Reflection asymmetry and triaxiality in even-even atomic nuclei

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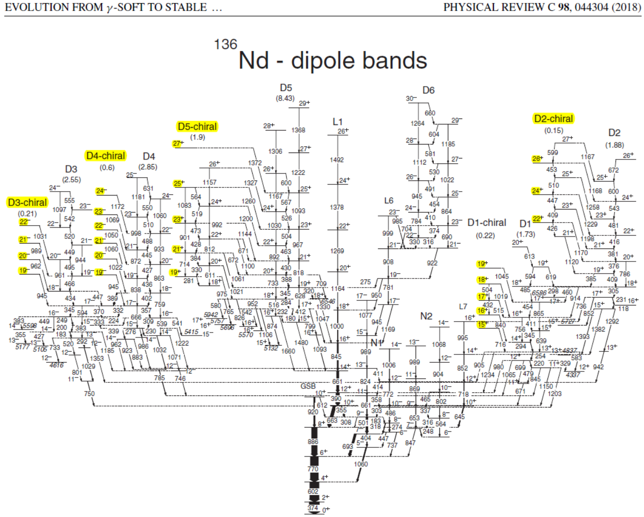
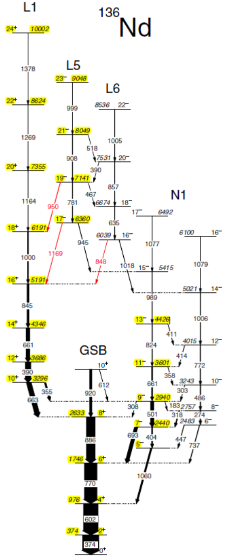
Chiral structure of experimental energy bands in ^{136}Nd



C. Petrache et al.

Alternating-parity band (APB) structures in ^{136}Nd



C. Petrache, N.M., et al. PRC **102**, 014311 (2020)

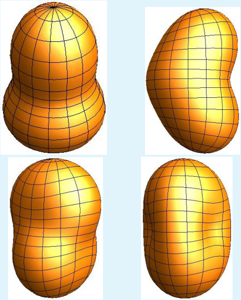
Coexistence of chiral/triaxial and octupole shapes

OBSERVATIONS:

- Chiral/triaxial and octupole shapes can coexist in certain nuclear regions.
- Octupole shapes in even-even nuclei manifest in the form of alternating-parity bands (APB).
- APBs are mainly attributed to axial (pear-shape) quadrupole-octupole (QO) deformation.

QUESTION: Could the APB indicate itself the presence of a **triaxial QO deformation**?

Quadrupole-Octupole Shapes



β_{20}=0.65 $\beta_{3\mu}$=0.35

[P. Butler and W. Nazarewicz, RMP **68**, 349 (1996)]

body-fixed coordinates

$\alpha_{\lambda\mu} \to \beta_{\lambda,\mu}$

$(\mu = 0, 1, 2, 3)$

$$R(\theta,\varphi) = R_0 \left(1 + \sum_{\lambda=2,3} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\varphi) \right)$$

Shell model origin of octupole deformation

- Coupling of orbitals with different parity and $(\Delta l = 3)$ near the Fermi level $\rightarrow (N, l, j) \otimes (N-1, l-3, j-3)$

- Particle numbers favouring strong octupole correlations

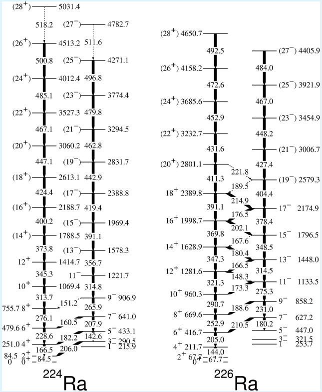
$$\mathbf{34}\ (g_{\frac{9}{2}} \otimes p_{\frac{3}{2}})\ \mathbf{56}\ (h_{\frac{11}{2}} \otimes d_{\frac{5}{2}})\ \mathbf{88}\ (i_{\frac{13}{2}} \otimes f_{\frac{7}{2}})\ \mathbf{134}\ (j_{\frac{15}{2}} \otimes g_{\frac{9}{2}})$$

$\Rightarrow$ regions of pronounced octupole deformations/collectivity

^{144}Ba $(Z = 56,\ N = 88) \Rightarrow$ Xe (Z=54) – Ba (Z=56) – Ce (Z=58)

^{222}Ra $(Z = 88,\ N = 134) \Rightarrow$ Rn (Z=86) – Ra (Z=88) – Th (Z=90)

Alternating-parity bands (APBs) at stable octupole mode



J. F. C. Cocks et al

PRL **78**, 2920 (1997)

Recent confirmations of GS octupole deformation:

^{224}Ra – L. Gaffney et al, Nature **497**, 199 (2013); P. Butler, JPG **43**, 073002 (2016)

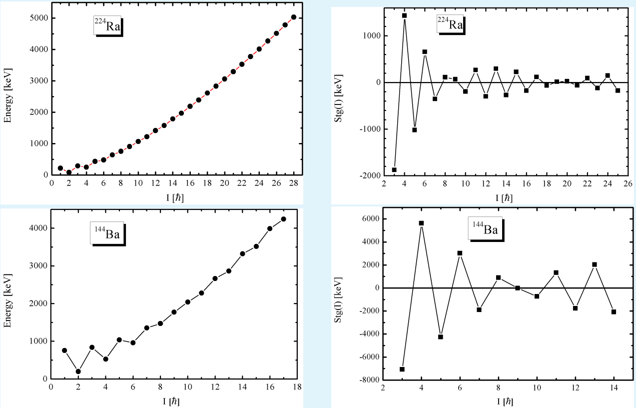
^{144,146}Ba – B. Bucher et al, PRL **116**, 112503 (2016); PRL **118**, 152504 (2017)

New data expected:

^{142}Ba, ^{222,224,226}Rn and ^{222,228}Ra – P. Butler, L. Gaffney et al – HIE-ISOLDE

^{140}Ba – T. Mertzimekis et al – IFIN-HH

Fine staggering effects in APB (octupole) bands



$\mathrm{Stg(I)} = 6\Delta E(I) - 4\Delta E(I-1) - 4\Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2)$

$\Delta E(I) = E(I+1) - E(I)$

$I < 10 - 12 \rightarrow$ octupole vibration and collective rotation

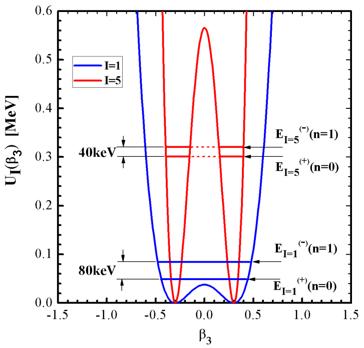
$I > 12 \rightarrow$ rotation of a stable quadrupole-octupole shape

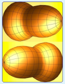
Model of octupole vibrations and quadrupole-octupole rotations

[N. M., S. Drenska, P. Yotov and W. Scheid, JPG 32, 497(2006)]

$$H_{osc}^{oct} = -\frac{\hbar^2}{2B_3}\frac{d^2}{d\beta_3{}^2} + U_I(\beta_3)$$

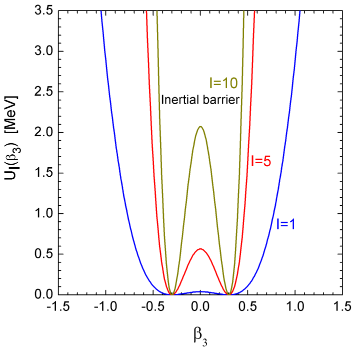
$$U(\beta_3, I) = \frac{1}{2}C(I)\beta_3{}^2 + \frac{I(I+1)}{2(d_2 + d_3\beta_3^2)} \qquad C(I) = \frac{2X(I)d_3}{(d_2 + d_3\beta_{3\ \min}^2)^2}$$





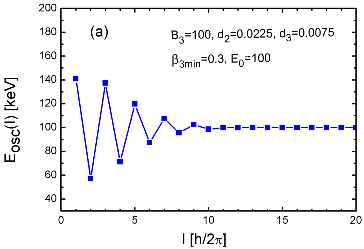
I=odd

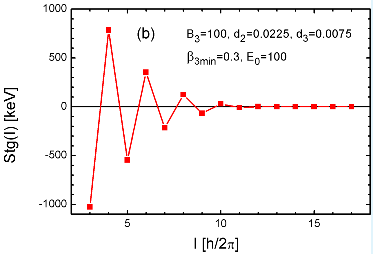
I=even



$\pi(-1)^I = 1$ $\qquad E_{vib}^{oct} = E_0 - \frac{1}{2}(-1)^I[E_I^{(-)} - E_I^{(+)}]$

Energy and parity shift from double-well potential





Low-spin parity shift effect

Parametrization of the octupole shape. Octahedron symmetry

$$V_3 = \sum_{\mu=-3}^{3} \alpha^{\text{fix}}_{3\,\mu} Y^*_{3\,\mu} \quad \rightarrow \quad \epsilon_0 A_2 + \sum_{i=1}^{3} \epsilon_1(i) F_1(i) + \sum_{i=1}^{3} \epsilon_2(i) F_2(i)$$ [I.Hamamoto et al, 1991]

$$A_2 = -\frac{i}{\sqrt{2}}(Y_{3\,2} - Y_{3\,-2}) = \frac{1}{r^3}\sqrt{\frac{105}{4\pi}}xyz$$

$$F_1(1) = Y_{3\,0} = \frac{1}{r^3}\sqrt{\frac{7}{4\pi}}z(z^2 - \frac{3}{2}x^2 - \frac{3}{2}y^2)$$

$$F_1(2) = -\frac{1}{4}\sqrt{5}(Y_{3\,3} - Y_{3\,-3}) + \frac{1}{4}\sqrt{3}(Y_{3\,1} - Y_{3\,-1})$$

$$= \frac{1}{r^3}\sqrt{\frac{7}{4\pi}}x(x^2 - \frac{3}{2}y^2 - \frac{3}{2}z^2)$$

$$F_1(3) = -i\frac{1}{4}\sqrt{5}(Y_{3\,3} + Y_{3\,-3}) - i\frac{1}{4}\sqrt{3}(Y_{3\,1} + Y_{3\,-1})$$

$$= \frac{1}{r^3}\sqrt{\frac{7}{4\pi}}y(y^2 - \frac{3}{2}z^2 - \frac{3}{2}x^2)$$

$$F_2(1) = \frac{1}{\sqrt{2}}(Y_{3\,2} + Y_{3\,-2}) = \frac{1}{r^3}\sqrt{\frac{105}{16\pi}}z(x^2 - y^2)$$

$$F_2(2) = \frac{1}{4}\sqrt{3}(Y_{3\,3} - Y_{3\,-3}) + \frac{1}{4}\sqrt{5}(Y_{3\,1} - Y_{3\,-1}) = \frac{1}{r^3}\sqrt{\frac{105}{16\pi}}x(y^2 - z^2)$$

$$F_2(3) = -i\frac{1}{4}\sqrt{3}(Y_{3\,3} + Y_{3\,-3}) + i\frac{1}{4}\sqrt{5}(Y_{3\,1} + Y_{3\,-1}) = \frac{1}{r^3}\sqrt{\frac{105}{16\pi}}y(z^2 - x^2)$$

Quadrupole-octupole rotation Hamiltonian. Octahedron symmetry

[N. M. et al, PRC **63**, 044305 (2001)]

$$\hat{H}_{\text{qorm}} = \hat{H}_{\text{quad}} + \hat{H}_{\text{oct}} + \hat{H}_{\text{qoc}}$$

$$\hat{H}_{\text{oct}} = \hat{H}_{A_2} + \sum_{r=1}^{2}\sum_{i=1}^{3}\hat{H}_{F_r(i)}$$

$$\hat{H}_{A_2} = a_2\frac{1}{4}[(\hat{I}_x\hat{I}_y + \hat{I}_y\hat{I}_x)\hat{I}_z + \hat{I}_z(\hat{I}_x\hat{I}_y + \hat{I}_y\hat{I}_x)]$$

$$\hat{H}_{F_1(1)} = \frac{1}{2}f_{11}(5\hat{I}_z^3 - 3\hat{I}_z\hat{I}^2) \qquad \hat{H}_{F_2(1)} = f_{21}\frac{1}{2}[\hat{I}_z(\hat{I}_x^2 - \hat{I}_y^2) + (\hat{I}_x^2 - \hat{I}_y^2)\hat{I}_z]$$

$$\hat{H}_{F_1(2)} = \frac{1}{2}f_{12}(5\hat{I}_x^3 - 3\hat{I}_x\hat{I}^2) \qquad \hat{H}_{F_2(2)} = f_{22}(\hat{I}_x\hat{I}^2 - \hat{I}_x^3 - \hat{I}_x\hat{I}_z^2 - \hat{I}_z^2\hat{I}_x)$$

$$\hat{H}_{F_1(3)} = \frac{1}{2}f_{13}(5\hat{I}_y^3 - 3\hat{I}_y\hat{I}^2) \qquad \hat{H}_{F_2(3)} = f_{23}(\hat{I}_y\hat{I}_z^2 + \hat{I}_z^2\hat{I}_y + \hat{I}_y^3 - \hat{I}_y\hat{I}^2)$$

$$\hat{H}_{\text{quad}} = A\hat{I}^2 + A'\hat{I}_z^2 + C'(\hat{I}_x^2 - \hat{I}_y^2) \qquad \hat{H}_{\text{qoc}} = f_{\text{qoc}}\frac{1}{I^2}(15\hat{I}_z^5 - 14\hat{I}_z^3\hat{I}^2 + 3\hat{I}_z\hat{I}^4)$$

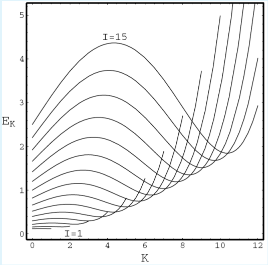
Quadrupole-octupole rotation Hamiltonian. Octahedron symmetry

$$\hat{H}_{\text{qorm}} = \hat{H}_{\text{quad}} + \hat{H}_{\text{oct}} + \hat{H}_{\text{qoc}}$$

$$\hat{H}_{\text{oct}} = \hat{H}_{A_2} + \sum_{r=1}^{2}\sum_{i=1}^{3}\hat{H}_{F_r(i)}$$

Point-symmetry contents: $\hat{H}_{F_1(1)} \to Y_{30} \to \mathrm{D}_\infty$; $\hat{H}_{F_1(2)} \to (Y_{3\,1} - Y_{3\,-1}) \to \mathrm{C}_{2v}$; $\hat{H}_{F_2(1)} \to (Y_{3\,2} + Y_{3\,-2}) \to \mathrm{T}_d$; $\hat{H}_{F_2(2)} \to (Y_{3\,3} - Y_{3\,-3}) \to \mathrm{D}_{3h}$ [Octahedron irreps]

$$\begin{aligned} E_K^{\text{qorm}}(I) &= AI(I+1) + A'K^2 + (1/2)f_{11}\left[5K^3 - 3KI(I+1)\right] \\ &+ f_{\text{qoc}}(1/I^2)\left[15K^5 - 14K^3I(I+1) + 3KI^2(I+1)^2\right] \end{aligned}$$



$K_{\min}$ → **high-spin “beat” staggering effect**
[N. M. et al, PRC **63**, 044305 (2001); JPG **32**, 497(2006)]

Total QO rotation and vibration energy

⇒ Octupole vibration and quadrupole-octupole rotation

$$\boxed{E_{\text{coll}} = E_{\text{vib}}^{\text{oct}} + E_{\text{qorm}}}$$

⇒ **strong parity shift effect** at low angular momenta

⇒ **beat staggering patterns** in the higher energy regions

Probing QORM description of ^{136}Nd APB [C. Petrache, N.M. et al. PRC 102, 014311 (2020)]

TABLE III. Experimental and QORM-calculated energies of the levels of the GSB (positive parity) and band $N1$ (negative parity) of ^{136}Nd.

I^π	E_{theory} (keV)	E_{exp} (keV)	$E_{\text{theory}}-E_{\text{exp}}$ (keV)
1^-	1740	–	–
2^+	367	374	−7
3^-	1945	–	–
4^+	985	976	9
5^-	2057	2036	21
6^+	1776	1746	30
7^-	2408	2440	−32
8^+	2540	2633	−93
9^-	2978	2940	38
10^+	3276	–	–
11^-	3666	3601	65
12^+	4013	–	–
13^-	4396	4426	−30

the model and its application are given in Ref. [32]). We chose

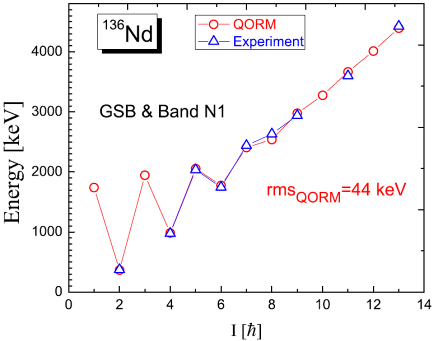
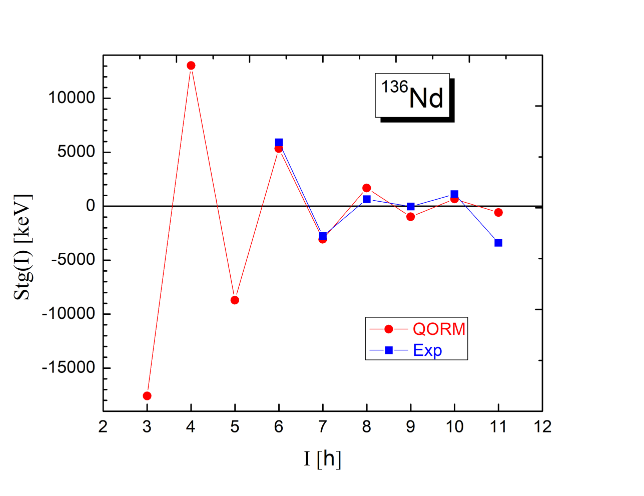
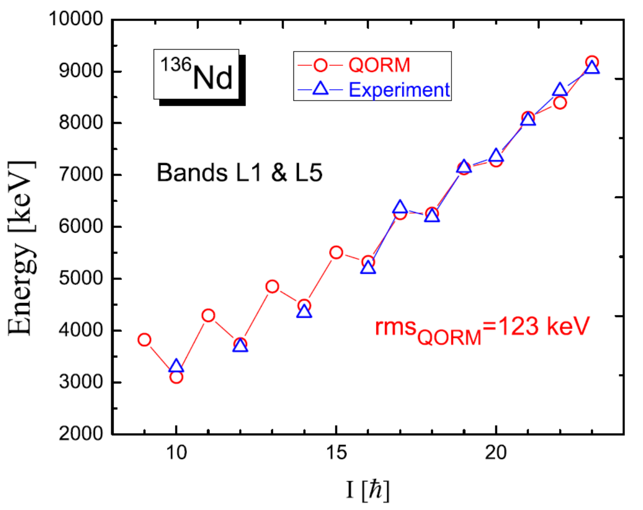


FIG. 6. QORM description of ^{136}Nd APB formed by GSB up to $I = 8^+$ and band $N1$ up to $I = 13^-$. See the text for the model parameters and explanation.

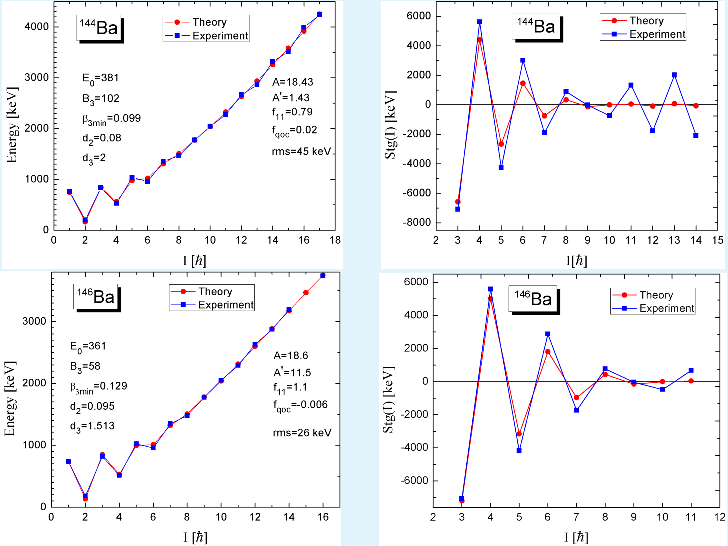
Experimental and theoretical staggering patterns for ^{136}Nd APB



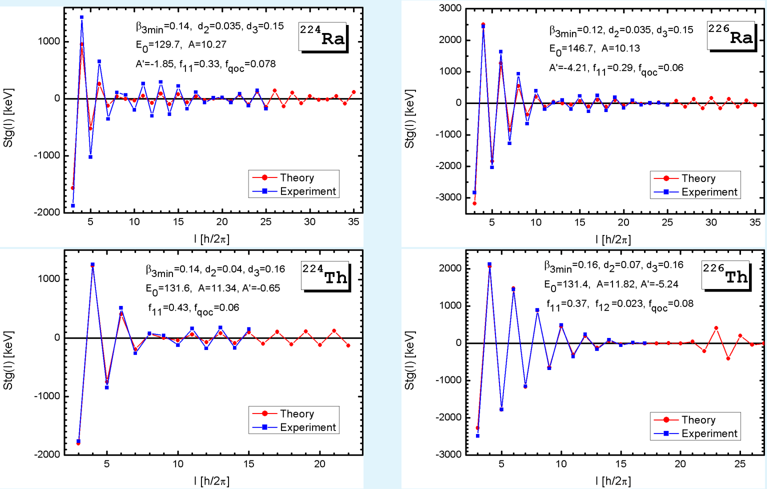
Experimental and theoretical levels of the non-yrast APB in ^{136}Nd



^{144,146}Ba: QORM description

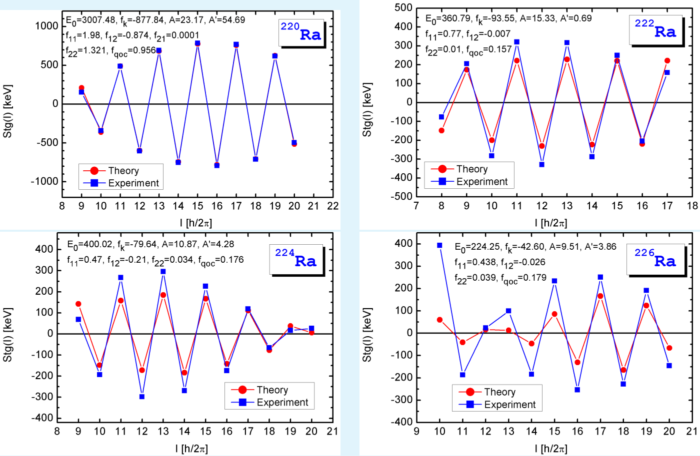


“Beat” staggering patterns in Ra and Th nuclei



[N. M., P. Yotov, S. Drenska and W. Scheid, JPG **32**, 497 (2006)]

QORM application to high-spin APB regions ⇒ QO triaxiality



- ✓ **Well reproduced high-spin beat staggering effect**
- ✓ **Rotation of stable quadrupole-octupole shape**
- ✓ **Contribution of non-axial Hamiltonian terms**

Conclusion

- ^{136}Nd data: Possible coexistence of quadrupole triaxial/chiral and quadrupole-octupole collective degrees of freedom.
- QORM description of ^{136}Nd APB: Support of its quadrupole-octupole character.
- QORM description of high-spin APB structure in Ra isotopes: Indications for high-spin quadrupole-octupole triaxiality.
- Need for detailed study of high-spin APB structure in wider range of nuclei.
- Need for unified study of triaxiality and reflection asymmetry.