## LABEX

Mathématique
Hadamard.

# Recent advances in precision holography 

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## Motivation

- AdS/CFT provides a gauge theory description of string or M-theory on asymptotically locally AdS backgrounds. Immediate consequence:

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Z_{\mathrm{CFT}}[\mathrm{~J}]=Z_{\text {string } / \mathrm{M}}[\phi]
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- The correspondence is meant to be valid at finite $N$. Schematically,

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- At strong coupling, CFT observables encode information about supergravity. Corrections teach us about string/M-theory beyond the low-energy limit.
- Studying and understanding corrections on both sides of the duality:

Precision Holography

## Precision holography

- Need concrete realizations where we can test and make predictions $\rightarrow$ use supersymmetry to gain computational mileage.


## SCFT

Localization gives exact results Solving the matrix models yields $1 / N$ expansion to any order

Various techniques available Analytic \& numeric

Supergravity
Exact results out of reach*
Work order-by-order in the derivative expansion

Can also study one-loop effects
LO, NLO, NNLO tests
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- New handle on AdS vacua of string/M-theory with non-trivial fluxes.
- This talk: recent progress in precision holography for $\mathrm{AdS}_{4} / \mathrm{CFT}_{3}$.


## $\mathrm{AdS}_{4} / \mathrm{CFT}_{3}$ from M 2 branes

- Study 3d $\mathcal{N} \geq 2$ SCFTs arising from low-energy limit of $N$ M2 branes.
- No $\lambda$ in M-theory $\rightarrow$ easier to study the correspondence.


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- No $\lambda$ in M-theory $\rightarrow$ easier to study the correspondence.
- M-theory engineers many dual pairs:

3d Chern-Simons-matter theories §
M-theory on $\mathrm{AlAdS}_{4} \times X_{7}$

Take $X_{7}$ to be Sasaki-Einstein $\rightarrow$ CSm

$$
\begin{gathered}
X_{7}=S^{7} / \mathbb{Z}_{k}(\text { free }) \rightarrow \text { ABJM } \\
X_{7}=S^{7} / \mathbb{Z}_{r}(\text { f.p. }) \rightarrow \text { ADHM } \\
X_{7}=N^{010}, V^{52}, Q^{111}, \ldots
\end{gathered}
$$

- Put the gauge theory on compact $M_{3}$ and study susy partition functions. Use localization to compute them exactly for various $\left(X_{7}, M_{3}\right)$.


## ABJM on the squashed sphere: $M_{3}=S_{b}^{3}$

- Let us start with simple $X_{7}=S^{7} / \mathbb{Z}_{k}$ that gives rise to ABJM theory $\rightarrow U(N)_{k} \times U(N)_{-k} C S m$ with bifundamental chirals and a superpotential.
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[Aharony, Bergman, Jafferis, Maldacena'08]
- The partition function on $S_{b}^{3}$ localizes to a matrix model.
- For $\mathcal{F}=\left\{\begin{array}{l}k \geq 1 \& b=1 \\ k=1 \& b=\sqrt{3}\end{array}\right.$ matrix model reduces to a free Fermi gas. $\quad$ [Mariño, Putrov' $11 ;$ Hatsuda '16]


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[Mariño,Putrov'11; Hatsuda'16]
- Fully solved using refined topological string on toric non-compact CYs.
[Grassi,Hatsuda,Mariño'15]
- All perturbative terms in the large $N$ limit resum to an Airy function:

$$
Z_{\mathcal{F}}(N)=e^{A} C^{-\frac{1}{3}} \mathrm{Ai}\left[C^{-\frac{1}{3}}(N-B)\right]+\mathcal{O}\left(e^{-\sqrt{N / k}}, e^{-\sqrt{N k}}\right)
$$

with $(k, b)$-dependent coefficients $(A, B, C)$ explicitly known.

## Dual supergravity

- Systematic expansion of the free energy at large $N$ :

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-\log Z_{\mathcal{F}}(N)=\frac{2}{3 \sqrt{C}} N^{\frac{3}{2}}-\frac{B}{\sqrt{C}} N^{\frac{1}{2}}+\frac{1}{4} \log N+\mathcal{O}\left(N^{0}\right)
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- Dual to 4 d minimal supergravity solutions $\left(g_{4}, A\right)$ with $\Lambda<0$ and $\partial \mathcal{M}=S_{b}^{3}$
- LO $N^{\frac{3}{2}}$ term matches the two-derivative regularized on-shell actions.
[Emparan, Johnson, Myers'99; Martelli, Passias, Sparks'11]
- NLO $N^{\frac{1}{2}}$ term reproduced by including bulk four-derivative terms.
[Bobev, Charles, Hristov, VR‘21]
- NNLO $\log N$ term is a one-loop effect from summing over the KK modes around the 11d backgrounds.
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- NNLO $\log N$ term is a one-loop effect from summing over the KK modes around the 11d backgrounds.
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- Precision holography $\rightarrow$ prediction for all higher-derivative and higher-loop effects in the bulk!


## Beyond the ABJM Fermi gas?

- For general $(k, b)$ there is no known Fermi gas.
- Conjecture: we again have an Airy function

$$
\begin{aligned}
Z_{k, b}(N) & =e^{A} C^{-\frac{1}{3}} \mathrm{Ai}\left[C^{-\frac{1}{3}}(N-B)\right]+\mathcal{O}\left(e^{-\sqrt{N / k}}, e^{-\sqrt{N k}}\right) \\
C & =\frac{32}{\pi^{2} k} Q^{-4} \quad \text { and } \quad B=\frac{k}{24}-\frac{2}{k}\left(\frac{1}{3}-2 Q^{-2}\right)
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- Consistent with dual supergravity results up to and including log terms.
- Consistent with ongoing detailed numerical studies of the matrix model. Can even access the large- $k$ expansion of the prefactor:

$$
A=\left[\left(b-b^{-1}\right)^{2}-4\right] \frac{\zeta(3)}{32 \pi^{2}} k^{2}+\mathcal{O}\left(k^{0}\right)
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- Question: what structure produces this Airy function in M-theory? Is there a Fermi gas or topological string picture? Something new?


## A word on correlators

- The conjecture has implications for ABJM dynamics since the squashing parameter $b$ couples to the stress tensor.
- $\mathcal{N}=2$ Ward identities imply
[Closset, Dumitrescu,Festuccia,Komargodski'12]

$$
\left\langle T_{\mu \nu}(x) T_{\rho \sigma}(0)\right\rangle=C_{T} \mathcal{T}_{\mu \nu \rho \sigma}(x) \quad \text { and } \quad C_{T}=\left.\frac{\partial^{2} \log Z_{k, b}}{\partial b^{2}}\right|_{b=1}
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- Taking more derivatives gives access to integrated correlators of stress tensors over the round 3 -sphere.
- Flat space limit gives a way to study scattering amplitudes in M-theory.
[Chester, Pufu, Yin'18]


## ABJM on $M_{3}=S^{1} \times \Sigma_{g}$

- Turn on flux for exact R-sym $\int_{\Sigma_{\mathfrak{g}}} d A_{R}=2 \pi(1-\mathfrak{g}) \rightarrow$ topological twist.
- Use localization to compute the Topologically Twisted Index (TTI).


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[Benini,Hristov, Zaffaroni'16; Closset,Kim,Willett'17]

$$
Z_{S^{1} \times \Sigma_{\mathfrak{g}}}(N)=\sum_{\left\{x_{i}, \tilde{x}_{j}\right\} \in \mathrm{BAE}} \mathcal{B}\left(x_{i}, \tilde{x}_{j}\right)
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Sum over solutions to transcendental eqs called Bethe Ansatz Equations.

- Amenable to numerical evaluation:
- Use known large $N$ solution as init
- Numerically solve BAE $\rightarrow x_{i}=x_{i}^{\star}$ and $\tilde{x}_{j}=\tilde{x}_{j}^{\star}$
- Evaluate $\mathcal{B}\left(x_{i}^{\star}, \tilde{x}_{j}^{\star}\right)$ numerically to high precision
- Fit with respect to ( $N, k$ ) and (hopefully) infer analytic expressions


## The ABJM twisted index

- Detailed numerics $\rightarrow$ we can propose an analytic formula: [Bobev, Hong, VR‘22]

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\begin{aligned}
-\log Z_{S^{1} \times \Sigma_{\mathfrak{g}}}(N)= & (1-\mathfrak{g}) \frac{\pi \sqrt{2 k}}{3}\left(\widehat{N}^{\frac{3}{2}}-\frac{3}{k} \widehat{N}^{\frac{1}{2}}\right)+\frac{1-\mathfrak{g}}{2} \log \widehat{N} \\
& +(1-\mathfrak{g}) f(k)+\mathcal{O}\left(e^{-\sqrt{N / k}}, e^{-\sqrt{N k}}\right)
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- All perturbative corrections in $1 / N$ resum to a (very!) simple form.

Can also access the large- $k$ expansion of the constant term:

$$
f(k)=\frac{3 \zeta(3)}{8 \pi^{2}} k^{2}-\frac{7}{6} \log k+\mathcal{O}\left(k^{0}\right)
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- Question: what is the interpretation of $\widehat{N}$ in M -theory?
- Expand at large $N \rightarrow N^{\frac{3}{2}}, N^{\frac{1}{2}}, \log N$ terms... and all perturbative corrections.
- Useful for precision holography.


## Dual supergravity

- The index captures the path integral of M-theory on the 11d background

$$
\begin{aligned}
d s_{11}^{2} & =\frac{L^{2}}{4} d s_{4}^{2}+L^{2} d s_{\mathbb{C P}_{3}}^{2}+L^{2}\left(d \psi+\sigma+\frac{1}{4} A\right)^{2} \\
G_{4} & =\frac{3 L^{3}}{8} \text { vol }_{4}-\frac{1}{4} \star_{4} F \wedge J
\end{aligned}
$$

with $\left(g_{4}, F=d A\right)$ the Euclidean Romans solution of $4 \mathrm{~d} \mathcal{N}=2$ minimal gauged supergravity. [Romans‘92; Genolini, Ipiña, Sparks ${ }^{\text {‘ } 19 ;}$ Bobev, Charles, Min ‘20]

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- When $\mathfrak{g}>1$, can Wick rotate to obtain a Lorentzian magnetic black hole interpolating between $\mathrm{AdS}_{4}$ and $\mathrm{AdS}_{2} \times \Sigma_{\mathfrak{g}}$ near-horizon.
- LO, NLO and NNLO for BH entropy match the corresponding terms in TTI.
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- Precision holography $\rightarrow$ prediction for the corrected Bekenstein-Hawking entropy of this BPS black hole to all orders in $1 / N$.


## Less supersymmetry

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- $\mathcal{N}=3$ with $X_{7}=N^{010} / \mathbb{Z}_{k} \rightarrow \widehat{N}=N+\frac{5 k}{48}+\frac{1}{3 k}$ and

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- In all cases, black hole solution exists $\rightarrow$ prediction for entropy at finite $N$.


## Conclusions

- Localized partition functions in SCFTs can be studied very precisely.
- Precision holography uncovers structures in M-theory: Airy, "shifted" N...
- Tests and predictions for AdS vacua, including AdS black holes.


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