Chirality and Wobbling in Atomic Nuclei (CWAN'23)
July 10 - July 14, 2023, Huizhou, China

Study of chirality and wobbling within CDFT+PRM

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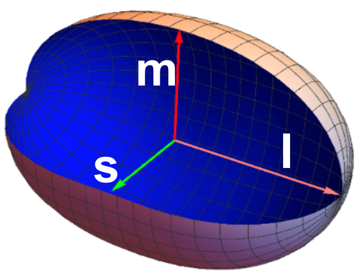
Outline

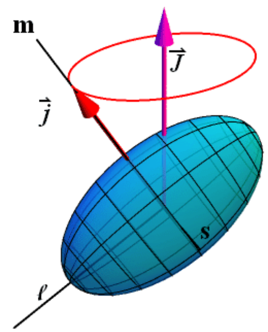
- Chirality and wobbling
- CDFT+PRM
- CDFT+PRM for chirality
- CDFT+PRM for wobbling
- Summary

Wobbling and chirality

- **Nuclear wobbling:**

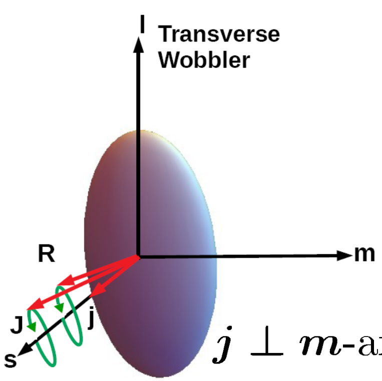
Bohr & Mottelson1975, Vol. II, page 190





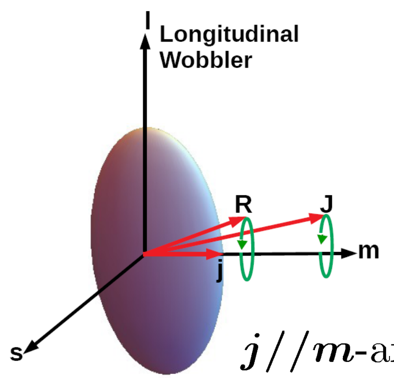
Wobbling in odd-mass nuclei

Frauendorf &Dönau, PRC 89, 014322 (2014)



$j \perp m$-axis

$\mathcal{J}_m > \mathcal{J}_s, \mathcal{J}_l$

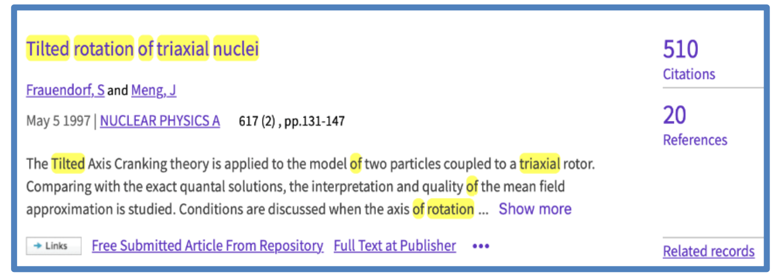


$j // m$-axis

$\mathcal{J}_m > \mathcal{J}_s, \mathcal{J}_l$

- **Nuclear chirality**

Frauendorf &Meng, NPA 617, 131 (1997)


Tilted rotation of triaxial nuclei

Frauendorf, S and Meng, J

May 5 1997 | NUCLEAR PHYSICS A 617 (2) , pp.131-147

The Tilted Axis Cranking theory is applied to the model of two particles coupled to a triaxial rotor. Comparing with the exact quantal solutions, the interpretation and quality of the mean field approximation is studied. Conditions are discussed when the axis of rotation ... Show more

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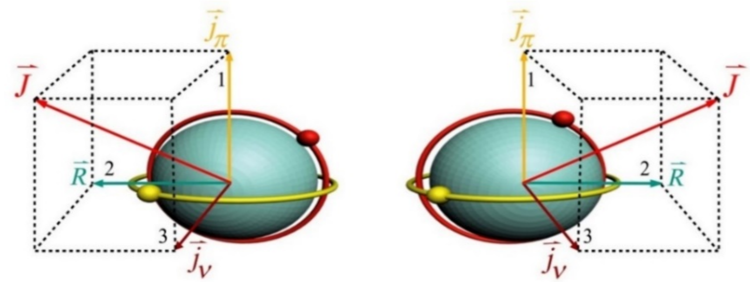
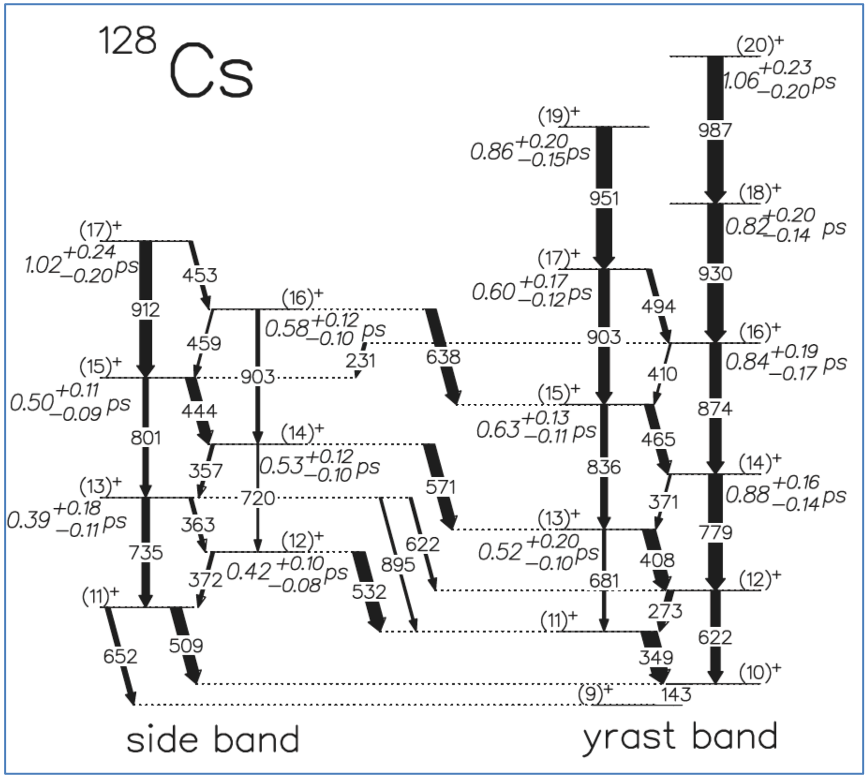


(a) Chiral objects

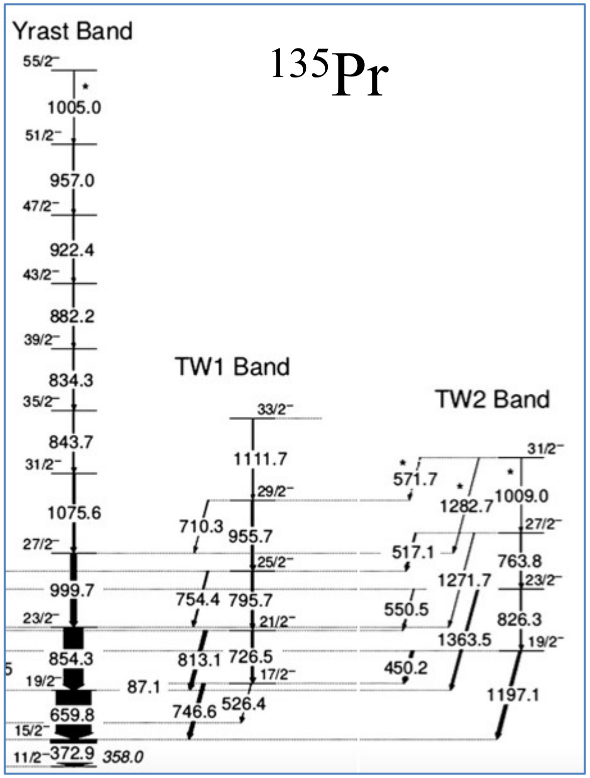
c.f. Prof. Frauendorf's and Prof. Meng's talk

Coupling and fluctuation?

Chiral doublet bands and wobbling multi-phonon bands: **How to deal with the coupling between the collective motion and particle motion? How to deal with the quantal fluctuation of angular momentum orientation?**



Chiral doublet bands



Wobbling bands

Particle rotor model

- The Nobel Prize in Physics 1975 was awarded jointly to Aage Niels Bohr, Ben Roy Mottelson and Leo James Rainwater "for the discovery of the connection between collective motion and particle motion in atomic nuclei and the development of the theory of the structure of the atomic nucleus based on this connection."

https://www.nobelprize.org/prizes/physics/1975/summary/

- PRM: the rotating nuclear system is described as a coupling system of a core (with different shape) and valence particles (one, two, …)

$$\hat{H}_{\mathrm{PRM}} = \hat{H}_{\mathrm{coll}} + \hat{H}_{\mathrm{intr}}$$

Bohr & Mottelson1975, Vol. II, Appendix 4A

$$\hat{H}_{\mathrm{coll}} = \sum_{i=1}^{3} \frac{\hat{R}_i^2}{2\mathcal{J}_i} = \sum_{i=1}^{3} \frac{(\hat{I}_i - \hat{j}_i)^2}{2\mathcal{J}_i} \qquad \hat{H}_{\mathrm{intr}} = \sum_{\nu} \varepsilon_\nu a_\nu^\dagger a_\nu$$

Achievements: simple, economic, quantal model

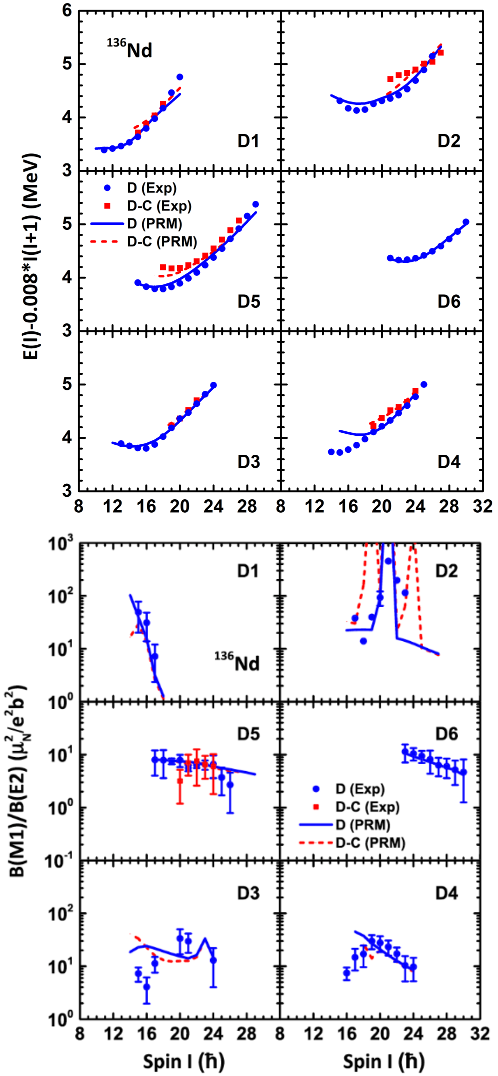
To describe the wobbling and chiral motion, triaxial particle rotor model is developed:

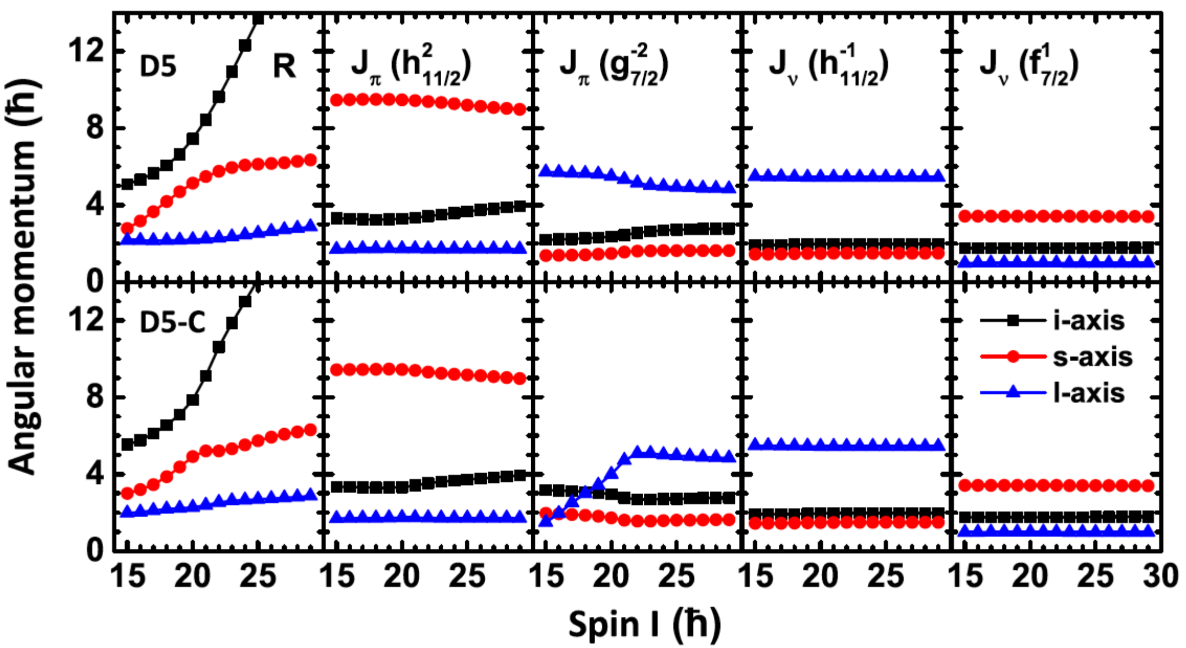
PRC 82, 067302 (2010); PRL 110, 1772504 (2013); PRC 94, 054308 (2016); PRC 98, 044314 (2018); PLB 782, 744 (2018); PRC 100, 061301(R) (2019); PLB 807, 135596 (2020); PLB 807, 135568 (2020); EPJA 58, 75 (2022)

Four-j shells PRM

- **Four-j shells PRM was developed to describe the chirality of ^{136}Nd**

QB&Lv&Petrache&Meng, PLB 782, 744 (2018)

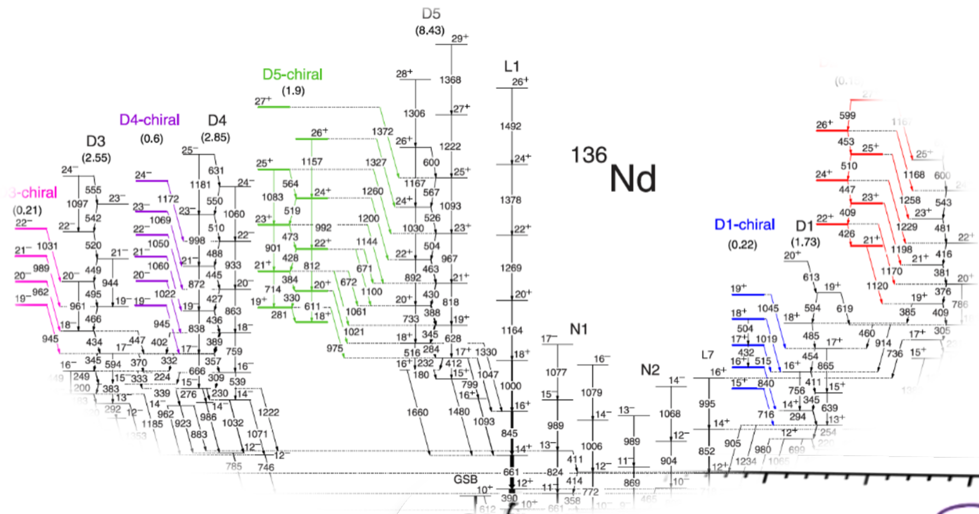




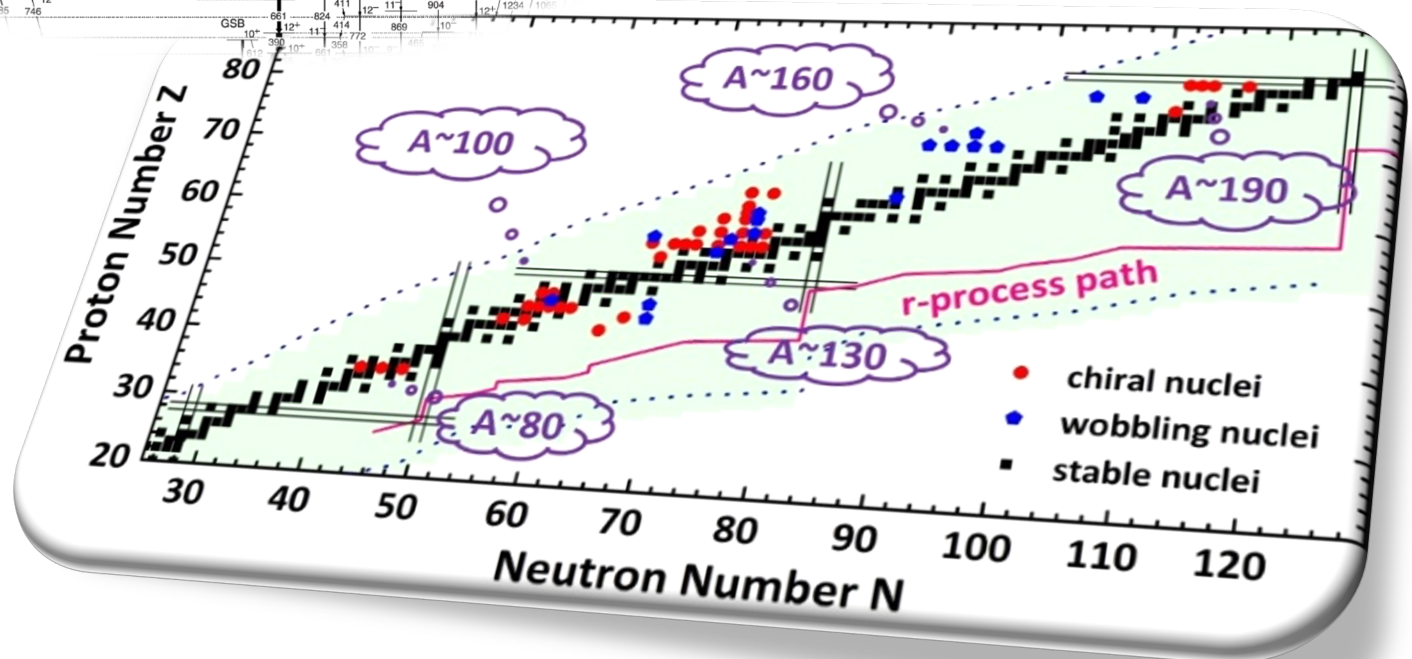
PRM describes the experimental energy spectra well and provides the chiral geometry picture.

c.f. Prof. Petrache's talk

Configuration and deformation?

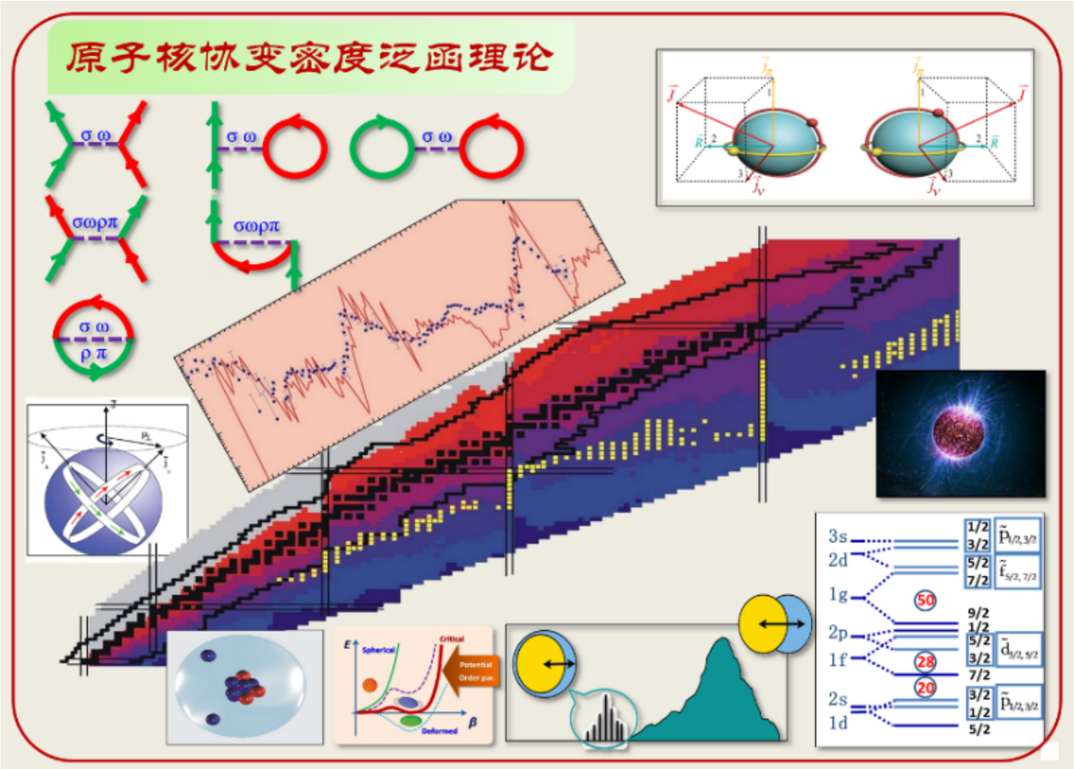


Chiral and wobbling bands locate in different mass regions accompanied with complex levels. How to know their deformation parameters and valence particle configuration?



Covariant density functional theory

- Covariant density functional theory: a relativistic quantum many-body theory that based on effective field theory and density functional theory.
- microscopic, universal, reliable, …

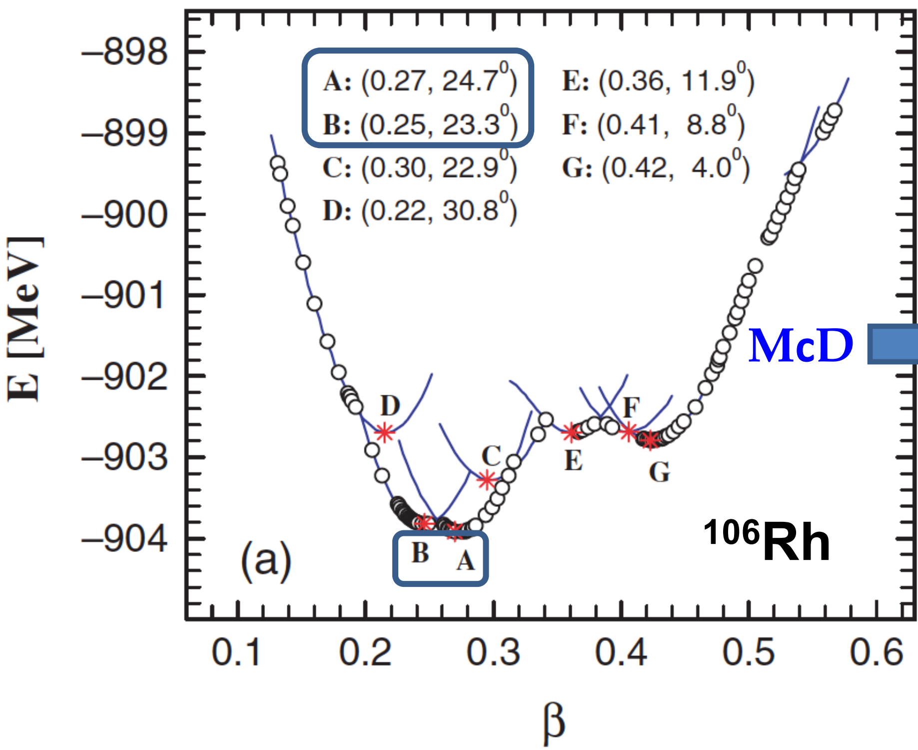


- Constraint calculations are developed to obtain deformation and nucleon occupation information. *PRC 73, 037303 (2006)*

Multiple chiral doublets (McD)

- **McD:** more than one pair of chiral doublet bands in **one** single nucleus

Meng&Peng&Zhang&Zhou, PRC73, 037303 (2006)



Config. A $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^{1}$

Config. B $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^{2}(gd)^{1}$

fingerprint of triaxial shape coexistence

Potential energy curve by constrained covariant density functional theory (CDFT)

c.f. Prof. Meng's talk

The investigation followed by:

- *Prediction for other odd-odd Rh isotopes*
  Peng et al., PRC77, 024309 (2008)
- *Confirmed with time-odd fields included*
  Yao et al., PRC79, 067302 (2009)
- *Odd-A Rh isotopes and other isotopes*
  Li et al., PRC83, 037301 (2011); 97, 034306 (2018)
  Qi et al., PRC98, 014305 (2018);
  Peng et al., PRC 98, 024320 (2018);
  Hu et al., PRC104, 064325 (2021)

How do we study

- **A combination of PRM (quantal descriptions) and CDFT (microscopic inputs) is used to investigate nuclear rotation; a standard processes is carried out step by step…**
    - **experimental information: energy level scheme, EM transition**

1. **Learn from data:** **analysis key physical quantity (rotational frequency, quasi-particle alignment…)**
2. **CDFT:** **constrained calculations to learn the occupations of valence nucleons (configuration), to get reliable deformation information**
3. **PRM:** **describe data, study the underlying physics**

balanced "efficiency" and "fairness"
work rather well for complicated level schemes

- ✓ *PRL 110, 172504 (2013)*
- ✓ *PRL 112, 202502 (2014)*
- ✓ *PRL 113, 032501 (2014)*
- ✓ *PRL 116, 112501 (2016)*
- ✓ *PRL 120, 022502 (2018)*
- ✓ *PRL 122, 062501 (2019)*
- ✓ *PRL 124, 052501 (2020)*
- ✓ *PRL 125, 132501 (2020)*
- ✓ *……*

- **In the following, some of results will introduced.**

Outline

- ❑ **CDFT+PRM for chirality**

McD in ^{133}Ce

- **McD in ^{133}Ce: first application, 2013, 10th anniversary**

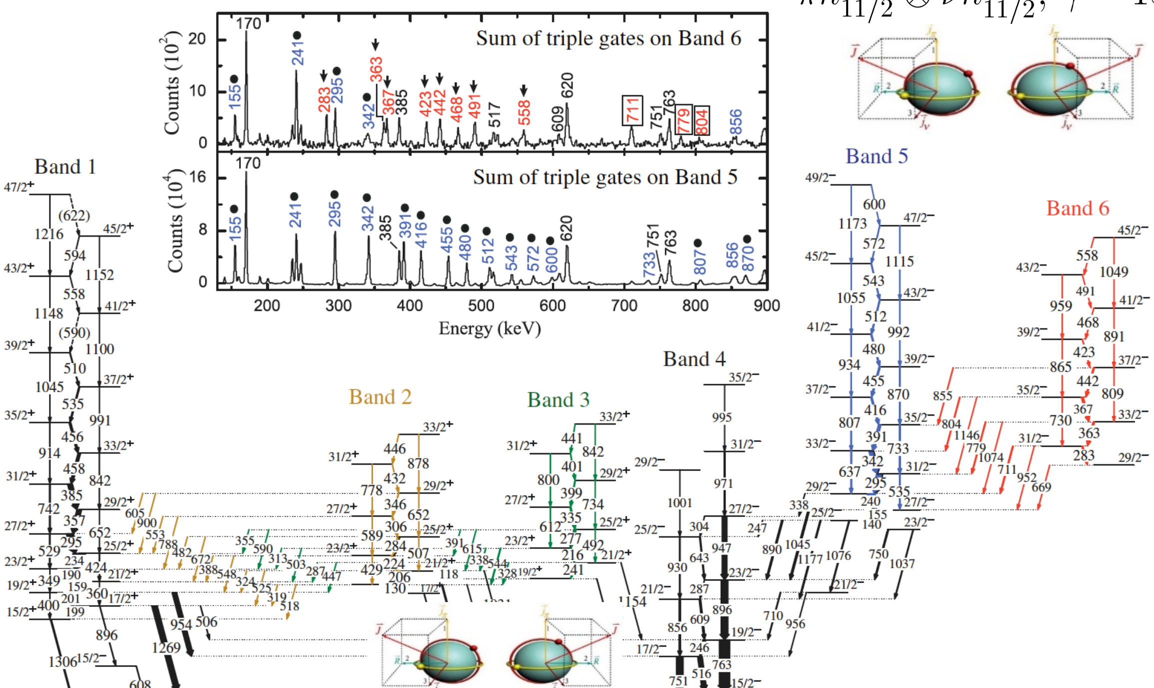
Exp@ANL; Theorey@CDFT+PRM

PRL **110,** 172504 (2013) PHYSICAL REVIEW LETTERS week ending 26 APRIL 2013

Evidence for Multiple Chiral Doublet Bands in ^{133}Ce

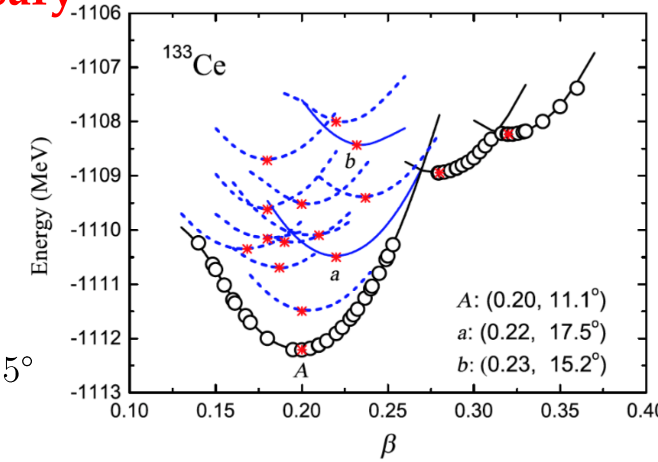
A. D. Ayangeakaa,[1] U. Garg,[1] M. D. Anthony,[1] S. Frauendorf,[1] J. T. Matta,[1] B. K. Nayak,[1,*] D. Patel,[1] Q. B. Chen (陈启博),[2] S. Q. Zhang (张双全),[2] P. W. Zhao (赵鹏巍),[2] B. Qi (亓斌),[3] J. Meng (孟杰),[2,4,5] R. V. F. Janssens,[6] M. P. Carpenter,[6] C. J. Chiara,[6,7] F. G. Kondev,[8] T. Lauritsen,[6] D. Seweryniak,[6] S. Zhu,[6] S. S. Ghugre,[9] and R. Palit[10,11]

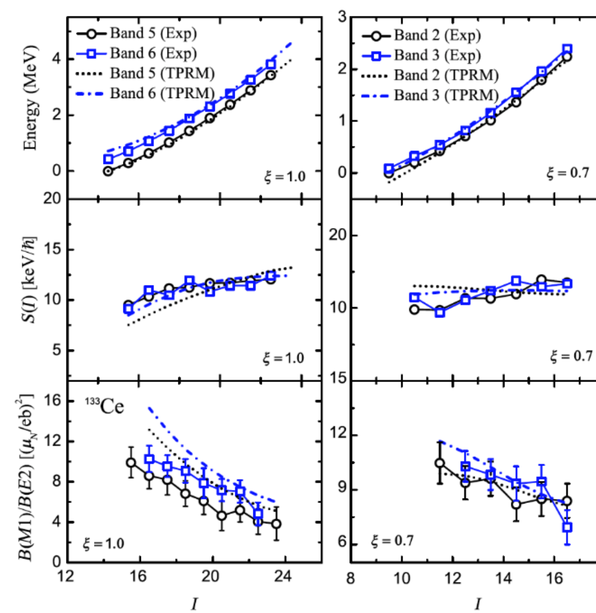
$\pi h^2_{11/2} \otimes \nu h^{-1}_{11/2}$, $\gamma \sim 15^\circ$



$\pi[g^{-1}_{7/2} h^1_{11/2}] \otimes \nu h^{-1}_{11/2}$, $\gamma \sim 18^\circ$

^{133}Ce





CDFT+PRM for chirality

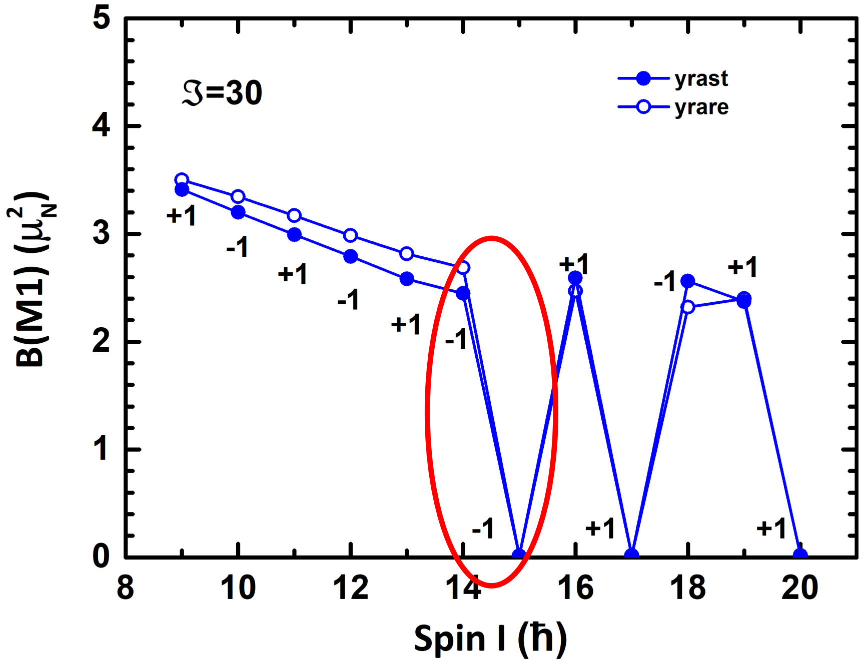
- ^{133}Ce: McD, triaxial shape coexistence. *PRL 110, 172504 (2013)*
- ^{103}Rh: McD with identical conf., robustness. *PRL 113, 032501 (2014)*
- ^{106}Ag: band-crossing, chiral conundrum. *PRL 112, 202502 (2014)*
- ^{78}Br: McD with octupole correlations. *PRL 116, 112501 (2016)*
- ^{133}La: chirality. *PRC 94, 064309 (2016)*
- ^{128}Cs: g-factor. *PRL 120, 022502 (2018); PRC 106, 014318 (2022)*
- ^{136}Nd: McD in e-e nucleus, 4-j PRM. *PLB 782, 744 (2018)*
- Co isotopes: McD prediction. *PRC 98, 024320 (2018)*
- ^{60}Ni: chirality prediction. *PLB 793, 303 (2019)*
- ^{135}Nd: McD. *PRC 100, 024314 (2019)*
- ^{195}Tl: chirality and planar rotation. *PLB 806, 135489 (2020)*
- ^{137}Nd: McD. *EPJA 56, 208 (2020)*
- ^{81}Kr: McD and pseudospin doublets. *PLB 827, 137006 (2022)*

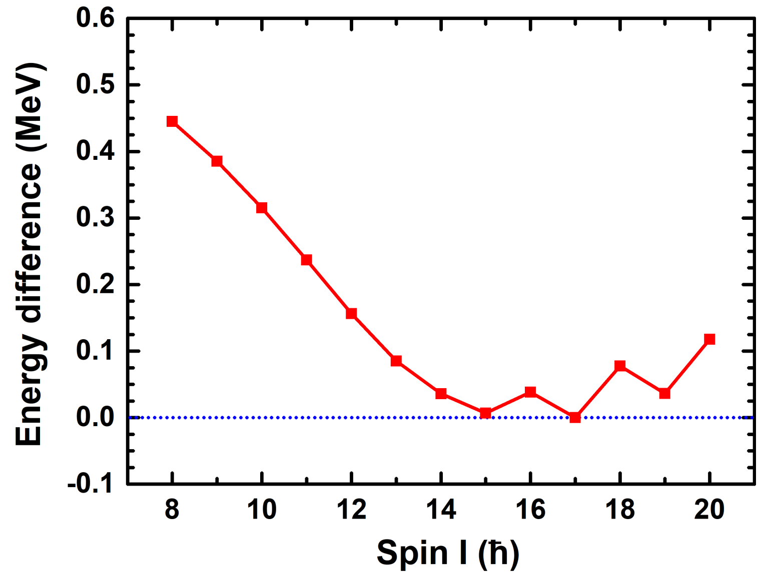
14 papers: 5PRL+4PLB+4PRC+1EPJA

A~60 | A~80 | A~100 | A~130 | A~190

B(M1) staggering

- **Particle-hole calculations with g=270: q with m-axis, φ with s-axis**



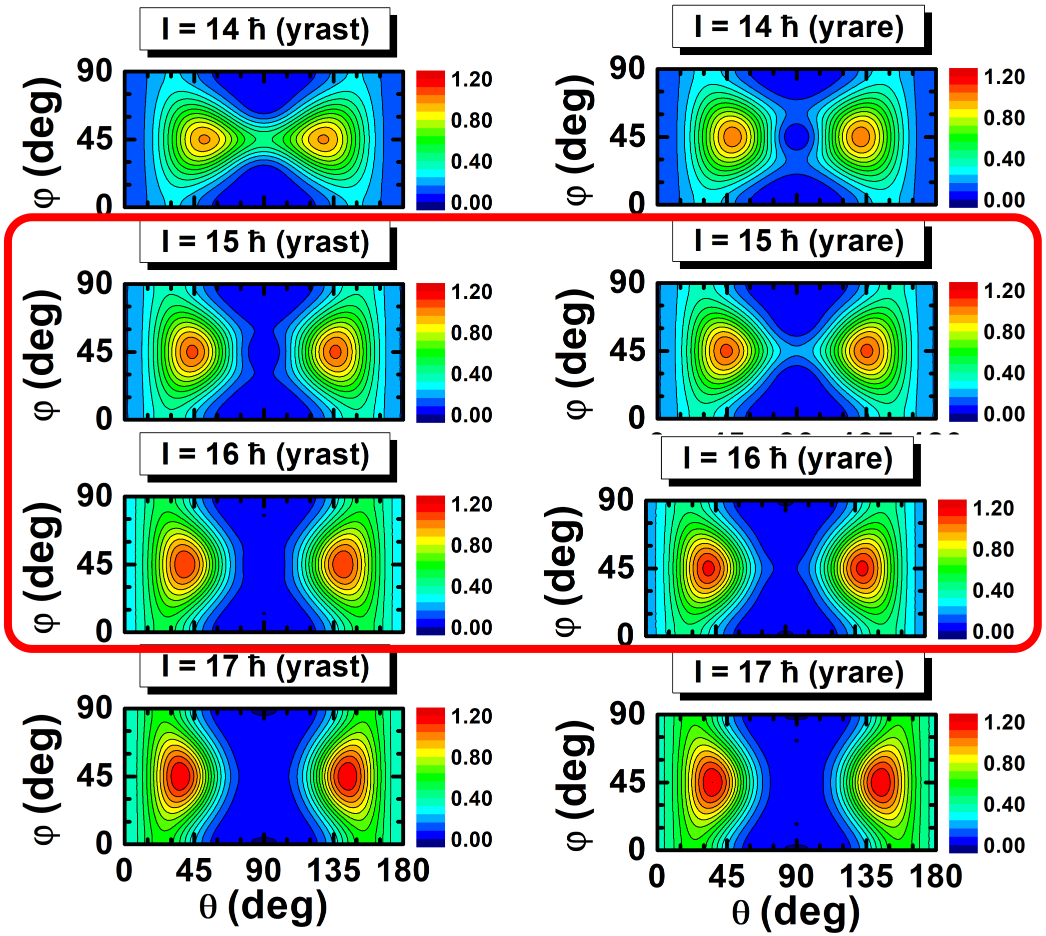


The values +1, -1, +1, -1, … are the eigenvalues of the A=R3(pi/2)C, introduced in the Koike2004PRL paper. The B(M1) is enhanced between opposite A values.

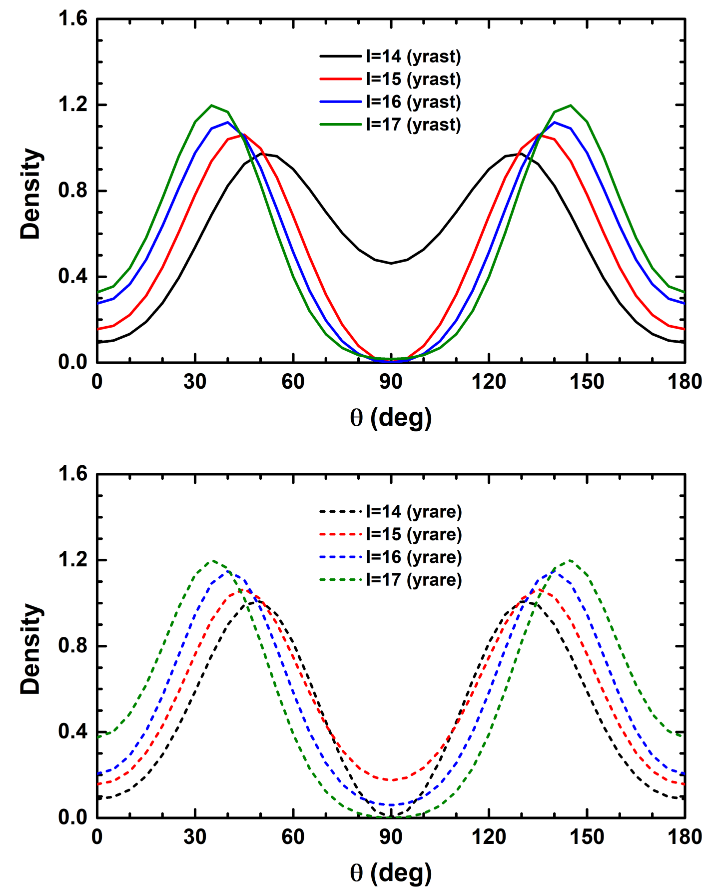
The energy difference at I=15 and 17 is very small (<6 keV)

B(M1) staggering

- Particle-hole calculations with g=270: q with m-axis, φ with s-axis



At φ=45

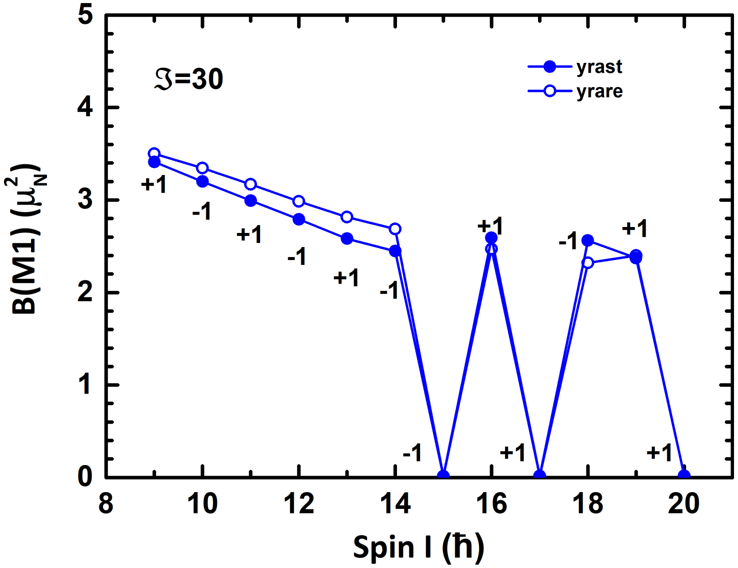


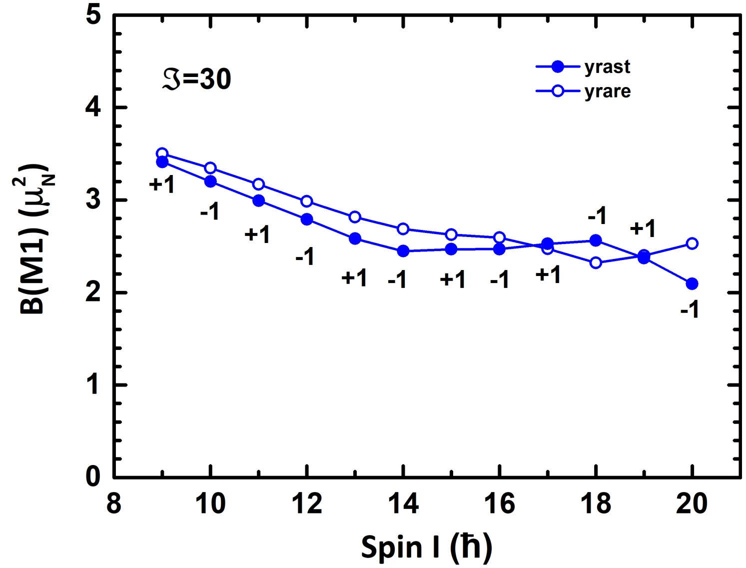
It looks that the yrast and yrare states at I=15, 16 should interchange

B(M1) staggering

- **Particle-hole calculations with g=270**

Interchange: organize the band according to the a-plot





The question: why a symmetric wave function corresponds to a higher energy, while an anti-symmetric wave function corresponds to a lower energy at I=15, 16? Not solved yet.

Outline

- **CDFT+PRM for wobbling**

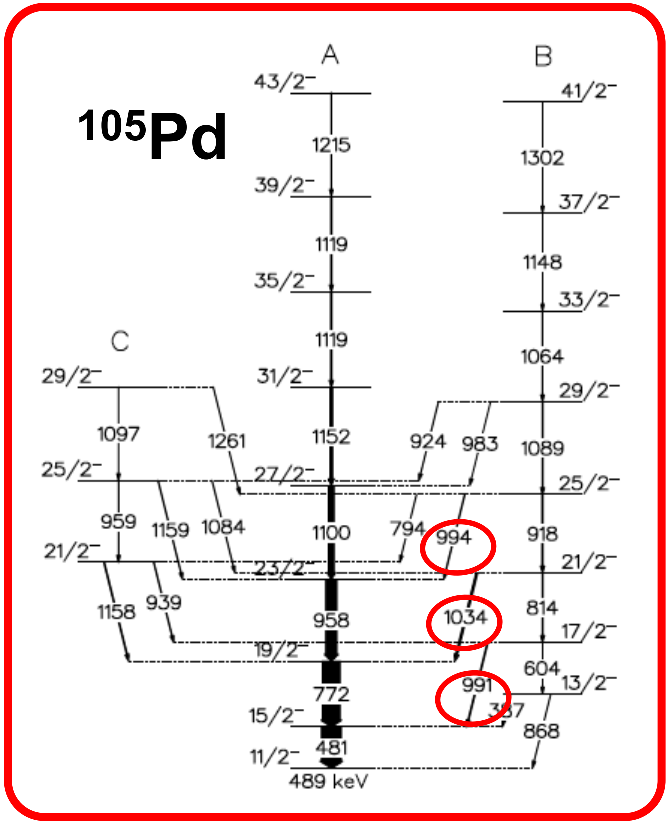
Wobbling in ^{105}Pd

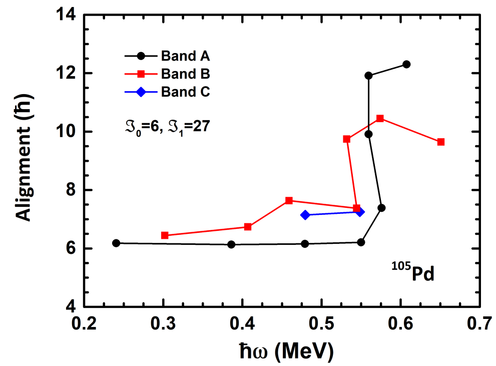
- **Wobbling in ^{105}Pd : first application, 2019**

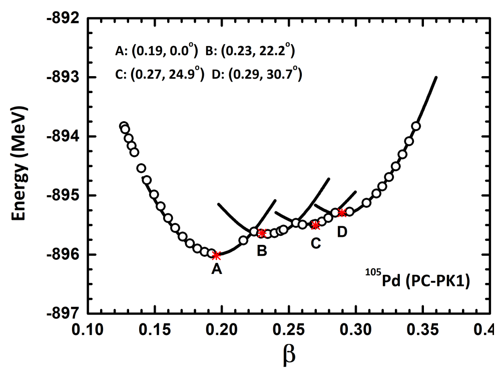
PHYSICAL REVIEW LETTERS 122, 062501 (2019)

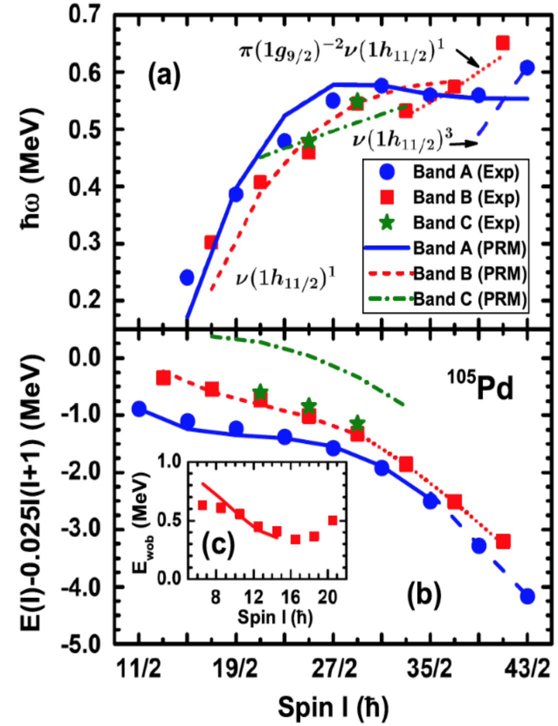
Experimental Evidence for Transverse Wobbling in ^{105}Pd

J. Timár,[1,*] Q. B. Chen,[2] B. Kruzsicz,[1] D. Sohler,[1] I. Kuti,[1] S. Q. Zhang,[3] J. Meng,[3] P. Joshi,[4] R. Wadsworth,[4] K. Starosta,[5] A. Algora,[1,6] P. Bednarczyk,[7] D. Curien,[8] Zs. Dombrádi,[1] G. Duchêne,[8] A. Gizon,[9] J. Gizon,[9] D. G. Jenkins,[4] T. Koike,[10] A. Krasznahorkay,[1] J. Molnár,[1] B. M. Nyakó,[1] E. S. Paul,[11] G. Rainovski,[12] J. N. Scheurer,[13] A. J. Simons,[4] C. Vaman,[14] and L. Zolnai[1]





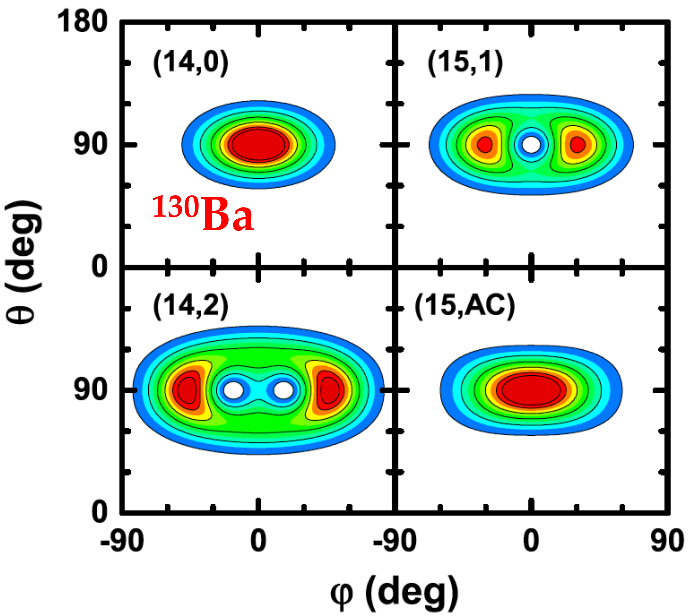


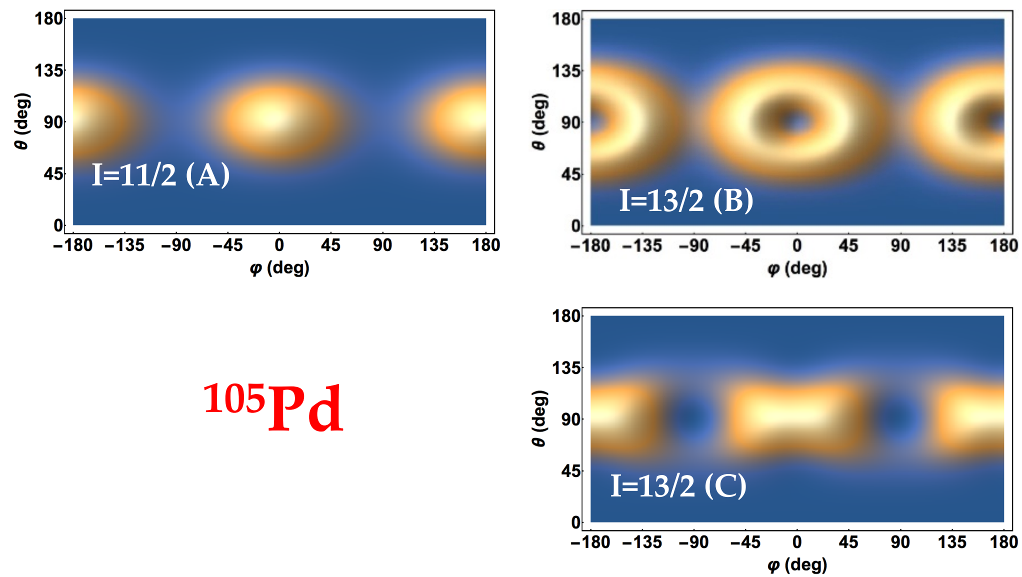


$I_i^\pi \rightarrow I_f^\pi$	E_γ (keV)	δ		$[B(M1)_{out}/B(E2)_{in}](\mu_N^2/e^2b^2)$		$[B(E2)_{out}/B(E2)_{in}]$	
		Expt	PRM	Expt	PRM	Expt	PRM
$17/2^- \rightarrow 15/2^-$	991	1.8 ± 0.5	2.38	0.162 ± 0.097	0.105	0.66 ± 0.18	0.736
$21/2^- \rightarrow 19/2^-$	1034	2.3 ± 0.3	2.30 [a]	0.089 ± 0.026	0.069	0.60 ± 0.09	0.465
$25/2^- \rightarrow 23/2^-$	994	2.7 ± 0.6	1.99	0.029 ± 0.016	0.057	0.34 ± 0.07	0.329

CDFT+PRM for wobbling

- ^{105}Pd: neutron wobbler, mixing ratio has positive (opposite) sign. *PRL 112, 062501 (2019)*
- ^{130}Ba: e-e transverse wobbler, mixing ratio is suppressed. *PRC 100, 061301(R) (2019)*
- ^{183}Au: another evidence for triaxial shape coexistence. *PRL 125, 132501 (2020)*
- ^{187}Au: along inter-mediate axis leads to longitudinal wobbling. *PRL 124, 052501 (2020)*
- Ni isotopes: wobbling prediction. *PRC 104, 064325 (2021)*





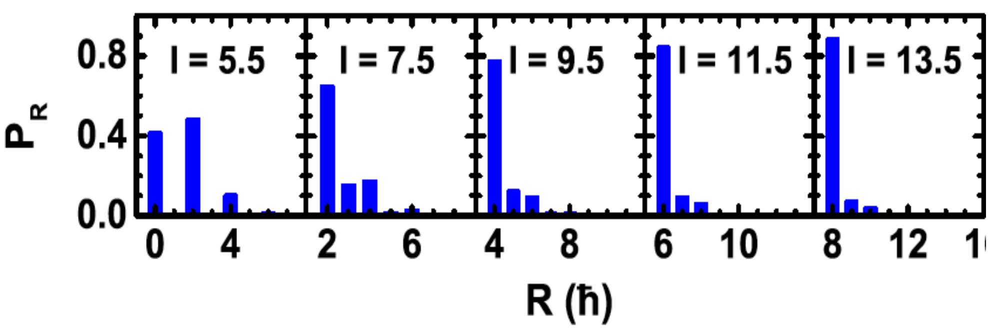
The TW picture in even-even nucleus is the same as the one in the odd-mass nucleus!

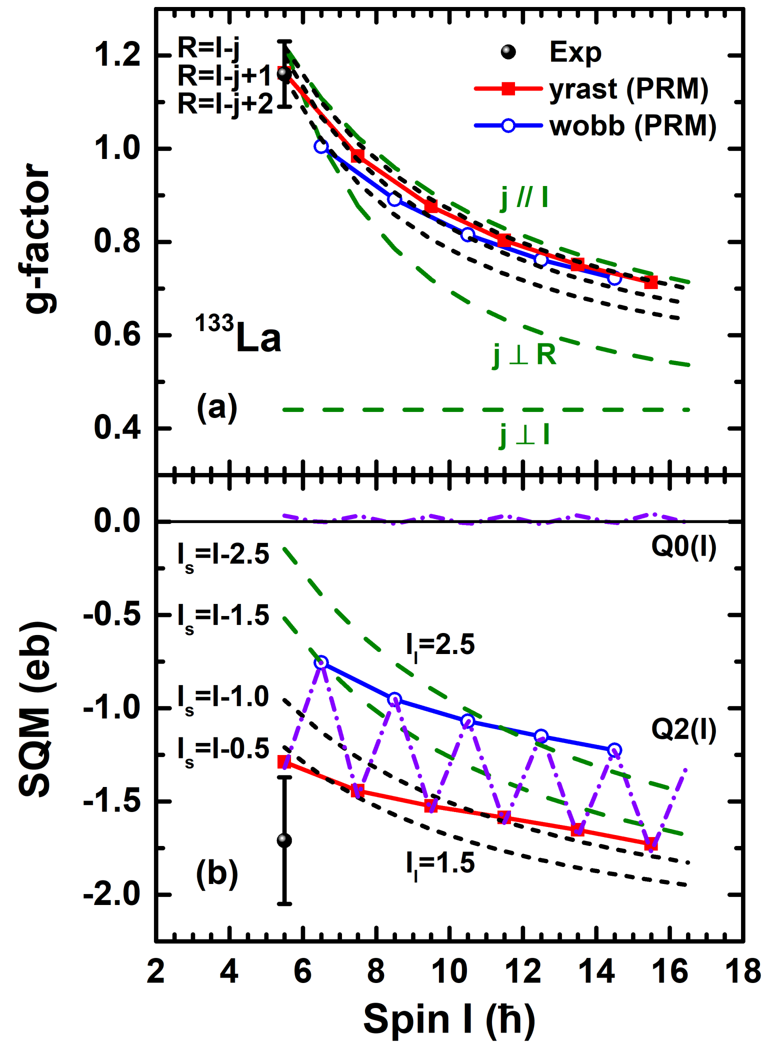
New observables

- **g-factor and static quadrupole moment**

Laskar et al., PRC 101, 034315 (2020): Exp
QB, Frauendorf, Kaiser, Meißner, J.Meng, PLB 807, 135596 (2020), **^{133}La**
Broocks, QB, Kaiser, Meißner, EPJA 57, 161 (2021), **^{135}Pr, ^{105}Pd, ^{187}Au**

$$
\begin{aligned}
g &= \frac{\langle g_p \boldsymbol{j}_p \cdot \boldsymbol{I} + g_R \boldsymbol{R} \cdot \boldsymbol{I} \rangle}{I(I+1)} \\
&= g_R + (g_p - g_R)\frac{\langle \boldsymbol{j}_p \cdot \boldsymbol{I} \rangle}{I(I+1)} \\
&= g_R + (g_p - g_R)\frac{j(j+1)}{I(I+1)} + (g_p - g_R)\frac{\langle \boldsymbol{j}_p \cdot \boldsymbol{R} \rangle}{I(I+1)} \\
&= \frac{1}{2}\left[(g_p + g_R) + (g_p - g_R)\frac{j(j+1) - \langle \boldsymbol{R}^2 \rangle}{I(I+1)}\right].
\end{aligned}
$$



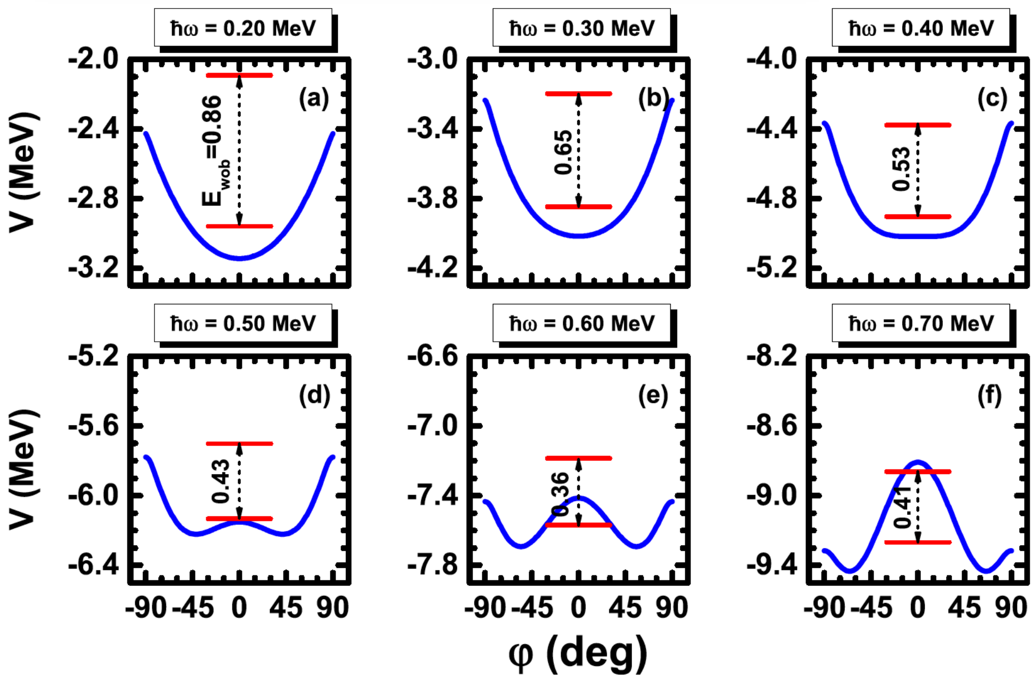


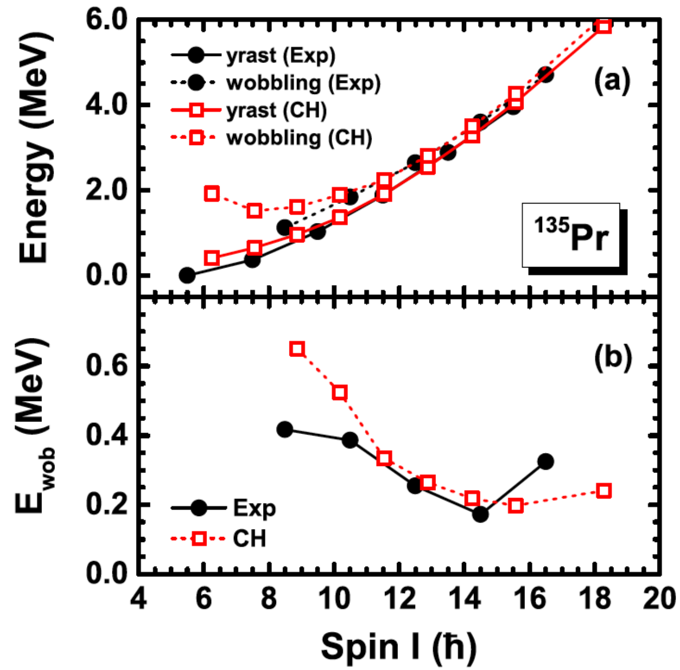
More experimental measurements?

Influences on E_{wob}: rotation

$$\hat{H}_{\text{coll}} = -\frac{\hbar^2}{2\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi}\frac{1}{\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi} + V(\varphi),$$

QB, S.Q.Zhang, P.W.Zhao, J.Meng, PRC 90, 044306 (2014)
QB, S.Q.Zhang, J.Meng, PRC 94, 054308 (2016)



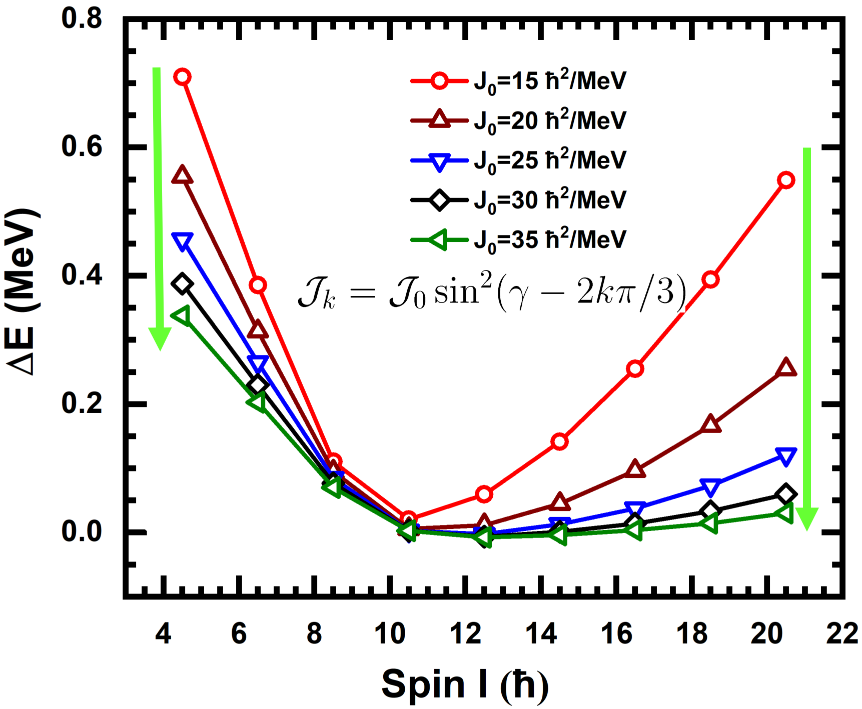


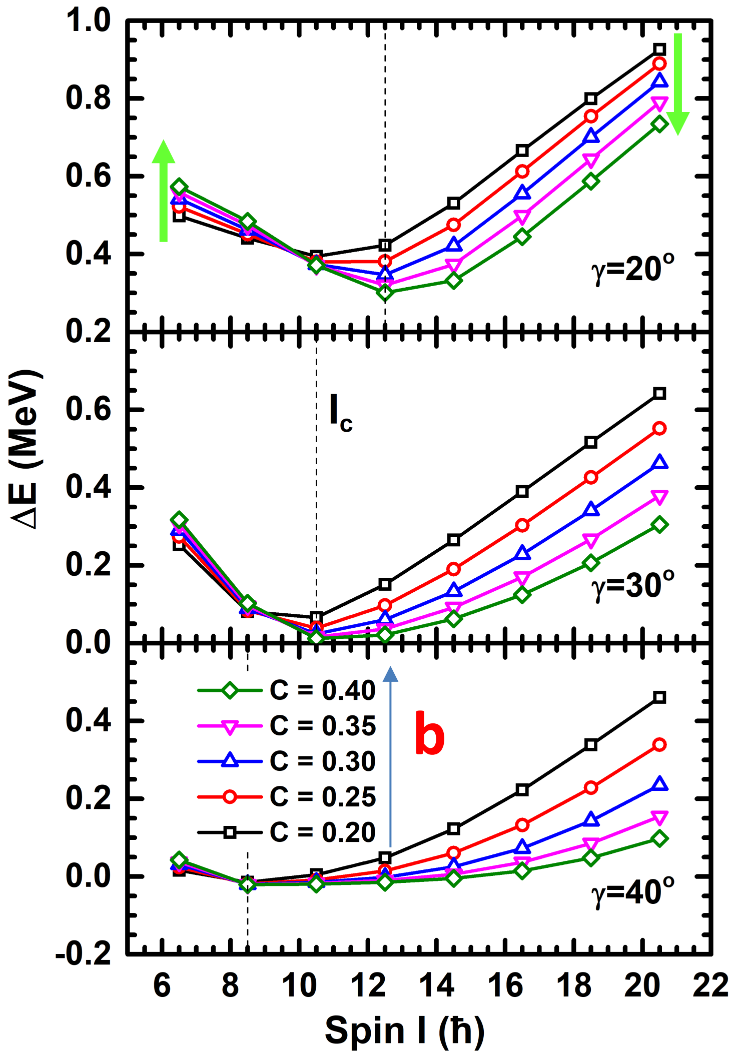
Collective Hamiltonian calculations: collective potential changes with rotation due to the Coriolis force (from one center to double centers to one center), Ewob decrease and then increase.

Influences on E_{wob}: deformation

HFA: *Frauendorf &Dönau, PRC 89, 014322 (2014)*

$$\hbar\omega_w = \frac{j}{\mathcal{J}_3}\left[\left(1+\frac{J}{j}\left(\frac{\mathcal{J}_3}{\mathcal{J}_1}-1\right)\right)\left(1+\frac{J}{j}\left(\frac{\mathcal{J}_3}{\mathcal{J}_2}-1\right)\right)\right]^{1/2}$$

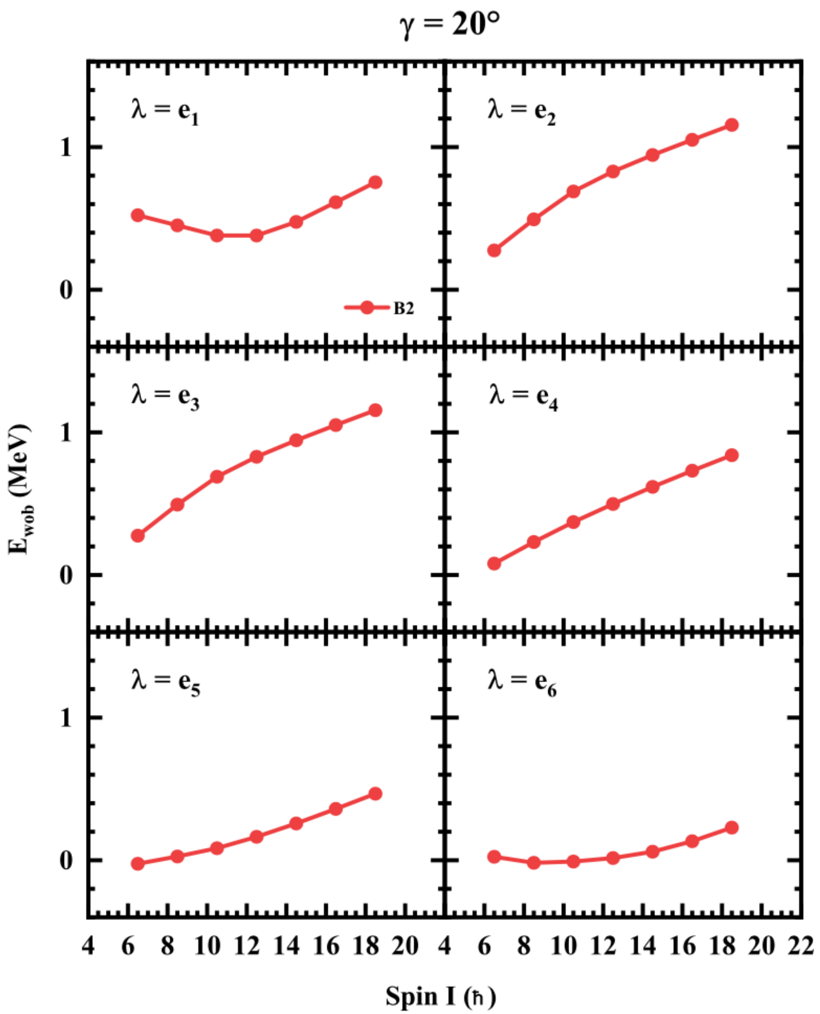


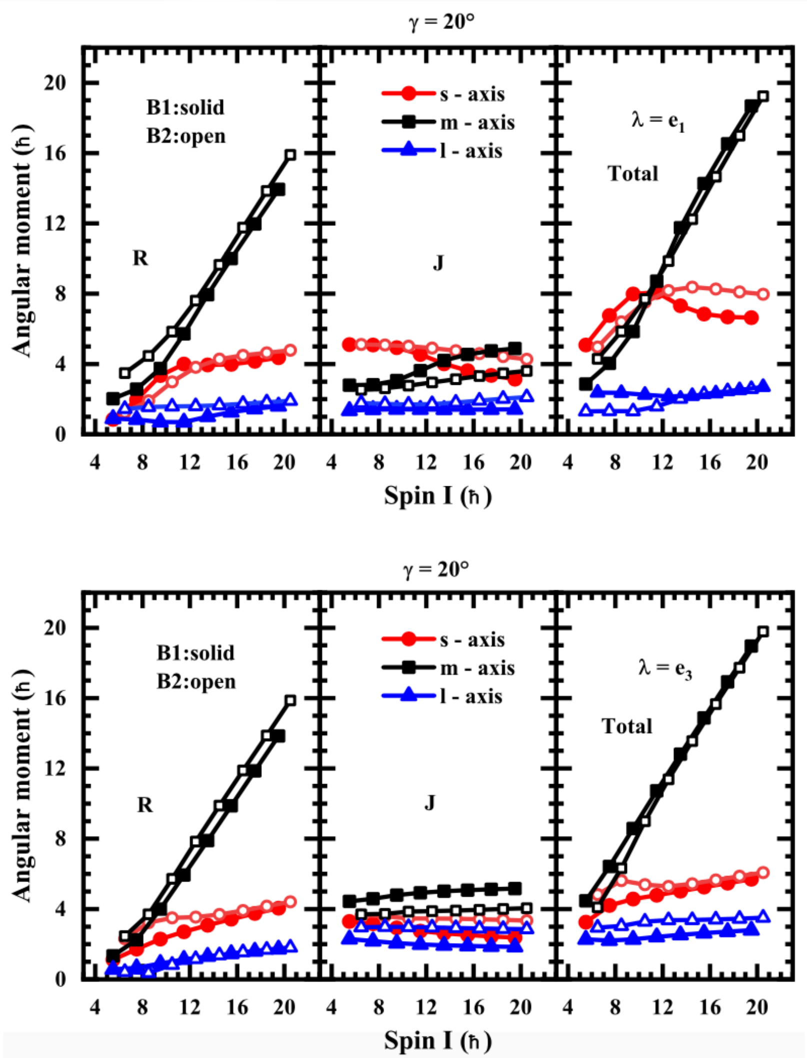


PRM calculations (w/o HFA): **triaxiality (MoI ratios) determines the critical spin, beta and MoI determine the magnitude.**

H.M.Dai, QB, in preparation

Influences on E_{wob}: Fermi surface



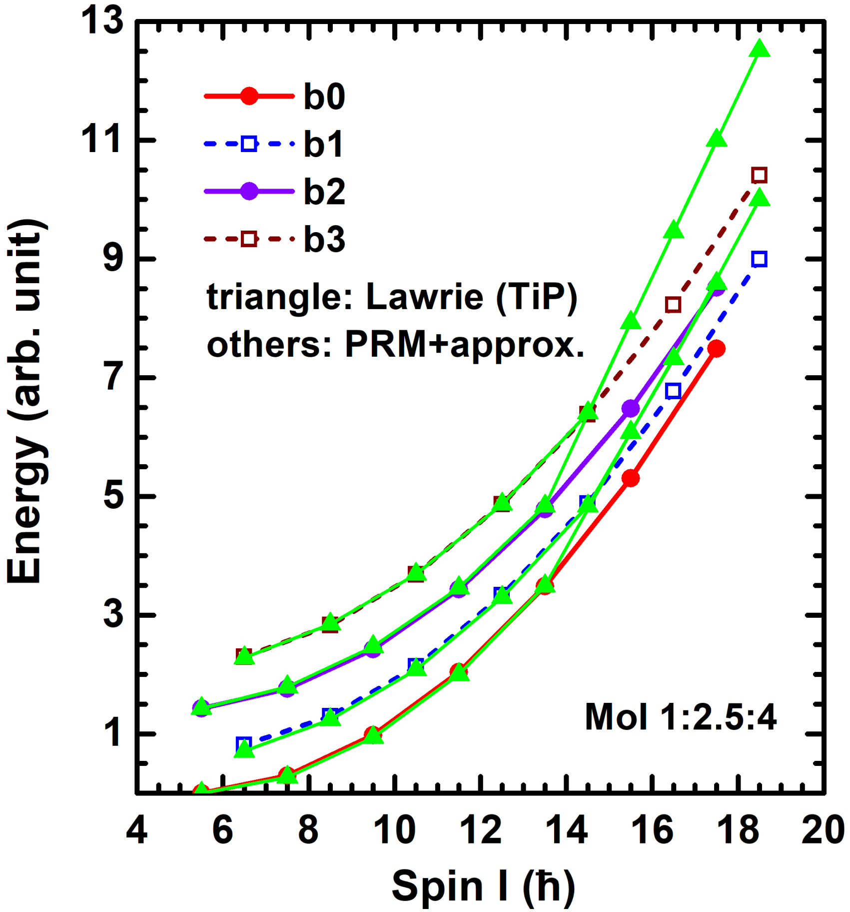


PRM calculations (w/o HFA): Particle a.m. is important; s-axis: decrease; m-axis: increase

S.H.Li, H.M.Dai, QB, in preparation

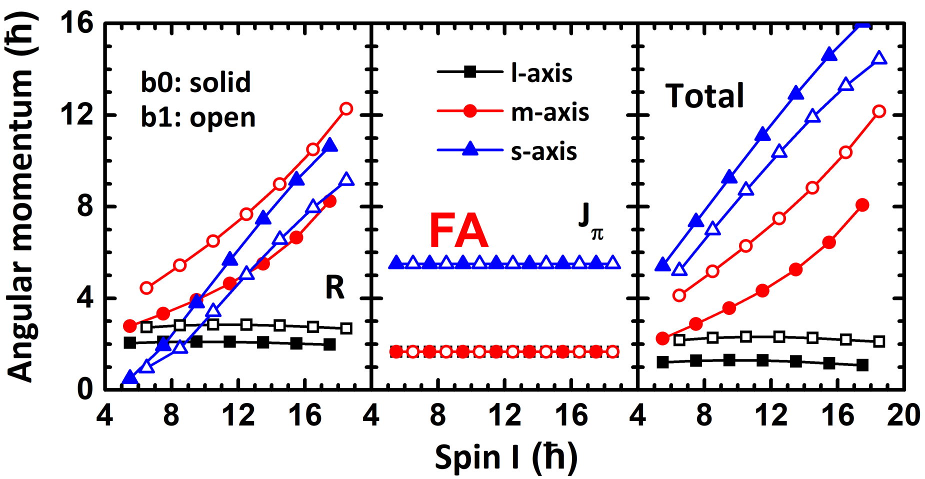
Compare with TiP

- **Tilted axis precession:** *Lawrie&Shirinda&PetrachePRC 101, 034306 (2020)*



Lawrie (TiP): from the paper

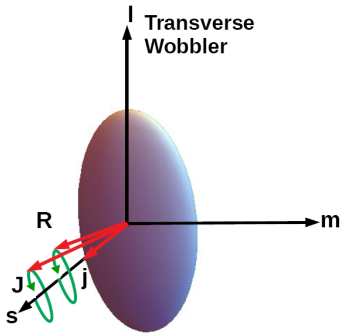
PRM+approx. (frozen alignment): Only |j=11/2, k_s=11/2> is used to couple the rotor. Basis no: (2I+1)/2. [Not a realistic PRM calculation]

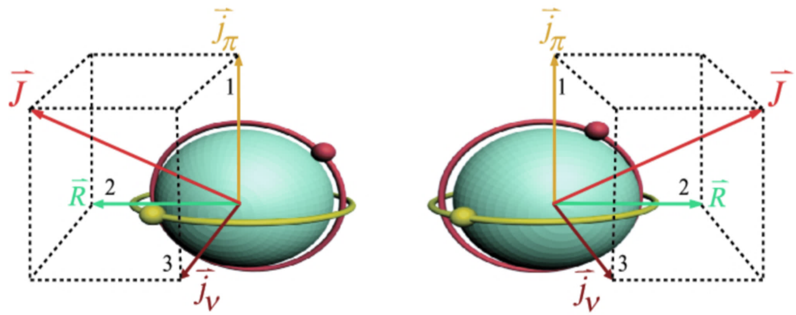


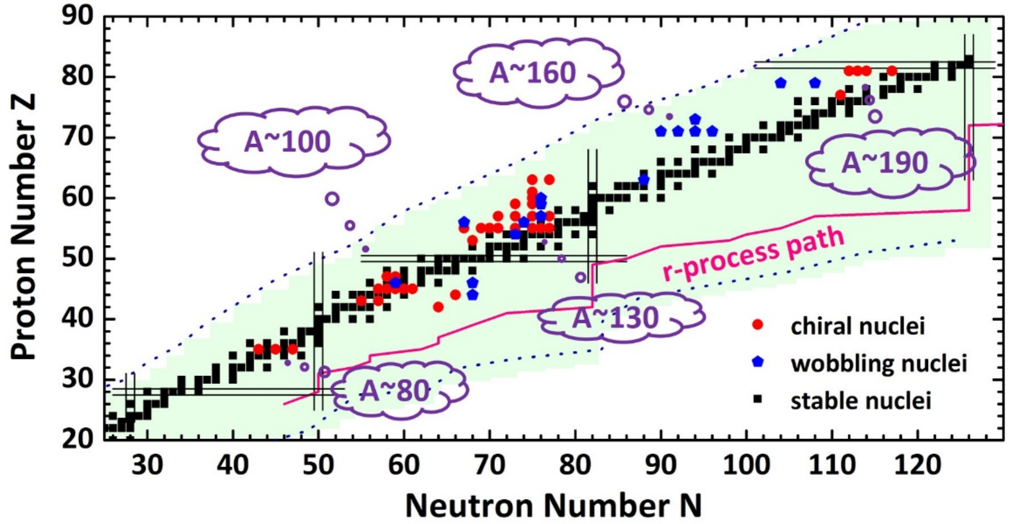
Disagreement in the high spin region, why?

Summary

- CDFT+PRM is a useful method to study the chiral and wobbling nuclei.
- 15+ candidates for wobbling bands in A~100, 130, 160, and 190 mass regions.
- 50+ candidates for chiral doublet bands in A~80, 100, 130, and 190 mass regions.







- ➢ Towards lighter and heavier masses
- ➢ Towards nuclei far from the beta-stability line
- ➢ Towards exotic deformations
- ➢ New observables

Collaborations

Shanghai
Huaming Dai
Shihua Li
Bin Hu
Bowen Sun
Yingzhi Ji

Beijing
Jie Meng
Shuangquan Zhang
Pengwei Zhao
Jing Peng
Zhenhua Zhang

Shandong
Shouyu Wang
Bin Qi
Chen Liu
Hui Zhang
Lin Mu

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Costel Petrache

South Africa
Rainer Lieder
Robert Bark

Hungary
Janos Timar
Istvan Kuti

Poland
Julian Srebrny
Ernest Grodner

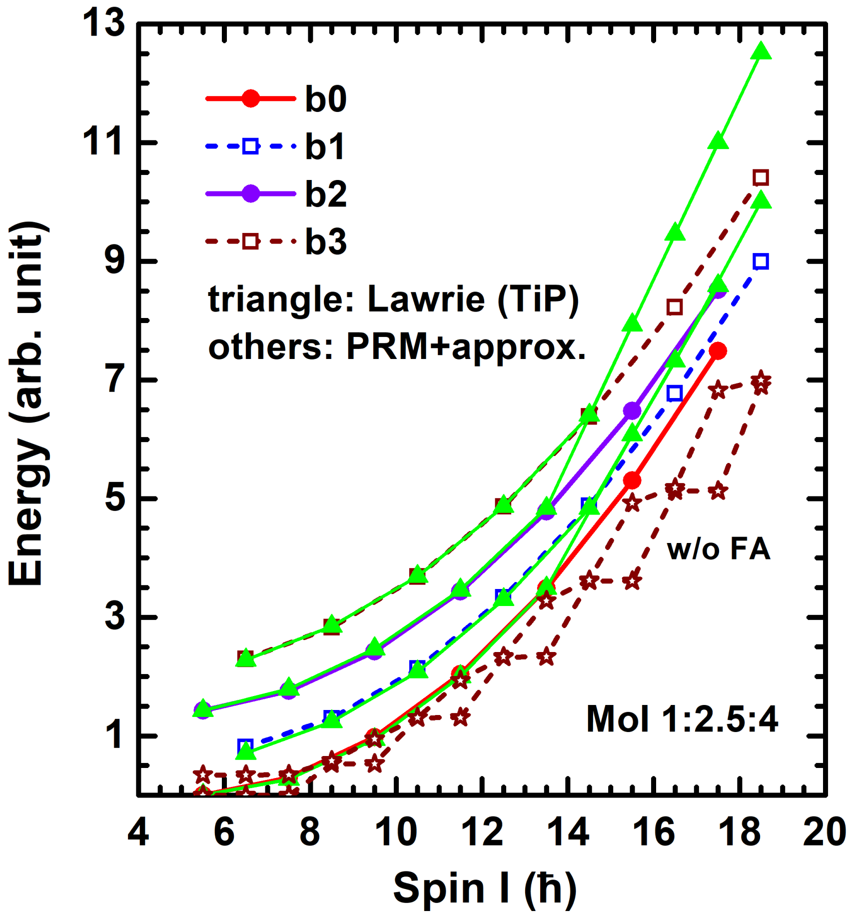
India
Gopal Mukherjee
Soumen Nandi

....

Thank you!

Compare with TiP

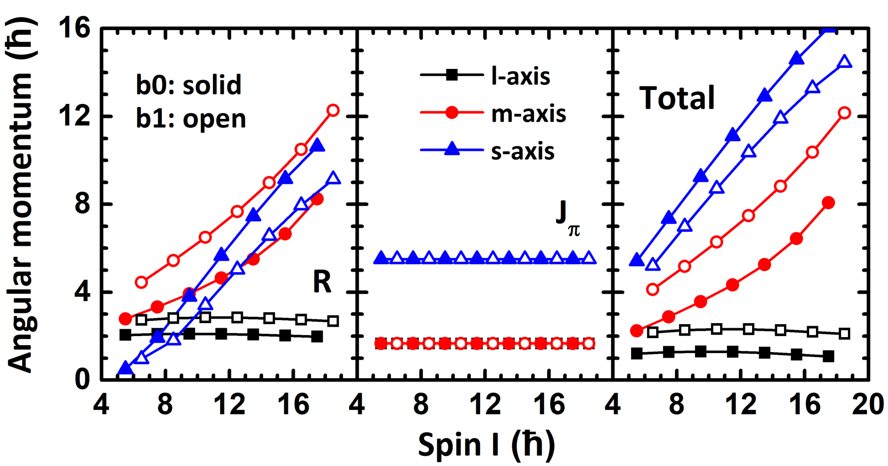
- Tilted axis precession: PRC 101, 034306 (2020)



Lawrie (TiP): from the paper

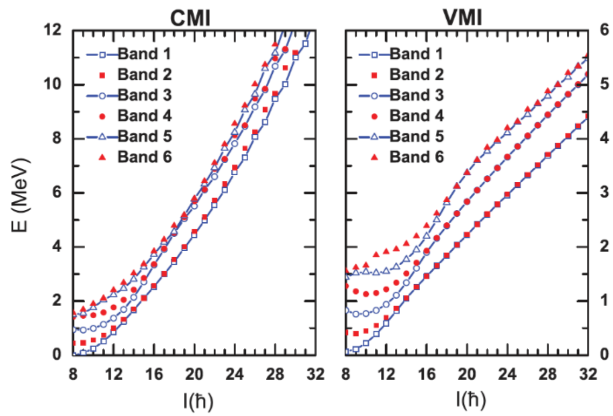
PRM+approx. (frozen alignment): Only |j=11/2, k_s=11/2> is used to couple the rotor. Basis no: (2I+1)/2. [Not an exact PRM calculation]

Disagreement: the high spin region?

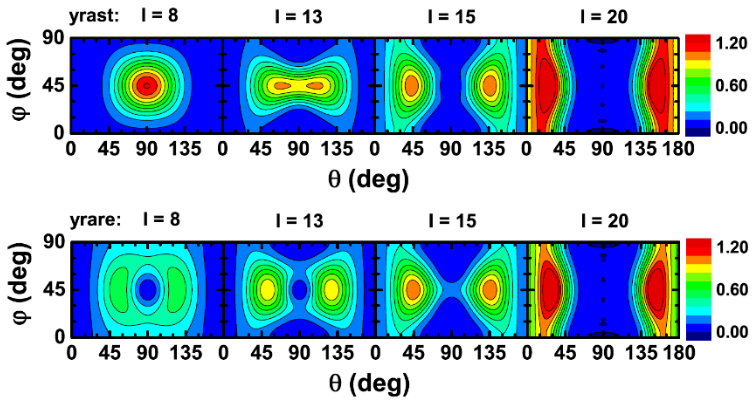


PRM for chirality

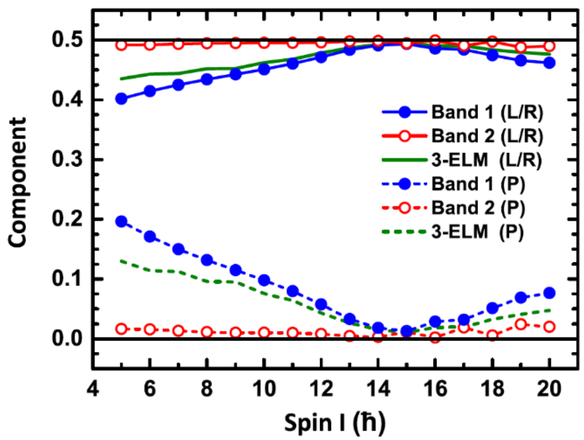
- **PRM for chirality:** excited chiral doublet bands, orientation evolution, rotor behavior, planar contribution



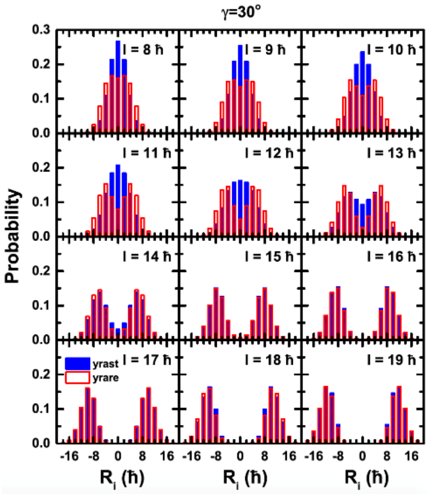
QB&Yao&Zhang&Qi, PRC 82, 067302 (2010)



QB&Meng, PRC 98, 031303(R) (2018)



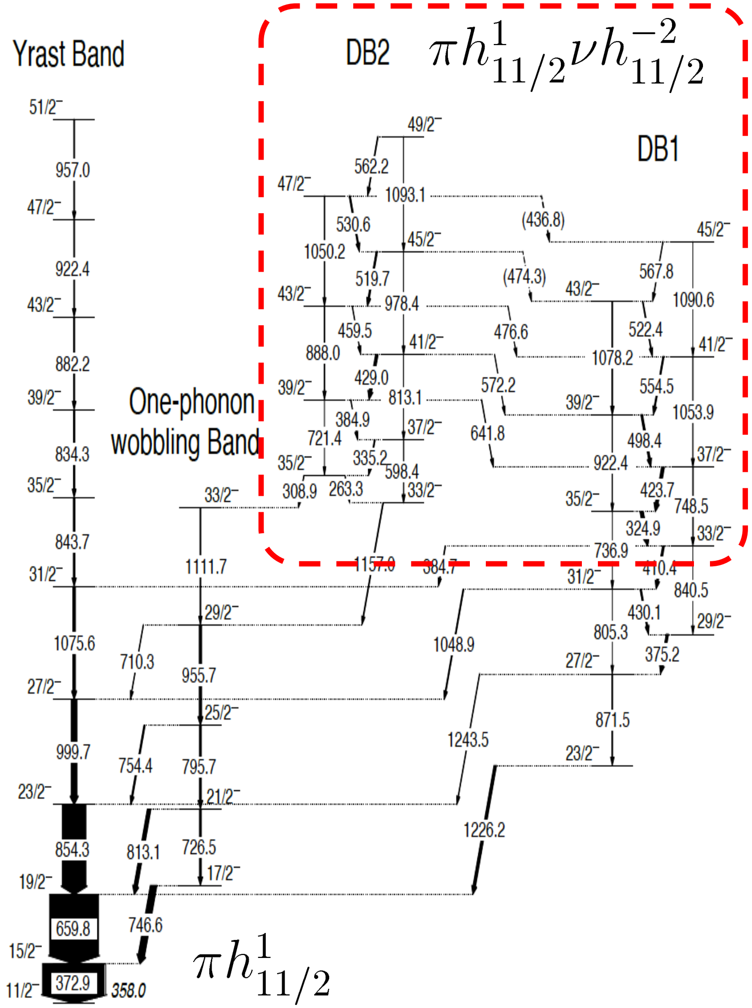
QB&Starosta&Koike, PRC 97, 041303(R) (2018)

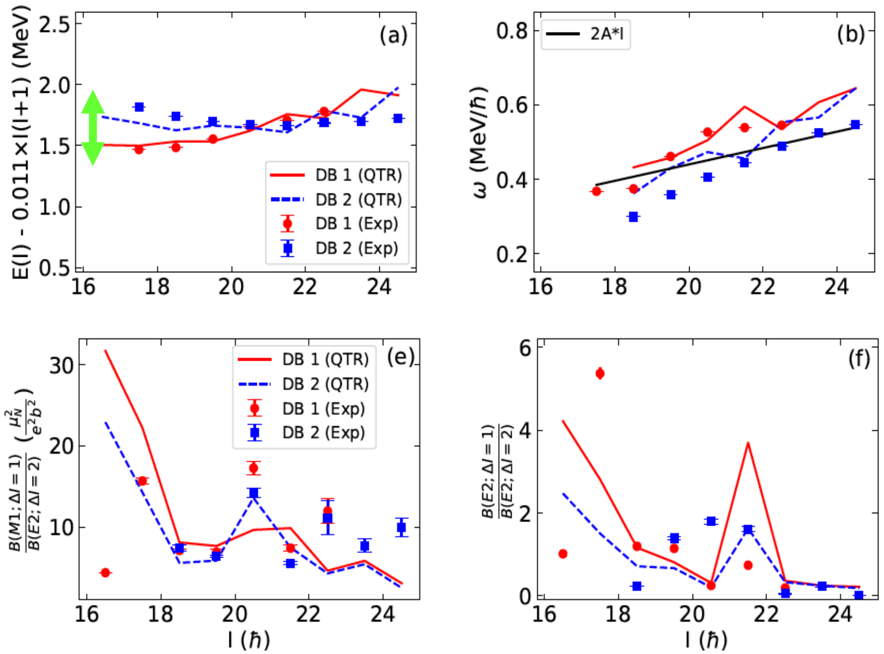


QB&Kaiser&Meissner&Meng, PRC 99, 064326 (2019)

Chirality in ^{135}Pr

- **Chirality in ^{135}Pr: coexistence with wobbling** *Sensharma et al., submitted*



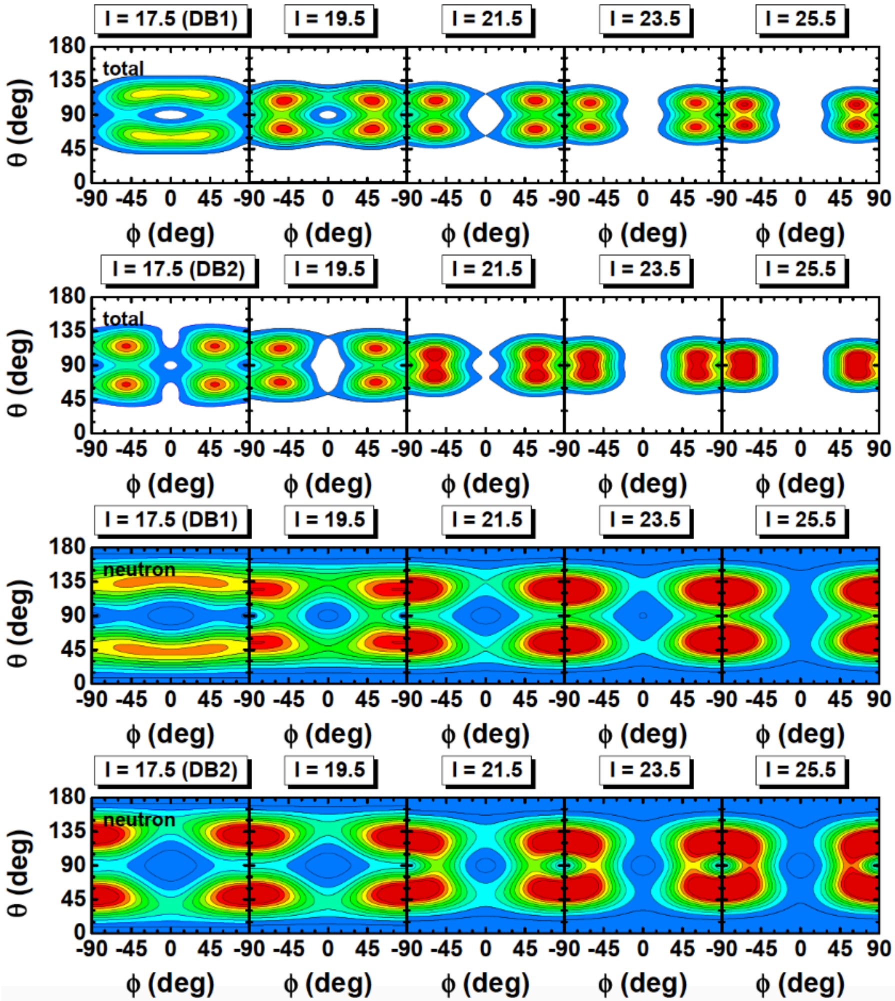


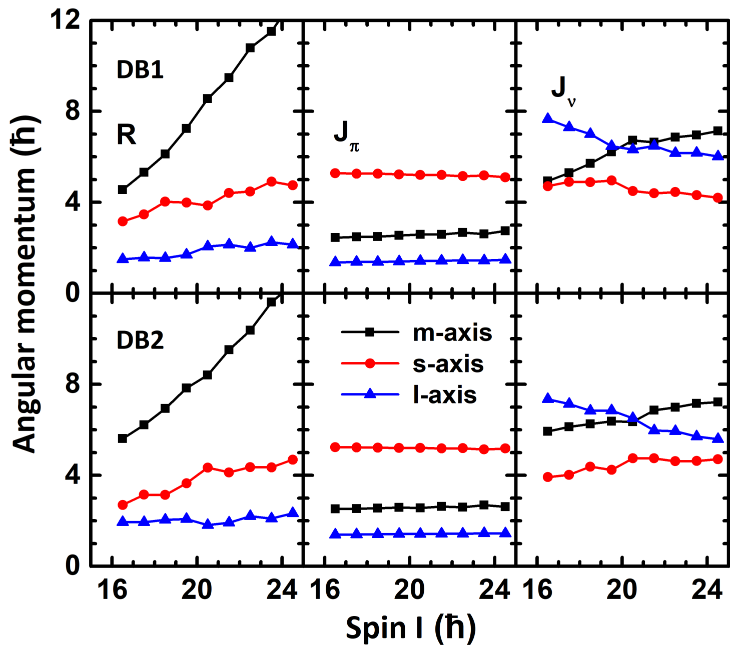
Small energy difference; similar B(M1)/B(E2).

Chirality in ^{135}Pr

- **Chirality in ^{135}Pr: coexistence with wobbling**

Sensharma et al., submitted





Similar distributions for bands DB1 and DB2. Neutron is not a pure hole and l-axis component is small. Aplanar orientation closed to s-m plane.