山东大学(威海) SHANDONG UNIVERSITY, WEIHAI

空间科学与物理学院
School of Space Science and Physics

Theoretical studies on the chirality and wobbling in SDU

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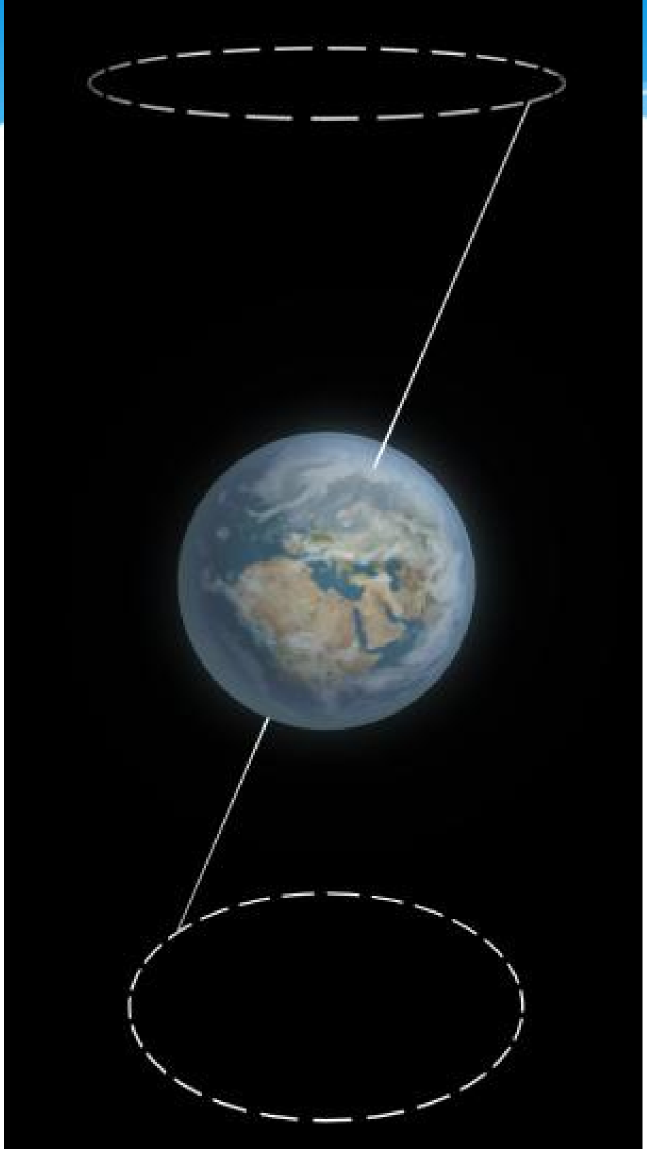
Outline

Wobbling

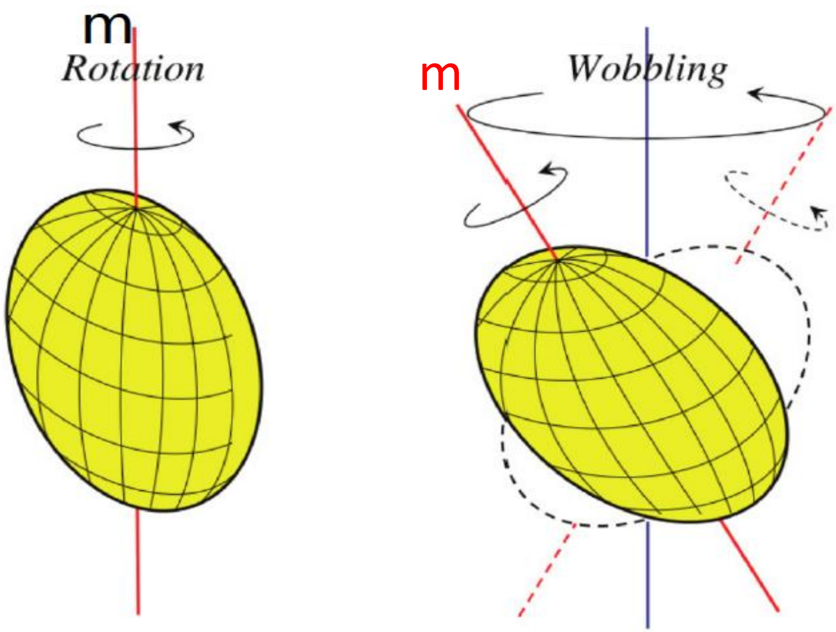
- in even-even nulclei:
  - Precession and tunneling
  - Quadrupole-octupole deformed nucleus
- in odd-A nuclei：
  - Transverse mode: sensitive to MOI
- in odd-odd nuclei :
  - Coexistence of Chirality and Wobbling

Even-Even nuclei



Precession(进动)

the angle of Precession is changing --Nutation（章动）

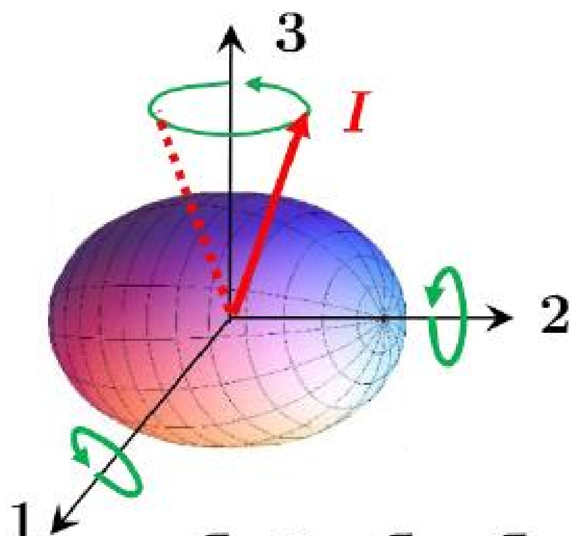


Rotational angular momentum for a triaxial nucleus is not aligned along the axis with the largest moment of inertia, but precesses and wobbles

A. Bohr and B. R. Mottelson, Nuclear Structure Vol. II. (1975)

Even-Even nuclei

In a triaxial deformed even-even nucleus



$\mathcal{J}_3 \gg \mathcal{J}_1, \mathcal{J}_2$

$$\hat{H}_{\text{rot}} = \frac{\hat{I}_1^2}{2\mathcal{J}_1} + \frac{\hat{I}_2^2}{2\mathcal{J}_2} + \frac{\hat{I}_3^2}{2\mathcal{J}_3}$$

Considering the **approximation**

$$[I_-, I_+] = 2I_3 \approx 2I \qquad (I_\pm = I_2 \pm iI_1)$$

$$E(I, \boldsymbol{n}) = \frac{I(I+1)}{2\mathcal{J}_3} + (\boldsymbol{n} + \frac{1}{2})\hbar\Omega_{\text{wob}}$$

$$\hbar\Omega_{\text{wob}} = 2I\sqrt{\left(\frac{\hbar^2}{2\mathcal{J}_1} - \frac{\hbar^2}{2\mathcal{J}_3}\right)\left(\frac{\hbar^2}{2\mathcal{J}_2} - \frac{\hbar^2}{2\mathcal{J}_3}\right)} \propto I$$

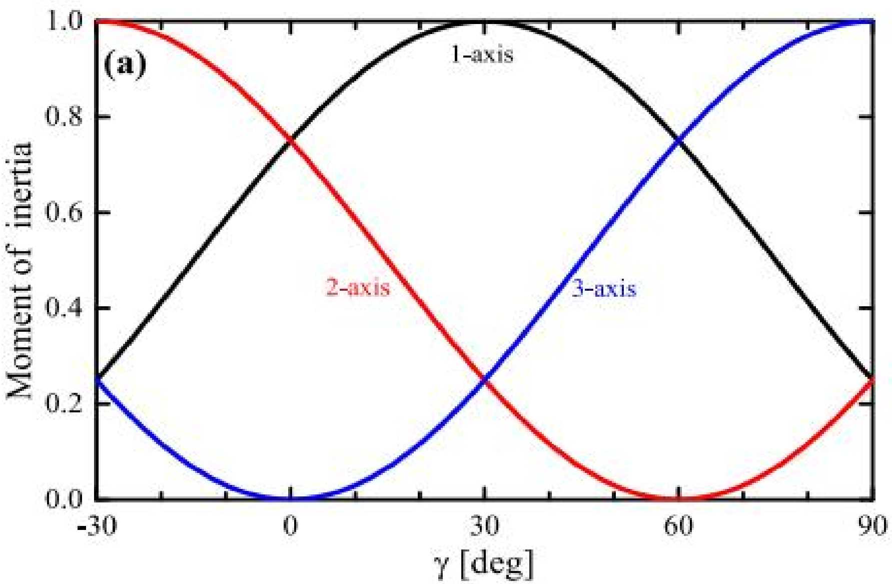
Harmonic approximation (HA)

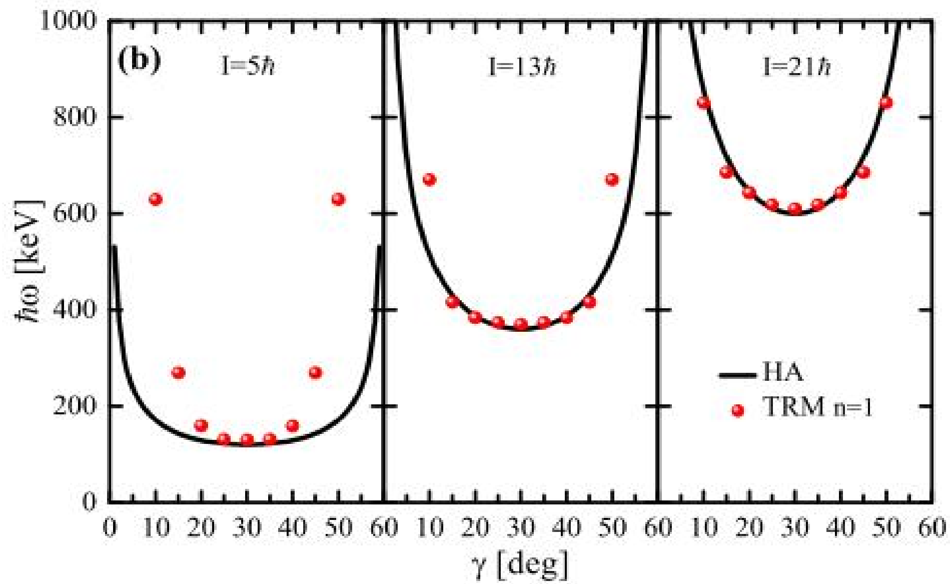
$$\boldsymbol{I} \uparrow, \quad \boldsymbol{\hbar\Omega_{\text{wob}}} \uparrow$$

A. Bohr and B. R. Mottelson, Nuclear Structure Vol. II. (1975)

Even-Even nuclei

$$\mathcal{J}_k = \mathcal{J}_0 \sin^2(\gamma - \frac{2}{3}\pi k)$$





Hydrodynamical MoI of the three principal axes as functions of triaxial parameter γ. The unit is taken as J_0.

$$\hbar\omega = I\left[\left(\frac{1}{\mathcal{J}_2} - \frac{1}{\mathcal{J}_1}\right)\left(\frac{1}{\mathcal{J}_3} - \frac{1}{\mathcal{J}_1}\right)\right]^{1/2}$$

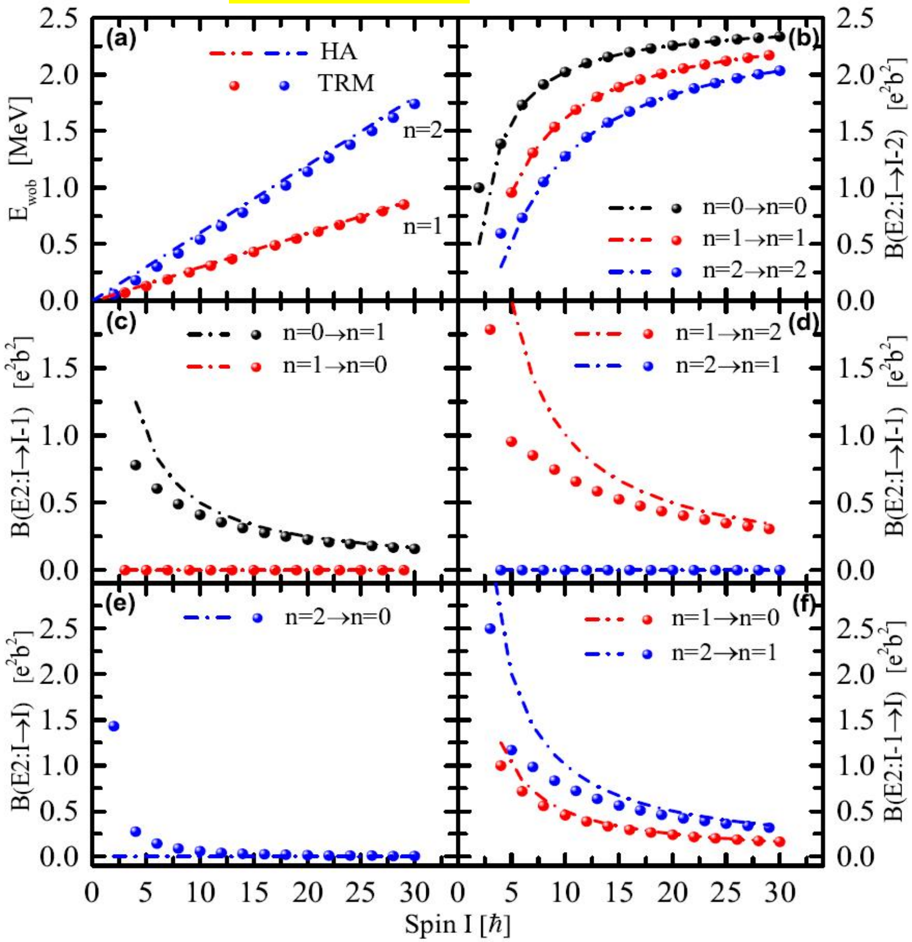
wobbling frequency as a function of γ calculated by HA equation (Line) and triaxial rotor model (Dot)

$$E_{\text{wob}} = E(n,I) - \frac{1}{2}[E(0,I-1) + E(0,I+1)]$$

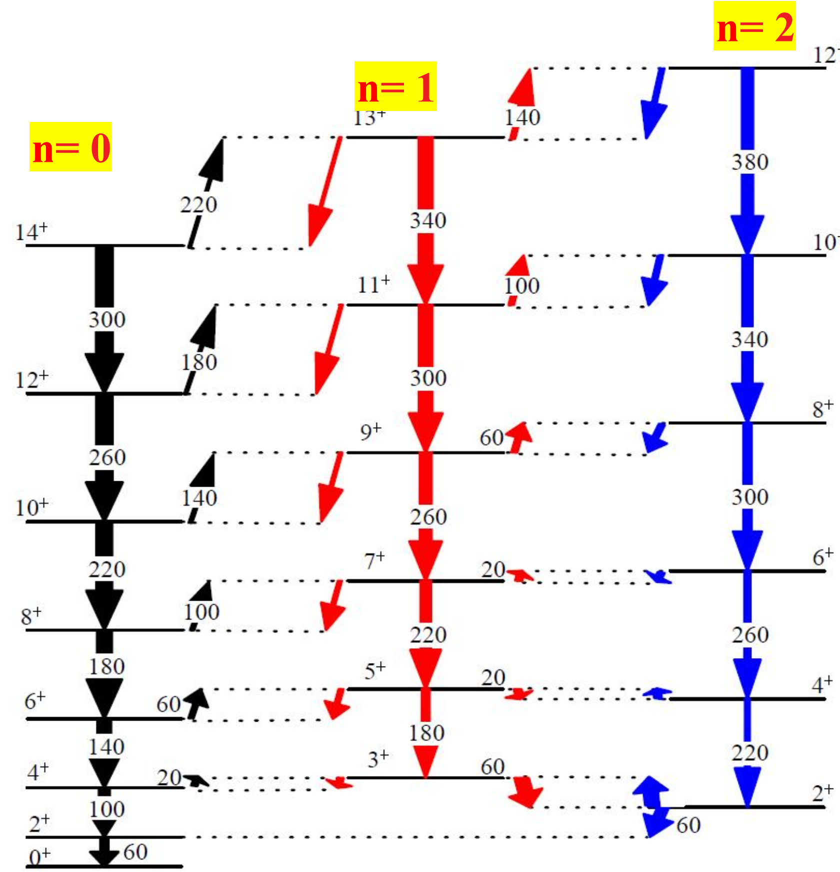
n: phonon number , here n=1

Even-Even nuclei

γ=30°



wobbling energies, intraband and interband B(E2) values calculated by HA (Line) and rotor model (Dot)

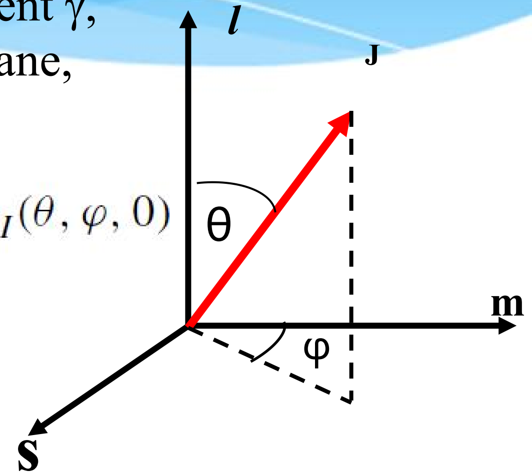


Energy level scheme calculated by the rotor model for ground band and n = 1, 2 wobbling bands

*BQ**, Zhang, Wang, Q.B.Chen*, JPG 48 (2021) 055102

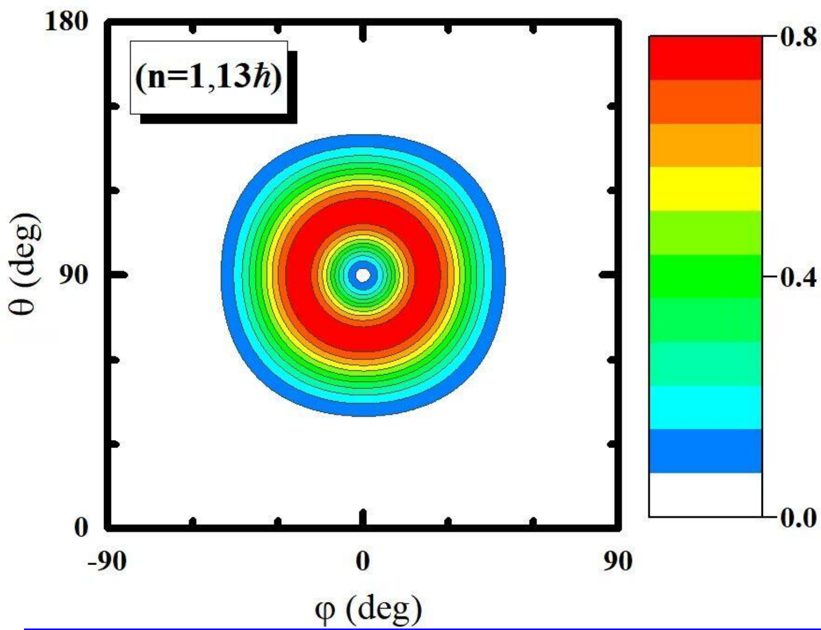
Even-Even nuclei

To further illustrate the angular momentum geometry with different γ, the probability distribution of angular momentum on the (θ,ϕ) plane, i.e., azimuthal plot, is calculated.

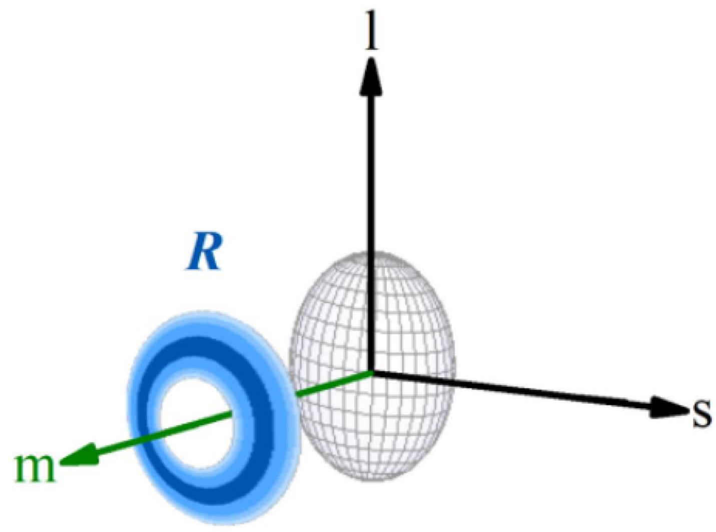


$$\mathcal{P}^{(\nu)}(\theta,\varphi) = \langle I,\theta\varphi \mid I I \nu\rangle^2 = \frac{2I+1}{8\pi}\sum_{KK'} D^{I*}_{KI}(\theta,\varphi,0)\rho^{(\nu)}_{KK'}D^{I}_{K'I}(\theta,\varphi,0)$$

S. Frauendorf, Report on "International conference on Chiral bands in nuclei, KTH, Stockholm", 2016.04
F. Q. Chen, Q. B. Chen, et al, PRC 96, 051303(R) (2017).
Q. B. Chen and J. Meng, PRC 98, 031303(R) (2018).



probability distribution of angular momentum



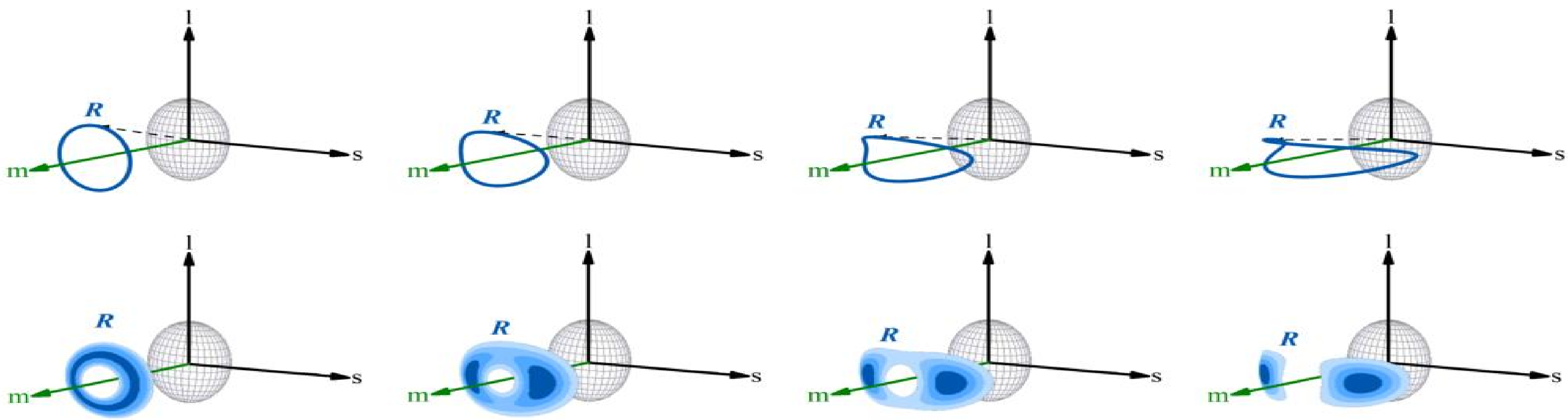
Schematic picture

Even-Even nuclei

Classical trajectory (upper panels) and quantum probability density distribution (lower panels) of angular momentum for a triaxial rotor at spin 13 $\hbar$ with differnent MOI (i.e., different γ).

Precession in classical mechnism ↔ intersection of

$$\begin{cases} R_m^2 + R_s^2 + R_l^2 = R(R+1) & \text{angular mom. sphere} \\ R_m^2/\mathcal{J}_m + R_s^2/\mathcal{J}_s + R_l^2/\mathcal{J}_l = 2E & \text{energy ellipsoid} \end{cases}$$



(a) (b) (c) (d)

$\mathcal{J}_m/\mathcal{J}_s/\mathcal{J}_l$: (a) 4/1/1 ($\gamma = 30^\circ$); (b) 5.6/1.8/1 ($\gamma = 25^\circ$); (c) 8.3/3.5/1 ($\gamma = 20^\circ$); (d) 13.9/7.5/1 ($\gamma = 15^\circ$)

Precession and tunneling are two aspects of the quantum wobbling motion.

Zhang, *BQ*, Wang, Jia, Wang, PRC 105, 034339 (2022)**

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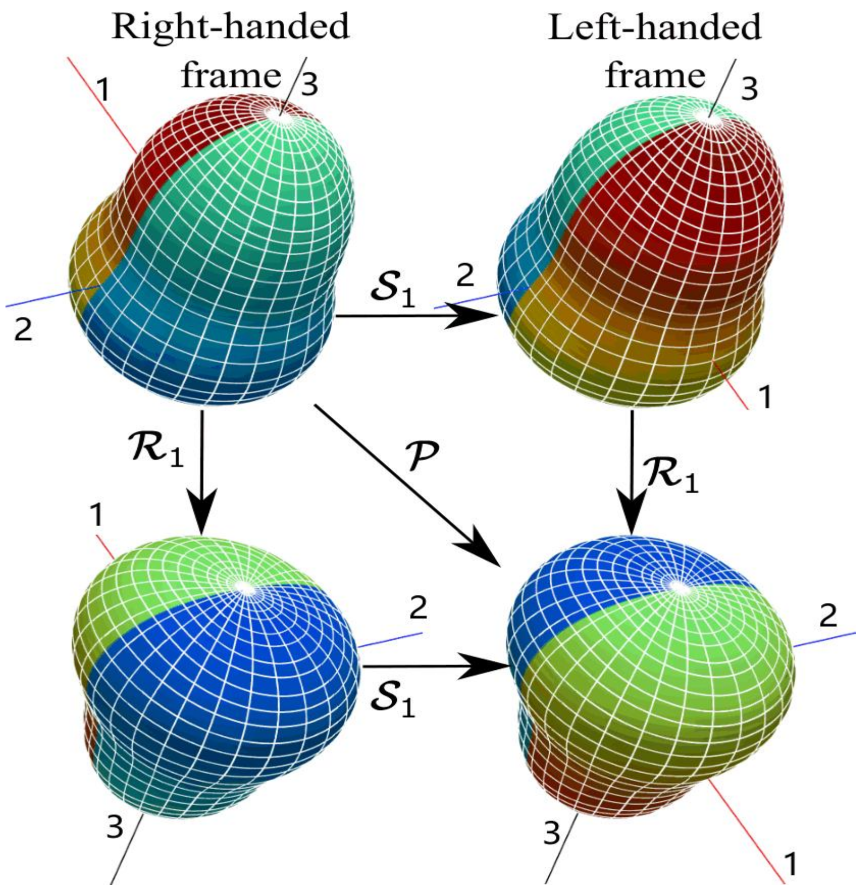
Even-Even nuclei

$Y_{20}+Y_{22}+Y_{30}$



- quadrupole-octupole deformed nucleus
- P: space reflection,
- R_1 : rotation through angle π about 1-axis,
- S_1: reflection with respect to the 2-3 plane.

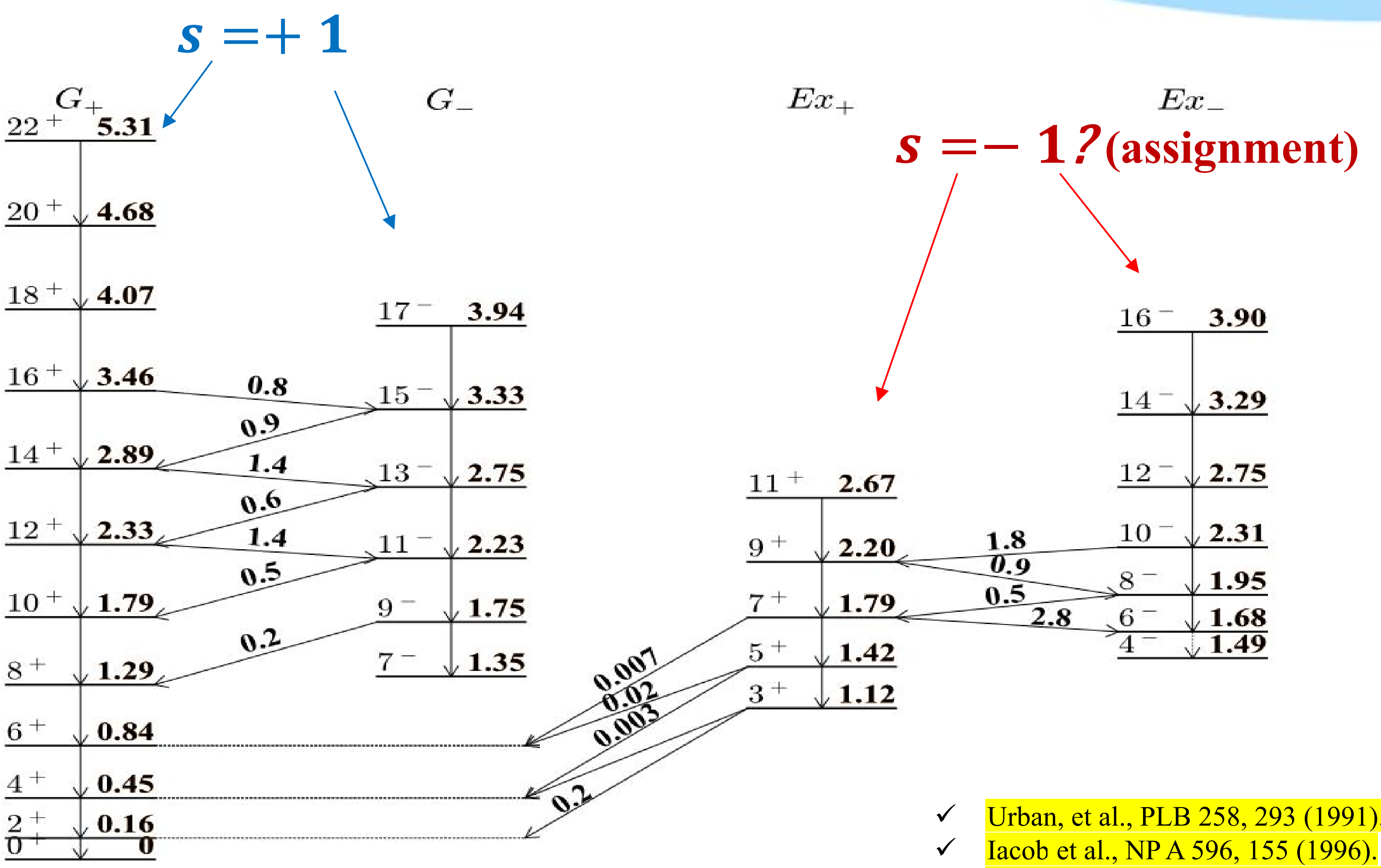
multiplication table of C_{2v}

	E	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{R}_3$
E	E	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{R}_3$
$\mathcal{S}_1$	$\mathcal{S}_1$	E	$\mathcal{R}_3$	$\mathcal{S}_2$
$\mathcal{S}_2$	$\mathcal{S}_2$	$\mathcal{R}_3$	E	$\mathcal{S}_1$
$\mathcal{R}_3$	$\mathcal{R}_3$	$\mathcal{S}_2$	$\mathcal{S}_1$	E

Symmetry of nuclear density distribution: C_{2v} point group

Even-Even nuclei

Experimental data in ^{148}Ce



- ✓ Urban, et al., PLB 258, 293 (1991).
- ✓ Iacob et al., NP A 596, 155 (1996).
- ✓ Y. J. Chen et al., PRC 73, 054316 (2006).
- ✓ Y. Huang et al., PRC 93, 064321 (2016).

Even-Even nuclei

	E	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{R}_3$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_3$	$\mathcal{P}$
E	E	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{R}_3$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_3$	$\mathcal{P}$
$\mathcal{R}_1$	$\mathcal{R}_1$	E	$\mathcal{R}_3$	$\mathcal{R}_2$	$\mathcal{P}$	$\mathcal{S}_3$	$\mathcal{S}_2$	$\mathcal{S}_1$
$\mathcal{R}_2$	$\mathcal{R}_2$	$\mathcal{R}_3$	E	$\mathcal{R}_1$	$\mathcal{S}_3$	$\mathcal{P}$	$\mathcal{S}_1$	$\mathcal{S}_2$
$\mathcal{R}_3$	$\mathcal{R}_3$	$\mathcal{R}_2$	$\mathcal{R}_1$	E	$\mathcal{S}_2$	$\mathcal{S}_1$	$\mathcal{P}$	$\mathcal{S}_3$
$\mathcal{S}_1$	$\mathcal{S}_1$	$\mathcal{P}$	$\mathcal{S}_3$	$\mathcal{S}_2$	E	$\mathcal{R}_3$	$\mathcal{R}_2$	$\mathcal{R}_1$
$\mathcal{S}_2$	$\mathcal{S}_2$	$\mathcal{S}_3$	$\mathcal{P}$	$\mathcal{S}_1$	$\mathcal{R}_3$	E	$\mathcal{R}_1$	$\mathcal{R}_2$
$\mathcal{S}_3$	$\mathcal{S}_3$	$\mathcal{S}_2$	$\mathcal{S}_1$	$\mathcal{P}$	$\mathcal{R}_2$	$\mathcal{R}_1$	E	$\mathcal{R}_3$
$\mathcal{P}$	$\mathcal{P}$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_3$	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{R}_3$	E

	E	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{R}_3$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_3$	$\mathcal{P}$
A_g	1	1	1	1	1	1	1	1
B_{1g}	1	1	−1	−1	1	−1	−1	1
B_{2g}	1	−1	1	−1	−1	1	−1	1
B_{3g}	1	−1	−1	1	−1	−1	1	1
A_u	1	1	1	1	−1	−1	−1	−1
B_{1u}	1	1	−1	−1	−1	1	1	−1
B_{2u}	1	−1	1	−1	1	−1	1	−1
B_{3u}	1	−1	−1	1	1	1	−1	−1

Wang, *BQ**, Liu, Aman, Zhang, PRC 106, 064325 (2022)

Symmetry group of Hamitonian / wave function:
$\{E,R_1,R_2,R_3,S_1,S_2,S_3,P\}$

considered as direct product of

- **$C_{2v}\times\{E,P\}$**

broken symmetries in the nuclear density distribution are restored in the laboratory frame

- **$D_2\times\{E,P\}$**

rotational states with one parity for the quadrupole-deformed nucleus are extended to the states with two parities for the octupole deformed nucleus

Even-Even nuclei

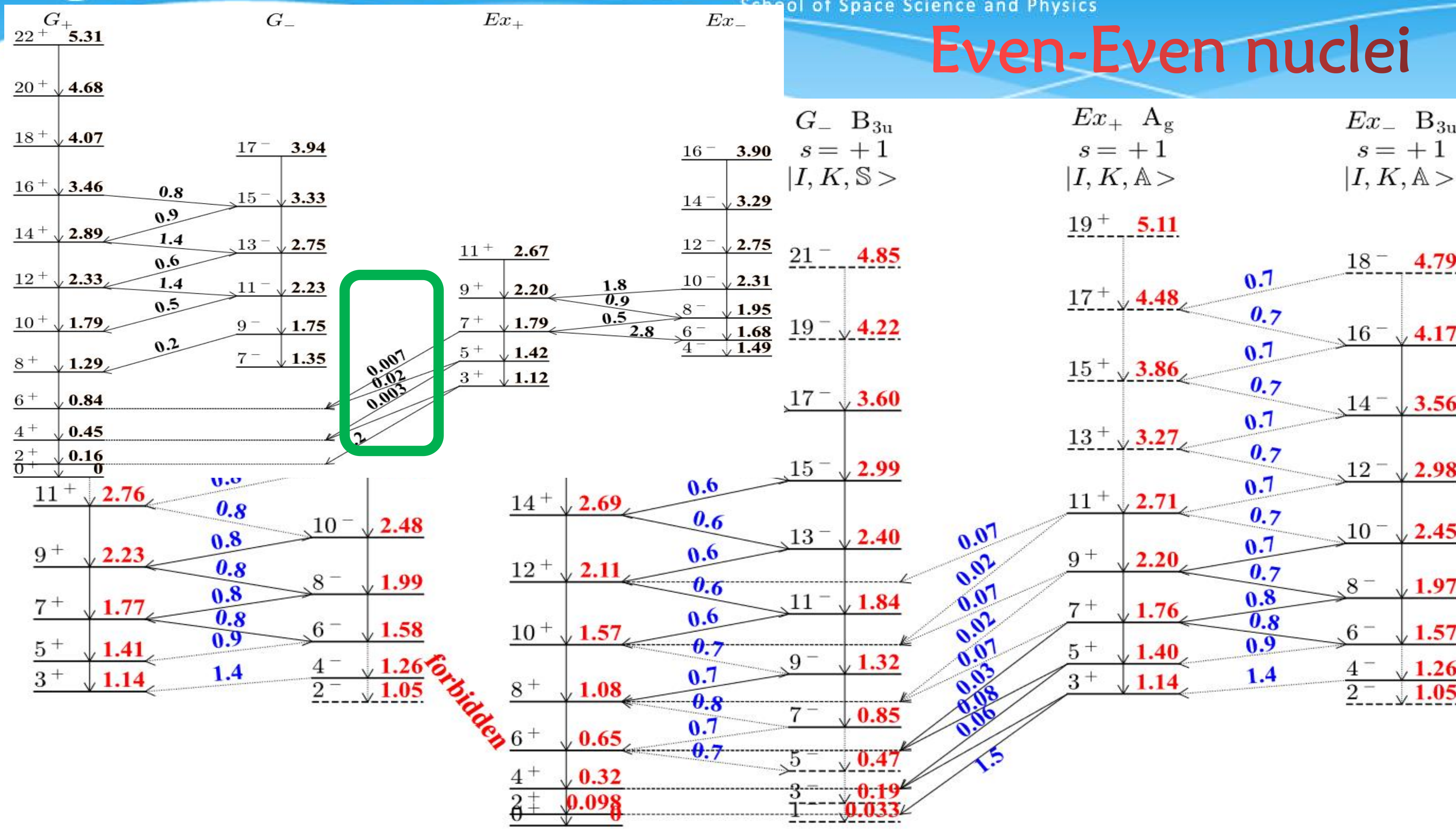
$$|I, K, \mathbb{S}\rangle = \frac{1}{\sqrt{1+\delta_{K0}}}\frac{1}{\sqrt{2}}(|IMK\rangle + |IM-K\rangle),$$

$$|I, K, \mathbb{A}\rangle = \frac{1}{\sqrt{2}}(|IMK\rangle - |IM-K\rangle),$$

(r_1, r_2, r_3)	D_2 group REP	$\|I, K, \mathbb{S}\rangle$		$\|I, K, \mathbb{A}\rangle$	
		I	K	I	K
$(+1, +1, +1)$	A	Even	Even	Odd	Even
$(+1, -1, -1)$	B_1	Even	Odd	Odd	Odd
$(-1, +1, -1)$	B_2	Odd	Odd	Even	Odd
$(-1, -1, +1)$	B_3	Odd	Even	Even	Even

Case	Wave function	K	Even I		Odd I	
			$p=+1$	$p=-1$	$p=+1$	$p=-1$
I	$\|I, K, \mathbb{S}\rangle$	Odd	B_{1g}	B_{1u}	B_{2g}	B_{2u}
II	$\|I, K, \mathbb{A}\rangle$	Odd	B_{2g}	B_{2u}	B_{1g}	B_{1u}
III	$\|I, K, \mathbb{A}\rangle$	Even	B_{3g}	B_{3u}	A_g	A_u
IV	$\|I, K, \mathbb{S}\rangle$	Even	A_g	A_u	B_{3g}	B_{3u}

Even-Even nuclei



s in the figure means s_1.
For two group representations
A_g and B_{3u}, $s_1=s_2$

The reasonable wave functions for excited band are selected by agreeing with the experimental transition properties.

Even-Even nuclei

- The reasonable wave functions for excited band are selected by agreeing with the experimental transition properties.

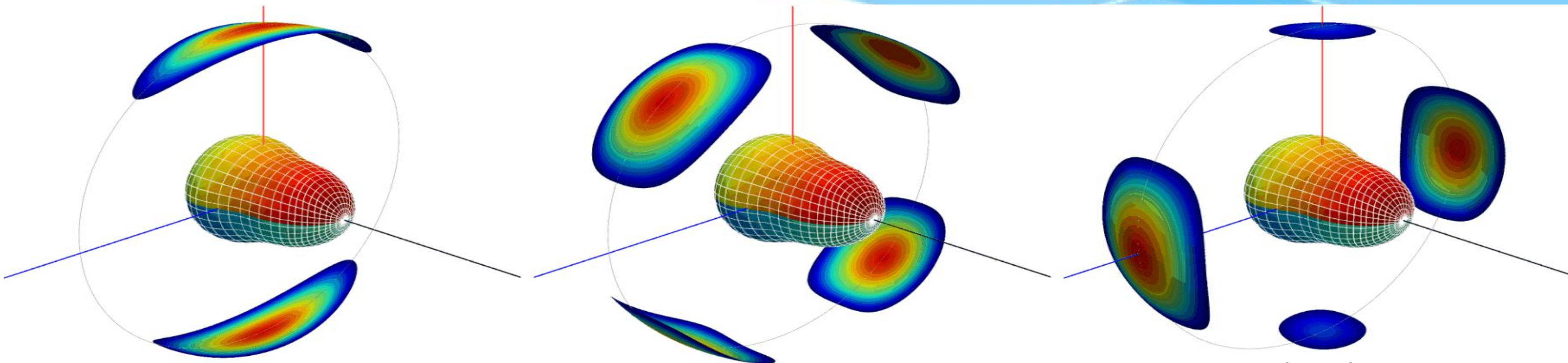
Parity + sequence in both ground and excited band: Ag, $(r_1,s_1,p)=(+1,+1,+1)$
Parity - sequence in both ground and excited band: B_{3u}, $(r_1,s_1,p)=(-1,+1,-1)$

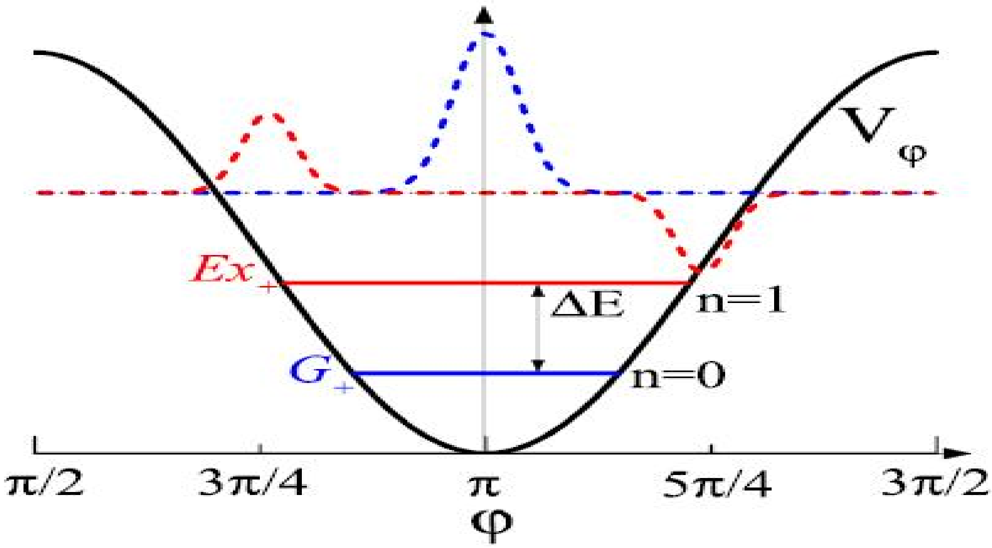
- The selected wave functions are also consistent with the conclusion obtained from the perspective of symmetry restoration

In such an assignment, the states occur with two values ± 1 of the quantum number r_1, r_2, s_3, and p, which corresponds to the violation of R_1, R_2, S_3, and P in the intrinsic frame.

While the states occur with single value of r_3, s_1, and s_2 in the laboratory frame which corresponds to R_3, S_1, and S_2 invariance in the intrinsic frame.

Even-Even nuclei





The excited band originates from the wobbling excitation of ground band.

Wobbling

in $Y_{20}+Y_{22}+Y_{30}$ nucleus

Wang, *BQ, Liu, Aman, Zhang， PRC 106, 064325 (2022)**

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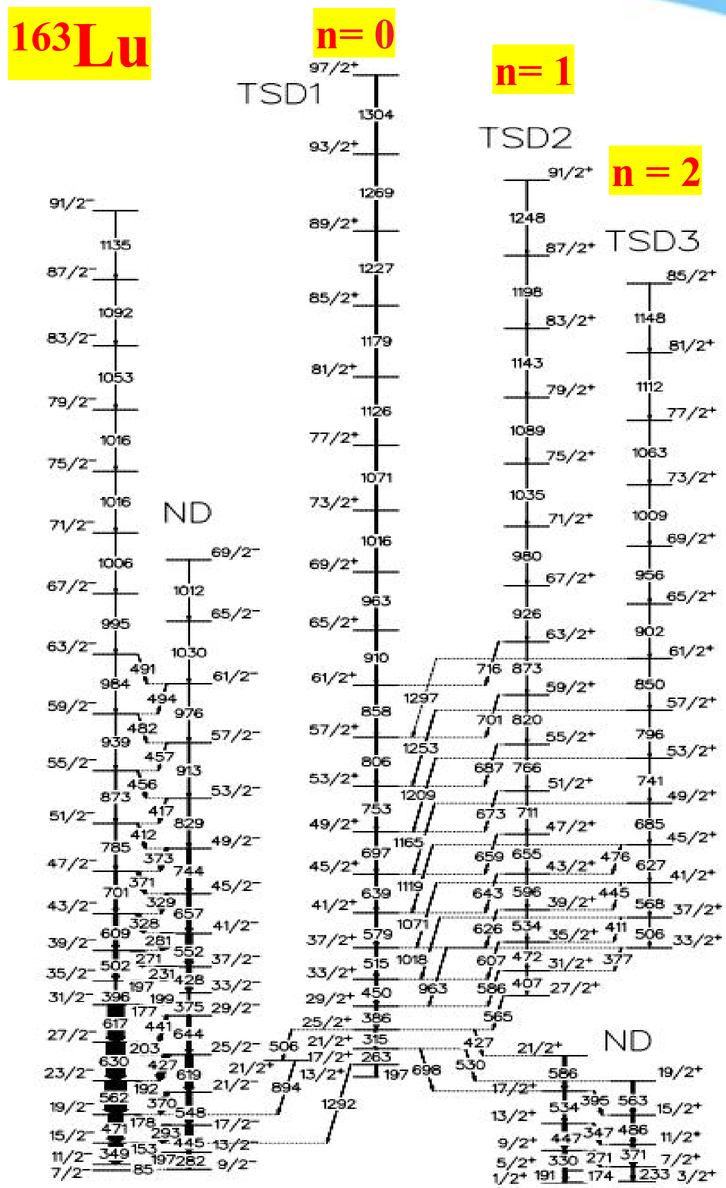
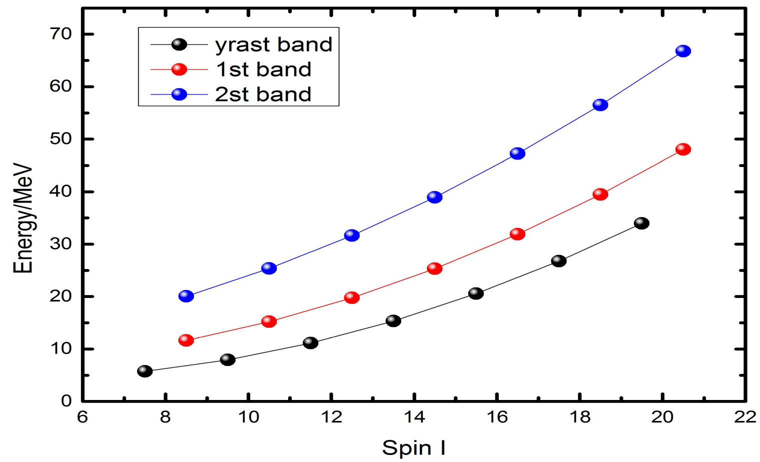
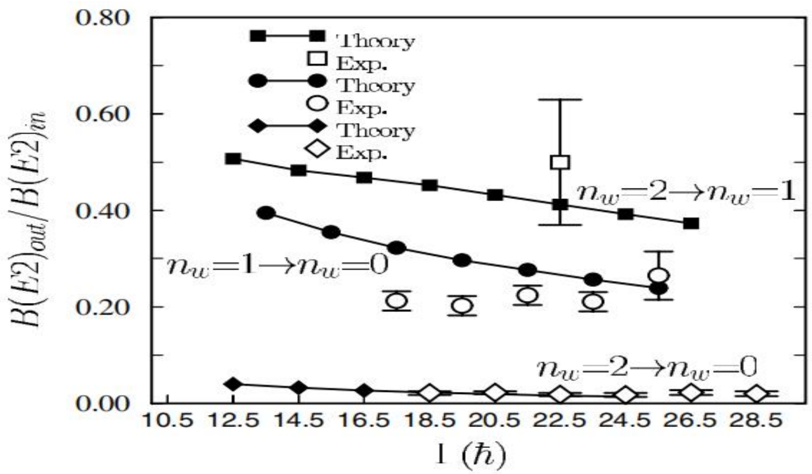
odd-A nuclei







Ødegård *et al.* PRL 86, 5866 (2001)

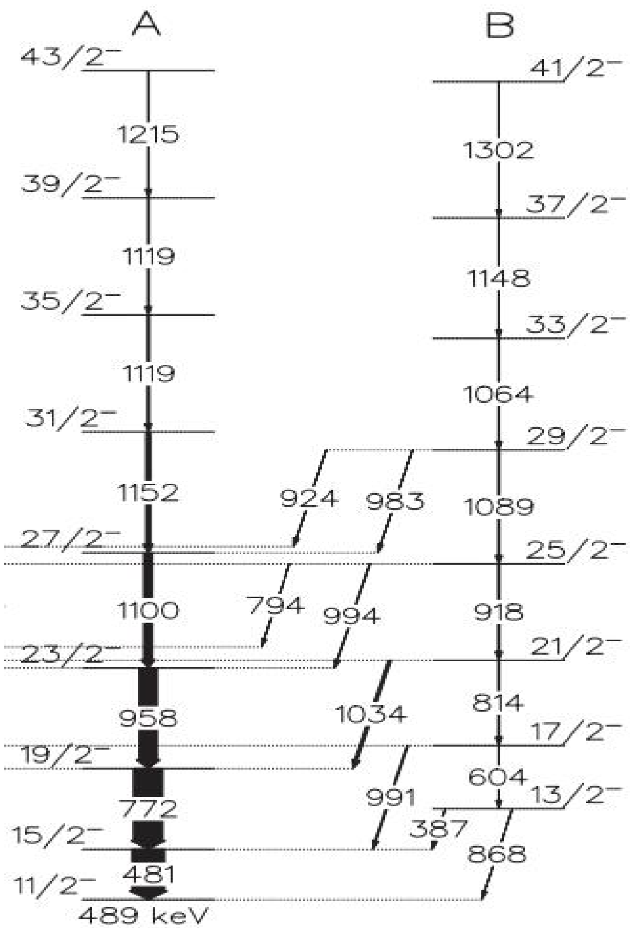
Jensen *et al.* PRL 89.142503 (2002)

odd-A nuclei

Nuclei	Z	N	configuration	β	γ	mass rigen	Reference
^{105}Pd	46	59	$\nu h_{11/2}$	0.27	25	100	J. Timár, *et al.*, PRL. 122, 062501 (2019).
^{127}Xe	54	73	$\nu h_{11/2}$			130	S. Chakraborty, et al. PLB 811, 135854 (2020).
^{130}Ba	56	74	$\pi(h_{11/2})^2$	0.24	21.5		Q. B. Chen, et al. PRC 100, 061301 (2019).
^{133}Ba	56	77	$\nu h_{11/2}$				D. K. Rojeeta, et al. PLB 823, 136756 (2021).
^{133}La	57	76	$\pi h_{11/2}$	0.17	26		S. Biswas, *et al.*, EPJA 55: 159(2019).
^{135}Pr	59	76	$\pi h_{11/2}$	0.17	26		J. T. Matta, *et al.*, PRL 114, 082501 (2015).
^{161}Lu	71	90	$\pi i_{13/2}$	0.42	20	160	P. Bringel, *et al.*, EPJA 24, 167 (2005).
^{163}Lu	71	92	$\pi i_{13/2}$	0.42	20		S. W. Ødegård, *et al.*, PRL. 86, 5866 (2001).
^{165}Lu	71	94	$\pi i_{13/2}$	0.42	20		G. Schönwaßer, *et al.*, PLB 552, 9 (2003).
^{167}Lu	71	96	$\pi i_{13/2}$	0.43	19		H. Amro, *et al.*, PLB553, 197 (2003).
^{167} Ta	73	94	$\pi i_{13/2}$	0.41	20		D. J. Hartley, *et al.*, PRC 80, 041304(R) (2009).
^{183}Au	79	104	$\pi i_{13/2}$	0.29	21.4	190	S. Nandi, *et al.*, PRL 125, 132501 (2020).
			$\pi h_{9/2}$	0.3	20		
^{187}Au	79	108	$\pi h_{9/2}$	0.23	23		N. Sensharma, *et al.*, PRL 124, 052501 (2020).

odd-A nuclei

n=0 n=1



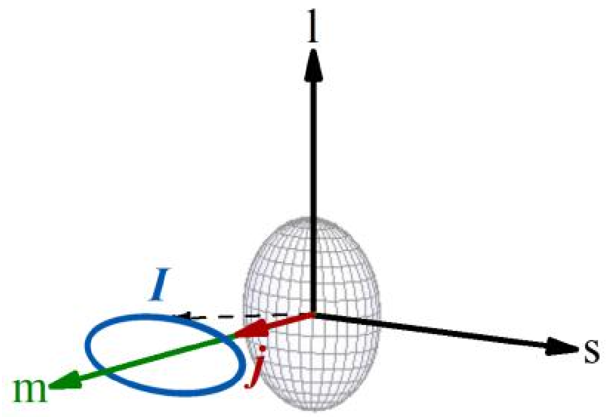
Fingerprint of wobbling band:

- Sequences of $\Delta I=2$ rotational bands
- Exhibit similar moments of inertia, spin alignments and in-band B(E2) values for ground (n=0) and excited (n=1,2) band
- the interband $\Delta I = 1, n \rightarrow n-1$ transitions are dominated by the E2 component
- the wobbling energy decreases with spin I, contrary to the behavior expected for even-even nuclei

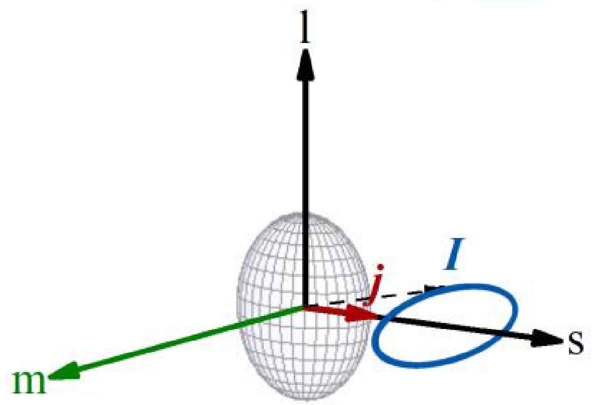
J. Timár, et al., Phys. Rev. Lett. 122, 062501 (2019)

odd-A nuclei

Longitudinal wobbler (LW)



Transverse wobbler (TW)



$$\hbar\Omega_{\text{wob}} = \frac{j}{\mathcal{J}_3}\left\{\left[1+\frac{J}{j}\left(\frac{\mathcal{J}_3}{\mathcal{J}_1}-1\right)\right]\left[1+\frac{J}{j}\left(\frac{\mathcal{J}_3}{\mathcal{J}_2}-1\right)\right]\right\}^{1/2}$$

$$\hbar\Omega_{\text{wob}} = \frac{j}{\mathcal{J}_1}\left\{\left[1+\frac{J}{j}\left(\frac{\mathcal{J}_1}{\mathcal{J}_3}-1\right)\right]\left[1+\frac{J}{j}\left(\frac{\mathcal{J}_1}{\mathcal{J}_2}-1\right)\right]\right\}^{1/2}$$

	the orientation of ***j*** (//***I***)	wobbling energy	quasiparticle orbital of j shell
LW	aligned **parallel** to the m-axis	**increases** with spin	middle (m-axis)
TW	aligned **perpendicular** to the m-axis	**decreases** with spin	bottom (s-axis) top (l-axis)

Frauendorf & Dönau, PRC 89, 014322 (2014)

odd-A nuclei

Stability of the wobbling motion in an odd-mass nucleus and the analysisof ^{135}Pr
Tanabe K and Sugawara-Tanabe K 2017 Phys. Rev. C 95 064315

Comment on "Stability of the wobbling motion in an odd-mass nucleus and the analysis of ^{135}Pr"
Frauendorf S 2018 Phys. Rev. C 97 069801

Reply to "Comment on 'Stability of the wobbling motion in an odd-mass nucleus and the analysis of ^{135}Pr'" Tanabe K 2018 Phys. Rev. C 97 069802

Tilted precession and wobbling in triaxial nuclei,
Lawrie, Shirinda and Petrache 2020 Phys. Rev. C 101 034306

Eur. Phys. J. A (2022) 58:75
https://doi.org/10.1140/epja/s10050-022-00727-5
THE EUROPEAN PHYSICAL JOURNAL A
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Regular Article - Theoretical Physics

Study of wobbling modes by means of spin coherent state maps

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[3] Physics Department, University of Notre Dame, Notre Dame, IN 46556, USA

odd-A nuclei

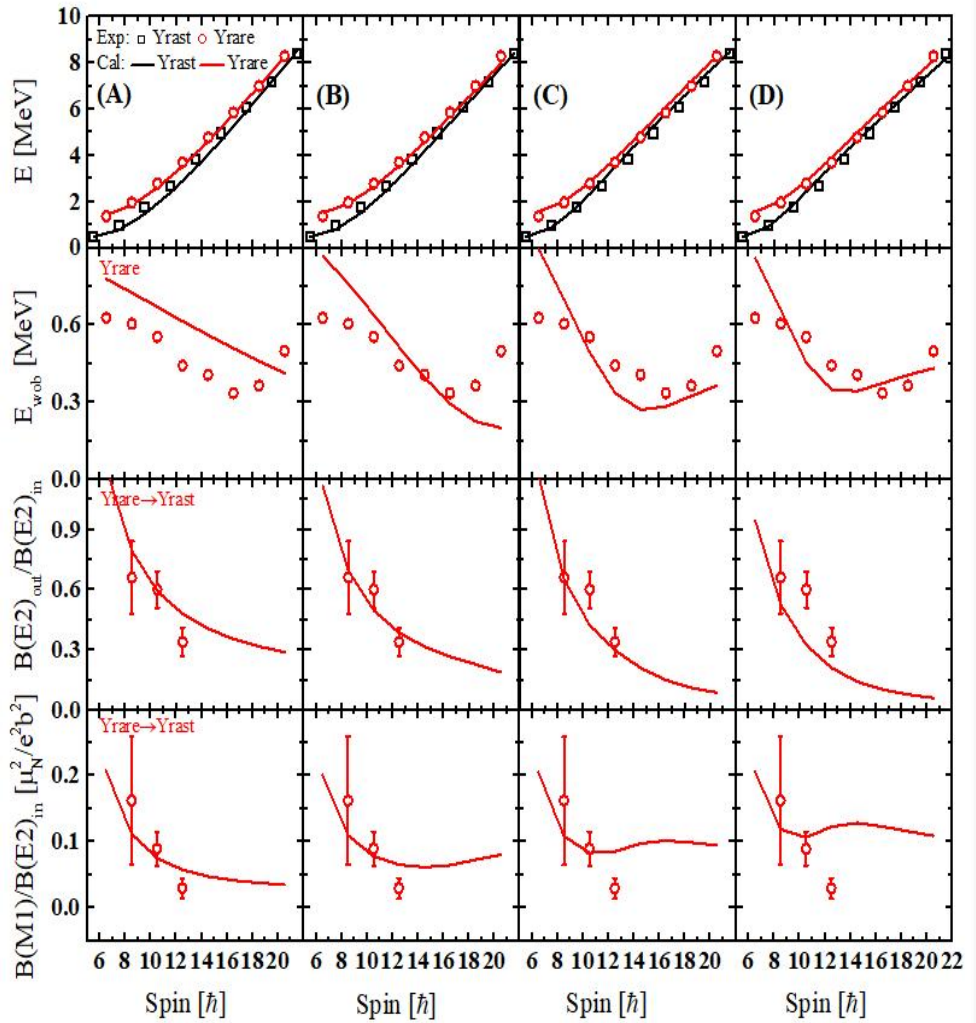
Employing different parameter sets of moment of inertia (MOI), several calculated results for ^{105}Pd could be in good agreement with the experimental data

configuration	$\nu(1h_{11/2})^1$
deformation	β~0.27; γ~25^{o}
g-factor	g_R=0.43
	g_n=-0.21
quadrupole moments	Q=3.0eb

$$\mathcal{J}_k = a_k\sqrt{1+bI(I+1)}$$

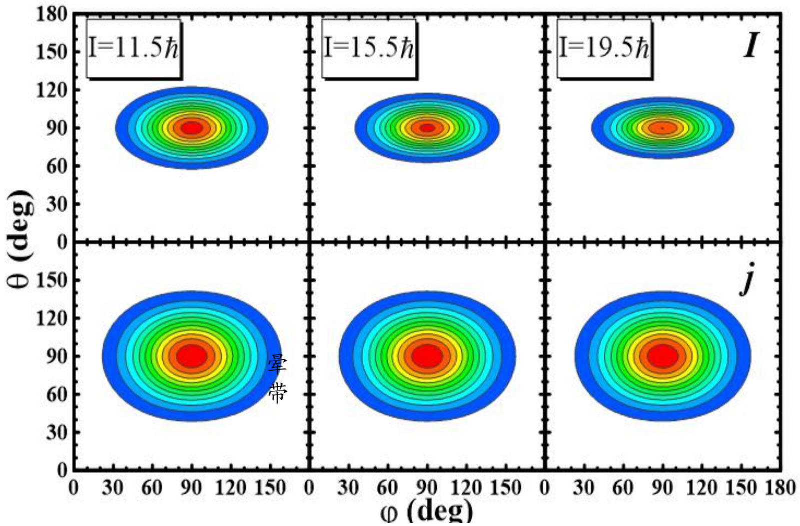
J. Timár, et al., Phys. Rev. Lett. 122, 062501 (2019)

参数组	a_m	a_s	a_l	b	$\mathcal{J}_m : \mathcal{J}_s : \mathcal{J}_l$
(A)	6.0	5.4	1.8	0.016	1 : 0.9 : 0.3
(B)	6.0	4.2	1.2	0.023	1 : 0.7 : 0.2
(C)	6.0	3.0	1.0	0.026	1 : 0.5 : 0.17
(D)	12.0	3.6	1.0	0.008	1 : 0.3 : 0.08



odd-A nuclei

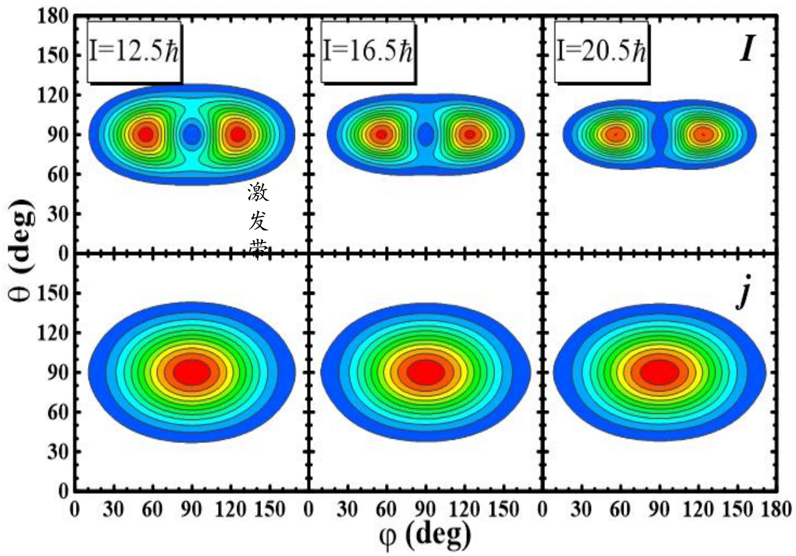
Take set (A) of MOI results as an example:

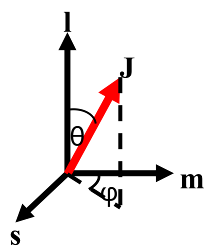


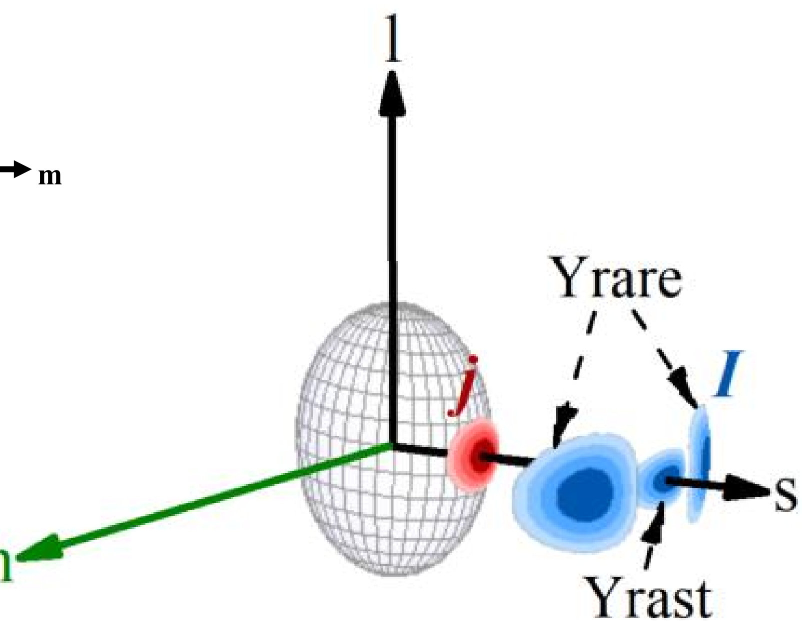
close to the rigid-body model

参数组	a_m	a_s	a_l	b	$\mathcal{J}_m : \mathcal{J}_s : \mathcal{J}_l$
(A)	6.0	5.4	1.8	0.016	1 : 0.9 : 0.3

The probability distribution of angular momentum on the (θ,ϕ) plane, i.e., azimuthal plot, is shown and corresponding schematic diagram is provided.







azimuthal plot see Refs. Chen, Chen, Luo, Meng and Zhang PRC 96 051303(2017)
Chen and Meng, PRC 98, 031303(2018)

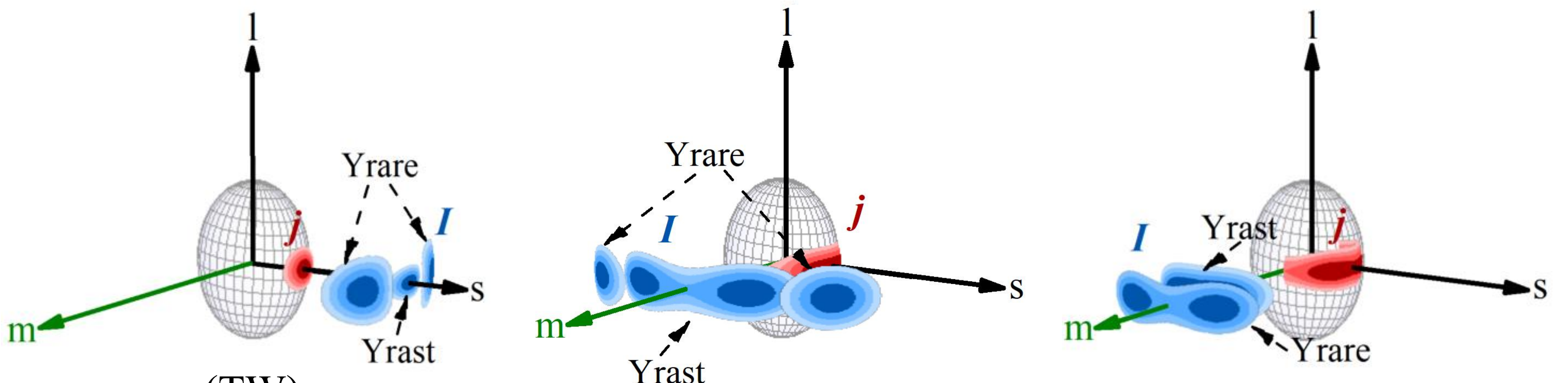
odd-A nuclei

参数组	a_m	a_s	a_l	b	$\mathcal{J}_m : \mathcal{J}_s : \mathcal{J}_l$
(A)	6.0	5.4	1.8	0.016	1 : 0.9 : 0.3
(B)	6.0	4.2	1.2	0.023	1 : 0.7 : 0.2
(C)	6.0	3.0	1.0	0.026	1 : 0.5 : 0.17
(D)	12.0	3.6	1.0	0.008	1 : 0.3 : 0.08

(A): close to the rigid-body model

(D): close to the hydrodynamical model

corresponding angular momentum geometry show distinct modes of rotational excitation



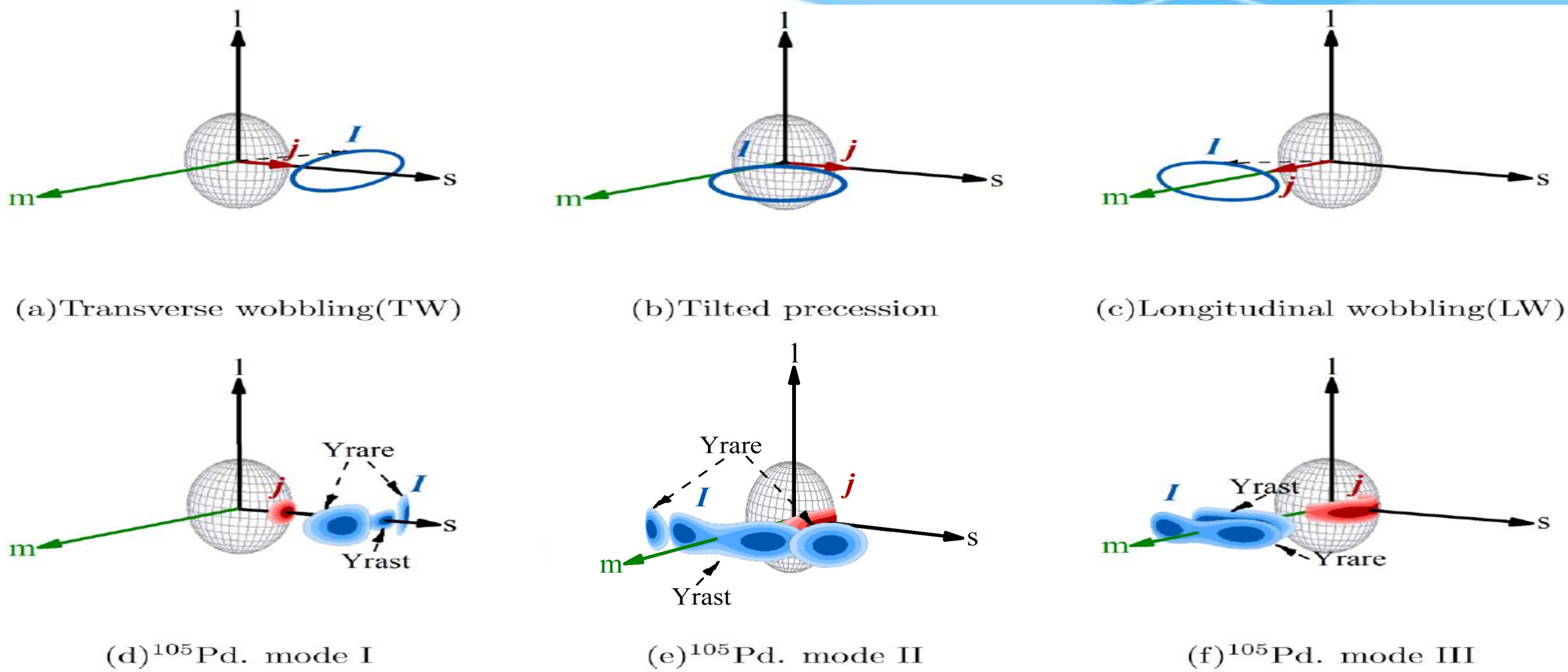
(TW)
Mode I

Mode II

(similar to LW)
Mode III

With the increasing of the ratio between the MOI at the *m* and *s* axis, namely $\mathcal{J}_m/\mathcal{J}_s$, the rotational modes gradually changes from Mode I to Mode II and then to Mode III.

odd-A nuclei



(a)Transverse wobbling(TW)

(b)Tilted precession

(c)Longitudinal wobbling(LW)

(d)^{105}Pd. mode I

(e)^{105}Pd. mode II

(f)^{105}Pd. mode III

- distinct modes of rotational excitation are shown when Employing different parameter sets of moment of inertia (MOI),
- TW mode is sensitive to the ratio $\mathcal{J}m/\mathcal{J}s$
- ideal precession does not appear, the tunneling between two orientations of angular momentum may be preferable

Zhang, *BQ, Wang,Liu, Wang， PRC105, 034339 (2022)**

Outline

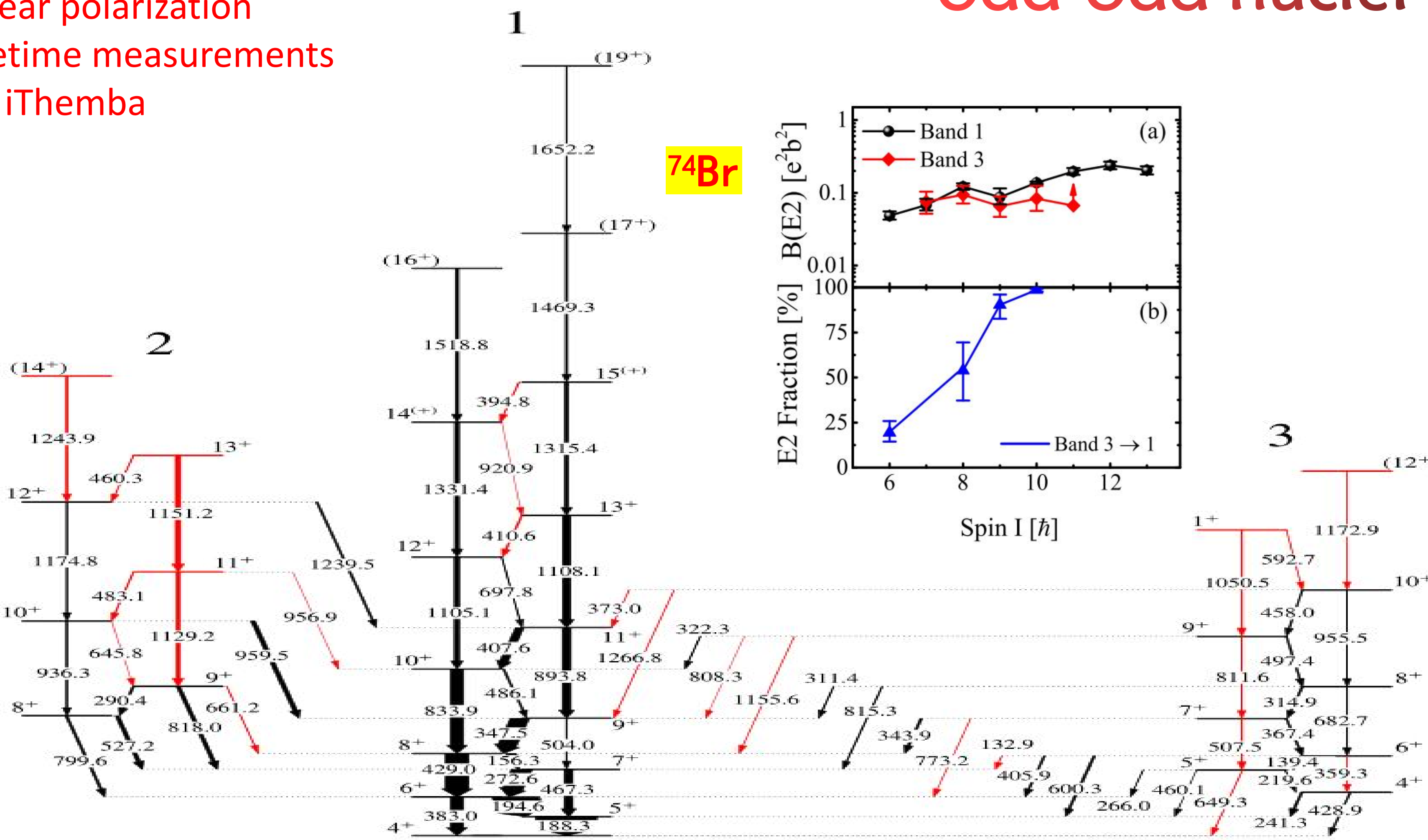
Wobbling

- in even-even nulclei:
  - Precession and tunneling
  - Quadrupole-octupole deformed nucleus
- in odd-A nuclei:
  - Transverse mode: sensitive to MOI
- in odd-odd nuclei :
  - Coexistence of Chirality and Wobbling

odd-odd nuclei

Angular distribution
Linear polarization
Lifetime measurements
@ iThemba

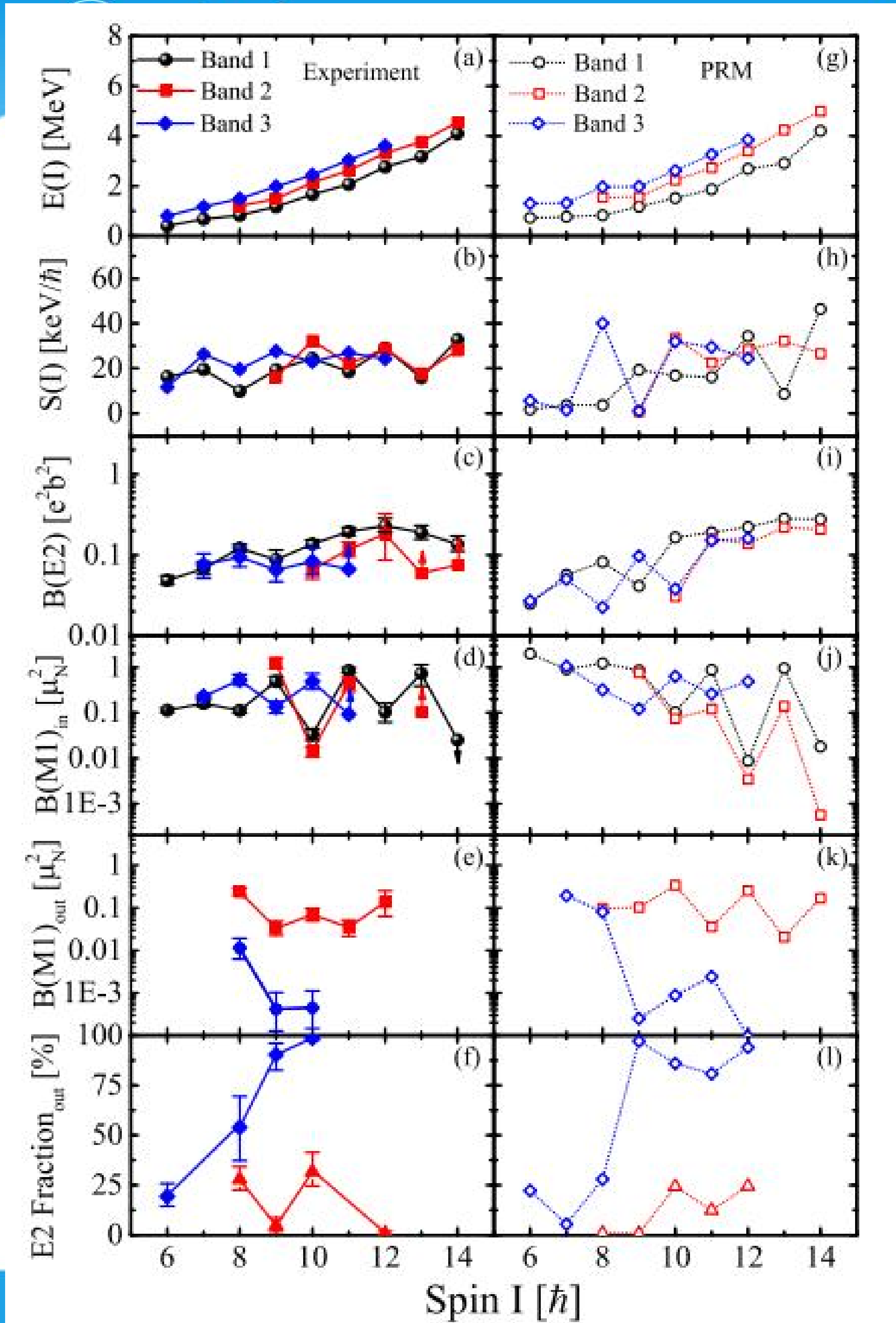
^{74}Br



chiral partner　　yrast band　　wobbling band

First Observation for Chiral-Wobbler in Nuclei, **R J Guo, et al. under Review**

odd-odd nuclei



Parameters of particle rotor model :

$\pi\, g_{9/2}$ (particle-like) $\nu\, g_{9/2}$(midle subshell)
pairing gap 1.40 MeV.

Deformation parameters
$(\beta, \gamma) = (0.45, 27.5^{o})$ from RMF

moments of inertia ~hydrodynamical

The calculated results reproduced the corresponding experimental data well

Wobbling

- in even-even nulclei:
  - Precession and tunneling
  - Quadrupole-octupole deformed nucleus
- in odd-A nuclei：
  - Transverse mode: sensitive to MOI
- in odd-odd nuclei :
  - Coexistence of Chirality and Wobbling

Thank you for your attention!

Appendix

理论框架

多粒子转子模型哈密顿量:

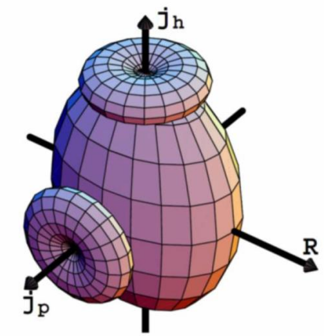
$$H = \hat{H}_{coll} + \hat{H}_{intr}$$

集体部分:

$$H_{coll} = \sum_{1}^{3} \frac{\hat{R}_i^2}{2\mathcal{J}_i} = \sum_{1}^{3} \frac{\hat{I}_i^2 - \hat{j}_i^2}{2\mathcal{J}_i}$$

内禀部分:

$$\hat{H}_{intr} = \sum_{\nu} \varepsilon_{p,\nu} a_{p,\nu}^{+} a_{p,\nu} + \sum_{\nu'} \varepsilon_{n,\nu'} a_{n,\nu'}^{+} a_{n,\nu'}$$



一粒子，一空穴与三轴转子耦合图像

价核子哈密顿量用单-j哈密顿量给出:

$$h_{sp} = \pm\frac{1}{2}C\{\cos\gamma(j_3^2 - \frac{j(j+1)}{3}) + \frac{\sin\gamma}{2\sqrt{3}}(j_+^2 + j_-^2)\}$$

对于z个质子和n个中子的体系，内禀波函数

$$|\varphi\rangle = (\prod_{i=1}^{z_1} a_{p,\nu_i}^{\dagger})(\prod_{i=1}^{z_2} a_{p,\bar{\mu}_i}^{\dagger})(\prod_{i=1}^{z_1} a_{p,\nu_i'}^{\dagger})(\prod_{i=1}^{z_2} a_{p,\bar{\mu}_i'}^{\dagger})|0\rangle$$

Bohr,Mottelson,Nuclear Structure, Vol. 2 (1975) ;QI,PLB (2009)

附录

理论框架

体系波函数: $|IM\rangle = \sum_{K\varphi} c_{K\varphi}|IMK\varphi\rangle$

$$|IMK\varphi\rangle = \frac{1}{\sqrt{2(1+\delta_{K0}\delta_{\varphi,\bar{\varphi}})}}(|IMK\rangle|\varphi\rangle + (-1)^{I-K}|IM-K\rangle|\bar{\varphi}\rangle)$$

约化电磁跃迁几率:

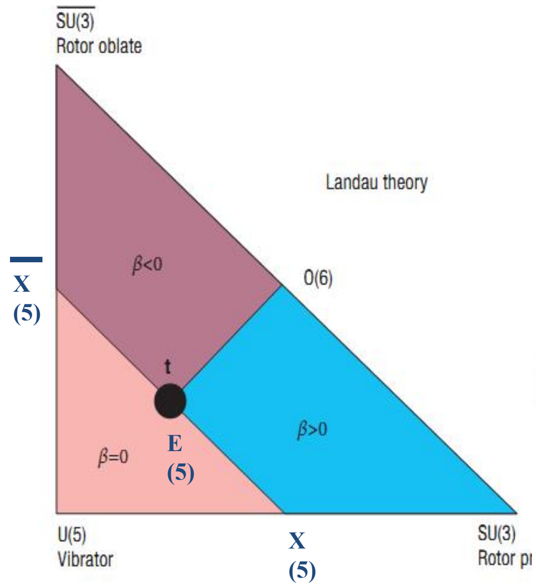
$$B(\sigma\lambda, I' \to I) = \frac{1}{2I+1}\sum_{\mu M}\left|\langle IM|\hat{M}(\sigma\lambda,\mu)|I'M'\rangle\right|^2$$

$$\hat{M}(M1,\mu) = \sqrt{\frac{3}{4\pi}}\frac{e}{2Mc}\left[(g_p - g_R)\hat{j}_{p\mu} + (g_n - g_R)\hat{j}_{n\mu}\right]$$

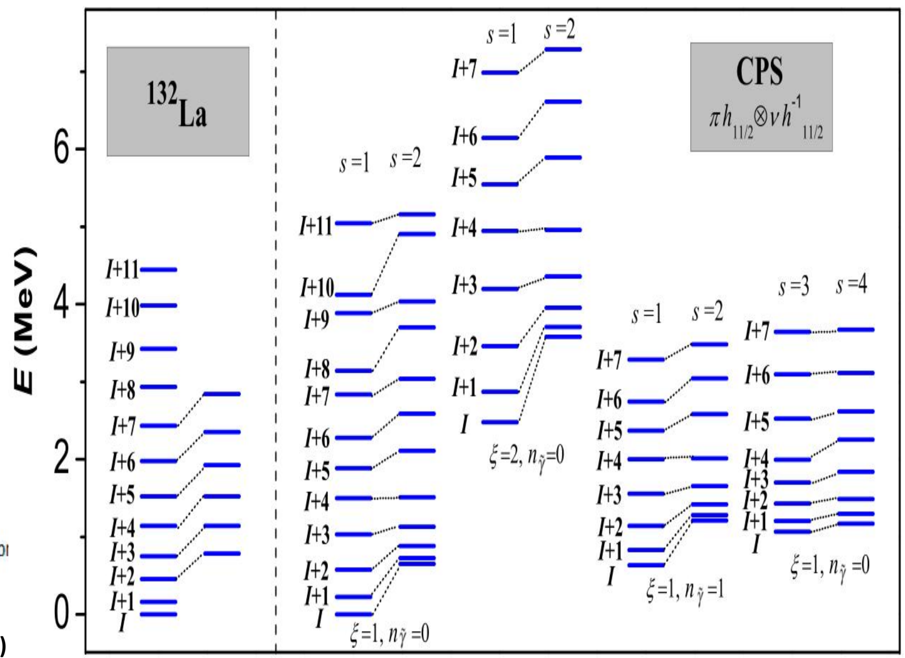
$$\hat{M}(E2,\mu) = \sqrt{\frac{5}{16\pi}}\hat{Q}_{2\mu}$$

B. Qi et al.,Phys. Lett. B 675, 175 (2009).
A. D. Ayangeakaa et al., Phys. Rev. Lett. 110, 172504 (2013).
I. Kuti et al., Phys. Rev. Lett. 113, 03 (2014)

形状相变+摇摆



Cejnar, Jolie, Casten, Rev. Mod. Phys. 82 (2010)



Critical point symmetry for odd-odd nuclei and collective multiple chiral doublet bands
Y. Zhang, BQ, and S.Q. Zhang
SCIENCE CHINA, 64 ,122011 (2021)

公式证明：

Introduce the parameter A_k : $\mathcal{J}_k = \frac{\hbar^2}{2A_k}$

$$H = A_1 I_1^2 + A_2 I_2^2 + A_3 I_3^2 = A_3 I^2 + H'$$

$$H' = \frac{1}{2}(A_2 + A_1 - 2A_3)(I_2^2 + I_1^2) + \frac{1}{2}(A_2 - A_1)(I_2^2 - I_1^2) = \frac{1}{2}\alpha\frac{I_2^2+I_1^2}{I} + \frac{1}{2}\beta\frac{I_2^2-I_1^2}{I}$$

$$\alpha = (A_2 + A_1 - 2A_3)I, \quad \beta = (A_2 - A_1)I$$

Considering the **approximation**

$$[I_-, I_+] = 2I_3 \approx 2I \quad (I_\pm = I_2 \pm iI_1)$$

We introduce the operator

$$c^\dagger = \frac{1}{\sqrt{2I}} I_+, c = \frac{1}{\sqrt{2I}} I_-$$

Then $H' = \frac{1}{2}\alpha(c^\dagger c + cc^\dagger) + \frac{1}{2}\beta(c^\dagger c^\dagger + cc)$

公式证明： Introduce

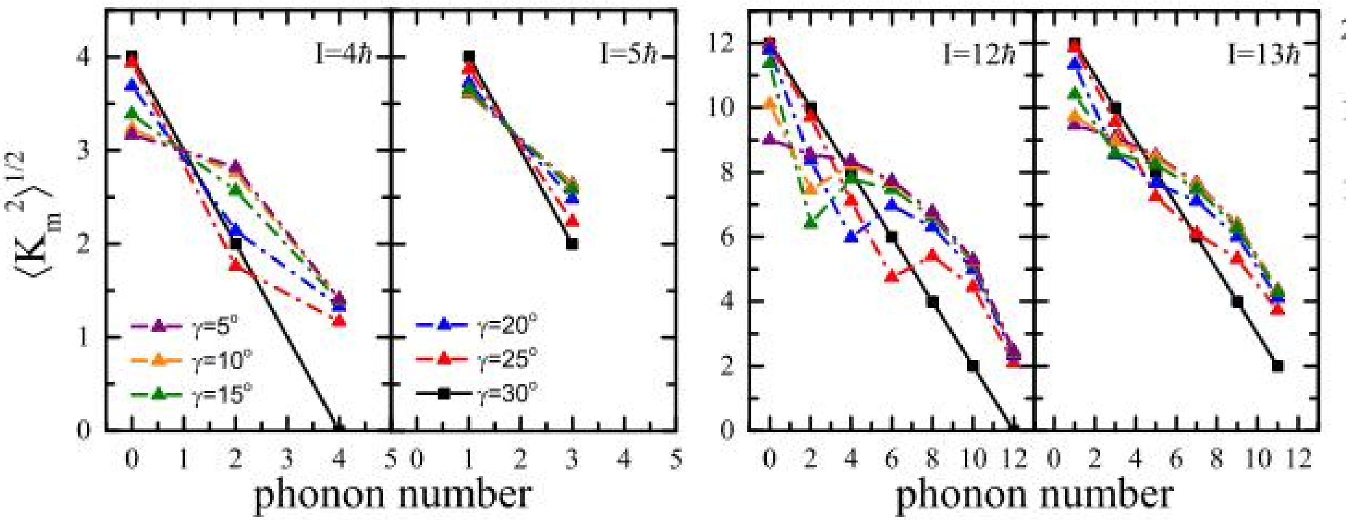
$$c^\dagger = x\hat{c}^\dagger + y\hat{c}, \quad \hat{c}^\dagger = xc^\dagger - yc$$

$$x, y = [\frac{1}{2}(\frac{\alpha}{(\alpha^2-\beta^2|})^{1/2} \pm 1)]^{1/2}, \quad x^2 - y^2 = 1$$

$$H' = \frac{1}{2}\alpha(c^\dagger c + cc^\dagger) + \frac{1}{2}\beta(c^\dagger c^\dagger + cc)$$
$$= \sqrt{\alpha^2 - \beta^2}[\hat{c}^\dagger\hat{c} + \frac{1}{2}] = \hbar\omega(\hat{n} + \frac{1}{2})$$

$$\hbar\omega = \sqrt{\alpha^2 - \beta^2} = \sqrt{(A_2 + A_1 - 2A_3)^2 I^2 - (A_2 - A_1)^2 I^2}$$
$$= 2I[(A_2 - A_3)(A_1 - A_3)]^{1/2}$$
$$= 2I[(\frac{\hbar^2}{2\mathcal{J}_2} - \frac{\hbar^2}{2\mathcal{J}_3})(\frac{\hbar^2}{2\mathcal{J}_1} - \frac{\hbar^2}{2\mathcal{J}_3})]^{1/2}$$

三轴自由度的影响



For TRM, K_m = I-n for γ=30° exactly

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1. *Collective Hamiltonian for wobbling modes* Q. B. Chen, S. Q. Zhang, P. W. Zhao, and J. Meng, Phys. Rev. C (2014) 90, 044306.
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5. *Two-dimensional collective Hamiltonian for chiral and wobbling modes,* Q. B. Chen, S. Q. Zhang, P. W. Zhao, R. V. Jolos, and J. Meng Phys. Rev. C (2016) 94, 044301.
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7. *Behavior of the collective rotor in wobbling motion,* E. Streck, Q. B. Chen, N. Kaiser, et al., Phys. Rev. C (2018) 98, 044314.
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9. Experimental Evidence for Transverse Wobbling in ^{105}Pd J. Timár, Q. B. Chen, et al., Phys. Rev. L (2019) 122, 062501.
10. *Transverse wobbling in an even-even nucleus,* Q. B. Chen, S. Frauendorf, and C. M. Petrache, Phys. Rev. C (2019) 100, 061301(R).
11. *First Observation of Multiple Transverse Wobbling Bands of Different Kinds in ^{183}Au,* Nandi, Mukherjee, Q. B. Chen, et al., Phys. Rev. L (2020) 125, 132501.
12. *g-factor and static quadrupole moment for the wobbling mode in ^{133}La,* Q.B. Chen, et al., Phys. Lett. B (2020) 807, 135596.
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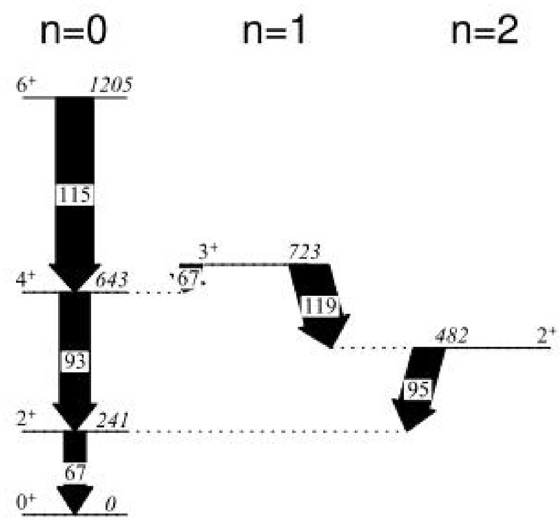
14. *Microscopic investigation on the existence of transverse wobbling under the effect of rotational alignment: The ^{136}Nd case* F. Q. Chen and C. M. Petrache, Phys. Rev. C (2021) 103, 064319.
15. *Study of wobbling modes by means of spin coherent state maps,* Q. B. Chen, S. Frauendorf, Eur. Phys. J. A (2022) 58, 75.
16. *Dynamics of rotation in chiral nuclei,* Z. X. Ren, P. W. Zhao, and J. Meng, Phys. Rev. C (2022) 105, L011301.

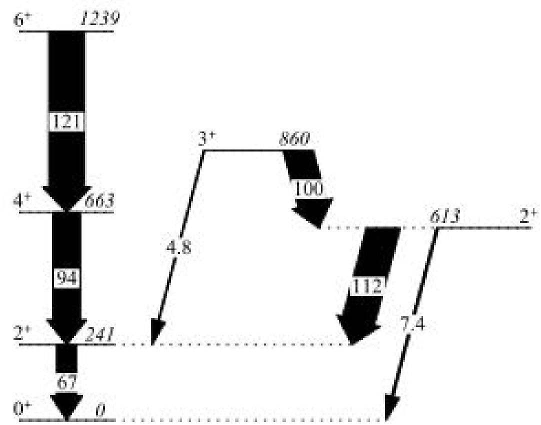
………, ………,

建议的摇摆带

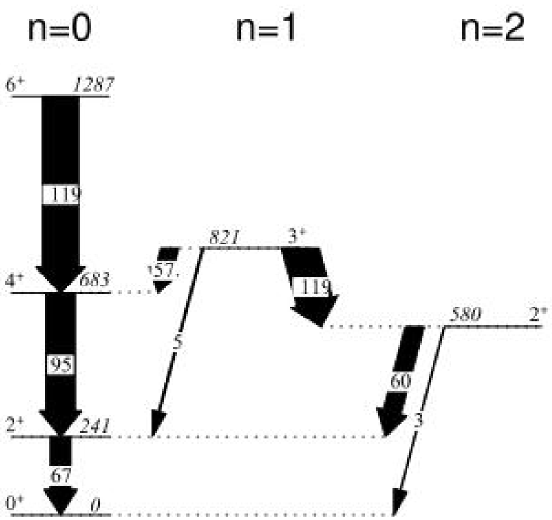
将3_1^+和2_2^+态建议为候选摇摆带的带首



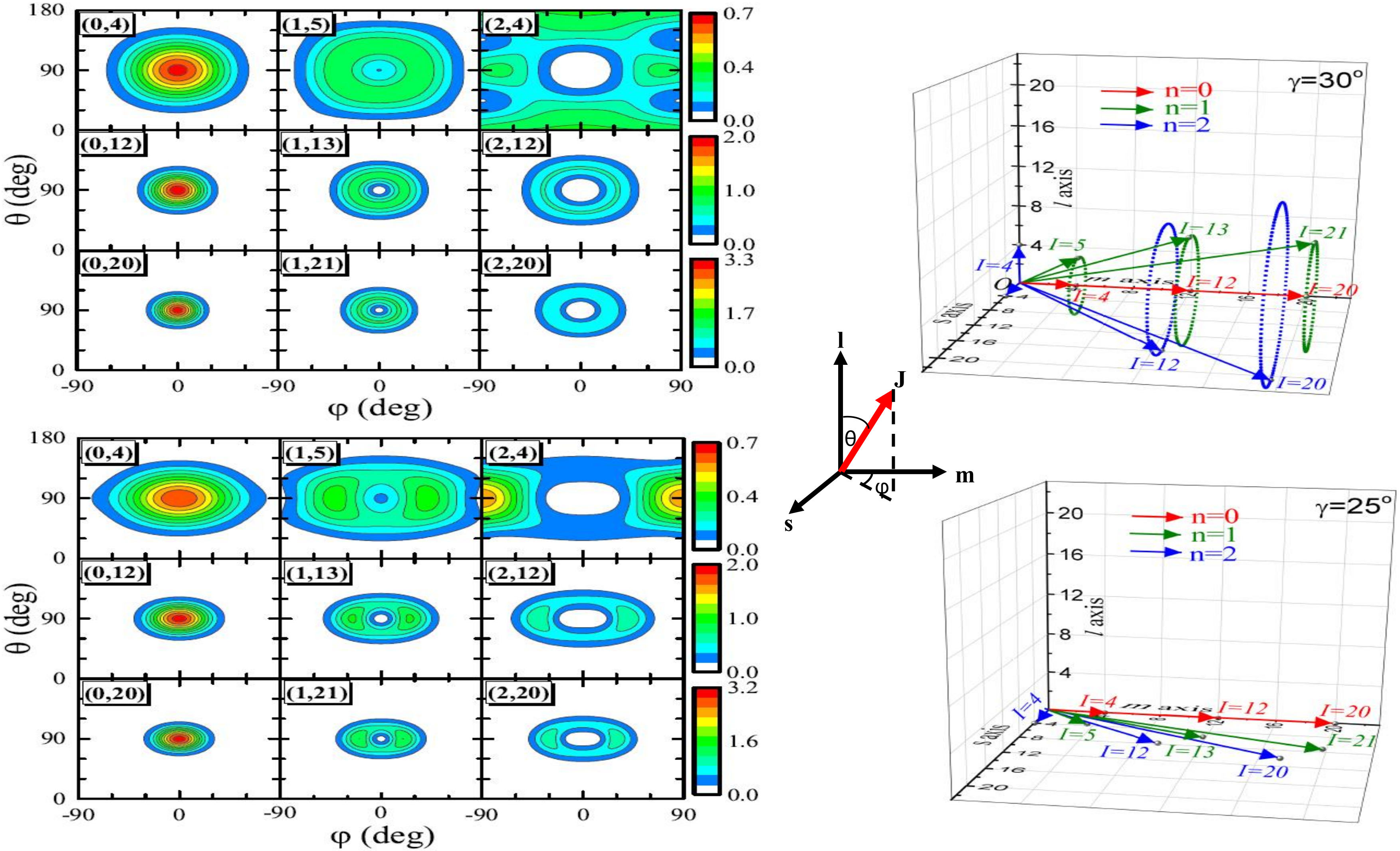
(a) γ=30° 模型结果



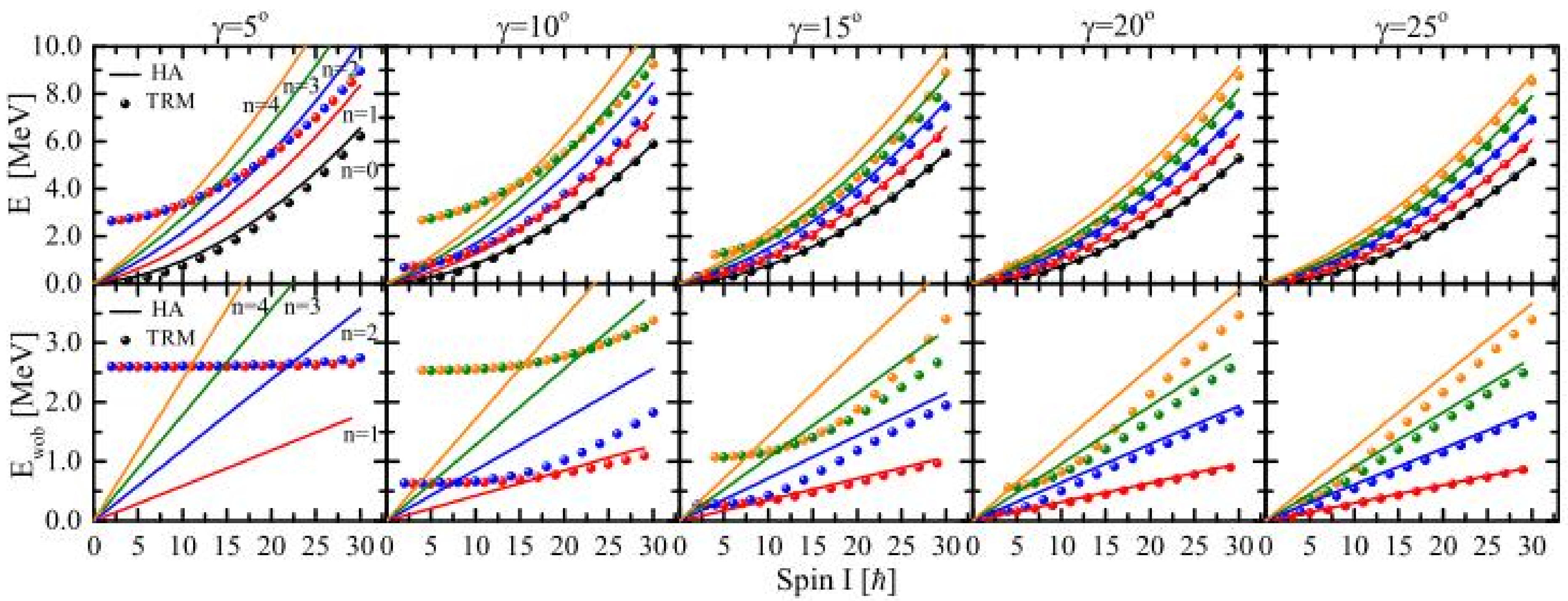
(b) ^{110}Ru 实验值



(c) γ=25° 模型结果



BQ*, Zhang, Wang, Q.B.Chen*, JPG 48 (2021) 055102


γ=5°
γ=10°
γ=15°
γ=20°
γ=25°
HA
TRM
n=0
n=1
n=2
n=3
n=4
E [MeV]
E_wob [MeV]
Spin I [ħ]

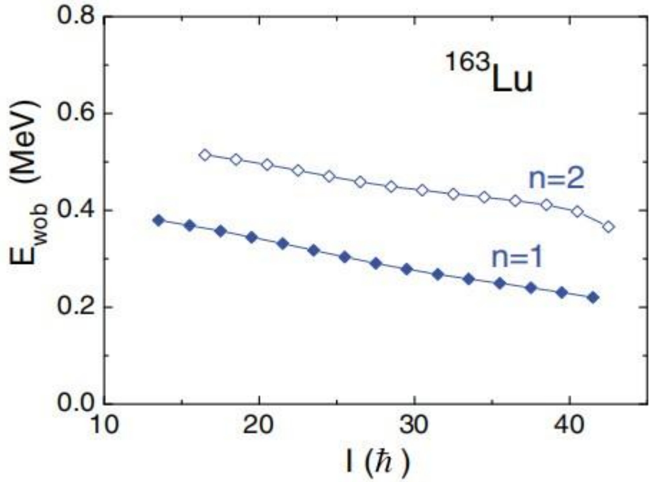
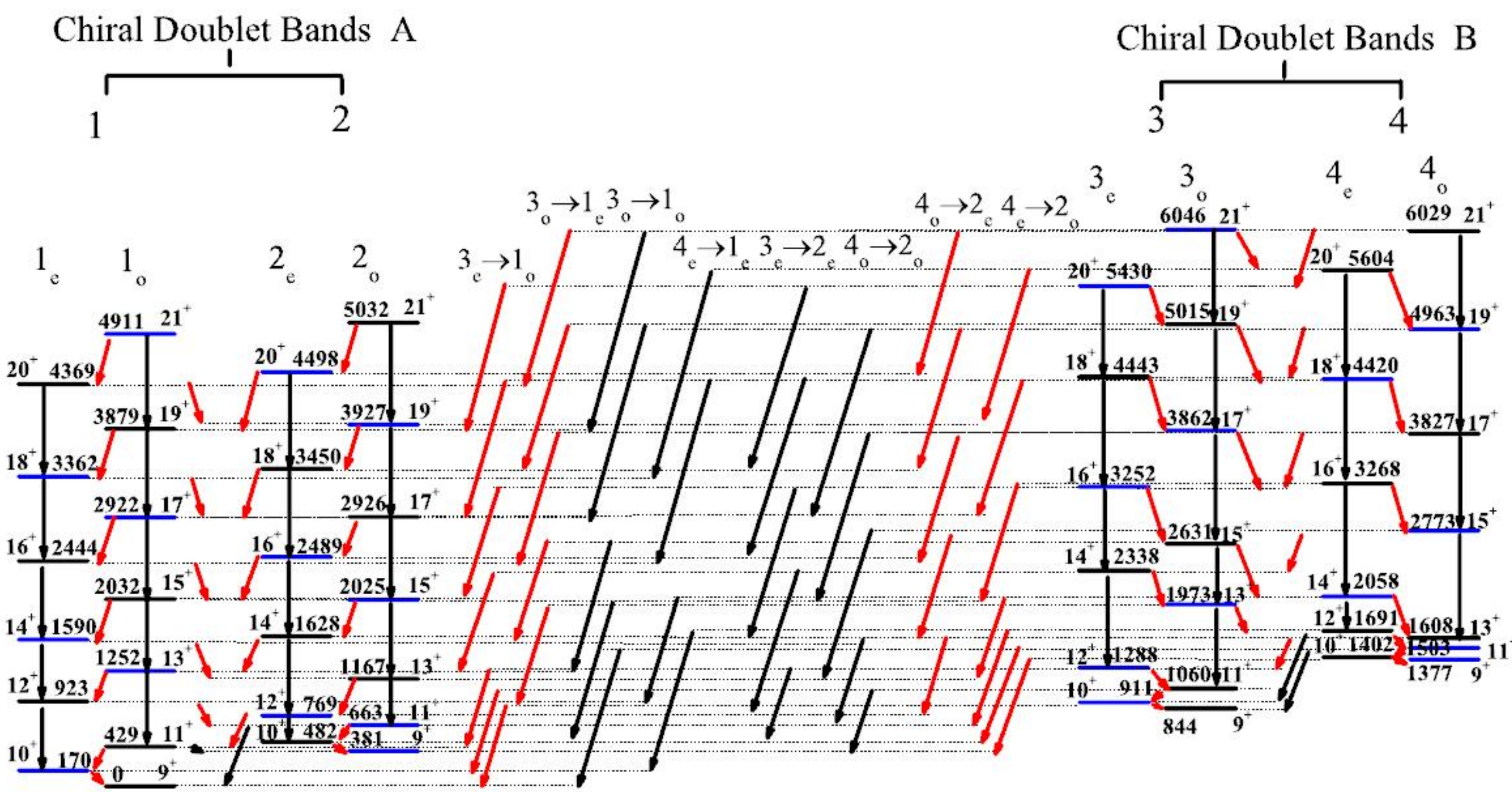


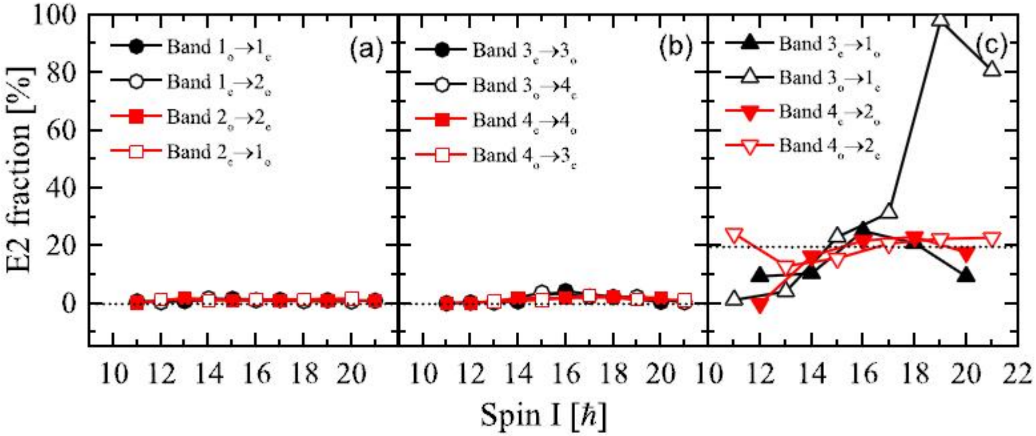
FIG. 3. (Color online) Experimental energies of the two lowest wobbling bands $n = 1, 2$ relative to the $\pi i_{13/2}$ $n = 0$ sequence (interpolated by a cubic spline) in ^{163}Lu. Data from [3].

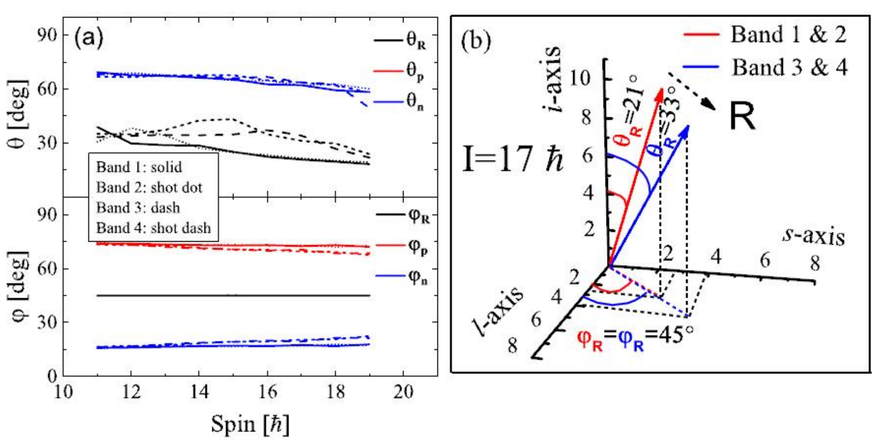
手征摇摆共存


Chiral Doublet Bands A
1
2
Chiral Doublet Bands B
3
4
1_e
1_o
2_e
2_o
$3_e \rightarrow 1_o$
$3_o \rightarrow 1_e$
$3_o \rightarrow 1_o$
$4_e \rightarrow 1_e$
$3_e \rightarrow 2_e$
$4_e \rightarrow 2_o$
$4_o \rightarrow 2_e$
$4_e \rightarrow 2_o$
3_e
3_o
4_e
4_o

odd-odd nuclei

手征摇摆共存





计算了E2和M1跃迁的混合比

带3&4 相比 带1&2
有摇摆激发的运动

手征和摇摆如何共存
还需讨论和澄清！

Jia, Wang*, *BQ*, Liu, Zhu, PLB 833, 137303 (2022)