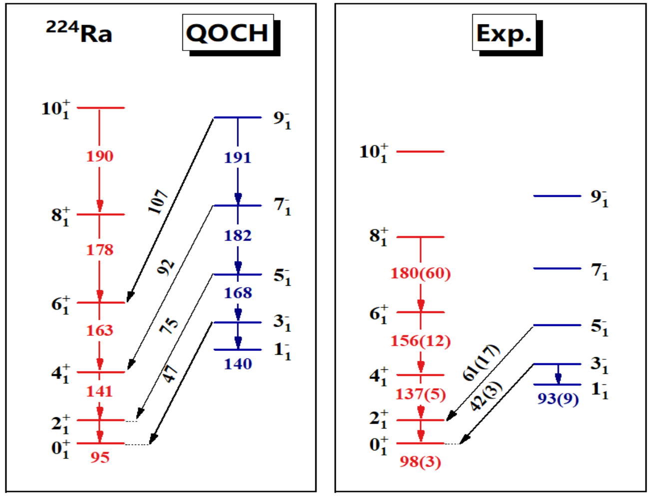




$$E_{\text{CDF}} = \int d^3(\mathbf{r})\varepsilon_{\text{CDF}}(\mathbf{r})$$

$$= \sum_k \int d\mathbf{r} v_k^2 \bar{\psi}_k(\mathbf{r})(-i\gamma\nabla + m)\psi_k(\mathbf{r})$$

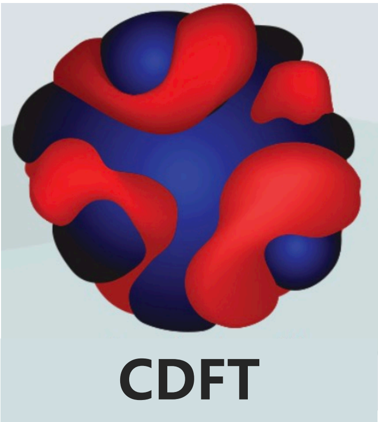
$$+ \int d\mathbf{r} \left(\frac{\alpha_S}{2}\rho_S^2 + \frac{\beta_S}{3}\rho^3 + \frac{\gamma_S}{3}\rho_S^4 + \frac{\delta_S}{2}\rho_S\Delta\rho_S + \frac{\alpha_V}{2}j_\mu j^\mu + \frac{\gamma_V}{4}(j_\mu j^\mu)^2 \right.$$

$$\left. + \frac{\delta_V}{2}j_\mu\Delta j^\mu + \frac{\alpha_{TV}}{2}\vec{j}^{\mu}_{TV}\cdot(\vec{j}_{TV})_\mu + \frac{\delta_{TV}}{2}\vec{j}^{\mu}_{TV}\cdot\Delta(\vec{j}_{TV})_\mu + \frac{e^2}{2}\rho_p A^0 \right)$$

PC-PK1 functional

Zhao, ZPLi, Yao & Meng, PRC 82, 054319 (2010).

Collective Hamiltonian based on CDFT



Construct

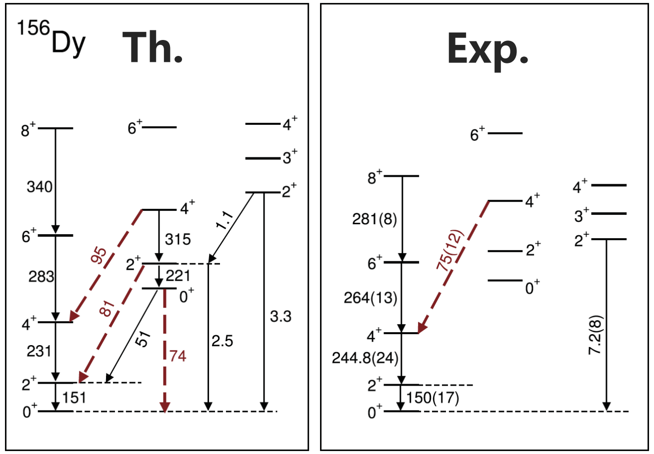
Collective parameters

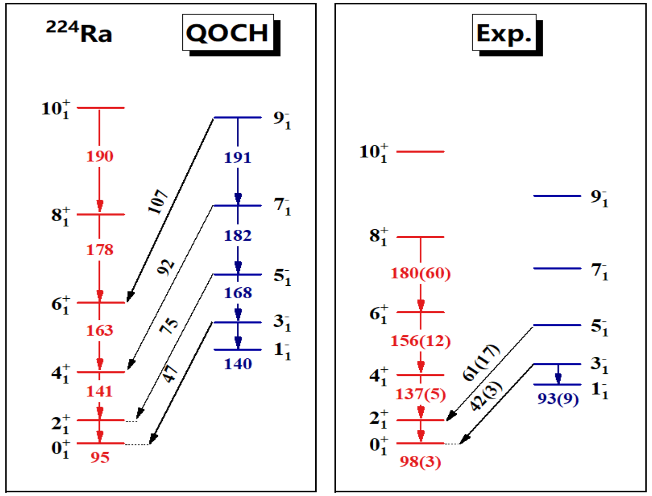
Collective Hamiltonian

$$\hat{H}_{\rm coll} = \hat{T}_{\rm vib} + \hat{T}_{\rm rot} + V_{\rm coll}$$

$(\beta_{20}, \beta_{22}, \beta_{30}, \beta_{32}, \ \Omega \ ...)$

Solve





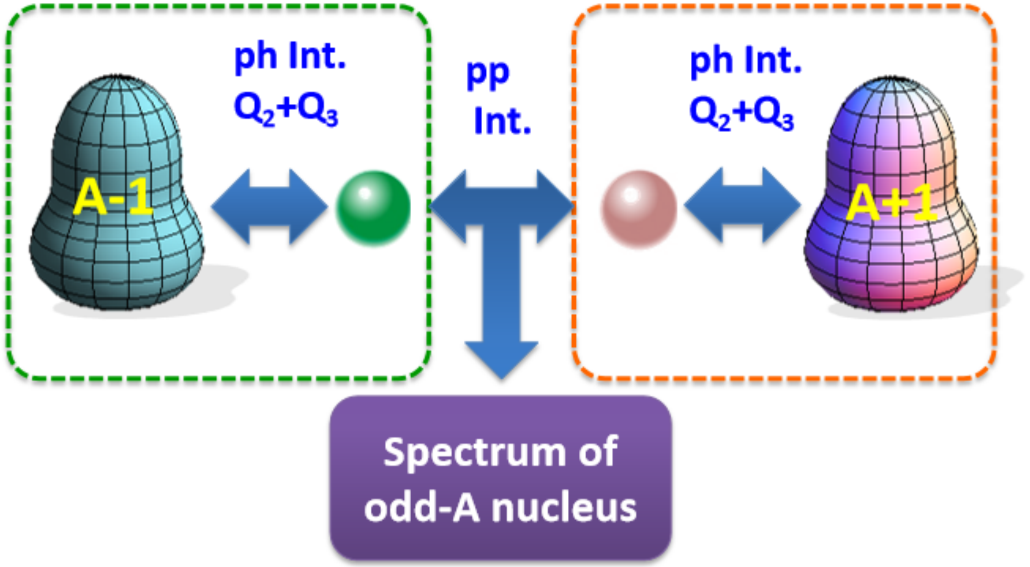
$$\hat{H}_{coll} = -\frac{\hbar^2}{2\sqrt{G}} \sum_{ij} \frac{\partial}{\partial \beta_i} \sqrt{G} \left(B^{-1}_{vib}\right)_{ij} \frac{\partial}{\partial \beta_j} + \sum_{k=1}^{3} \frac{\hat{I}_k^2}{2\mathcal{J}_k} + V_{\rm coll}(\beta)$$

Mass tensor:

$$B_{vib} = \begin{pmatrix} B_{\beta_{20}\beta_{20}} & B_{\beta_{20}\beta_{22}} & B_{\beta_{20}\beta_{30}} & B_{\beta_{20}\beta_{32}} \\ B_{\beta_{22}\beta_{20}} & B_{\beta_{22}\beta_{22}} & B_{\beta_{22}\beta_{30}} & B_{\beta_{22}\beta_{32}} \\ B_{\beta_{30}\beta_{20}} & B_{\beta_{30}\beta_{22}} & B_{\beta_{30}\beta_{30}} & B_{\beta_{30}\beta_{32}} \\ B_{\beta_{32}\beta_{20}} & B_{\beta_{32}\beta_{22}} & B_{\beta_{32}\beta_{30}} & B_{\beta_{32}\beta_{32}} \end{pmatrix}$$

Odd-A nuclei

➢ Microscopic core-quasiparticle coupling model (CQC)



Sun, Quan, ZPLi, et al, PRC100, 044319 (2019)

$$|\alpha J M_J \pi\rangle^A = \sum_{\mu\Omega} \left\{ U_{\mu\Omega} \left[a^\dagger_\mu |\Omega\rangle\right]^{A-1}_{JM_J\pi} + \mathrm{V}_{\mu\Omega} \left[a_\mu |\Omega\rangle\right]^{A+1}_{JM_J\pi} \right\}$$

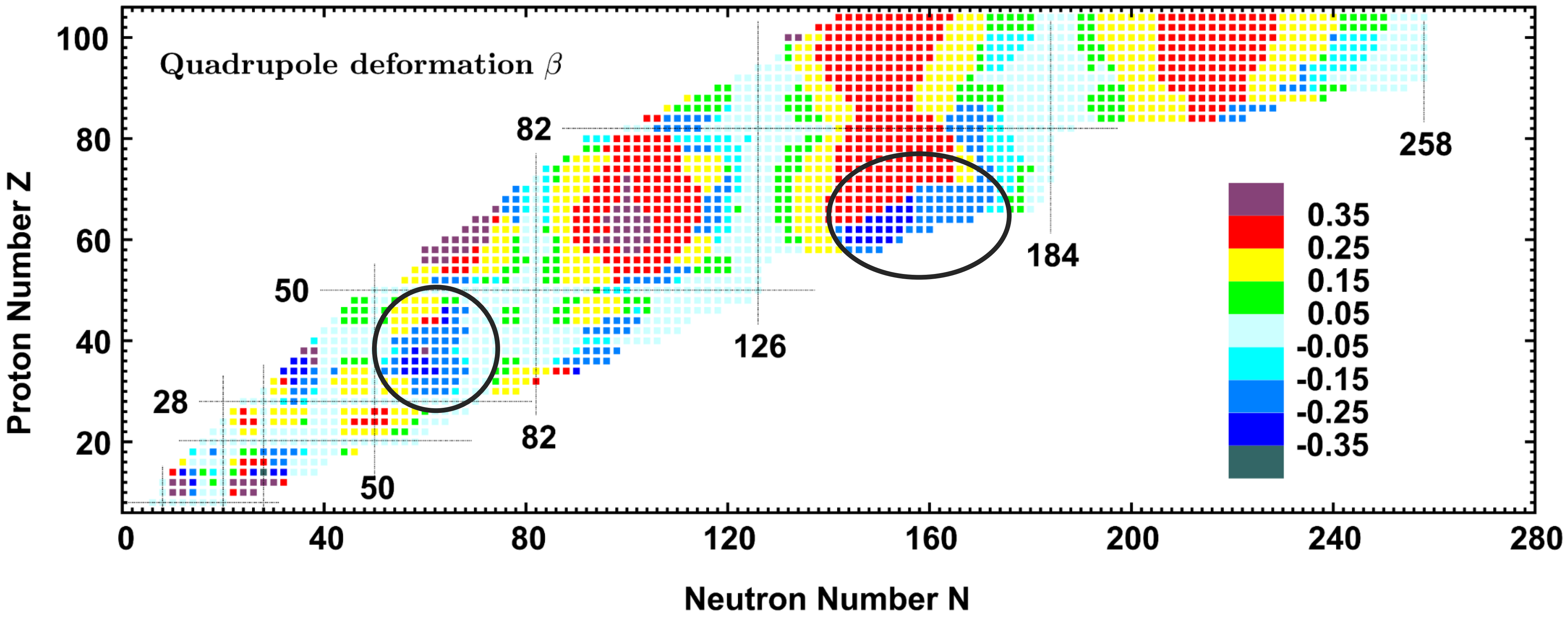
s.p. levels | Multipole int. | Pairing gap | Collective levels

$$H = H_{\mathrm{qp}} + H_{\mathrm{c}}$$

$$= \begin{pmatrix} (\varepsilon^{A-1} - \varepsilon_f) + \Gamma^{A-1} & \Delta^{A+1} \\ \Delta^{\dagger A-1} & -(\varepsilon^{A+1} - \varepsilon_f) - \Gamma^{A+1} \end{pmatrix} + \begin{pmatrix} E^{A-1} & 0 \\ 0 & E^{A+1} \end{pmatrix}$$

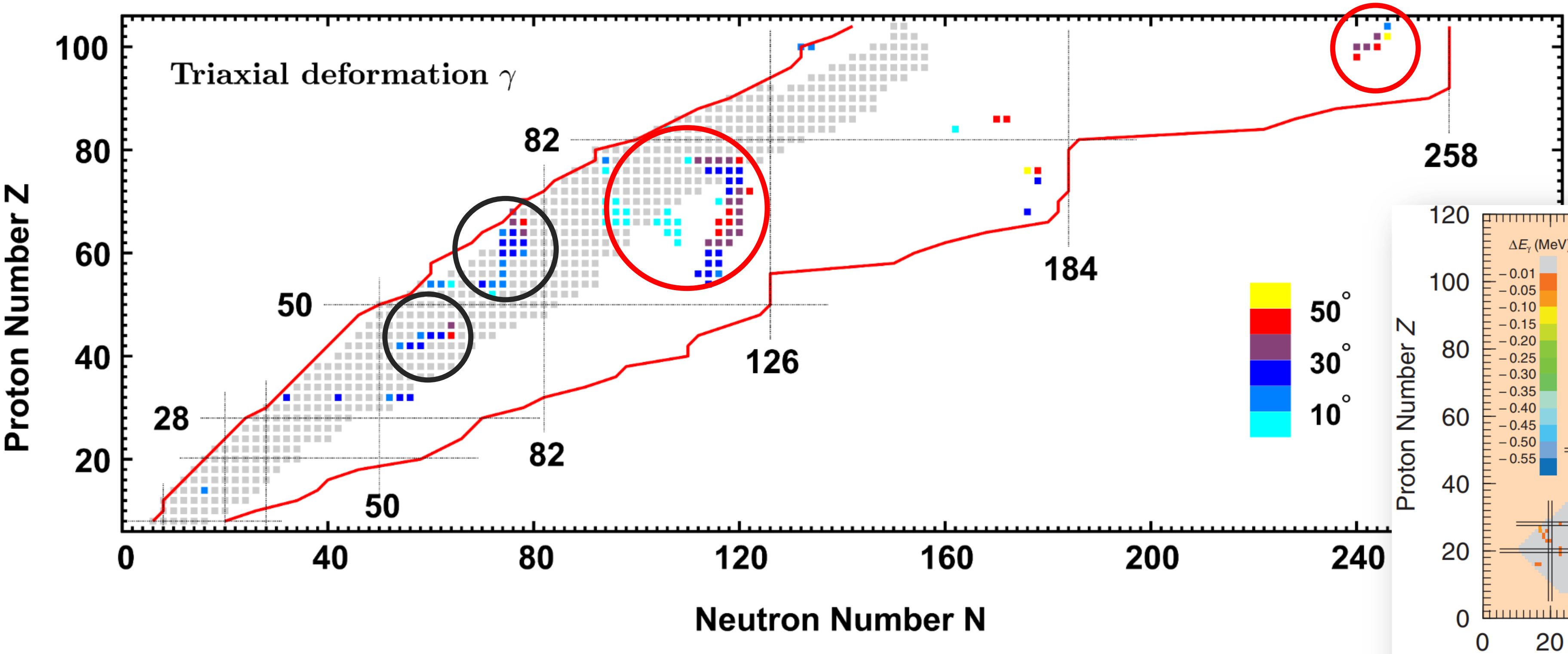
Outline

Static quadrupole deformation β

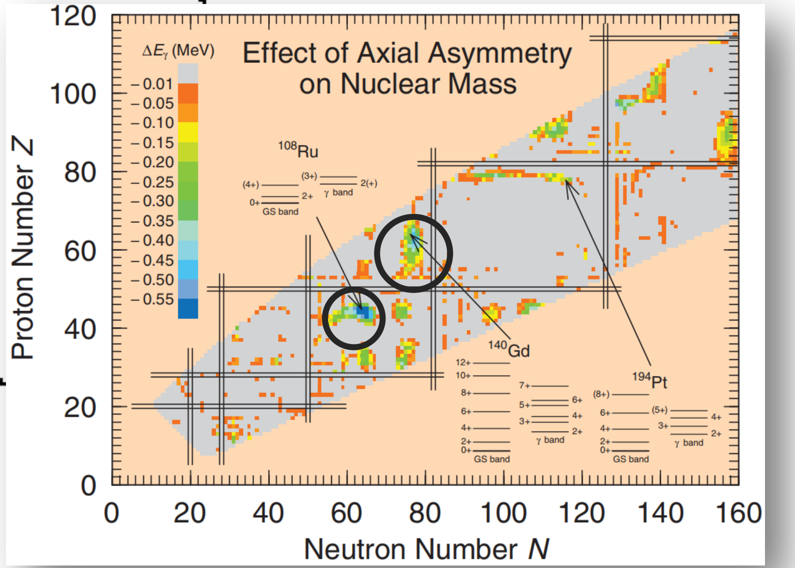


- Most of the deformed nuclei are prolate ($\beta > 0$). The nuclei with oblate deformation are mainly distributed in (N, Z)~ (60,40), (150, 60).
- The quadrupole deformation of most nuclei is in the range of $-0.35 < \beta < 0.35$.

Static triaxial deformation γ



P. Möller *et al.*, *Phys. Rev. Lett.* **97**, 162502 (2006)



- ➢108 triaxially deformed even-even nuclei; Triaxial deformation energy: 0~1.89 MeV
- ➢Finite-range liquid-drop model:

  >700 triaxially deformed nuclei; Triaxial deformation energy : 0~0.63 MeV

Online data base

http://nuclearmap.jcnp.org

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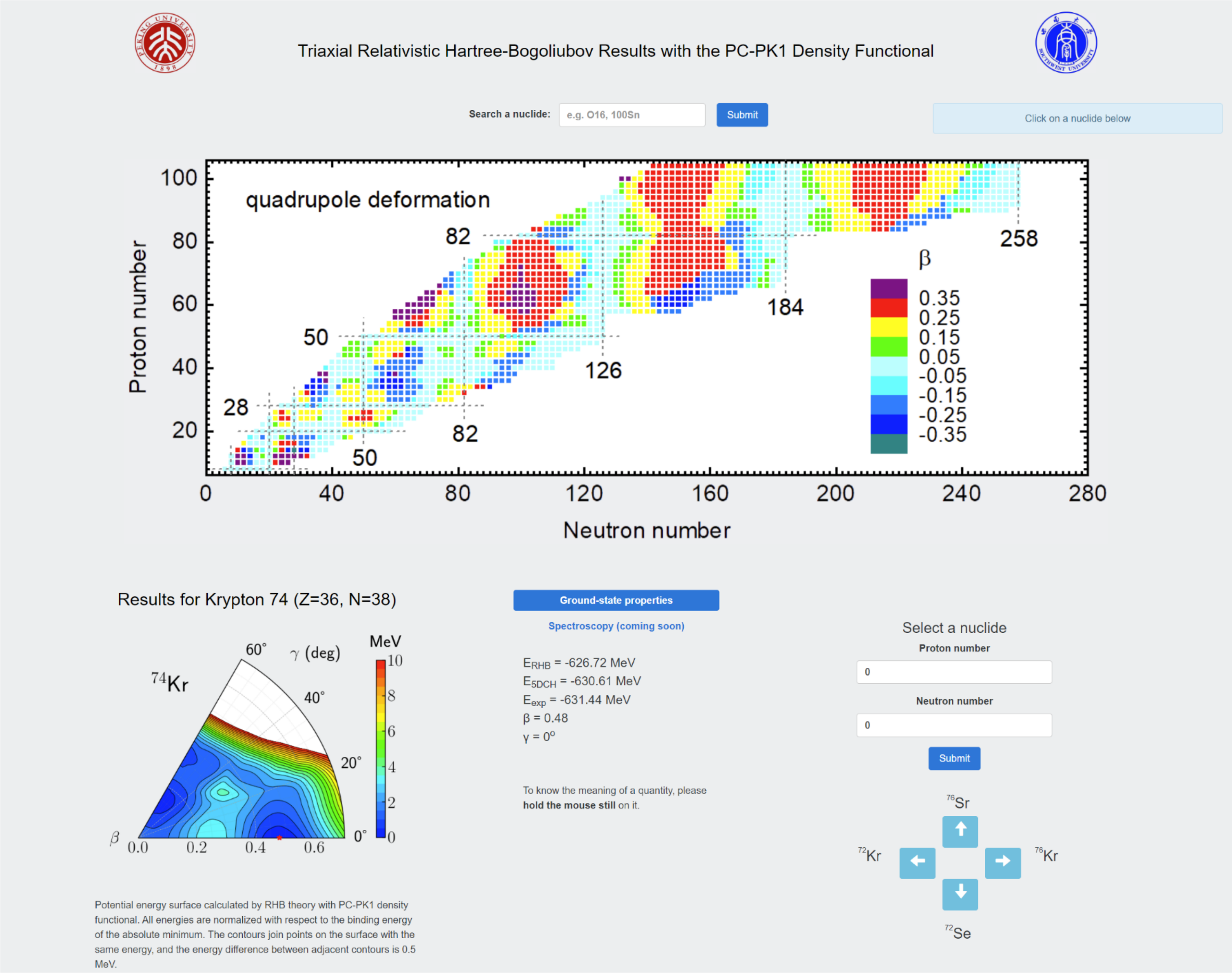
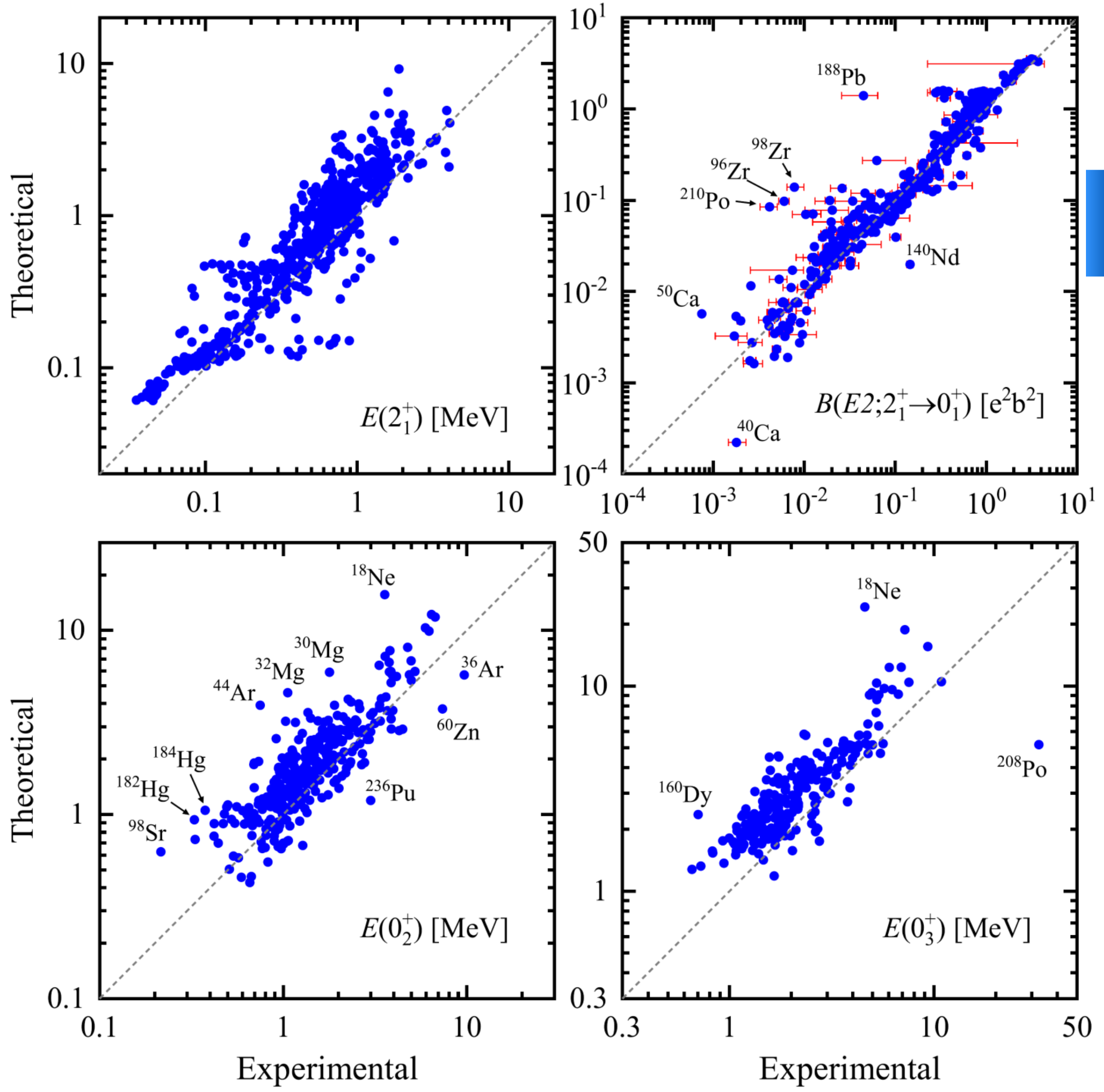


Quadrupole excitations

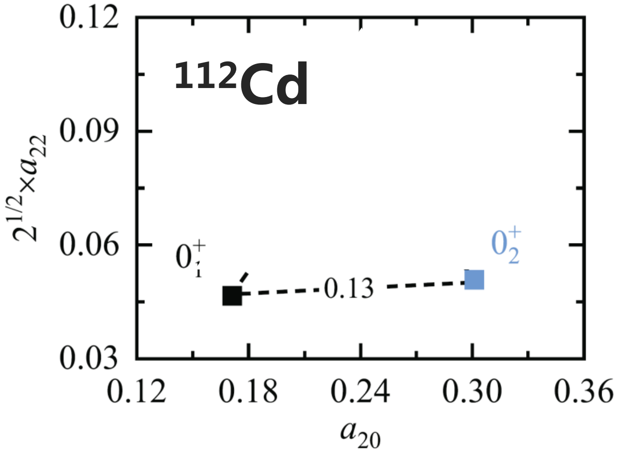


$$q_2(0_i^+) = \sum_j \langle 0_i^+||\hat{Q}_2||2_j^+\rangle\langle 2_j^+||\hat{Q}_2||0_i^+\rangle$$

$$= \left(\frac{3ZeR^2}{4\pi}\right)^2 [(a_{20}^{\text{eff}})^2 + 2(a_{22}^{\text{eff}})^2]$$

$$q_3(0_i^+) = -\sqrt{\frac{7}{10}} \sum_{jk} \langle 0_i^+||\hat{Q}_2||2_j^+\rangle\langle 2_j^+||\hat{Q}_2||2_k^+\rangle\langle 2_k^+||\hat{Q}_2||0_i^+\rangle$$

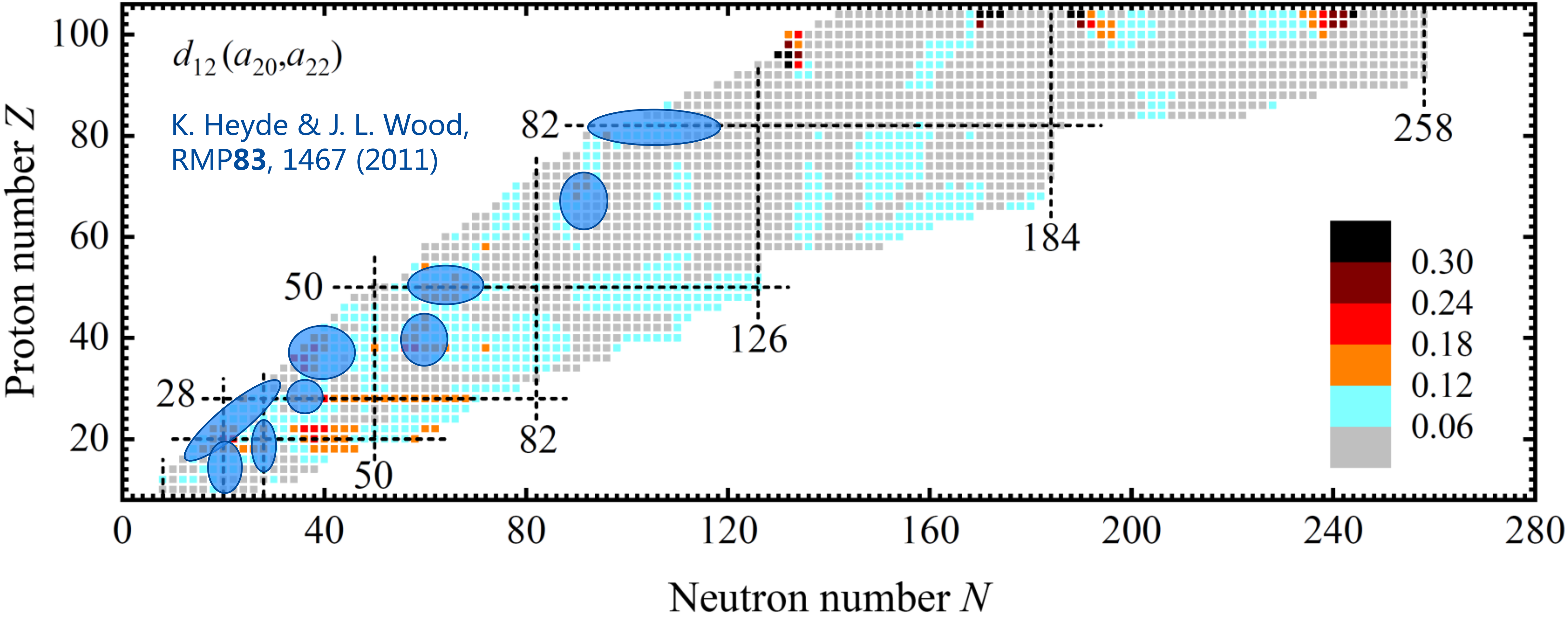
$$= \left(\frac{3ZeR^2}{4\pi}\right)^3 [(a_{20}^{\text{eff}})^3 - 6a_{20}^{\text{eff}}(a_{22}^{\text{eff}})^2]$$



- **0^+ states have different shapes**

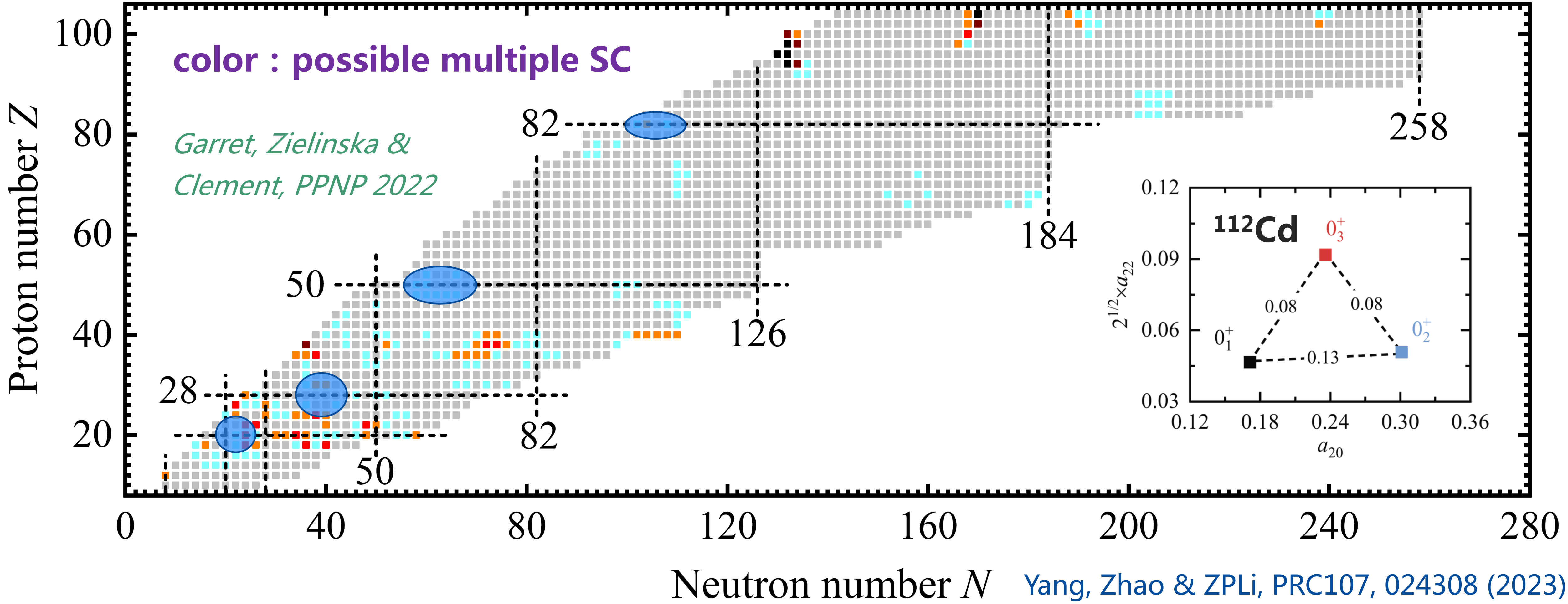
Garrett et al., PRL 2019

Nuclear shape coexistence



- Z ~ shell or sub-shell; transitional regions
- New regions in neutron-rich side and heavy nuclei

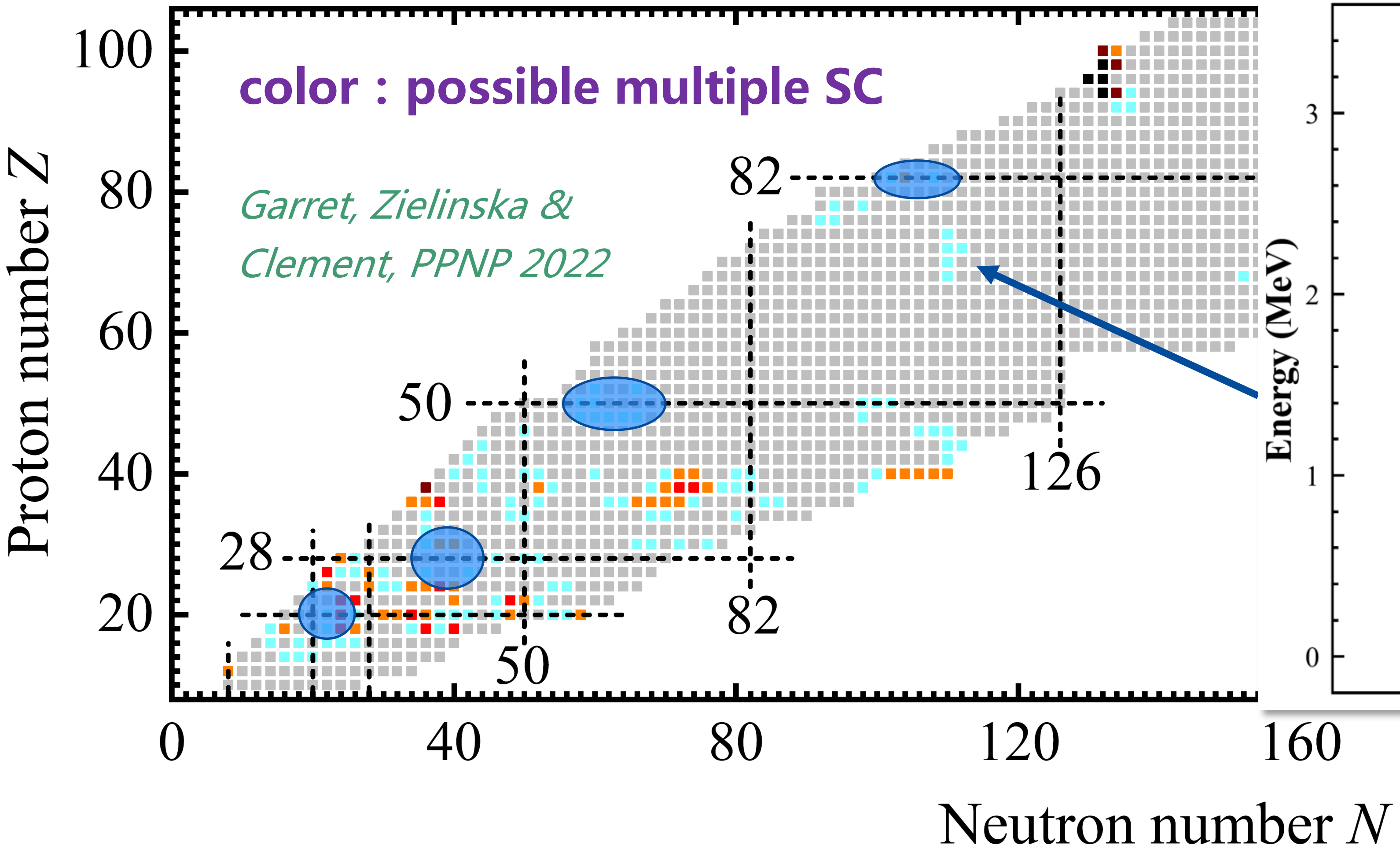
Multiple shape coexistence

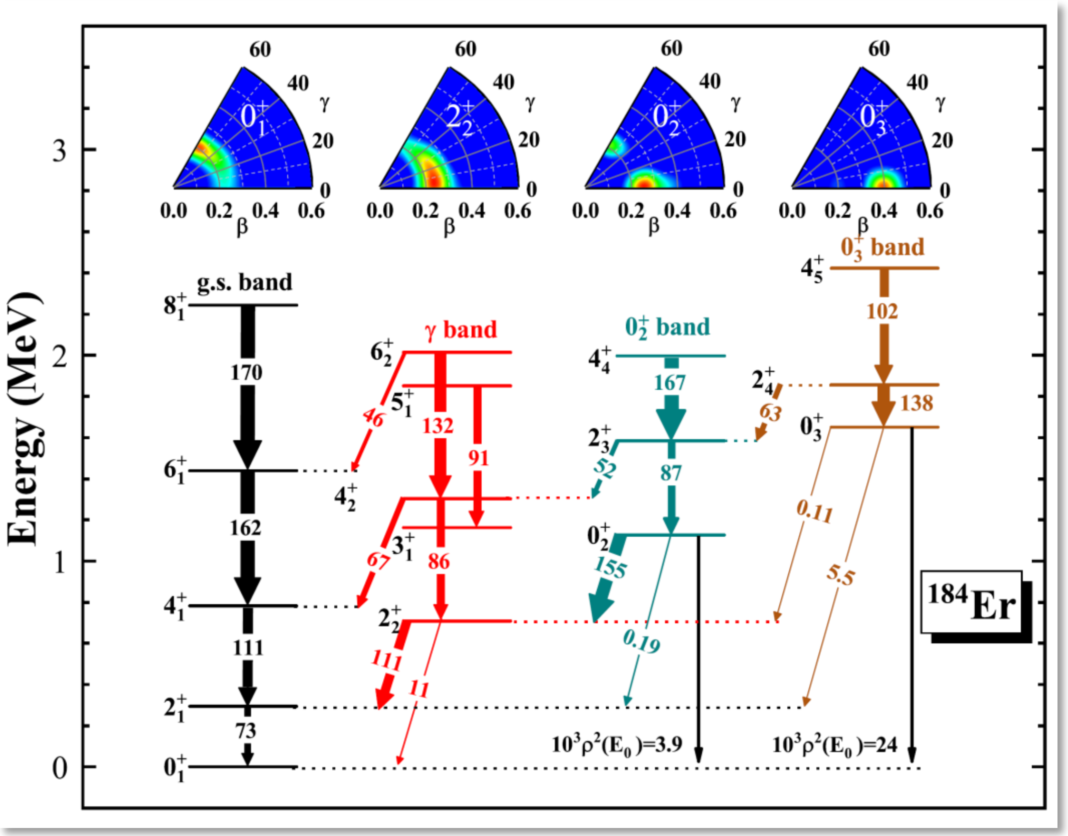


Yang, Zhao & ZPLi, PRC107, 024308 (2023)

- **Z~20, 28, 40, 50, 82; transitional regions**

Multiple shape coexistence





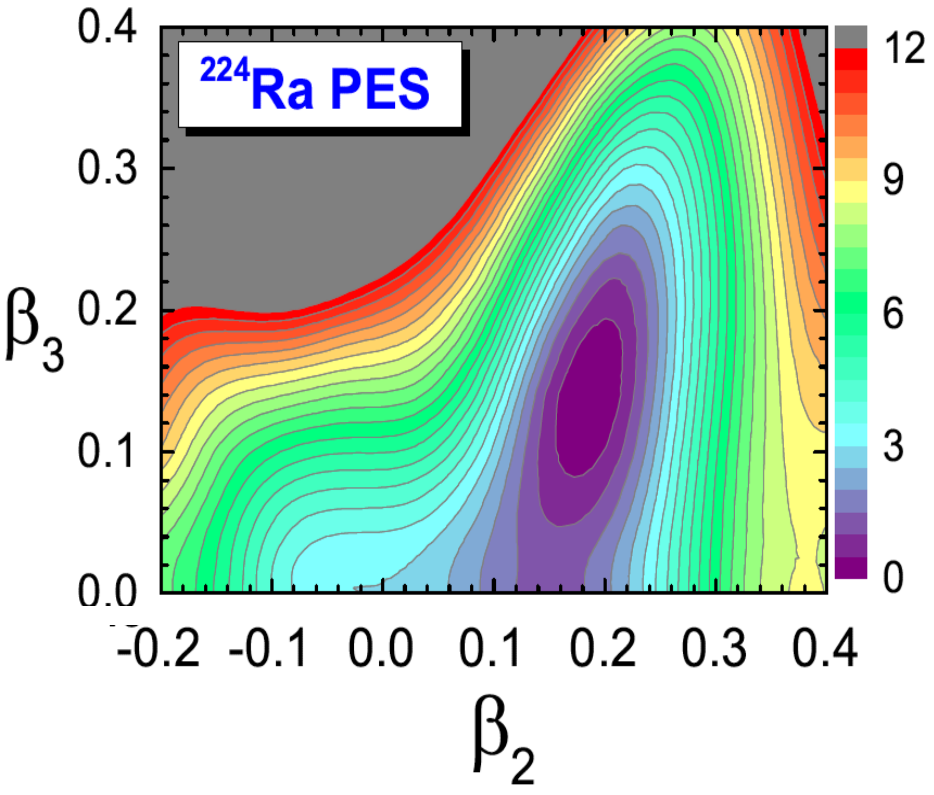
Yang et al., PRC(2021)

- ➢ **Z~20, 28, 40, 50, 82; transitional regions**

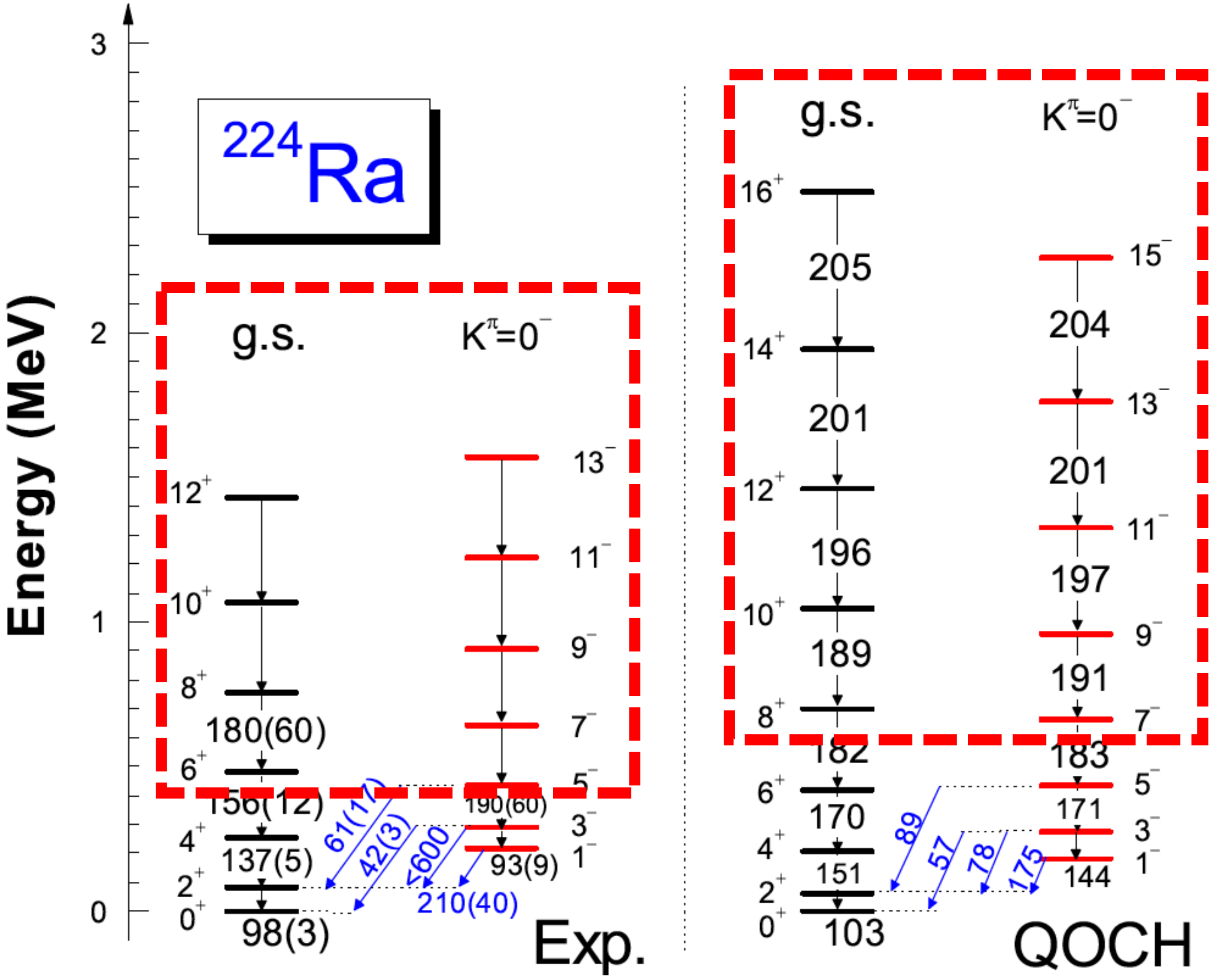
Outline

Octupole deformation in ^{224}Ra

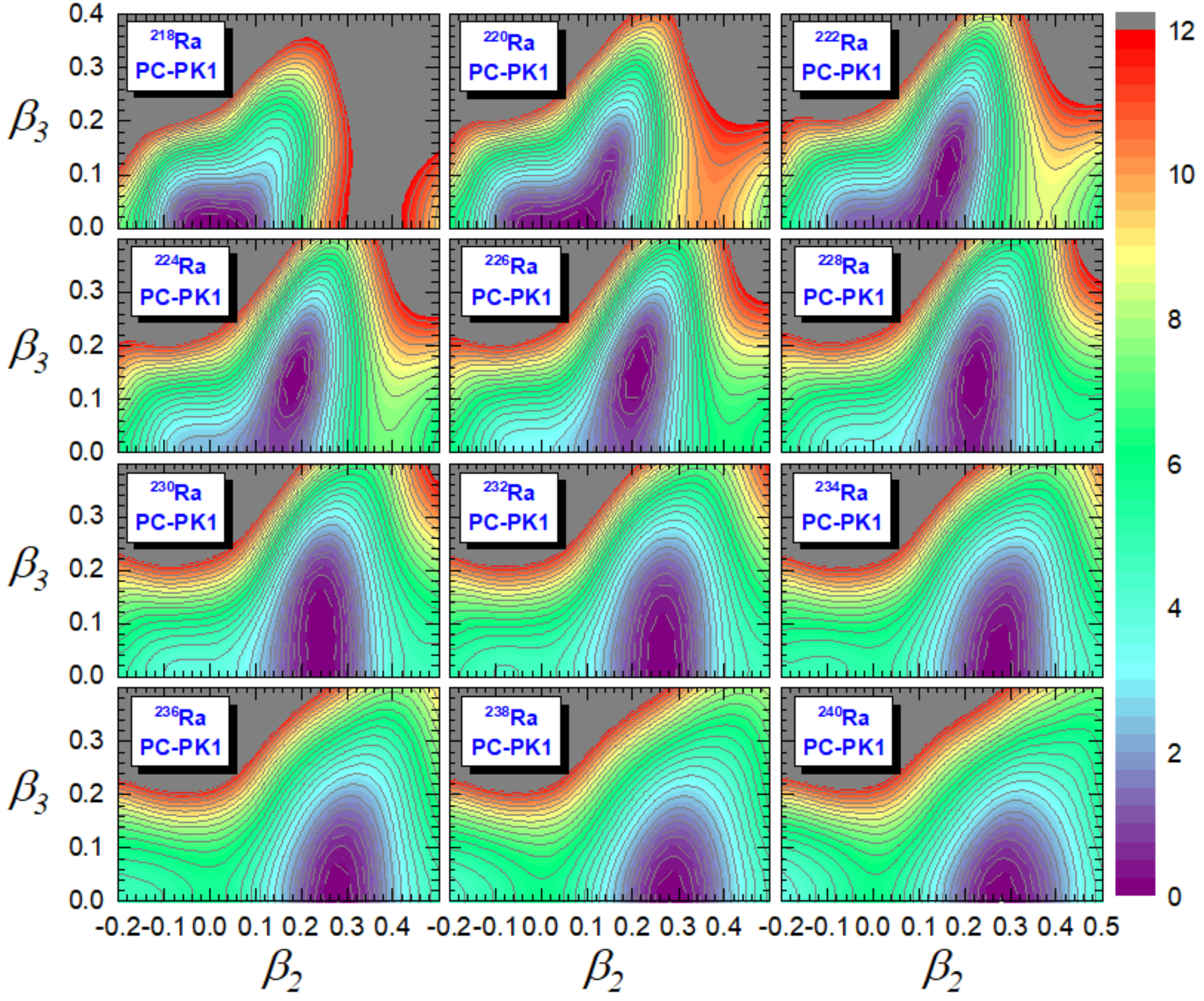
- PES of ^{224}Ra

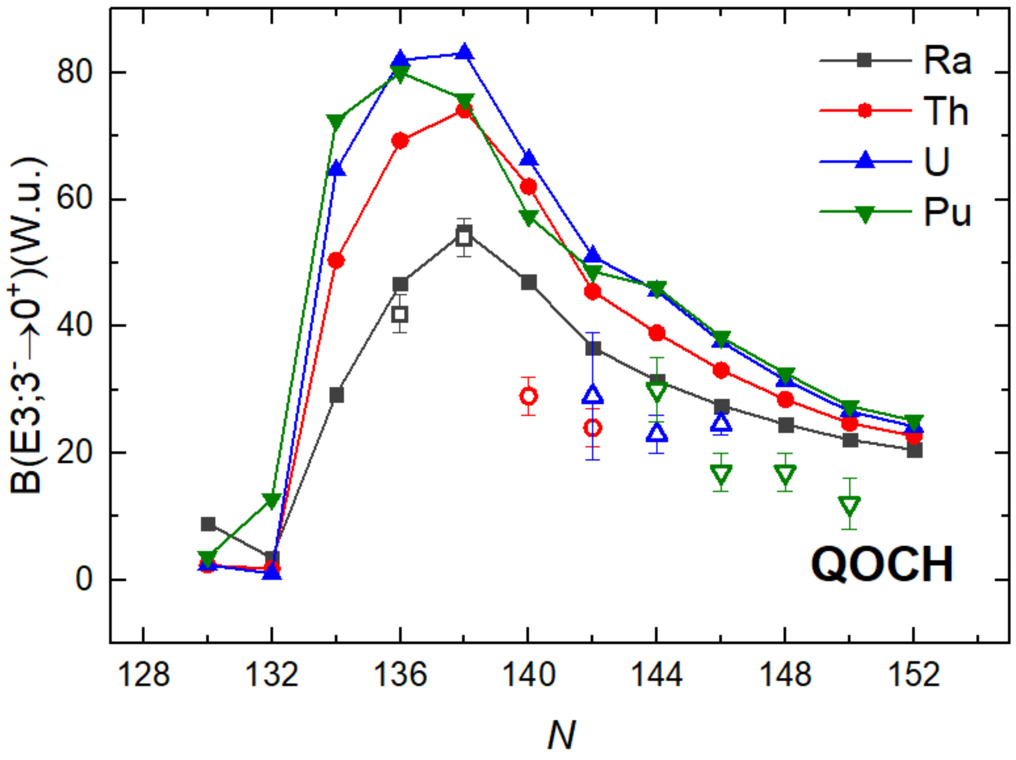


- Low-lying spectrum of ^{224}Ra



Octupole shape transition



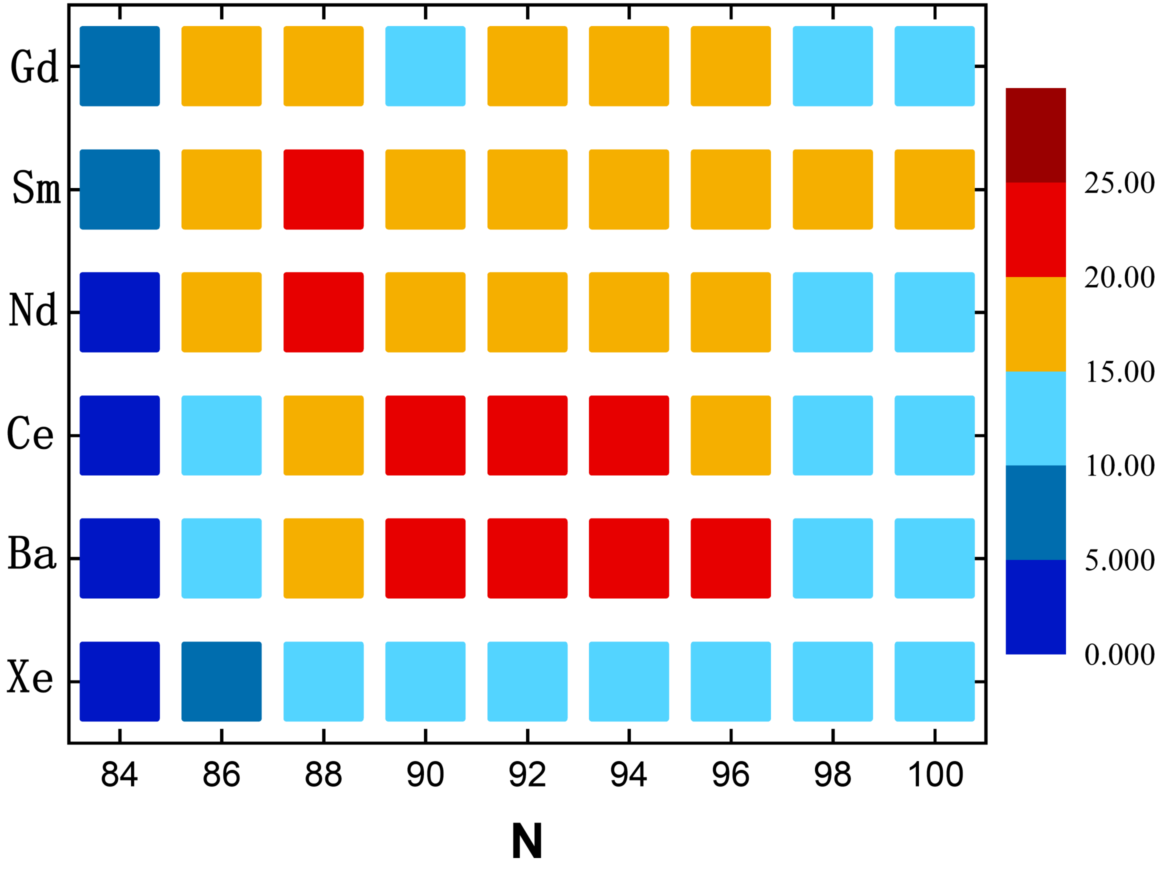


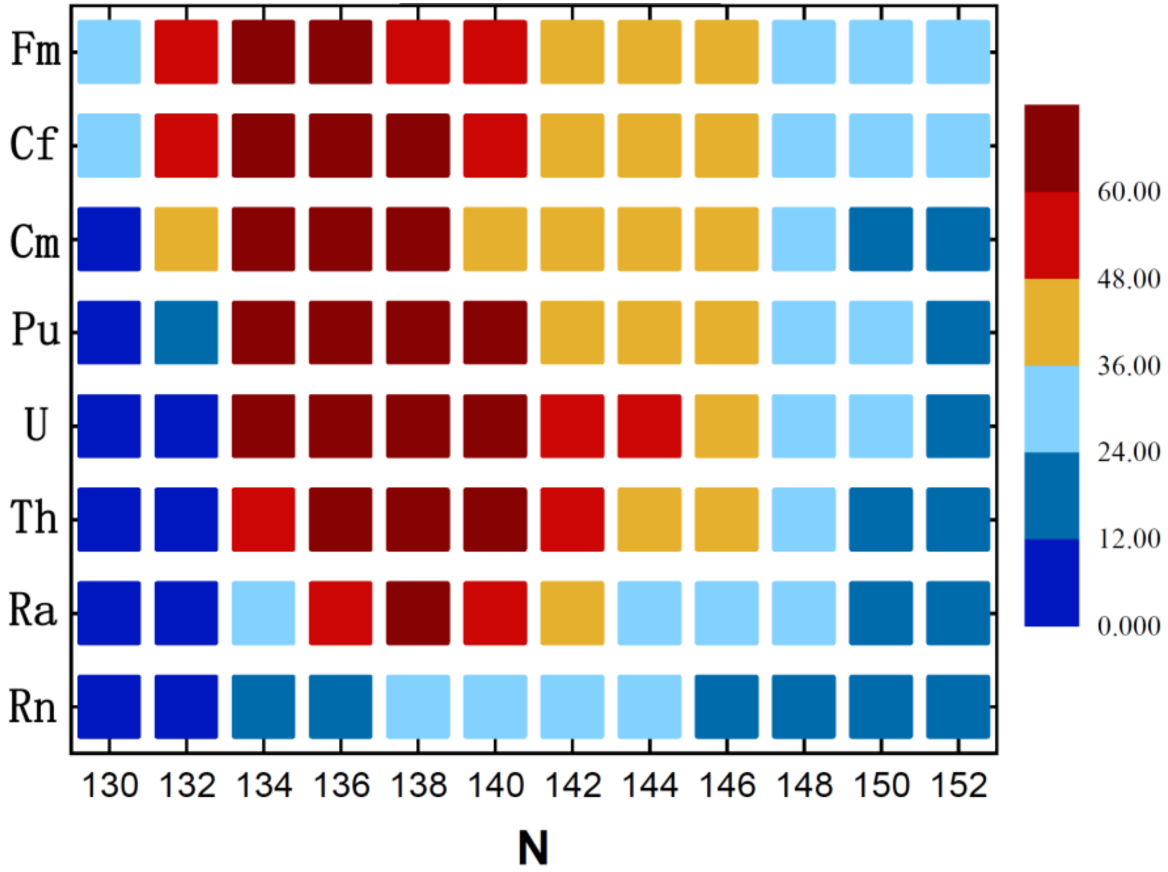
- Ra, Th, U, Pu (N=130~152)
  near sph. → octupole def.
  → octupole soft → quad. def.

- Largest B(E3) at N~136

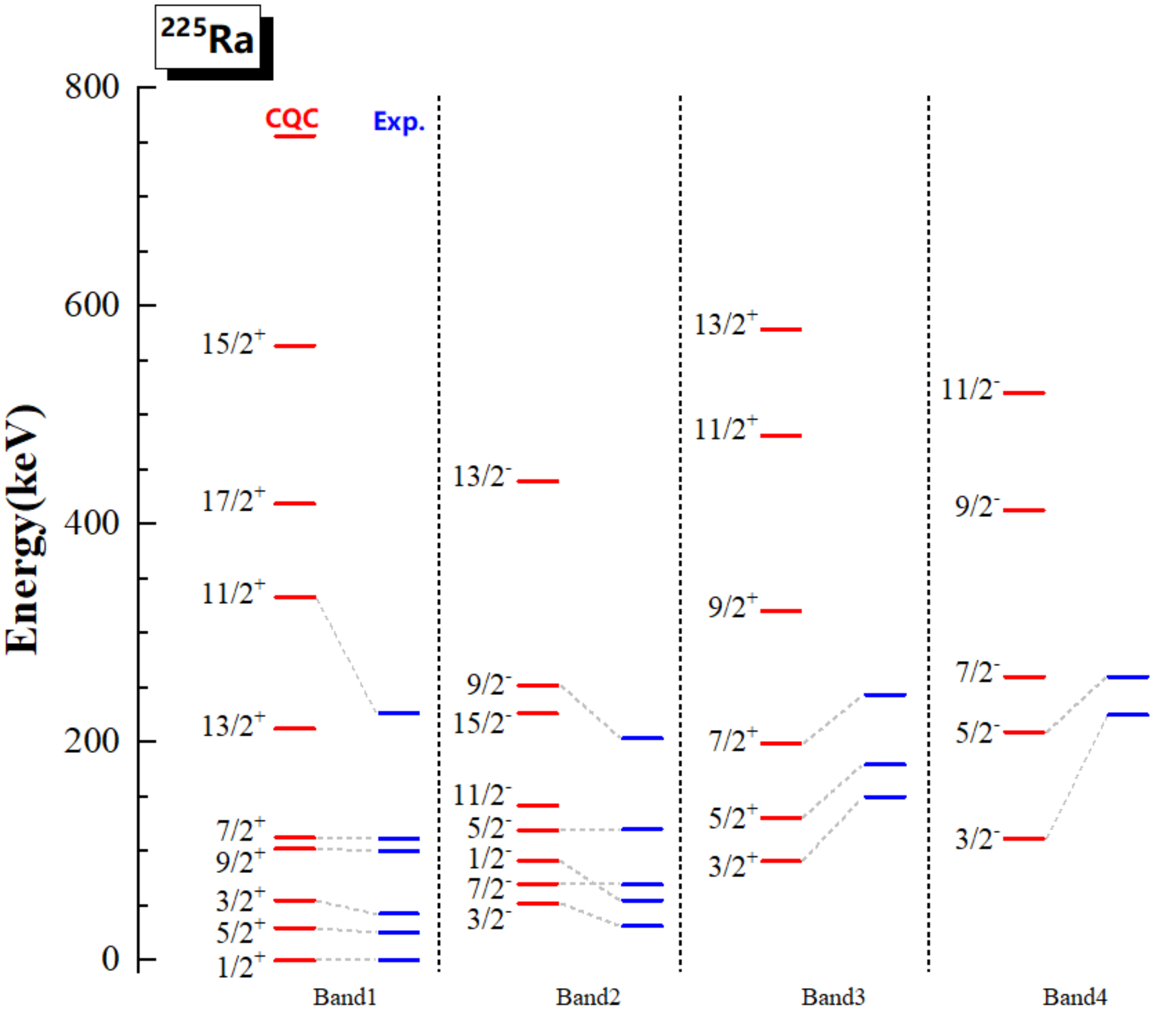
Octupole shape transition

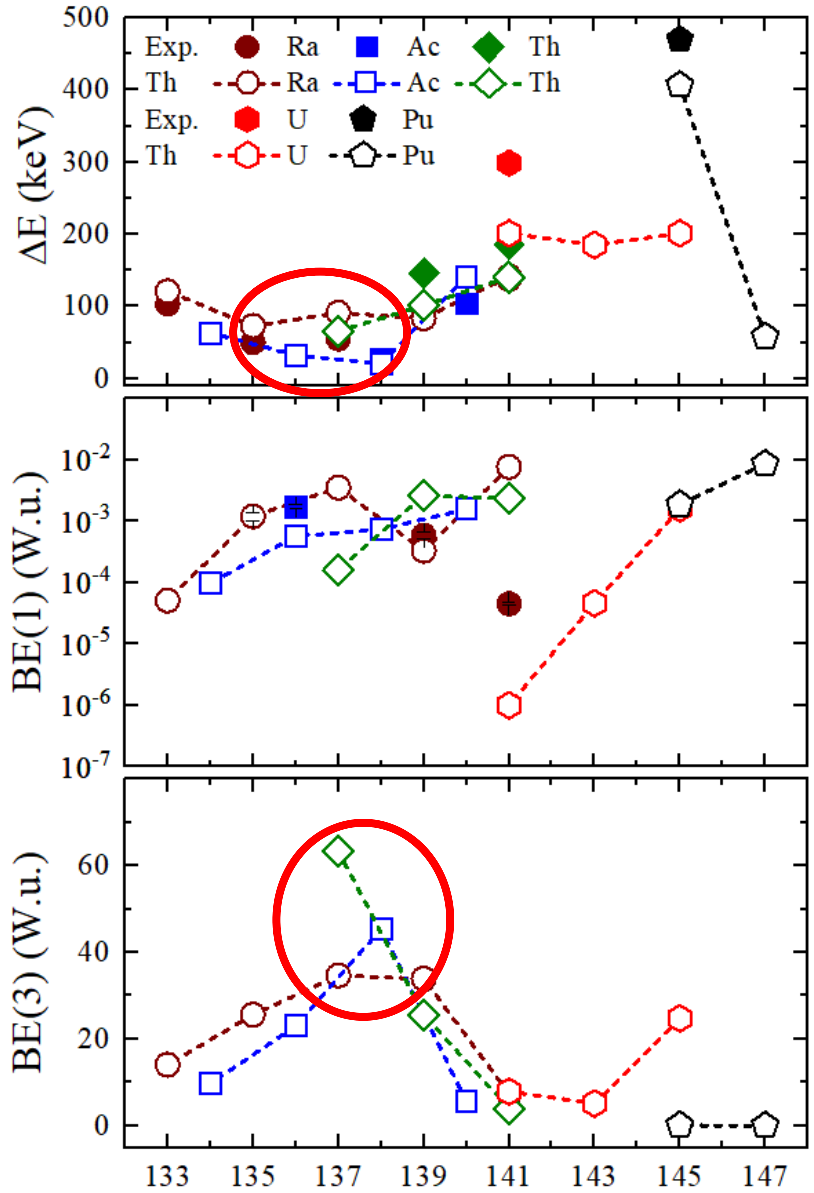
➤ B(E3;3⁻ → 0⁺)





Parity doublets in odd-A Ra and actinides

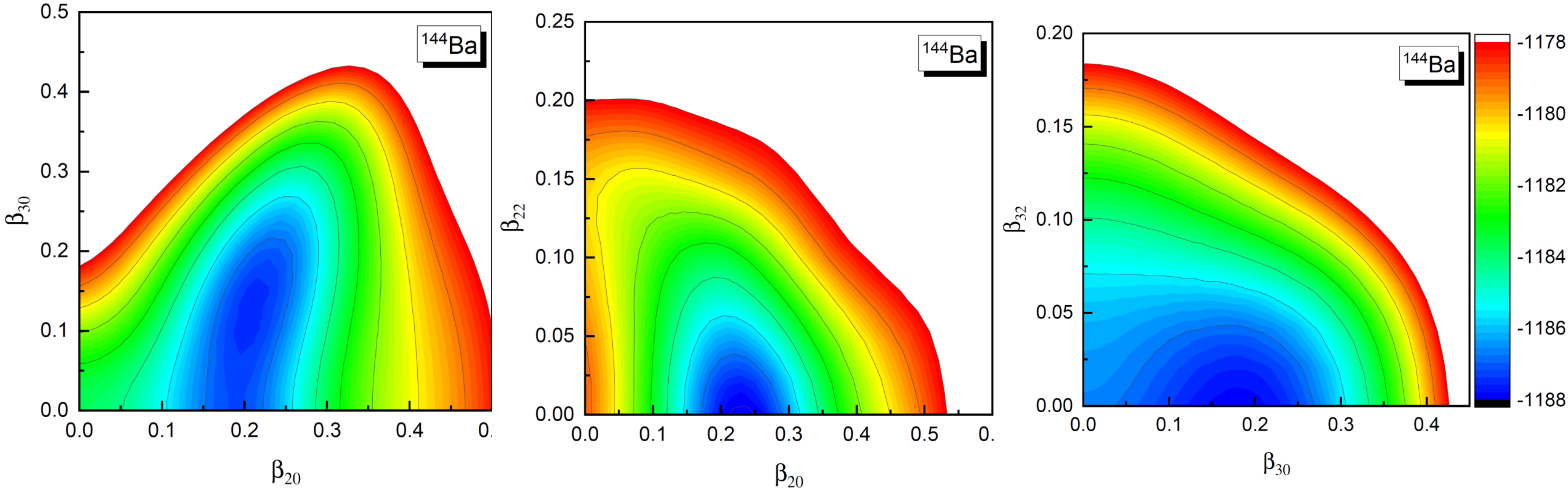




PRL 118, 152504 (2017)

➢ **Parity doublets in ^{225}Ra is reproduced.**

Triaxial octupole d.o.f.



➢ β_{30} **minimum;** β_{22}**: quite stiff** β_{32}**: quite soft**

➢ **7D collective Hamiltonian (7DCH) has been constructed, checking …**

Summary and outlook

西南大學

◆ Summary

- Global description of nuclear deformations, low-lying states and B(E2)
- Using quadrupole shape invariants to analyze shape coexistence: reproduce the known regions and **predict new regions**
- Octupole shapes around **N~90** and **N~136**; parity doublets in odd-A actinides.

◆ Outlook: triaxial octupole, chiral bands, …

Thank you for your attention!

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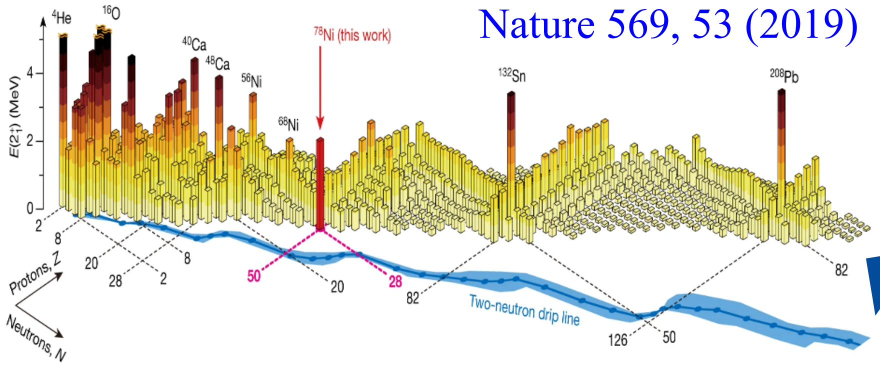
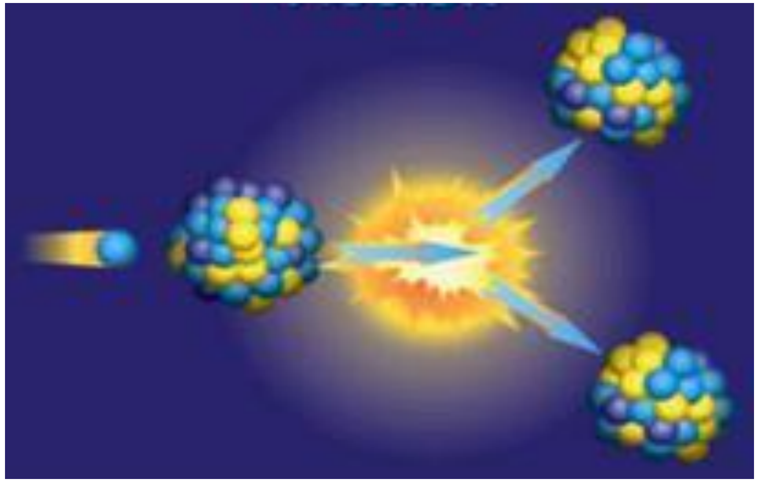
Nuclear shapes and excitations

Nature 569, 53 (2019)

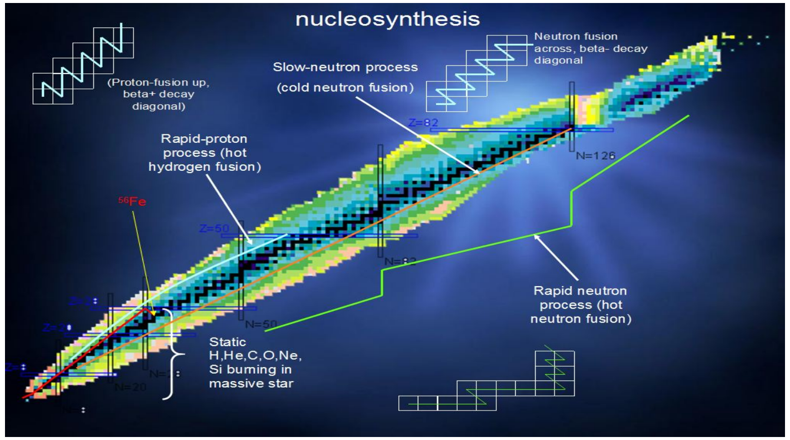


Structure: shell, shell evolution, effective interactions …

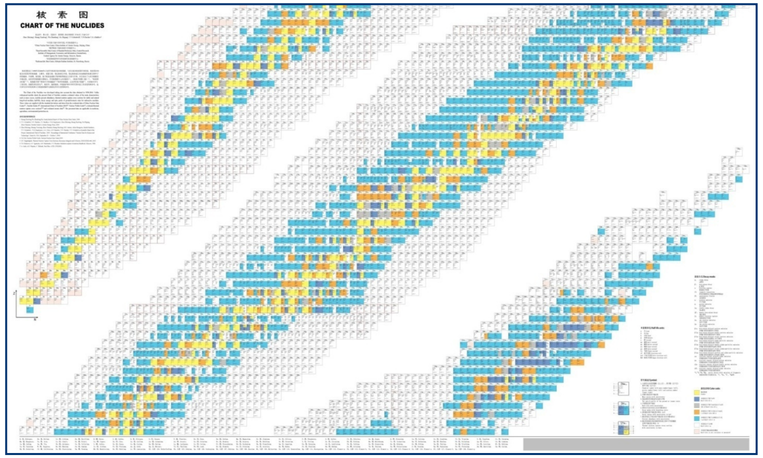


Reaction: fission, reaction dynamics …

Nuclear shapes and excitations



Astrophysics: r process, rp process, supernova …



Nuclear data, engineering