Spin dynamics of triaxial nuclei with quasiparticle alignments

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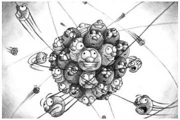
Theoretical background

Wobbling - phenomenological interpretation of the excited rotational states [A. Bohr and B. R. Mottelson, *Nuclear Structure* (1975)]

Chirality - handedness of the trihedral corners formed by three spin vectors with respect to the total angular momentum vector. [S. Frauendorf and J. Meng, Nucl. Phys. A **617**, 131 (1997)]

Particle-rotor Hamiltonian

$$H = H_R + H_{sp}$$



$$H_R = \sum_{k=1,2,3} A_k(\hat{I}_k - \hat{j}_k - \hat{j}'_k)^2$$ triaxial rotor Hamiltonian

$\vec{R} = \vec{I} - \vec{j} - \vec{j}'$ core angular momentum

$j \,\&\, j'$ single-particle spins $j(j') = 11/2$

A_k the inertial parameters related to the MOI by $A_k = \frac{1}{2\mathcal{J}_k}$

- Hydrodynamic MOI assumption: $\mathcal{J}_k = \frac{4}{3}\mathcal{J}_0 \sin^2\left(\gamma - \frac{2}{3}k\pi\right)$
- Axes 1,2,3 become long, short, intermediate $\Longrightarrow \gamma \in (60^\circ, 120^\circ)$

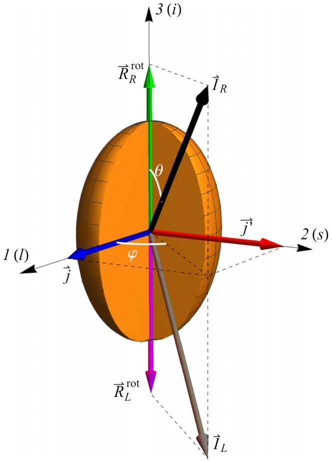
$$R_k = R_0\left[1 + \sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma - \frac{2\pi}{3}k\right)\right]$$

Alignment geometry

(1) long axis - alignment of the hole angular momentum

(2) short axis - alignment of the particle angular momentum

(3) medium axis - core rotation with maximal MOI



Rigidly aligned quasiparticles

$$\hat{j}_1 = j_1 \cos\alpha, \ \hat{j}'_1 = 0,$$
$$\hat{j}_2 = 0, \ \hat{j}'_2 = j' \cos\alpha',$$
$$\hat{j}_3 = j \sin\alpha, \ \hat{j}'_3 = j' \sin\alpha'$$

The relevant Hamiltonian to be treated

$$H_{align} =$$
$$A_1 \hat{I}_1^2 + A_2 \hat{I}_2^2 + A_3 \hat{I}_3^2$$
$$-2A_1 j \cos\alpha \hat{I}_1 - 2A_2 j' \cos\alpha' \hat{I}_2$$
$$-2A_3 (j \sin\alpha + j' \sin\alpha') \hat{I}_3$$

Transverse wobbling ☞ $j = 0$ and small α'

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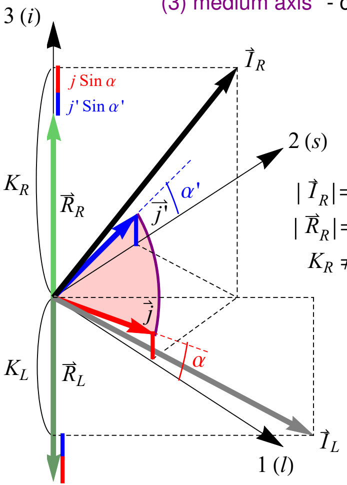

$$|\vec{I}_R| = |\vec{I}_L|$$
$$|\vec{R}_R| = |\vec{R}_L|$$
$$K_R \neq K_L$$

Rigidly aligned quasiparticles

$$\begin{aligned}
\hat{j}_1 &= j_1 \cos\alpha, \;\; \hat{j'}_1 = 0, \\
\hat{j}_2 &= 0, \;\; \hat{j'}_2 = j' \cos\alpha', \\
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Transverse wobbling ☞ $j = 0$ and small α'

- A description by means of only few classical variables associated to some particular dynamics of the quantum system - **is desired**

$\Downarrow$ Solution ☞ **The semiclassical approach**

[**RB**, Phys. Rev. C **97**, 024302 (2018); **98**, 014303 (2018), **RB**, Phys. Lett. B **797**, 134853 (2019); **817**, 136308 (2021)]

- Relies on a time-dependent variational principle applied to a variational state which is constructed according to the problem.

$$\delta \int_0^t \langle\psi|H_{align} - \frac{\partial}{\partial t'}|\psi\rangle dt' = 0$$

- The variational principle provides the time-dependence of some restricted set of variables which parametrize the variational state:

$$|\psi_{IM}(\theta,\varphi)\rangle = \sum_{K=-I}^{I} \frac{1}{2^I}\sqrt{\frac{(2I)!}{(I-K)!(I+K)!}}(1+\cos\theta)^{\frac{I-K}{2}}(1-\cos\theta)^{\frac{I+K}{2}} e^{i\varphi(I+K)}|IMK\rangle$$

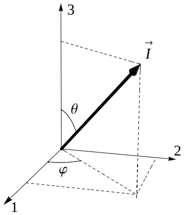
$$= \frac{1}{\left(1+\tan^2\frac{\theta}{2}\right)^I} e^{\tan\frac{\theta}{2}e^{i\varphi}\hat{I}_-}|IMI\rangle$$

Stereographic representation:

$0 \le \theta < \pi,\ \ 0 \le \varphi < 2\pi$

Projection variable $x = I\cos\theta,\ \ -I < x \le I$

$\equiv K$ projection on axis 3



Semiclassical description

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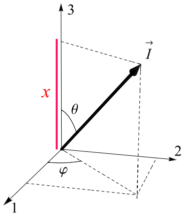


- Solving the equations provided by the variational principle is equivalent to solving the eigenvalue equation associated to the quantum Hamiltonian H_{align}.

The full structure of the classical system is reproduced if the variables are canonical:

$$\frac{\partial \mathcal{H}}{\partial x} = \dot{\varphi}, \quad \frac{\partial \mathcal{H}}{\partial \varphi} = -\dot{x} \quad \text{or} \quad \{x, \mathcal{H}\} = \dot{x}, \ \{\varphi, \mathcal{H}\} = \dot{\varphi}, \quad \{f, g\} = \frac{\partial f}{\partial \varphi}\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial g}{\partial \varphi}$$

$$\{\varphi, x\} = 1$$

φ the generalized coordinate

x the generalized momentum

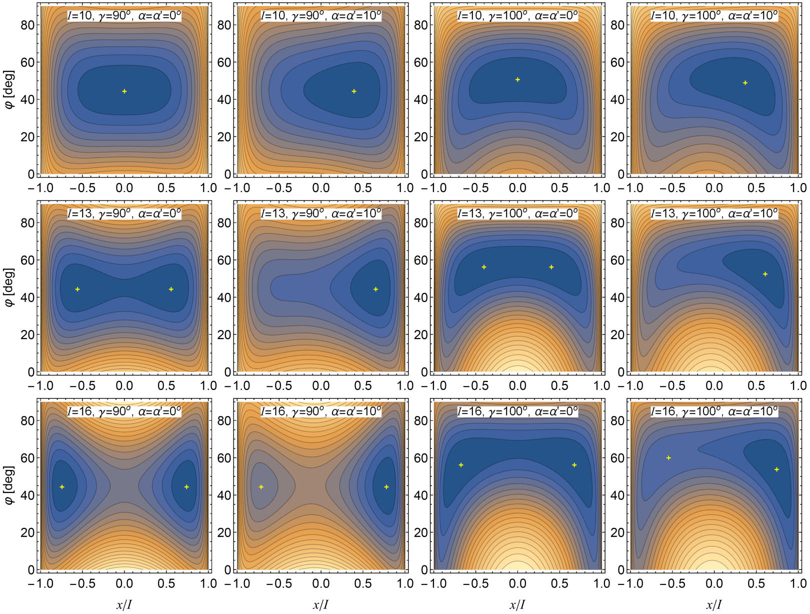
Poisson bracket

Classical energy function in terms of the canonical variables:

$$\begin{aligned}\mathcal{H}(x,\varphi) &= \frac{I}{2}(A_1 + A_2) + A_3 I^2 \\ &+ \frac{(2I-1)(I^2 - x^2)}{2I}(A_1 \cos^2\varphi + A_2 \sin^2\varphi - A_3) \\ &-2\sqrt{I^2 - x^2}(A_1 j \cos\alpha \cos\varphi + A_2 j' \cos\alpha' \sin\varphi) \\ &-2A_3 x(j \sin\alpha + j' \sin\alpha').\end{aligned}$$

- The classical orbits are closed curves in the phase space of the canonical coordinates which are concentrically positioned around the stationary points of the constant energy surface.

Classical energy function (chiral configuration $j = j' = 11/2$)



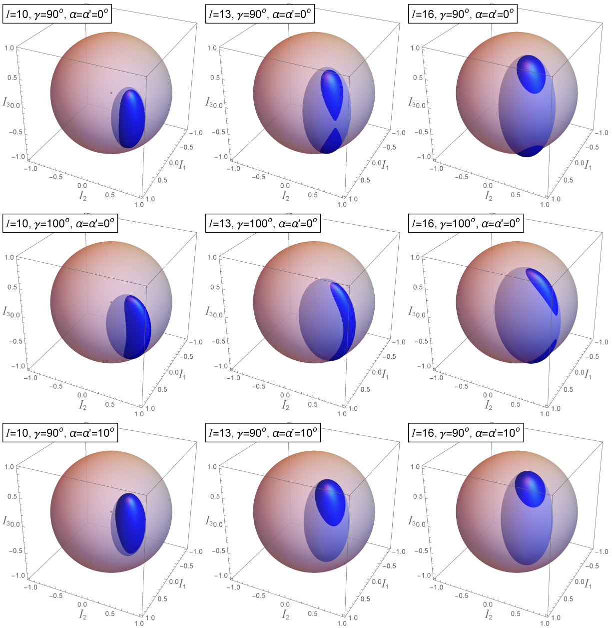
- The classical trajectory of the angular momentum vector $\vec{I}$ is a curve in the space of its classical projections

$$
\begin{aligned}
\mathcal{I}_1 &= \langle \hat{I}_1 \rangle = \sqrt{I^2 - x^2}\cos\varphi, \\
\mathcal{I}_2 &= \langle \hat{I}_2 \rangle = \sqrt{I^2 - x^2}\sin\varphi, \\
\mathcal{I}_3 &= \langle \hat{I}_3 \rangle = x. \\
\sum_k \langle \hat{I}_k^2 \rangle &= I(I+1)
\end{aligned}
$$

- It is determined by the intersection of the constant energy surfaces provided by the constants of motions:

$$
\begin{aligned}
\text{Shifted ellipsoid} \quad \mathcal{H} &= A_1\mathcal{I}_1^2 + A_2\mathcal{I}_2^2 + A_3\mathcal{I}_3^2 \\
&\quad -2A_1\mathcal{I}_1 j\cos\alpha - 2A_2\mathcal{I}_2 j'\cos\alpha' \\
&\quad -2A_3\mathcal{I}_3(j\sin\alpha + j'\sin\alpha'), \\
\text{Sphere} \quad I^2 &= \mathcal{I}_1^2 + \mathcal{I}_2^2 + \mathcal{I}_3^2.
\end{aligned}
$$

Classical trajectories (chiral configuration $j = j' = 11/2$)



Stationary points for chiral configuration (no tilting)

- The stationary points where $\dot{\varphi} = \dot{x} = 0$ and are stable against fluctuations are those which minimize the classical energy.

Planar

$$x = 0, \quad \frac{(2I-1)}{2}(A_2 - A_1)\cos\varphi_p \sin\varphi_p = A_2 j' \cos\varphi_p - A_1 j \sin\varphi_p$$

$$\mathcal{I}_1 = I\cos\varphi_p, \quad \mathcal{I}_2 = I\sin\varphi_p, \quad \mathcal{I}_3 = 0.$$

Aplanar

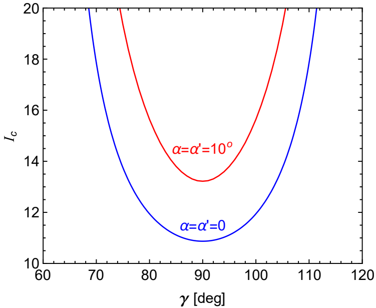
$$x_{\substack{R\\L}} = \pm I\cos\theta_a^I, \quad \sin\varphi_a = \frac{A_2 j'(A_1 - A_3)}{\sqrt{A_1^2 j^2 (A_2 - A_3)^2 + A_2^2 j'^2 (A_1 - A_3)^2}}$$

$$\mathcal{I}_1 = I\sin\theta_a^I \cos\varphi_a = \frac{2IA_1 j}{(2I-1)(A_1 - A_3)}, \quad \mathcal{I}_2 = I\sin\theta_a^I \sin\varphi_a = \frac{2IA_2 j'}{(2I-1)(A_2 - A_3)}$$

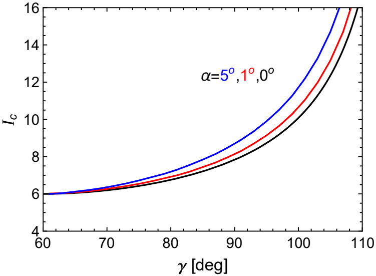
$$\mathcal{I}_3 = \pm I\cos\theta_a^I, \quad \sin\theta_a^I = \frac{2\sqrt{A_1^2 j^2 (A_2 - A_3)^2 + A_2^2 j'^2 (A_1 - A_3)^2}}{(2I-1)(A_1 - A_3)(A_2 - A_3)}$$

Separatrix

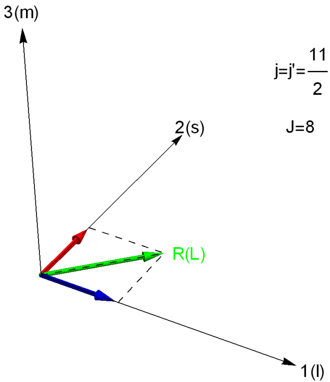
Chiral configuration
$j = j' = 11/2$



Transverse alignment (wobbling)
$j' = 11/2$, α tilting from the s axis

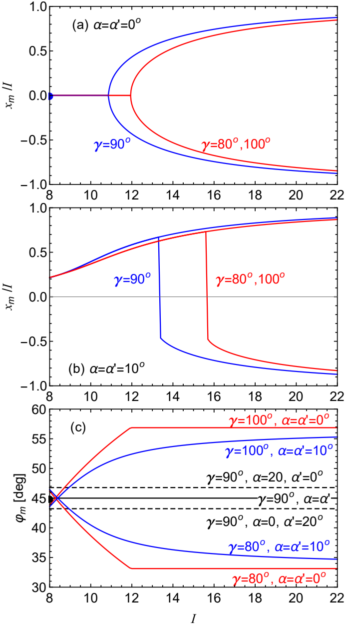


Stationary points

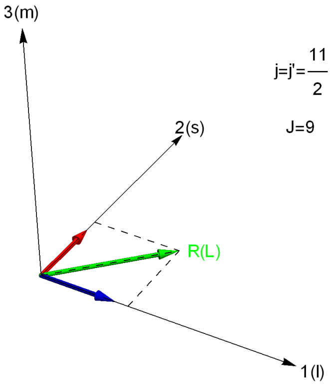


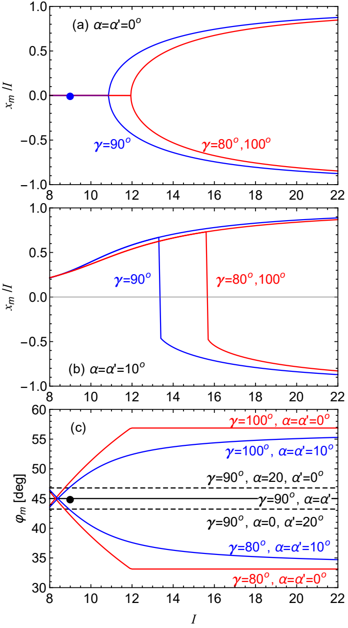
$j=j'=\frac{11}{2}$

J=8



Stationary points





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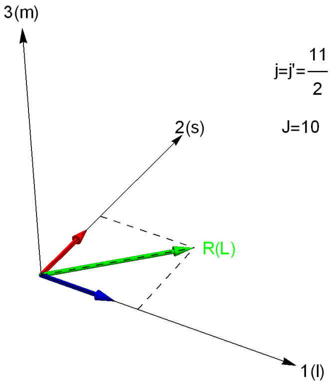
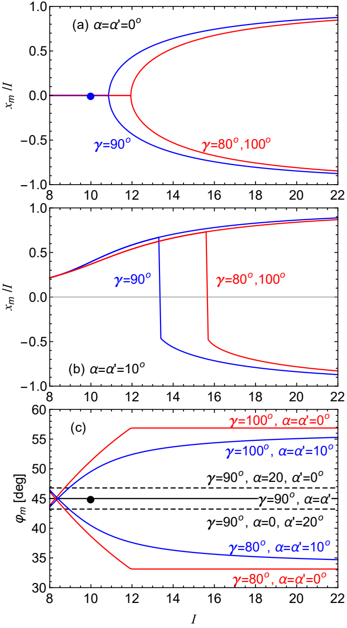
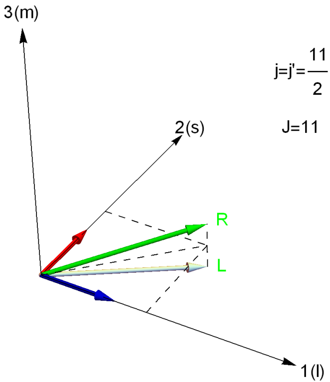


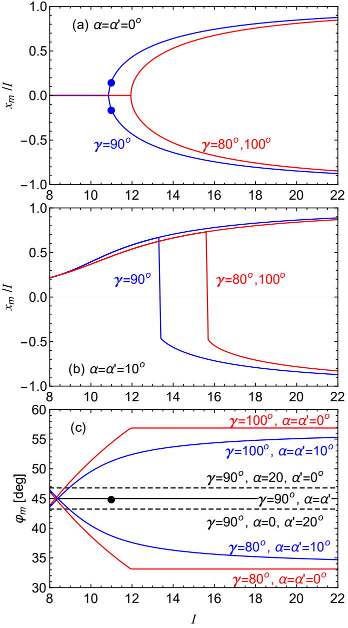


Stationary points

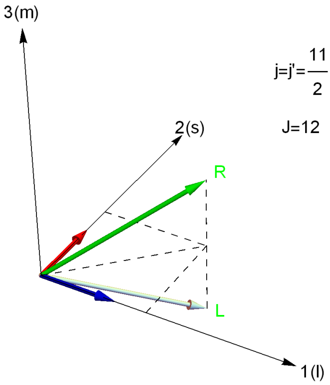


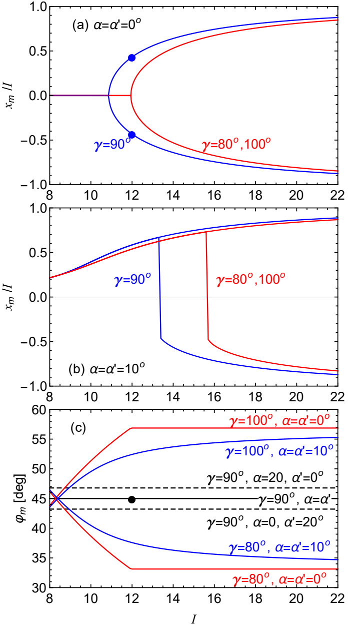
$j = j' = \frac{11}{2}$

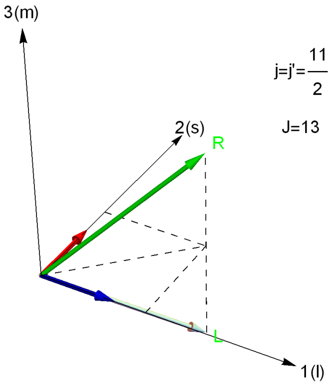
$J = 11$



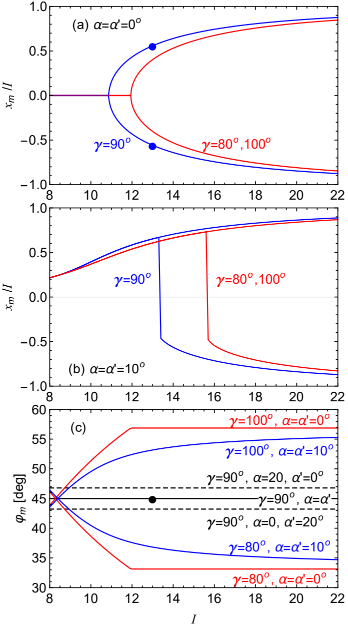
Stationary points





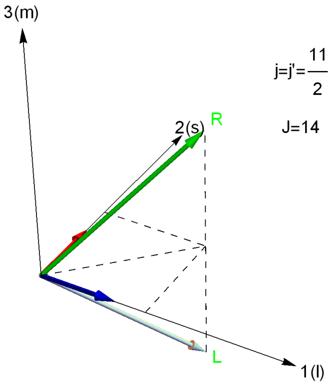


3(m)
2(s)
R
L
1(l)
j=j'=11/2
J=13



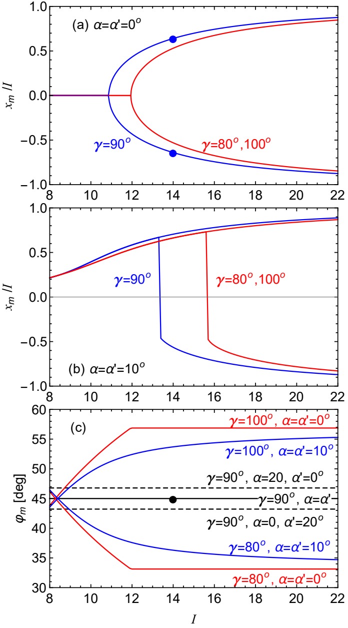
(a) α=α'=0°
γ=90°
γ=80°,100°
(b) α=α'=10°
γ=90°
γ=80°,100°
x_m/I
(c)
γ=100°, α=α'=0°
γ=100°, α=α'=10°
γ=90°, α=20, α'=0°
γ=90°, α=α'
γ=90°, α=0, α'=20°
γ=80°, α=α'=10°
γ=80°, α=α'=0°
φ_m [deg]
I

Stationary points

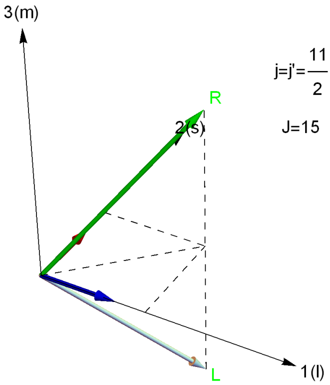


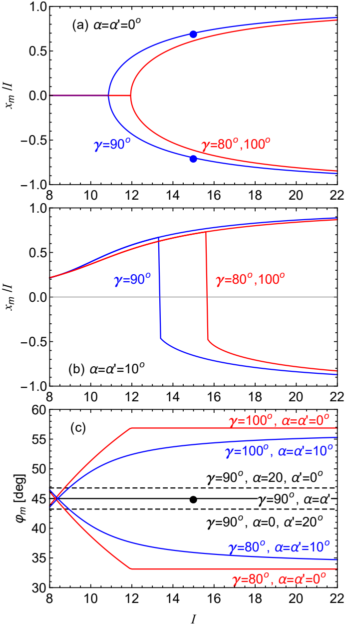
$j = j' = \frac{11}{2}$

$J = 14$



Stationary points





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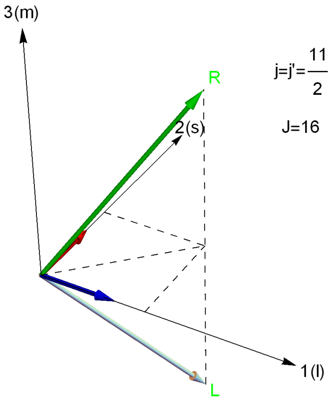

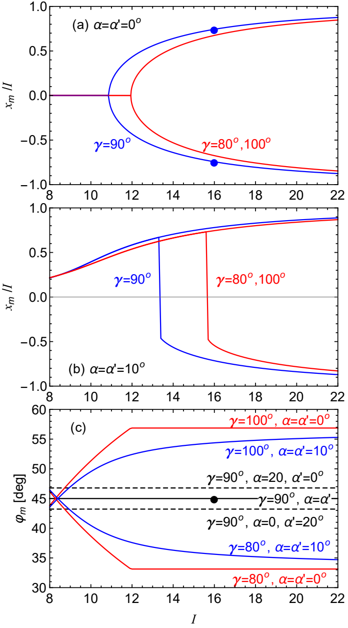

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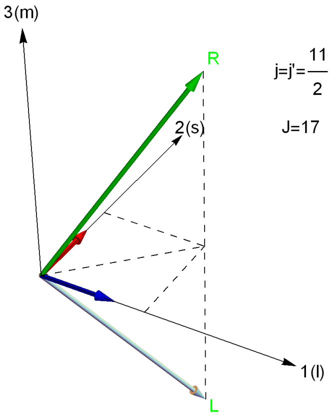

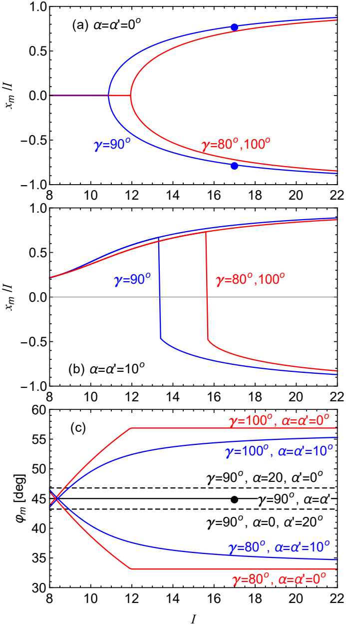

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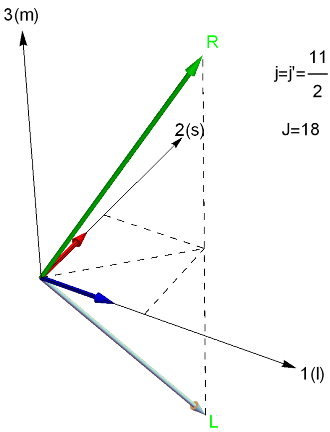
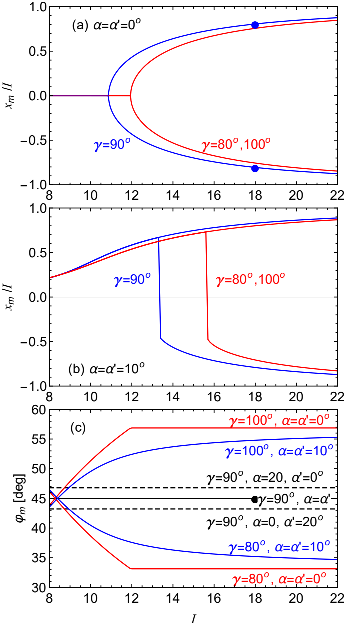




Emergence of chiral and wobbling bands

- Energy function is expanded around the corresponding single minimum in $\varphi_0(x)$ for fixed values of x:

$$\tilde{\mathcal{H}}(x,\varphi) \approx \mathcal{H}(r,\varphi_0(x)) + \frac{1}{2}\left(\frac{\partial^2\mathcal{H}}{\partial\varphi^2}\right)_{\varphi_0(x)} [\varphi - \varphi_0(x)]^2 \,,$$

- Quantization of the properly symmetrized $\tilde{\mathcal{H}}(x,\varphi)$ with $\varphi \to i\frac{d}{dx}$

$$\hat{H}_c = -\frac{1}{2}\frac{1}{\sqrt{B(x)}}\frac{d}{dx}\frac{1}{\sqrt{B(x)}}\frac{d}{dx} + V(x),$$

Effective mass $B(x) = \left[\frac{\partial^2\mathcal{H}(x,\varphi)}{\partial\varphi^2}\right]^{-1}_{\varphi_0(x)}$

Chiral(wobbling) potential $V(x) = \mathcal{H}(x,\varphi_0(x)) + \frac{B''(x)}{8\,[B(x)]^2} - \frac{9\,[B'(x)]^2}{32\,[B(x)]^3}.$

$\varphi_0(x)$ {

wobbling configuration (axial&planar rotation)

$= 0(Mod\,\pi/2) = const.$ [**RB**, C. M. Petrache, Phys. Rev. C **106**, 014313 (2022)]

chiral configuration (planar&aplanar rotation)

$= \frac{j'\cos\alpha'}{j\cos\alpha} = const.\;\; \gamma = 90°,\;\; A_1 = A_2$ [**RB**, Phys. Lett. B **817**, 136308 (2021)]

$= f(x),\;\; \gamma \neq 90°\;\; f(x)$ – numerically detrmined [**RB**, Phys. Lett. B **797**, 134853 (2019)]

Quantum energies and wave functions

Eigensystem is bounded by $|x| \leq I$ ☞ Particle in the box basis states

$$f_I^s(x) = \frac{[B(x)]^{-\frac{1}{4}}}{\sqrt{I}} \left\{ \sum_{n=1}^{n_{Max}} A_n^s \cos\left[\frac{(2n-1)\pi x}{2I}\right] + \sum_{n=1}^{n_{Max}} B_n^s \sin\left[\frac{2n\pi x}{2I}\right] \right\},$$

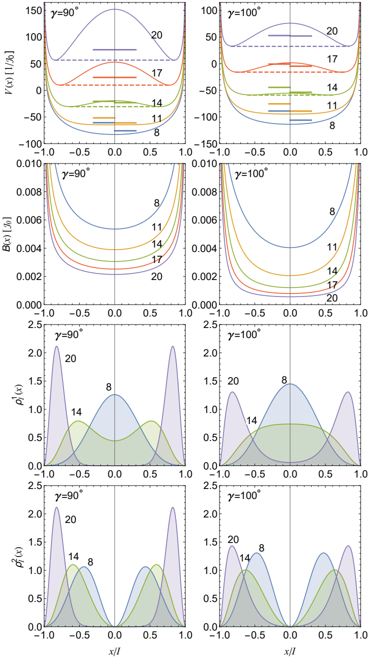
$\Downarrow$ Solutions $F_{Is}(x) = f_I^s(x)\,[B(x)]^{\frac{1}{4}}$

Quantum number of the bands $n = s - 1$, s - solution's order.

- Total energy $E(I,n) = E_{diag}(\mathcal{J}_0, \gamma, \alpha, \alpha'; Is) + CI(I+1) + E_0$
- Probability distribution from the chiral Hamiltonian $\rho_s^I(x) = |F_{Is}(x)|^2$
- Total wave function $|\Psi_{IMs}\rangle = \sum_{K=-I}^{I} \left[\sum_{K=-I}^{I} F_{Is}(K)^2\right]^{-1/2} F_{Is}(K)|IKM\rangle$
- For $\alpha = \alpha' = 0$, s plays the role of a parity-like quantum number associated to the $\pm x$ symmetry of the problem.

Potential →

Effective mass →



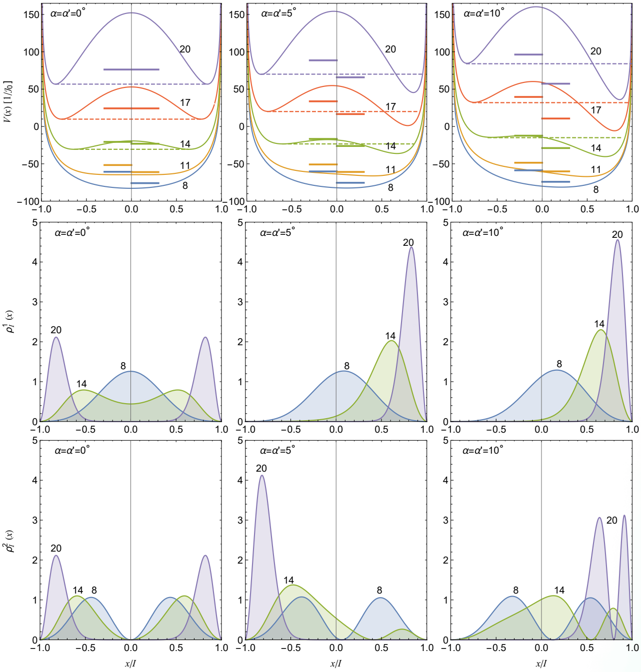
$j = j' = 11/2$

$\alpha = \alpha' = 0^\circ$ No tilting

Principal axis alignment

← ground band

← excited band



$j = j' = 11/2$
$\gamma = 90°$

← ground band

← excited band

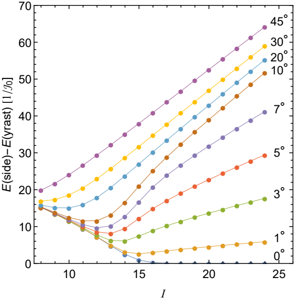
[**RB**, Phys. Lett. B **817**, 136308 (2021)]

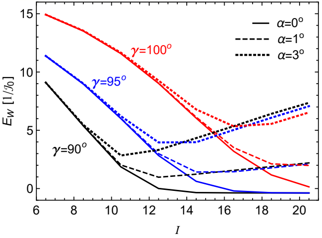
Chiral Geometry

$j = j' = 11/2$

$\gamma = 90^\circ \longrightarrow$

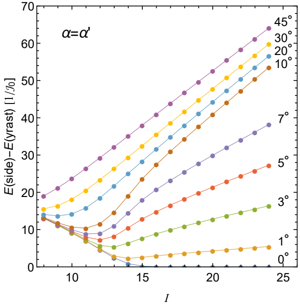
$\alpha = \alpha'$

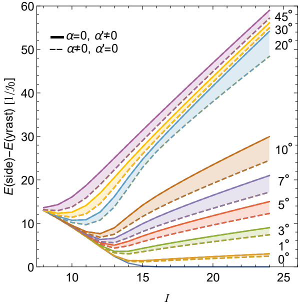




Transverse wobbling $j' = 11/2, j = 0$ $\uparrow$

$$E_W(I) = E(I) - [E(I+1) + E(I-1)]/2$$

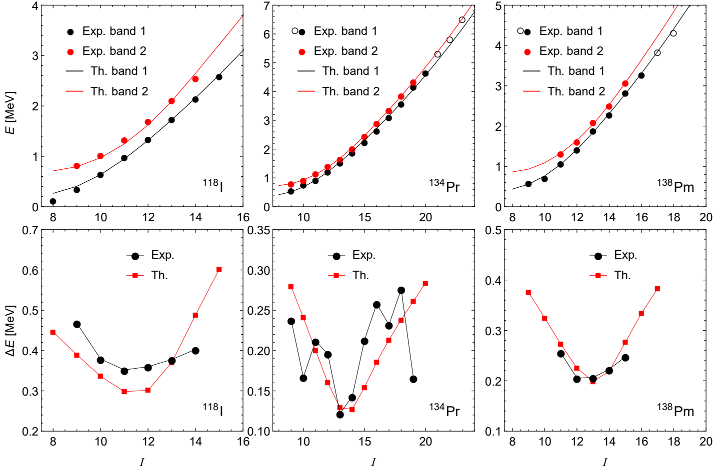




Chiral geometry $j = 9/2, \quad j' = 11/2, \quad \gamma = 90^\circ$ $\nearrow$

Applications - Chiral configuration with tilted alignments and $\gamma = 90^\circ$

Nucl.	j (hole)	j' (particle)	α	α'	E_0 [MeV]	$\mathcal{J}_0$ [Mev^{-1}]	C [keV]	rms [keV]
^{118}I	$9/2\ (\pi)$	$11/2\ (\nu)$	3°	10°	2.293	29.294	0.19	61.4
^{134}Pr	$11/2\ (\nu)$	$11/2(\pi)$	3°	3°	1.759	48.076	3.24	43.9
^{138}Pm	$11/2\ (\nu)$	$11/2(\pi)$	0°	8°	2.303	35.681	3.34	31.4



[**RB**, Phys. Lett. B **817**, 136308 (2021); Front. Phys., submitted (2023)]

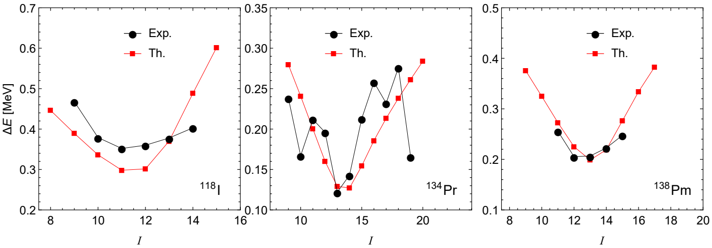
Nucl.	j (hole)	j' (particle)	α	α'	E_0 [MeV]	$\mathcal{J}_0$ [Mev^{-1}]	C [keV]	rms [keV]
^{118}I	$9/2$ (π)	$11/2$ (ν)	3°	10°	2.293	29.294	0.19	61.4
^{134}Pr	$11/2$ (ν)	$11/2(\pi)$	3°	3°	1.759	48.076	3.24	43.9
^{138}Pm	$11/2$ (ν)	$11/2(\pi)$	0°	8°	2.303	35.681	3.34	31.4

^{134}Pr Small tilting $\rightarrow$ still the best candidate for chiral symmetry.

^{138}Pm PRM calculations

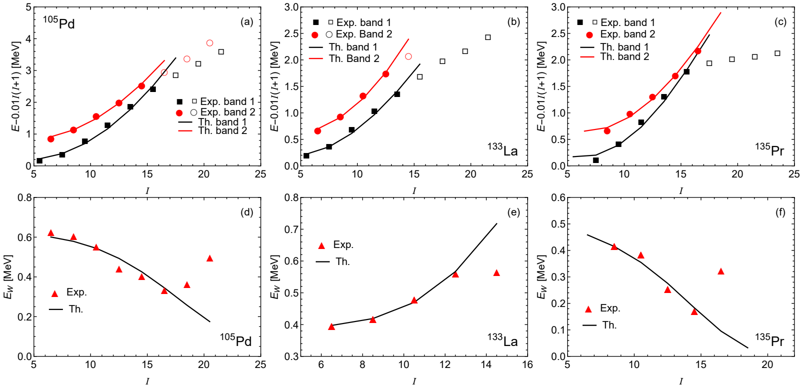
[P. Siwach, P. Arumugam, L. S. Ferreira, E. Maglione, Phys. Lett. B **811**, 135937 (2020).]

Only the dominant proton quasiparticle configuration has a sizable medium axis component.



[**RB**, Phys. Lett. B **817**, 136308 (2021); Front. Phys., submitted (2023)]

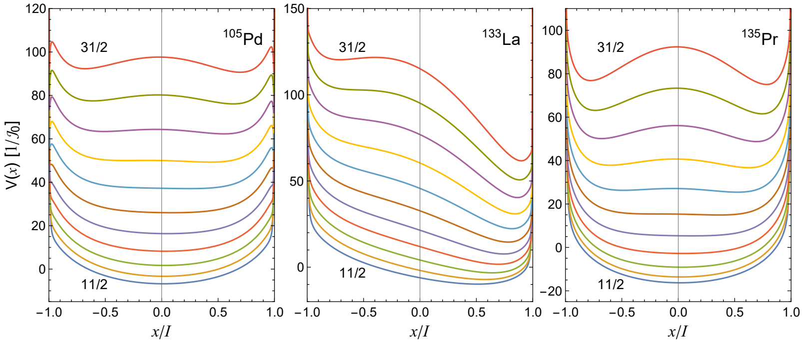
Nucl.	γ	α	$1/\mathcal{J}_0$ [keV]	E_0 [keV]	C [keV]	I_c
^{105}Pd	104°	1°	30.49	272.28	6.891	12.52
^{133}La	101°	32°	24.40	325.89	8.049	14.88
^{135}Pr	100°	1°	30.99	679.23	3.822	10.64



[**RB**, A. I. Budaca, Eur. Phys. J. A, submitted (2023)]

Applications - Wobbling configuration with tilted alignments

Nucl.	γ	α	$1/\mathcal{J}_0$ [keV]	E_0 [keV]	C [keV]	I_c
^{105}Pd	104°	1°	30.49	272.28	6.891	12.52
^{133}La	101°	32°	24.40	325.89	8.049	14.88
^{135}Pr	100°	1°	30.99	679.23	3.822	10.64



[**RB**, A. I. Budaca, Eur. Phys. J. A, submitted (2023)]

Nucl.	γ	α	$1/\mathcal{J}_0$ [keV]	E_0 [keV]	C [keV]	I_c
^{105}Pd	104°	1°	30.49	272.28	6.891	12.52
^{133}La	101°	32°	24.40	325.89	8.049	14.88
^{135}Pr	100°	1°	30.99	679.23	3.822	10.64

^{133}La

- Tilted axis wobbling.
- Still small tilting, to be considered as a hole quasiparticle.

  [T. M. Semkov *et al.*, Phys. Rev. C **34**, 523 (1986)]

  [L. Hildingson, W. Klamra, Th. Lindblad, C. G. Lindén, G. Sletten, G. Székely, Z. Phys. A **338**, 125 (1991)]

  [C. M. Petrache *et al.*, Phys. Rev. C **94**, 064309 (2016)]
- PRM calculations

  [Q. B. Chen, S. Frauendorf, N. Kaiser, Ulf-G. Meißner, J. Meng, Phys. Lett. B **807**, 135596 (2020)]

$$\frac{\langle j_{3(m)}\rangle}{\langle j_{2(s)}\rangle} \approx 0.5 \approx \sin\alpha$$

- A particle-rotor system with rigid single-particle spin alignments is studied in a semiclassical approach.
- The existence conditions were identified for distinct dynamic phases.
- A Schrödinger equation is obtained from the classical picture for a continuous projection variable (x), which allows the phenomenological interpretation of the quantum states in terms of oscillations and tilted axis rotations.

References:

- A. A. Raduta, RB, C. M. Raduta, Phys. Rev. C **76**, 064309 (2007).
- RB, Phys. Rev. C **98**, 014303 (2018).
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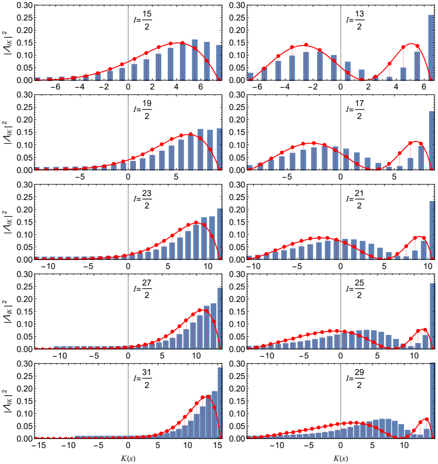
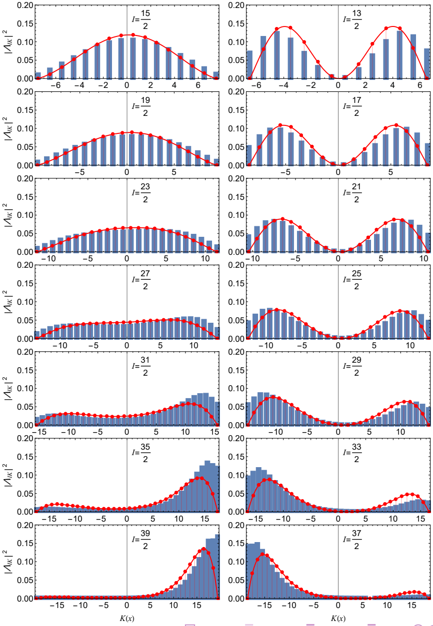
Wobbling K-plots

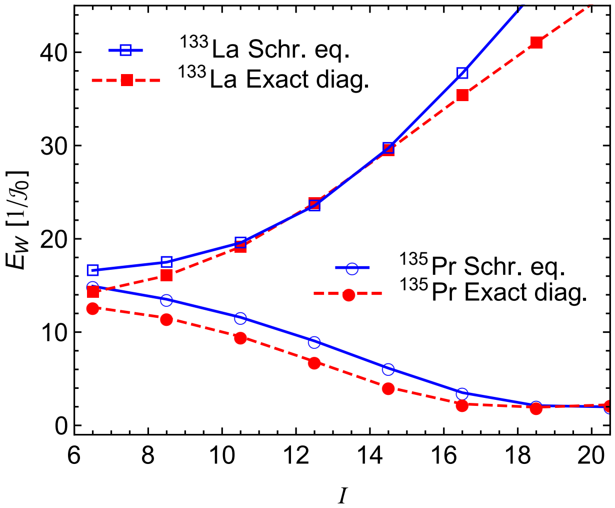


↑

^{133}La



^{135}Pr ⟶


^{133}La Schr. eq.
^{133}La Exact diag.
^{135}Pr Schr. eq.
^{135}Pr Exact diag.
E_W [1/$\mathcal{J}_0$]
I
0
10
20
30
40
6
8
10
12
14
16
18
20

Second order expansion

$$T_{2\mu}(E2) = t_1 q_{2\mu} + t_2 \left[q \times q\right]_{2\mu}$$

in quadrupole moments

$$q_{2\mu} = \beta \left[\cos\gamma D^2_{\mu 0} + \frac{\sin\gamma}{\sqrt{2}}\left(D^2_{\mu 2} + D^2_{\mu -2}\right)\right]$$