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Quaternionic Transform:

A new Light on the Solar Power Spectrum

A series harmonic can be described by a sum of sines and cosines

If the series is not harmonic, it cannot be guaranteed that the Discrete Fourier transform is the least squares approximation to the time series

Here, we present the effect of extending the Fourier kernel to a particular quaternion and exploring the impact when it is applied to the best time series we know: GOLF/SoHO from the closest star, our Sun.

GOLF_22y_MEAN

method

The kernel of the new quaternionic transform reads

$$\mathcal{F}^{ker}_{mod}(\theta) = e^{\mathbf{i}\pi\hat{\theta}} \cdot e^{\mathbf{j}\pi\theta}$$

Can be expanded in the quaternion components:

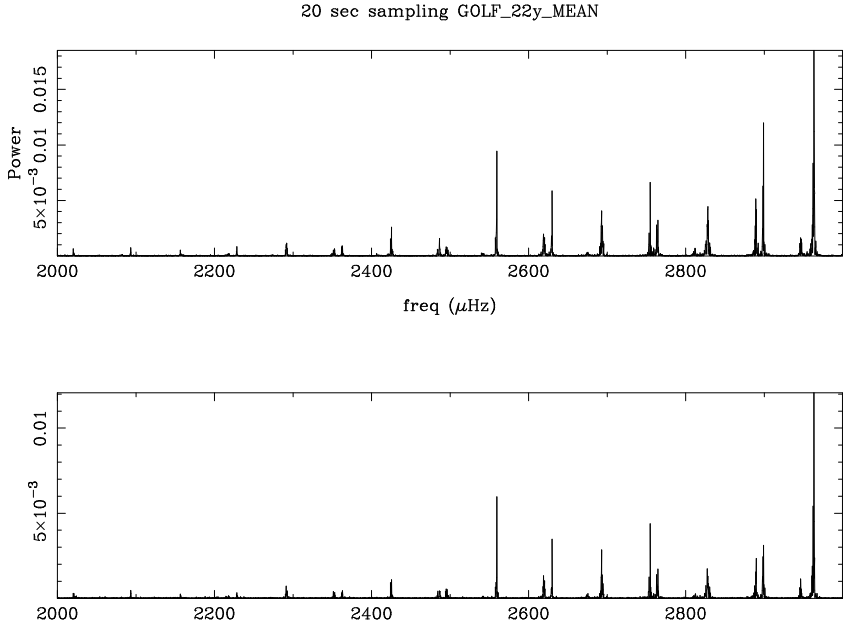
$$\mathcal{F}^{ker}_{mod}(\theta) = Re(co(\theta)) + \mathbf{i}\, Re(si(\theta)) + \mathbf{j}\, Im(co(\theta)) + \mathbf{k}\, Im(si(\theta))$$

method

We study the underlying statistics that the modified DFT produces. We will follow a similar procedure as Scargle (1982) for the a standard DFT where the author proved that the Lomb-Scargle periodogram has a chi-square distribution with two degrees of freedom.

We concluded that if the 4 variables have equal variances the distribution of P mod (ω j) is a chi-square with 4 degrees of freedom (Papoulis, 1965).

method.



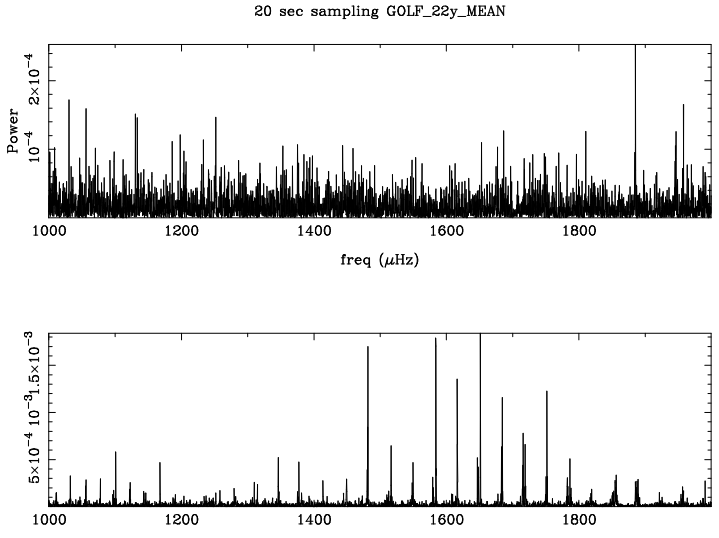
Modified (down) and Classical (up) Fourier Transforms of th SoHO data obtained during 16 days, at the region between 2,000 and 4,000 μHz.

Note that the main differences between both transforms in this range is the higher amplitude in the case of the quaternion Fourier transform, compatible with a better signa to- noise ratio.

But...

RESULTS: ANALYSIS OF SOHO DATA

We performed our new analysis on a time series of radial velocities, corresponding to 16 days (with 20 s of sampling time, and not 80 s as cited by official website) from the GOLF instrument aboard the SoHO satellite (Gabriel et al., 1995). Although there are much longer observations from SoHO/GOLF, we limited the total duration of our dataset to 16 days as a trade-off between avoiding the influence of rotation and preserving frequency resolution.



- Modified (down) and Classical (up) Fourier Transforms of the SoHO data obtained during 16 days, at the region between 1,000 and 2,000 μHz.
- The Gaussian envelope of the structure visible in the red plot could be a hint of the presence of not identified modes in the common Fourier analysis but further analysis is required to confirm this.

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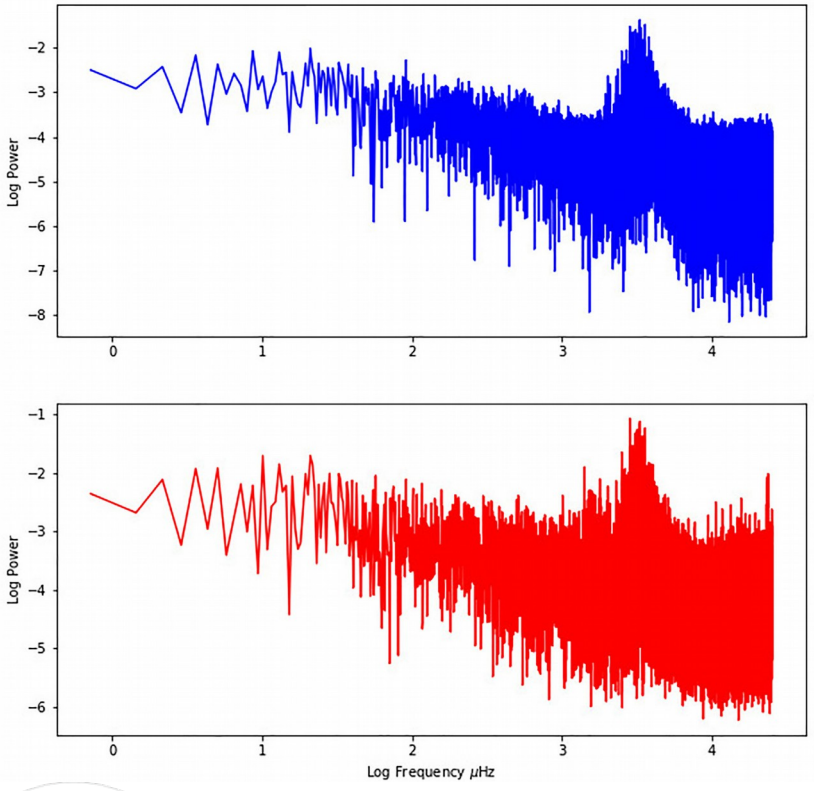

Modified (red) and Classical (blue) Fourier Transforms of the SoHO data obtained during 16 days, from 0 to the Nyquist frequency (i.e., 25 mHz) in loglog scale.

- The results are consistent with a notable improvement in the signal-to-noise ratio in the low frequency range.
- This opens the possibility of detecting the elusive g modes of the Sun in future works.

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9th Iberian Meeting on Asteroseismology

Laying the groundwork for the road ahead