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Rotational excitations in rare-earth nuclei: a comparative study within different cranking models

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Backbendings in odd-A Tm isotopes

The experimental evidence of a sharp backbending in the ground state band $\pi 1/2^+[411]$ of ^{169}Tm has been observed for the first time.

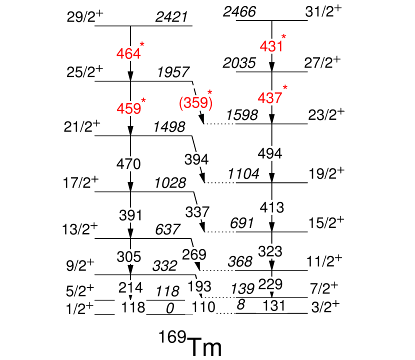


FIG. 2. Proposed level scheme of ^{169}Tm in the present work. The new γ rays are marked by asterisks (*).

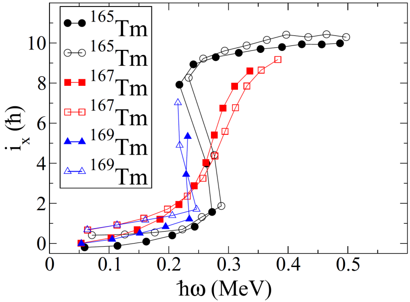


FIG. 3. Quasiparticle aligned angular momenta (i_x) as a function of rotational frequency ($\hbar\omega$) for odd-A Tm isotopes. Haris parameters $J_0 = 35\,\hbar^2\,\text{MeV}^{-1}$ and $J_1 = 43\,\hbar^4\,\text{MeV}^{-3}$ are taken.

A sharper up-bend than that in ^{167}Tm is expected in ^{171}Tm.

Md. A. Asgar et al., PRC, 95 (2017) 031304R.

Calculated interaction strengths

TABLE III. Experimental and calculated crossing frequencies (ω_c) and calculated interaction strengths (V) at the band crossings for the $[411]1/2^+$ band in Tm isotopes.

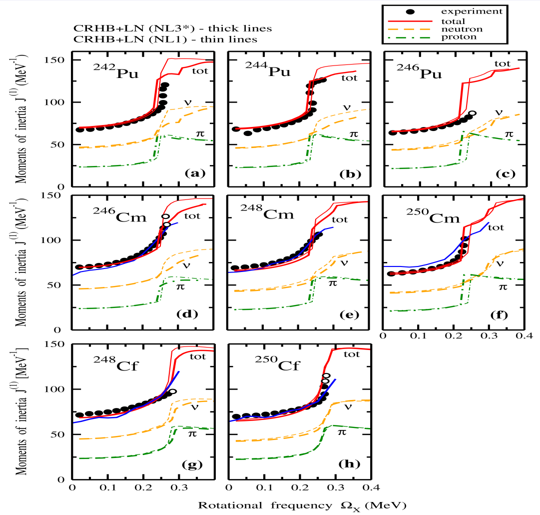
A	ω_c(exp) (MeV)	ω_c(cal) (MeV)	V (keV)
165	0.29	0.30	20
167	$\sim$ 0.27	0.28	125
169	0.24	0.25	10

Using the cranked shell model, they investigated the crossing frequencies, the interaction strengths and the following conclusions are obtained

- The $N = 98$ shell gap significantly affects the nature of the alignment of the nuclei in the $A \sim 170$ region.
- The calculated interaction strength for ^{171}Tm is 75 keV and a less smooth up-bend than that in ^{167}Tm is expected.

Md. A. Asgar et al., PRC, 95 (2017) 031304R.

Predicted upbendings by different cranking models



CRHB+LN always predicts sharp upbendings, while PNC-CSM predicts both sharp and gradual upbendings.

A. V. Afanasjev, PS, 89 (2014) 054001

It is necessary to evaluate the accuracy of different cranking models when describing the experimental data.

The goals of this study are

(i) to evaluate the weak and strong points of different cranking models;

(ii) to evaluate typical accuracy of the description of experimental data by these methods.

Three models are adopted

(i) The cranked relativistic Hartree-Bogoliubov approach with pairing correlations treated by the Lipkin-Nogami method (abbreviated as CRHB+LN).

A. V. Afanasjev et al., NPA, 676 (2000) 196

(ii) The cranking covariant density functional theory with pairing correlations treated by the shell-model-like approach (abbreviated as cranking CDFT-SLAP).

Z. Shi et al., PRC, 97 (2018) 034317; B. W. Xiong, PRC, 101 (2020) 054305

(iii) The particle-number conserving method based on the cranked shell model in which the phenomenological Nilsson potential is adopted (abbreviated as PNC-CSM).

J. Y. Zeng, et al., PRC, 50 (1994) 1388

Cranking models with particle-number conserving method

The cranking many-body Hamiltonian with pairing correlations can be written as

$$\hat{H} = \hat{H}_0 + \hat{H}_{\mathrm{P}} = \sum \hat{h}_0 + \hat{H}_{\mathrm{P}} = \sum (h_{\mathrm{s.p.}} - \omega_x j_x) + \hat{H}_{\mathrm{P}}$$

$h_{\mathrm{s.p.}}$ can be chosen any mean field Hamiltonian, $-\omega_x j_x$ is the Coriolis interaction, H_{P} is the pairing interaction.

Particle-number conserving method (Shell-model-like approach) is used to treat the pairing correlations, in which the particle number is conserved and blocking effects are taken into account exactly.

1. Diagonalize H_0 to get the cranked single-particle basis.
2. Construct the cranked many particle configuration (CMPC) space using the cranked single-particle basis.
3. Diagonalize $\hat{H}$ in a sufficiently large truncated CMPC space.

J. Y. Zeng, et al., PRC, 50 (1994) 1388

Z. Shi et al., PRC, 97 (2018) 034317
Z. H. Zhang et al., PRC, 101 (2020) 054303
B. W. Xiong, PRC, 101 (2020) 054305
Y. P. Wang and J. Meng, PLB 841 (2023) 137923

Cranking models with particle-number conserving method

The eigenstate of $\hat{H}$ is:

$$|\psi\rangle = \sum_i C_i |i\rangle \quad (C_i \text{ is real}) ,$$

where $|i\rangle$ is a CMPC.

In the present work, two kinds of mean-field are adopted

- microscopic CDFT approach with point coupling functional PC-PK1 and meson-exchange functional NL5(E);

  P.W. Zhao et al., PRC, 82 (2010) 82   S. E. Agbemava et al., PRC, 99 (2019) 014318

- phenomenological Nilsson potential.

  S. G. Nilsson et al., NPA, 131 (1969) 1

In the cranking CDFT-SLAP, the occupation probabilities ($n_\mu = \sum_i |C_i|^2 P_{i\mu}$) will be iterated back into the densities and currents to achieve self-consistency.

The CRHB+LN approach

The cranked relativistic Hartree-Bogoluibov (CRHB) equations with approximate particle number projection by means of the Lipkin-Nogami (LN) method are given by

$$\begin{pmatrix} \hat{h}_{\mathrm{D}}(\eta) - \lambda(\eta) - \omega_x \hat{J}_x & \hat{\Delta}(\eta) \\ -\hat{\Delta}^*(\eta) & -\hat{h}^*_{\mathrm{D}}(\eta) + \lambda(\eta) + \omega_x \hat{J}^*_x \end{pmatrix} \begin{pmatrix} U(\mathbf{r}) \\ V(\mathbf{r}) \end{pmatrix}_k = E_k(\eta) \begin{pmatrix} U(\mathbf{r}) \\ V(\mathbf{r}) \end{pmatrix}_k$$

where

$$\begin{aligned} \hat{h}_{\mathrm{D}}(\eta) &= \hat{h}_{\mathrm{D}} + 2\lambda_2 \left[(1+\eta)\rho - \mathrm{Tr}(\rho)\right], \\ \hat{\Delta}(\eta) &= \hat{\Delta} - 2\lambda_2(1-\eta)\kappa, \\ \lambda(\eta) &= \lambda_1 + \lambda_2 \left[1+\eta\right], \\ E_k(\eta) &= E_k - \eta\lambda_2. \end{aligned}$$

$\hat{h}_{\mathrm{D}}$ is the single-nucleon Dirac Hamiltonian. $\hat{\Delta}$ is the pairing potential, U_k and V_k are quasiparticle Dirac spinors and E_k denote the quasiparticle energies.

A. V. Afanasjev et al., NPA, 676 (2000) 196

In the CRHB theory the finite range Gogny interaction is aodpted

$$V^{pp}(1,2) = f \sum_{i=1,2} e^{-[(\mathbf{r}_1-\mathbf{r}_2)/\mu_i]^2} \times (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau)$$

Numerical details

For cranking CDFT-SLAP:

- The point coupling functional PC-PK1 and meson-exchange functional NL5(E) are adopted.
- The Dirac spinors are expanded into 3DHO basis with 14 major shells. When using NL5(E) functional, 20 major shells are used for mesons.
- The dimensions of the CMPC space are 1000 for both protons and neutrons.
- Monopole pairing is adopted and the effective pairing strengths are $G_p = G_n = 1.5$ MeV.

For CRHB+LN:

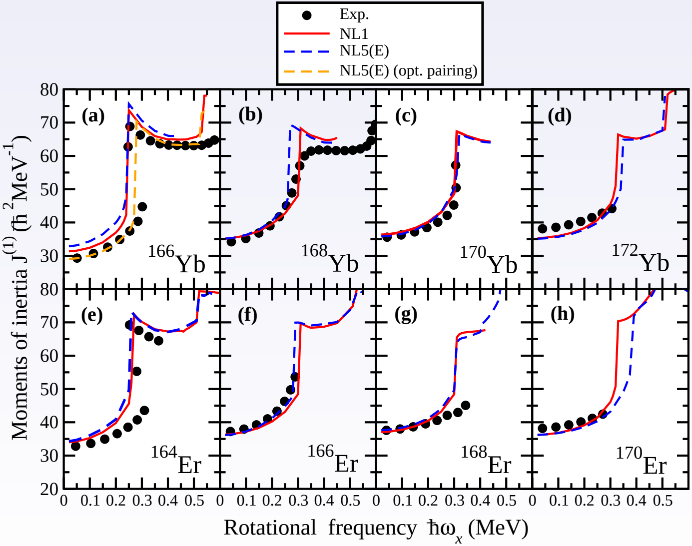
- The meson-exchange functional NL1 and NL5(E) are adopted.
- The Dirac spinors are expanded into 3DHO basis with 14 major shells and 20 major shells are used for mesons.
- The scaling factor f of the Gogny pairing is $f = 0.957$ and $f = 0.950$ for the NL1 and NL5(E) functionals, respectively.

Numerical details

For PNC-CSM:

- Deformation parameters (ε_2, ε_4) are taken from the Lund systematics.
  - R. Bengtsson et al., ADNDT, 35 (1986) 15
- The Nilsson parameters (κ, μ) are obtained from the parameters of those fitted for $A = 150$ mass region with some modifications in proton subsystem.
  - T. Bengtsson, NPA 512 (1990) 124.
- The dimensions of the CMPC space are 1000 both for protons and neutrons.
- For even-even Er and Yb isotopes the proton monopole pairing strengths $G_{0\mathrm{p}} = 0.35$ MeV. The neutron monopole pairing strengths $G_{0\mathrm{n}} = 0.40$ MeV for $N = 96$ and 98 isotopes and $G_{0\mathrm{n}} = 0.25$ MeV for $N = 100$ and 102 isotopes. For odd-A nuclei ^{165,167,169,171}Tm, the monopole pairing strengths are $G_{0\mathrm{p}} = 0.31$ MeV and $G_{0\mathrm{n}} = 0.33$ MeV. The quadrupole pairing strengths are chosen as $G_{2\mathrm{p}} = G_{2\mathrm{n}} = 0.006$ MeV.

CRHB+LN method for the even-even Er and Yb isotopes



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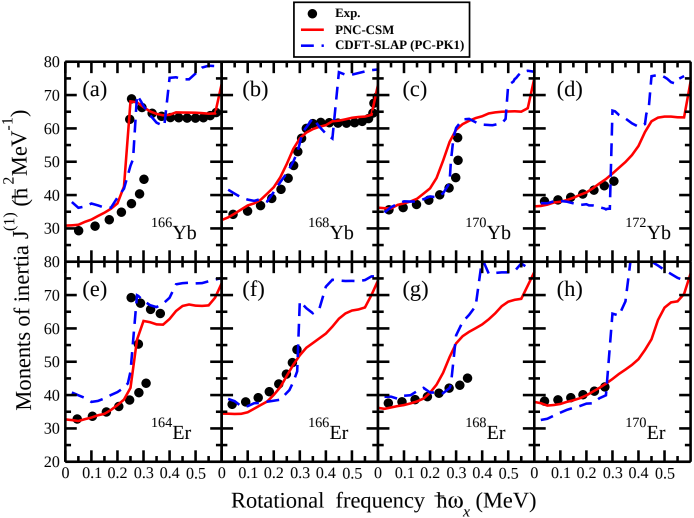
PNC methods for the even-even Er and Yb isotopes



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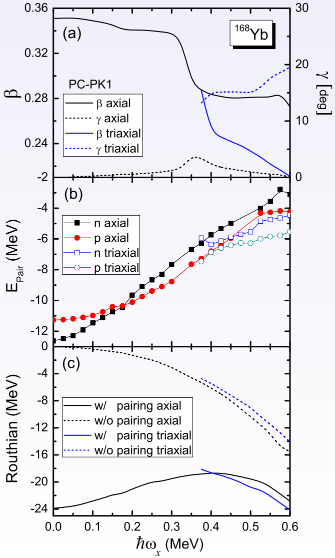
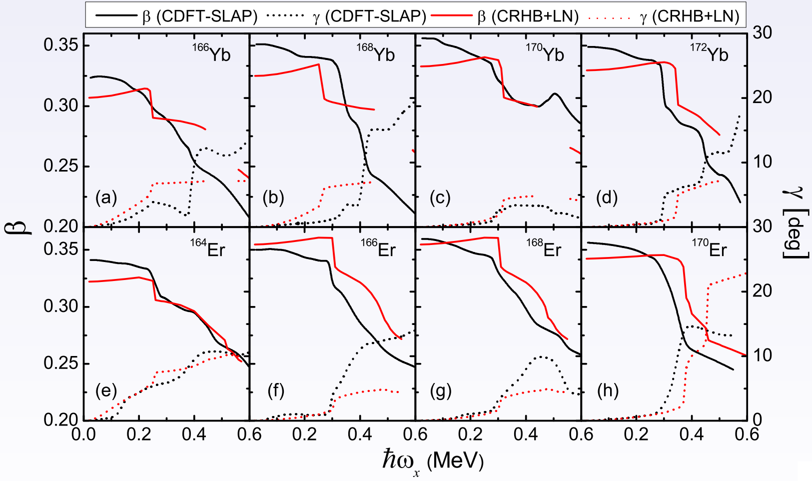
The deformation parameters, pairing energies, and total Routhians obtained in the cranking CDFT-SLAP



- A triaxial minima appears at $\hbar\omega \approx 0.35$ MeV after the first band crossing.
- The pairing energies in the triaxial minima are larger than those in the near-axial one.
- The total Routhian of the near-axial minimum is energetically favored without pairing, while the triaxial minimum becomes energetically favoured at $\hbar\omega > 0.4$ MeV when pairing is considered.
- The energies of these two minima are very close to each other in some rotational frequency range.

The evolution of deformation parameters



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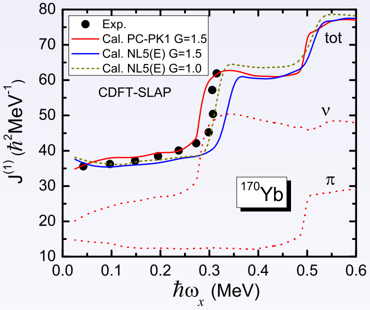
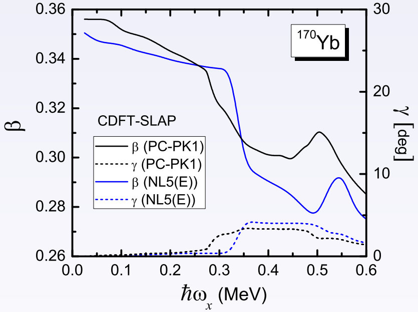
The MOIs and the deformation parameters of the GSB in ^{170}Yb by cranking CDFT-SLAP with PC-PK1 and NL5(E)





- Smaller paring strengthes are needed in NL5(E).
- The equilibrium deformations are rather close to each other with PC-PK1 and NL5(E).

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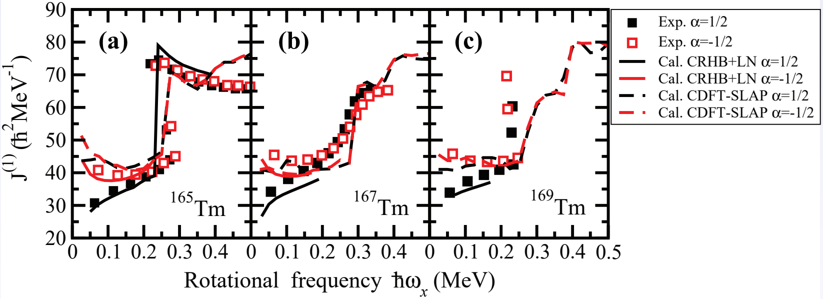
The MOIs of the GSB $\pi 1/2^+[411]$ in odd-A Tm isotopes



- The signature splitting is well reproduced in CRHB+LN calculations. However, it converge only at low frequencies for most cases.
- For cranking CDFT-SLAP, the convergence can be obtained up to very high frequency. However, MOIs are somewhat overestimated at low frequency and the signature splitting is not reproduced.
- Both cranking CDFT-SLAP and CRHB+LN are all quite time-consuming when calculating these odd-A nuclei.

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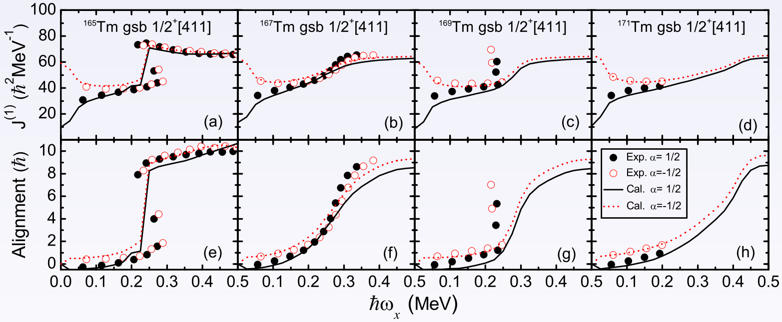
The PNC-CSM results for odd-A Tm isotopes



- The MOIs and level-crossings are well reproduced by the PNC-CSM.
- A gradual alignment is predicted in the ground state band of ^{171}Tm.

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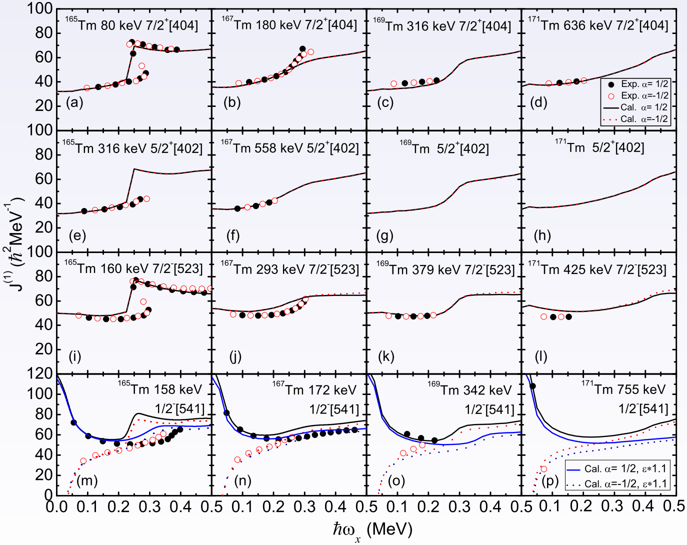
The PNC-CSM results for the excited bands



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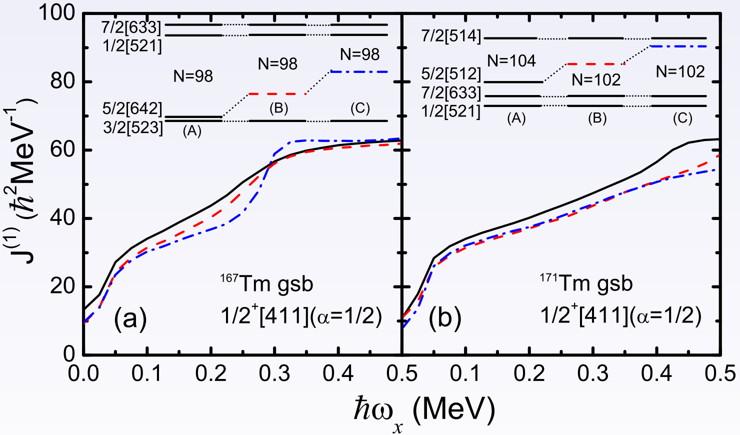
The influence of $N = 98$ and $N = 102$ neutron shell gaps



- The band crossing features not only depends on the shell structure close to the Fermi level, but also on specific high-j orbital located in the vicinity of the Fermi surface.

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Summary

Three cranking models (CRHB+LN, cranking CDFT-SLAP, PNC-CSM) are adopted to investigate the rotational properties of the rare-earth nuclei. The comparison of these models reveals the following features

- The calculated results obtained with different CEDFs within the framework of one model are in general close to each other.
- The LN method allows us to avoid the pairing collapse for most of the cases in the frequency range of interest. However, there is still no numerical convergence in some cases in the vicinity of second band crossings and at extremely high rotational frequencies.
- There is no pairing collapse in the PNC (SLAP) method, so the convergence can be obtained even at very high spins.
- The CDFT-based models predict sharper band crossing features as compared with PNC-CSM calculations.
- The calculations with PNC-CSM for odd-A nuclei are much easier and the results are good.
- The influence of $N = 98$ and $N = 102$ neutron shell gaps are analyzed.

Thank you !