Gyroscopic Memory Effects in Gravity and Electrodynamics

Blagoje Oblak
(École polytechnique)

May 2023

arXiv : 2112.04535, 2203.16216, 2304.12348
with Ali Seraj

Intro

(How a top sees gravitational waves)

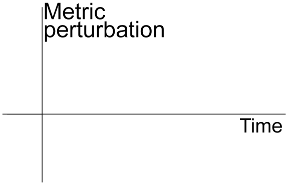
Two motivations

Two motivations

1. Observable **gravitational waves** !

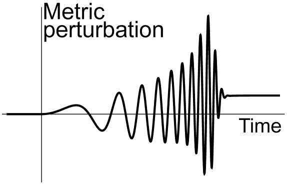
Two motivations

1. Observable **gravitational waves** !



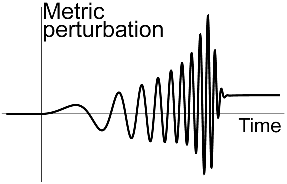
Two motivations

1. Observable **gravitational waves** !



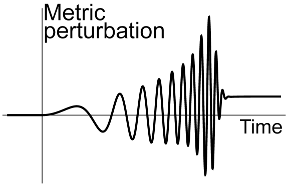
Two motivations

1. Observable **gravitational waves** !
   - Measurable implications ?



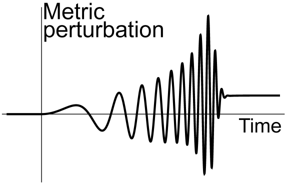
Two motivations

1. Observable **gravitational waves** !
   - Measurable implications ?
   - **Memory effect** :



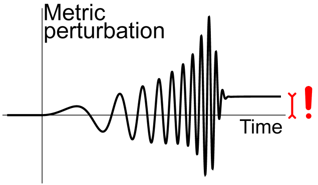
Two motivations

1. Observable **gravitational waves** !
   - Measurable implications ?
   - **Memory effect** :
     Metric offset after wave



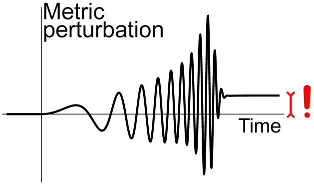
Two motivations

1. Observable **gravitational waves** !
   - Measurable implications ?
   - **Memory effect** :
     Metric offset after wave



Two motivations

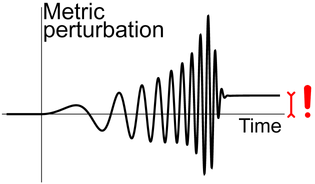
1. Observable **gravitational waves** !
   - Measurable implications ?
   - **Memory effect** :
     Metric offset after wave



2. Semiclassical gravity ?

Two motivations

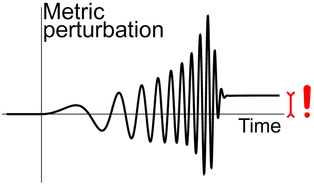
1. Observable **gravitational waves** !
   - Measurable implications ?
   - **Memory effect** :
     Metric offset after wave



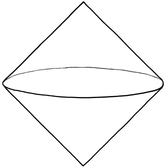
2. Semiclassical gravity ?
   - Large distance **asymptotic symmetries**

Two motivations

1. Observable **gravitational waves** !
   - Measurable implications ?
   - **Memory effect** :
     Metric offset after wave

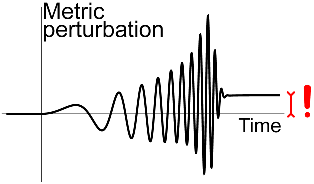


2. Semiclassical gravity ?
   - Large distance **asymptotic symmetries**

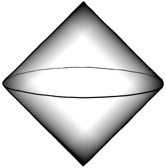


Two motivations

1. Observable **gravitational waves** !
   - Measurable implications ?
   - **Memory effect** :
     Metric offset after wave

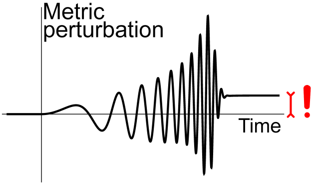


2. Semiclassical gravity ?
   - Large distance **asymptotic symmetries**

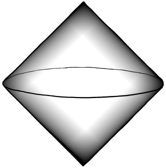


TWO MOTIVATIONS

1. Observable **gravitational waves** !
   - Measurable implications ?
   - **Memory effect** :
     Metric offset after wave

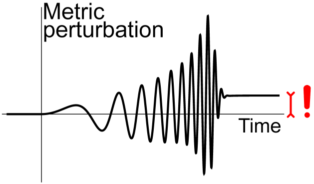


2. Semiclassical gravity ?
   - Large distance **asymptotic symmetries**
   - Memory probes currents

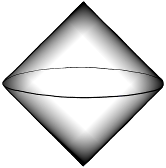


TWO MOTIVATIONS

1. Observable **gravitational waves** !
   - Measurable implications ?
   - **Memory effect** :
     Metric offset after wave

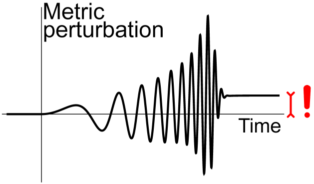


2. Semiclassical gravity ?
   - Large distance **asymptotic symmetries**
   - Memory probes currents
     (Noether currents of boundary theory)

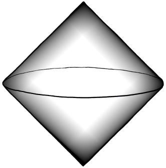


TWO MOTIVATIONS

1. Observable **gravitational waves** !
   - Measurable implications ?
   - **Memory effect** :
     Metric offset after wave



2. Semiclassical gravity ?
   - Large distance **asymptotic symmetries**
   - Memory probes currents
     (Noether currents of boundary theory)



Memory = **observable** effect of gravitational **symmetries**

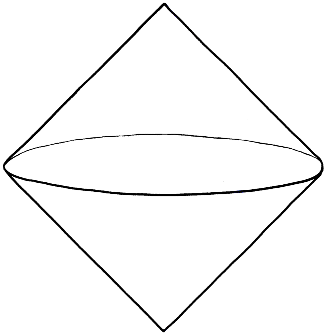
This Talk in a Nutshell

This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes**

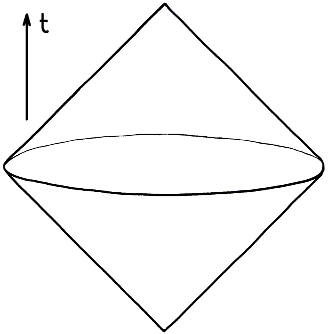
This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :



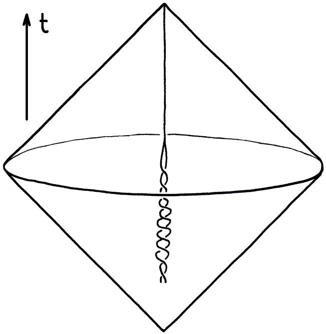
THIS TALK IN A NUTSHELL

Memory effects seen with freely falling **gyroscopes** :



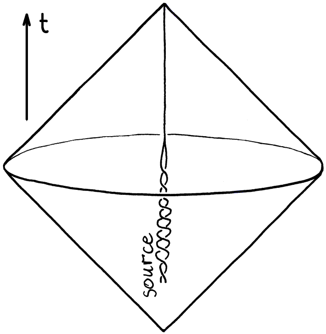
THIS TALK IN A NUTSHELL

Memory effects seen with freely falling **gyroscopes** :



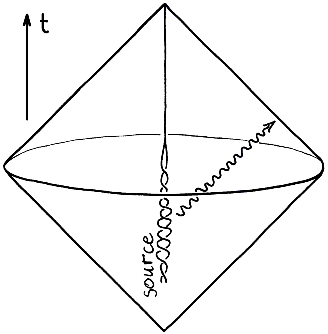
This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :



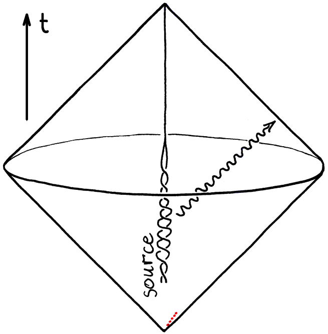
This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :



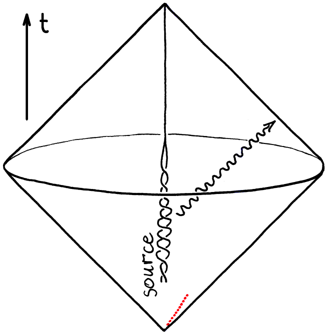
This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :



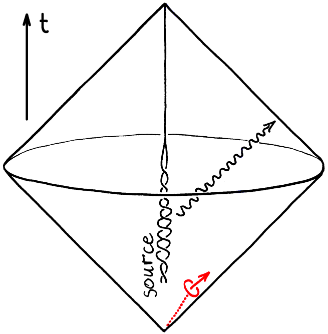
This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :



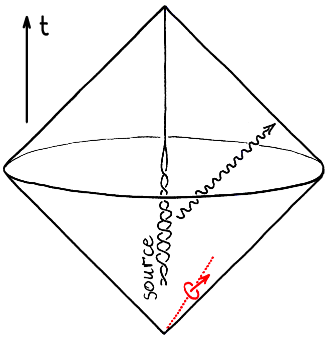
THIS TALK IN A NUTSHELL

Memory effects seen with freely falling **gyroscopes** :



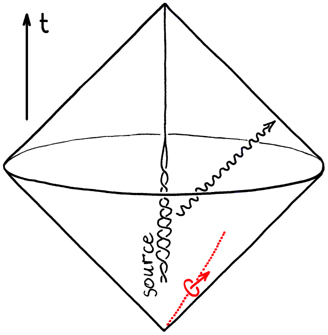
This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :



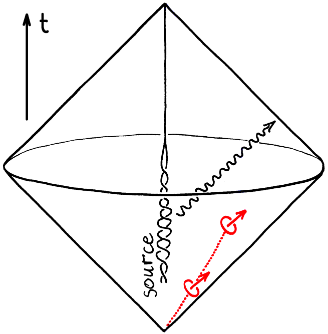
THIS TALK IN A NUTSHELL

Memory effects seen with freely falling **gyroscopes** :



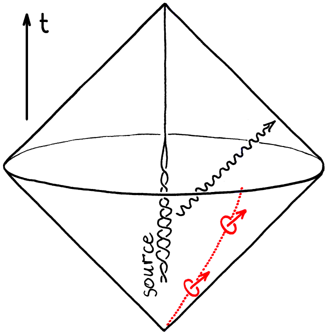
This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :



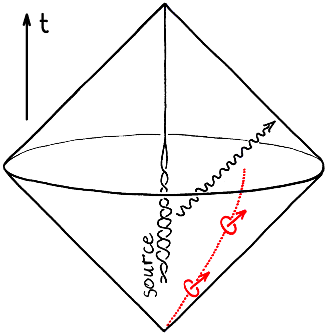
This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :



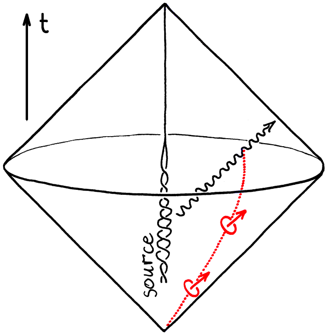
This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :



This Talk in a Nutshell

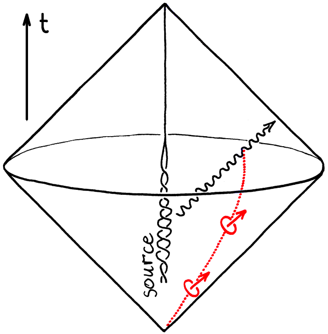
Memory effects seen with freely falling **gyroscopes** :



This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :

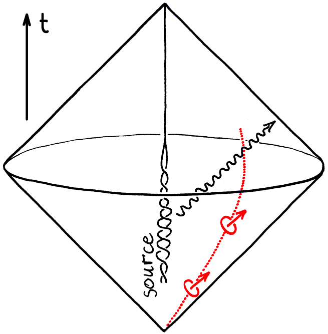
Waves cause precession



This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :

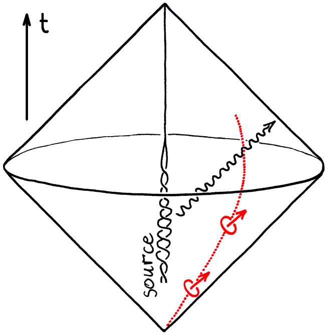
Waves cause precession



THIS TALK IN A NUTSHELL

Memory effects seen with freely falling **gyroscopes** :

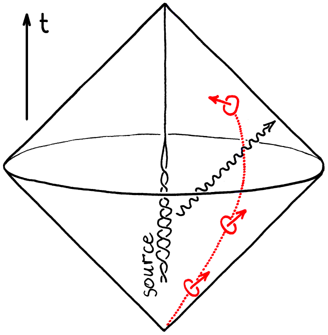
Waves cause precession



This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :

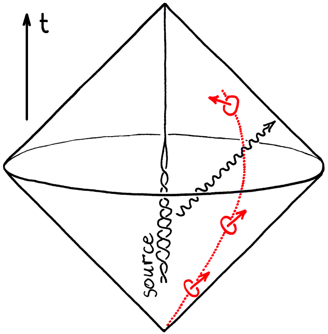
Waves cause precession



This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :

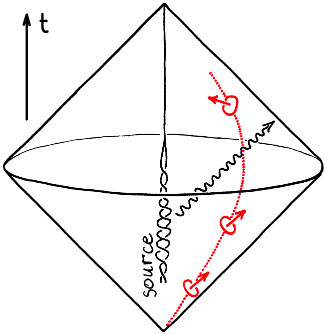
Waves cause precession



THIS TALK IN A NUTSHELL

Memory effects seen with freely falling **gyroscopes** :

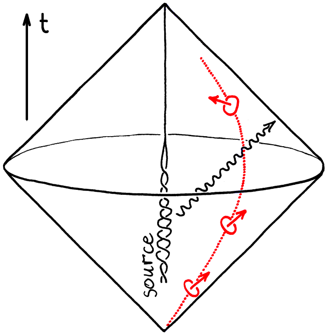
Waves cause precession



THIS TALK IN A NUTSHELL

Memory effects seen with freely falling **gyroscopes** :

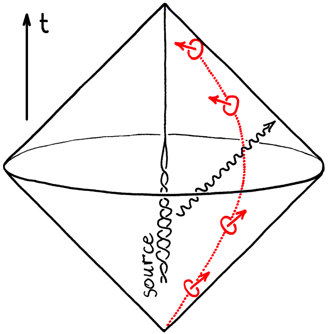
Waves cause precession



This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :

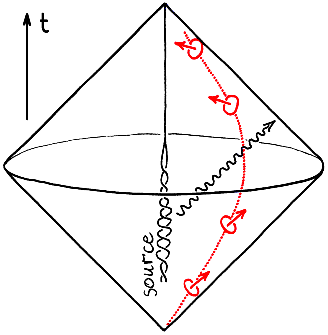
Waves cause precession



This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :

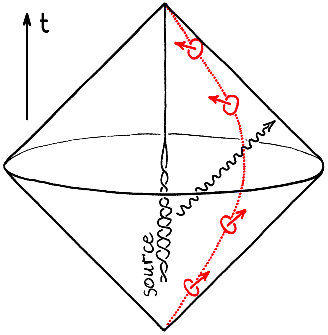
Waves cause precession



This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :

Waves cause precession

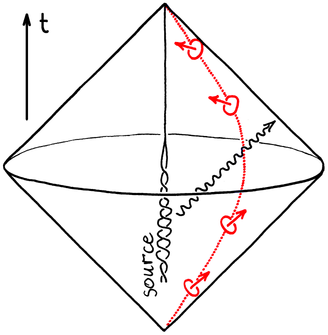


This Talk in a Nutshell

Memory effects seen with freely falling **gyroscopes** :

Waves cause precession

- Orientation memory !



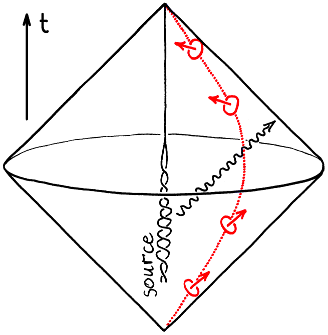
THIS TALK IN A NUTSHELL

Memory effects seen with freely falling **gyroscopes** :

Waves cause precession

- Orientation memory !

Underlying symmetries :



This Talk in a Nutshell

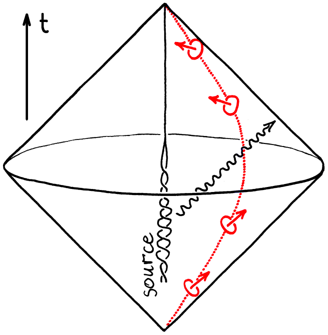
Memory effects seen with freely falling **gyroscopes** :

Waves cause precession

- Orientation memory !

Underlying symmetries :

- Precession rate = current of **dual aspt symmetries**



This Talk in a Nutshell

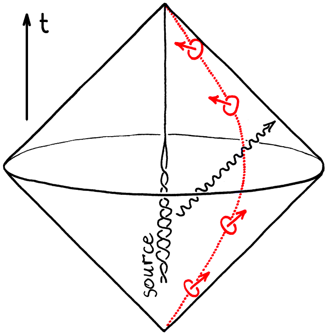
Memory effects seen with freely falling **gyroscopes** :

Waves cause precession

- Orientation memory !

Underlying symmetries :

- Precession rate = current of **dual aspt symmetries**
- Memory involves **electric-magnetic duality**



PLAN

1. Orientation memory in electrodynamics

2. Frames tied to distant stars

3. Orientation memory in gravity

Plan

1. **Orientation memory in electrodynamics**

2. Frames tied to distant stars

3. Orientation memory in gravity

PLAN

1. Orientation memory in electrodynamics

2. Frames tied to distant stars

3. Orientation memory in gravity

PLAN

1. Orientation memory in electrodynamics

2. Frames tied to distant stars... in radiative metric

3. Orientation memory in gravity

PLAN

1. Orientation memory in electrodynamics
2. Frames tied to distant stars... in radiative metric
3. **Orientation memory in gravity**

1. Orientation Memory in Electrodynamics

Electrodynamics at null infinity

Electrodynamics at null infinity

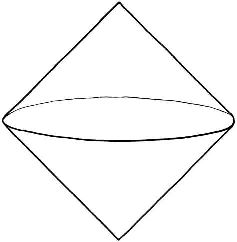
Minkowski in polar coordinates :

$$ds^2 = -dt^2 + \quad dr^2 \quad + r^2 h_{ab} d\theta^a d\theta^b$$

Electrodynamics at null infinity

Minkowski in polar coordinates :

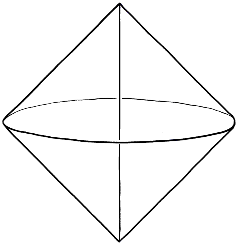
$$ds^2 = -dt^2 + \quad dr^2 \quad + r^2 h_{ab} d\theta^a d\theta^b$$



Electrodynamics at null infinity

Minkowski in polar coordinates :

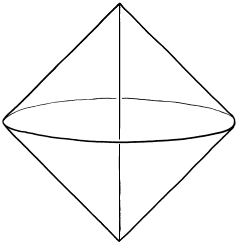
$$ds^2 = -dt^2 + \quad dr^2 \quad + r^2 h_{ab} d\theta^a d\theta^b$$



ELECTRODYNAMICS AT NULL INFINITY

Minkowski in **Bondi** coordinates :

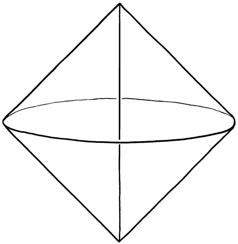
$$ds^2 = -dt^2 + \quad dr^2 \quad + r^2\, h_{ab} d\theta^a d\theta^b$$



ELECTRODYNAMICS AT NULL INFINITY

Minkowski in **Bondi** coordinates $(u = t - r)$:

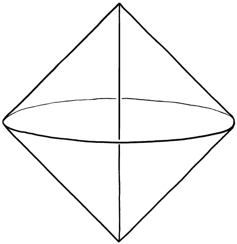
$$ds^2 = -dt^2 + \quad dr^2 \quad + r^2\, h_{ab} d\theta^a d\theta^b$$



ELECTRODYNAMICS AT NULL INFINITY

Minkowski in **Bondi** coordinates $(u = t - r)$:

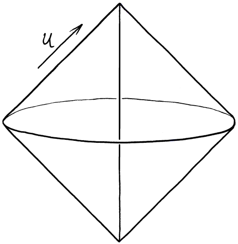
$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$



Electrodynamics at null infinity

Minkowski in **Bondi** coordinates $(u = t - r)$:

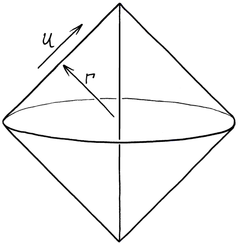
$$ds^2 = -\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\,h_{ab}\mathrm{d}\theta^a\mathrm{d}\theta^b$$



Electrodynamics at null infinity

Minkowski in **Bondi** coordinates $(u = t - r)$:

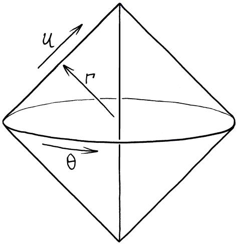
$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$



Electrodynamics at null infinity

Minkowski in **Bondi** coordinates $(u = t - r)$:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

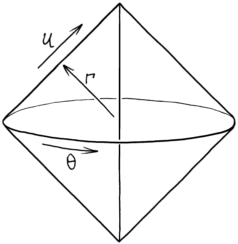


Electrodynamics at null infinity

Minkowski in **Bondi** coordinates $(u = t - r)$:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$

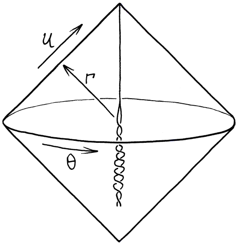


Electrodynamics at null infinity

Minkowski in **Bondi** coordinates $(u = t - r)$:

$$\mathrm{d}s^2 = -\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\,h_{ab}\mathrm{d}\theta^a\mathrm{d}\theta^b$$

- Null infinity at $r \to \infty$

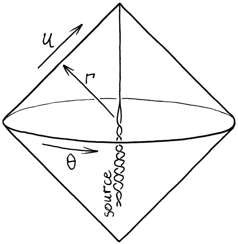


ELECTRODYNAMICS AT NULL INFINITY

Minkowski in **Bondi** coordinates $(u = t - r)$:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$

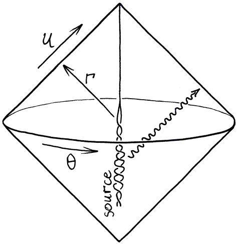


Electrodynamics at null infinity

Minkowski in **Bondi** coordinates $(u = t - r)$:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$

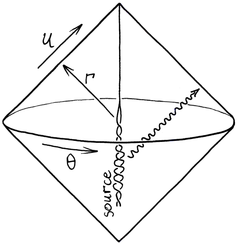


ELECTRODYNAMICS AT NULL INFINITY

Minkowski in **Bondi** coordinates $(u = t - r)$:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



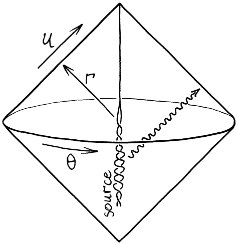
Electromagnetic field $\mathcal{A}$

Electrodynamics at null infinity

Minkowski in **Bondi** coordinates $(u = t - r)$:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



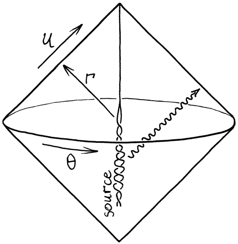
Electromagnetic field $\mathcal{A}$ [Radial gauge $\mathcal{A}_r \equiv 0$]

Electrodynamics at null infinity

Minkowski in **Bondi** coordinates ($u = t - r$) :

$$\mathrm{d}s^2 = -\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\, h_{ab}\mathrm{d}\theta^a\mathrm{d}\theta^b$$

- Null infinity at $r \to \infty$



Electromagnetic field $\mathcal{A}$

[Radial gauge $\mathcal{A}_r \equiv 0$]

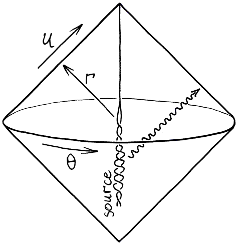
- $\mathcal{A} \sim C_a \mathrm{d}\theta^a + \frac{1}{r} D_a C^a + \cdots$

Electrodynamics at null infinity

Minkowski in **Bondi** coordinates $(u = t - r)$:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Electromagnetic field $\mathcal{A}$

[Radial gauge $\mathcal{A}_r \equiv 0$]

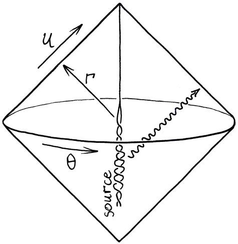
- $\mathcal{A} \sim C_a d\theta^a + \frac{1}{r} D_a C^a + \cdots$

ELECTRODYNAMICS AT NULL INFINITY

Minkowski in **Bondi** coordinates $(u = t - r)$:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab} d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Electromagnetic field $\mathcal{A}$

[Radial gauge $\mathcal{A}_r \equiv 0$]

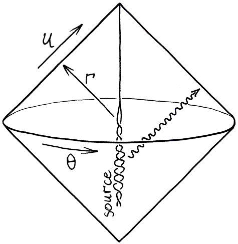
- $\mathcal{A} \sim C_a d\theta^a + \frac{1}{r} D_a C^a + \cdots$
- $C_a(u, \theta)$

Electrodynamics at null infinity

Minkowski in **Bondi** coordinates $(u = t - r)$:

$$\mathrm{d}s^2 = -\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\, h_{ab}\mathrm{d}\theta^a\mathrm{d}\theta^b$$

- Null infinity at $r \to \infty$



Electromagnetic field $\mathcal{A}$

[Radial gauge $\mathcal{A}_r \equiv 0$]

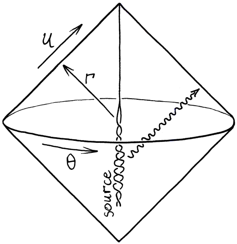
- $\mathcal{A} \sim C_a \mathrm{d}\theta^a + \frac{1}{r} D_a C^a + \cdots$
- $C_a(u, \theta) =$ electromagnetic boundary data

ELECTRODYNAMICS AT NULL INFINITY

Minkowski in **Bondi** coordinates $(u = t - r)$:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Electromagnetic field $\mathcal{A}$ [Radial gauge $\mathcal{A}_r \equiv 0$]

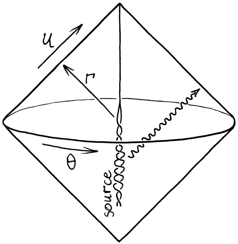
- $\mathcal{A} \sim C_a d\theta^a + \frac{1}{r} D_a C^a + \cdots$
- $C_a(u, \theta) =$ electromagnetic boundary data
- Radiation = **news** tensor $\partial_u C_a$

ELECTRODYNAMICS AT NULL INFINITY

Minkowski in **Bondi** coordinates $(u = t - r)$:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Electromagnetic field $\mathcal{A}$ [Radial gauge $\mathcal{A}_r \equiv 0$]

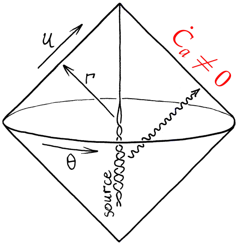
- $\mathcal{A} \sim C_a d\theta^a + \frac{1}{r} D_a C^a + \cdots$
- $C_a(u, \theta)$ = electromagnetic boundary data
- Radiation = **news** tensor $\partial_u C_a \equiv \dot{C}_a$

Electrodynamics at null infinity

Minkowski in **Bondi** coordinates $(u = t - r)$:

$$\mathrm{d}s^2 = -\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\,h_{ab}\mathrm{d}\theta^a\mathrm{d}\theta^b$$

- Null infinity at $r \to \infty$



Electromagnetic field $\mathcal{A}$ [Radial gauge $\mathcal{A}_r \equiv 0$]

- $\mathcal{A} \sim \mathbf{C_a}\mathrm{d}\theta^a + \frac{1}{r}D_aC^a + \cdots$
- $C_a(u,\theta) =$ electromagnetic boundary data
- Radiation = **news** tensor $\partial_u C_a \equiv \dot{C}_a$

Radiation causes precession

RADIATION CAUSES PRECESSION

Static observer with **magnetic dipole** $\mathbf{M}$

RADIATION CAUSES PRECESSION

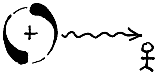
Static observer with **magnetic dipole** $\mathbf{M}$

Radiation causes precession

Static observer with **magnetic dipole** $\mathbf{M}$

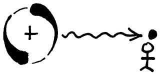
RADIATION CAUSES PRECESSION

Static observer with **magnetic dipole** $\mathbf{M}$



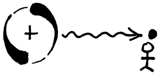
Radiation causes precession

Static observer with **magnetic dipole** $\mathbf{M}$



RADIATION CAUSES PRECESSION

Static observer with **magnetic dipole** $\mathbf{M}$

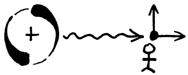


- Orientation in Cartesian frame e_1, e_2, e_3

RADIATION CAUSES PRECESSION

Static observer with **magnetic dipole** $\mathbf{M}$

- Orientation in Cartesian frame e_1, e_2, e_3



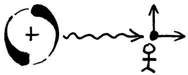
Radiation causes precession

Static observer with **magnetic dipole** $\mathbf{M} = M^i \, \boldsymbol{e}_i$

- Orientation in Cartesian frame $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$

RADIATION CAUSES PRECESSION

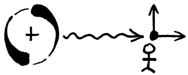
Static observer with **magnetic dipole** $\mathbf{M} = M^i \boldsymbol{e}_i$



- Orientation in Cartesian frame $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$
- Precession $\dot{\mathbf{M}} = \gamma \, \boldsymbol{B} \wedge \mathbf{M}$ [γ = gyromagnetic ratio]

Radiation causes precession

Static observer with **magnetic dipole** $\mathbf{M} = M^i \boldsymbol{e}_i$



- Orientation in Cartesian frame $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$
- Precession $\dot{\mathbf{M}} = \gamma \boldsymbol{B} \wedge \mathbf{M}$ [γ = gyromagnetic ratio]
- Solution = time-ordered exponential

Radiation causes precession

Static observer with **magnetic dipole** $\mathbf{M} = M^i \, \boldsymbol{e}_i$

- Orientation in Cartesian frame $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$
- Precession $\dot{\mathbf{M}} = \gamma \, \boldsymbol{B} \wedge \mathbf{M}$ [γ = gyromagnetic ratio]
- Solution = time-ordered exponential = expansion in $1/r$

Radiation causes precession

Static observer with **magnetic dipole** $\mathbf{M} = M^i \boldsymbol{e}_i$

- Orientation in Cartesian frame $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$
- Precession $\dot{\mathbf{M}} = \gamma \boldsymbol{B} \wedge \mathbf{M}$ [γ = gyromagnetic ratio]
- Solution = time-ordered exponential = expansion in $1/r$

Use **source-oriented frame**

Radiation causes precession

Static observer with **magnetic dipole** $\mathbf{M} = M^i \boldsymbol{e}_i$

- Orientation in Cartesian frame $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$
- Precession $\dot{\mathbf{M}} = \gamma \boldsymbol{B} \wedge \mathbf{M}$ [γ = gyromagnetic ratio]
- Solution = time-ordered exponential = expansion in $1/r$

Use **source-oriented frame** $\boldsymbol{e}_r, \{\boldsymbol{e}_a\}$

Radiation causes precession

Static observer with **magnetic dipole** $\mathbf{M} = M^i \boldsymbol{e}_i$

- Orientation in Cartesian frame $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$
- Precession $\dot{\mathbf{M}} = \gamma \boldsymbol{B} \wedge \mathbf{M}$ [γ = gyromagnetic ratio]
- Solution = time-ordered exponential = expansion in $1/r$

Use **source-oriented frame** $\boldsymbol{e}_r, \{\boldsymbol{e}_a\}$

- $\mathcal{F}_{ra}$ = precession in longitudinal plane [unimportant for us]

RADIATION CAUSES PRECESSION

Static observer with **magnetic dipole** $\mathbf{M} = M^i\, \boldsymbol{e}_i$

- Orientation in Cartesian frame $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$
- Precession $\dot{\mathbf{M}} = \gamma\, \boldsymbol{B} \wedge \mathbf{M}$ [γ = gyromagnetic ratio]
- Solution = time-ordered exponential = expansion in $1/r$

Use **source-oriented frame** $\boldsymbol{e}_r, \{\boldsymbol{e}_a\}$

- $\mathcal{F}_{ra}$ = precession in longitudinal plane [unimportant for us]
- $\mathcal{F}_{ab}$ + quadratic term = precession in transverse plane

Radiation causes precession

Static observer with **magnetic dipole** $\mathbf{M} = M^i\, \boldsymbol{e}_i$

- Orientation in Cartesian frame $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$
- Precession $\dot{\mathbf{M}} = \gamma\, \boldsymbol{B} \wedge \mathbf{M}$ [γ = gyromagnetic ratio]
- Solution = time-ordered exponential = expansion in $1/r$

Use **source-oriented frame** $\boldsymbol{e}_r, \{\boldsymbol{e}_a\}$

- $\mathcal{F}_{ra}$ = precession in longitudinal plane [unimportant for us]
- $\mathcal{F}_{ab}$ + quadratic term = precession in transverse plane :

$$\Omega_{\parallel} \sim \frac{\gamma}{2r^2}\Big(D_a\widetilde{C}^a - \gamma\,\dot{C}_a\widetilde{C}^a\Big)$$

Radiation causes precession

Static observer with **magnetic dipole** $\mathbf{M} = M^i \, \boldsymbol{e}_i$

- Orientation in Cartesian frame $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$
- Precession $\dot{\mathbf{M}} = \gamma \, \boldsymbol{B} \wedge \mathbf{M}$ [γ = gyromagnetic ratio]
- Solution = time-ordered exponential = expansion in $1/r$

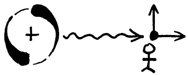
Use **source-oriented frame** $\boldsymbol{e}_r, \{\boldsymbol{e}_a\}$

- $\mathcal{F}_{ra}$ = precession in longitudinal plane [unimportant for us]
- $\mathcal{F}_{ab}$ + quadratic term = precession in transverse plane :

$$\Omega_{\|} \sim \frac{\gamma}{2r^2}\Big(D_a \widetilde{C}^a - \gamma \, \dot{C}_a \widetilde{C}^a\Big)$$

Radiation causes precession

Static observer with **magnetic dipole** $\mathbf{M} = M^i \, e_i$



- Orientation in Cartesian frame e_1, e_2, e_3
- Precession $\dot{\mathbf{M}} = \gamma \, \boldsymbol{B} \wedge \mathbf{M}$ [γ = gyromagnetic ratio]
- Solution = time-ordered exponential = expansion in $1/r$

Use **source-oriented frame** $e_r, \{e_a\}$

- $\mathcal{F}_{ra}$ = precession in longitudinal plane [unimportant for us]
- $\mathcal{F}_{ab}$ + quadratic term = precession in transverse plane :

$$\Omega_{\|} \sim \frac{\gamma}{2r^2}\left(D_a \widetilde{C}^a - \gamma \, \dot{C}_a \widetilde{C}^a\right)$$

Radiation causes precession

Static observer with **magnetic dipole** $\mathbf{M} = M^i\, e_i$

- Orientation in Cartesian frame e_1, e_2, e_3
- Precession $\dot{\mathbf{M}} = \gamma\, \boldsymbol{B} \wedge \mathbf{M}$ [γ = gyromagnetic ratio]
- Solution = time-ordered exponential = expansion in $1/r$

Use **source-oriented frame** $e_r, \{e_a\}$

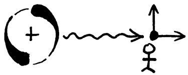
- $\mathcal{F}_{ra}$ = precession in longitudinal plane [unimportant for us]
- $\mathcal{F}_{ab}$ + quadratic term = precession in transverse plane :

$$\Omega_{\|} \sim \frac{\gamma}{2r^2}\left(D_a \widetilde{C}^a - \gamma\, \dot{C}_a \widetilde{C}^a\right)$$

with $\widetilde{C}_a = \epsilon_a{}^b\, C_b$ dual boundary data

RADIATION CAUSES PRECESSION

Static observer with **magnetic dipole** $\mathbf{M} = M^i\, e_i$



- Orientation in Cartesian frame e_1, e_2, e_3
- Precession $\dot{\mathbf{M}} = \gamma\, \boldsymbol{B} \wedge \mathbf{M}$ [γ = gyromagnetic ratio]
- Solution = time-ordered exponential = expansion in $1/r$

Use **source-oriented frame** $e_r, \{e_a\}$

- $\mathcal{F}_{ra}$ = precession in longitudinal plane [unimportant for us]
- $\mathcal{F}_{ab}$ + quadratic term = precession in transverse plane :

$$\Omega_{\|} \sim \frac{\gamma}{2r^2}\Big(D_a \widetilde{C}^a - \gamma\, \dot{C}_a \widetilde{C}^a\Big)$$

with $\widetilde{C}_a = \epsilon_a{}^b\, C_b$ dual boundary data

RADIATION CAUSES PRECESSION

Static observer with **magnetic dipole** $\mathbf{M} = M^i\,\boldsymbol{e}_i$

- Orientation in Cartesian frame $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$
- Precession $\dot{\mathbf{M}} = \gamma\,\boldsymbol{B} \wedge \mathbf{M}$ [γ = gyromagnetic ratio]
- Solution = time-ordered exponential = expansion in $1/r$

Use **source-oriented frame** $\boldsymbol{e}_r, \{\boldsymbol{e}_a\}$

- $\mathcal{F}_{ra}$ = precession in longitudinal plane [unimportant for us]
- $\mathcal{F}_{ab}$ + **quadratic term** = precession in transverse plane :

$$\Omega_{\parallel} \sim \frac{\gamma}{2r^2}\Big(D_a\widetilde{C}^a - \gamma\,\dot{C}_a\widetilde{C}^a\Big)$$

with $\widetilde{C}_a = \epsilon_a{}^b\,C_b$ dual boundary data

Radiation causes precession

$$\Omega_{\parallel} \sim \frac{\gamma}{2r^2}\left(D_a\widetilde{C}^a - \gamma\,\dot{C}_a\widetilde{C}^a\right)$$

Radiation causes precession

Orientation memory $= \int \mathrm{d}u\, \Omega_{\|}$

$$\Omega_{\|} \sim \frac{\gamma}{2r^2}\left(D_a\widetilde{C}^a - \gamma\, \dot{C}_a\widetilde{C}^a\right)$$

Radiation causes precession

Orientation memory $= \int \mathrm{d}u\, \Omega_{\|}$

- Involves $\int \mathrm{d}u\, \dot{C}_a \widetilde{C}^a$

$$\Omega_{\|} \sim \frac{\gamma}{2r^2}\left(D_a \widetilde{C}^a - \gamma\, \dot{C}_a \widetilde{C}^a\right)$$

Radiation causes precession

Orientation memory $= \int \mathrm{d}u\, \Omega_{\|}$

- Involves $\int \mathrm{d}u\, \dot{C}_a \widetilde{C}^a =$ canonical generator of celestial **electric-magnetic duality** $\delta C_a = \varepsilon(\theta)\, \widetilde{C}_a$

$$\Omega_{\|} \sim \frac{\gamma}{2r^2}\left(D_a \widetilde{C}^a - \gamma\, \dot{C}_a \widetilde{C}^a\right)$$

Radiation causes precession

Orientation memory $= \int \mathrm{d}u\,\Omega_{||}$

- Involves $\int \mathrm{d}u\,\dot{C}_a \widetilde{C}^a =$ canonical generator of celestial **electric-magnetic duality** $\delta C_a = \varepsilon(\theta)\,\widetilde{C}_a$

Occurs for **simple sources** !

$$\Omega_{||} \sim \frac{\gamma}{2r^2}\left(D_a\widetilde{C}^a - \gamma\,\dot{C}_a\widetilde{C}^a\right)$$

RADIATION CAUSES PRECESSION

Orientation memory $= \int \mathrm{d}u\, \Omega_{\parallel}$

- Involves $\int \mathrm{d}u\, \dot{C}_a \widetilde{C}^a =$ canonical generator of celestial **electric-magnetic duality** $\delta C_a = \varepsilon(\theta)\, \widetilde{C}_a$

Occurs for **simple sources** !

- Rotating point charge

$$\Omega_{\parallel} \sim \frac{\gamma}{2r^2} \left(D_a \widetilde{C}^a - \gamma\, \dot{C}_a \widetilde{C}^a \right)$$

Radiation causes precession

Orientation memory $= \int \mathrm{d}u\, \Omega_{\parallel}$

- Involves $\int \mathrm{d}u\, \dot{C}_a \widetilde{C}^a$ = canonical generator of celestial **electric-magnetic duality** $\delta C_a = \varepsilon(\theta)\, \widetilde{C}_a$

Occurs for **simple sources** !

- Rotating point charge : $D_a \widetilde{C}^a = 0$

$$\Omega_{\parallel} \sim \frac{\gamma}{2r^2}\left(D_a \widetilde{C}^a - \gamma\, \dot{C}_a \widetilde{C}^a\right)$$

Radiation causes precession

Orientation memory $= \int \mathrm{d}u\, \Omega_{||}$

- Involves $\int \mathrm{d}u\, \dot{C}_a \widetilde{C}^a$ = canonical generator of celestial **electric-magnetic duality** $\delta C_a = \varepsilon(\theta)\, \widetilde{C}_a$

Occurs for **simple sources** !

- Rotating point charge : $D_a \widetilde{C}^a = 0$ but $\dot{C}_a \widetilde{C}^a \sim \omega \frac{v^2}{c^2} \cos\theta$

$$\Omega_{||} \sim \frac{\gamma}{2r^2} \left(D_a \widetilde{C}^a - \gamma\, \dot{C}_a \widetilde{C}^a \right)$$

Radiation causes precession

Orientation memory $= \int \mathrm{d}u \, \Omega_{||}$

- Involves $\int \mathrm{d}u \, \dot{C}_a \widetilde{C}^a$ = canonical generator of celestial **electric-magnetic duality** $\delta C_a = \varepsilon(\theta) \, \widetilde{C}_a$

Occurs for **simple sources** !

- Rotating point charge : $D_a \widetilde{C}^a = 0$ but $\dot{C}_a \widetilde{C}^a \sim \omega \frac{v^2}{c^2} \cos\theta$
- Analogue for black hole mergers ?

$$\Omega_{||} \sim \frac{\gamma}{2r^2} \left(D_a \widetilde{C}^a - \gamma \, \dot{C}_a \widetilde{C}^a \right)$$

2. Frames Tied to Distant Stars

Metric at infinity

Metric at infinity

Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

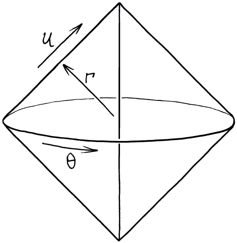
- Null infinity at $r \to \infty$

Metric at infinity

Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$

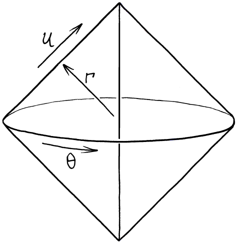


Metric at infinity

Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



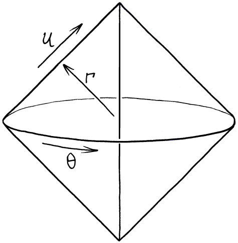
Observer sees **asymptotically flat** metric :

Metric at infinity

Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Observer sees **asymptotically flat** metric :

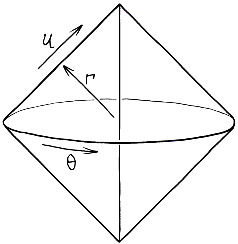
$$ds^2 \sim -(1 + \cdots)du^2$$

Metric at infinity

Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Observer sees **asymptotically flat** metric :

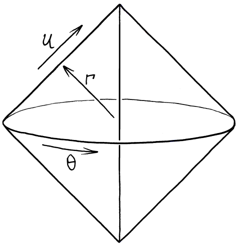
$$ds^2 \sim -(1 + \overset{\text{mass}}{\cdots})du^2$$

Metric at infinity

Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Observer sees **asymptotically flat** metric :

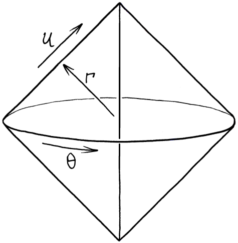
$$ds^2 \sim -(1 + \overset{\text{mass}}{\cdots})du^2 - (2 + \cdots)du\,dr$$

Metric at infinity

Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\, h_{ab} d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Observer sees **asymptotically flat** metric :

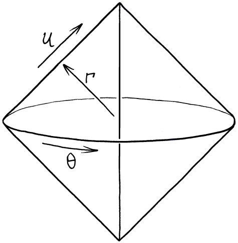
$$ds^2 \sim -(1 + \overset{\text{mass}}{\cdots})du^2 - (2 + \cdots)du\,dr + (\cdots)du\,d\theta^a$$

Metric at infinity

Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Observer sees **asymptotically flat** metric :

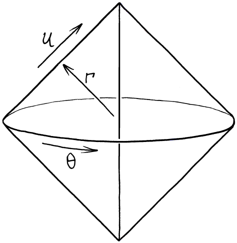
$$ds^2 \sim -(1 + \underbrace{\cdots}_{\text{mass}})du^2 - (2 + \cdots)du\,dr + (\underbrace{\cdots}_{\text{angular mom}})du\,d\theta^a$$

METRIC AT INFINITY

Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Observer sees **asymptotically flat** metric :

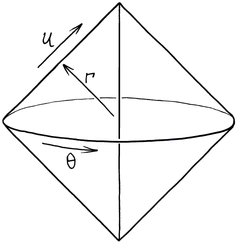
$$ds^2 \sim -(\overset{\text{mass}}{1 + \cdots})du^2 - (2 + \cdots)du\,dr + \overset{\text{angular mom}}{(\cdots)}du\,d\theta^a + (r^2 h_{ab} + r\,C_{ab} + \cdots)d\theta^a d\theta^b$$

METRIC AT INFINITY

Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Observer sees **asymptotically flat** metric :

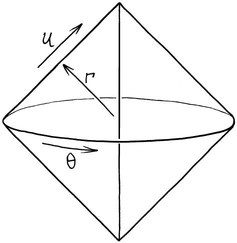
$$ds^2 \sim -(1 + \overset{\text{mass}}{\cdots})du^2 - (2 + \cdots)du\,dr + \overset{\text{angular mom}}{(\cdots)}du\,d\theta^a$$
$$+ (r^2 h_{ab} + r\,C_{ab} + \cdots)d\theta^a d\theta^b$$

METRIC AT INFINITY

Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Observer sees **asymptotically flat** metric :

$$ds^2 \sim -(1 + \overset{\text{mass}}{\cdots})du^2 - (2 + \cdots)du\,dr + \overset{\text{angular mom}}{(\cdots)}du\,d\theta^a$$
$$+ (r^2 h_{ab} + r\,C_{ab} + \cdots)d\theta^a d\theta^b$$

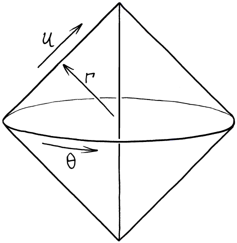
- $C_{ab}(u, \theta)$

Metric at infinity

Minkowski in Bondi coord $(u = t - r)$:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Observer sees **asymptotically flat** metric :

$$ds^2 \sim -(1 + \underbrace{\cdots}_{\text{mass}})du^2 - (2 + \cdots)du\,dr + (\underbrace{\cdots}_{\text{angular mom}})du\,d\theta^a + (r^2 h_{ab} + r\,C_{ab} + \cdots)d\theta^a d\theta^b$$

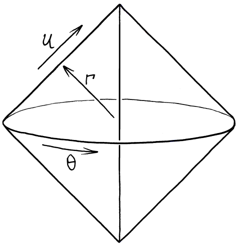
- $C_{ab}(u,\theta)$ = aspt **shear** of null rays

Metric at infinity

Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Observer sees **asymptotically flat** metric :

$$ds^2 \sim -(1 + \underbrace{\cdots}_{\text{mass}})du^2 - (2 + \cdots)du\,dr + (\underbrace{\cdots}_{\text{angular mom}})du\,d\theta^a + (r^2 h_{ab} + r\,C_{ab} + \cdots)d\theta^a d\theta^b$$

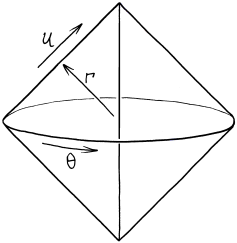
- $C_{ab}(u, \theta)$ = aspt **shear** of null rays
- Radiation = **news** tensor $\partial_u C_{ab}$

Metric at infinity

Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$



Observer sees **asymptotically flat** metric :

$$ds^2 \sim -\big(1 + \underbrace{\cdots}_{\text{mass}}\big)du^2 - \big(2 + \cdots\big)du\,dr + \big(\underbrace{\cdots}_{\text{angular mom}}\big)du\,d\theta^a + \big(r^2 h_{ab} + r\,C_{ab} + \cdots\big)d\theta^a d\theta^b$$

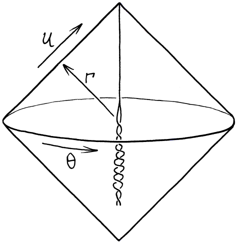
- $C_{ab}(u,\theta)$ = aspt **shear** of null rays
- Radiation = **news** tensor $\partial_u C_{ab} \equiv \dot{C}_{ab}$

Metric at infinity

Minkowski in Bondi coord ($u = t - r$) :

$$\mathrm{d}s^2 = -\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\, h_{ab}\mathrm{d}\theta^a \mathrm{d}\theta^b$$

- Null infinity at $r \to \infty$

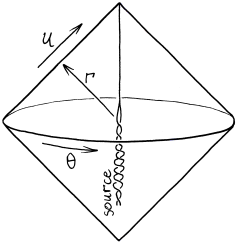


Observer sees **asymptotically flat** metric :

$$\mathrm{d}s^2 \sim -\big(1 + \underbrace{\cdots}_{\text{mass}}\big)\mathrm{d}u^2 - \big(2 + \cdots\big)\mathrm{d}u\,\mathrm{d}r + \big(\underbrace{\cdots}_{\text{angular mom}}\big)\mathrm{d}u\,\mathrm{d}\theta^a + \big(r^2 h_{ab} + r\, C_{ab} + \cdots\big)\mathrm{d}\theta^a \mathrm{d}\theta^b$$

- $C_{ab}(u,\theta)$ = aspt **shear** of null rays
- Radiation = **news** tensor $\partial_u C_{ab} \equiv \dot{C}_{ab}$

Metric at infinity



Minkowski in Bondi coord $(u = t - r)$:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

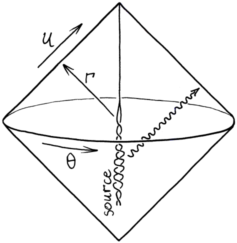
- Null infinity at $r \to \infty$

Observer sees **asymptotically flat** metric :

$$ds^2 \sim -(1 + \underbrace{\cdots}_{\text{mass}})du^2 - (2 + \cdots)du\,dr + (\underbrace{\cdots}_{\text{angular mom}})du\,d\theta^a + (r^2 h_{ab} + r\,C_{ab} + \cdots)d\theta^a d\theta^b$$

- $C_{ab}(u, \theta)$ = aspt **shear** of null rays
- Radiation = **news** tensor $\partial_u C_{ab} \equiv \dot{C}_{ab}$

Metric at infinity



Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

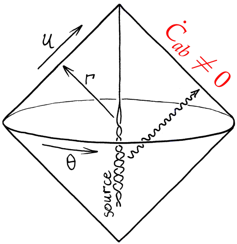
- Null infinity at $r \to \infty$

Observer sees **asymptotically flat** metric :

$$ds^2 \sim -(1 + \overset{\text{mass}}{\cdots})du^2 - (2 + \cdots)du\,dr + (\overset{\text{angular mom}}{\cdots})du\,d\theta^a + (r^2 h_{ab} + r\,C_{ab} + \cdots)d\theta^a d\theta^b$$

- $C_{ab}(u, \theta)$ = aspt **shear** of null rays
- Radiation = **news** tensor $\partial_u C_{ab} \equiv \dot{C}_{ab}$

Metric at infinity



Minkowski in Bondi coord ($u = t - r$) :

$$ds^2 = -du^2 - 2\,du\,dr + r^2\,h_{ab}d\theta^a d\theta^b$$

- Null infinity at $r \to \infty$

Observer sees **asymptotically flat** metric :

$$ds^2 \sim -(\underbrace{1 + \cdots}_{\text{mass}})du^2 - (2 + \cdots)du\,dr + (\underbrace{\cdots}_{\text{angular mom}})du\,d\theta^a + (r^2 h_{ab} + r\,C_{ab} + \cdots)d\theta^a d\theta^b$$

- $C_{ab}(u, \theta)$ = aspt **shear** of null rays
- Radiation = **news** tensor $\partial_u C_{ab} \equiv \dot{C}_{ab}$

Local Frame

Local Frame

Freely falling observer

Local Frame

Freely falling observer

Local Frame

Freely falling observer

Local Frame

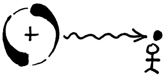
Freely falling observer

Local Frame

Freely falling observer with gyroscope

LOCAL FRAME

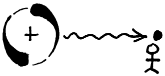
Freely falling observer with gyroscope



Local Frame

Freely falling observer with gyroscope

Measure orientation ?



Local Frame

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**

Local Frame

Freely falling observer with gyroscope

Measure orientation :

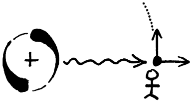
- Build **source-oriented frame**

Local Frame

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**

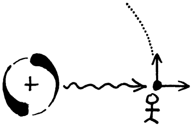


Local Frame

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**

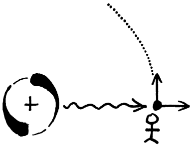


Local Frame

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**

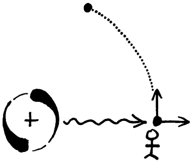


Local Frame

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**

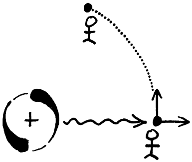


LOCAL FRAME

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**

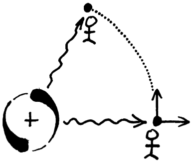


Local Frame

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**

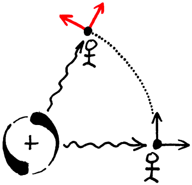


Local Frame

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**

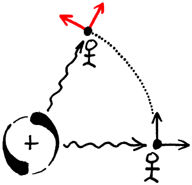


Local Frame

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !



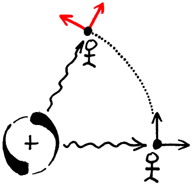
LOCAL FRAME

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead



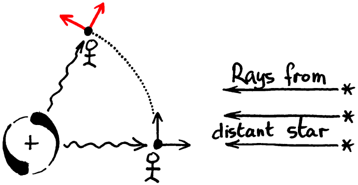
Local Frame

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead



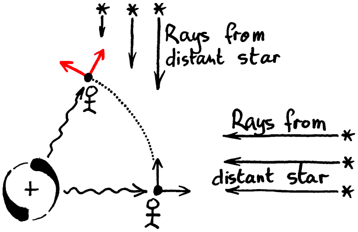
Local Frame

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead



LOCAL FRAME

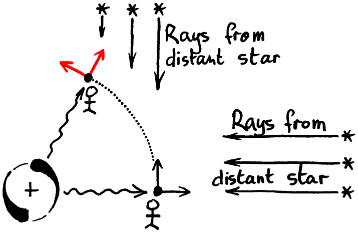
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



Local Frame

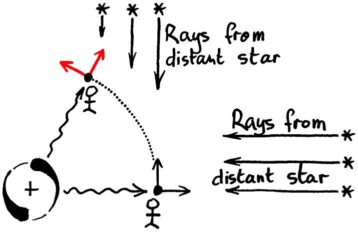
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



Local Frame

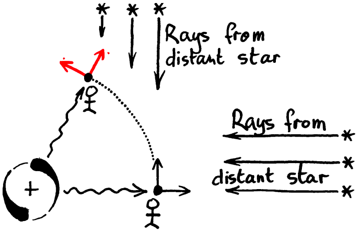
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



Local Frame

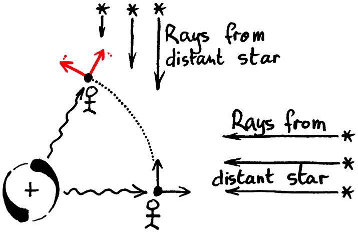
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



Local Frame

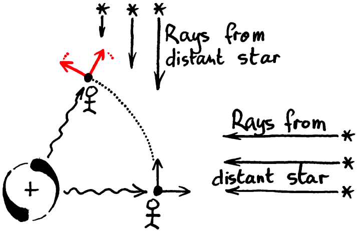
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



Local Frame

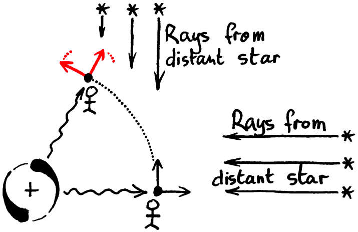
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



Local Frame

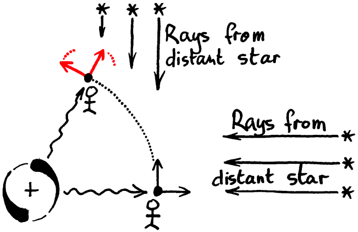
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



LOCAL FRAME

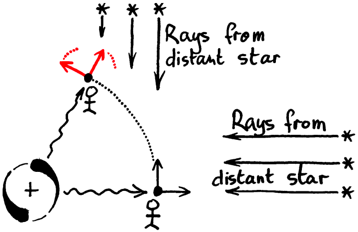
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



Local Frame

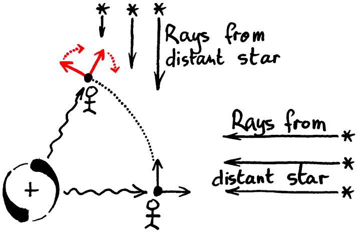
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



Local Frame

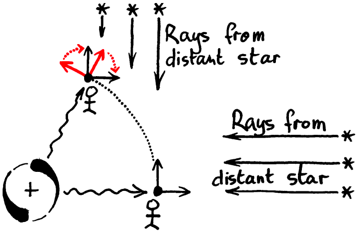
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



Local Frame

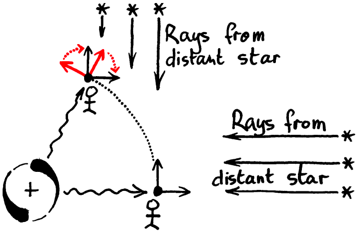
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



Local Frame

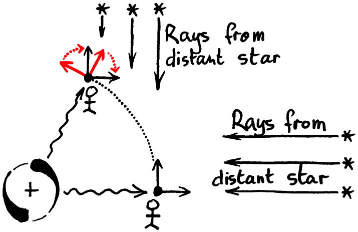
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



Local Frame

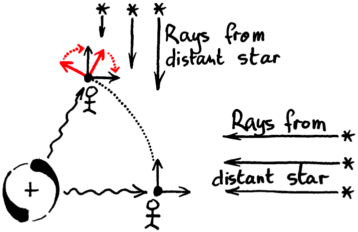
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



Local Frame

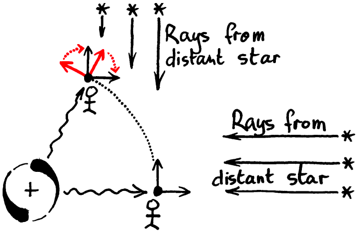
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation



Local Frame

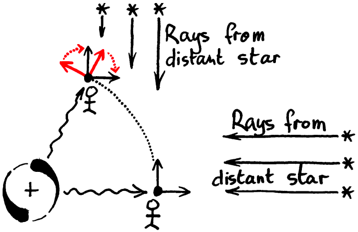
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation
- Frame e_1, e_2, e_3



Local Frame

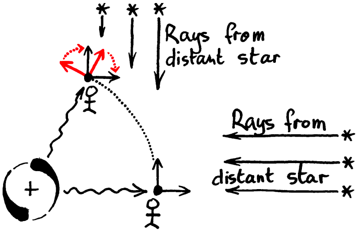
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation
- Frame e_1, e_2, e_3
- Gyroscope's spin $\mathbf{S} = S^i\, e_i$

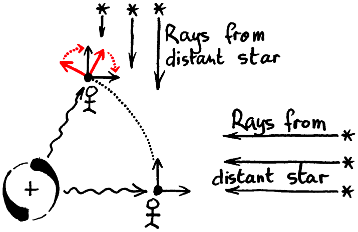


Local Frame

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !



Choose **distant stars** instead

- Compensating rotation
- Frame $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$
- Gyroscope's spin $\mathbf{S} = S^i\, \boldsymbol{e}_i$

Parallel transport $\leftrightarrow$ **precession** $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

Local Frame

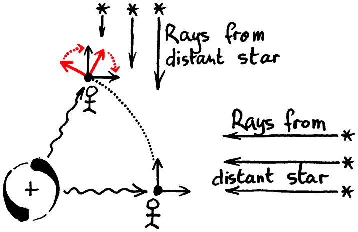
Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !

Choose **distant stars** instead

- Compensating rotation
- Frame $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$
- Gyroscope's spin $\mathbf{S} = S^i \boldsymbol{e}_i$



Parallel transport $\leftrightarrow$ **precession** $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

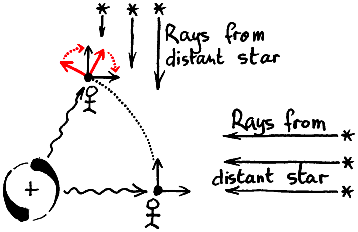
- **Find $\boldsymbol{\Omega}$!**

Local Frame

Freely falling observer with gyroscope

Measure orientation :

- Build **source-oriented frame**
- Spurious precession !



Choose **distant stars** instead

- Compensating rotation
- Frame e_1, e_2, e_3
- Gyroscope's spin $\mathbf{S} = S^i\, e_i$

Parallel transport $\leftrightarrow$ **precession** $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- **Find $\boldsymbol{\Omega}$** as fct of radiative data C_{ab}

3. Orientation Memory in Gravity

Radiation Causes Precession

Radiation Causes Precession

Parallel transport $\leftrightarrow$ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

Radiation Causes Precession

Parallel transport $\leftrightarrow$ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

Radiation Causes Precession

Parallel transport $\leftrightarrow$ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection

Radiation Causes Precession

Parallel transport ↔ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection :

$$\Omega_\perp \sim \mathcal{O}(r^{-3})$$

Radiation Causes Precession

Parallel transport $\leftrightarrow$ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection :

$$\Omega_\perp \sim \mathcal{O}(r^{-3}), \qquad \Omega_\parallel \sim \frac{1}{r^2}\left(D_a D_b \widetilde{C}^{ab} - \tfrac{1}{2}\dot{C}_{ab}\widetilde{C}^{ab}\right)$$

Radiation Causes Precession

Parallel transport $\leftrightarrow$ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection :

$$\Omega_\perp \sim \mathcal{O}(r^{-3}), \qquad \Omega_\| \sim \frac{1}{r^2}\left(D_a D_b \widetilde{C}^{ab} - \tfrac{1}{2}\dot{C}_{ab}\widetilde{C}^{ab}\right)$$

Radiation Causes Precession

Parallel transport $\leftrightarrow$ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection :

$$\Omega_{\perp} \sim \mathcal{O}(r^{-3}), \qquad \Omega_{\parallel} \sim \frac{1}{r^2}\left(D_a D_b \widetilde{C}^{ab} - \frac{1}{2}\dot{C}_{ab}\widetilde{C}^{ab}\right)$$

- Precession along ray axis

Radiation Causes Precession

Parallel transport $\leftrightarrow$ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection :

$$\Omega_\perp \sim \mathcal{O}(r^{-3}), \qquad \Omega_\parallel \sim \frac{1}{r^2}\left(D_a D_b \widetilde{C}^{ab} - \tfrac{1}{2}\dot{C}_{ab}\widetilde{C}^{ab}\right)$$

- Precession along ray axis
- Linear term

Radiation Causes Precession

Parallel transport $\leftrightarrow$ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection :

$$\Omega_\perp \sim \mathcal{O}(r^{-3}), \qquad \Omega_\parallel \sim \frac{1}{r^2}\left(D_a D_b \widetilde{C}^{ab} - \frac{1}{2}\dot{C}_{ab}\widetilde{C}^{ab}\right)$$

- Precession along ray axis
- Linear term

Radiation Causes Precession

Parallel transport $\leftrightarrow$ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection :

$$\Omega_{\perp} \sim \mathcal{O}(r^{-3}), \qquad \Omega_{\parallel} \sim \frac{1}{r^2}\left(D_a D_b \widetilde{C}^{ab} - \frac{1}{2}\dot{C}_{ab}\widetilde{C}^{ab}\right)$$

- Precession along ray axis
- Linear term has dual shear $\widetilde{C}_{ab} \equiv \epsilon_{ca} C_b{}^c$

Radiation Causes Precession

Parallel transport $\leftrightarrow$ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection :

$$\Omega_\perp \sim \mathcal{O}(r^{-3}), \qquad \Omega_\parallel \sim \frac{1}{r^2}\left(D_a D_b \widetilde{C}^{ab} - \tfrac{1}{2}\dot{C}_{ab}\widetilde{C}^{ab} \right)$$

- Precession along ray axis
- Linear term has dual shear $\widetilde{C}_{ab} \equiv \epsilon_{ca} C_b{}^c$
- Quadratic term

Radiation Causes Precession

Parallel transport $\leftrightarrow$ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection :

$$\Omega_{\perp} \sim \mathcal{O}(r^{-3}), \qquad \Omega_{\parallel} \sim \frac{1}{r^2}\left(D_a D_b \widetilde{C}^{ab} - \frac{1}{2}\dot{C}_{ab}\widetilde{C}^{ab}\right)$$

- Precession along ray axis
- Linear term has dual shear $\widetilde{C}_{ab} \equiv \epsilon_{ca} C_b{}^c$
- Quadratic term

Radiation Causes Precession

Parallel transport $\leftrightarrow$ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection :

$$\Omega_\perp \sim \mathcal{O}(r^{-3}), \qquad \Omega_\parallel \sim \frac{1}{r^2}\left(D_a D_b \widetilde{C}^{ab} - \frac{1}{2}\dot{C}_{ab}\widetilde{C}^{ab}\right)$$

- Precession along ray axis
- Linear term has dual shear $\widetilde{C}_{ab} \equiv \epsilon_{ca} C_b{}^c$
- Quadratic term has news $\dot{C}_{ab}$

Radiation Causes Precession

Parallel transport $\leftrightarrow$ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection :

$$\Omega_{\perp} \sim \mathcal{O}(r^{-3}), \qquad \Omega_{\parallel} \sim \frac{1}{r^2}\left(D_a D_b \widetilde{C}^{ab} - \frac{1}{2}\dot{C}_{ab}\widetilde{C}^{ab}\right)$$

- Precession along ray axis
- Linear term has dual shear $\widetilde{C}_{ab} \equiv \epsilon_{ca} C_b{}^c$
- Quadratic term has news $\dot{C}_{ab}$
- **As in electrodynamics** !

Radiation Causes Precession

Parallel transport ↔ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection :

$$\Omega_\perp \sim \mathcal{O}(r^{-3}), \qquad \Omega_\parallel \sim \frac{1}{r^2}\left(D_a D_b \widetilde{C}^{ab} - \tfrac{1}{2}\dot{C}_{ab}\widetilde{C}^{ab}\right)$$

- Precession along ray axis
- Linear term has dual shear $\widetilde{C}_{ab} \equiv \epsilon_{ca} C_b{}^c$
- Quadratic term has news $\dot{C}_{ab}$
- **As in electrodynamics** ! [But universal by equivalence principle]

Radiation Causes Precession

Parallel transport ↔ precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$

- $\boldsymbol{\Omega}$ found from spin connection :

$$\Omega_\perp \sim \mathcal{O}(r^{-3}), \qquad \Omega_\parallel \sim \frac{1}{r^2}\Big(D_a D_b \widetilde{C}^{ab} - \tfrac{1}{2}\dot{C}_{ab}\widetilde{C}^{ab}\Big)$$

- Precession along ray axis
- Linear term has dual shear $\widetilde{C}_{ab} \equiv \epsilon_{ca} C_b{}^c$
- Quadratic term has news $\dot{C}_{ab}$
- **As in electrodynamics !** [But universal by equivalence principle]

$DD\widetilde{C} - \frac{1}{2}\dot{C}\widetilde{C} =$ **dual mass aspect** [Porrati+, Godazgar+, Freidel+,...]

Orientation Memory

Orientation Memory

Precession $\dot{\mathbf{S}} = \mathbf{\Omega} \times \mathbf{S}$ during radiation burst

Orientation Memory

Precession $\dot{\mathbf{S}} = \mathbf{\Omega} \times \mathbf{S}$ during radiation burst

- Net orientation change

Orientation Memory

Precession $\dot{\mathbf{S}} = \mathbf{\Omega} \times \mathbf{S}$ during radiation burst

- Net orientation change $\Delta\mathbf{S} \sim \int \mathrm{d}u\, \mathbf{\Omega} \times \mathbf{S}_{\text{initial}}$

Orientation Memory

Precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$ during radiation burst

- Net orientation change $\Delta \mathbf{S} \sim \int \mathrm{d}u \, \boldsymbol{\Omega} \times \mathbf{S}_{\text{initial}}$
- Orientation memory $= \int \mathrm{d}u \, \Omega_{||}$

Orientation Memory

Precession $\dot{\mathbf{S}} = \mathbf{\Omega} \times \mathbf{S}$ during radiation burst

- Net orientation change $\Delta \mathbf{S} \sim \int du\, \mathbf{\Omega} \times \mathbf{S}_{\text{initial}}$
- Orientation memory $= \int du\, \Omega_{\|}$
- Involves $\int du\, \dot{C}_{ab} \widetilde{C}^{ab}$

Orientation Memory

Precession $\dot{\mathbf{S}} = \mathbf{\Omega} \times \mathbf{S}$ during radiation burst

- Net orientation change $\Delta\mathbf{S} \sim \int \mathrm{d}u\, \mathbf{\Omega} \times \mathbf{S}_{\text{initial}}$
- Orientation memory $= \int \mathrm{d}u\, \Omega_{\|}$
- Involves $\int \mathrm{d}u\, \dot{C}_{ab}\widetilde{C}^{ab}$ = canonical generator of celestial **electric-magnetic duality** $\delta C_{ab} = \varepsilon(\theta)\, \widetilde{C}_{ab}$

Orientation Memory

Precession $\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$ during radiation burst

- Net orientation change $\Delta\mathbf{S} \sim \int \mathrm{d}u\, \boldsymbol{\Omega} \times \mathbf{S}_{\text{initial}}$
- Orientation memory $= \int \mathrm{d}u\, \Omega_{\|}$
- Involves $\int \mathrm{d}u\, \dot{C}_{ab}\widetilde{C}^{ab}$ = canonical generator of celestial **electric-magnetic duality** $\delta C_{ab} = \varepsilon(\theta)\, \widetilde{C}_{ab}$

As in electrodynamics again !

SUMMARY

Summary

Free **gyroscopes** react to **gravitational waves**

SUMMARY

Free **gyroscopes** react to **gravitational waves** by changing **orientation** wrt faraway stars

SUMMARY

Free **gyroscopes** react to **gravitational waves** by changing **orientation** wrt faraway stars

- Compute spin connection in Bondi metric

Summary

Free **gyroscopes** react to **gravitational waves** by changing **orientation** wrt faraway stars

- Compute spin connection in Bondi metric
- Involves gravitomagnetic "dual" currents

Summary

Free **gyroscopes** react to **gravitational waves** by changing **orientation** wrt faraway stars

- Compute spin connection in Bondi metric
- Involves gravitomagnetic "dual" currents

Order of magnitude $\Phi \sim 10^{-39} \left(\frac{M/M_\odot}{r/1\,\mathrm{Mpc}} \right)^2$

SUMMARY

Free **gyroscopes** react to **gravitational waves**
by changing **orientation** wrt faraway stars

- Compute spin connection in Bondi metric
- Involves gravitomagnetic "dual" currents

Order of magnitude $\Phi \sim 10^{-39} \left(\frac{M/M_\odot}{r/1\,\text{Mpc}} \right)^2$

- Observable for supermassive black hole mergers ?

SUMMARY

Free **gyroscopes** react to **gravitational waves** by changing **orientation** wrt faraway stars

- Compute spin connection in Bondi metric
- Involves gravitomagnetic "dual" currents

Order of magnitude $\Phi \sim 10^{-39} \left(\frac{M/M_{\odot}}{r/1\,\mathrm{Mpc}} \right)^2$

- Observable for supermassive black hole mergers ?
- To be continued !

Merci !