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Selection rules of electromagnetic transitions for chirality-parity (ChP) violation in atomic nuclei

Yuanyuan Wang (王媛媛)

Mathematics and Physics Department, North China Electric Power University

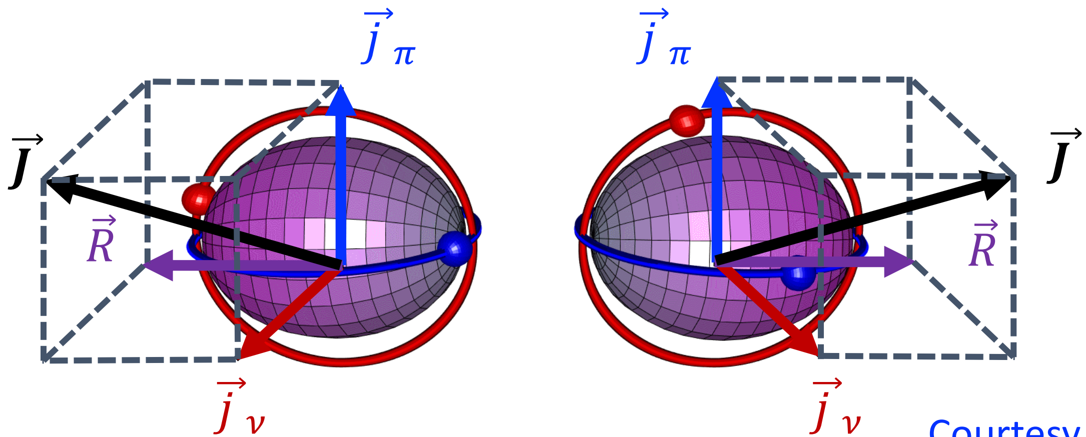
Outline

- Introduction
- Reflection-asymmetric triaxial particle rotor model
- MχD with octupole correlations
- Chirality-Parity (ChP) quartet bands
- Summary and perspective

Nuclear chirality

- Chirality is a subject of general interest in natural science.
- The nuclear chirality (experimental signal is **chiral doublet bands**) was first proposed in 1997.

S. Frauendorf and J. Meng, NPA (1997) 617:131



Courtesy By X. H. Wu

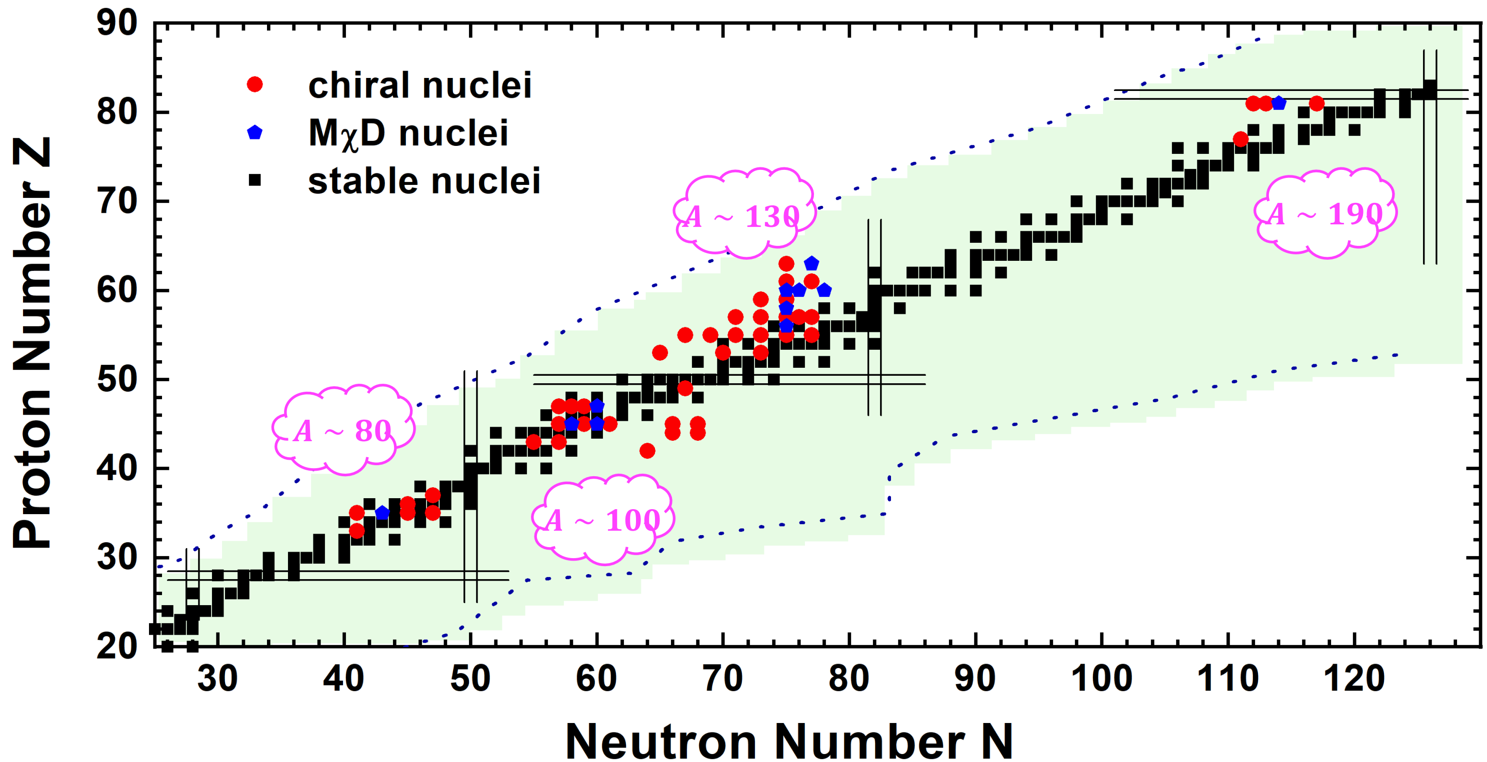
- The **multiple chiral doublets** (**MχD**) was suggested in 2006, and the experimental evidence for MχD have been reported in ^{133}Ce, ^{103}Rh, ^{78}Br, ^{136}Nd, ^{135}Nd, and ^{131}Ba, etc.

J. Meng et al , PRC (2006) 73: 037303
A. D. Ayangeakaa et al., PRL (2013) 110: 172504; I.Kuti et al., PRL (2014) 113: 032501
C. Liu et al., PRL (2016) 116: 112501; C. M. Petrache et al., PRC (2018) 97: 041304
B. F. Lv et al., PRC (2019) 100: 024314; S. Guo et al., PLB (2020) 807: 135572

Experimental progress

- Up to now, **71** candidate chiral doublet bands in **65** nuclei (including **11** nuclei with **MχD**) have been reported in A ~ 80, 100, 130, and 190 mass regions.

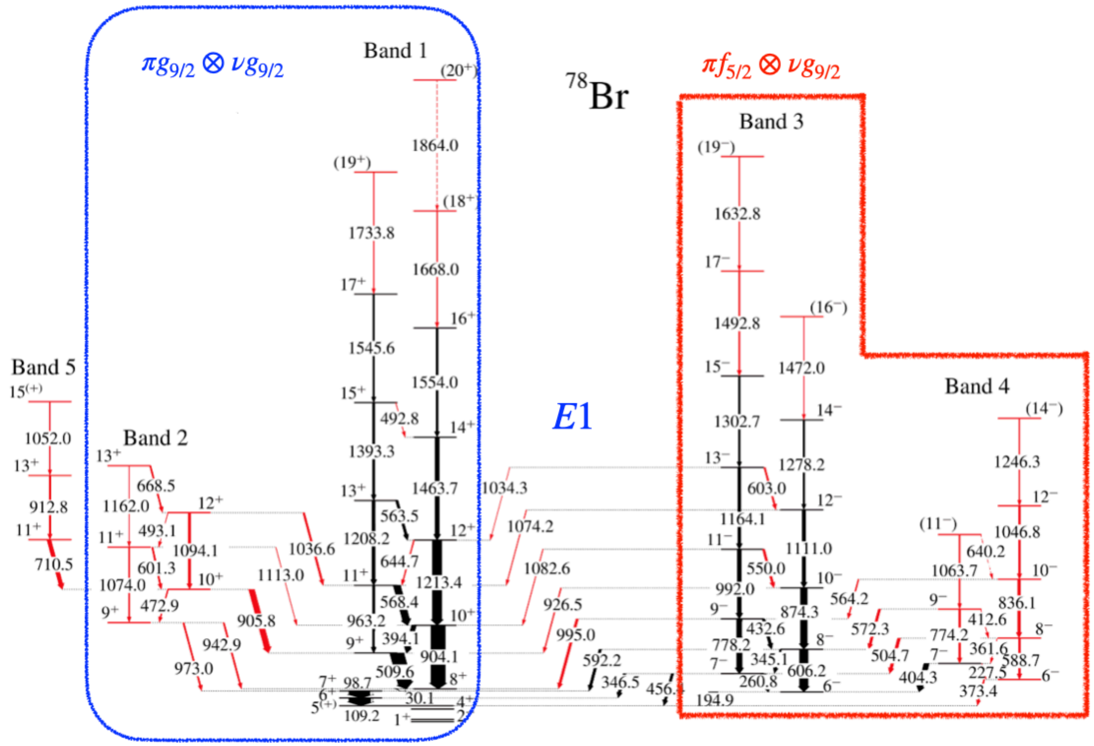
Xiong and **Y. Y. Wang**, ADNDT (2019) 125:193

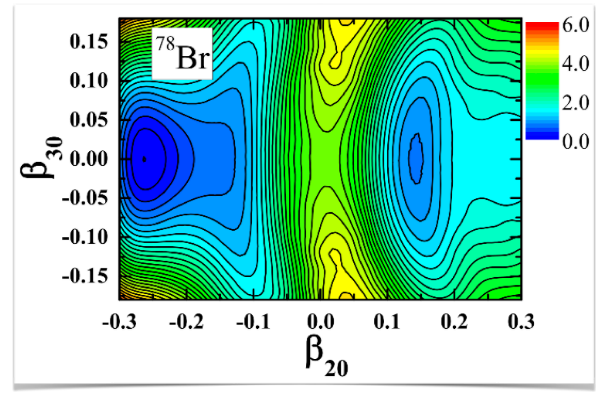


12 candidate chiral doublets in 8 nuclei are newly observed from 2019 to 2023

MχD with octupole correlations

- **MχD with octupole correlations** in ^{78}Br provides an example of chiral geometry in octupole soft nuclei. C. Liu et al., PRL (2016) 116: 112501





- ✓ Possibility of the Chirality-Parity (ChP) violation, i.e., the simultaneous breaking of **chiral** and **reflection** symmetries !
- ✓ A model with both **triaxial** and **octupole** degrees of freedom !

Theoretical progress

- **Particle rotor model (PRM)** Frauendorf&Meng1997NPA; Peng2003PRC; Koike2004PRL; Zhang2007PRC; Qi2009PLB; Chen2018PLB
  - ✓ a quantal model coupling the collective rotation and the single-particle motions in the laboratory reference frame
  - ✓ describes directly the quantum tunneling and energy splitting between the doublets
- **Tilted axis cranking (TAC)** Frauendorf&Meng1997NPA; Dimitrov2000PRL; Olbratorwski2004PRL; Zhao2017PLB
- TAC + random phase approximation Mukhopadhyay2007PRL; Almehed2011PRC
- TAC + collective Hamiltonian Chen2013PRC; Chen2016PRC; Wu2018PRC
- Interaction boson-fermion-fermion model Tonev2006PRL; Brant2008PRC,2009PRC
- Generalized coherent state model Raduta2015JPG,2016JPG,2017JPG
- Projected shell model Bhat2012PLB, 2014NPA; Chen2017PRC,2018PLB Wang2019PRC
- …

Reflection-Asymmetric Triaxial Particle Rotor Model

- The total RAT-PRM Hamiltonian： Y. Y. Wang, Zhang, Zhao, and Meng, PLB (2019) 792: 454

$$\hat{H} = \hat{H}^{p}_{\text{intr.}} + \hat{H}^{n}_{\text{intr.}} + \hat{H}_{\text{core}}$$

✓ The intrinsic Hamiltonian $\hat{H}^{p(n)}_{\text{intr.}}$

ε_ν: Nilsson levels

$$\begin{aligned}\hat{H}^{p(n)}_{\text{intr.}} &= \hat{H}^{p(n)}_{\text{s.p.}} + \hat{H}_{\text{pair}} \\ &= \sum_{\nu>0}(\varepsilon^{p(n)}_{\nu} - \lambda)(a^\dagger_\nu a_\nu + a^\dagger_{\bar{\nu}} a_{\bar{\nu}}) - \frac{\Delta}{2}\sum_{\nu>0}(a^\dagger_\nu a^\dagger_{\bar{\nu}} + a_{\bar{\nu}} a_\nu)\end{aligned}$$

✓ The core Hamiltonian $\hat{H}_{\text{core.}}$:

G. A. Leander and S. K. Sheline, NPA (1984) 413: 375

$$\hat{H}_{\text{core}} = \sum_{i=1}^{3}\frac{\hat{R}^2_i}{2\mathcal{J}_i} + \frac{1}{2}E(0^-)(1-\hat{P}_c)$$

where $\hat{\mathbf{R}}_i$, $\mathcal{J}_i$ are the angular momentum operators and the moments of inertia for the core; $E(0^-)$ is the core parity splitting parameter; $\hat{P}_c = \hat{P}\,\hat{\pi}_p\hat{\pi}_n$ is the parity operator for the core.

$$\mathcal{J}_i = \mathcal{J}_0 \sin^2(\gamma - \frac{2\pi}{3}i), \;\; i = 1,2,3$$

Solution of the RAT-PRM Hamiltonian

- The symmetrized strong-coupled basis is constructed by considering the symmetry of the core rotor concerning the reflection of the intrinsic 1-3 plane

$$|\Psi^{\nu}_{IMK\pm}\rangle = \frac{1}{2\sqrt{1+\delta_{K0}}}(1+\hat{S}_2)|IMK\rangle\psi^{\nu}_{\pm},$$

where $\widehat{S}_2 = \widehat{P}_c\,\widehat{R}_2$ is the reflection operator with respect to the 1-3 plane; $|\mathrm{IMK}\rangle$ is the Wigner function; $\psi^{\nu}_{\pm}$ are the intrinsic wavefunctions with good parity,

$$\psi^{\nu}_{+} = (1+\hat{P})\tilde{\chi}^{\nu}_{p}\tilde{\chi}^{\nu}_{n}\Phi_a = (1+\hat{P}_c\hat{\pi}_p\hat{\pi}_n)\tilde{\chi}^{\nu}_{p}\tilde{\chi}^{\nu}_{n}\Phi_a,$$
$$\psi^{\nu}_{+} = (1-\hat{P})\tilde{\chi}^{\nu}_{p}\tilde{\chi}^{\nu}_{n}\Phi_a = (1-\hat{P}_c\hat{\pi}_p\hat{\pi}_n)\tilde{\chi}^{\nu}_{p}\tilde{\chi}^{\nu}_{n}\Phi_a,$$

where $\widehat{P}$ is the total parity operator; $\widetilde{\chi}^{\,\nu}_{p(n)}$ is the BCS quasiparticle states; Φ_a represents the intrinsic orientation of the core.

- The diagonalization of the $\widehat{H}$ gives rise to the eigenstates $|IMp\rangle$:

$$|IMp\rangle = \sum_{\nu K} c^{\nu}_{IKp}|\Psi^{\nu}_{IMKp}\rangle\,.$$

Electromagnetic transitions

- The reduced electromagnetic transition probabilities $\boldsymbol{B(\sigma\lambda)}$:

$$B(\sigma\lambda, I_i \to I_f') = \frac{1}{2I+1} \sum_{\mu M'} |\langle I'M'p'|\hat{\mathscr{M}}(\sigma\lambda\mu)|IMp\rangle|^2$$

✓ the magnetic dipole (M1) transition operator $\hat{\mathcal{M}}(\boldsymbol{M1\mu})$

$$\hat{\mathscr{M}}(M1,\mu) = \sqrt{\frac{3}{4\pi}}\frac{e\hbar}{2Mc}\left[(g_p - g_R)\hat{j}^p_{1\mu} + (g_n - g_R)\hat{j}^n_{1\mu}\right],$$

✓ the electric multipole transition operator $\hat{\mathcal{M}}(\boldsymbol{E\lambda\mu})$

$$\begin{aligned}\hat{\mathscr{M}}(E\lambda,\mu) &= \hat{q}^{(c)}_{\lambda\mu} + \hat{q}^{(p)}_{\lambda\mu} \\ &= \frac{3Ze}{4\pi}R_0^\lambda\beta_{\lambda\mu} + e_{\rm eff}\sum_{i=1}^{n}(\frac{1}{2} - t_3^{(i)})r_i^\lambda Y^*_{\lambda\mu},\end{aligned}$$

effective charge:

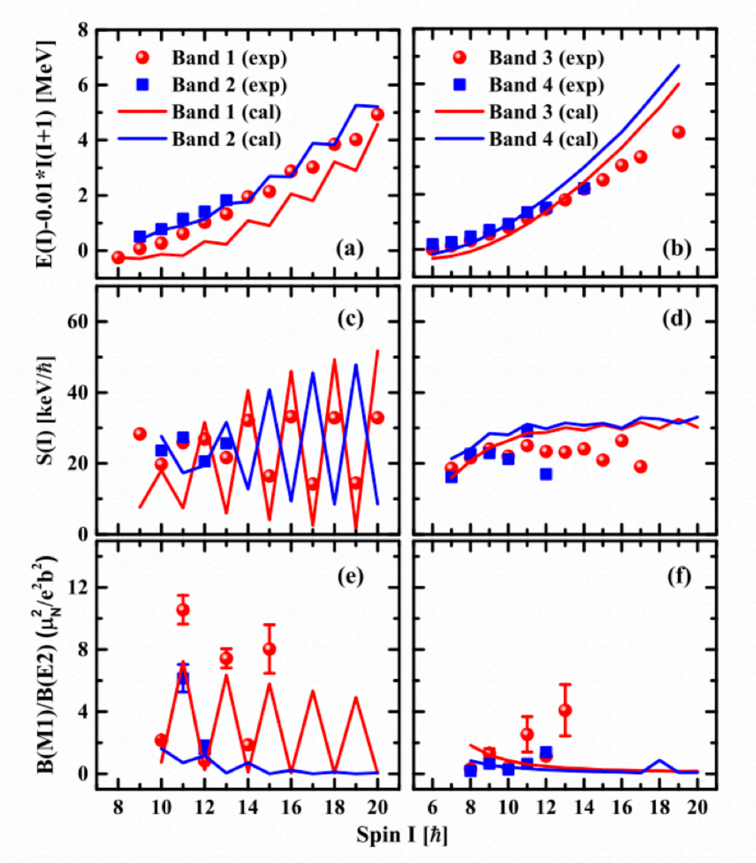
$$e_{\rm eff} = e\left[\frac{N-Z}{2A} - t_3^{(i)}\right] \times \left[1 - 0.7\frac{(\hbar\omega)^2}{(\hbar\omega)^2 - E_\gamma^2}\right].$$

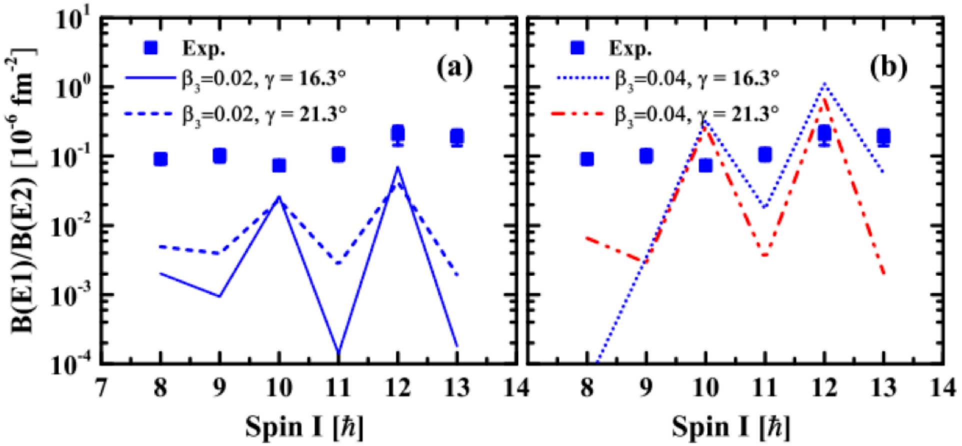
A. Bohr, B. R. Mottelson, Nuclear Structure Vol II (1975)

MχD with octupole correlations within RAT-PRM

- In 2019, a reflection-asymmetric triaxial PRM (RAT-PRM) is developed and applied to investigate the MχD with octupole correlations in ^{78}Br.

Y. Y. Wang, Zhang, Zhao, and Meng, PLB (2019) 792: 454

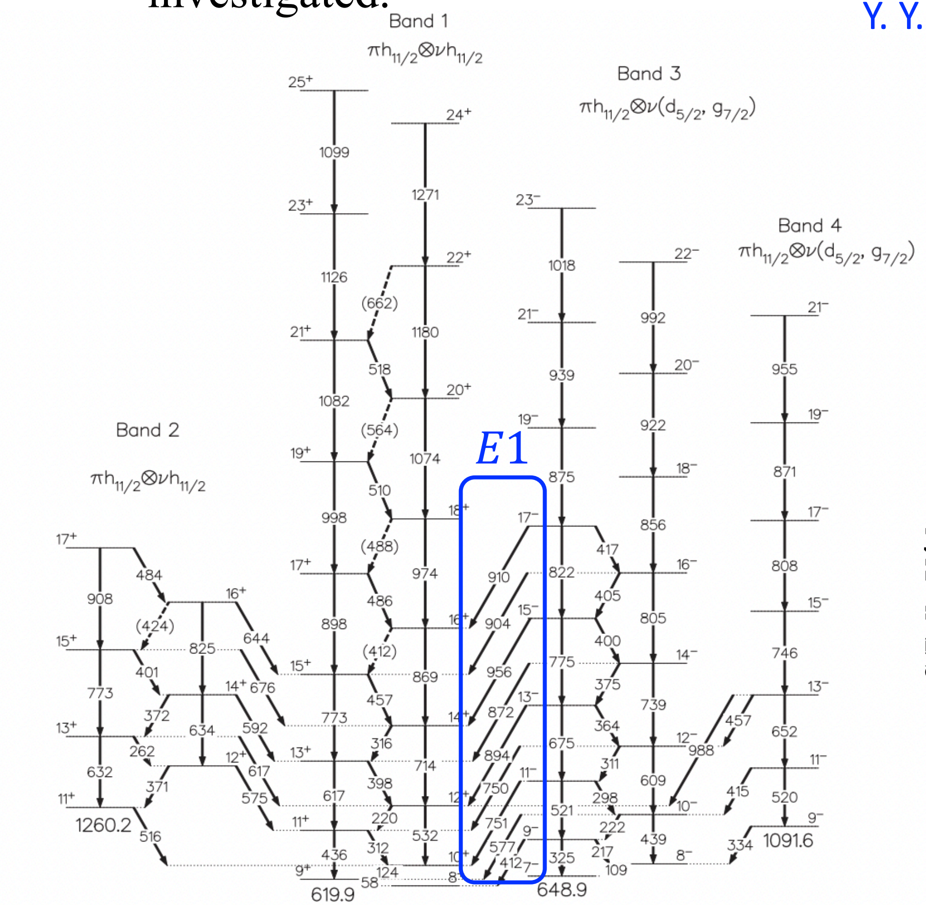




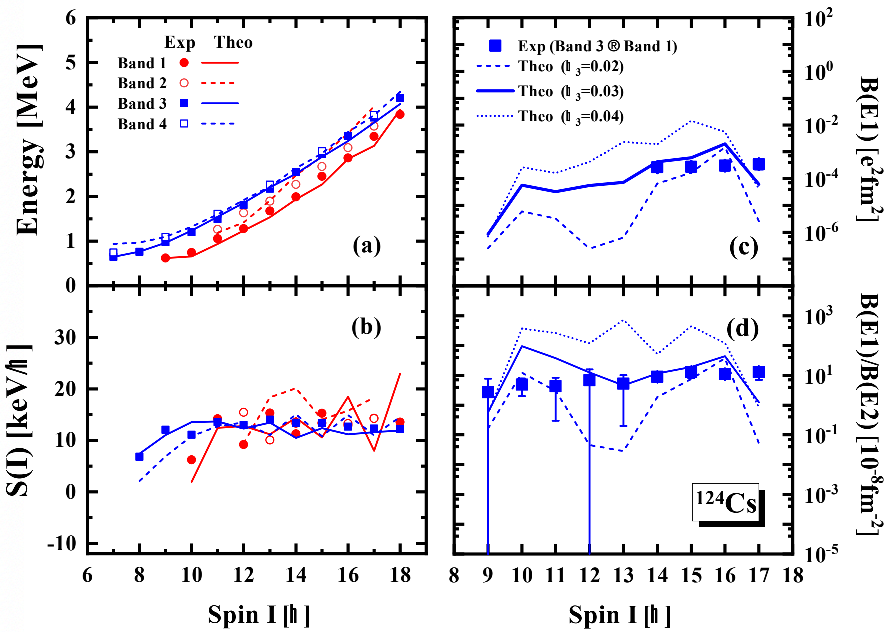
MχD with octupole correlations within RAT-PRM

- In 2020, by using the developed RAT-PRM, the possibility of the observed positive- and negative-parity bands in ^{124}Cs as MχD with octupole correlations has been investigated.

Y. Y. Wang and S. Q. Zhang, PRC (2020) 102: 034303



Data from [PRC (2015) 92:064307]

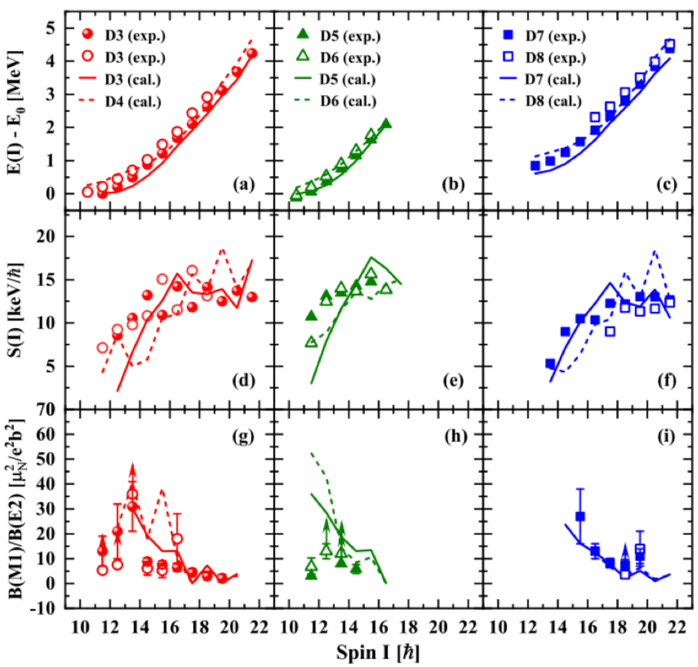


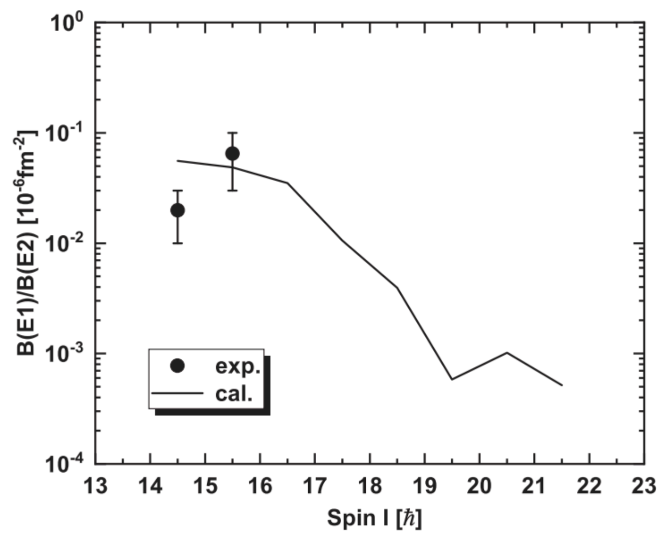
MχD with octupole correlations within RAT-PRM

- In 2020, two pairs of positive-parity bands and one pair of negative-parity bands, as well as the $E1$ transitions among them in nucleus ^{131}Ba, also provides the evidence of MχD candidates with octupole correlations. S. Guo et al., PLB (2020) 807 : 135572

- To investigate the observed MχD with octupole correlations and possible pseudospin-chiral quartet bands, a RAT-PRM with three quasiparticles and a reflection-asymmetric triaxial rotor has been developed.

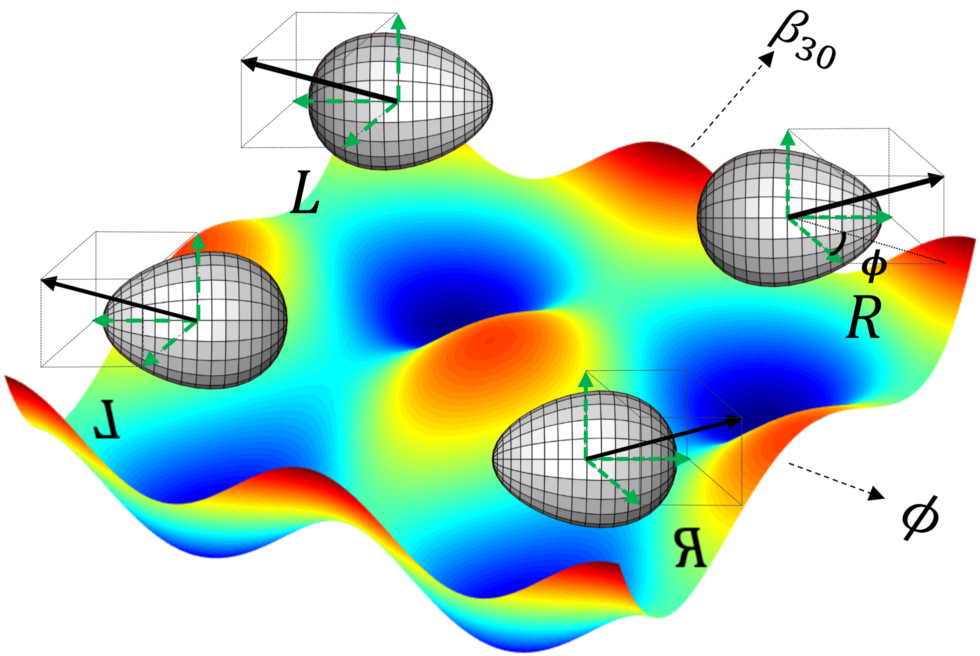
Y. P. Wang, Y. Y. Wang, and J. Meng, PRC (2020) 102: 024313





ChP violation in atomic nuclei

- For the nucleus with both triaxial and octupole deformations, the ChP violation, i.e., the simultaneous breaking of chiral and reflection symmetries may occur in the intrinsic frame.



Nuclear ChP violation has not been observed experimentally !

What's the fingerprint of the nuclear ChP violation ?

- The ChP quartet bands, i.e., four nearly degenerate ΔI=1 bands may be established in the laboratory frame.

S. Frauendorf RMP (2001) 73: 463
C. Liu et al., PRL (2016) 116: 112501

RAT-PRM Hamiltonian in two-j shell

- A special case of RAT-PRM : a rotational core with $\gamma = 90°$ coupled with one particle and one hole in a two-j shell with $\Delta l = \Delta j = 3\hbar$

Y. Y. Wang, X. H. Wu, S. Q. Zhang, P. W. Zhao, and J. Meng, Sci. Bull. (2020) 65: 2001

$$\hat{H} = \hat{H}^{p}_{\rm s.p.} + \hat{H}^{n}_{\rm s.p.} + \hat{H}_{\rm core}$$

✓ The intrinsic Hamiltonian $\hat{\boldsymbol{H}}^{p(n)}_{s.p.}$:

$$\hat{H}^{p}_{\rm s.p.} = +\hat{h}_{lj} + \hbar\omega_0\Big[\beta_{10}Y_{10} - \beta_{30}Y_{30} - \beta_2\cos\gamma Y_{20} - \frac{\beta_2\sin\gamma}{\sqrt{2}}(Y_{22}+Y_{2-2})\Big],$$
$$\hat{H}^{n}_{\rm s.p.} = -\hat{h}_{lj} - \hbar\omega_0\Big[\beta_{10}Y_{10} - \beta_{30}Y_{30} - \beta_2\cos\gamma Y_{20} - \frac{\beta_2\sin\gamma}{\sqrt{2}}(Y_{22}+Y_{2-2})\Big],$$

where $\hat{h}_{lj}$ is the spherical single-particle Hamiltonian with l the orbital angular momentum and j the total angular momentum.

✓ The core Hamiltonian $\hat{\boldsymbol{H}}_{core}$

$$\hat{H}_{\rm core} = \frac{1}{2\mathcal{J}_0}\Big[\hat{R}_3^2 + 4(\hat{R}_1^2 + \hat{R}_2^2)\Big] + \frac{1}{2}E(0^-)(1-\hat{P}_c),$$

New symmetry for ideal ChP violation

- The RAT-PRM Hamiltonian $\hat{H}$ has good total parity P, good angular momentum I, and V_4 symmetry.

- Moreover, if the two-j shell for the particle and hole is the same, the total Hamiltonian is invariant under the operation

$$\hat{\mathcal{A}} = \hat{\mathcal{R}}_3(\frac{\pi}{2})\hat{C}\hat{\pi}$$

T. Koike et al., PRL (2004) 93:172502

  - ✓ $\hat{\mathcal{R}}_3(\frac{\pi}{2})$: the rotation by $\frac{\pi}{2}$ around 3-axis
  - ✓ $\hat{C}$: the exchange of particle and hole
  - ✓ $\hat{\pi}$: the intrinsic space-reflection

- Quantum number of operation $\hat{\mathcal{A}}$: *chiture* $\mathcal{A}$

- Quantum number of operation $\hat{\mathcal{B}} = \hat{P}\hat{\mathcal{A}}$: *chiplex* $\mathcal{B}$

$$\mathcal{R}_3(\pi) = e^{i\pi\hat{R}_3} \quad \to r \quad \textit{signature}$$
$$\mathcal{S} = \hat{P}\mathcal{R}_3(\pi) \quad \to s \quad \textit{simplex}$$

- The energy eigenstates are simultaneous eigenstates of the $\hat{\mathcal{B}}$ operator having eigenvalues of $\mathcal{B} = +1$ or -1.

Selection rules of EM transitions

- The eigenstates of the RAT-PRM Hamiltonian can be characterized by the total parity $\boldsymbol{P}$ and the *chiplex* $\boldsymbol{\mathcal{B}}$.
  - ✓ $\boldsymbol{V_4}$ symmetry, which leads to $R_3 = 0, \pm 2, \pm 4, \cdots$

$$\hat{\mathcal{B}} = \hat{P}\hat{\mathcal{A}} = \hat{P}\hat{\mathcal{R}}_3(\frac{\pi}{2})\hat{C}\hat{\pi}$$

$$e^{\frac{\pi}{2}R_3} = \cos\frac{\pi}{2}R_3 + i\sin\frac{\pi}{2}R_3$$

$\boldsymbol{P}$	$\boldsymbol{\mathcal{B}}$	$\mathcal{A}$	R_3	$C\pi$
$\boldsymbol{+1}$	$\boldsymbol{+1}$	$+1$	$0, \pm 4, \pm 8 \cdots$	$+1$
			$\pm 2, \pm 6 \cdots$	-1
	$\boldsymbol{-1}$	-1	$0, \pm 4, \pm 8 \cdots$	-1
			$\pm 2, \pm 6 \cdots$	$+1$
$\boldsymbol{-1}$	$\boldsymbol{+1}$	-1	$0, \pm 4, \pm 8 \cdots$	-1
			$\pm 2, \pm 6 \cdots$	$+1$
	$\boldsymbol{-1}$	$+1$	$0, \pm 4, \pm 8 \cdots$	$+1$
			$\pm 2, \pm 6 \cdots$	-1

- Chiral doublets: states with same parity $\boldsymbol{P}$ but different *chiture* $\boldsymbol{\mathcal{A}}$
- Parity doublets: states with same *chiture* $\boldsymbol{\mathcal{A}}$ but different parity $\boldsymbol{P}$

Selection rules of EM transitions

- The $\boldsymbol{B(E2, I_i \to I_f)}$ can be calculated from operator

$$\hat{\mathcal{M}}(E2) = \sum_{\mu=0,\pm 2} \hat{q}^{(c)}_{2\mu}$$

Suppose that the parity and *chiplex* of the initial and final states are $(\boldsymbol{P_i}, \boldsymbol{\mathcal{B}_i})$ and $(\boldsymbol{P_f}, \boldsymbol{\mathcal{B}_f})$, respectively.

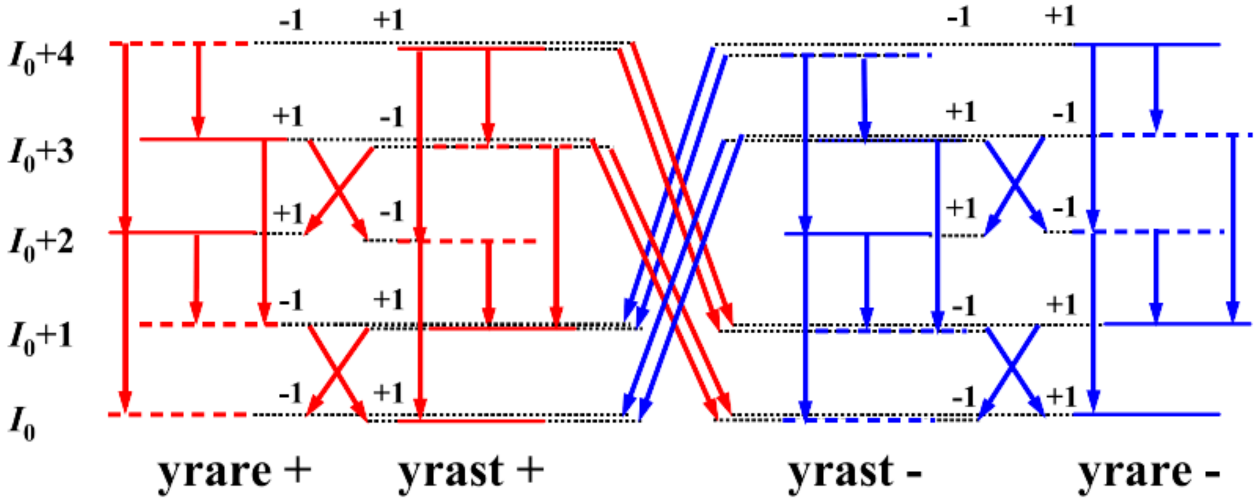
$E2$ transitions link states with same parity	→	$P_i = P_f$
the intrinsic part of $\hat{\mathcal{M}}(\boldsymbol{E2})$ is neglected	→	$C_i = C_f \quad \pi_i = \pi_f$
For $\gamma = 90°, \hat{q}_{20} \propto \beta_2 \cos\gamma$ vanish	→	$(R_3)_i = (R_3)_f \pm 2$

$$\mathcal{B}_i = P_i e^{i\frac{\pi}{2}(R_3)_i} C_i \pi_i = P_f e^{i\frac{\pi}{2}(R_3)_f \pm 2} C_f \pi_f = -\mathcal{B}_f$$

The *E2* matrix elements occur only between states with different *chiplex* $\boldsymbol{\mathcal{B}}$

ChP quartet bands

- Starting from the yrast and yrare states for the positive and negative parity at given I_0, two pairs of chiral doublet bands are organized with the allowed *E2* transitions between $\Delta I = 2$ states, which constitute the ChP quartet bands.



- ✓ *Chiplex* $\mathcal{B}$ changes the sign every $2\hbar$ in a band, no interband *E2* transitions;
- ✓ Intraband and interband $M1$ transitions exhibit staggering behavior with spin;
- ✓ $E3$ transitions between $(yrast + \to yrast -;\ yrare + \to yrare -)$ and $(yrast + \to yrare -;\ yrare + \to yrast -)$ alternating with spin

The *E2, M1, E3* transitions occur only between states with different *chiplex* $\mathcal{B}$

Summary and Perspective

- Development of a RAT-PRM with both **triaxial** and **octupole** degrees of freedom
- Interpretation of the observed **MχD with octupole correlations** in octupole soft nuclei
- Predication of the fingerprints of the **ChP violation** in atomic nuclei

IN THE FOLLOWING:

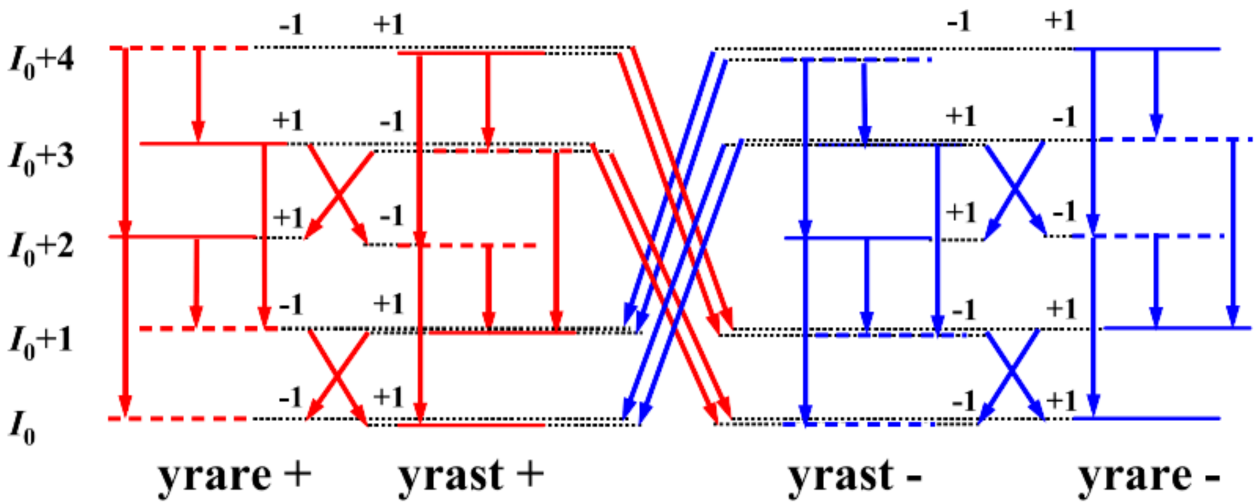
- **ChP violation in realistic nuclei**
- **Prediction of ChP violation candidates**
- **…**

Thank you !

Appendix

ChP quartet bands

- Starting from the yrast and yrare states for the positive and negative parity at given I_0, two pairs of chiral doublet bands are organized with the allowed *E2* transitions between $\Delta I = 2$ states, which constitute the ChP quartet bands.



- ✓ *Chiplex* $\mathcal{B}$ changes the sign every $2\hbar$ in a band, no interband *E2* transitions;
- ✓ The intraband and interband $B(M1)$ for chiral doublet bands exhibit staggering behavior;

Selection rules of EM transitions

	$E2$	$M1$		$E3$	
Intra-band	$I \to I-2$	$I \to I-1$	$I+1 \to I$	$I \to I-3$	$I+1 \to I-2$
yrast+ → yrast+ yrare+ → yrare+ yrast− → yrast− yrare− → yrare−	√	√	×	−	−
Inter-band	$I \to I-2$	$I \to I-1$	$I+1 \to I$	$I \to I-3$	$I+1 \to I-2$
yrast+ ↔ yrare+ yrast− ↔ yrare−	×	×	√	−	−
yrast+ ↔ yrast− yrare+ ↔ yrare−	−	−	−	√	×
yrast+ ↔ yrare− yrare+ ↔ yrast−	−	−	−	×	√

√：允许的跃迁　　×： *chiplex* $\mathcal{B}$ 禁戒的跃迁　　−：宇称 ***P*** 禁戒的跃迁

$M1$ 跃迁选择定则

- 对于磁偶极跃迁 $\boldsymbol{B(M1, I_i \to I_f)}$，跃迁算符

$$g_p - g_R = +1$$
$$g_n - g_R = -1$$

$$\hat{\mathcal{M}}(M1) = \sum_{\mu=0,\pm} \sqrt{\frac{3}{4\pi}} \frac{e\hbar}{2Mc} \left[(g_p - g_R) \hat{j}^p_{1\mu} + (g_n - g_R) \hat{j}^n_{1\mu} \right]$$

假设初、末态的宇称和 chiplex 分别为 $(P_i, \mathcal{B}_i)$ 和 $(P_f, \mathcal{B}_f)$

$M1$ 跃迁连接相同宇称态	→	$P_i = P_f$
$\hat{\mathcal{M}}(M1)_\mu \propto \hat{j}^p_{1\mu} - \hat{j}^n_{1\mu}$ 不改变内禀宇称	→	$C_i = -C_f \quad \pi_i = \pi_f$
连接 $\Delta R_3 = 0, \pm 1$ 的态	→	$(R_3)_i = (R_3)_f$

$$\mathcal{B}_i = P_i e^{i\frac{\pi}{2}(R_3)_i} C_i \pi_i = P_f e^{i\frac{\pi}{2}(R_3)_f} (-C_f) \pi_f = -\mathcal{B}_f$$

初末态 $\mathcal{B}$ 相反时存在 $M1$ 跃迁

$E3$ 跃迁选择定则

- 对于电八极跃迁 $\boldsymbol{B(E3, I_i \to I_f)}$，跃迁算符

$$Q_{3\mu} = \frac{3Ze}{4\pi} R_0^3 \beta_{3\mu}$$

$$\hat{\mathcal{M}}(E3) = \sum_{\mu=-3}^{3} Q_{3\mu} D_{3\mu}$$

假设初、末态的宇称和 chiplex 分别为 $(\boldsymbol{P_i}, \boldsymbol{\mathcal{B}_i})$ 和 $(\boldsymbol{P_f}, \boldsymbol{\mathcal{B}_f})$

$E3$ 跃迁连接不同宇称态	→	$P_i = -P_f$
仅考虑集体部分贡献	→	$C_i = C_f \quad \pi_i = \pi_f$
$Q_{30} \neq 0$, 连接 $\Delta R_3 = 0$ 的态	→	$(R_3)_i = (R_3)_f$

$$\mathcal{B}_i = P_i e^{i\frac{\pi}{2}(R_3)_i} C_i \pi_i = (-P_f) e^{i\frac{\pi}{2}(R_3)_f} C_f \pi_f = -\mathcal{B}_f$$

初末态 $\mathcal{B}$ 相反时存在 $E3$ 跃迁

$E1$ 跃迁选择定则

- 对于电偶极跃迁 $\boldsymbol{B(E1, I_i \rightarrow I_f)}$，跃迁算符

$\beta_{10} \propto \beta_2\beta_{30}\cos\gamma$

$$\hat{\mathcal{M}}(E1) = \hat{q}_{10}^{(c)} + \hat{q}_{10}^{(p)}$$
$$= \frac{3Ze}{4\pi} R_0\beta_{10} + e_{\text{eff}} \sum_{i=1}^{n} r_i Y_{10}^*$$

A. Bohr & B. R. , Nuclear structure Vol. II (1975)

✓ 集 体 部 分：

$\gamma = 90°$ 时，$\hat{q}_{10}^{(c)} \propto \beta_2\beta_{30} cos\gamma = 0$

✓ 单粒子部分：

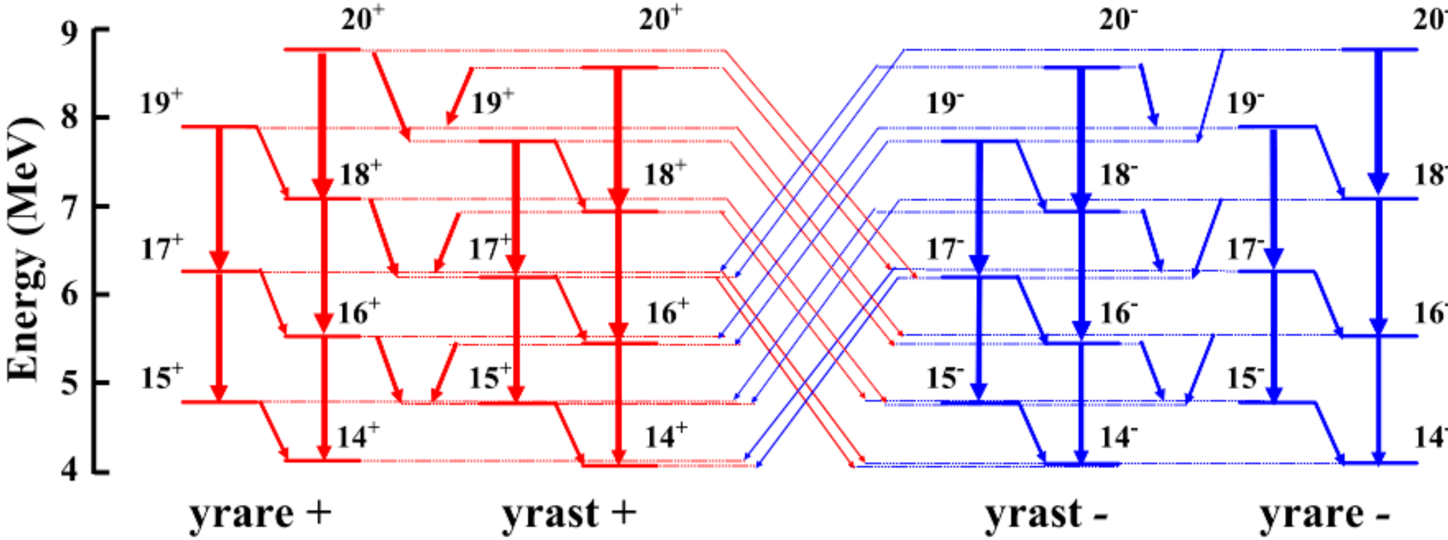
$\Delta l = \Delta j = 3\hbar$ 双 j 壳下，$\hat{q}_{10}^{(p)}$ 的 rY_{10} 矩阵元为 0

初末态 $\mathcal{B}$ 相反时存在 $E2$、$M1$、$E3$ 跃迁

V_4 symmetry

- In the group theory, the group members of V_4 group include
  - ✓ $\boldsymbol{S_1}$**:** the reflection operation with respect to the 2-3 plane
  - ✓ $\boldsymbol{S_2}$**:** the reflection operation with respect to the 1-3 plane
  - ✓ $\boldsymbol{S_3}$**:** a rotation by $\mathbf{180}^\circ$ about the 3-axis
  - ✓ $\boldsymbol{E}$ **:** unit group member

ChP violation in two-j shell



- ✓ Two-j shell: $\boldsymbol{h_{11/2}, d_{5/2}}$
- ✓ Quadrupole deformations: $\boldsymbol{\beta_2 = 0.3, \gamma = 90^\circ}$
- ✓ Octupole deformation: $\boldsymbol{\beta_3 = 0.1}$
- ✓ Core-parity splitting parameter: $\boldsymbol{E(0^-) = 0\ \mathrm{MeV}}$
- ✓ Moment of inertia: $\boldsymbol{J_0 = 18.0\ \hbar^2/\mathrm{MeV}}$