Impact of Renormalization on Order Parameter

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QCD phase diagram

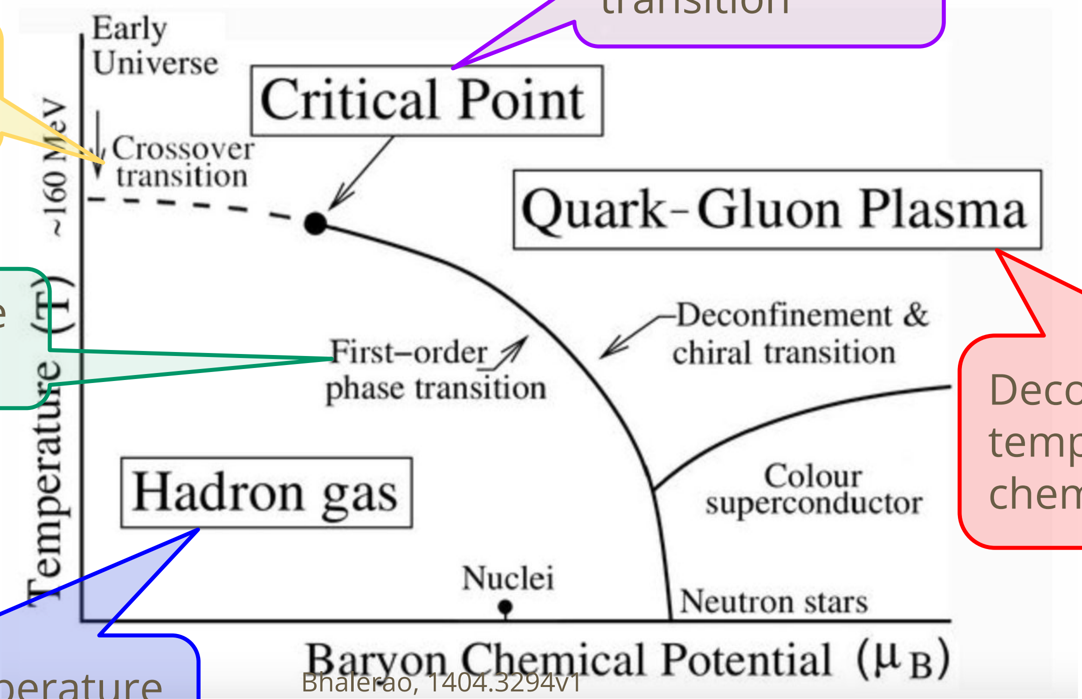
From lattice QCD

From effective models

Confined, low temperature and low chemical potential

End of 1st order transition

Deconfined, high temperature and high chemical potential



Bhalerao, 1404.3294v1

Fluid dynamics and Fluctuations

Successful at describing the spacetime evolution of systems created in heavy ion collisions

- ➔ Describes QGP-HG phase transition by including an adapted equation of state
- ➔ Current models limited to event-averaged quantities

Describing the impact of fluctuations is important, especially at phase transition; near the critical point, dynamical evolution and critical phenomena intertwine

2 approaches

Deterministic (Hydro+)

Stephanov, Yin 1712.10305
Rajagopal, Ridgway, Weller, Yin 1908.08539
An, Basar, Stephanov, Yee 1902.09517, 1912.13456
Pradeep, Rajagopal, Stephanov, Yin 2204.00639

Stochastic Fluid Dynamics

Challenge: lattice spacing dependence introduced by noise

Chiral Fluid Dynamics

Couple fluid dynamic evolution of fireball to fluctuations of chiral order parameter

$$\frac{\partial^2 \varphi(\vec{x},t)}{\partial t^2} - \nabla^2 \varphi(\vec{x},t) + \eta \frac{\partial \varphi(\vec{x},t)}{\partial t} + \frac{\partial V_{\text{eff}}[\varphi]}{\partial \varphi(\vec{x},t)} = \xi(\vec{x},t)$$

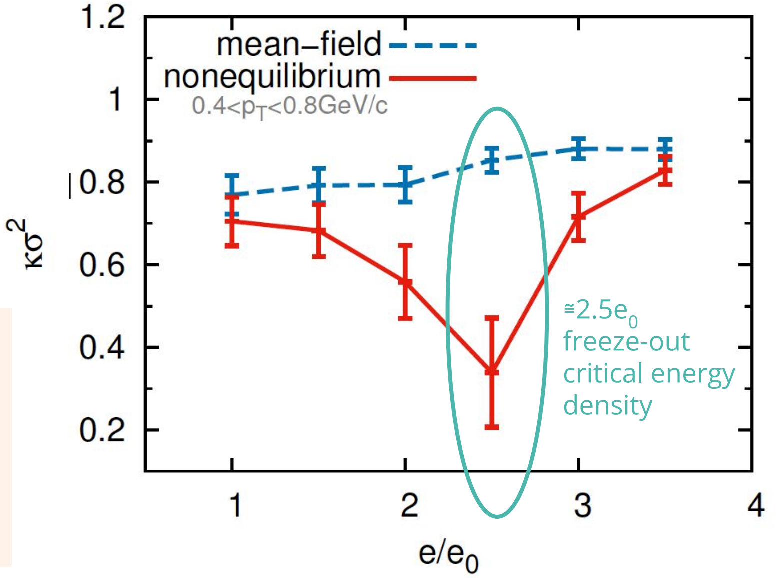
noise

$$\partial_\mu T^{\mu\nu} = -\partial_\mu T^{\mu\nu}_{\varphi}$$

Large dip in kurtosis of net-proton number where crossover is expected

Herold, Nahrgang, Yan and Kobdaj 1601.04839v1
Nahrgang, Leupold, Herold, Bleicher 1105.0622
Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, Wink 1808.01377v1

Lattice spacing dependence in simulations has been treated by coarse-graining the noise. We try to improve the approach with proper renormalization



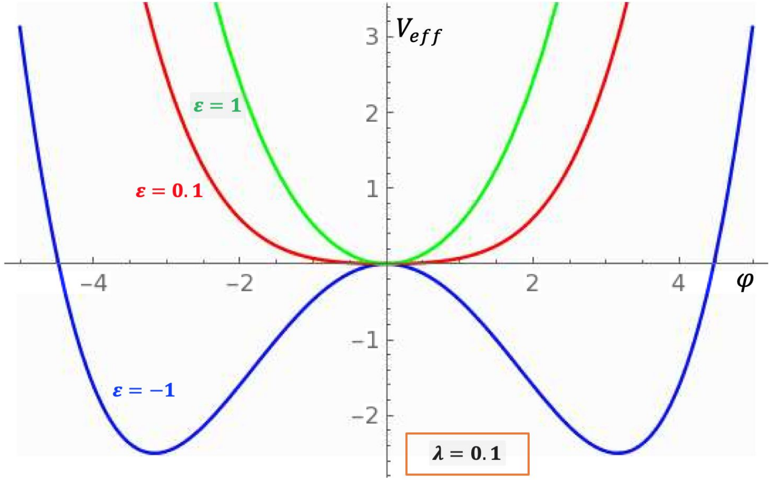
Relaxation Model

Decouple the equations and consider a simpler model:
Stochastic Relaxation Equation

$$\frac{\partial^2 \varphi(\vec{x},t)}{\partial t^2} - \nabla^2 \varphi(\vec{x},t) + \eta \frac{\partial \varphi(\vec{x},t)}{\partial t} + \frac{\partial V_{\text{eff}}[\varphi]}{\partial \varphi(\vec{x},t)} = \xi(\vec{x},t)$$

Effective potential

$$V_{\text{eff}}(\varphi) = \frac{1}{2}\varepsilon\varphi^2 + \frac{1}{4}\lambda\varphi^4$$



The noise ξ is defined by

$$\langle \xi(\vec{x},t) \rangle = 0$$

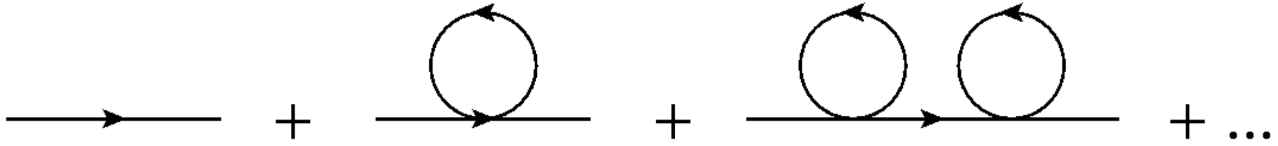
$$\langle \xi(\vec{x},t)\xi(\vec{x}',t') \rangle = 2\eta T\ \delta(\vec{x}-\vec{x}')\delta(t-t')$$

Lattice Spacing Dependence

- ➔ UV divergences caused by the noise translate as non-physical lattice spacing dependence in numerical simulations

$$\delta(\vec{x}' - \vec{x}) \rightarrow \frac{1}{dx^3}$$

- ➔ Loop corrections in the $\boldsymbol{\varphi}^4$ theory also introduce UV divergences



Jansen and Nickel

The tadpole diagram in the expansion of 2-point function gives a correction term (form depends on the regularisation/renormalization scheme)

Improved solution: lattice regularisation

Linear Approximation of V_{eff} (ε=1, λ=0)

3D system (cube with sides L=20) static at fixed temperature

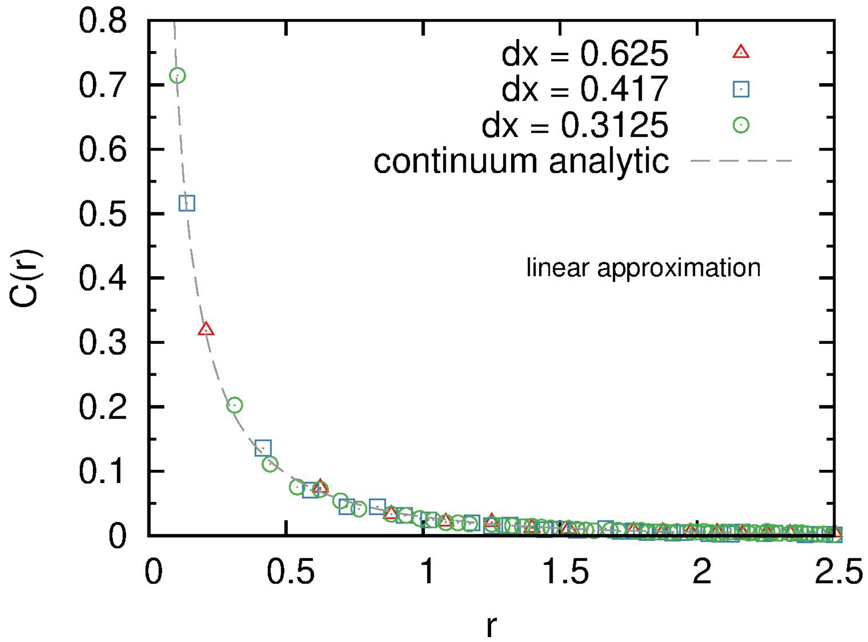
Code on GPU: input equations and parameters➞evaluate the dynamical variable ➞derive relevant observables (correlation function, different moments, etc.)

The 2-point function is

$$C(r) = \frac{T}{4\pi r} e^{-\frac{r}{r_c}}$$

Where $r_c = \sqrt{1/\epsilon}$ $\quad r = |\vec{x} - \vec{x}'|$

- Reproduced analytic result
- Benchmarked correlation function in our code
- No dx dependence for finite distances: introduced close to 0



Linear Approximation of V_{eff} (ε=1, λ=0)

Integrate correlation function in 1d to benchmark a logarithmic dx dependence

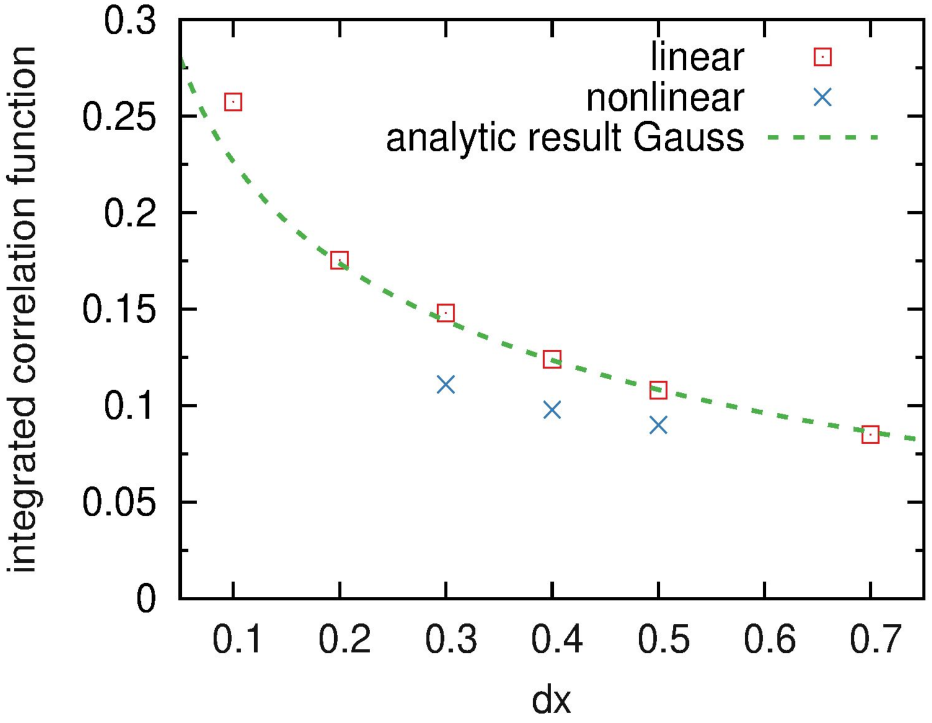
$$C_{int} = \int_{dx}^{X} \frac{T}{4\pi r} e^{-\frac{r}{r_c}} \, \mathrm{d}r =$$

$$\frac{T}{4\pi} \int_{dx}^{X} \left(\frac{1}{r} - \frac{1}{r_c} + \frac{r}{2r_c^2} - \frac{r^2}{3!r_c^3} + \ldots\right) dr =$$

$$\frac{T}{4\pi} \left[ln(r) - \frac{r}{r_c} + \frac{r^2}{4r_c^2} - \frac{r^3}{18r_c^3} + \ldots \right]_{dx}^{X}$$

- Reproduced analytic result
- Benchmarked dx dependence

➥include nonlinear terms



Observables: Mean and Variance

➔ Mean

$$\langle \varphi_V(t) \rangle = \frac{1}{V} \int d^3x \varphi(\vec{x}, t)$$

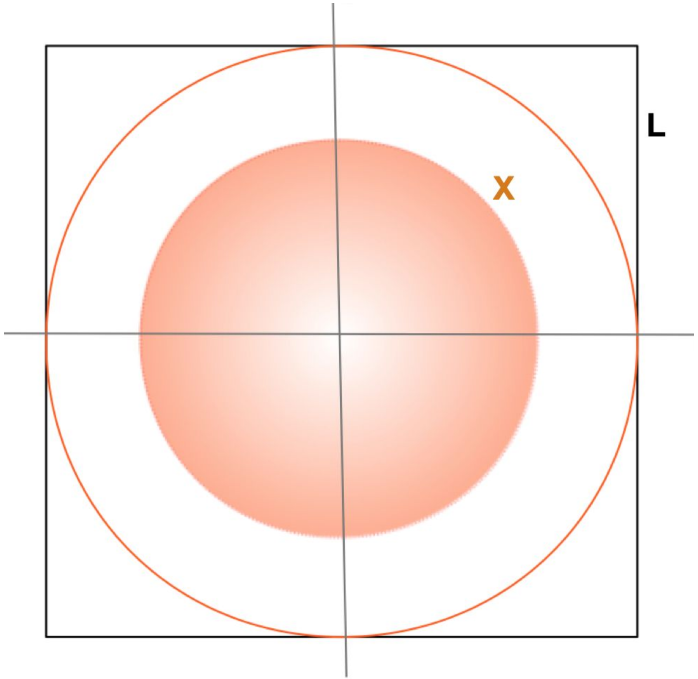
We are interested in fluctuation observables

➔ Variance

$$\langle \varphi_V^2(t) \rangle = \frac{1}{V} \int \int d^3x_1 d^3x_2 C(\vec{x_1} - \vec{x_2})$$

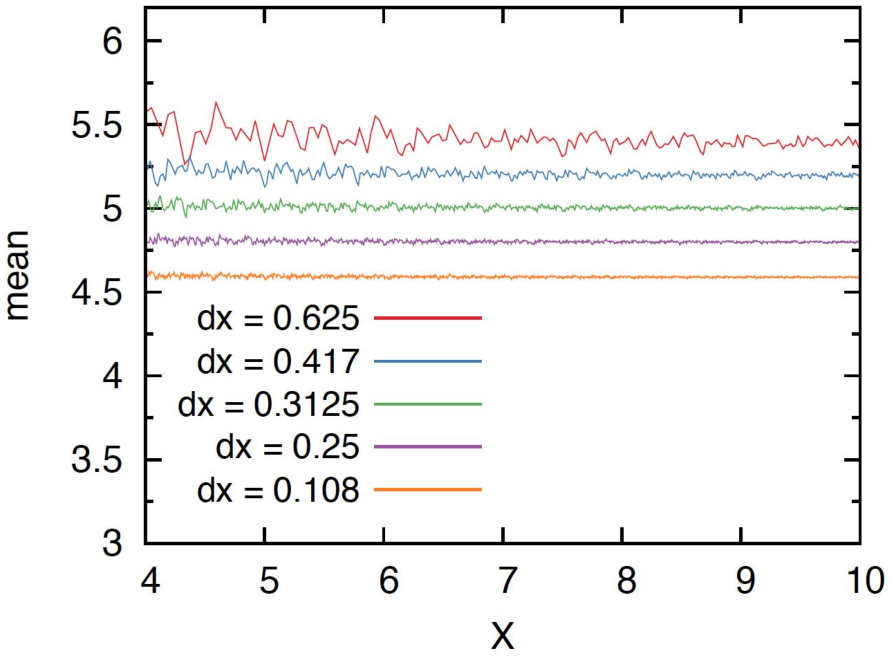
Where

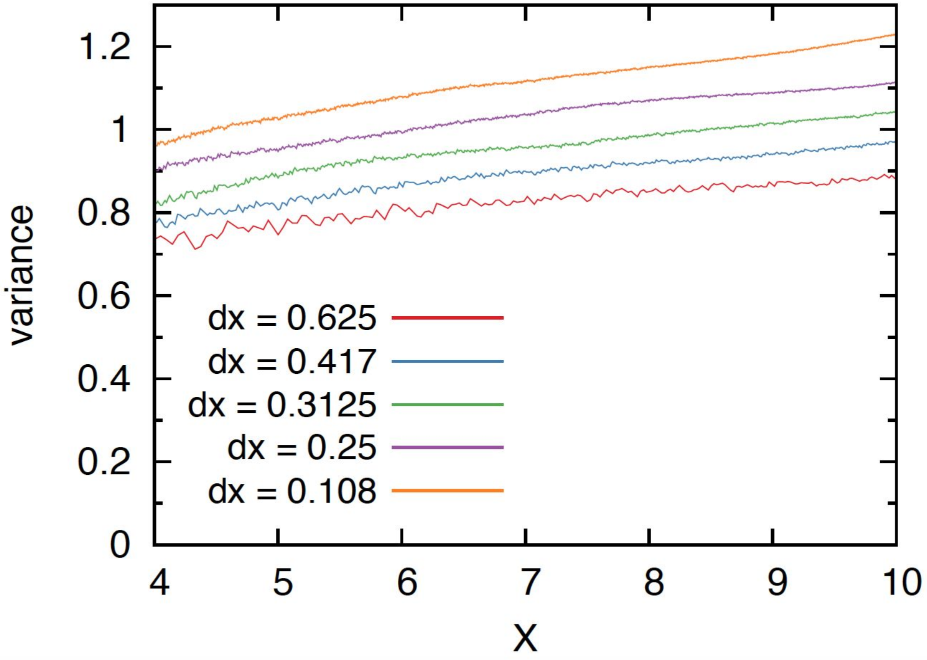
$$X \leq \frac{L}{2} \Rightarrow V \propto V_{sphere}$$



Mean and Variance at Equilibrium (ε=-1)

$$L = 20$$
$$\epsilon = -1$$
$$\lambda = 0.1$$
$$\frac{T}{\sqrt{|\varepsilon|}} = \frac{\eta}{\sqrt{|\varepsilon|}} = 1$$





Both observables are clearly dx dependent, but are volume independent

Lattice Regularisation

Equilibrium counterterm to correct V_{eff} from mass renormalization procedure

$$\mathcal{V}_{ct} = \{-\frac{3\lambda\Sigma}{4\pi}\frac{T}{dx} + \frac{3\lambda^2 T^2}{8\pi^2}[ln(\frac{6}{dxM}) + \zeta]\}\frac{\varphi^2}{2}$$

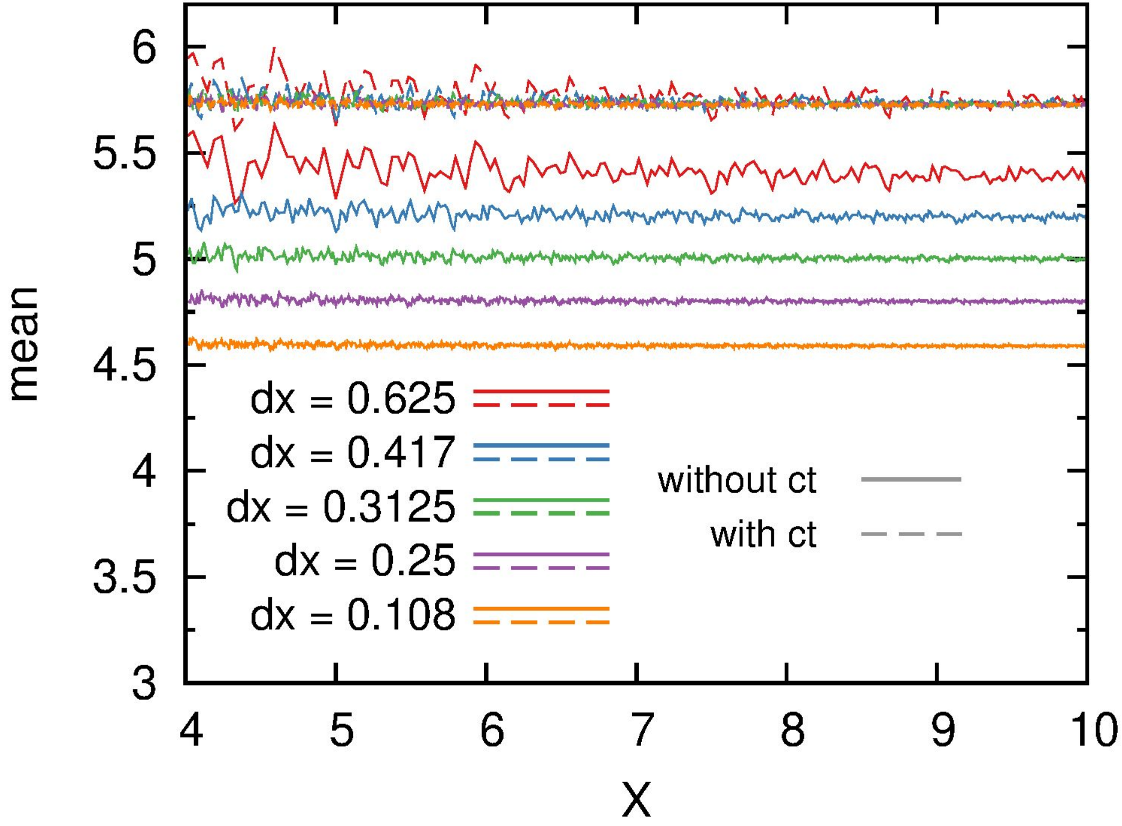
Cassol-Seewald et al. 0711.1866
Farakos et al. 9412091, 9404201v1
Gleiser, Ramos 9311278v

- $\Sigma \approx 3.1759$, $\zeta \approx 0.09$
- M renormalization scale
- Leading 1/dx dependence

$L = 20$
$\lambda = 0.1$
$\varepsilon = -1$ (broken symmetry)
$\varepsilon = 0.1$ (near critical point)

$$\frac{T}{\sqrt{|\varepsilon|}} = \frac{\eta}{\sqrt{|\varepsilon|}} = \frac{M}{\sqrt{|\varepsilon|}} = 1$$

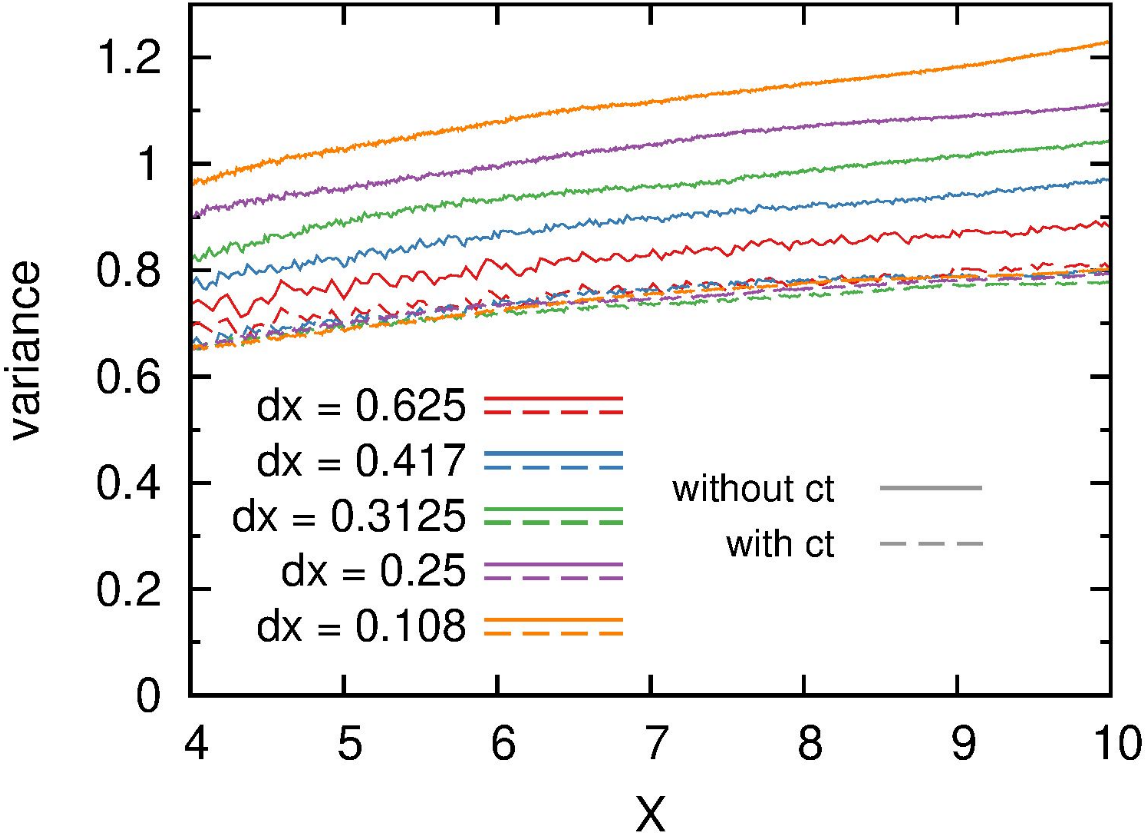
Mean at Equilibrium (ε=-1)



Lattice spacing dependence corrected

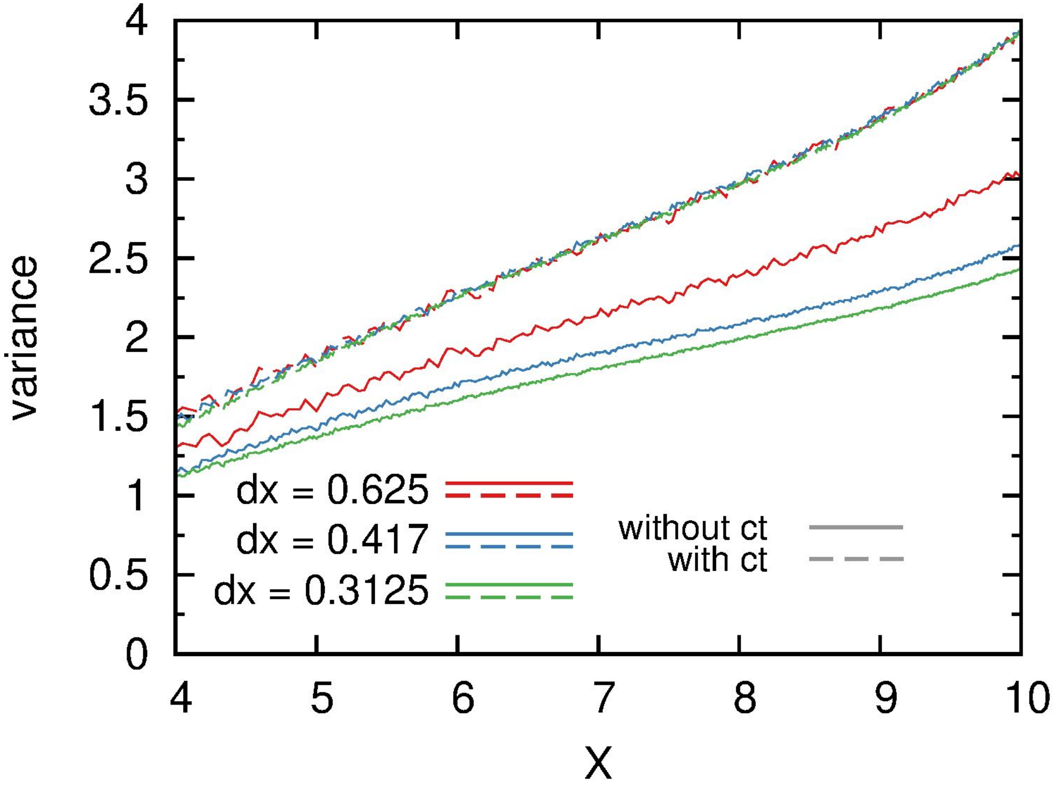
Consistent with previous equilibrium results
(Cassol-Seewald et al. 0711.1866 and references therein)

Variance at Equilibrium (ε=-1)



The same counterterm cures lattice spacing dependence at equilibrium for both the mean and the variance

Variance at Equilibrium (ε=0.1)



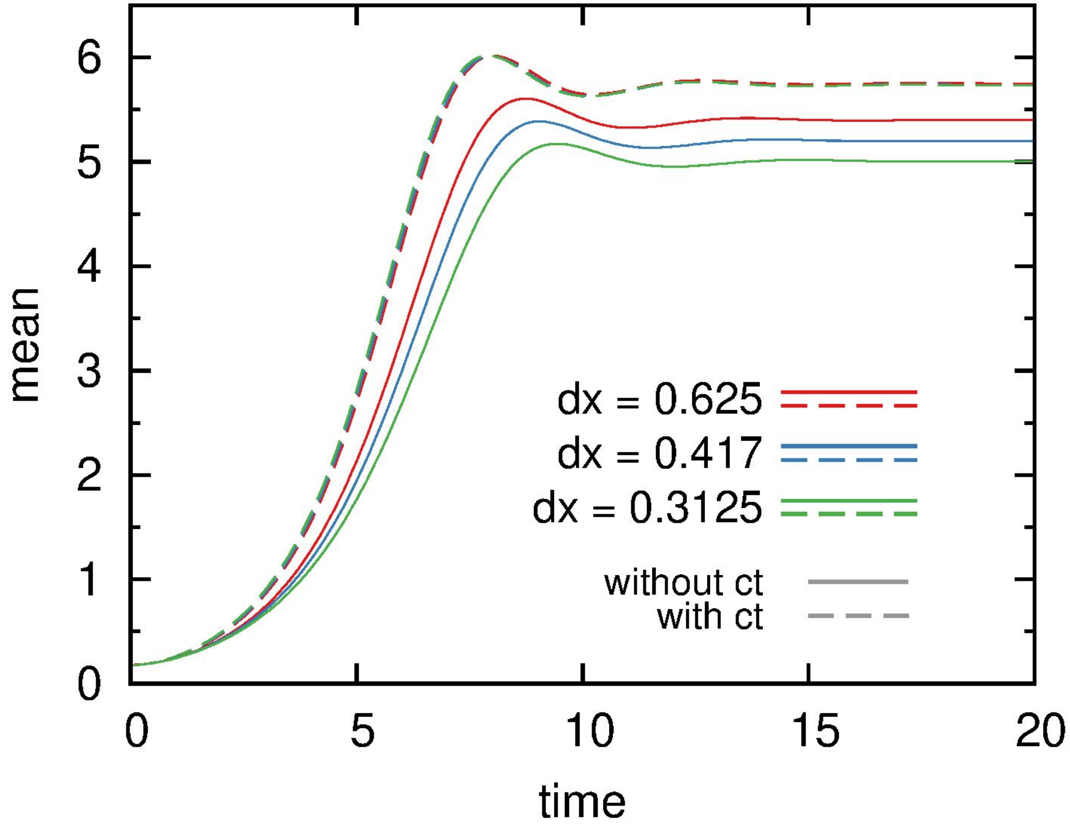
Close to the critical point, as ε decreases, the correlation length diverges

$$r_c = \sqrt{1/\epsilon}$$

Long-range fluctuations add up and the variance increases with the volume

➜Same counterterm corrects lattice spacing dependence close to critical point

Dynamical Evolution: Mean (ε=-1)

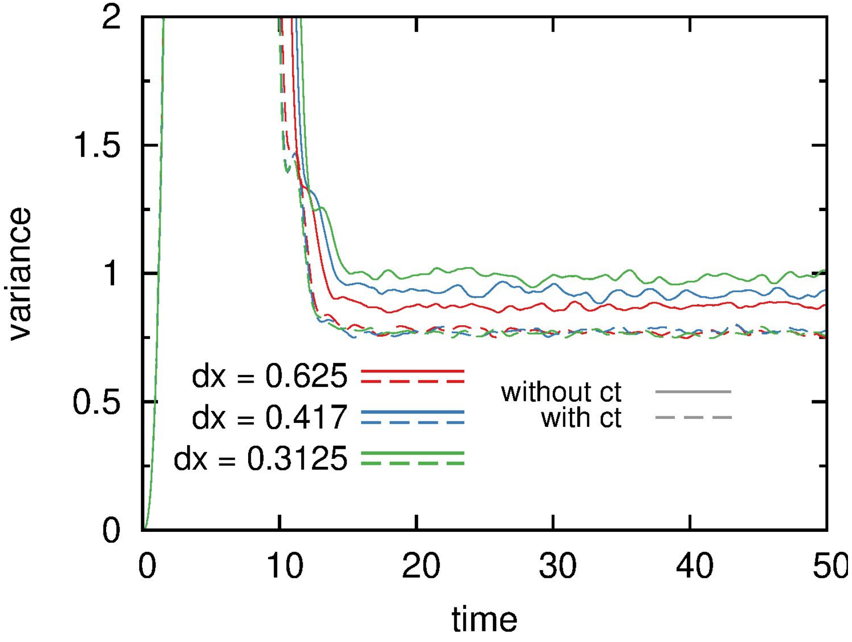


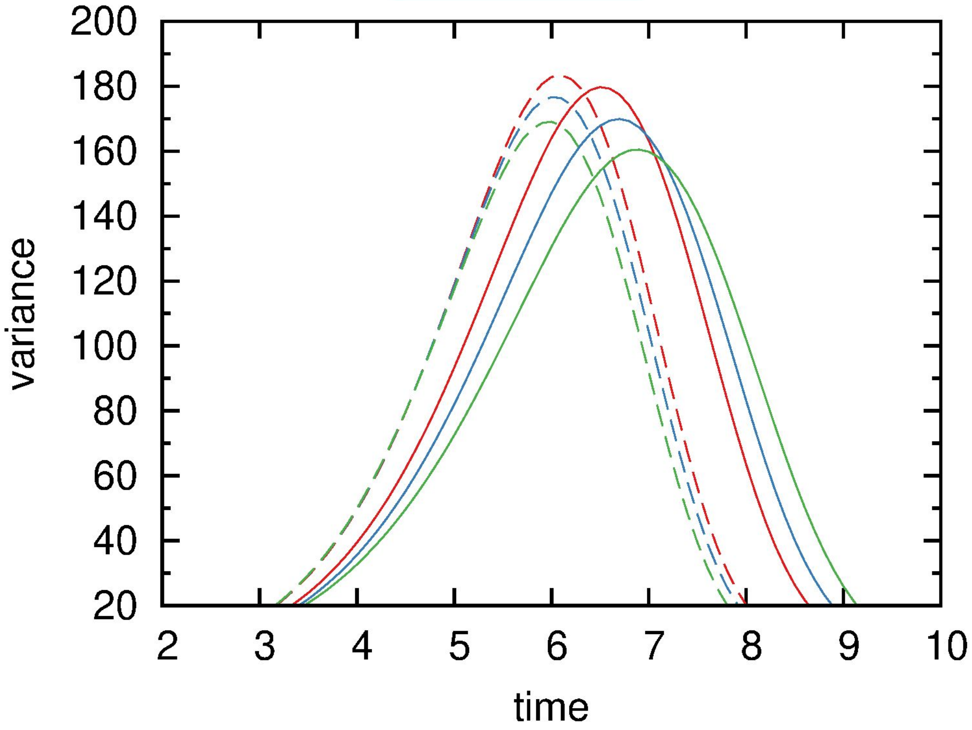
$\langle\varphi\rangle_{initial} = 0.1$
$\langle\varphi^2\rangle_{initial} = 0$

At early times, the mean shows very little dx dependence, despite the counterterm being calculated for equilibrium

Dynamical Evolution: Variance (ε=-1)

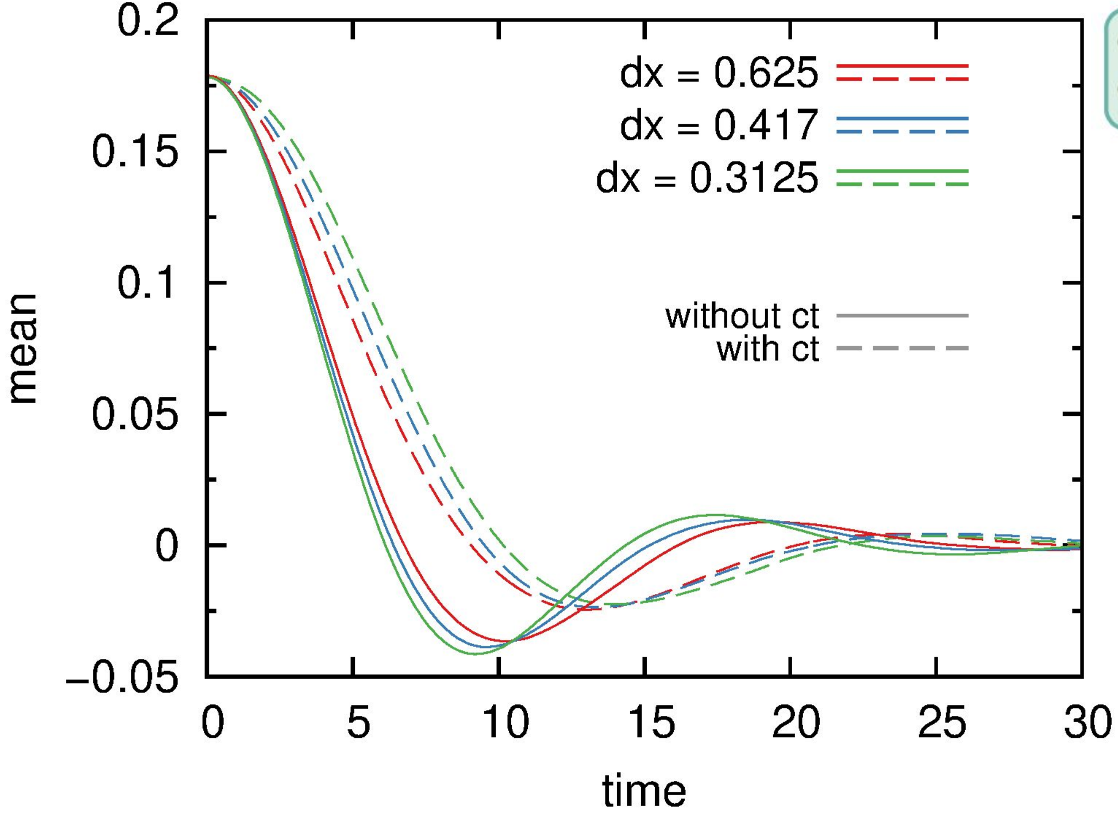
$\langle\varphi\rangle_{initial} = 0.1$
$\langle\varphi^2\rangle_{initial} = 0$





At early times, the counterterm doesn't seem to have a notable effect on the dx dependence

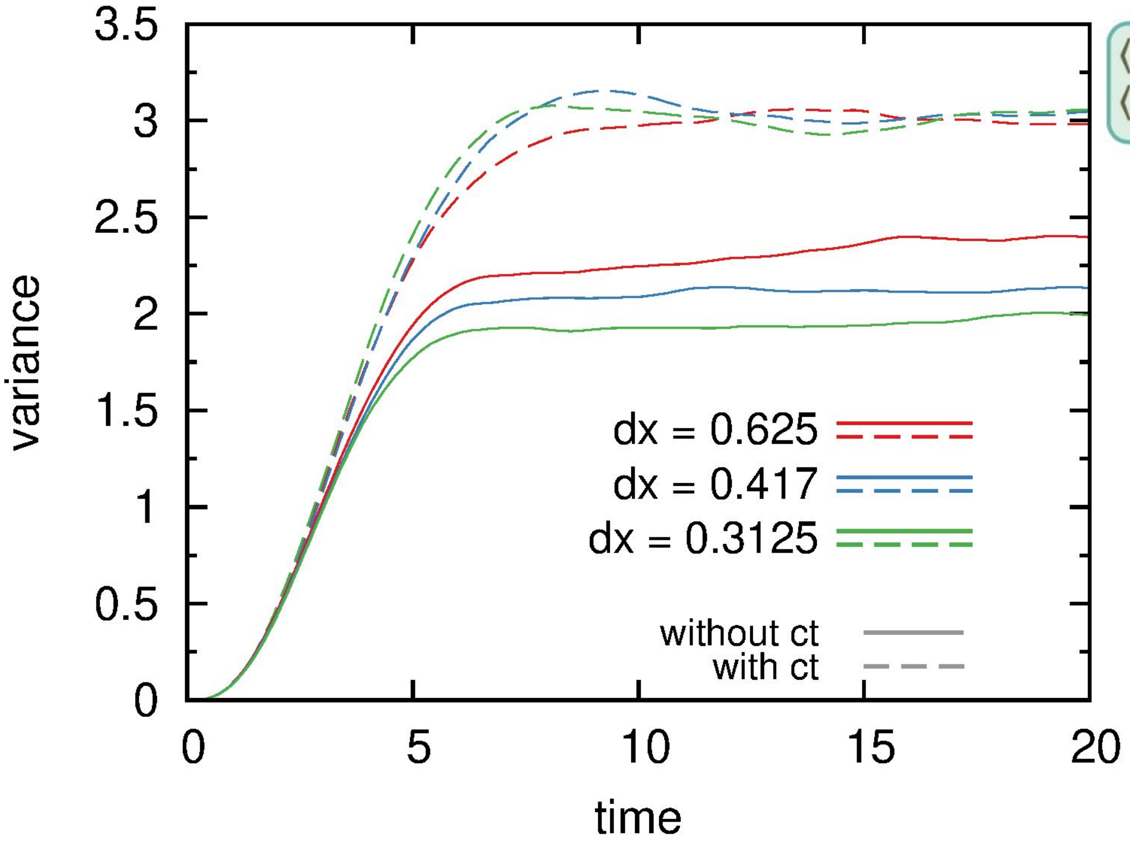
Dynamical Evolution: Mean (ε=0.1)



$\langle\varphi\rangle_{initial} = 0.1$
$\langle\varphi^2\rangle_{initial} = 0$

Close to the critical point, the dx dependence persists at early times for the mean

Dynamical Evolution: Variance (ε=0.1)



$\langle \varphi \rangle_{initial} = 0.1$
$\langle \varphi^2 \rangle_{initial} = 0$

The counterterm doesn't seem to have a noticeable effect on the variance in early times either

More statistics would be beneficial

Summary and Outlook

- Properly benchmarked the fluctuations in the code
- Demonstrated lattice spacing dependence of mean and variance
- Same counterterm works in equilibrium for mean and fluctuations both close to the critical point and away from it
- Equilibrium counterterm does not improve dynamical results at early times

➔ Compare relaxation and diffusion models which are expected to have different dynamical evolutions

➔ Derive dynamics for chiral field (V_{eff} and possibly η) in decoupled system first

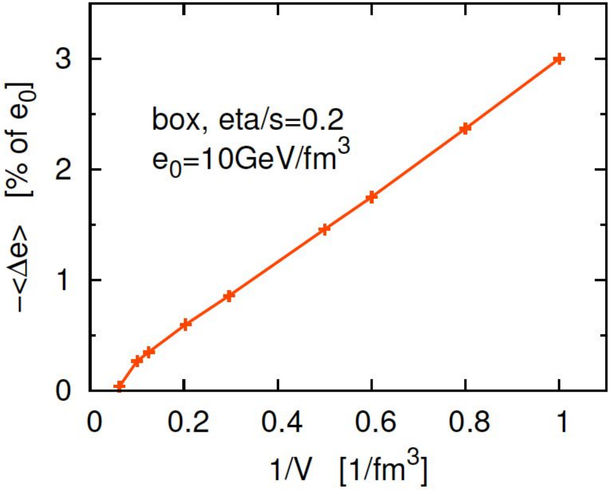
➔ Apply approach to the coupled chiral fluid dynamics: derive proper counterterm(s)

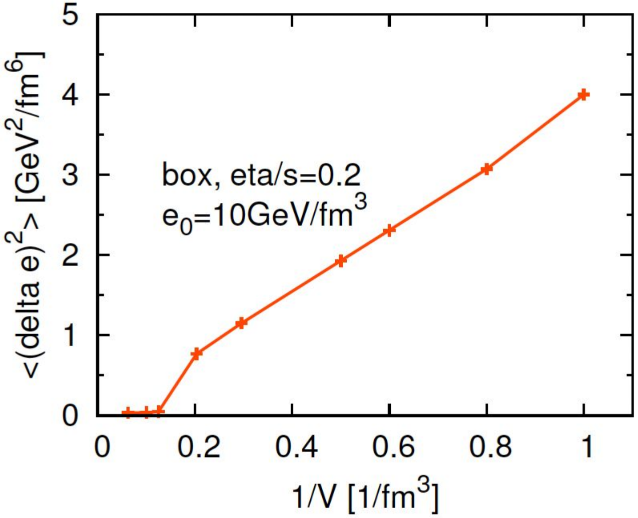
Backup: parameter file

```
Param(
  actions: [
    (Moments(["sig"]), TotalInterval(from: 0, to: 60, total: 1000)),
    (Window(["sig"]), At(60)),
    (StaticStructureFactor(["sig"], Radial), At(60)),
    (Correlation(["sig"],Radial), At(60)),
  ],
  config: (t_0: Some(0), t_max: 60, dim: D3S((32,20))),
  integrator: RK4(dt: 8.0000e-04),
  noises: Some([Normal(name: "xi")]),
  symbols: [
    "eta = 1
    eps = -1
    lam = 0.1
    T = 1
    sig' = psi
    psi' = #^2 sig - eta*psi - eps*sig -lam*sig^3 - sqrt(2*eta*T*ivdxyz*ivdt)*xi
    *sig = 0.1",
  ],
)
```

$\langle\varphi\rangle_{initial} = 0.1$
$\langle\varphi^2\rangle_{initial} = 0$
$(\partial_t\varphi)_{initial} = 0$
$10000\ simulations$

Backup: coarse-graining and filtering noise





Nahrgang, Bluhm, Schaefer, Bass
1704.03553v1

- ➢ Coarse-graining over grid of same scale
- ➢ Filtering large momenta modes in Fourier space
- ➢ Smearing by a Gauss distribution

2001.08831v1 [nucl-th] and references therein, 1704.03553

Backup: some equations

3d integral of correlation function in linear approximation

$$\int \frac{T}{4\pi r} e^{-\frac{r}{rc}} d\vec{r} = \frac{T}{4\pi} \int_{dx}^{X} \int_{0}^{\pi} \int_{0}^{2\pi} \frac{r^2 sin\theta}{r} e^{-\frac{r}{rc}} dr d\theta d\phi = T[(dx + r_c)e^{-\frac{dx}{rc}} - (X + r_c)e^{-\frac{X}{rc}}]$$

Volume of integration sphere

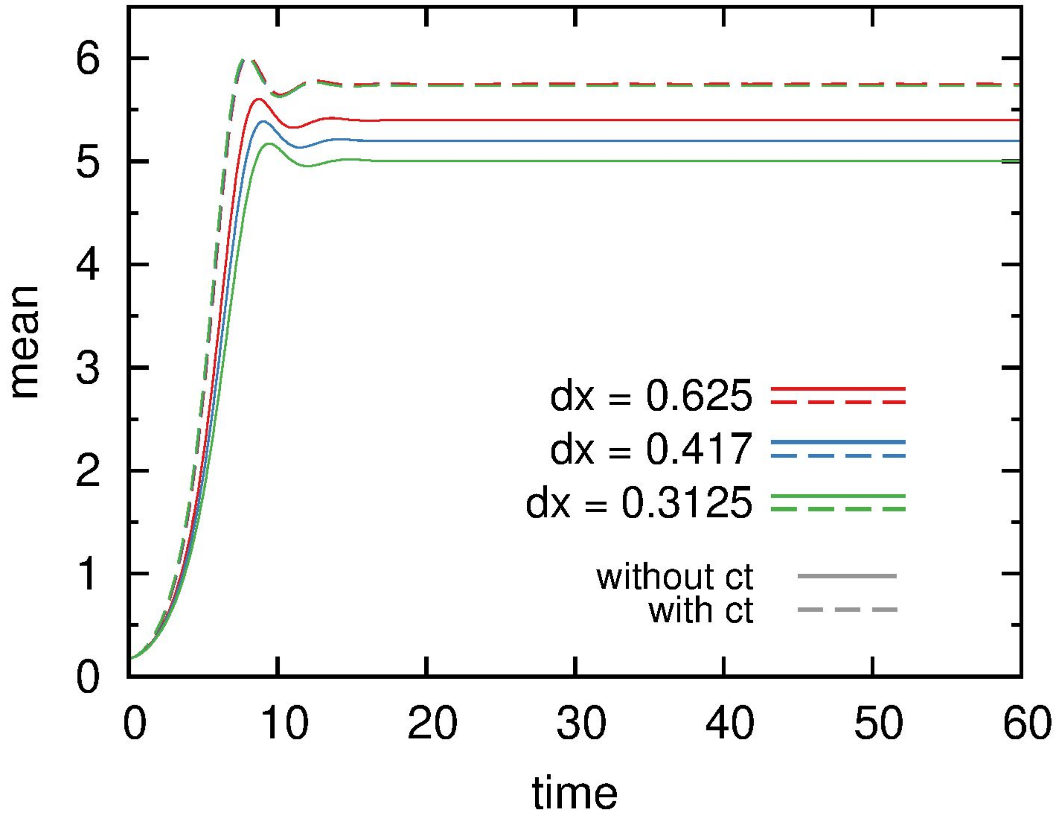
$$\frac{L}{2} < X < \frac{L\sqrt{3}}{2} \Rightarrow V \propto \frac{4}{3}X^3 - 2\pi(X - \frac{L}{2})^2(2X - \frac{L}{2})$$

$$X = \frac{L\sqrt{3}}{2} \Rightarrow V = L^3$$

Variance in linear approximation (before /V)

$$\langle \varphi^2 \rangle = VTr_c(r_c - r_c e^{-\frac{X}{rc}} - Xe^{-\frac{X}{rc}})$$

Backup: full plots ε=-1



Backup: ε=0.1 full plots