Integrability and Form Factors

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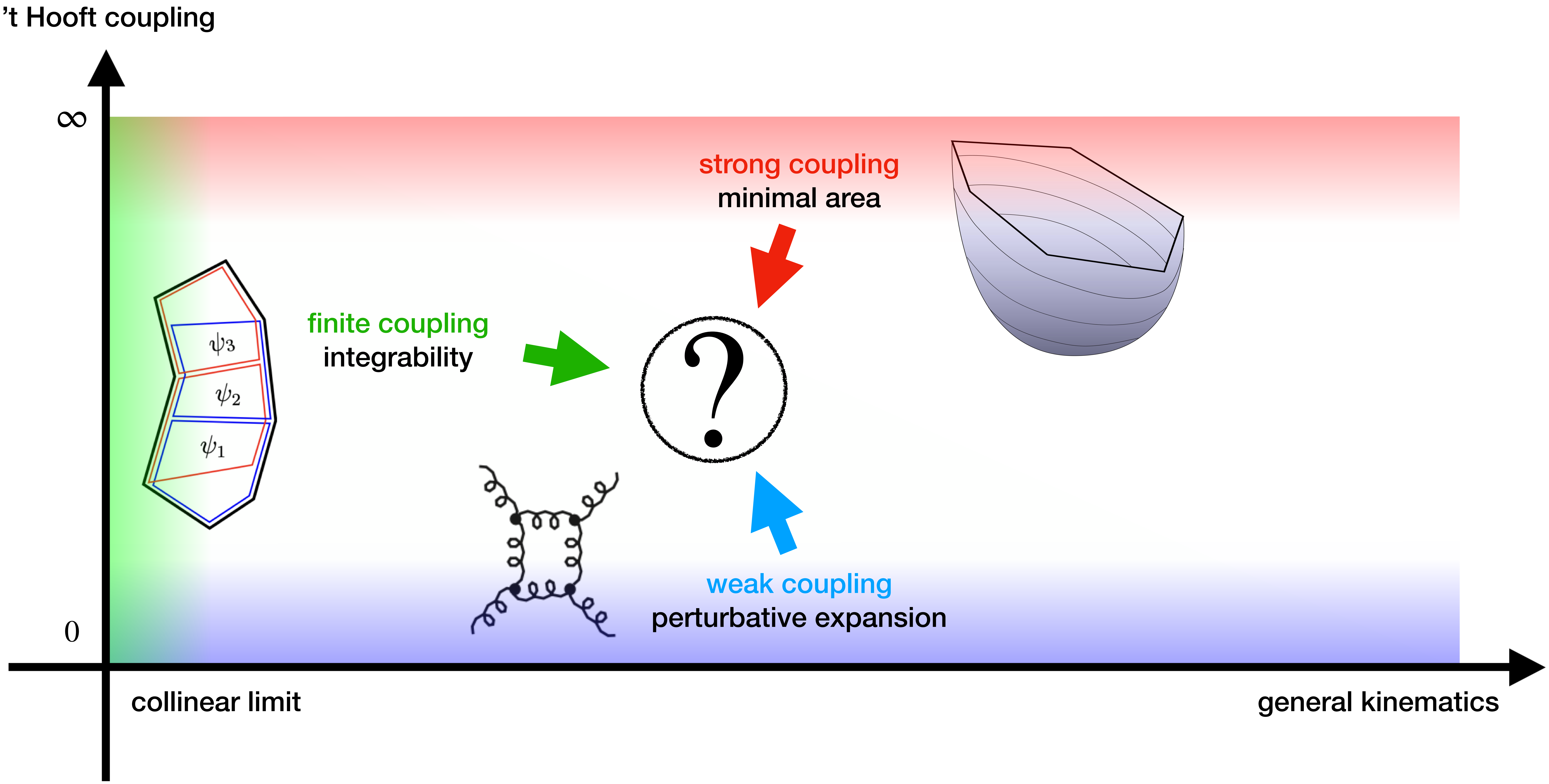
Based on [2009.11297, 2105.13367, 2112.10569] with **Amit Sever** and **Matthias Wilhelm**
and three upcoming papers with **Benjamin Basso** and **Lance Dixon**

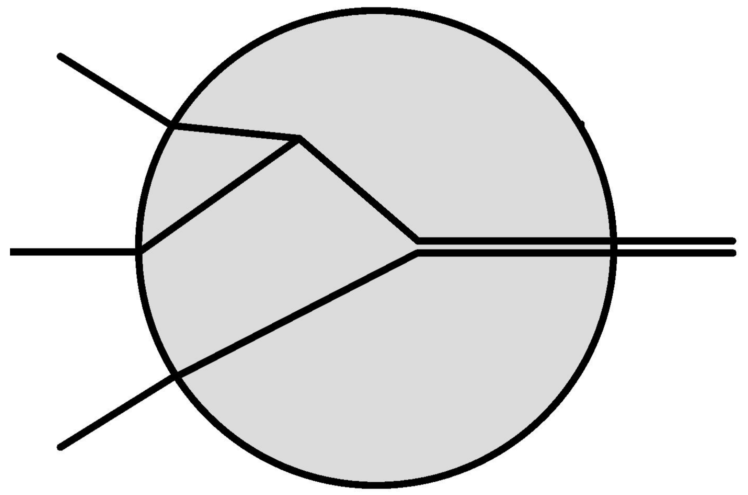
LAPTh, 23.05.23

Can we compute scattering amplitudes non-perturbatively?



Form Factors

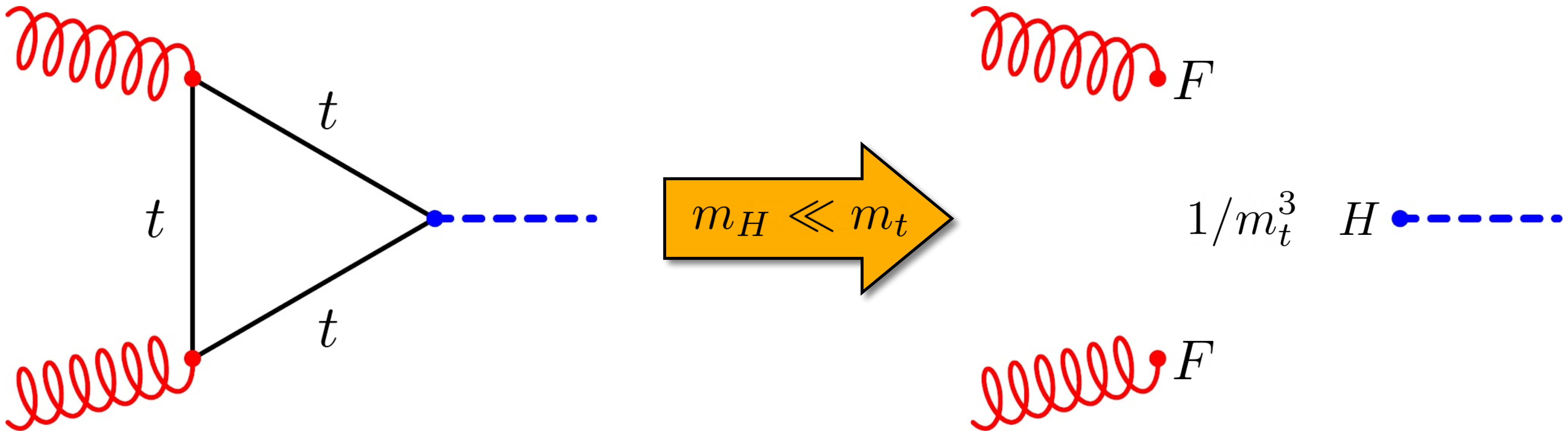
$$F_{\mathcal{O}}(k_1, \ldots, k_n) = \langle k_1, \ldots, k_n | \mathcal{O}(q) | 0 \rangle$$



The simplest non-trivial amplitude is **6pt** and parametrised by **3 dual conformal cross-ratios**

The simplest non-trivial form factor is **3pt** and parametrised by **2 dual conformal cross-ratios**

Experimental significance: deep inelastic scattering and Higgs production



$$\langle g, \ldots, g | H \rangle \qquad \langle g, \ldots, g | \mathrm{Tr}\{F^2\} | 0 \rangle$$

Operator dependence

Protected operators (BPS)

Unprotected operators

1/2-BPS multiplets:

$$\mathcal{O} = \mathrm{Tr}\left\{\left(\Phi^{++}\right)^2\right\} \longleftrightarrow \mathrm{Tr}\left\{\mathcal{L}\right\}$$

$$\mathcal{O} = \mathrm{Tr}\left\{\left(\Phi^{++}\right)^{2+\ell}\right\} \longleftrightarrow \mathrm{Tr}\left\{\mathcal{L}\left(\Phi^{++}\right)^{\ell}\right\}$$

Konishi multiplet:

$$\mathcal{O} = \delta^{IJ}\,\mathrm{Tr}\{\Phi_I\Phi_J\}$$

$\mathrm{Tr}\,F^n$ multiplets:

$$\mathcal{O} = \mathrm{Tr}\left\{F^{2+\ell}\right\} \longleftrightarrow \mathrm{Tr}\left\{\mathcal{L}\,F^{\ell}\right\}$$

any ℓ

[Basso, AT to appear]

$\ell = 0$

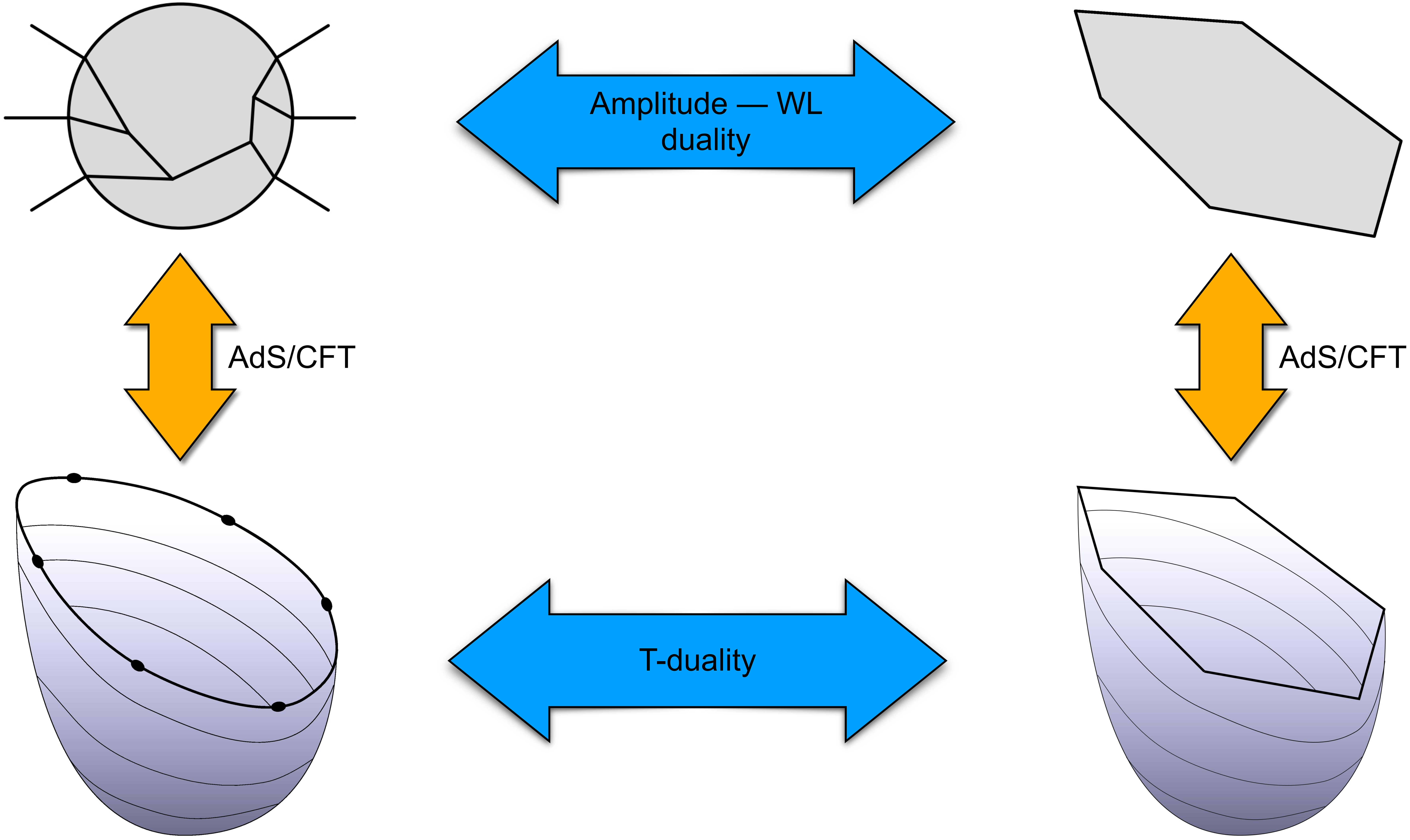
[Sever, AT, Wilhelm ’20 - ’21]

?

Form Factor Operator Product Expansion

Amplitude — Wilson Loop duality

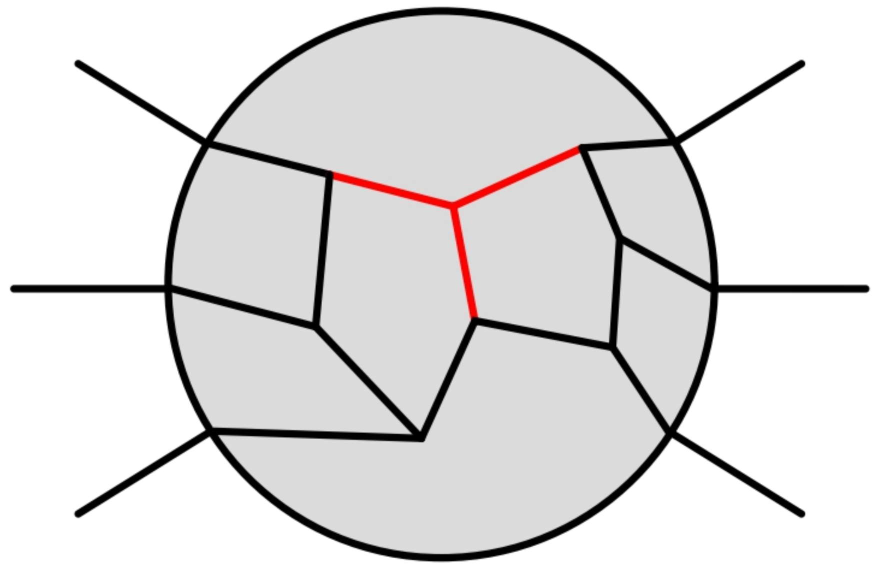
[Alday, Maldacena ’07]



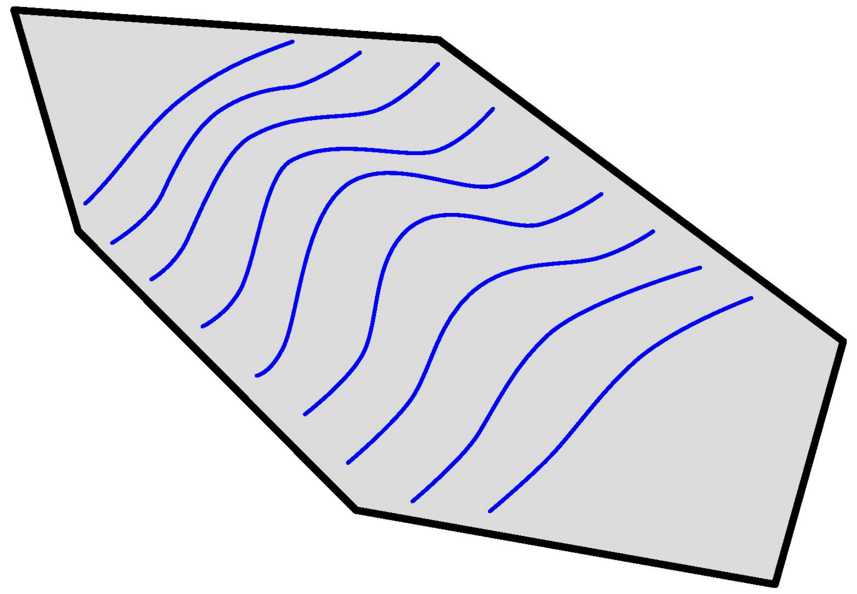
Wilson Loop OPE

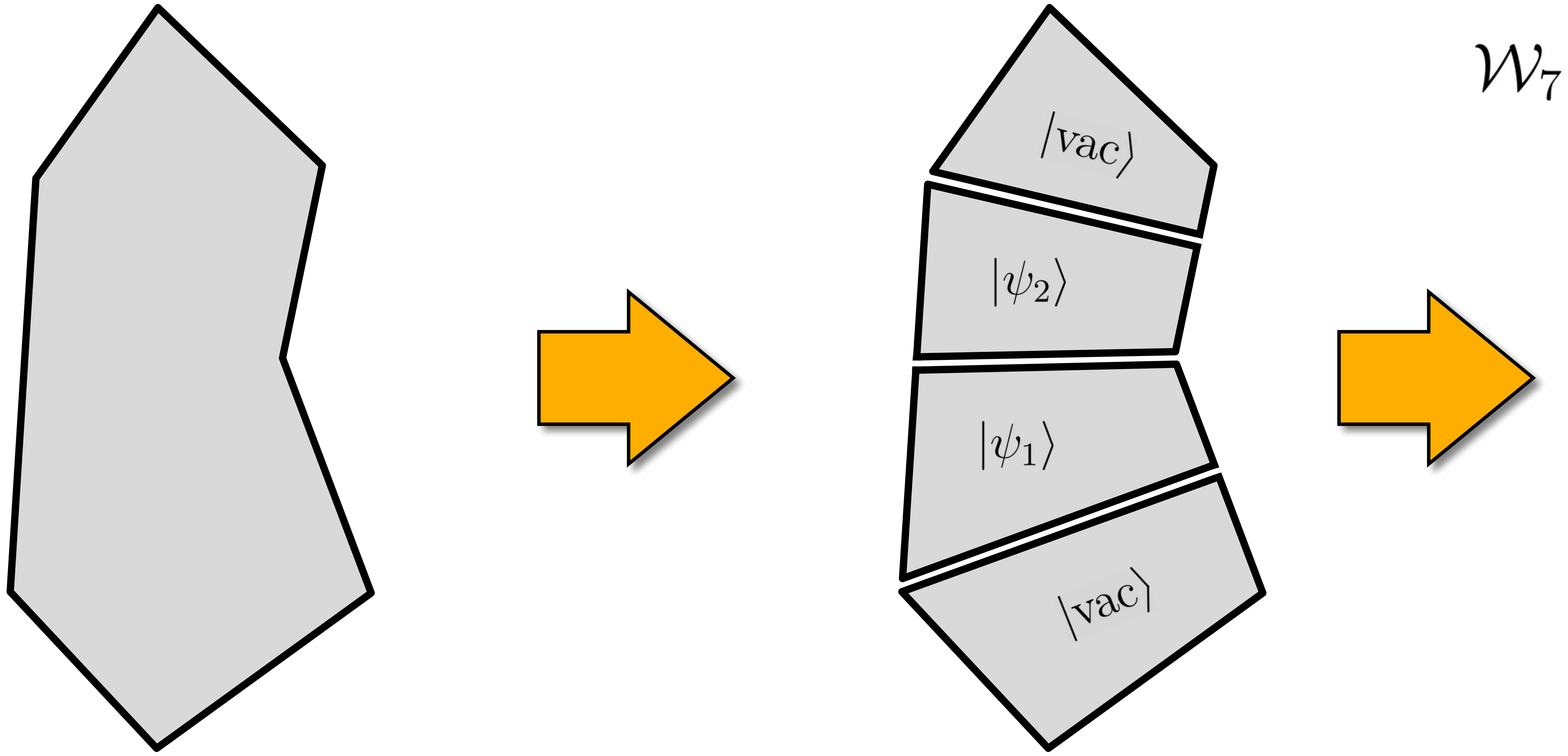
[Alday, Gaiotto, Maldacena, Sever, Vieira '11]

$$\sum_{L=0}^{\infty} g^{2L}$$



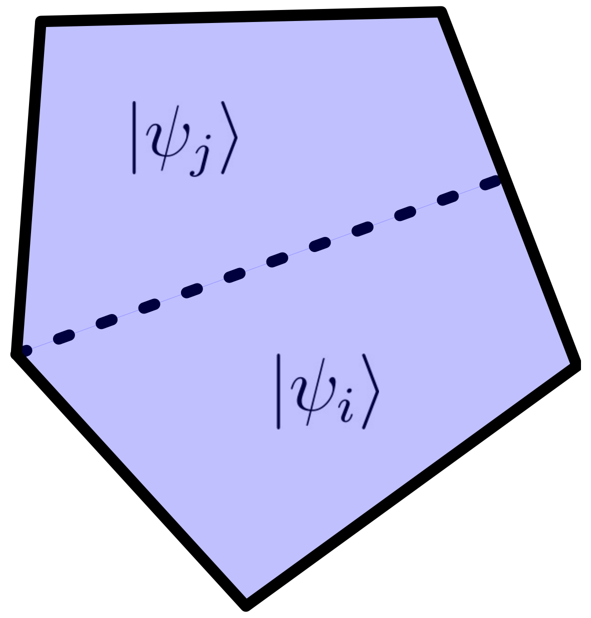
$$= \sum_{\psi} e^{-E(\psi)\tau}$$





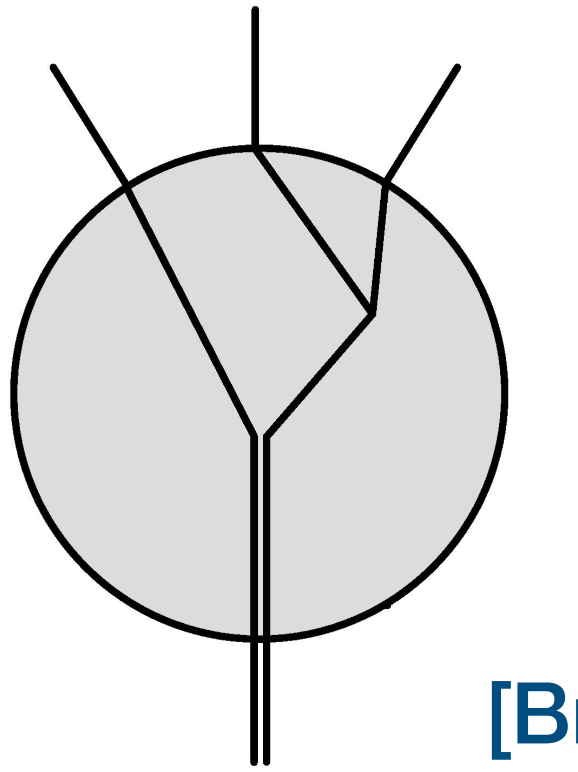
$$\mathcal{W}_7 \sim \sum_{\psi_i} P(0|\psi_1)\, P(\psi_1|\psi_2)\, P(\psi_2|0)$$

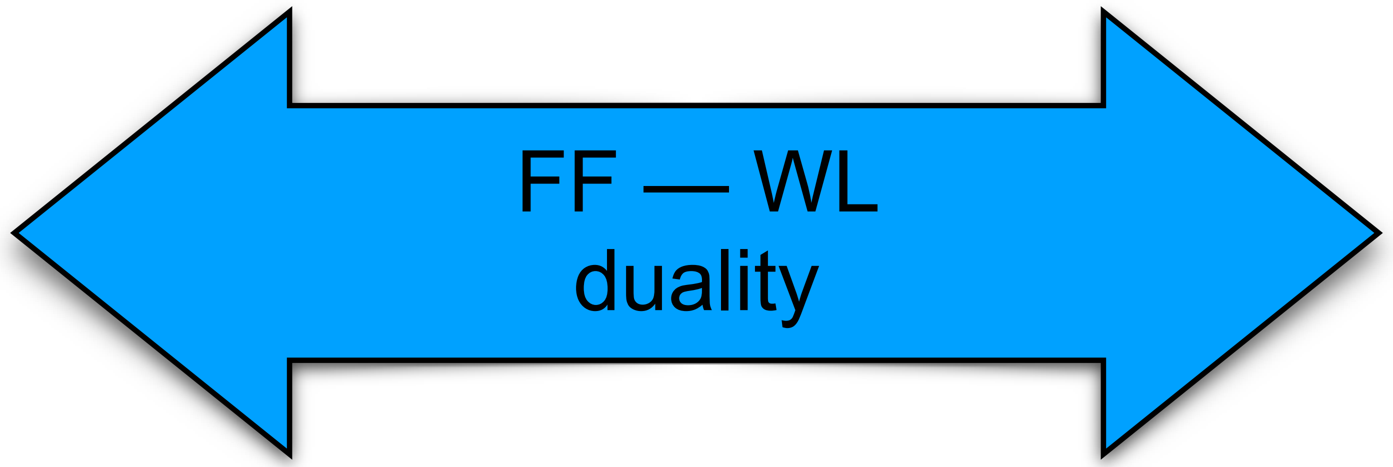
Pentagon transition

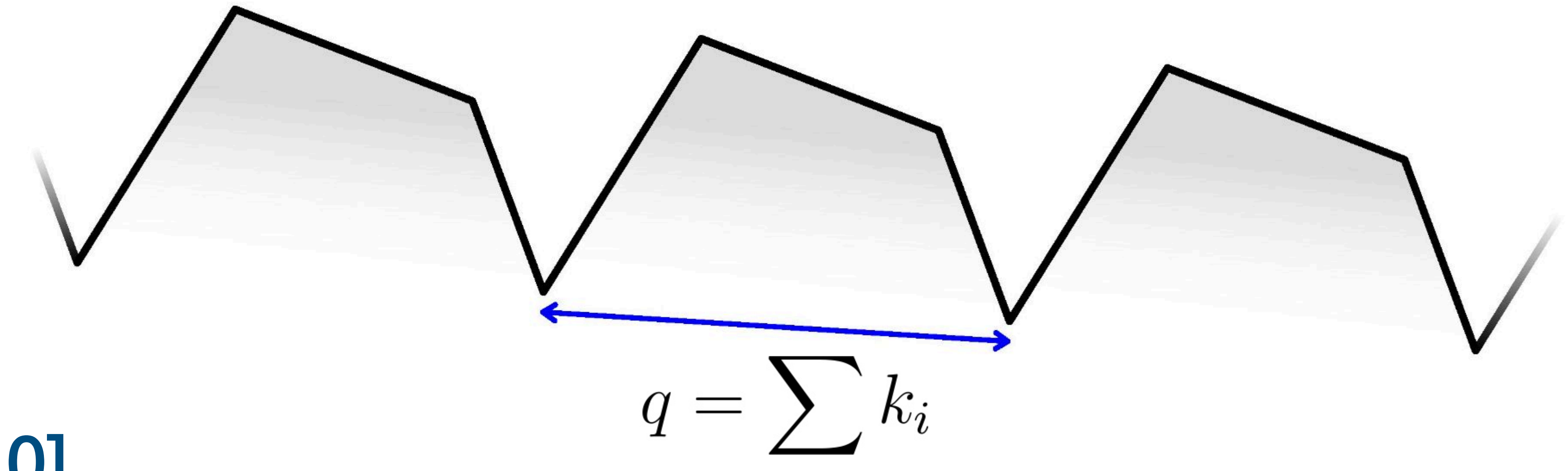


[Basso, Sever, Vieira '13 - '14]

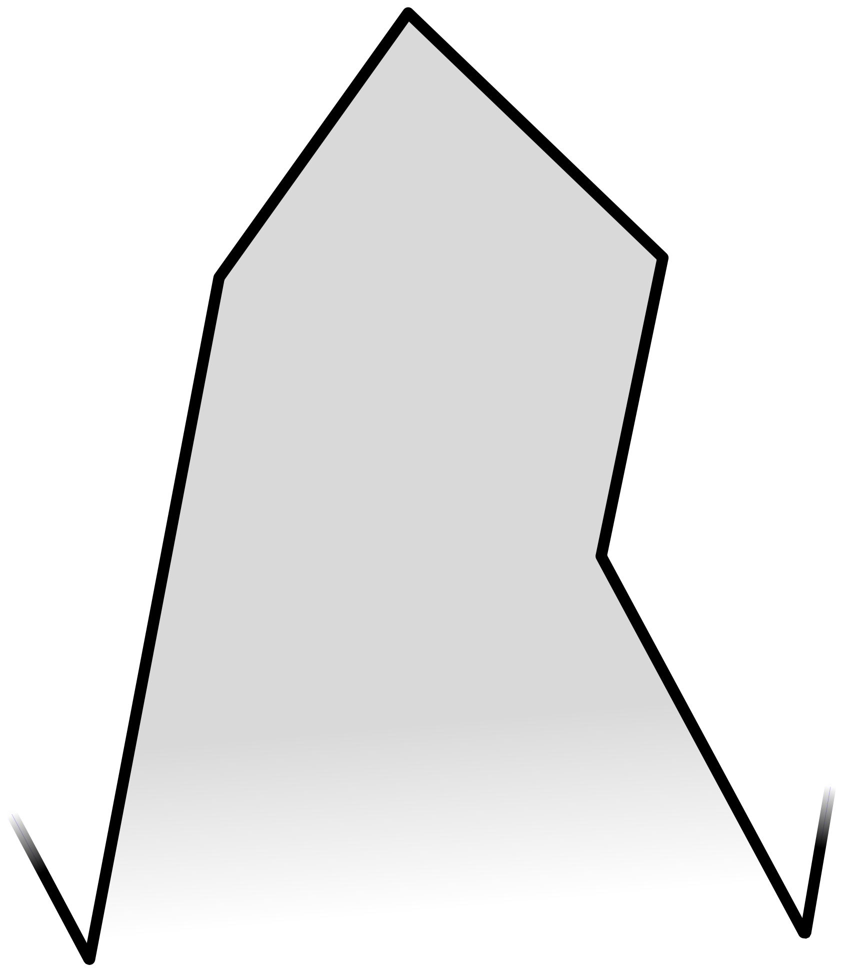
Form Factor OPE

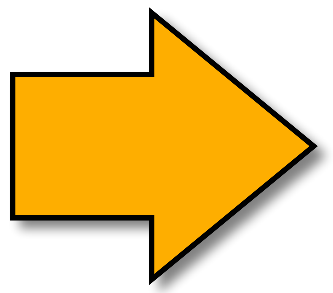


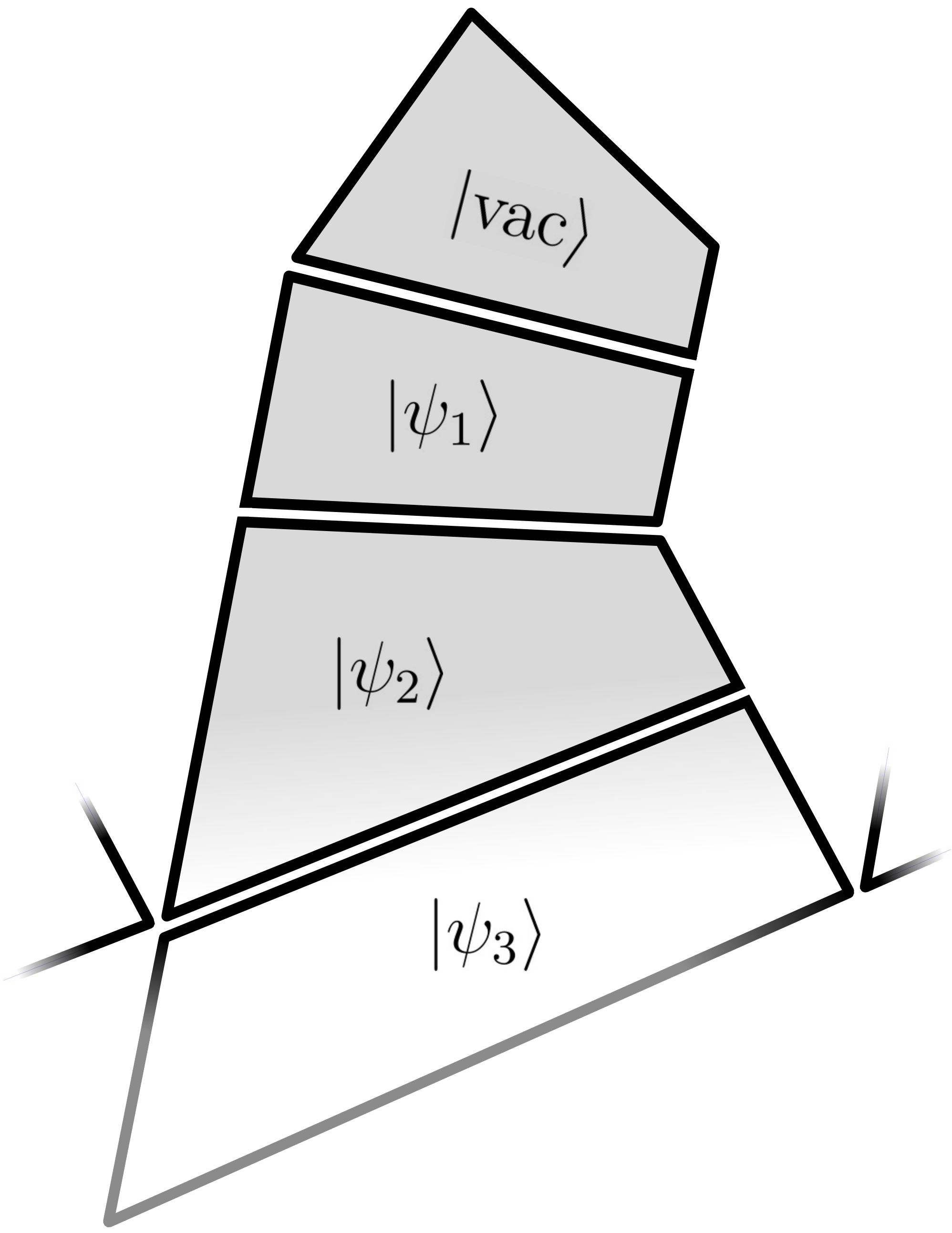




[Brandhuber, Spence, Travaglini, Yang ’10]





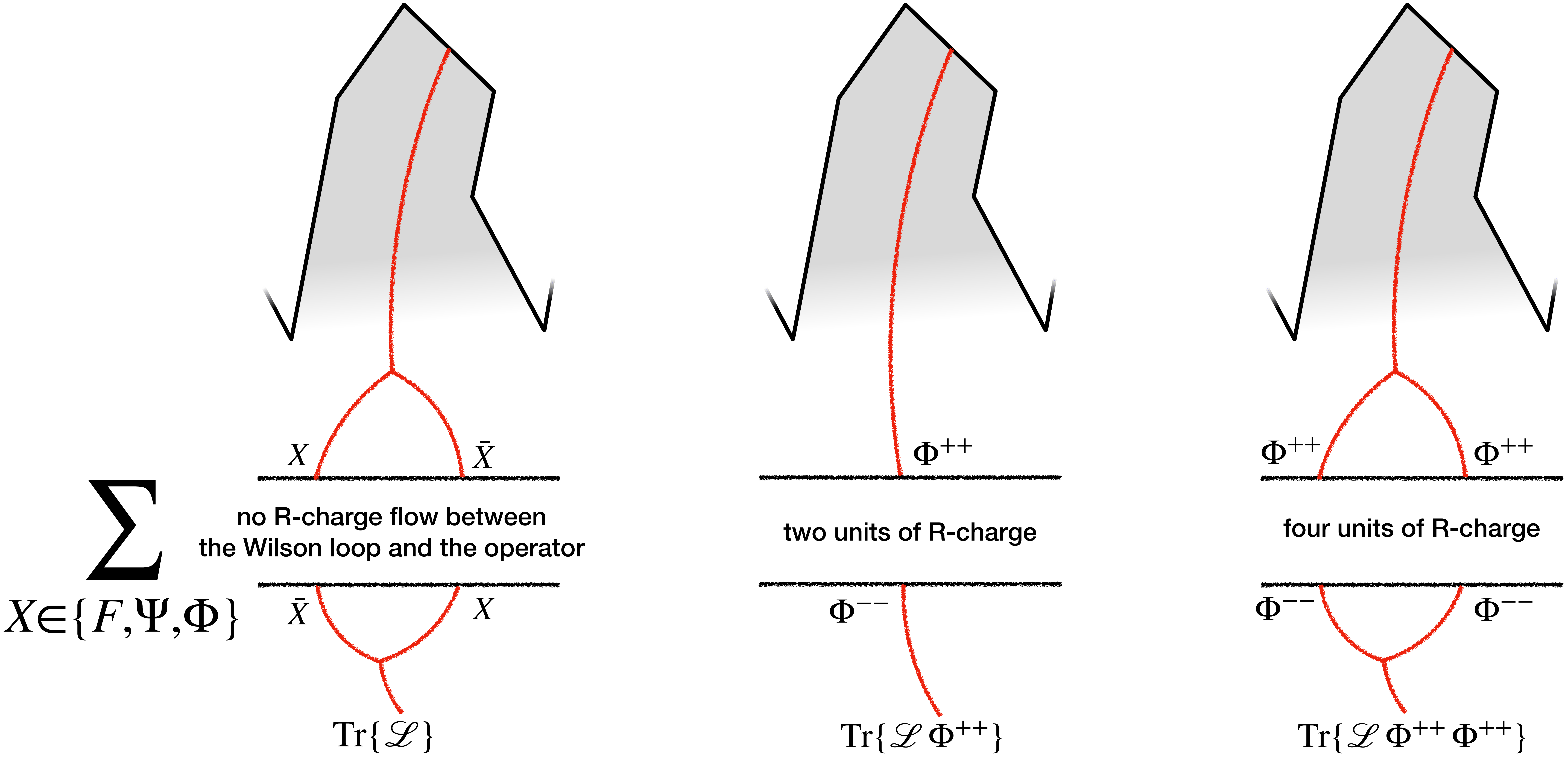


$$\mathcal{F}_5 \sim \sum_{\psi_i} P(0|\psi_1)\, P(\psi_1|\psi_2)\, P(\psi_2|\psi_3)\, F_{\mathcal{O}}(\psi_3)$$

Form factor transition

[Sever, AT, Wilhelm ’20 - ’21]

What is a Form Factor Transition?



[Sever, AT, Wilhelm ’20 - ’21]

[Basso, AT to appear]

Tilted Bessel Kernels

$$\mathbb{K}_{ij} = 2j(-1)^{ij+j}\int_0^\infty \frac{dt}{t}\frac{J_i(2gt)J_j(2gt)}{e^t-1}$$

[Beisert, Eden, Staudacher ’07]

$\alpha-$tilt

$$\mathbb{K}(\alpha) = 2\cos(\alpha)\begin{pmatrix}\cos(\alpha)\mathbb{K}_{\circ\circ} & \sin(\alpha)\mathbb{K}_{\circ\bullet}\\ \sin(\alpha)\mathbb{K}_{\bullet\circ} & \cos(\alpha)\mathbb{K}_{\bullet\bullet}\end{pmatrix}$$

[Basso, Dixon, Papathansiou ’20]

$\alpha = 0$ — octagon kernel

Pentagon transitions

$P(\psi\,|\,0)$

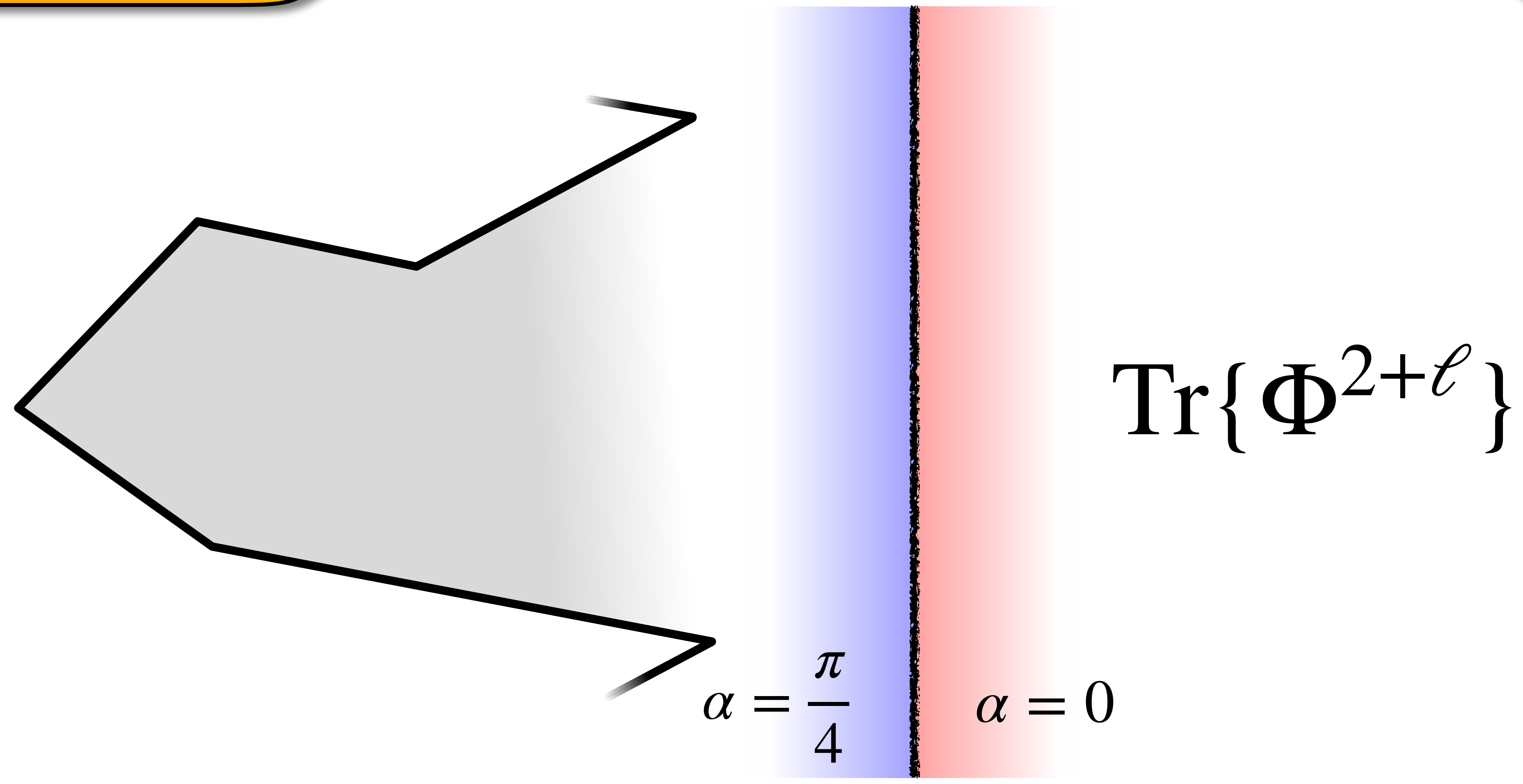
$\alpha = \frac{\pi}{4}$

$\alpha-$tilt

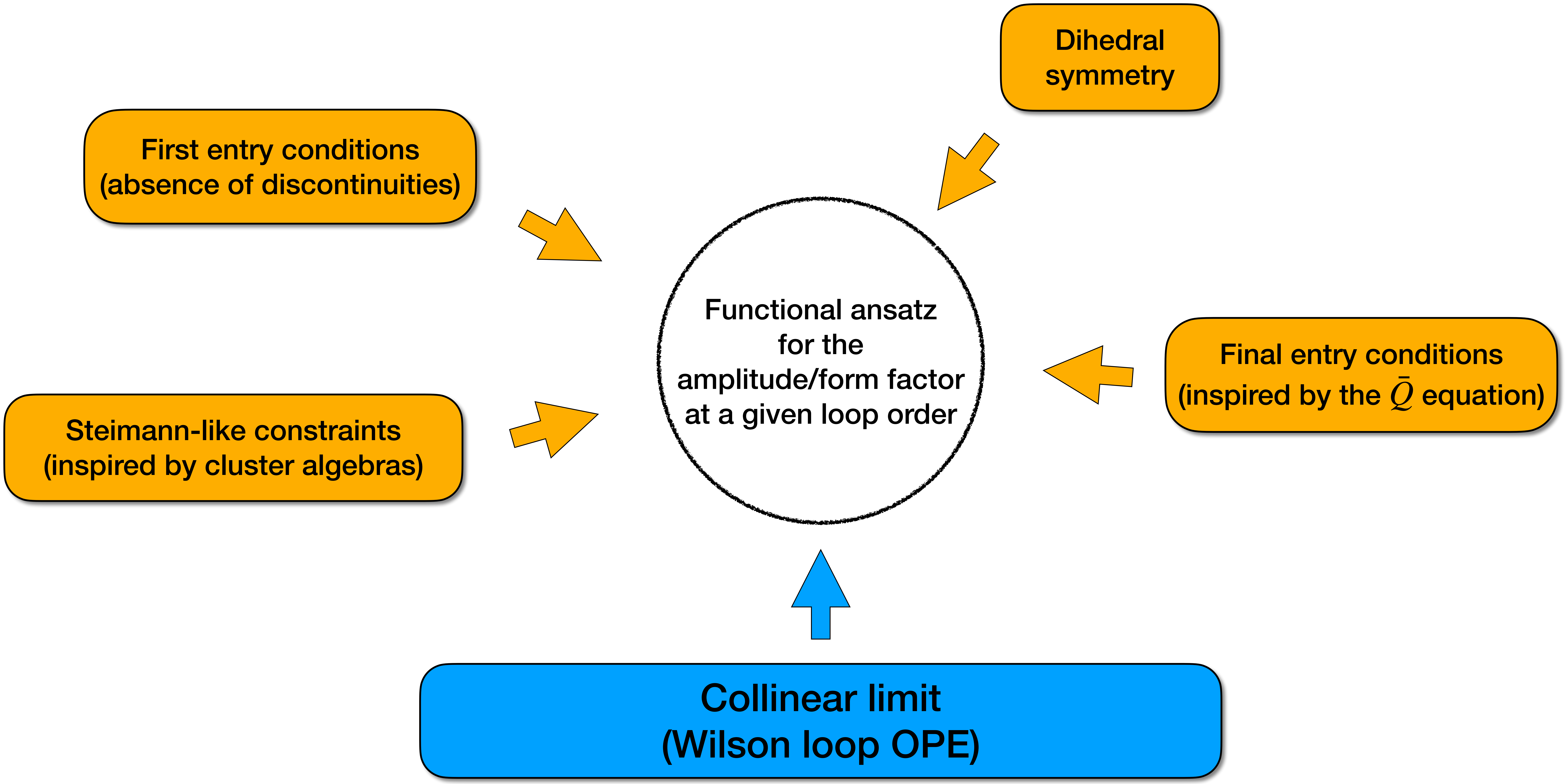
Form factor transitions

$F_{\mathcal{O}}(\psi)$

$\alpha = 0$



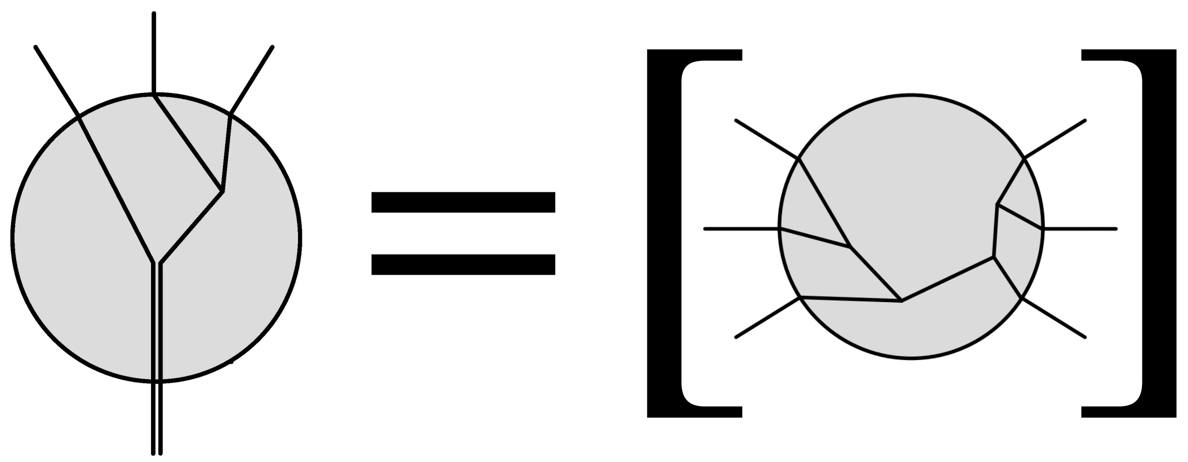
Perturbative bootstrap



Perturbative bootstrap

Three-point form factor of $\mathcal{O} = \mathrm{Tr}\{\Phi^2\}$ to 5 loops [Dixon, McLeod, Wilhelm '21]

to 8 loops [Dixon, Gurdogan, McLeod, Wilhelm '23]


antipodal map

[Dixon, Gurdogan, Liu, McLeod, Wilhelm '22]

Three-point form factor of $\mathcal{O} = \mathrm{Tr}\{\Phi^3\}$ to 5 loops [Dixon, AT, Basso to appear]

Perturbative bootstrap results

The function space is described by 6 “letters” (a, b, c, d, e, f).

In the case of $\mathcal{O} = \mathrm{Tr}\{\Phi^2\}$, the function space is additionally restricted by some “unexpected” constraints:

a-not-next-to-d, d-not-next-to-e

In the case of $\mathcal{O} = \mathrm{Tr}\{\Phi^3\}$, we see a much smaller number of constraints:

d-not-next-to-e

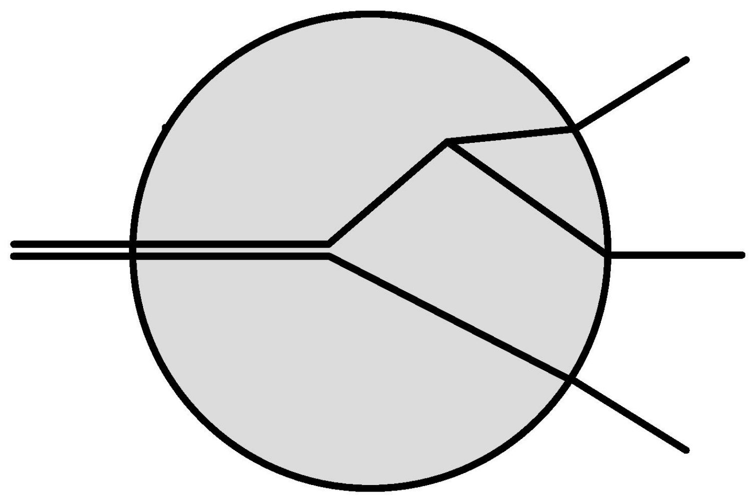
How many parameters does the OPE need to fix?

$\mathcal{O}$ \ L	2	3	4	5
$\mathrm{Tr}\{\Phi^2\}$	48	249	1290	6654
	2	5	17	38
$\mathrm{Tr}\{\Phi^3\}$	94	854	7699	~ 23000
	4	26	217	~ 2100

Form factors at tree level

$\mathcal{O} = \mathrm{Tr}\{(\Phi^{++})^{2+\ell}\}$

R-charge ℓ



Helicity k

N^{k}MHV

$$(a\,b\,c) = \frac{\delta^{0|2}\left(\langle ab\rangle\,\eta_c + \langle bc\rangle\,\eta_a + \langle ca\rangle\,\eta_b\right)}{\langle ab\rangle\,\langle bc\rangle\,\langle ca\rangle}$$

$$[a\,b\,c\,d\,e] = \frac{\delta^{0|4}\left(\langle[abcd\rangle\,\eta_{e]}\rangle\right)}{\langle abcd\rangle\,\langle bcde\rangle\,\langle cdea\rangle\,\langle deab\rangle\,\langle eabc\rangle}$$

$$\frac{\mathcal{F}^{\mathrm{MHV}(0)}_{\ell=1,n}}{\mathcal{F}^{\mathrm{MHV}(0)}_{\ell=0,n}} = \sum_{i=1}^{n} (\star\, i\, i+1)$$

$$\frac{\mathcal{F}^{\mathrm{MHV}(0)}_{\ell=2,n}}{\mathcal{F}^{\mathrm{MHV}(0)}_{\ell=0,n}} = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} (\star\, i\, i+1)\,(\star\, j\, j+1)$$

$\ldots$

[Basso, AT to appear]

N$^{(k,\ell)}$MHV

$$\frac{\mathcal{F}^{\mathrm{NMHV}(0)}_{\mathcal{T}_2,n}}{\mathcal{F}^{\mathrm{MHV}(0)}_{\mathcal{T}_2,n}} = \sum_{i=1}^{n}\sum_{j=i+2}^{i+n-1} [\star\, i\, i+1\, j\, j+1]$$

[Brandhuber, Gurdogan, Mooney, Travaglini, Yang '11]

This is the $m=2$ amplituhedron!

This is similar to the $m=4$ amplituhedron
(more complicated due to
the lack of total momentum conservation)

also appeared in [Caron-Huot, Coronado, Muhlmann '23]

Conclusions

We developed the **Form factor OPE** method, that allows us to compute form factors of all protected operators as expansions around the their collinear limits.

Using the **perturbative bootstrap** technique, we computed the form factor of $\mathcal{O} = \mathrm{Tr}\{\Phi^3\}$ up to 5 loop orders in the perturbative expansion for general kinematics.

Future directions

Constructing the Form factor OPE for unprotected operators, like the **Konishi**, and getting first non-perturbative results for form factors of unprotected operators.

Applying the **perturbative bootstrap** techniques to form factors of other protected and unprotected operators and searching for **antipodal dualities** that these objects might satisfy.

Understanding the **tree-level helicity structure** of the 1/2-BPS form factors