Review on S-matrix bootstrap

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Snowmass White Paper: S-matrix Bootstrap

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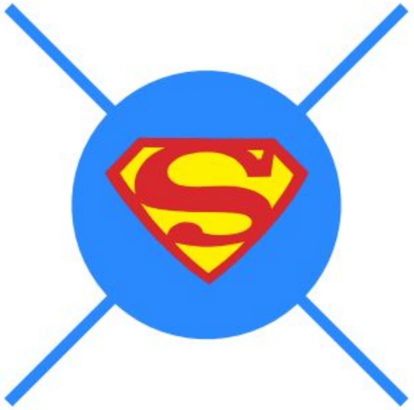
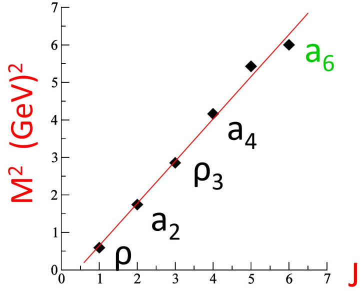


Old S-matrix bootstrap

before quark model: ideas developed for understanding strong interaction



Regge trajectory

$$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$$

Veneziano amplitude

⇨ string theory

Old S-matrix bootstrap

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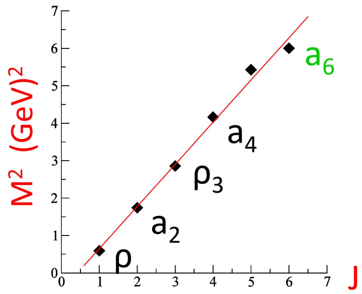

Regge trajectory

$$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} \Longrightarrow \text{string theory}$$

Veneziano amplitude

S-matrix bootstrap: solve strong interaction in a self-consistent way

symmetries+analyticity+crossing+unitarity

hindsight: too optimistic

infinitely many consistent QFTs compatible with bootstrap principles

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Modern perspective:

symmetries+analyticity+crossing+unitarity

+

convex optimization

hindsight: too optimistic

infinitely many consistent QFTs compatible with bootstrap principles

Modern perspective:

symmetries+analyticity+crossing+unitarity

+

convex optimization

- explore the landscape of QFTs non-perturbatively
- bound physical quantities
- extremal theories

2-to-2 S-matrix

$$\mathbf{S} = U(+\infty, -\infty) \qquad \mathbf{S}^\dagger \mathbf{S} = \mathbb{I}$$

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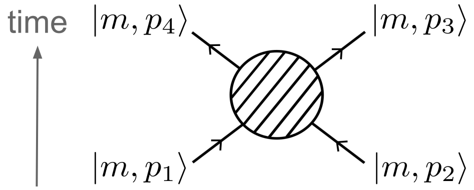
2-to-2 scattering lightest particle in massive QFT

$$s = (p_1 + p_2)^2 \qquad t = (p_1 - p_3)^2 \qquad u = (p_1 - p_4)^2$$

$$p_i^2 = m^2 \qquad s + t + u = 4m^2$$



$$\langle p_3, p_4 | \mathbf{S} | p_1, p_2 \rangle = \boxed{S(s,t,u)} (2\pi)^d \delta^{(d)}\left(p_3 + p_4 - p_1 - p_2\right)$$

scattering amplitude

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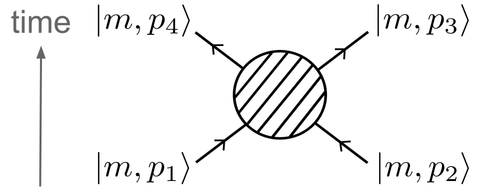

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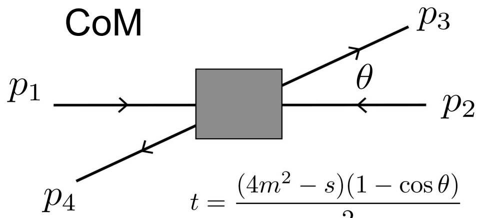
scattering amplitude

CoM



$$t = \frac{(4m^2 - s)(1 - \cos\theta)}{2}$$

total energy $\sqrt{s}$

2-particle irreps states $\quad |p_1, p_2\rangle \to |p_1, \ell\rangle$

$$\langle p', \ell | \mathbf{S} | p, \ell \rangle = \boxed{S_\ell(s)} \delta_{\ell \ell'} (2\pi)^d \delta^{(d)} (p - p')$$

partial amplitude

physical kinematics: $\quad s > 4m^2 \quad 4m^2 - s < t < 0$

2-to-2 S-matrix

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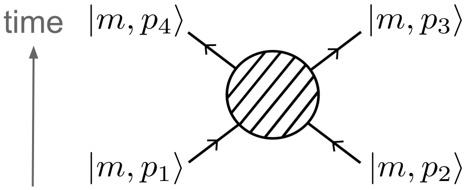

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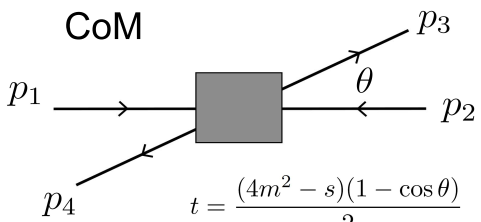

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total energy $\sqrt{s}$

3+1d: $S_\ell(s) = \frac{\pi}{4}\sqrt{\frac{s-4}{s}} \int_{-1}^{1} d\cos\theta P_\ell(\cos\theta) S(s^+, t)$

physical kinematics: $s > 4m^2 \quad 4m^2 - s < t < 0$

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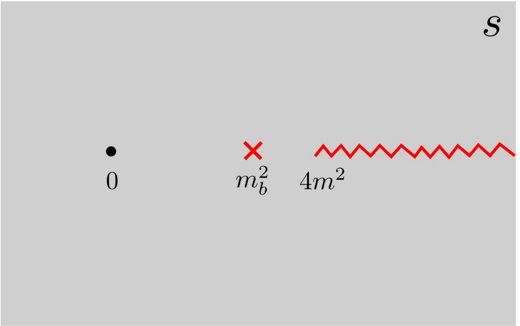
partial amplitude

Physical constraints: analyticity and crossing

analyticity: $S(s,t,u)$ analytic function in s,t,u

singularities ⟷ on-shell processes

poles: bound states
cuts: multiple particle states



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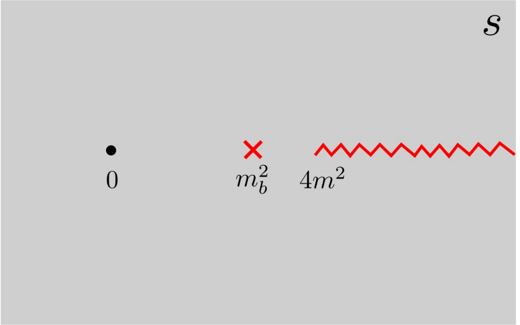

crossing: $S(s,t,u) = S(s,u,t) = S(u,t,s)$

e.g. single flavor case

Physical constraints: unitarity

$$\mathbf{S}^\dagger\mathbf{S} = \mathbb{I}$$

take a subsector D of state space, and a state $|\alpha\rangle \in D$

$$\langle\alpha|\mathbf{S}^\dagger\mathbf{S}|\alpha\rangle = \langle\alpha|\alpha\rangle$$

insert complete basis

$$\sum_{|\beta\rangle\in D} \langle\alpha|\mathbf{S}^\dagger|\beta\rangle\langle\beta|\mathbf{S}|\alpha\rangle + \sum_{|\beta\rangle\notin D} \langle\alpha|\mathbf{S}^\dagger|\beta\rangle\langle\beta|\mathbf{S}|\alpha\rangle = \langle\alpha|\alpha\rangle$$

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take D $\rightarrow$ subsector of two particle states $|p,\ell\rangle$

$$S_\ell^*(s)S_\ell(s) + \big(\text{positive stuff}\big) = 1$$

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$$\begin{pmatrix} 1 & S_\ell(s) \\ S_\ell^*(s) & 1 \end{pmatrix} \succeq 0$$

positive semidefinite → convex space

unitarity: $|S_\ell(s)|^2 \leq 1 \qquad s > 4m^2 \qquad \forall \ell$

1+1 d

no scattering angle $u = 0$ $t = 4m^2 - s$ *one independent Mandelstam variable* s $S(s)$

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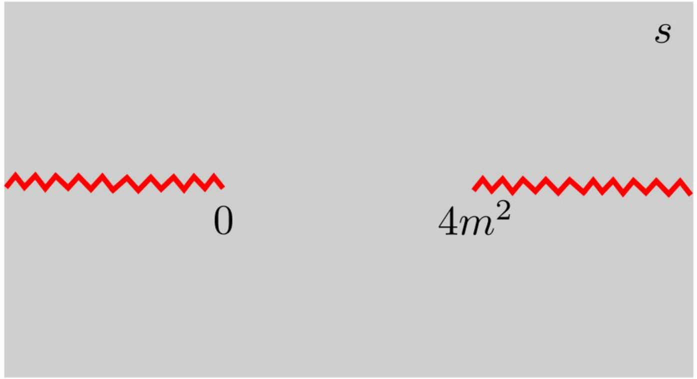

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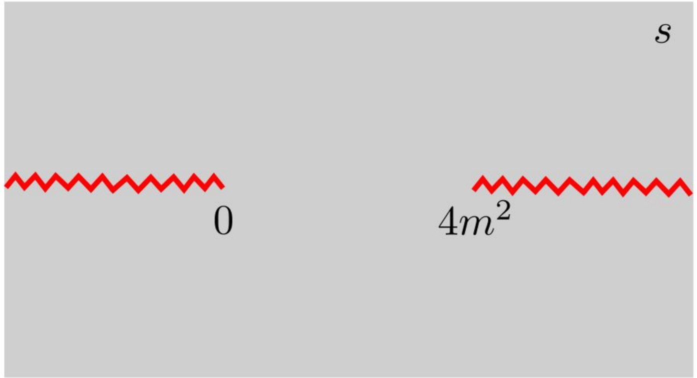

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specialty in 1+1d:

(Zamolodchikov ×2, 1970s)

analyticity+crossing+unitarity+factorization $\longrightarrow$ exactly solvable integrable S-matrices

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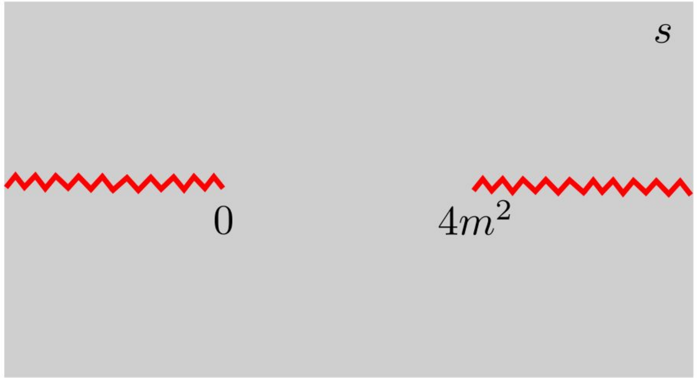

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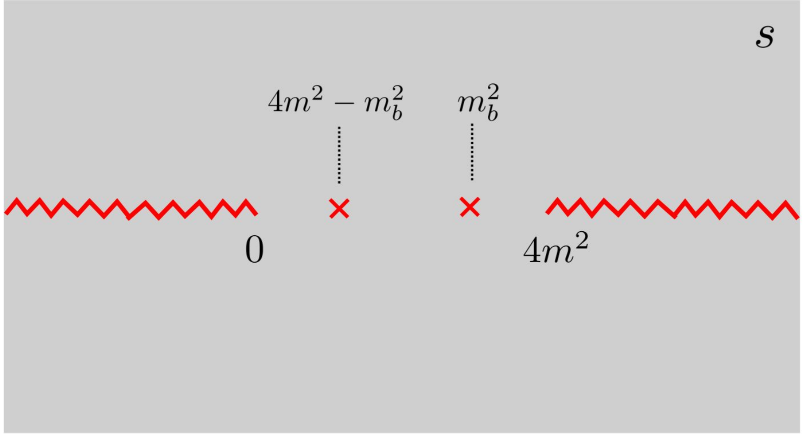
test ground for developing modern S-matrix bootstrap program

Modern S-matrix bootstrap program

[Paulos, Penedones, Toledo, van Rees, Vieira, 2016]

1+1d, single flavor, one bound state

$$S(s) = -\frac{g}{s - m_b^2} - \frac{g}{4m^2 - s - m_b^2} + \tilde{S}(s)$$



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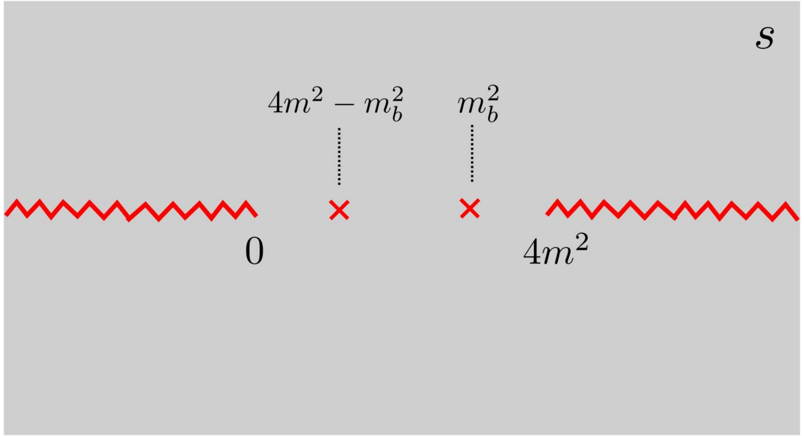

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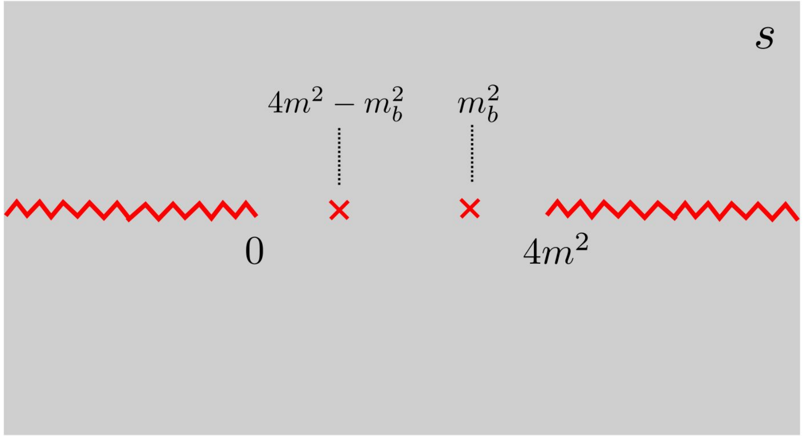
$$\boxed{\text{fix } m_b,\ g^{\max} = ?}$$

bounding the cubic coupling



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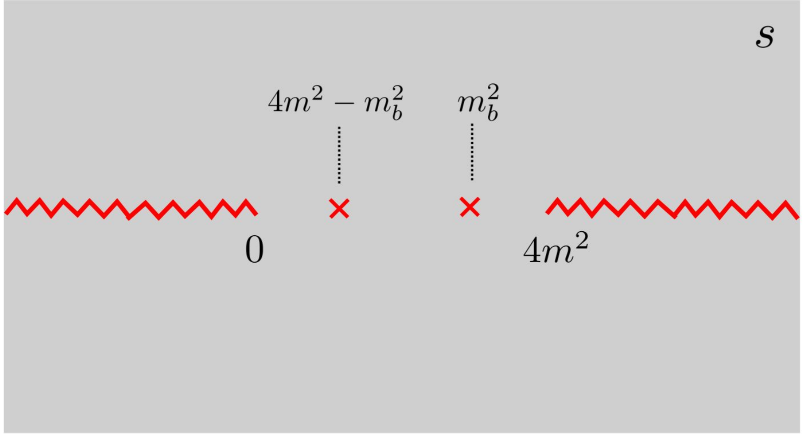
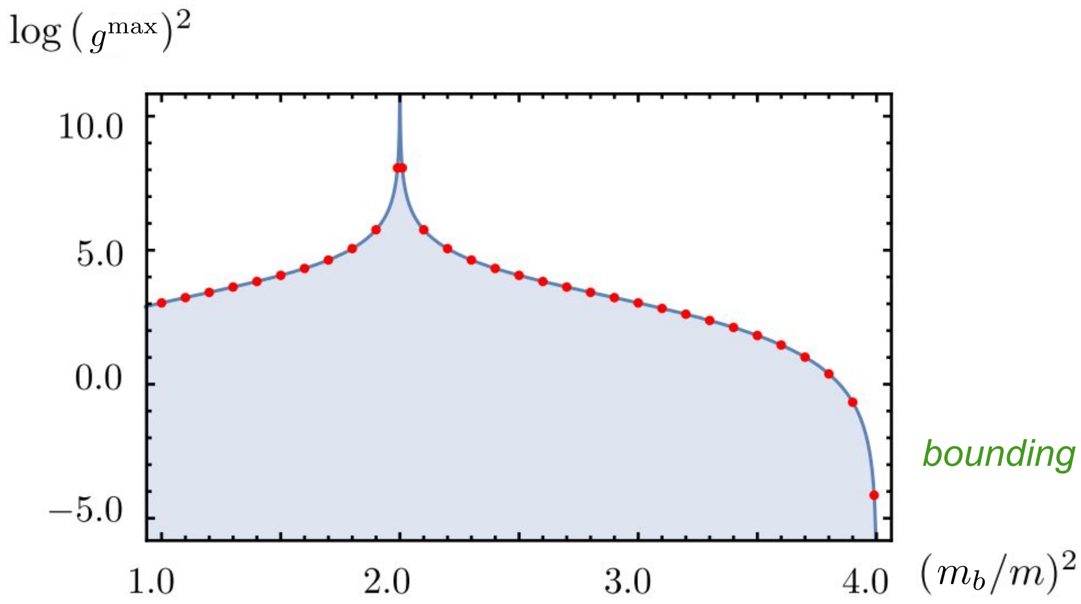
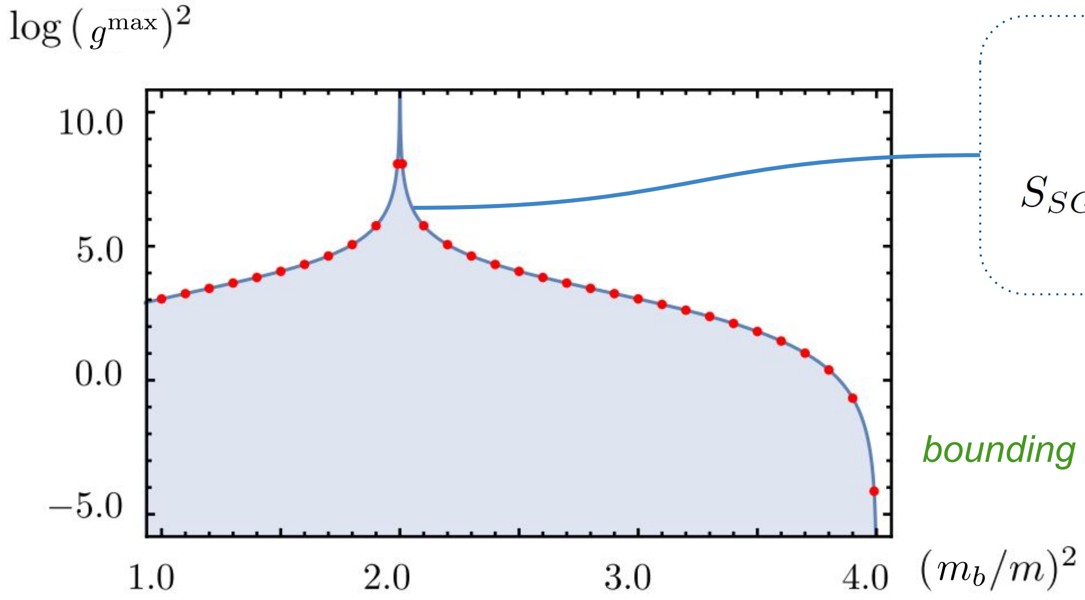


once the spectrum of states is given, it may not be possible to make a particular coupling constant too large without introducing new "bound" states



bounding physical quantity

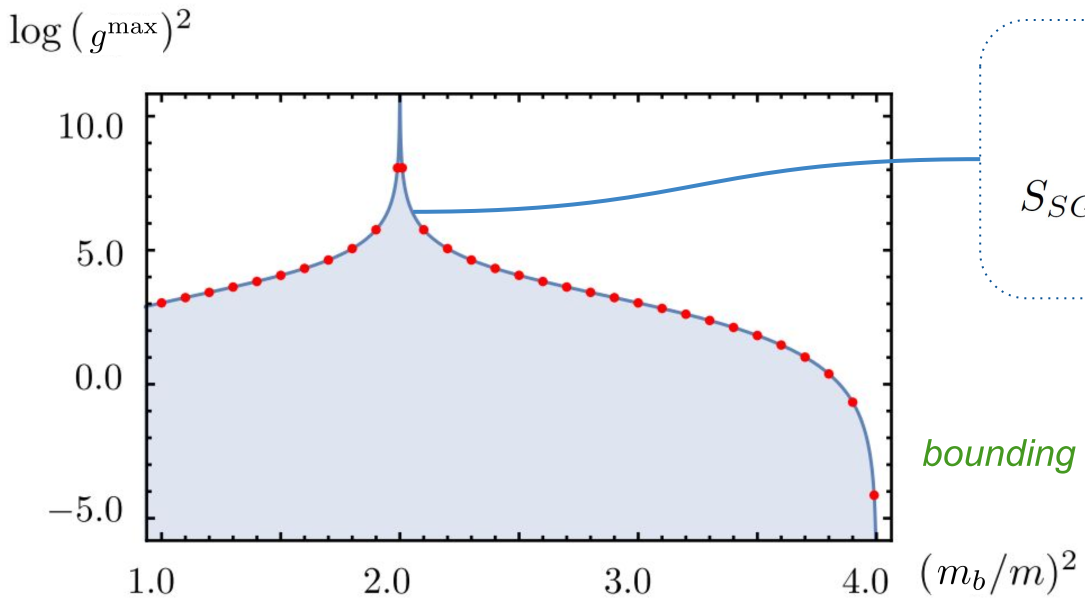


Sine-Gordon breather S-matrix

$$S_{SG}(s) = \frac{\sqrt{s}\sqrt{4m^2 - s} + m_b\sqrt{4m^2 - m_b^2}}{\sqrt{s}\sqrt{4m^2 - s} - m_b\sqrt{4m^2 - m_b^2}}$$

obtaining extremal amplitude

bounding physical quantity can be obtained analytically



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PHYSICAL REVIEW D VOLUME 6, NUMBER 10 15 NOVEMBER 1972

Rigorous Bounds on Coupling Constants in Two-Dimensional Field Theories*

Michael Creutz

Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

(Received 13 July 1972)

We show that renormalized three-particle coupling constants in a field theory with one space and one time dimension are bounded. This bound depends on the particle spectrum and assumes only analyticity, crossing, unitarity, and polynomial boundedness of the *S* matrix at infinity.

Bounding the space of S-matrices

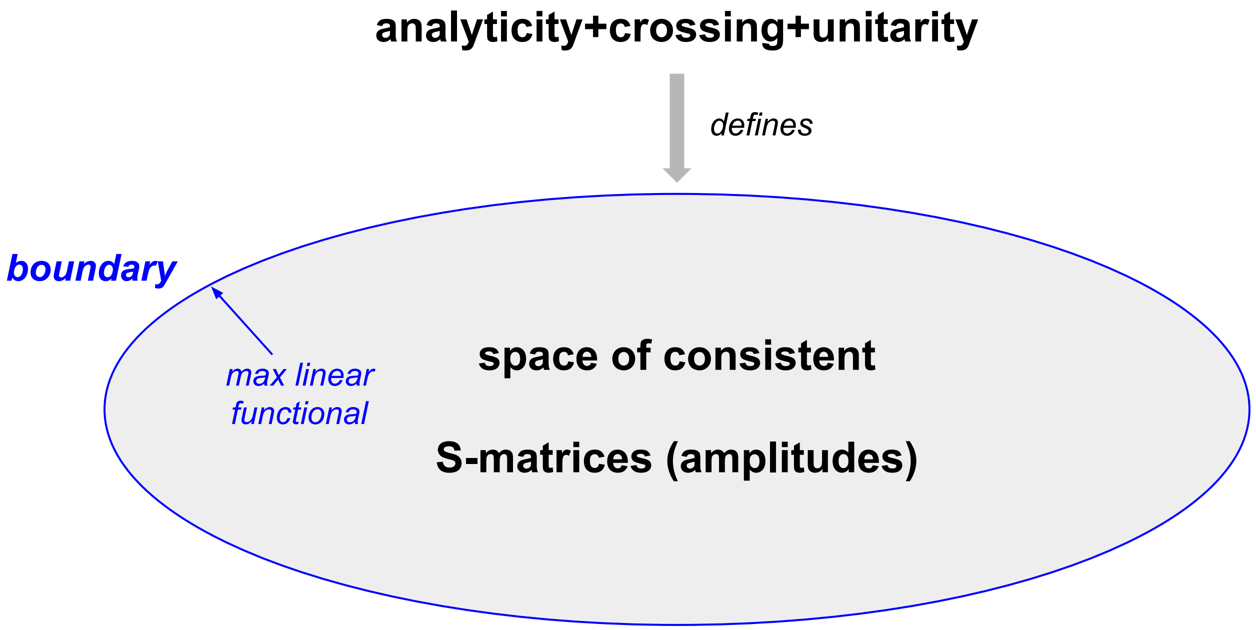
analyticity+crossing+unitarity

defines

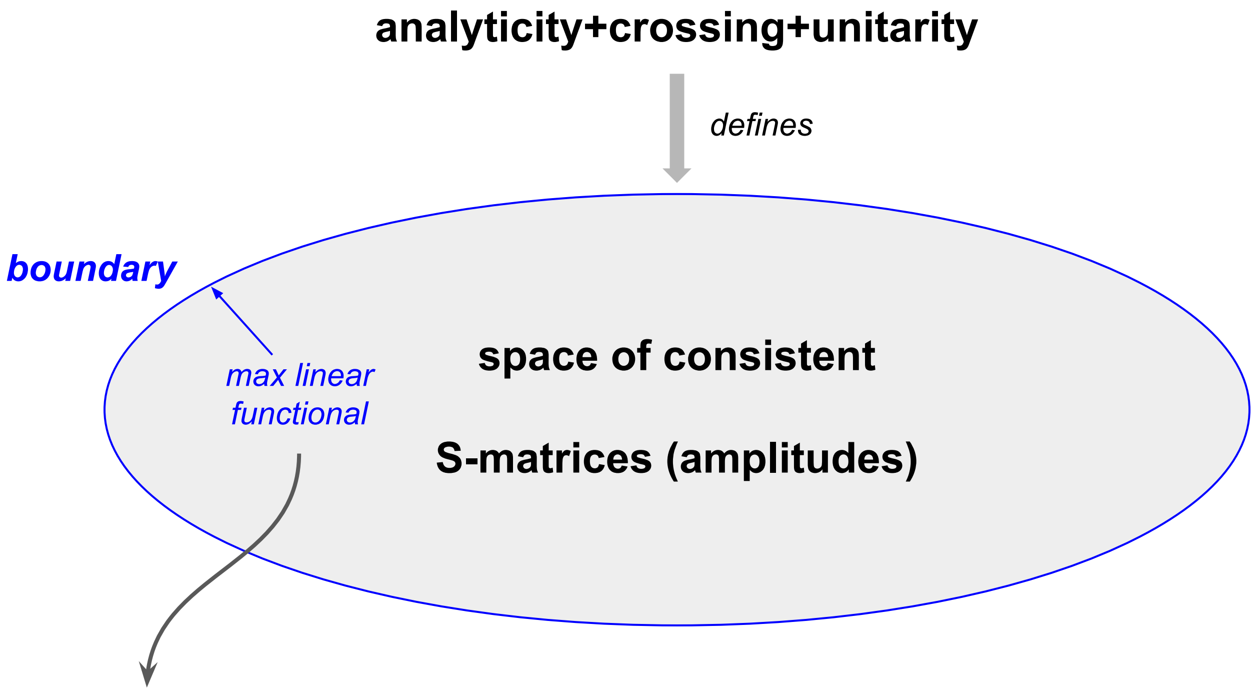
space of consistent

S-matrices (amplitudes)

Bounding the space of S-matrices



Bounding the space of S-matrices

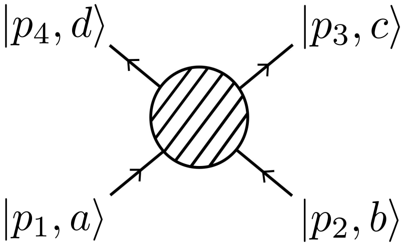


obtain the amplitude non-perturbatively

bound the physical quantities and obtain non-perturbative amplitudes at the same time

O(N) global symmetry

tension between crossing and unitarity



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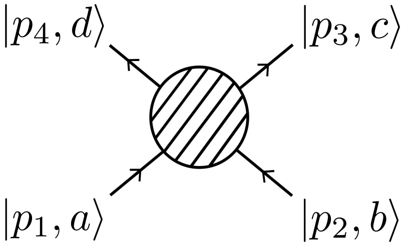
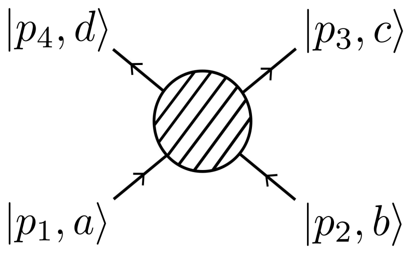


$$\langle p_3, c; p_4, d|\mathbf{S}|p_1, a; p_2, b\rangle = (2\pi)^2 \delta^{(2)}(p_3 + p_4 - p_1 - p_2)$$
$$\times \left\{ S_{\text{singlet}}(s) \frac{\delta_{ab}\delta_{cd}}{N} + S_{\text{sym}}(s) \left(\frac{\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}}{2} - \frac{\delta_{ab}\delta_{cd}}{N} \right) + S_{\text{antisym}}(s) \frac{\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}}{2} \right\}$$

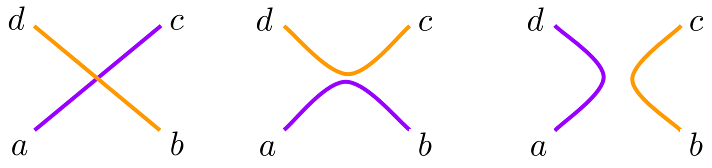
unitarity: $|S_a(s + i\epsilon)| \leq 1, \; s \geq 4m^2 \qquad a = \text{singlet, sym, antisym}$

O(N) global symmetry

tension between crossing and unitarity



crossing: $S_T(s) = S_T(4m^2 - s) \qquad S_A(s) = S_R(4m^2 - s)$



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$$\times\Big(S_T(s)\delta_{ac}\delta_{bd} + S_A(s)\delta_{ab}\delta_{cd} + S_R(s)\delta_{ad}\delta_{bc}\Big)$$

$$\times\left\{S_{\text{singlet}}(s)\frac{\delta_{ab}\delta_{cd}}{N} + S_{\text{sym}}(s)\left(\frac{\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}}{2} - \frac{\delta_{ab}\delta_{cd}}{N}\right) + S_{\text{antisym}}(s)\frac{\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}}{2}\right\}$$

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O(N) S-matrix bootstrap

[YH, Irrgang, Kruczenski; Cordova, Vieira; Paulos, Zheng, 2018]

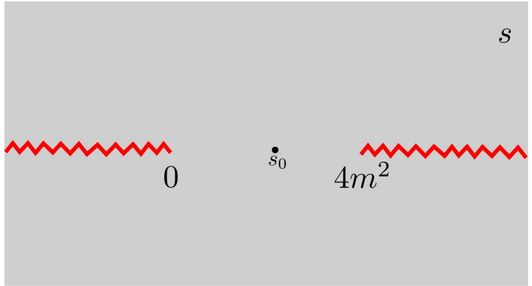
three analytic functions $S_a(s)$ $\quad a = \text{singlet, sym, antisym}$

assume no bound states

parametrized by boundary values

numerics: discretize

$$S_a(s_i), \quad i = 1, \ldots, M$$



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assume no bound states

$$S_a(4m^2 - s) = C_{ab} S_b(s) \qquad C_{ab} = \begin{pmatrix} \frac{1}{N} & -\frac{N}{2} + \frac{1}{2} & \frac{N}{2} + \frac{1}{2} - \frac{1}{N} \\ -\frac{1}{N} & \frac{1}{2} & \frac{1}{2} + \frac{1}{N} \\ \frac{1}{N} & \frac{1}{2} & \frac{1}{2} - \frac{1}{N} \end{pmatrix}$$

crossing

linear constraints

parametrized by boundary values

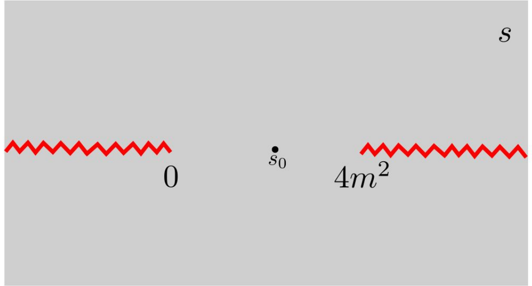
numerics: discretize

$$S_a(s_i), \quad i = 1, \ldots, M$$

convex space

$$\begin{pmatrix} 1 & S_a(s) \\ S_a^*(s) & 1 \end{pmatrix} \succeq 0 \qquad s \geq 4m^2$$

unitarity



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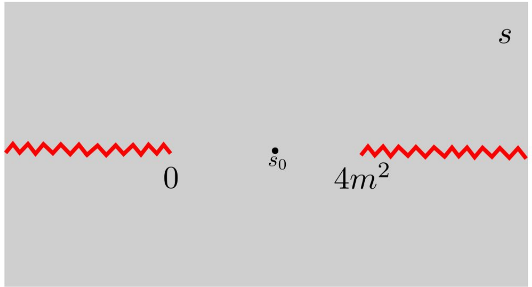
$$S_a(s_i), \quad i = 1, \dots, M$$



$$S_a(4m^2 - s) = C_{ab} S_b(s) \qquad C_{ab} = \begin{pmatrix} \frac{1}{N} & -\frac{N}{2} + \frac{1}{2} & \frac{N}{2} + \frac{1}{2} - \frac{1}{N} \\ -\frac{1}{N} & \frac{1}{2} & \frac{1}{2} + \frac{1}{N} \\ \frac{1}{N} & \frac{1}{2} & \frac{1}{2} - \frac{1}{N} \end{pmatrix}$$

crossing

linear constraints

convex optimization

convex space

$$\begin{pmatrix} 1 & S_a(s) \\ S_a^*(s) & 1 \end{pmatrix} \succeq 0 \qquad s \geq 4m^2$$

unitarity

linear functional

maximize $\mathcal{F}\left[S_a(s)\right]$

NLSM

O(N) nonlinear sigma model

$$\mathcal{L} = \frac{1}{g_0}\sum_{i=1}^{N}(\partial_\mu n_i)^2 \qquad \vec{n}^2 = 1$$

N *scalar particles with mass* m

no bound states *no free parameters*

asymptotic freedom

[Polyakov, 1975]

exact S-matrix obtained using integrability [Zamolodchikov ×2, 1979]

NLSM @ a vertex

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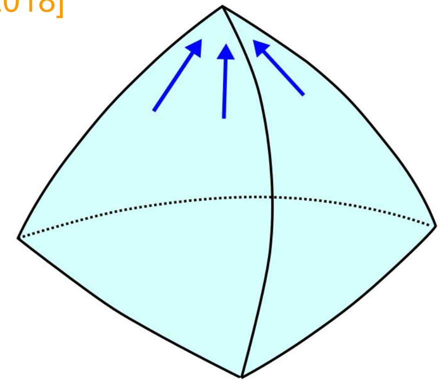
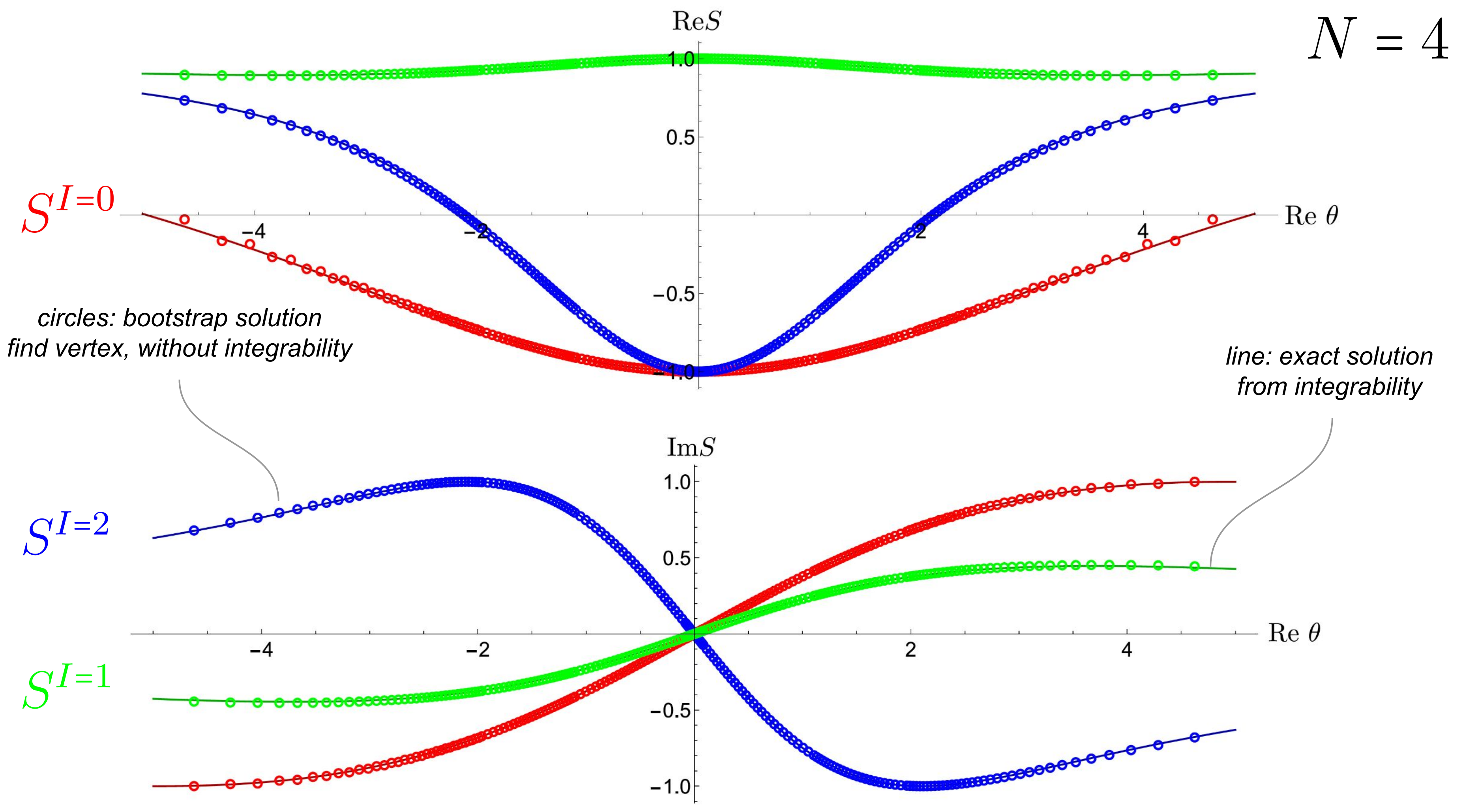
NLSM @ a vertex [YH, Irrgang, Kruczenski, 2018]

unitarity saturation → boundary of the space

no free parameters → rigid point → vertex



numerical evidence: many functionals lead to it


N = 4
ReS
1.0
0.5
-0.5
-1.0
Re θ
-4
-2
2
4
S^{I=0}
circles: bootstrap solution
find vertex, without integrability
line: exact solution
from integrability
ImS
1.0
0.5
-0.5
-1.0
Re θ
-4
-2
2
4
S^{I=2}
S^{I=1}

Map out the space of O(N) theories

rich geometrical structure

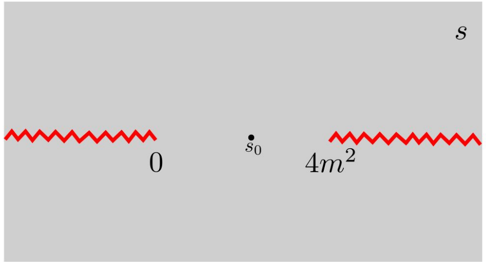
infinite dimensional space of functions $S_a(s)$

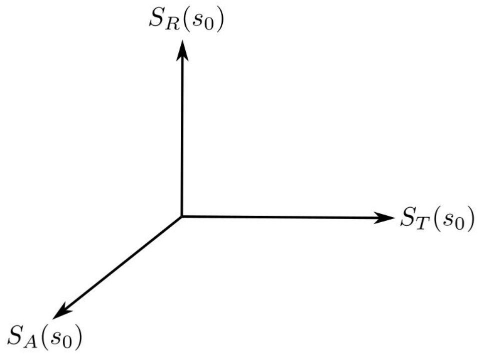
Map out the space of O(N) theories

rich geometrical structure

infinite dimensional space of functions $S_a(s)$

3-dimensional projection: $\Big(S_A(s_0), S_T(s_0), S_R(s_0)\Big)$





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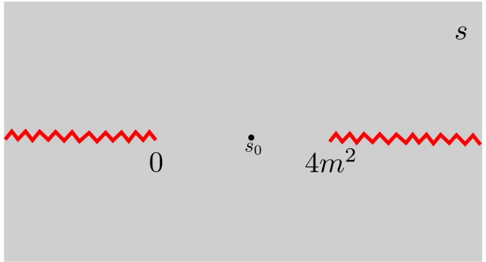
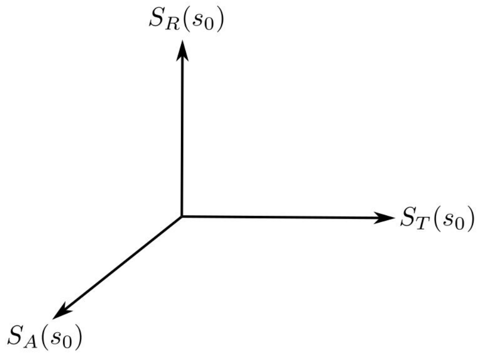
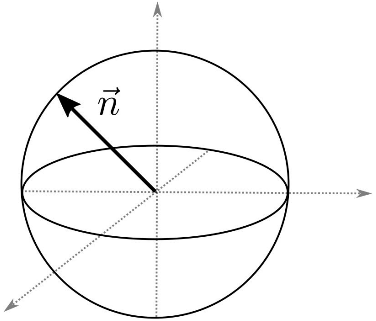




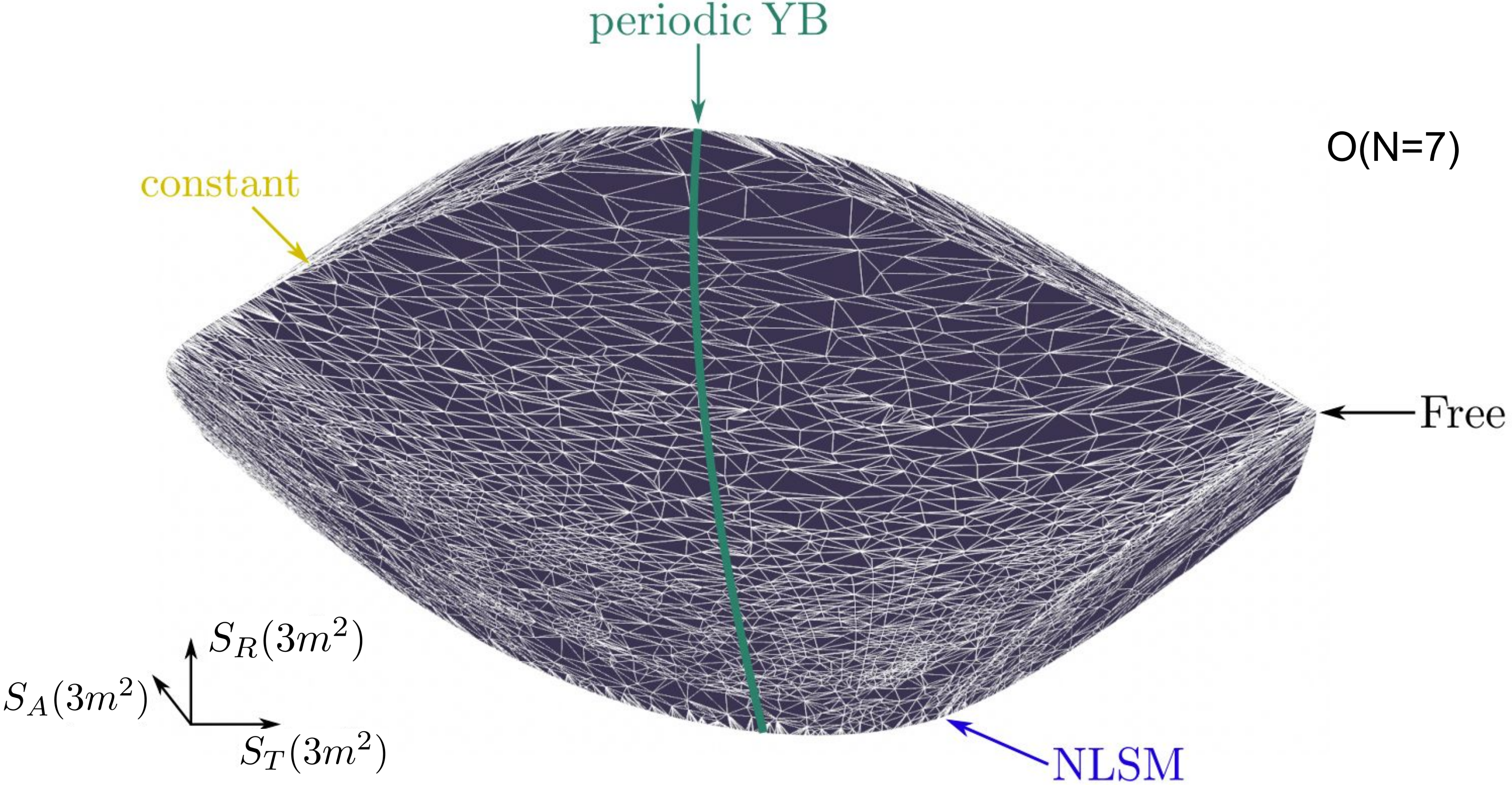


$$\mathcal{F}\big[S_a(s)\big] = \sum_a n_a S_a(s_0)$$

choose the vector $\vec{n} = (n_A, n_T, n_R)$

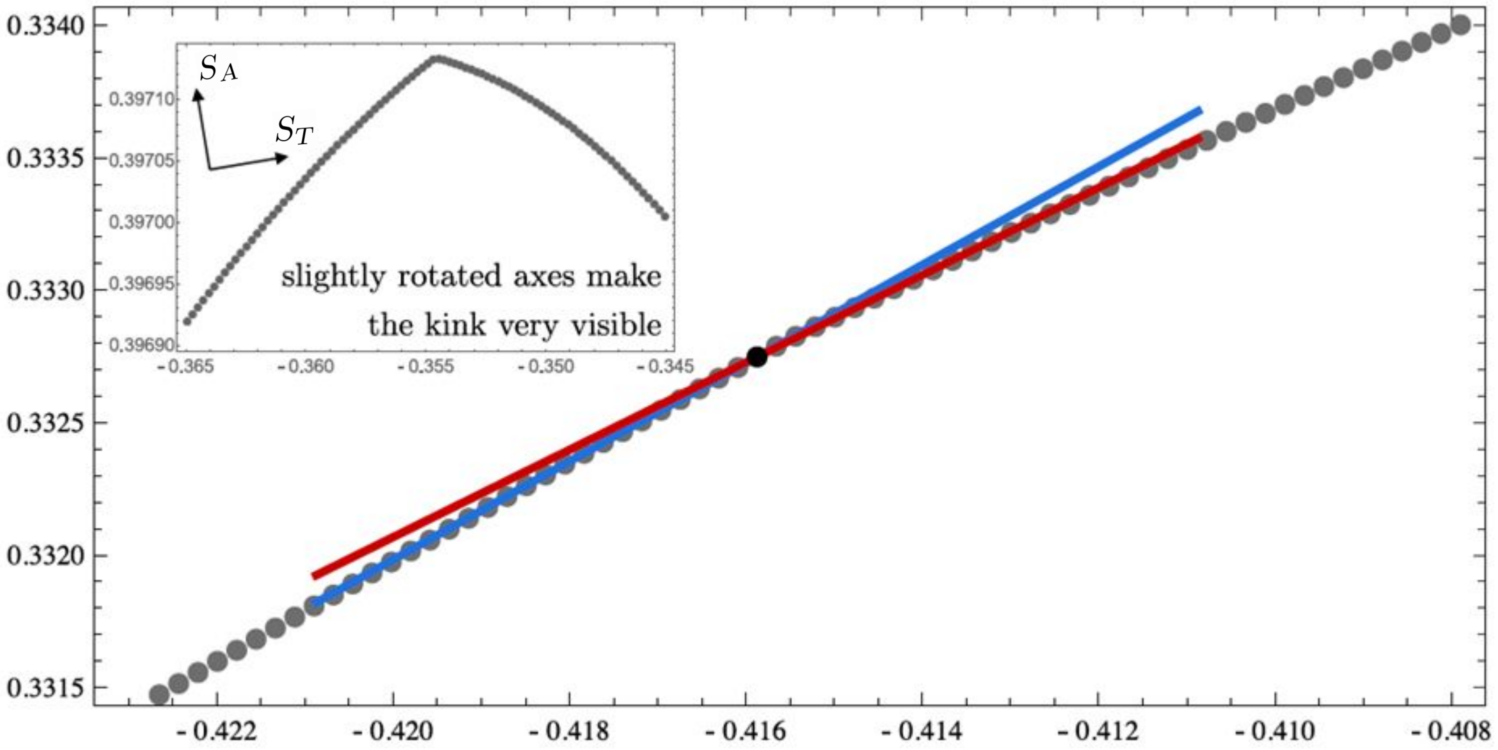
uniformly distributed on a unit sphere and scan

1+1d O(N) monolith



[Cordova, YH, Kruczenski, Vieira, 2019]

The NLSM kink



Dual S-matrix bootstrap

explicitly construct scattering amplitudes satisfying all the constraints

parameter space $S(s)$

[Cordova, YH, Kruczenski, Vieira, 2019; Guerrieri, Homrich, Vieira, 2020; Elias, Miro, Guerrieri, 2021]

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$$\max_{\{S(s)\}} \mathcal{F}_P \leq \min_{\{K(s)\}} \mathcal{F}_D$$

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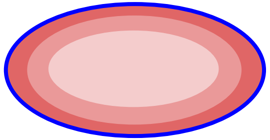
primal approach

solve constraints
increase parameter space
approach the boundary from inside



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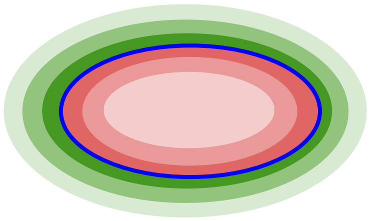


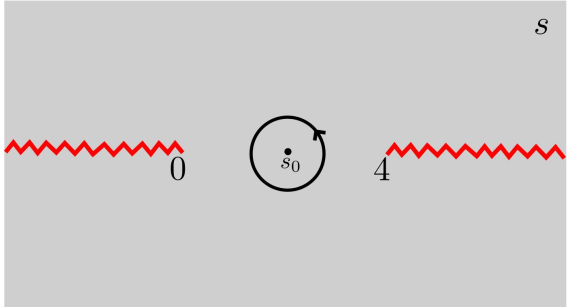
dual approach

increase space of Lagrange multipliers
impose more and more constraints
approach the boundary from outside

strict bounds

[Cordova, YH, Kruczenski, Vieira, 2019; Guerrieri, Homrich, Vieira, 2020; Elias, Miro, Guerrieri, 2021]

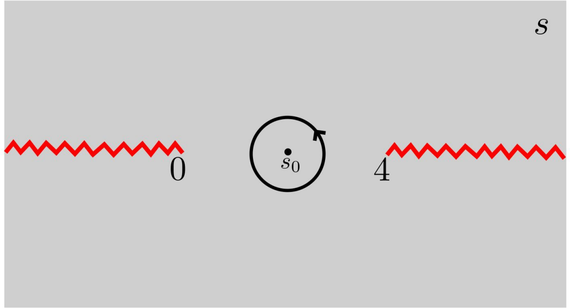
1+1d dual problem



$$S(s_0) = \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{S(s)}{s - s_0} ds = \frac{1}{2\pi i}\oint_{\mathcal{C}} S(s)K(s)ds$$

$K(s)$ has pole at s_0 with residue 1

1+1d dual problem



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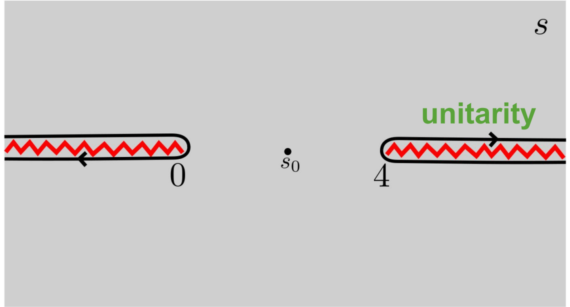
blow up contour
drop infinity
assume properties

$$= \underbrace{\frac{2}{\pi}\int_4^\infty \mathrm{Im}\left[S(s)K(s)\right]ds}_{\text{physical region integral}}$$



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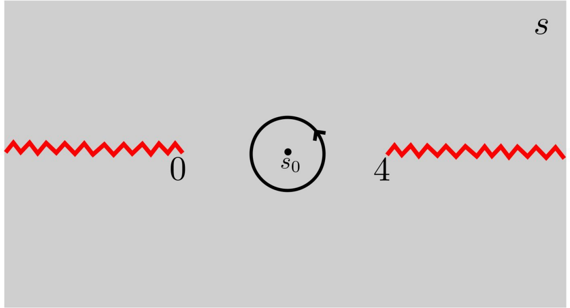

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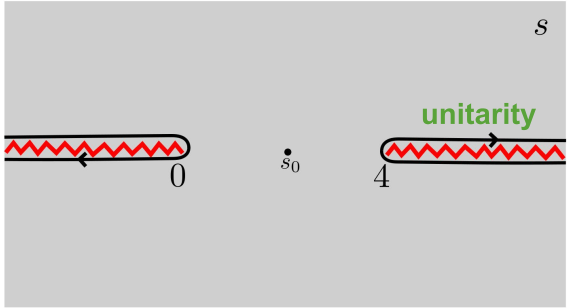
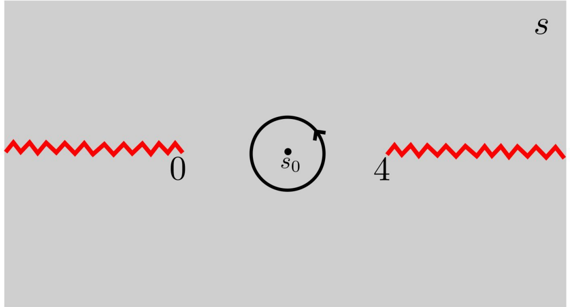


$$= \underbrace{\frac{2}{\pi}\int_4^\infty \mathrm{Im}\left[S(s)K(s)\right]ds}_{\text{physical region integral}} \leq \frac{2}{\pi}\int_4^\infty |S(s^+)K(s^+)|\,ds \leq \frac{2}{\pi}\int_4^\infty |K(s^+)|ds$$

unitarity

1+1d dual problem



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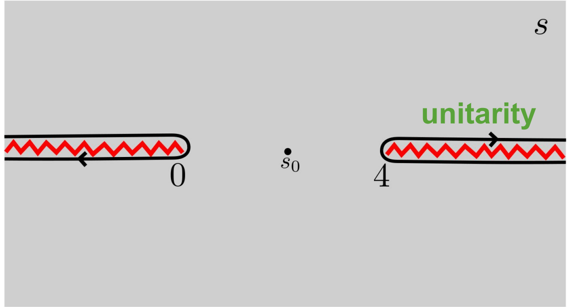

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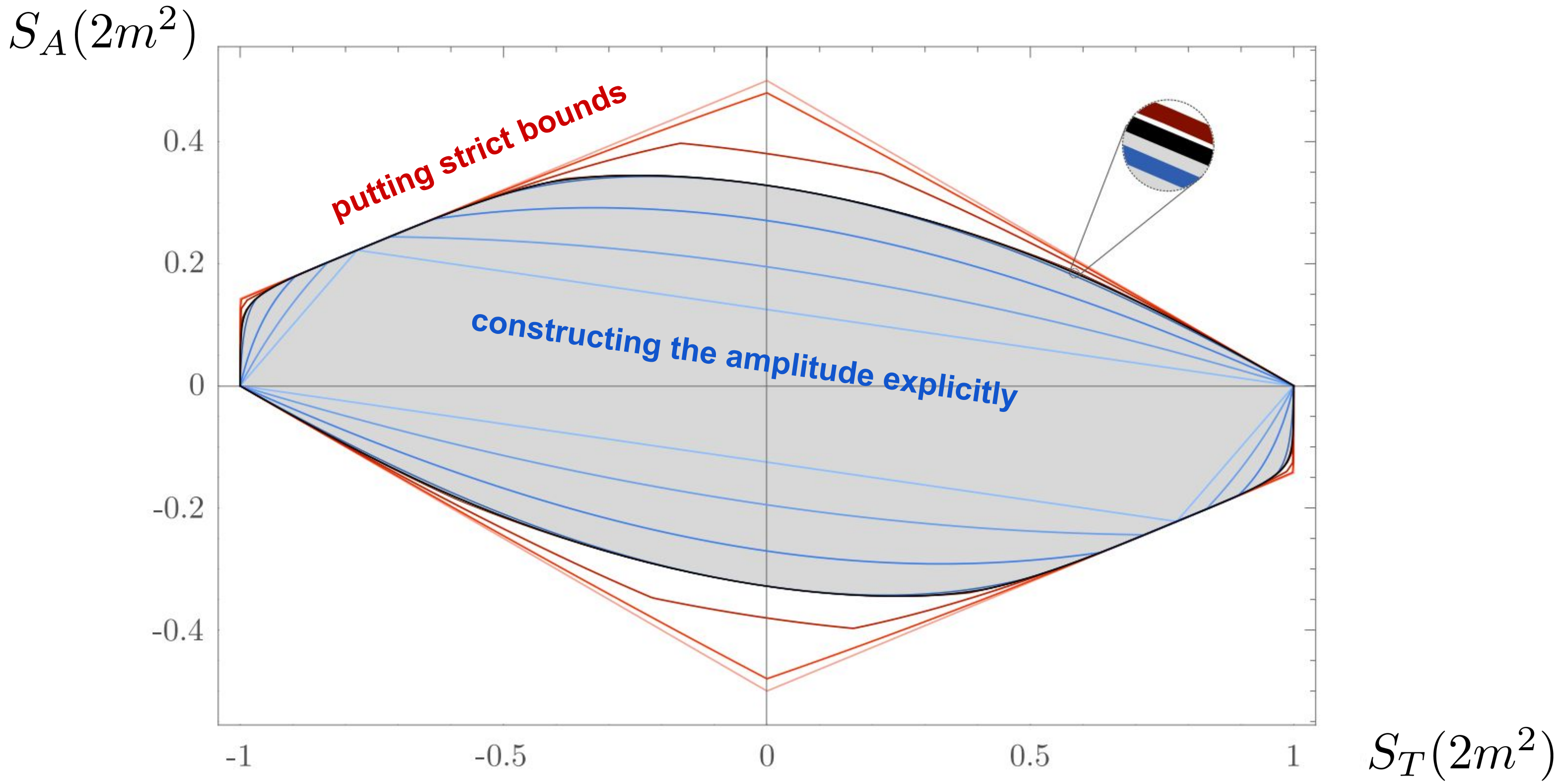
optimal bound by minimizing with given properties

Primal vs Dual



3+1d amplitudes

single flavor pion scattering: identical scalar particles, no bound state

$$\pi(p_1) + \pi(p_2) \to \pi(p_3) + \pi(p_4)$$

$$\mathbf{S} = \mathbb{I} + i\mathbf{T}$$

$$\langle p_3, p_4|\mathbf{T}|p_1, p_2\rangle = \boxed{T(s,t,u)}(2\pi)^4\delta^{(4)}(p_3 + p_4 - p_1 - p_2)$$

interacting amplitude

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analyticity: $T(s,t,u)$ cuts $s > 4m^2, t > 4m^2, u > 4m^2$ recall $s+t+u = 4m^2$

analytic functions of two variables

crossing: $T(s,t,u) = T(s,u,t) = T(u,t,s)$

unitarity: $|S_\ell(s)|^2 \leq 1 \quad s > 4m^2 \quad \forall \ell$

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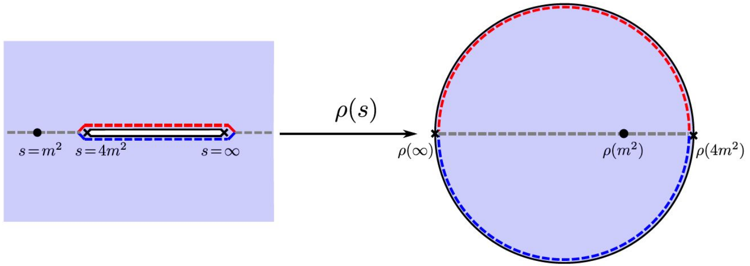
more severe tension between crossing and unitarity

$$S_\ell(s) = \frac{\pi}{4}\sqrt{\frac{s-4}{s}}\int_{-1}^{1} d\cos\theta P_\ell(\cos\theta) S(s^+, t)$$

3+1d primal bootstrap

[Paulos, Penedones, Toledo, van Rees, Vieira, 2017]



$$\rho_s = \frac{\sqrt{4m^2 - s_0} - \sqrt{4m^2 - s}}{\sqrt{4m^2 - s_0} + \sqrt{4m^2 - s}}$$

1, analyticity+crossing

$$T = \text{poles} + \sum_{a+b+c \leq N} \alpha_{(abc)} \rho_s^a \rho_t^b \rho_u^c$$

2, unitarity

$$S_\ell(s) = 1 + i\frac{\pi}{4}\sqrt{\frac{s-4}{s}} \int_{-1}^{1} d\cos\theta \, P_\ell(\cos\theta) T(s^+, t, u)$$

$$|S_\ell(s)|^2 \leq 1 \qquad s > 4m^2 \qquad \forall \ell$$

3, maximize a linear functional, extrapolate the results, bounds

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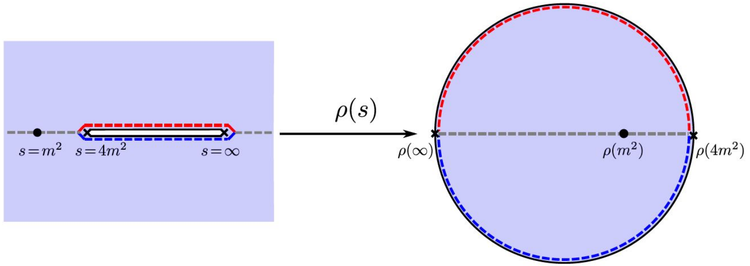

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generalize to various contexts:

isospin [Guerrieri, Penedones, Vieira, 2018]

spinning particles [Hebbar, Karateev, Penedones, 2020]

…

Semi-phenomenological pion bootstrap

pion: pseudo-Goldstone boson of chiral symmetry breaking

O(3) setup: $T^{(0)}, \quad T^{(1)}, \quad T^{(2)}$

[Guerrieri, Penedones, Vieira, 2018]

Semi-phenomenological pion bootstrap

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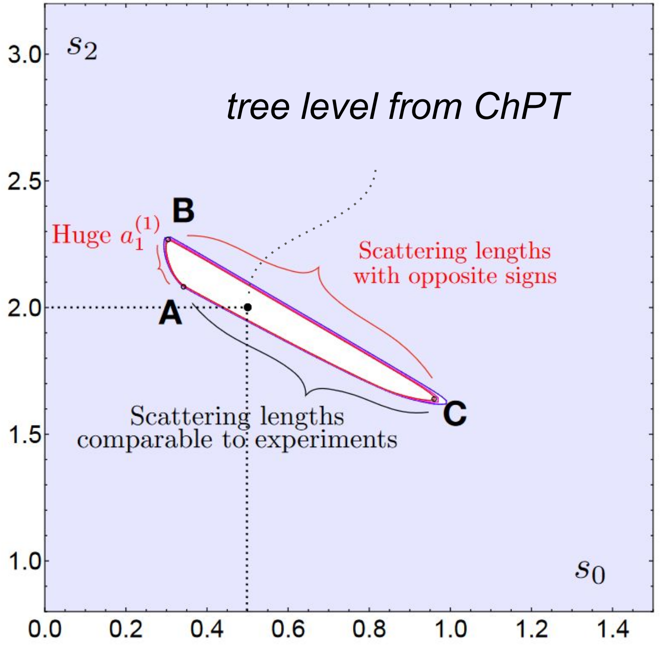
O(3) setup: $T^{(0)}, \;\; T^{(1)}, \;\; T^{(2)}$

e.g.

imposing rho resonance from experimental data: $S^{(1)}_{\ell=1}(m_\rho^2) = 0$

explore the allowed space of Adler zeros:

$$S^{(0)}_{\ell=0}(s_0) = 1 \qquad\qquad S^{(2)}_{\ell=0}(s_2) = 1$$



[Guerrieri, Penedones, Vieira, 2018]

Bounding non-perturbative coupling

[Paulos, Penedones, Toledo, van Rees, Vieira, 2017]

analyticity+crossing+unitarity max/min $\lambda \equiv T\left(\frac{4}{3}m^2, \frac{4}{3}m^2, \frac{4}{3}m^2\right)$ *quartic coupling of single pion scattering*

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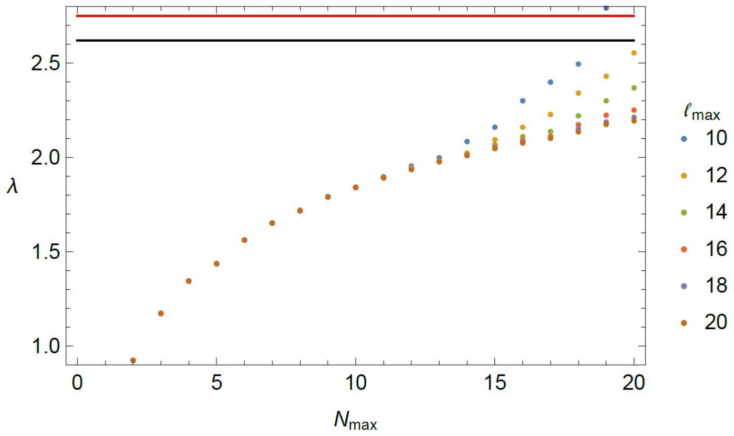

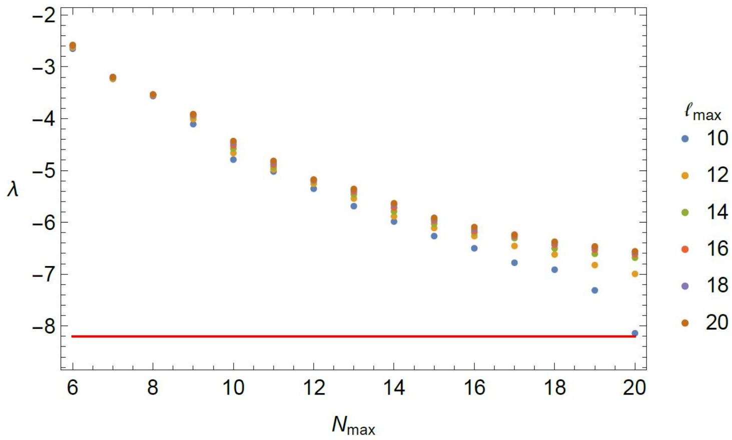

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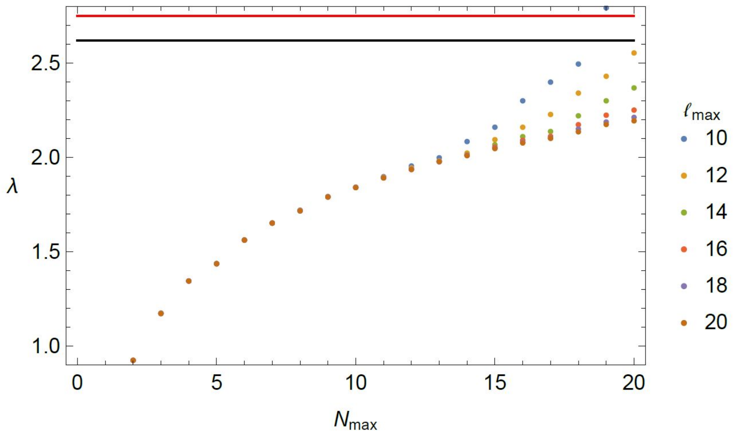

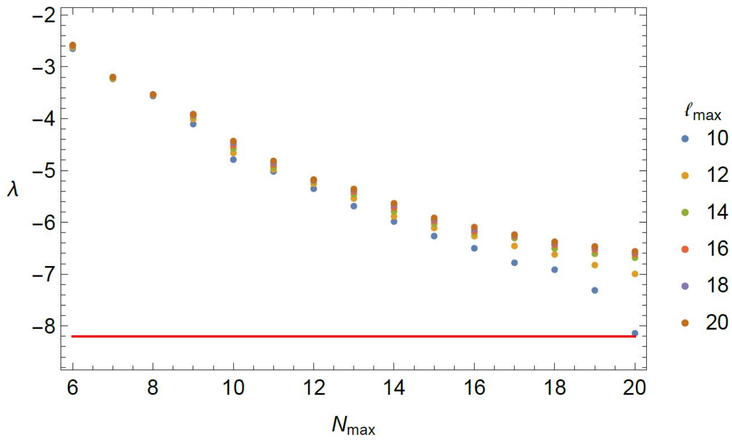

extremal amplitudes has a pole at multiple particle threshold

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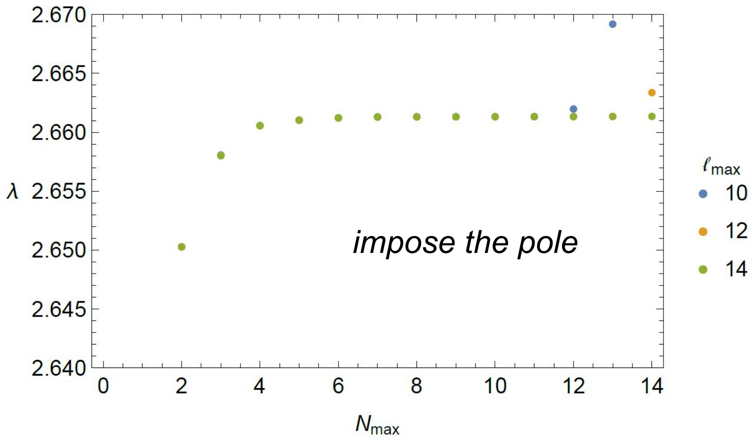

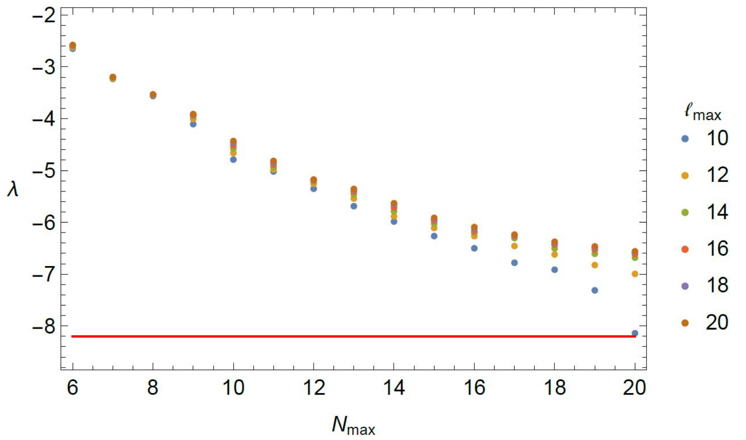

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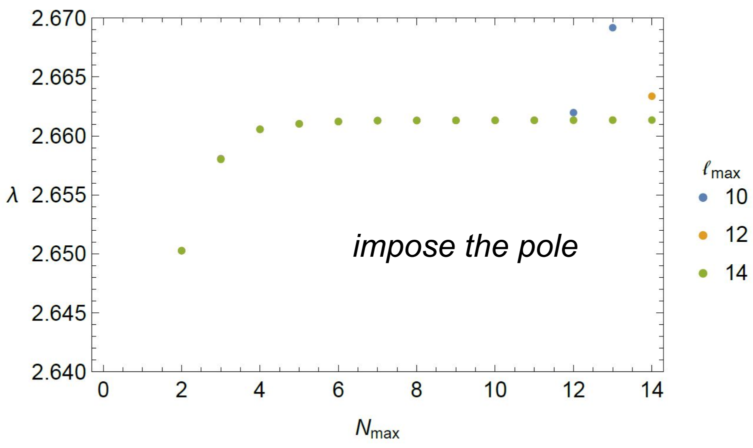
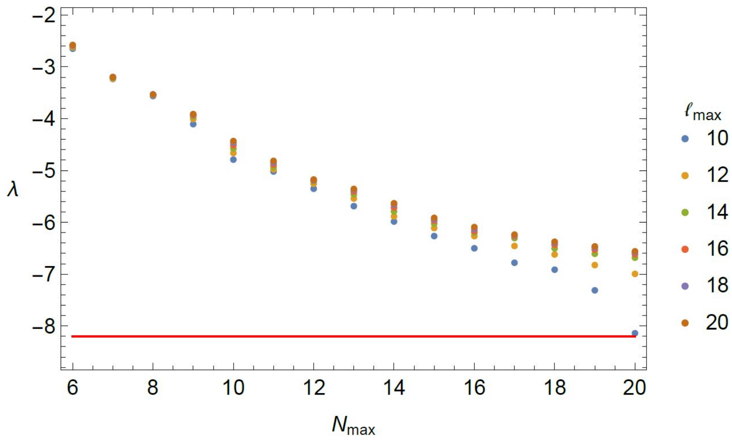




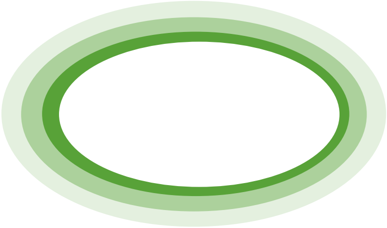
extremal amplitudes has a pole at multiple particle threshold

explore the space of theories not assuming such dynamical structure → non-constructive dual approach

3+1d dual approach

[YH, Kruczenski, 2021]

dual amplitude $$K(s,t) = -\frac{1}{(s-s_0)(t-t_0)} + \frac{i}{\pi^2}\int_{4m^2}^{\infty} dx \int_{4m^2-x}^{0} dy \frac{\bar{k}(x,y)}{(s-x)(t-y)}$$



dual partial waves: $$k_\ell(s) = \frac{(2\ell+1)}{\pi^3}\sqrt{s(s-4m^2)}\int_{-1}^{1} d\cos\theta P_\ell(\cos\theta) k(s,t)$$

"Lagrange multipliers", space of constraints, parametrize the dual problem

dual partial waves encodes the information about 2-to-2 scattering

3+1d dual approach

[YH, Kruczenski, 2021]

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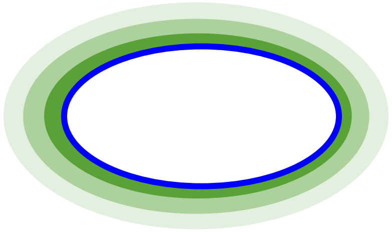

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"Lagrange multipliers", space of constraints, parametrize the dual problem

3+1d dual

$$\min_{\{k_\ell(s)\}} \mathcal{F}_D = \sum_{\ell \text{ even}} \int_{4m^2}^{\infty} ds \Big(|k_\ell(s)| - \mathrm{Re}k_\ell(s)\Big) + M_{\text{reg}}||\mathrm{Re}K||$$

boundary of the space

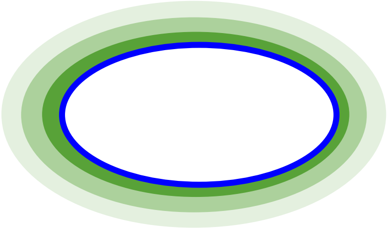
$$S_\ell(s) = \frac{k_\ell(s)}{|k_\ell(s)|}$$

dual partial waves encodes the information about 2-to-2 scattering

3+1d dual approach

[YH, Kruczenski, 2021]

dual amplitude $$K(s,t) = -\frac{1}{(s-s_0)(t-t_0)} + \frac{i}{\pi^2}\int_{4m^2}^{\infty} dx \int_{4m^2-x}^{0} dy \frac{\bar{k}(x,y)}{(s-x)(t-y)}$$



***dual partial waves*:** $$k_\ell(s) = \frac{(2\ell+1)}{\pi^3}\sqrt{s(s-4m^2)}\int_{-1}^{1} d\cos\theta P_\ell(\cos\theta) k(s,t)$$

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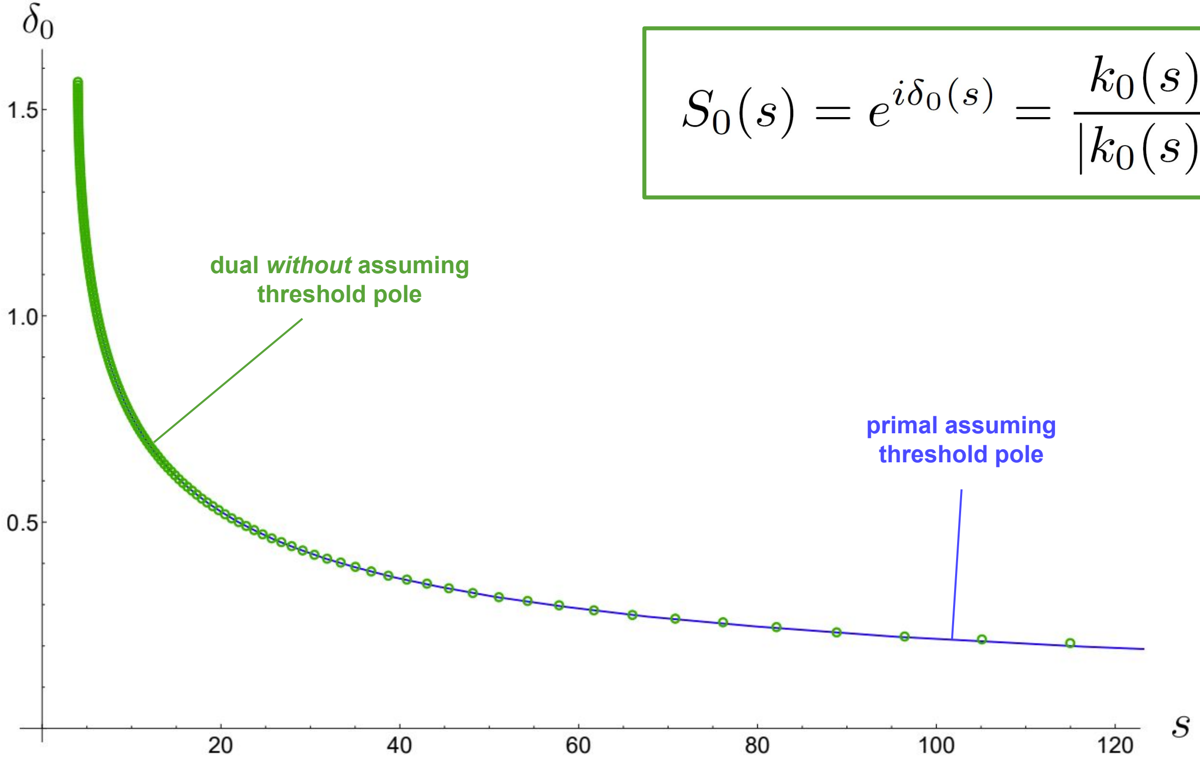
dual partial waves encodes the information about 2-to-2 scattering

dual partial waves are constraints free
after optimization, recover the analyticity+crossing+unitarity of physical amplitudes

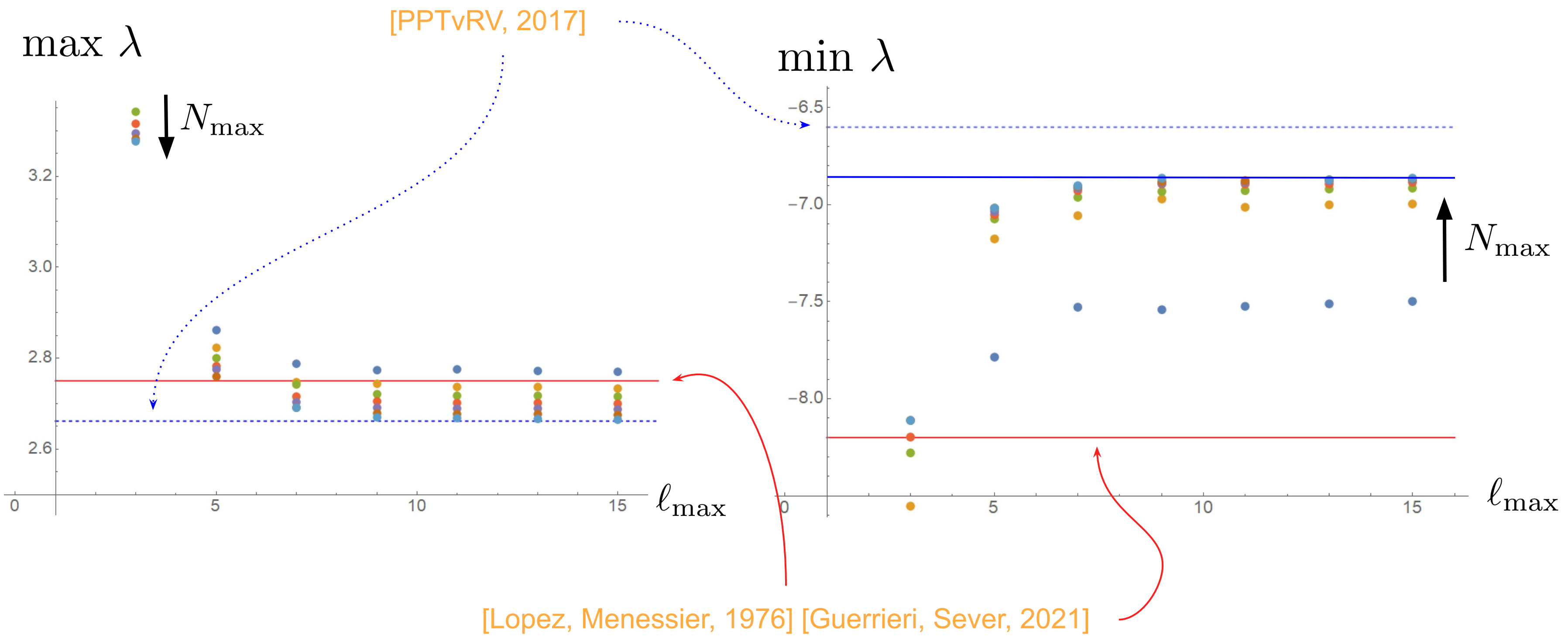
S-wave: phase shift

max $\quad \lambda \equiv T\left(\frac{4}{3}m^2, \frac{4}{3}m^2, \frac{4}{3}m^2\right)$

$$\boxed{S_0(s) = e^{i\delta_0(s)} = \frac{k_0(s)}{|k_0(s)|}}$$



Quartic coupling bounds – primal vs dual



Expanding the space of theories

1+1 d

$$\Lambda \equiv T(s = 2m^2)$$

quartic coupling bounds

max/min extremal amplitudes

higher d

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[Chen, Fitzpatrick, Karateev, 2021 & 2022; Miro, Guerrieri, Gumus, 2022]

Expanding the space of theories

1+1 d		*higher d*
$\Lambda \equiv T(s = 2m^2)$	quartic coupling bounds max/min extremal amplitudes	$\lambda \equiv T\left(\frac{4}{3}m^2, \frac{4}{3}m^2, \frac{4}{3}m^2\right)$
$\Lambda^{(n)} \equiv \lim_{s \to 2m^2} \partial_s^n T(s)$	expansion coeff around symmetric point	$m^{d-4} T(s,t) = \sum_{k,l=0}^{\infty} \lambda_{k,l}\, m^{-2(k+l)} (s - 4m^2/3)^k (t - 4m^2/3)^l$

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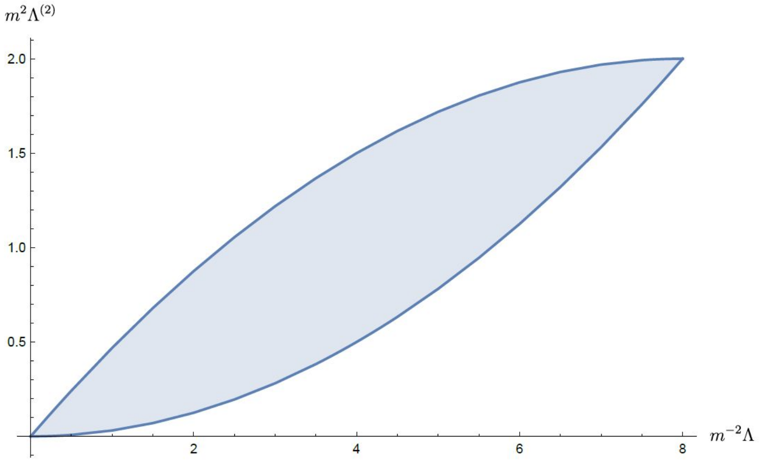

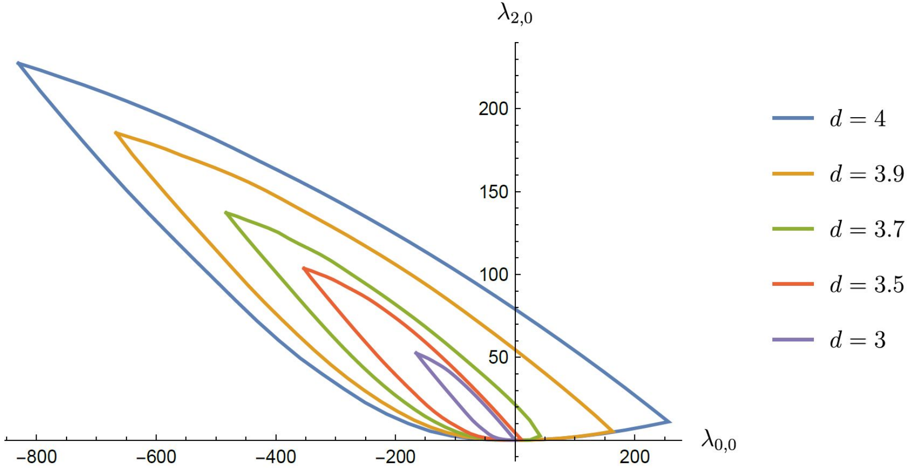

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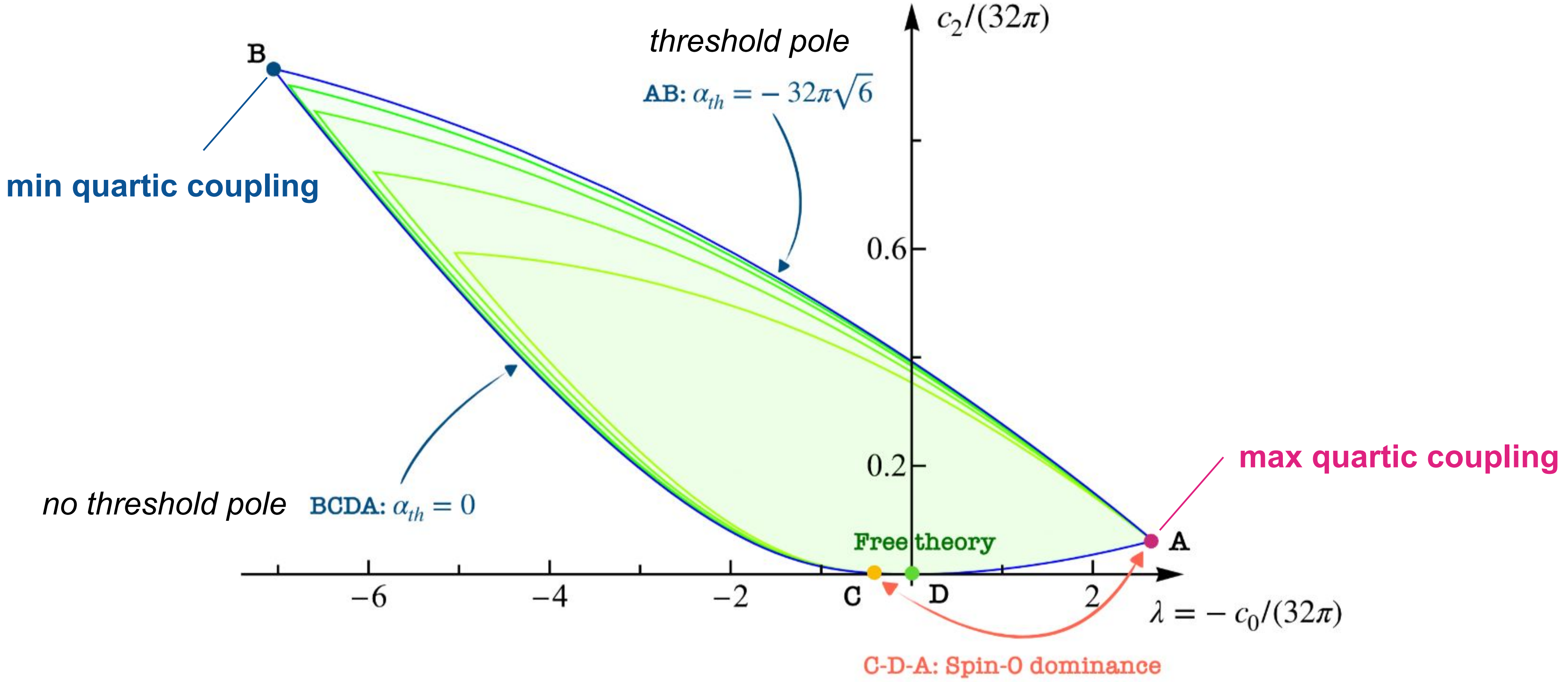

examine details of extremal amplitudes

Connect with UV: form factor bootstrap

[Karateev, Kuhn, Penedones, 2019]

nonperturbative definition of QFT as RG flow from UV to IR fixed point

$$|\psi_1\rangle = |p_1, p_2\rangle_{in}\,, \qquad |\psi_2\rangle = |p_1, p_2\rangle_{out}\,, \qquad |\psi_3\rangle = \int dx e^{i(p_1+p_2)\cdot x}\mathcal{O}(x)|0\rangle$$

local operator, connect to the UV

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form factor: $${}_{out}\langle\mathbf{n}|\,\mathcal{O}(x)\,|0\rangle = e^{-ip\cdot x}\mathcal{F}_n(p_1, \ldots, p_n)$$

two-particle form factor: $\mathcal{F}_2(p_1, p_2) = \mathcal{F}_2(s)$

$$\mathcal{F}_2(s) = -\frac{g^*\mathcal{F}_1^*}{s - m^2} + \ldots .$$

analytic function in cut plane $s \geq 4m^2$

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spectral density

$$\int d^dx\, e^{-ik\cdot x}\langle 0|\mathcal{O}^\dagger(x)\mathcal{O}(0)|0\rangle = (2\pi)\theta(k^0)\rho(-k^2)$$

$$\rho(s) = \rho_1(s) + \rho_2(s) + \ldots$$

$\delta(s-m^2)$ $\qquad$ $\theta(s-4m^2)$

with this setup, one can ask questions relating UV and IR

1+1d, trace of stress tensor: $\Theta(z,\bar{z}) \equiv 4T_{z\bar{z}}(z,\bar{z})$

$$c_{UV} - c_{IR} = (2\pi)^2 \times \frac{3}{4\pi} \int d^2x_E \, x_E^2 \, \langle 0|\Theta(x_E)\Theta(0)|0\rangle_T$$

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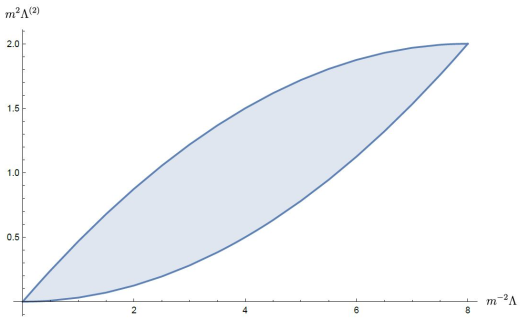
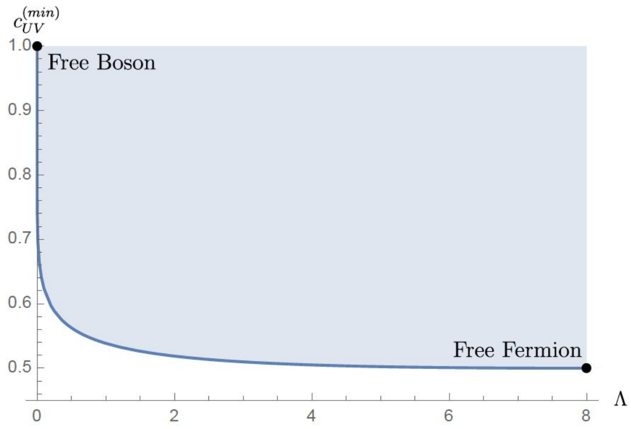
e.g. single flavor no bound states in 1+1d





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Explorations in Ising Field Theory

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IFT: 2d Ising model near critical point: magnetic deformation

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$$T(s) = -\frac{g^2}{s-m^2} + \dots \qquad \mathcal{F}(s) = -\frac{\mathcal{F}_1^\Theta g}{s-m^2} + \dots$$

$S(m^2(1-x)) = 0$ *related to strength of magnetic field*

tune the location of the zero x

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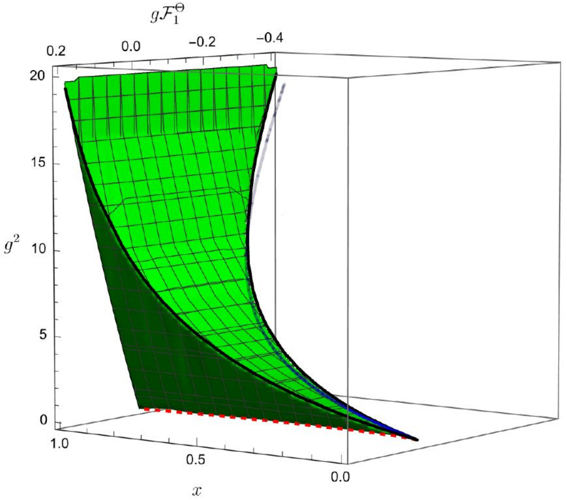

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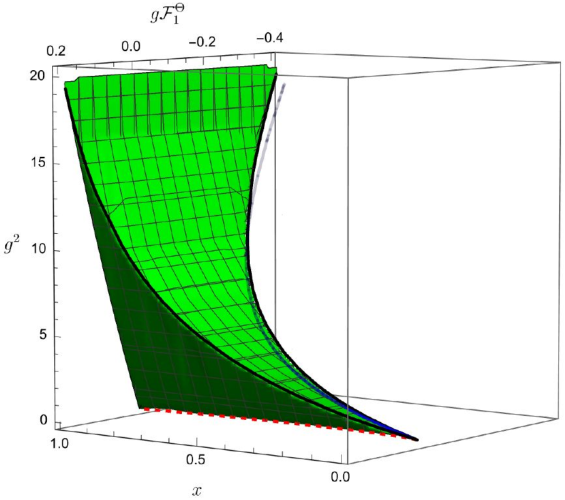

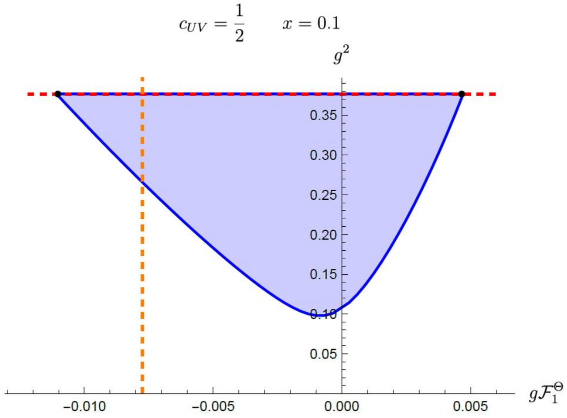

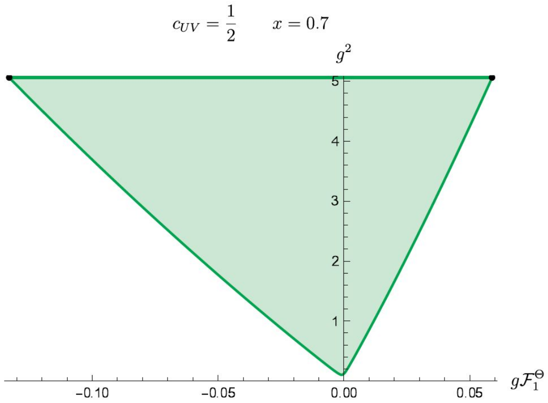

Bootstrap with Hamiltonian truncation data

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isolate a specific theory, instead of constructing generic bounds

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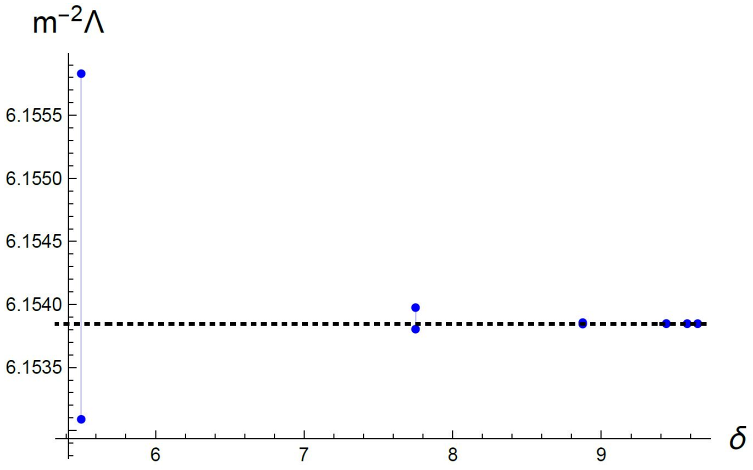
input

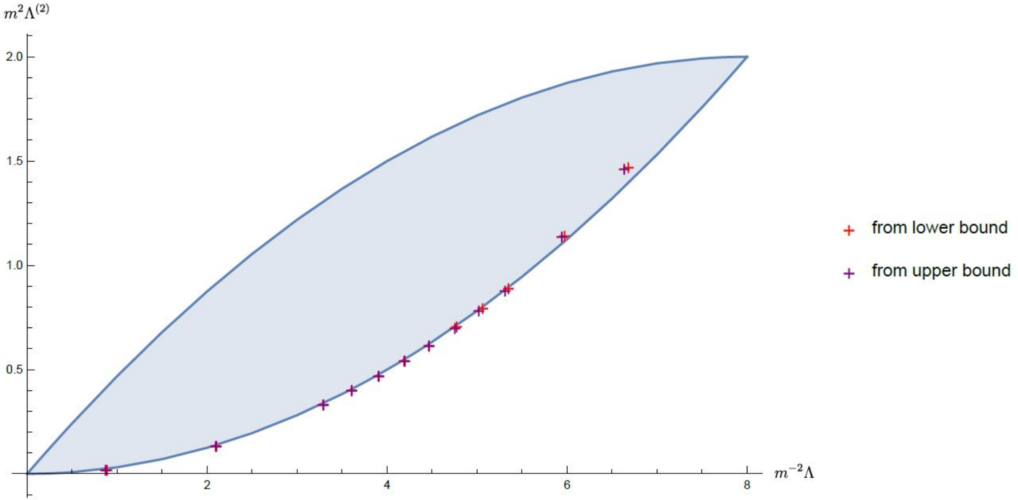
match δ

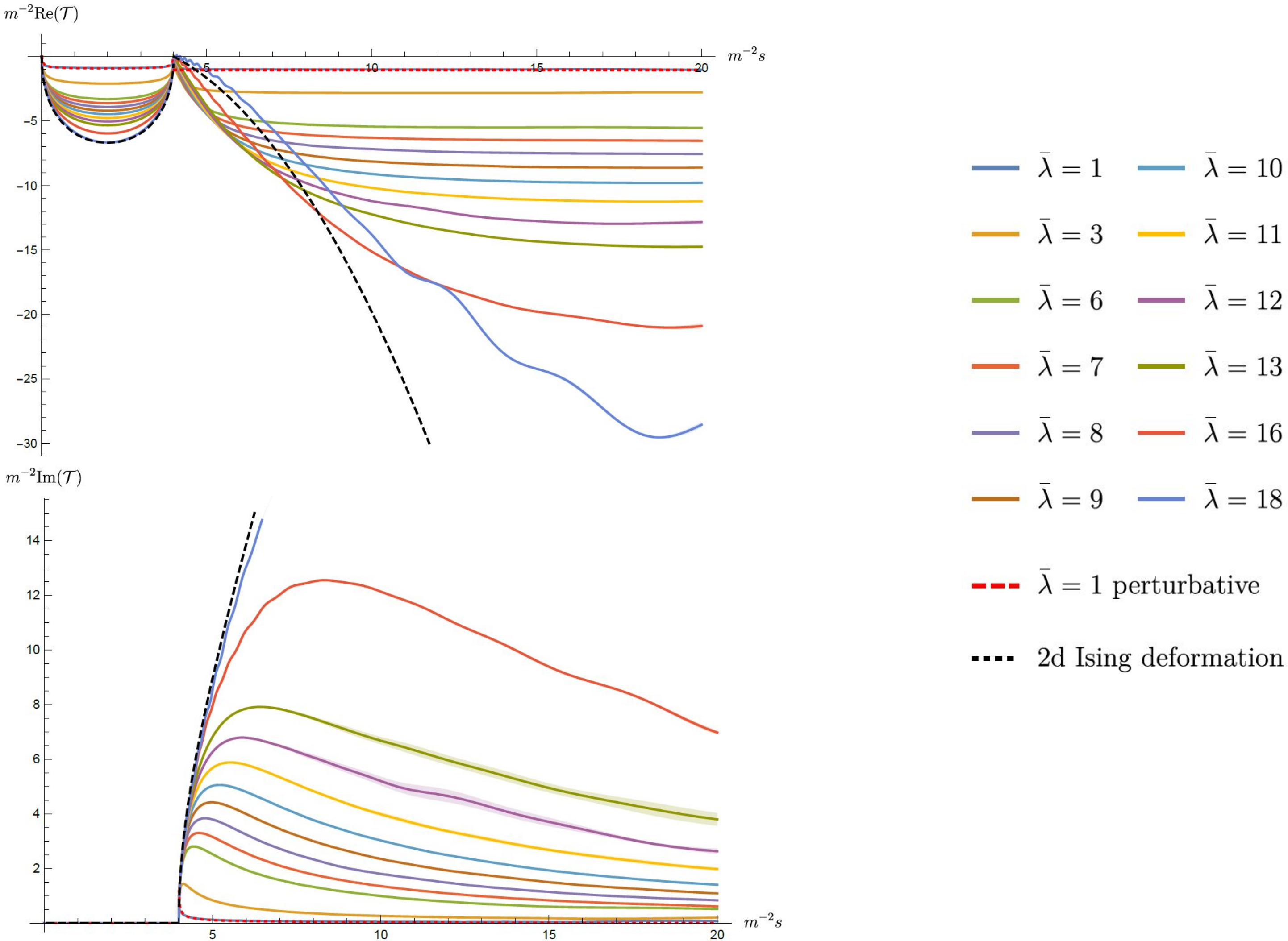
S-matrix/form-factor bootstrap: $\begin{pmatrix} 1 & \mathcal{S}^* \\ \mathcal{S} & 1 \end{pmatrix} \succeq 0$ $\qquad \begin{pmatrix} 1 & \mathcal{S}^* & \mathcal{F}_2^{*\Theta} \\ \mathcal{S} & 1 & \mathcal{F}_2^\Theta \\ \mathcal{F}_2^\Theta & \mathcal{F}_2^{*\Theta} & \rho \end{pmatrix} \succeq 0$

2d ϕ^4

maximize/minimize Λ







m^{-2}Re(T)
m^{-2}s
5
10
15
20
-5
-10
-15
-20
-25
-30
m^{-2}Im(T)
14
12
10
8
6
4
2
λ̄ = 1
λ̄ = 3
λ̄ = 6
λ̄ = 7
λ̄ = 8
λ̄ = 9
λ̄ = 10
λ̄ = 11
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λ̄ = 13
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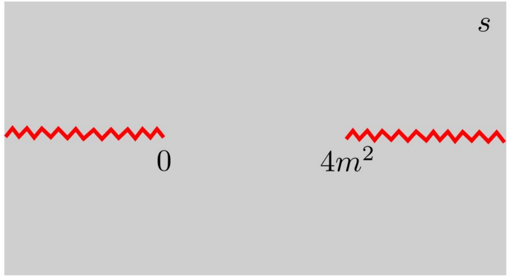
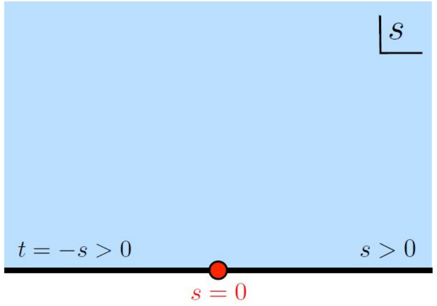
2d Ising deformation

Massless scattering

1+1d:



$m \to 0$



[Elias Miro, Guerrieri, Hebbar, Penedones, Vieira, 2019]

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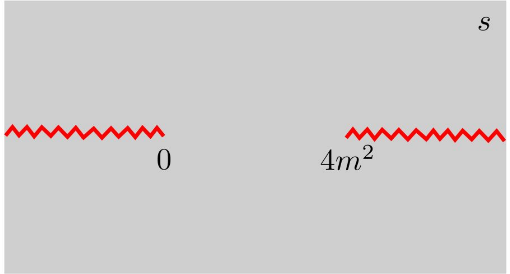

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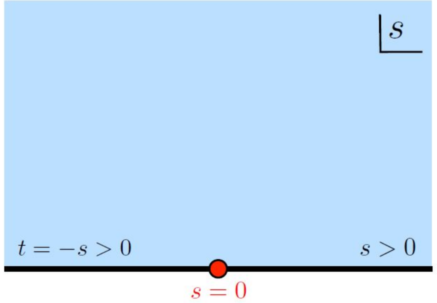

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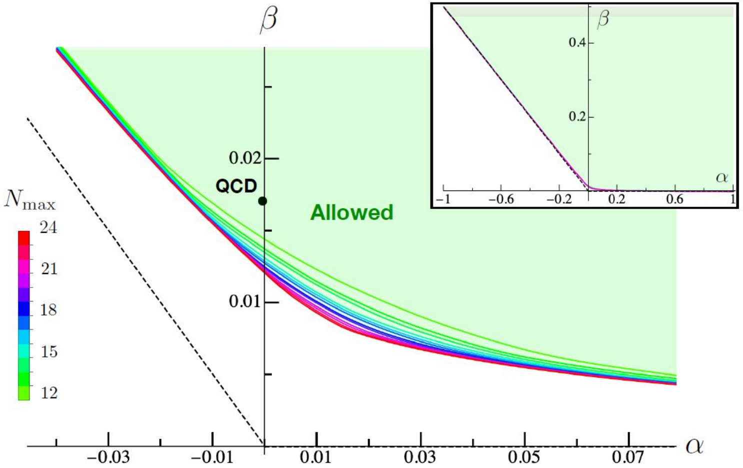

$$A(s|t,u) = \frac{s}{f_\pi^2} + \frac{1}{f_\pi^4}\left[\alpha s^2 + \beta(t^2+u^2) - \frac{s^2}{32\pi^2}\log\frac{-s}{f_\pi^2} - \frac{t-u}{96\pi^2}\left(t\log\frac{-t}{f_\pi^2} - u\log\frac{-u}{f_\pi^2}\right)\right] + \dots$$

[Guerrieri, Penedones, Vieira, 2020]

Other related approaches

- scattering from production: [Tourkine, Zhiboedov, 2021 & 2023]

Atkinson's method to construct amplitudes satisfying

analyticity+crossing+**unitarity(elastic+inelastic+multiparticle)** from production input

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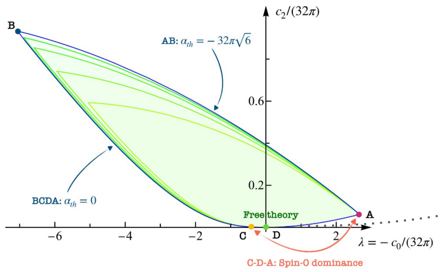

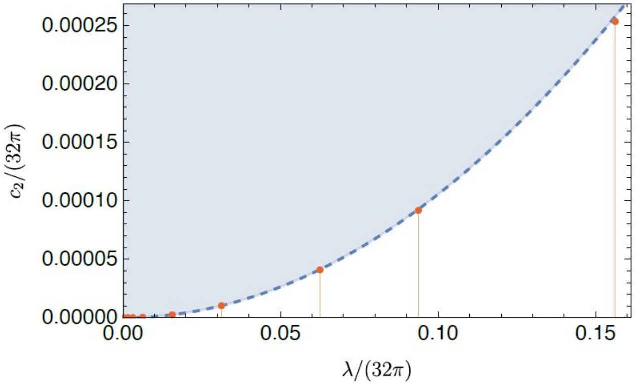

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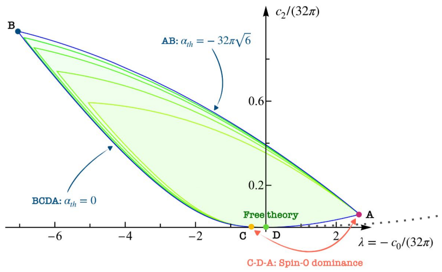

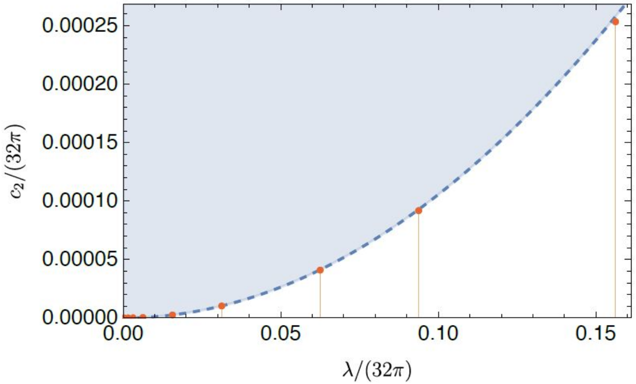

- EFT bootstrap: bounding Wilson coefficients with positivity — a dual approach

(Bellazini, Elias Miro, Rattazzi, Riembau, Riva, Tolley, Wang, Zhou, Caron-Huot, Van Duong, Sinha, Zahed, Arkani-Hamed, Huang, Huang, ……)

Thank you!