Effective masses in Relativistic Brueckner-Hartree-Fock Theory

Huizhou, 10.07.2023

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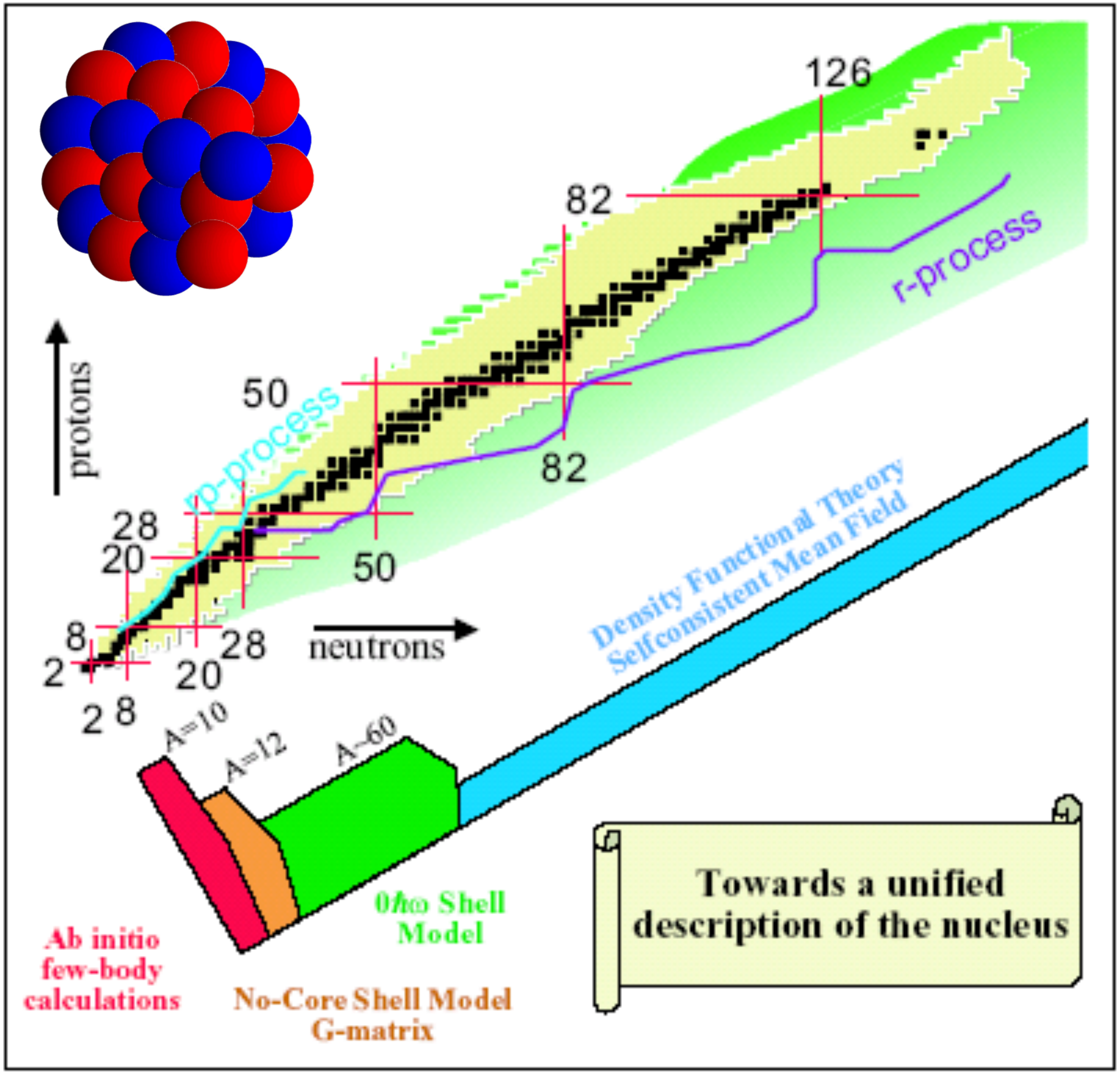
Content

- Motivation
- Relativistic Brueckner-Hartree-Fock Theory
- Applications in finite systems
- Full solution for nuclear matter
- Effective masses and their isospin dependence

Motivation

the nuclear many-body problem:



- Light nuclei:

  exact solution by config. mixing
  ab-initio possible
  complicated results
  non-relativistic

- Heavy, superheavy nuclei, fission etc.

  density functional threory
  simple universal description
  easy to visualize
  phenomenological

- Goal: Ab-initio derivation of density functional

- Ab-initio derivation of density functional theory

  first attempts of ab-initio go back to the fifties:

  Brueckner theory:

  based on the mean-field concept

  effective density-dep. interaction: G[ρ]

  mother of nuclear density functional theory
- Non-relativistic BHF fails: Three-body forces
- 1980: Relativistic BHF: no NNN necessary

problems:

a) no exact solution of RBHF in nuclear matter

many different approximations

b) no solution of RBHF in finite nuclei (tensor?)

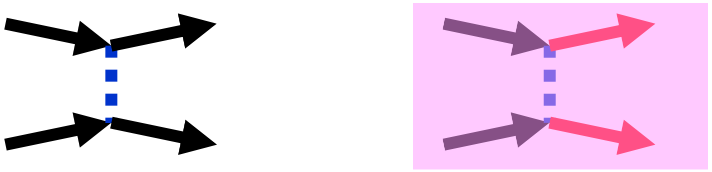
Why covariant ?

1) Large fields V ≈ 350 MeV , S ≈ – 400 MeV
2) Large spin-orbit splittings in nuclei
3) Success of relativistic Brueckner calculations
4) Success of intermediate energy proton scattering
5) Relativistic saturation mechanism
6) Consistent treatment of time-odd fields
7) Natural explanation of pseudospin symmetry
8) Connection to underlying theories ?
9) Use as many symmetries as possible in phenomenology

Brueckner theory (1958):

Brueckner,Gammel, Phys. Rev. 109, 1023 (1958)

- The nucleons in the interior of the nuclear medium do not feel the same bare force V, as the nucleons feel in free space.

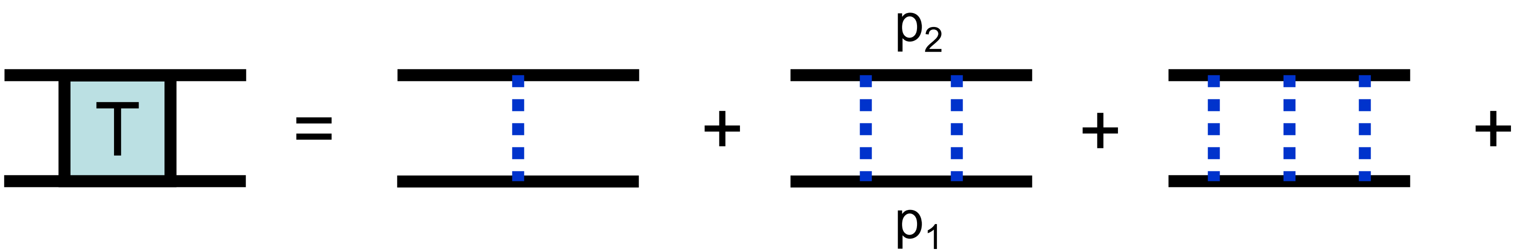


- They feel an effective force G.
- The Pauli principle prohibits the scattering into states, which are already occupied in the medium.
- Therefore this force G(ρ) depends on the density
- This force G is much weaker than the bare force V.
- Nucleons move nearly free in the nuclear medium and feel only a strong attraction at the surface (shell model)

Free nucleon-nucleon scattering:

Lippmann-Schwinger-Eq.

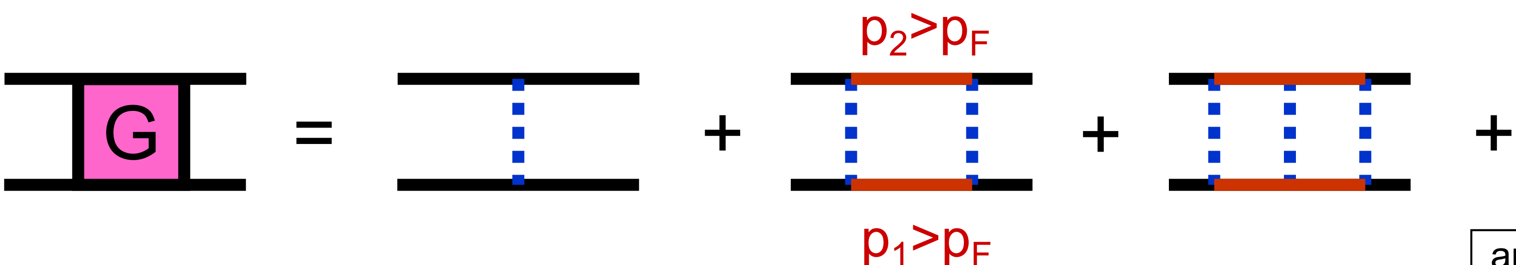
$$\langle \mathbf{k}_1\mathbf{k}_2|T|\mathbf{k}'_1\mathbf{k}'_2\rangle = \langle \mathbf{k}_1\mathbf{k}_2|V|\mathbf{k}'_1\mathbf{k}'_2\rangle + \sum_{\mathbf{p}_1\mathbf{p}_2}\langle \mathbf{k}_1\mathbf{k}_2|V|\mathbf{p}_1\mathbf{p}_2\rangle \frac{1}{E-\frac{\mathbf{p}_1^2}{2m}-\frac{\mathbf{p}_2^2}{2m}+i\eta}\langle \mathbf{p}_1\mathbf{p}_2|T|\mathbf{k}'_1\mathbf{k}'_2\rangle$$



exact

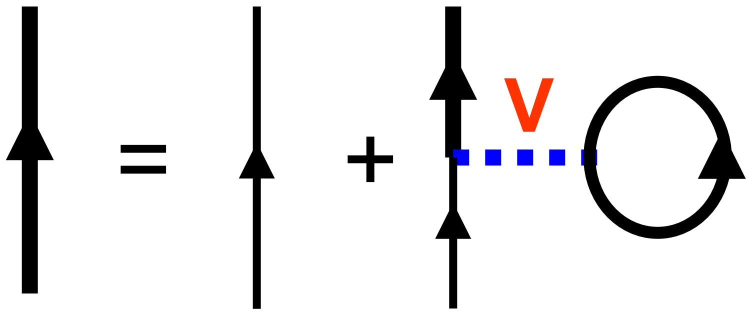
Scattering in the nuclear medium:

Bethe-Goldstone-Eq.

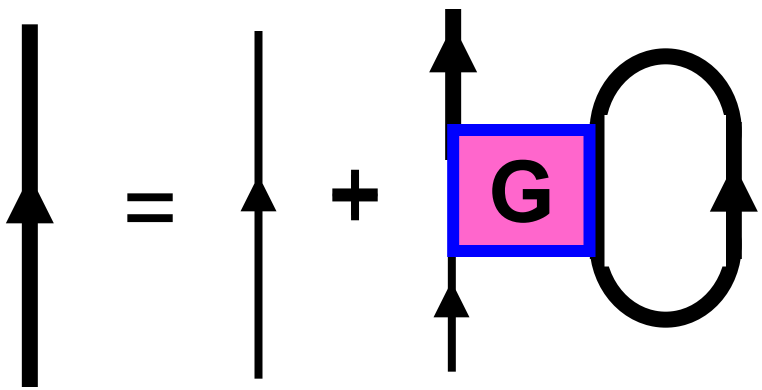


approximation

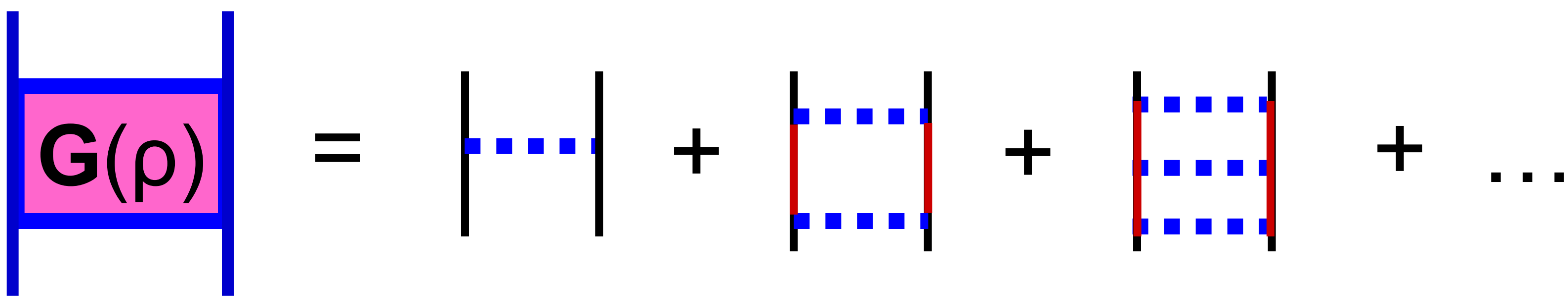
Ab-initio: Relativistic Brueckner Hartree-Fock:



Hartree-Fock

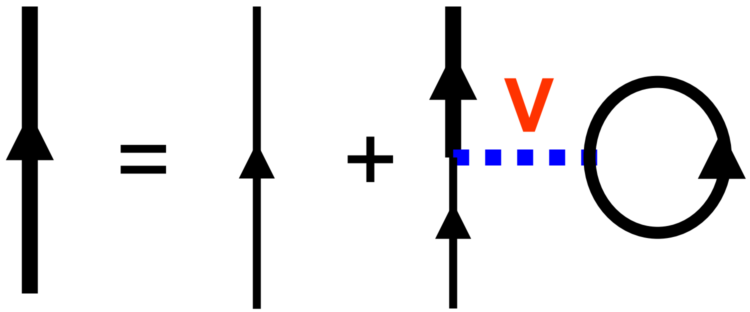


Brueckner Hartree-Fock

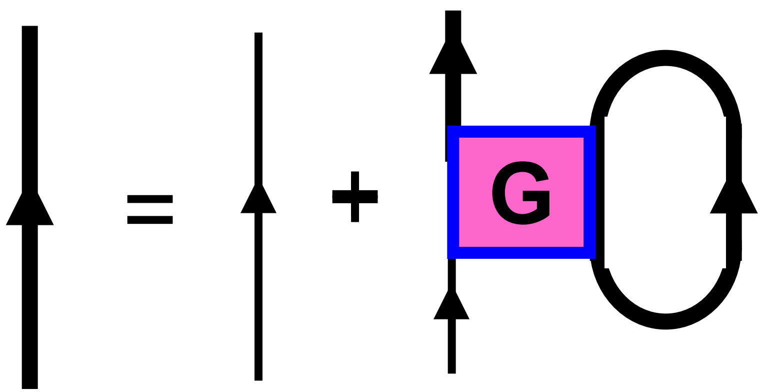


Summing up all ladder diagramms

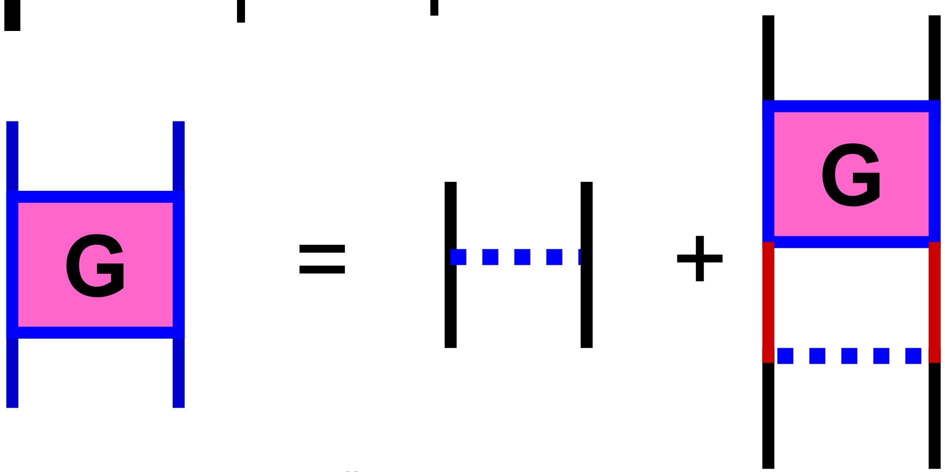
Ab-initio: Relativistic Brueckner Hartree-Fock:



Hartree-Fock



Brueckner Hartree-Fock

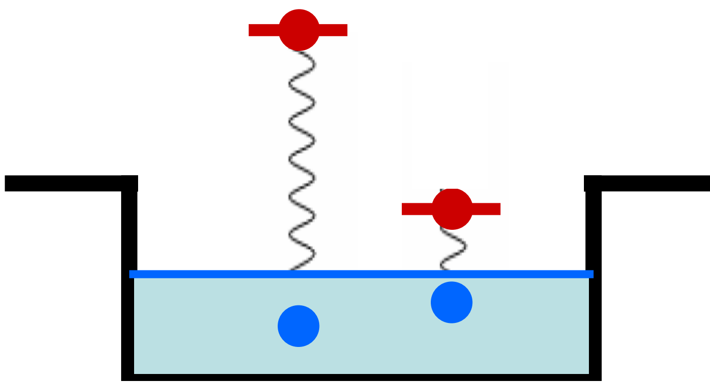


Bethe-Goldstone

Bethe-Goldstone equation:

- ω is the starting energy
- V is realistic interaction
- Q_F is the Pauli operator

$$G(\omega) = V + VQ_F \frac{1}{\omega - H_{HF}} Q_F G(\omega)$$



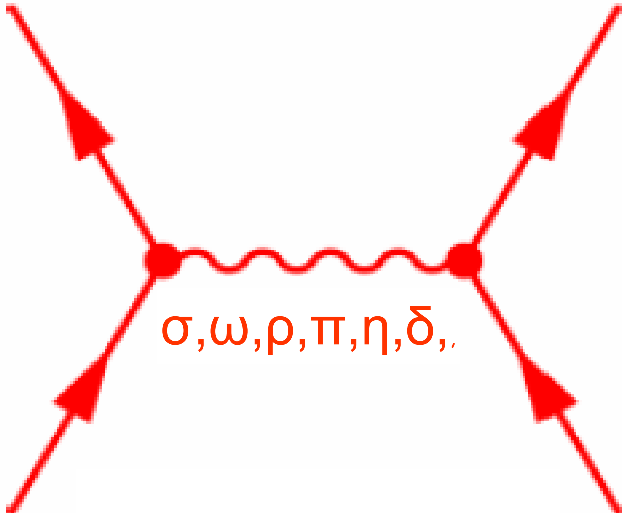
$$G(\omega) = V + V P_F(\omega) G(\omega)$$

$$P_F(\omega) = \sum_{m_1 m_2 > \varepsilon_F} |m_1 m_2\rangle \frac{1}{\omega - \varepsilon_{m_1} - \varepsilon_{m_2}} \langle m_1 m_2|$$

$$G(\omega) = \frac{1}{1 - V P_F(\omega)} V$$

Is solved in each step of the iteration

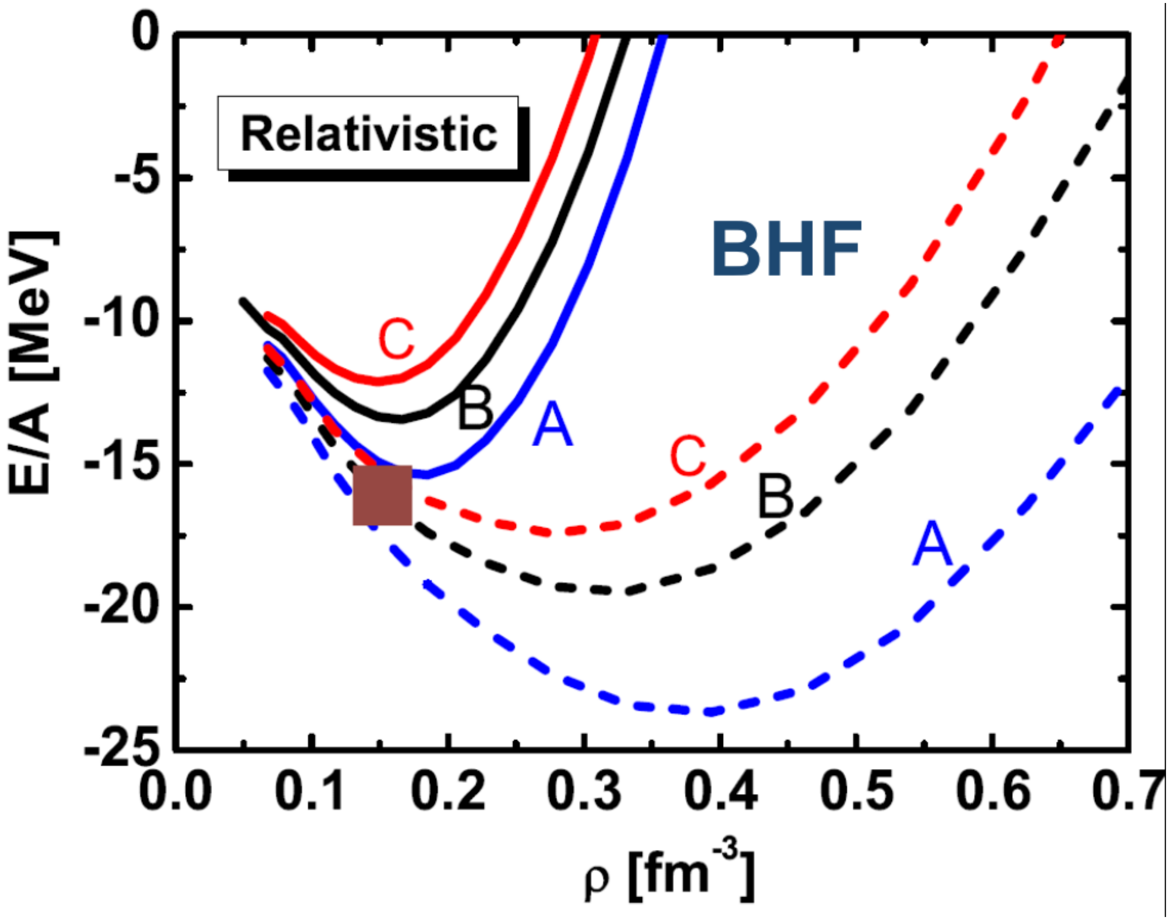
Bare nucleon-nucleon force:



Brockmann and Machleidt, PRC **42**, 1965 (1990).

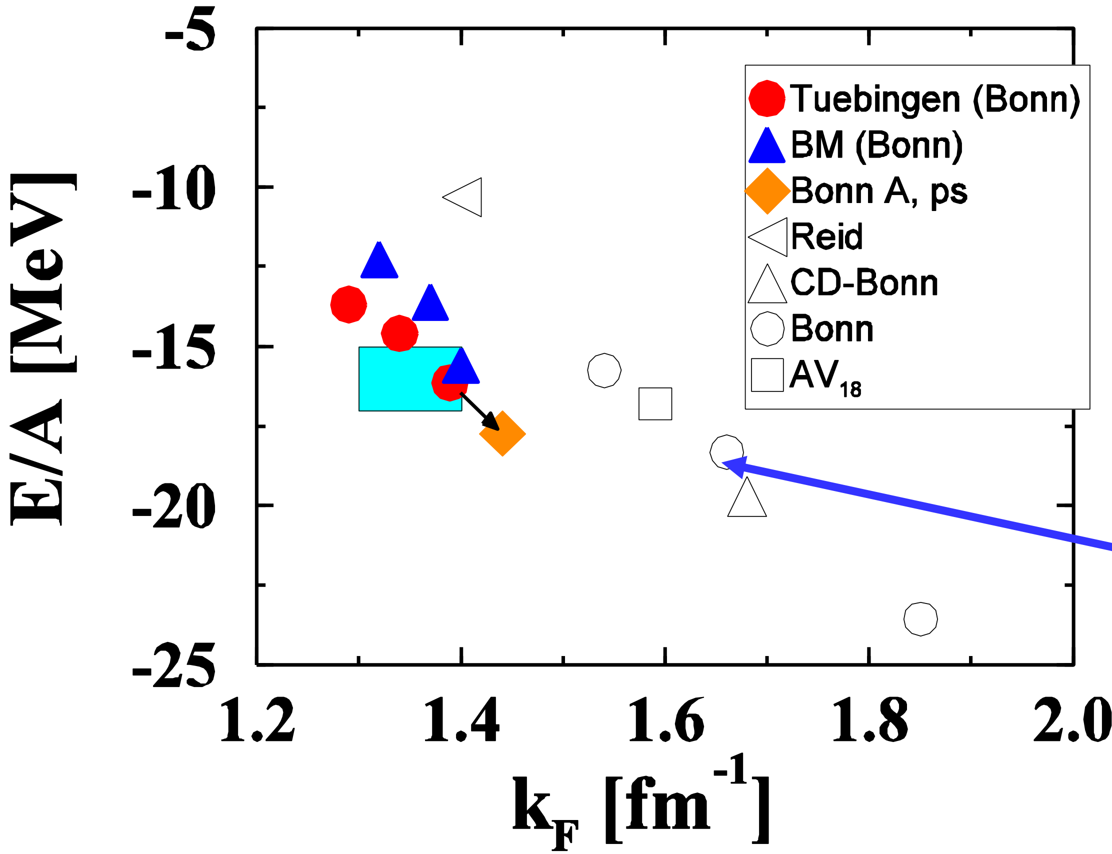
Meson Parameters	m_α (MeV)	Potential A $g_\alpha^2/4\pi$	Potential A Λ_α (GeV)	Potential B $g_\alpha^2/4\pi$	Potential B Λ_α (GeV)	Potential C $g_\alpha^2/4\pi$	Potential C Λ_α (GeV)
π	138.03	14.9	1.05	14.6	1.2	14.6	1.3
η	548.8	7	1.5	5	1.5	3	1.5
ρ	769	0.99	1.3	0.95	1.3	0.95	1.3
ω	782.6	20	1.5	20	1.5	20	1.5
δ	983	0.7709	2.0	3.1155	1.5	5.0742	1.5
σ	550	8.3141	2.0	8.0769	2.0	8.0279	1.8

Dirac-Brueckner-Hartree-Fock in nuclear matter



Brockmann and Machleidt, PRC **42**, 1965 (1990).

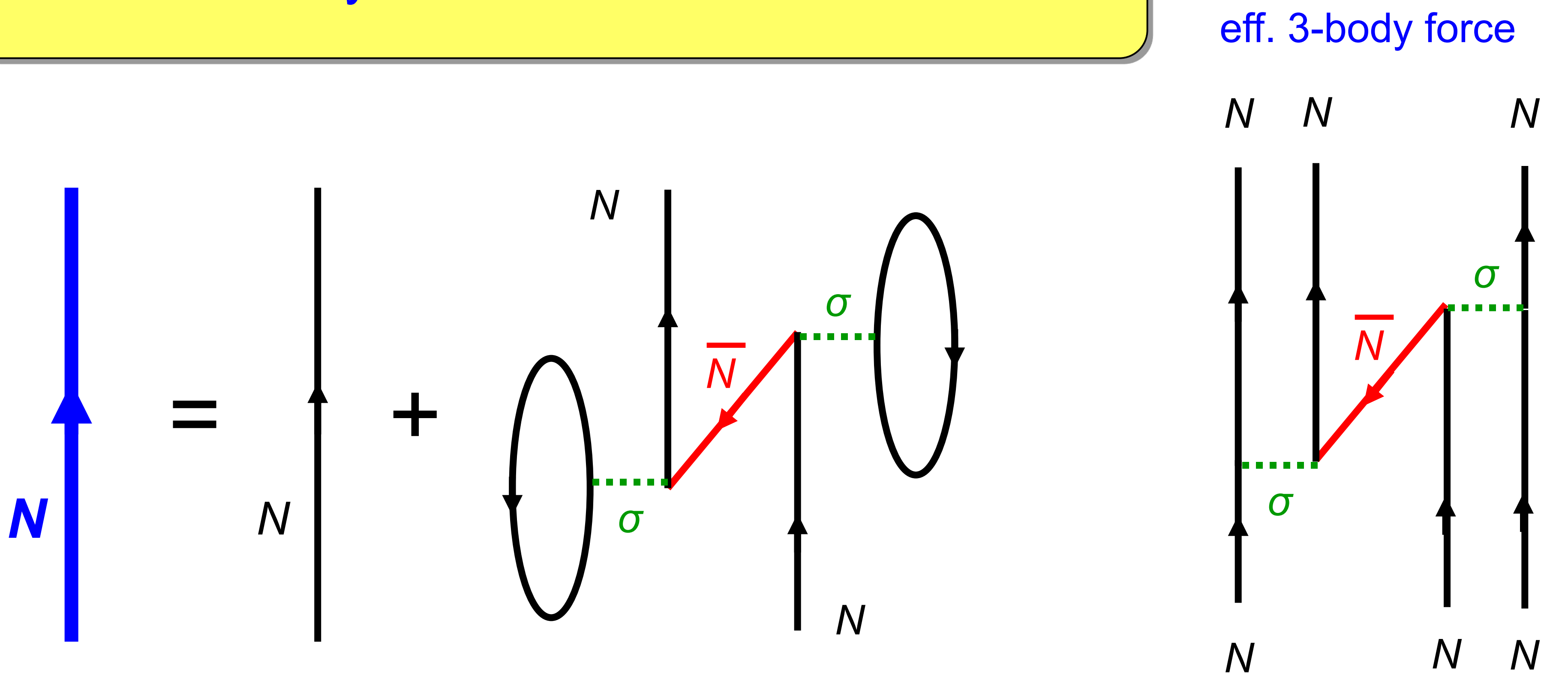
Dirac-Brueckner-Hartree-Fock in nuclear matter



C. Fuchs, LNP (2004)

Coester-line

Non-rel. 3-body-forces and relativistic effects



$$|\boldsymbol{u}(\boldsymbol{k},\lambda,m^*)\rangle = \alpha(m^*)|u(\boldsymbol{k},\lambda,m)\rangle + \beta(m^*)|\boldsymbol{v}(-\boldsymbol{k},-\lambda,m)\rangle$$

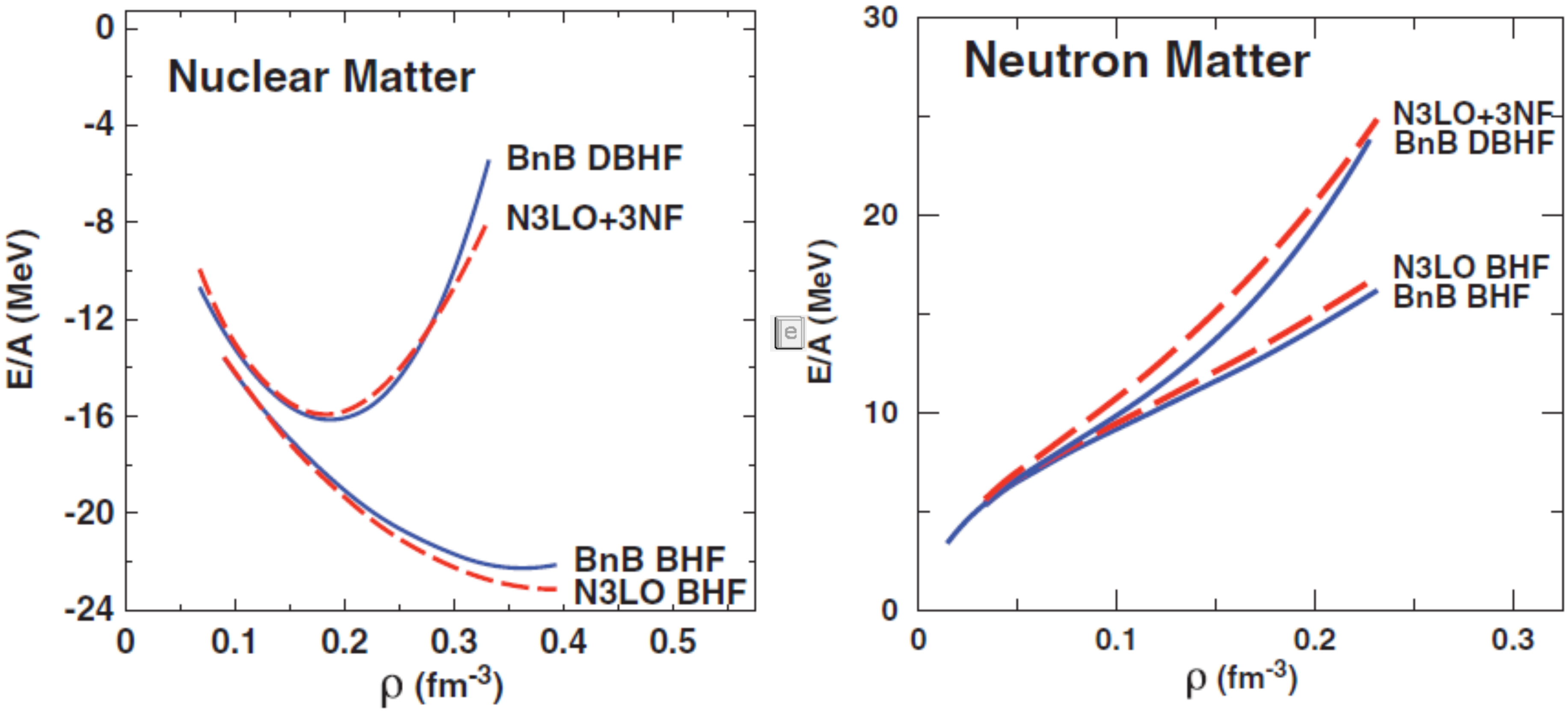
Dressed spinor

free spinor E>0

free spinor E<0

$$\rightarrow \quad \frac{\Delta E}{A} \approx 4.2\ \text{MeV} \quad \left(\frac{\rho}{\rho_0}\right)^{\frac{8}{3}}$$

Anastasio et al., PRC **23**, 2606 (1981) non-relativistic calculations with 3-body-forces vs. relativistic calculations without 3-body forces



Sammarruca, Chen, Coraggio, Itaco, Machleidt, PRC 86 , 054317 (2012)

Finite Nuclei: Local density approximation (LDA):

1. solve the Brueckner-Hartree-Fock equations
   in nuclear matter at various densities ρ

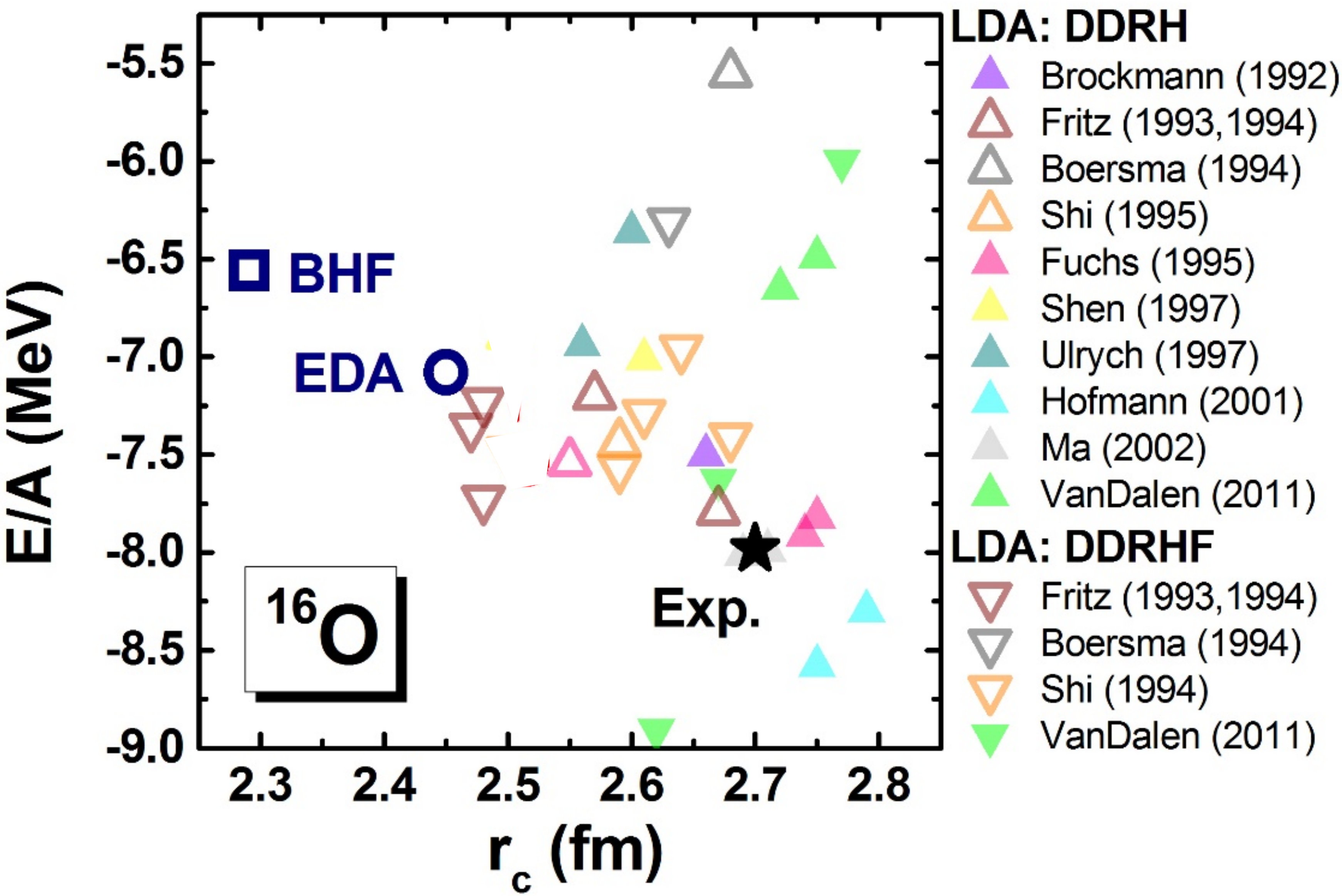
2. map the density dependent results on a
   Walecka model with density dependent couplings

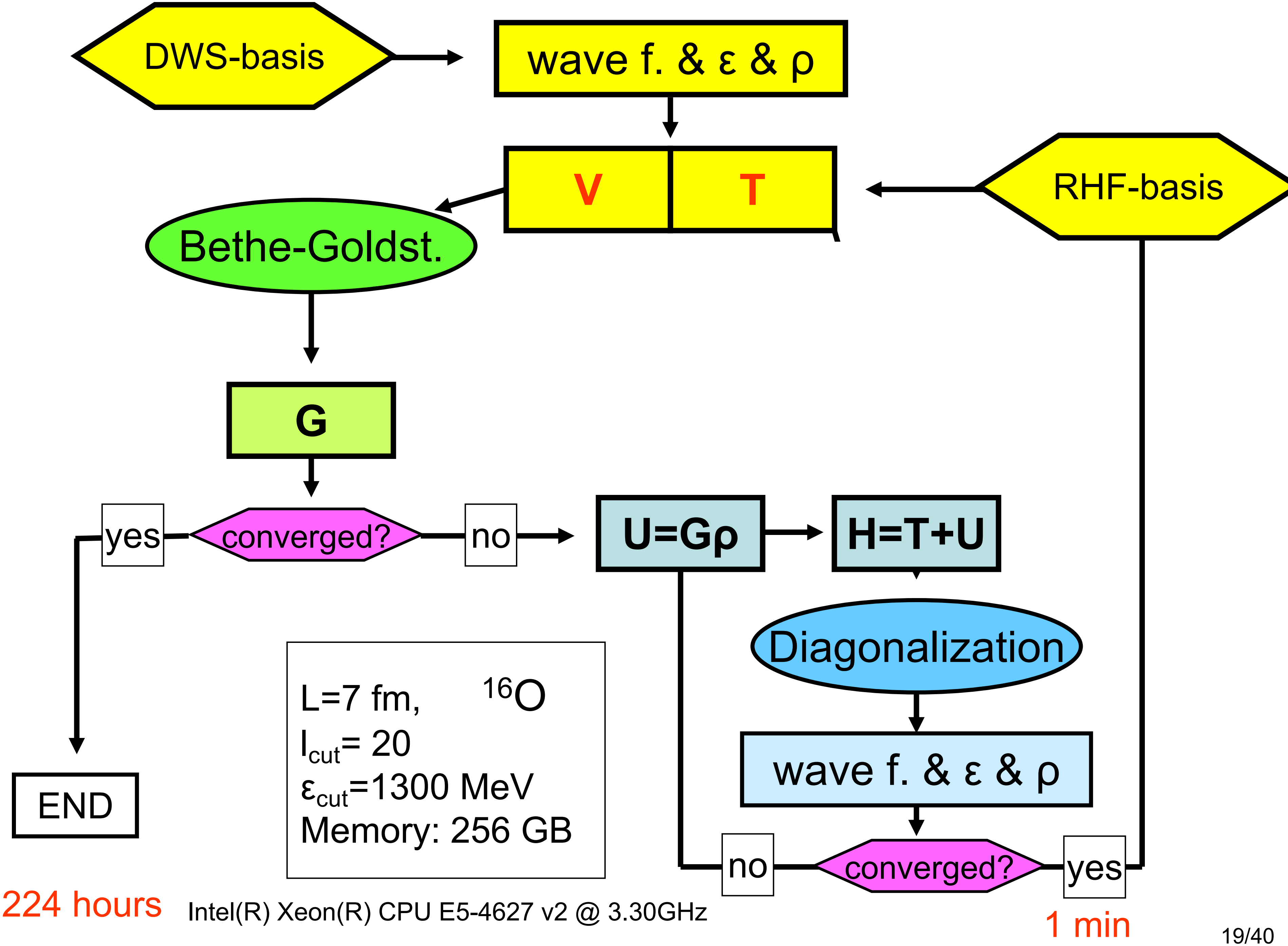
3. this yields → $g_\sigma(\rho)$, $g_\omega(\rho)$, ….

4. but: this mapping is not unique !

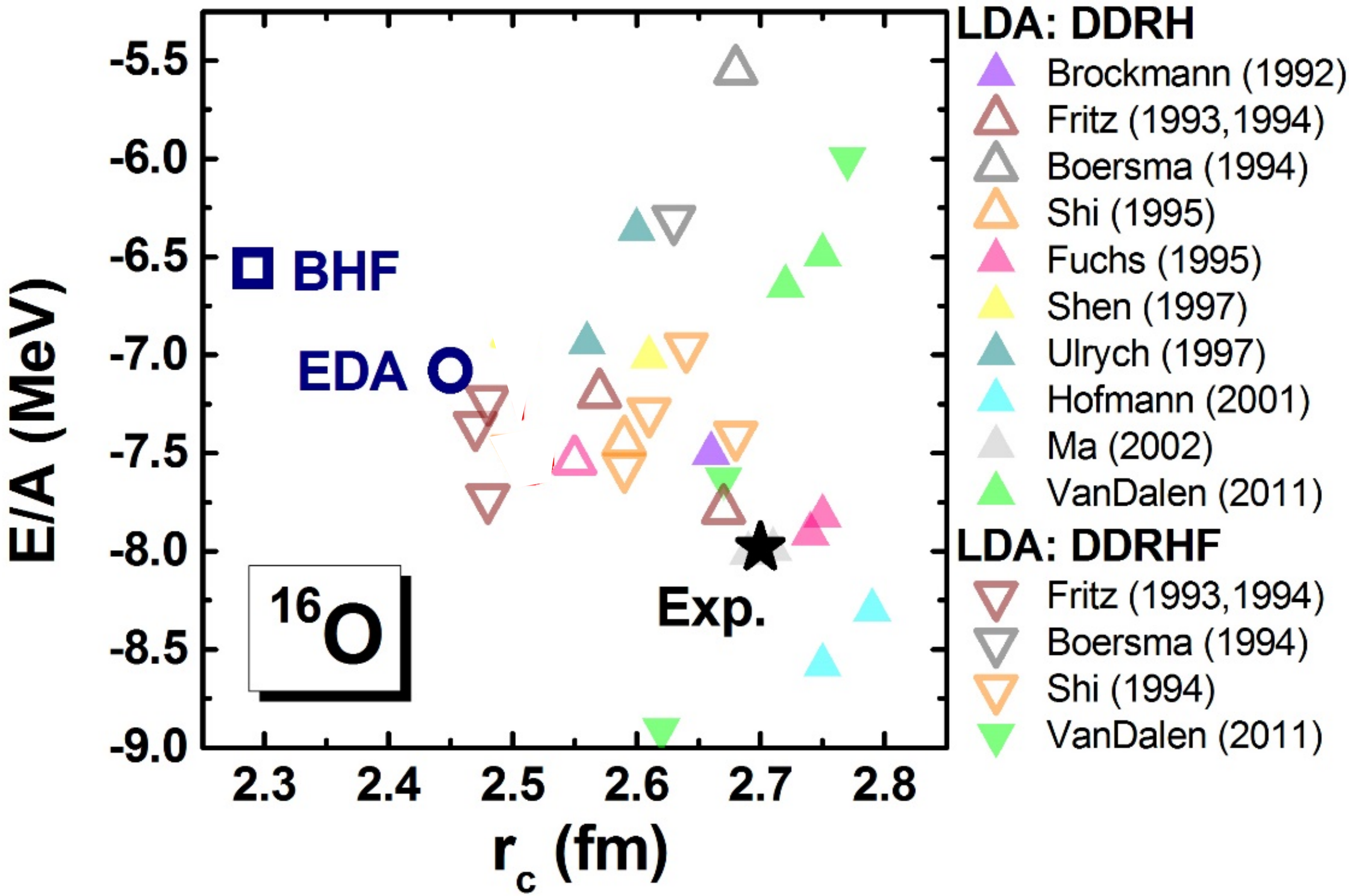
Brockmann and Toki, PRL **68**, 3408 (1992).

Relativistic BHf for finite nuclei:



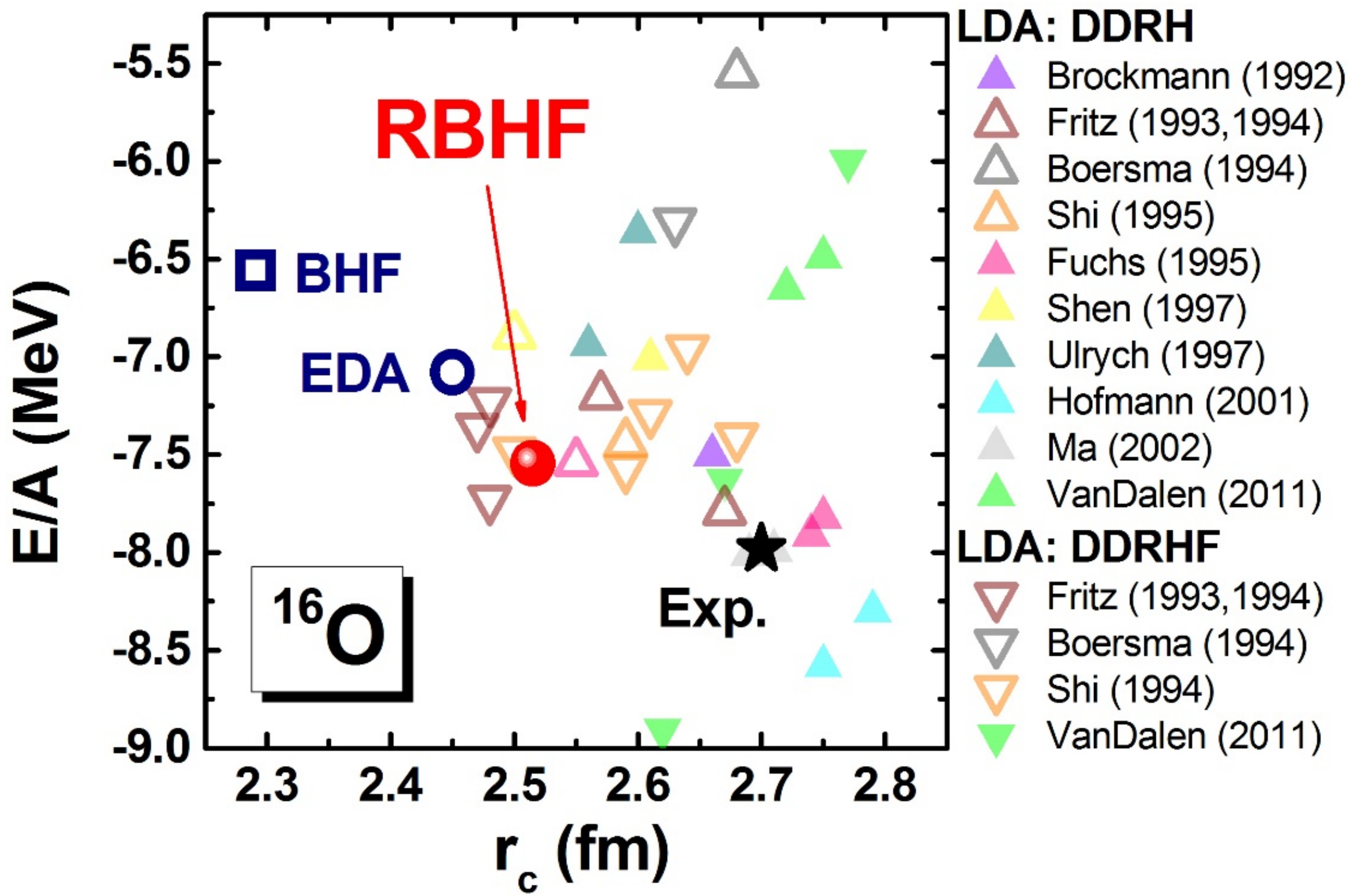

DWS-basis
wave f. & ε & ρ
V
T
RHF-basis
Bethe-Goldst.
G
yes
converged?
no
U=Gρ
H=T+U
Diagonalization
wave f. & ε & ρ
no
converged?
yes
END
L=7 fm, ^{16}O
l_{cut}= 20
ε_{cut}=1300 MeV
Memory: 256 GB
224 hours
Intel(R) Xeon(R) CPU E5-4627 v2 @ 3.30GHz
1 min

Relativistic BHf for finite nuclei:



Relativistic BHF for finite nuclei:

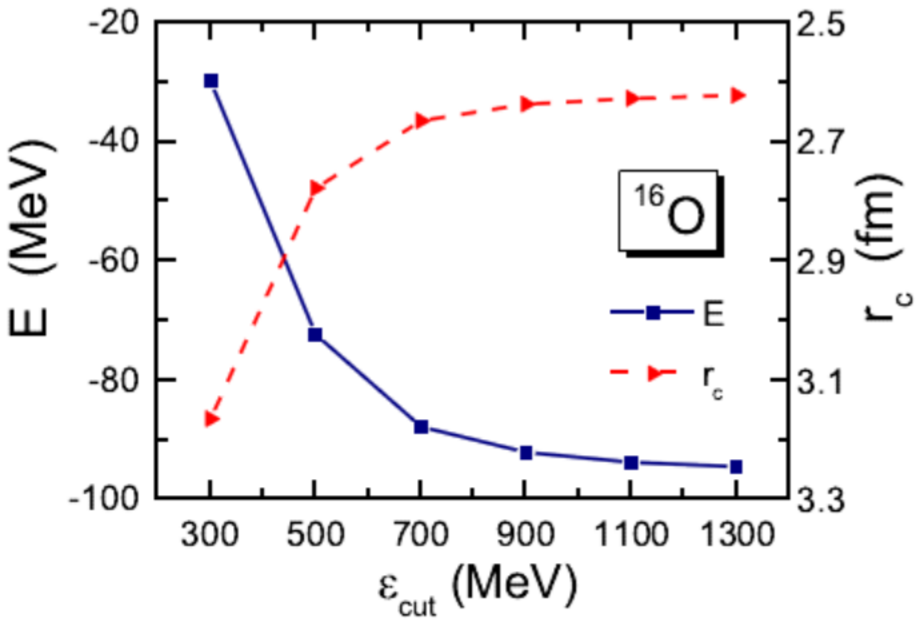
S.H. Shen et al (2017)

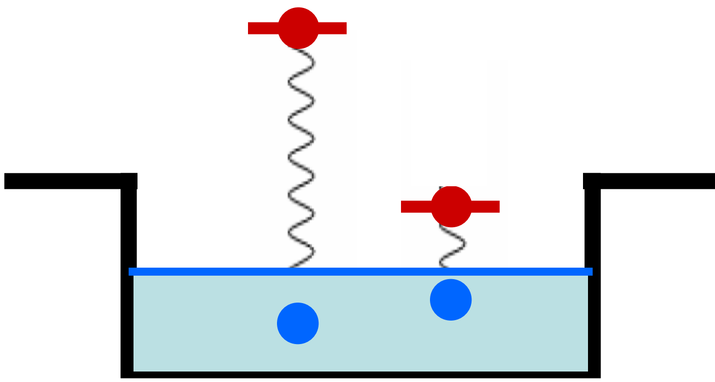


Results for ^{16}O

S.H. Shen et al, PRC 96, 014316 (2017)

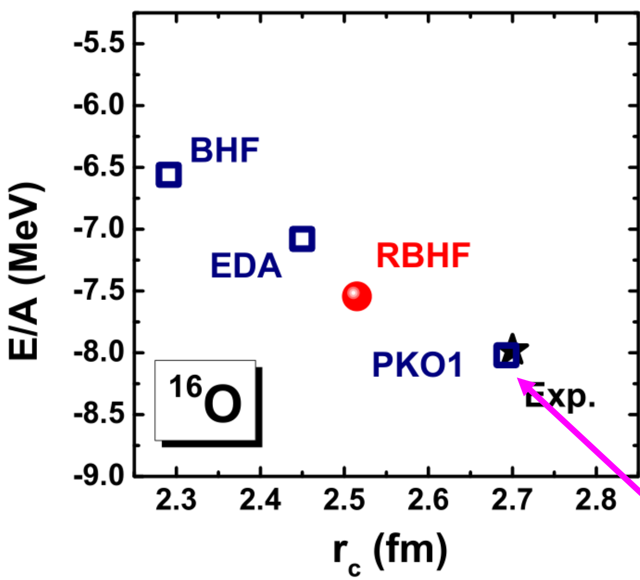
Convergenge with the cuf-off in single particle energy:

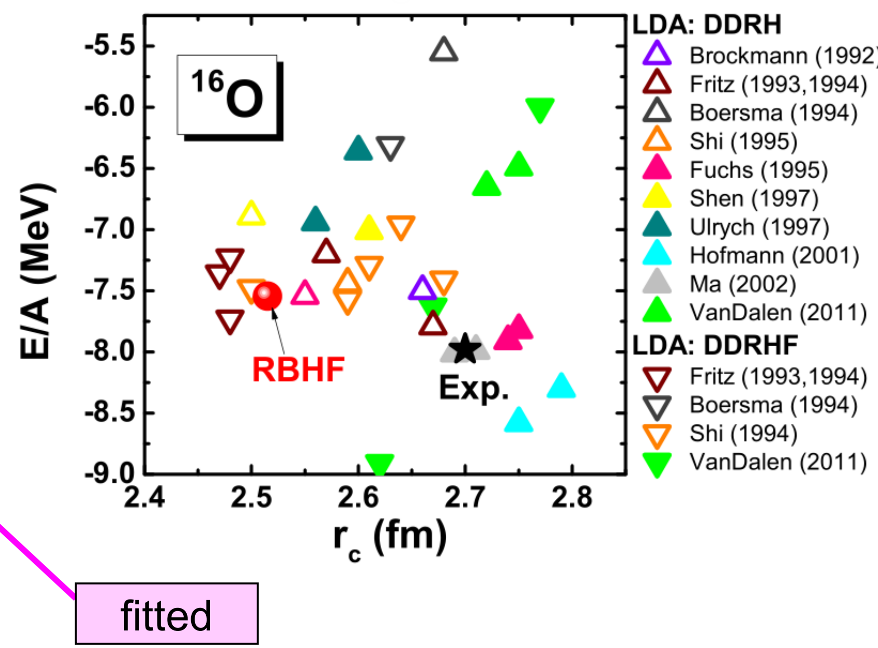




Bulk properties of ^{16}O:

- Energy per particle and charge radius of ^{16}O calculated by RBHF, non-relativistic BHF, BHF with EDA Müther1990PRC, RHF with PKO1 Long2010PLB (left); and RBHF with LDA (right) :



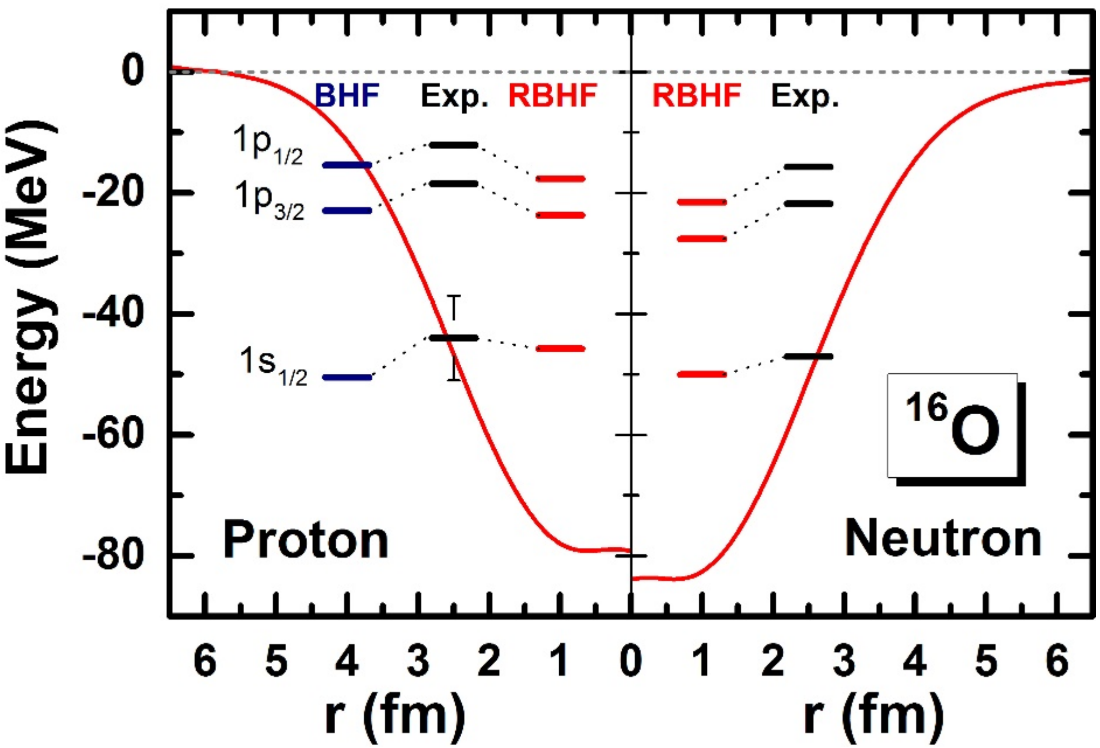


- Relativistic effect is very important to improve the description.
- There is a big uncertainty between different LDA calculations.

Results for ^{16}O:

	E (MeV)	r_c (fm)	r_m (fm)	$\Delta E^{ls}_{\pi 1p}$ (MeV)	
Exp.	-127.6	2.70	2.54	6.3	
DDRHF, PKO1	-128.3	2.68	2.54	6.4	Long et al, (2006)
DDRHF, PKA1	-127.0	2.80	2.67	6.0	Long et al, (2007)
RBHF, Bonn A	-120.2	2.53	2.39	5.3	S.H. Shen et al (2017).
RBHF (DWS)	-120.7	2.52	2.38	6.0	
BHF, AV18	-134.2	–	1.95	13.0	Hu et al, (2017)
CC, N^3LO	-120.9	–	2.30	–	Hagen et al, (2009)
IM-SRG, N^3LO	-122.9	–	–	–	Hergert et al, (2013)
NCSM, N^3LO	-119.7	–	–	–	Roth et al, (2011)
SCGF, N^3LO	-122.0	–	–	–	Cipollone et al, (2013)
NLEFT, N^2LO	-121.4	–	–	–	Laehde et al, (2014)
QMC, N^2LO	-87.0	2.76	–	–	Lonardoni et el, (2018

Single particle spectrum:



Shen, Hu, HZL, Meng, Ring, Zhang, *Chin. Phys. Lett.* **33**, 102103 (2016)

- a first *ab initio* calculations for finite nuclei in the **relativistic** scheme
- **Spin-orbit** splitting is reproduced well from the bare interaction
- **benchmark** for various LDA calculations

Different ab initio Methods for ^{40}Ca and ^{48}Ca

- Energies, charge radii, matter radii, and π1d spin-orbit splittings of ^{40}Ca and ^{48}Ca calculated by RBHF with Bonn A, comparing with data, CC with N^3LO G. Hagen, et al., *PRC* **82**, 034330 (2010) and with AV18 G. Hagen, et al., *PRC* **76**, 044305 (2007), BHF B. Hu, et al., *PRC* **95**, 034321 (2017), NCSM R. Roth, et al., *PRL* **99**, 092501 (2007).

	^{40}Ca				^{48}Ca		
	E (MeV)	r_c (fm)	r_m (fm)	$\Delta E_{\pi 1d}^{ls}$ (MeV)	E (MeV)	r_c (fm)	$\Delta E_{\pi 1d}^{ls}$ (MeV)
Exp.	**-342.1**	**3.48**	**-**	**6.6 ± 2.5**	**-416.1**	**3.48**	**4.7**
RBHF, Bonn A	-306.1	3.22	3.10	5.9	-357.3	3.25	2.7
CC, N^3LO	-345.2	-	-	-	-396.5	-	-
CC, AV18	-502.9	-	-	-	-	-	-
BHF, AV18	-552.1	-	2.20	24.9	-	-	-
NCSM, AV18	-461.8	-	2.27	-	-	-	-

For RBHF

Storage: 1100 GB
CPU time: 1720 h (=72d)

Storage: 1800 GB
CPU time: 4900 h (=204 d)

- Results for ^{40}Ca and ^{48}Ca given by RBHF are similar as for ^{16}O.
- CC with N^3LO reproduce the binding energy well, while other non-relativistic calculations give too much binding and too small radii.

Problems of RBHF in finite nuclei:

1. Limitation to light spherical nuclei (^{16}O, Ca, …)
   limitation in memory
   limitation in time (no parallelization for inversion)

2. Future goal: Softening of the bare relativistic force
   relativistic V_{lowk} (derived in nuclear matter)

3. Problem (since 40 years):
   There is no full solution of RBHF in nuclear matter !

Relativistic Hartree-Fock in nucl. matter

$$H = H_0 + \Sigma = \beta M + \vec{\alpha}\vec{k} + \Sigma$$

Self-energy Σ in the Walecka model:

$$\Sigma = \beta S + V_0 + \vec{\alpha}\vec{V} = \begin{pmatrix} S + V_0 & \vec{\sigma}\vec{V} \\ \vec{\sigma}\vec{V} & -S + V_0 \end{pmatrix}$$

Self-energy Σ in BHF:

$$\Sigma_{12} = \sum_{34} G[\rho]_{1324}\rho_{43} = \begin{pmatrix} \Sigma^{++} & \Sigma^{+-} \\ \Sigma^{-+} & \Sigma^{--} \end{pmatrix}$$

Conventional solution of RBHF in nucl. Matter:

Thompson-equation: (3D reduction of the Bethe-Salpeter Equation)

$$T^{++++}(E) = V^{++++} + V^{++++}\frac{1}{E - E_{kin}}T^{++++}(E)$$

Bethe-Goldstone equation

$$G^{++++}(W) = V^{++++} + V^{++++}\frac{Q}{W - E_{56}}G^{++++}(W)$$

Self energy:

$$\Sigma_{12}^{++} = \sum_{34} G_{1324}^{++++}\rho_{43}^{++} \qquad \Sigma^{-+} = ???, \qquad \Sigma^{--} = ???$$

Approximations for Σ^{-+}, Σ^{--} …

Perturbation theory: Anastasio et al, PRC 23 (1981)

Projection onto Lorentz invariants: Horowitz et al NPA 464 (1987)

Greens-function techniques: Weigel et al, PRC 38 (1988)

Momentum dependence of $\Sigma^{++}(p)$ is used to determine S and V_0
Brockmann et al, PRC 42 (1990)

Effective DBHF-method, Schiller et al, EPJA 11 (2001)

Full solution ????, Katayama et al, PLB 747 (2015)

Full solution Spectator-Method de Jong, Lenske, PRC 890 (1998)

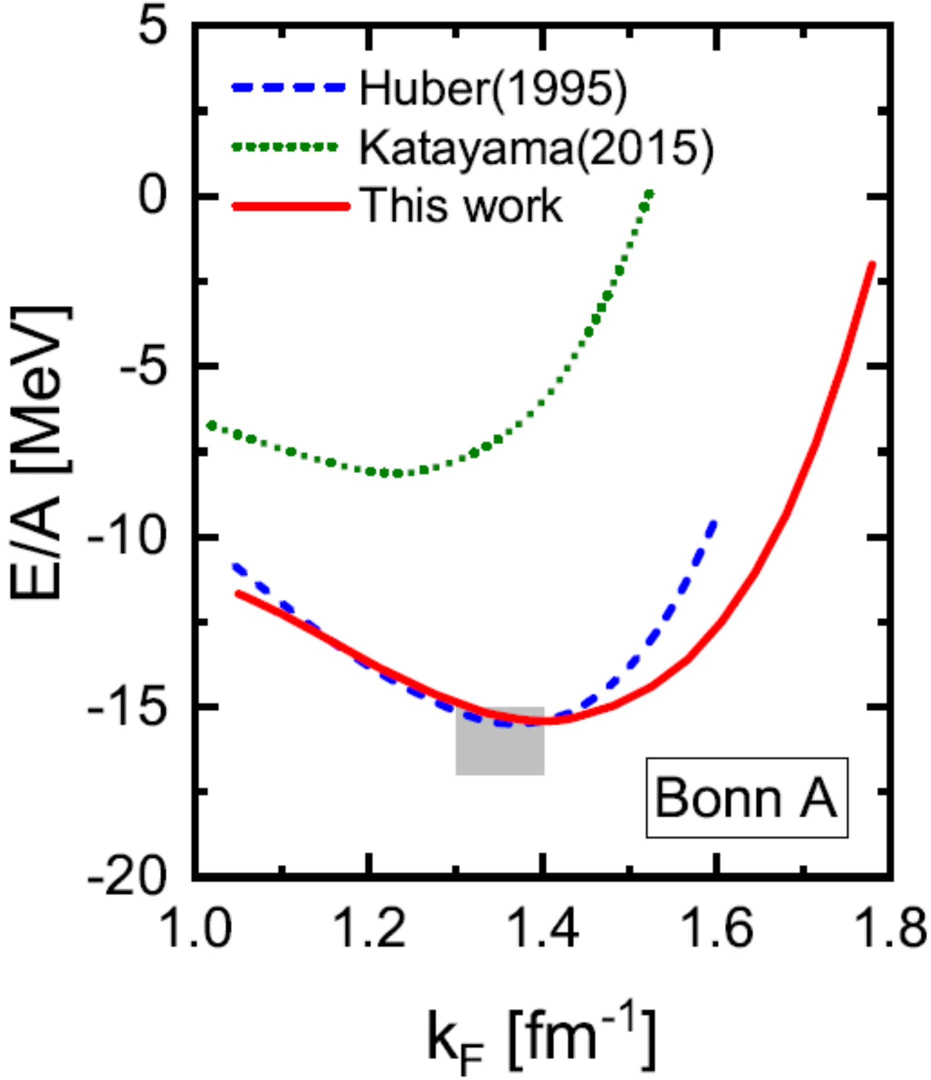
Full solution for G^{++++}, G^{+-++}, G^{--++}, …

$$G^{-+++}(W) = V^{-+++} + V^{-+++}\frac{Q}{W - E_{56}}G^{++++}(W)$$

$$
\begin{aligned}
{}^0G_J^{-+++} &= {}^0V_J^{-+++} + \int \frac{M_{\rm av}^{*2}}{E_{\rm av}^{*2}} \frac{Q_{\rm av}}{W - 2E_{\rm av}} \left[{}^0V_J^{-+++} \cdot {}^0G_J^{++++} + {}^2V_J^{-+++} \cdot {}^3G_J^{++++}\right],\\
{}^1G_J^{-+++} &= {}^1V_J^{-+++} + \int \frac{M_{\rm av}^{*2}}{E_{\rm av}^{*2}} \frac{Q_{\rm av}}{W - 2E_{\rm av}} \left[{}^3V_J^{-+++} \cdot {}^2G_J^{++++} + {}^1V_J^{-+++} \cdot {}^1G_J^{++++}\right],\\
{}^2G_J^{-+++} &= {}^2V_J^{-+++} + \int \frac{M_{\rm av}^{*2}}{E_{\rm av}^{*2}} \frac{Q_{\rm av}}{W - 2E_{\rm av}} \left[{}^0V_J^{-+++} \cdot {}^2G_J^{++++} + {}^2V_J^{-+++} \cdot {}^1G_J^{++++}\right],\\
{}^3G_J^{-+++} &= {}^3V_J^{-+++} + \int \frac{M_{\rm av}^{*2}}{E_{\rm av}^{*2}} \frac{Q_{\rm av}}{W - 2E_{\rm av}} \left[{}^3V_J^{-+++} \cdot {}^0G_J^{++++} + {}^1V_J^{-+++} \cdot {}^3G_J^{++++}\right],
\end{aligned}
$$

Sibo Wang, Ziang Zhao, P.R. and Jie Meng, PRC 103, 054301 (2021)

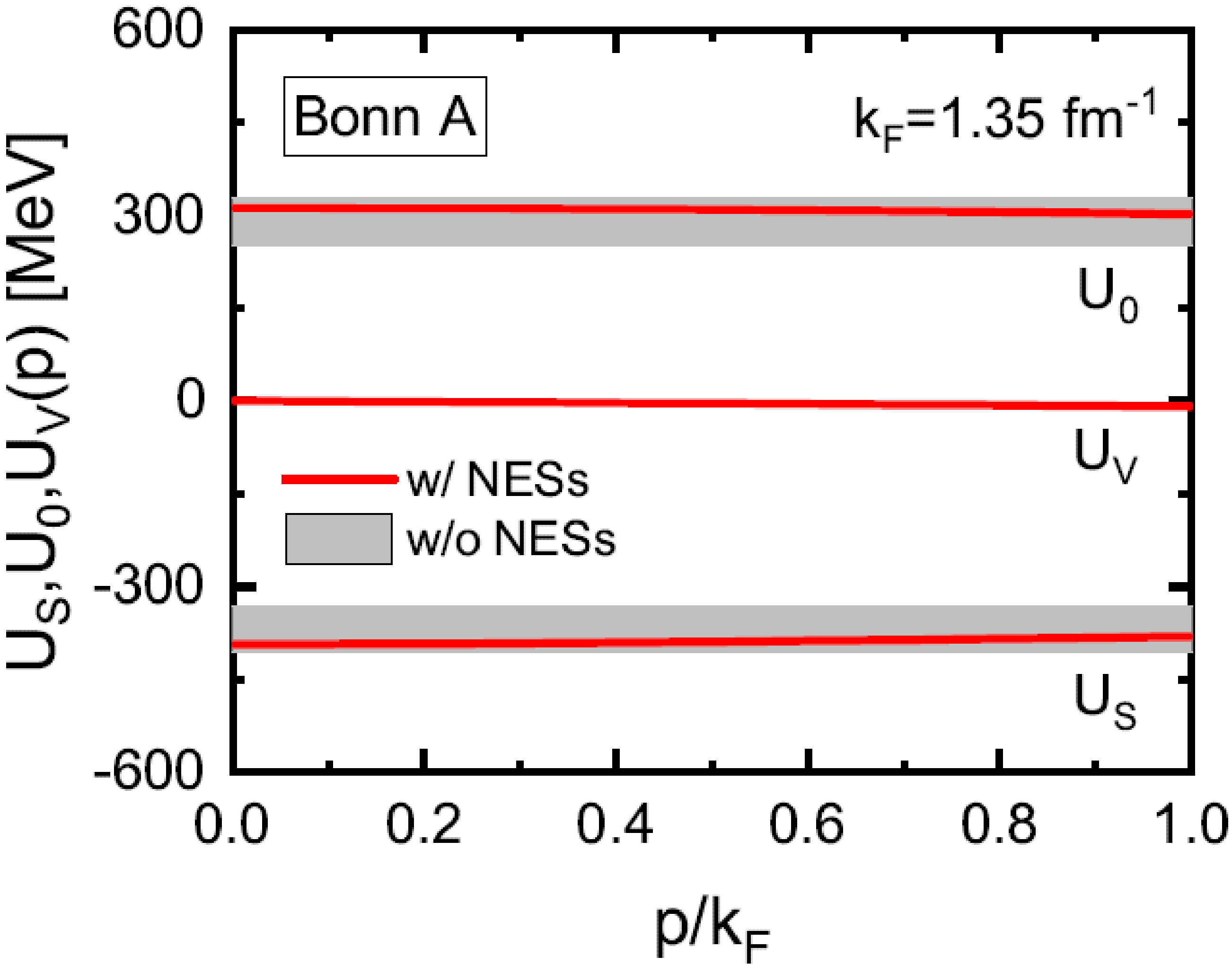
Results for symmetric nuclear matter:



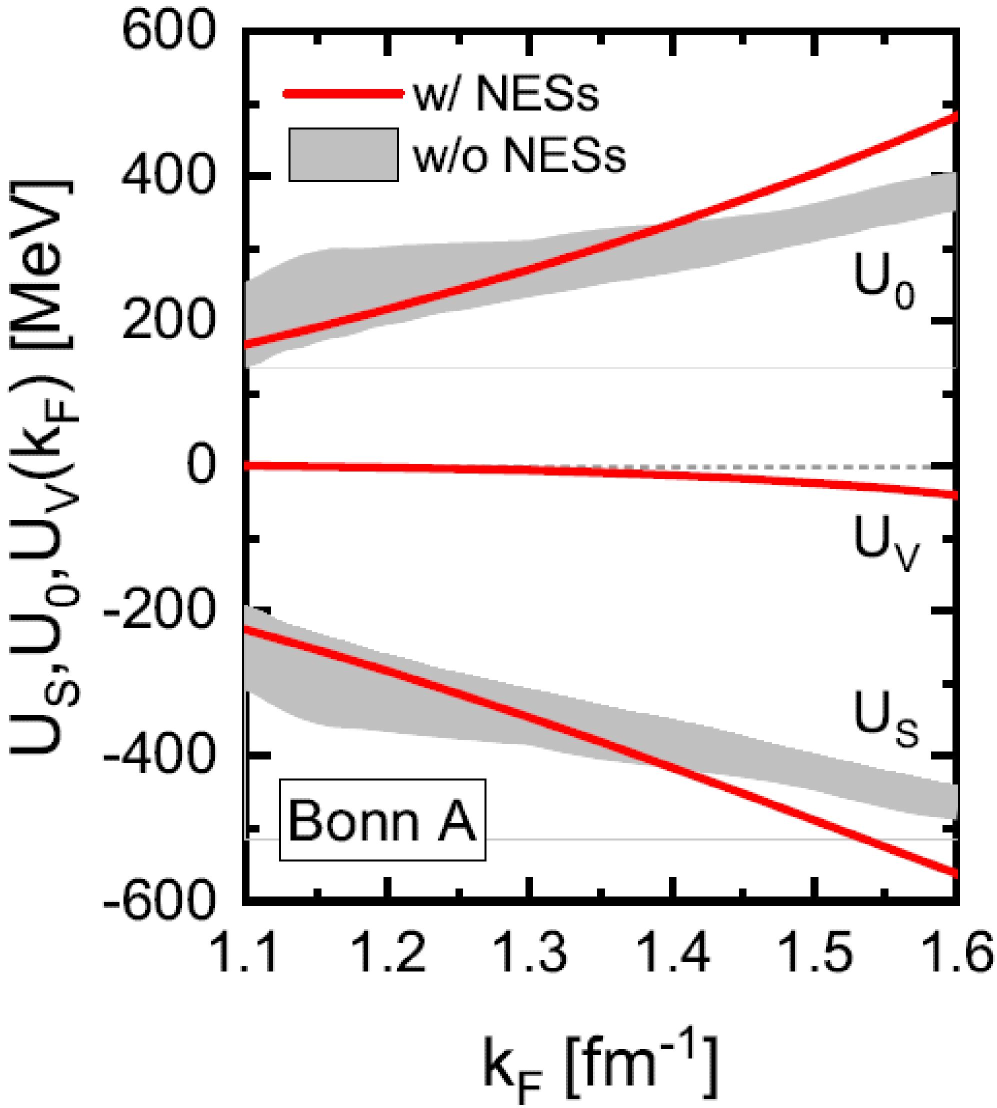
Sibo Wang, Ziang Zhao, P.R. and Jie Meng, PRC 103, 054301 (2021)

Momentum dependence of the effective mass:

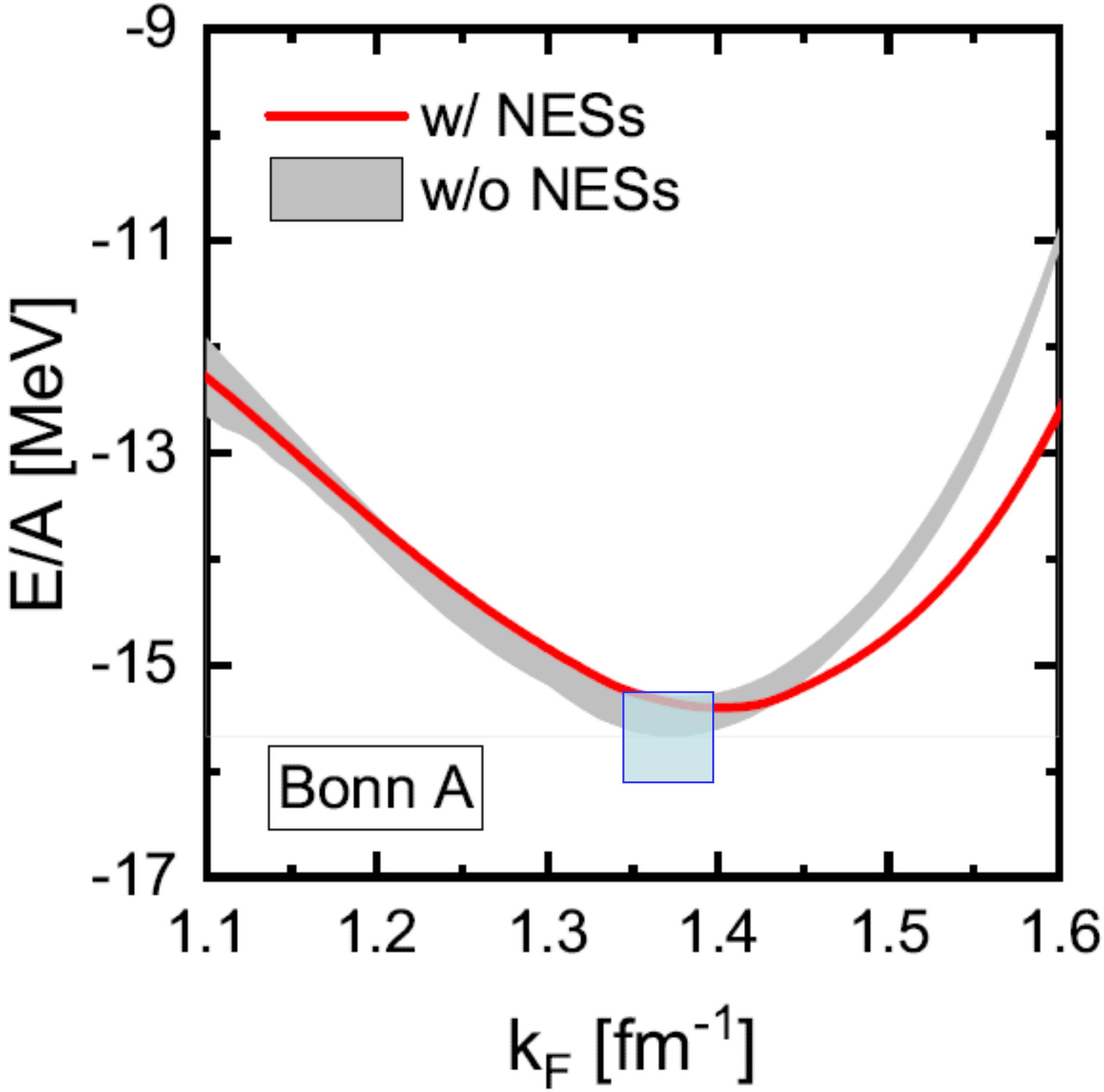
$$\Sigma(p) \;=\; \beta U_S(p) + U_0(p) + \vec{\alpha}\hat{\vec{p}}U_V(p)\ldots$$



Density dependence:



Equation of state:



Properties of symmetric nuclear matter:

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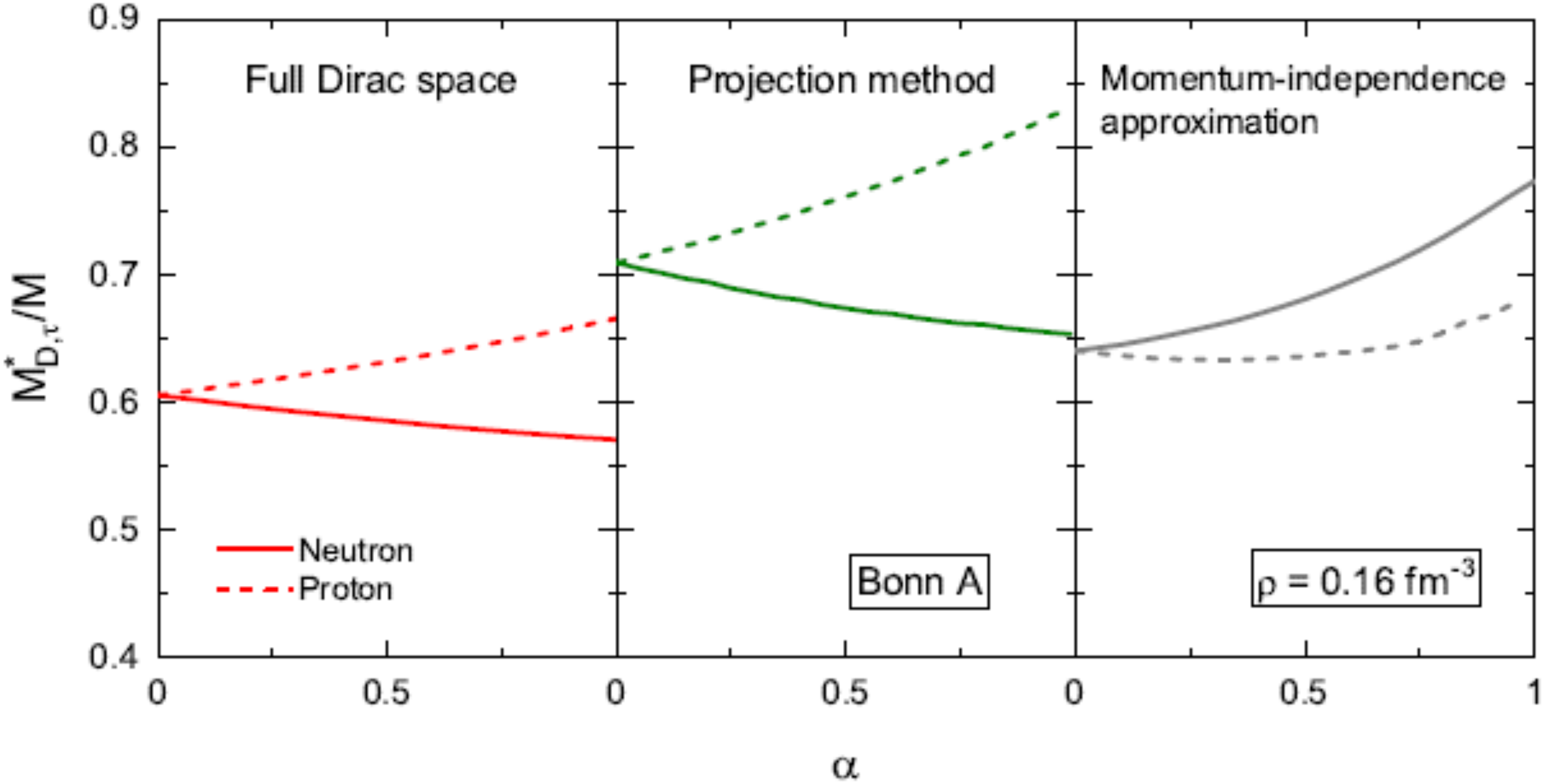
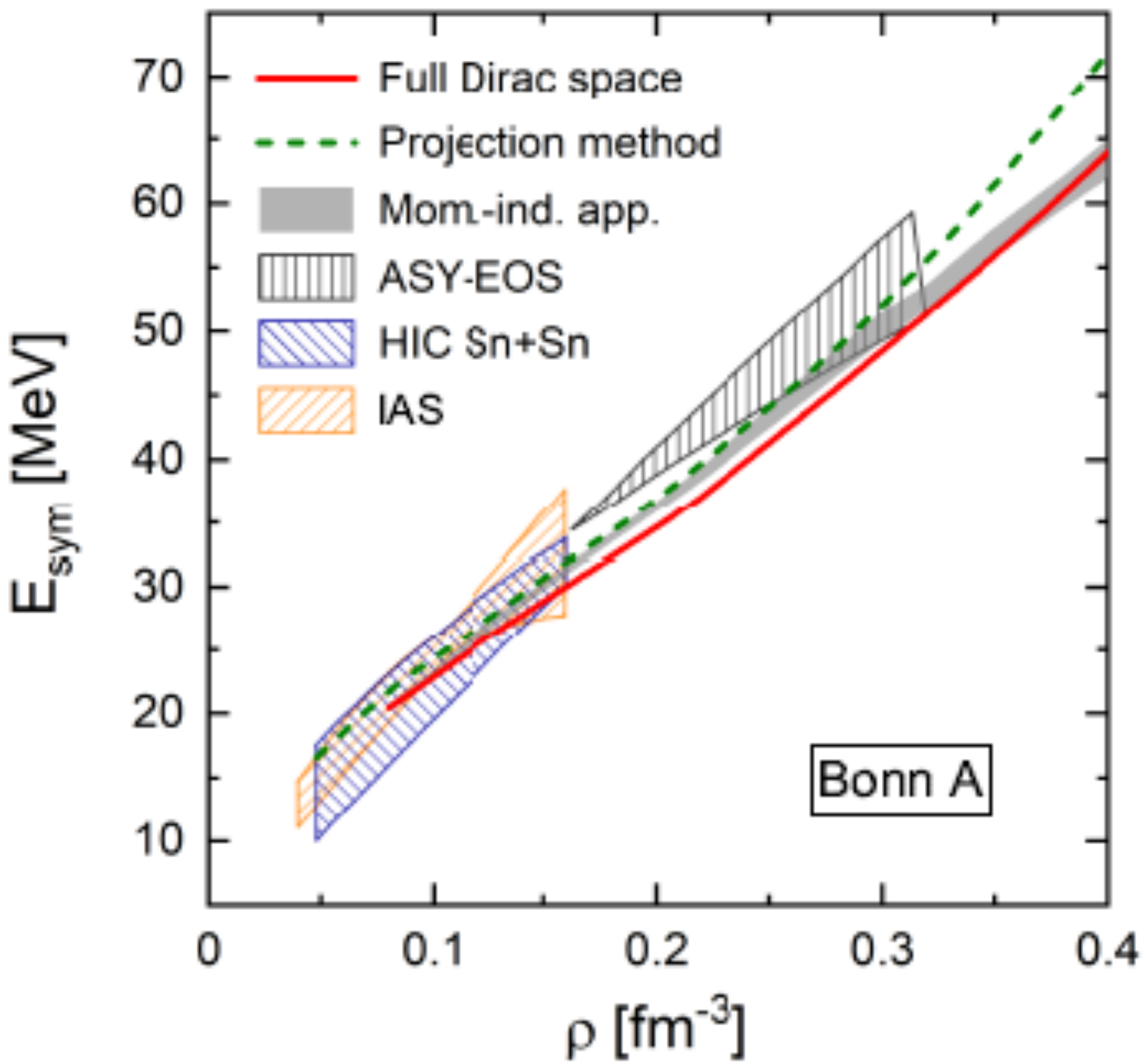
Potential	ρ_0 [fm^{-3}]	E/A [MeV]	K_∞ [MeV]	M_D^*/M
RBHF Bonn A	0.188	-15.40	258	0.55
RBHF Bonn B	0.164	-13.36	206	0.61
RBHF Bonn C	0.144	-12.09	150	0.65
BHF Bonn A	0.428	-23.55	204	
BHF Bonn B	0.309	-18.30	160	
BHF Bonn C	0.247	-15.75	103	
NL3	0.148	-16.30	272	0.60
DD-ME2	0.152	-16.14	251	0.57
DD-PC1	0.152	-16.06	230	0.58
PC-PK1	0.154	-16.12	238	0.59
PKO1	0.152	-16.00	250	0.59
Empirical	0.16 ±0.01	-16±1	240±20	

Isospin dependence of the effective Dirac mass:

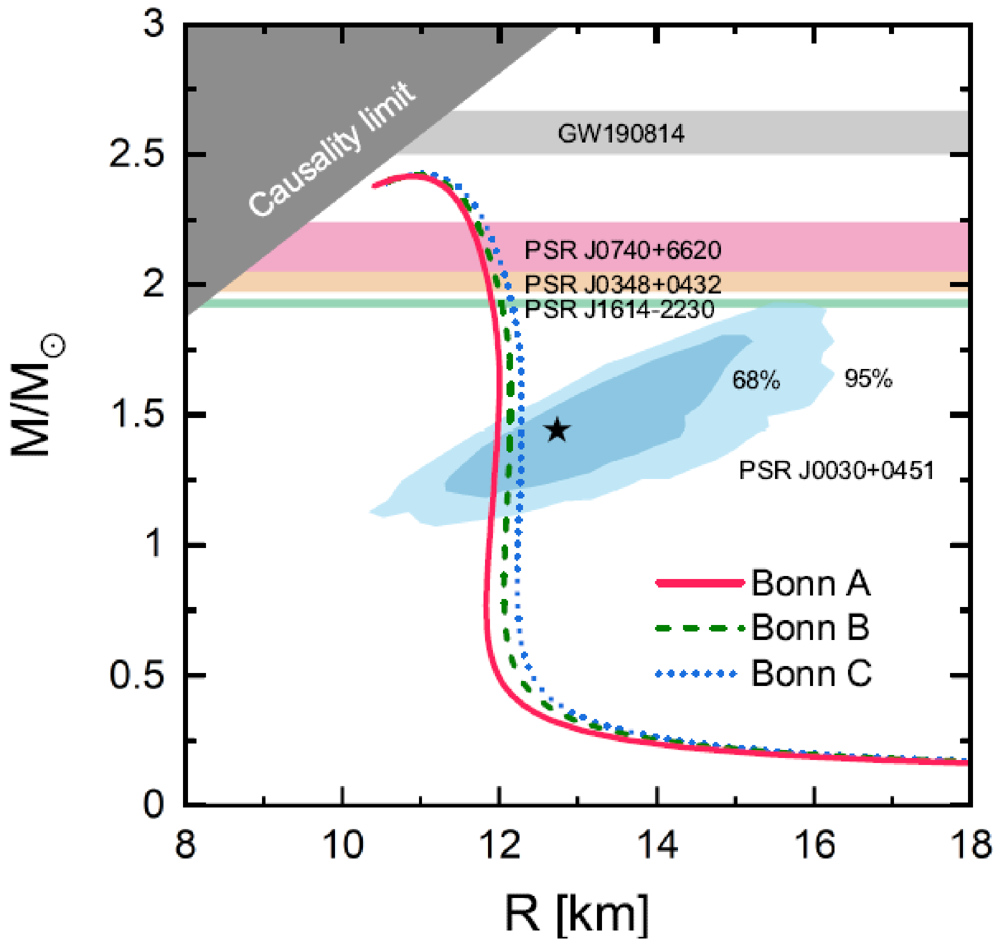


asymmetry parameter: $\alpha = (\rho_n - \rho_p)/\rho$ at saturation

Symmetry energy:



mass-radius relations for neutron stars:



Conclusions:

- RBHF is a successful microscopic tool
- Full solution in nuclear matter was missing since 40 years

  This gap is now solved

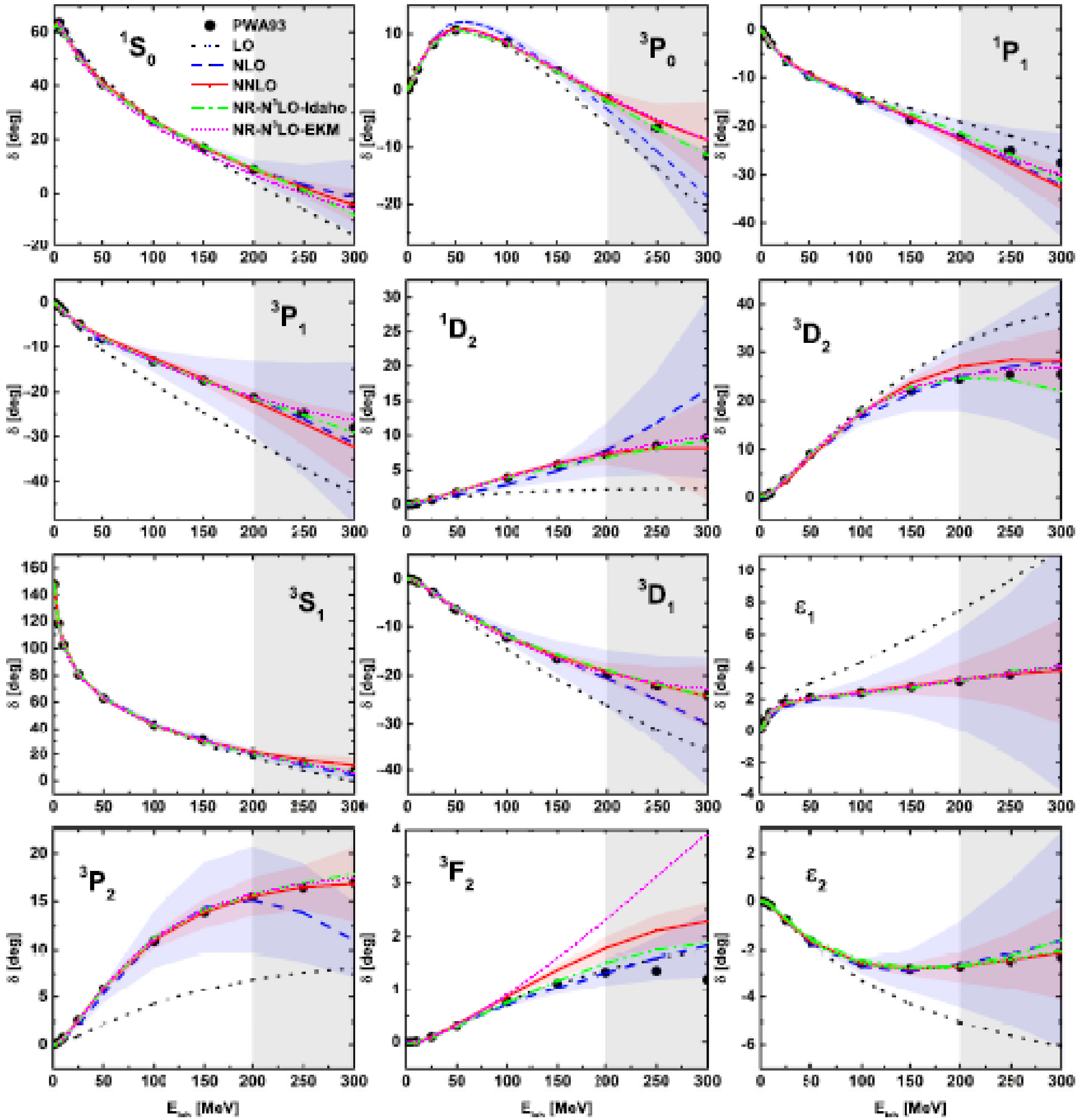
  Exact results are in agreement with earlier approximations

How to improve the results?

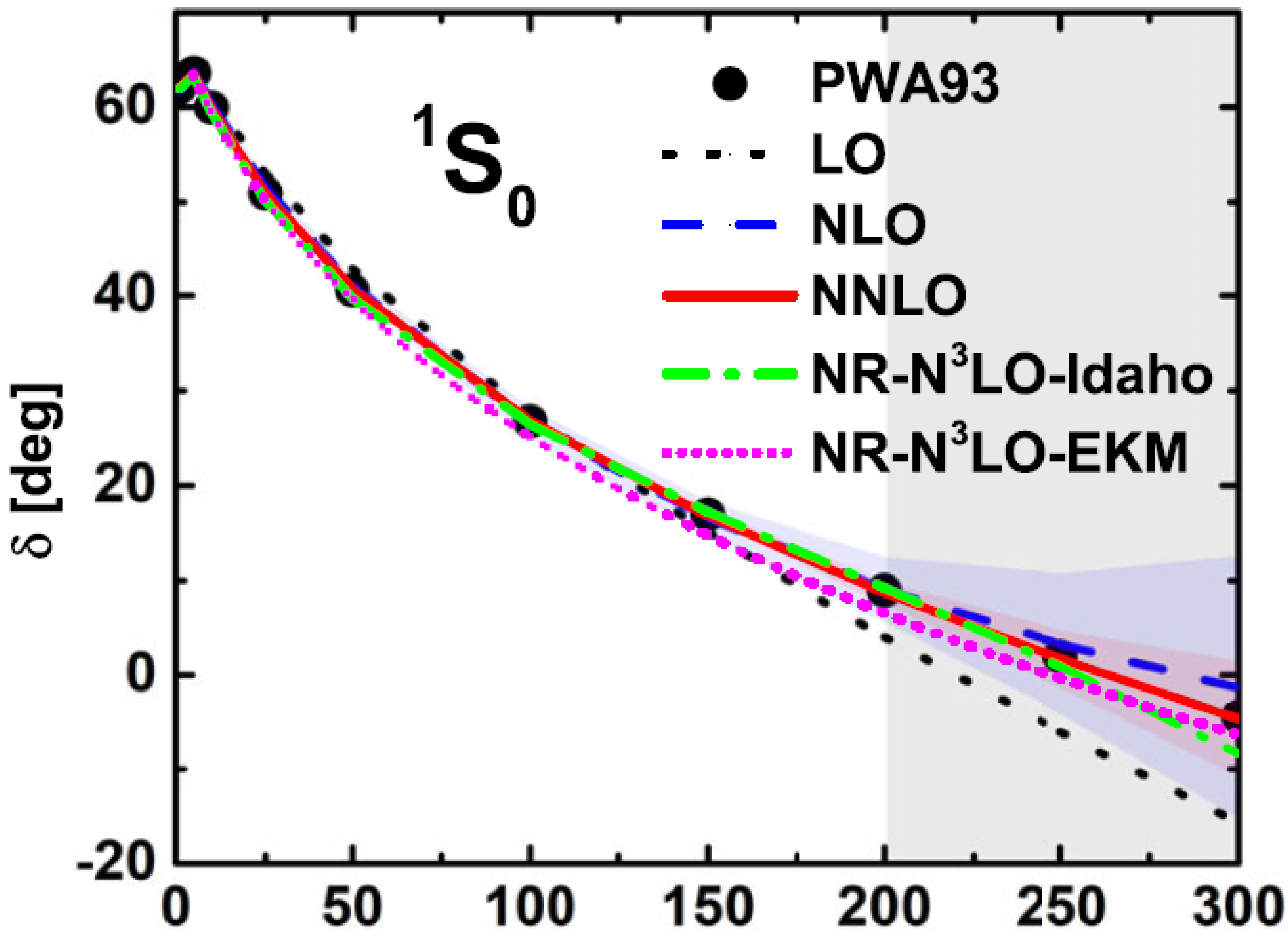
- Relativistic V_{lowk} ?
- Other relativistic NN-forces ?
- Relativistic NNN-forces ?
- Extended Brueckner theory (3 hole lines …)?

Chiral rel. NNLO force:

Lu, et al., PRC 103, 054301 (2021)



Chiral rel. NNLO force:



Outlook for the future:

- simplify the calculations:

  Brueckner theory with renormalized forces ($V_{low\ k}$) …

  Local density approximation under control

- heavy nuclei and the tensor force
- open shell nuclei: pairing, deformation
- optical potential
- short range correlations

Thanks to my collaborators:

Shihang Shen (Jülich)
S.Q. Zhang (Beijing)
Jie Meng (Beijing)
Haozhao Liang (RIKEN)
Sibo Wang (Beijing)
Hui Tong (Beijing)
Qiang Zhao (Beijing)
Chencan Wang (Beijing)

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Thank you