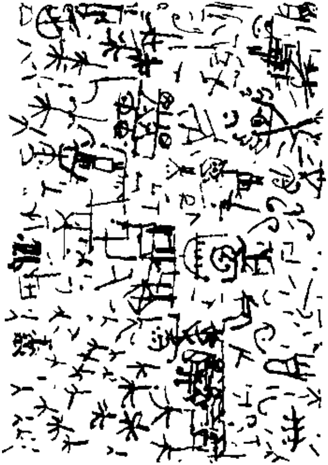
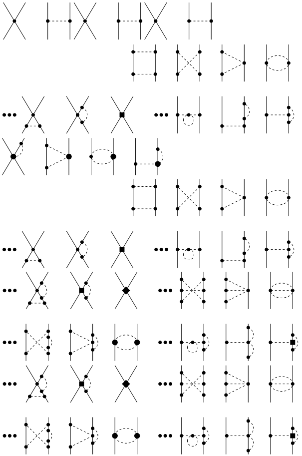
SU3 symmetry and link to clusterization

Frédéric Nowacki

The nuclear interaction: the complex view

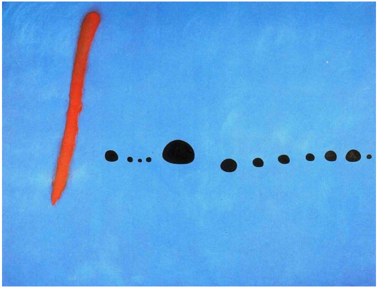


P. Klee, art

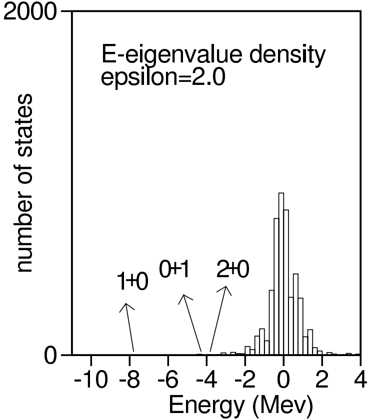


E. Epelbaum, physics

The nuclear interaction: the simple view



J. Miro, art



A. Zuker, physics

Separation of the effective Hamiltonian

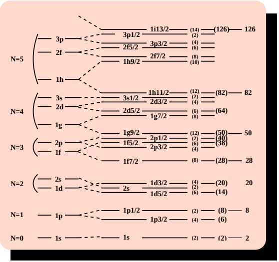
Monopole and multipole

Multipole expansion:

$$H = H_{monopole} + H_{multipole}$$

$H_{monopole}$:

- *Spherical mean-field*
- *Evolution of the spherical single particle levels*



$H_{multipole}$:

- *Correlations*
- *Energy gains*

☞ pairing, quadrupole

M. Dufour and A. Zuker (PRC 54 1996 1641)

The monopole hamiltonian

$$V = \sum_{JT} V_{ijkl}^{JT} \left[(a_i^+ a_j^+)^{JT} (\tilde{a}_k \tilde{a}_l)^{JT} \right]^{00}$$

In order to express the number of articles operators $n_i = a_i^+ a_i \propto (a_i^+ \tilde{a}_i)^0$,

→ particle-hole recoupling :

$$V = \sum_{\lambda\tau} W_{ikjl}^{\lambda\tau} \left[(a_i^+ \tilde{a}_k)^{\lambda\tau} (a_j^+ \tilde{a}_l)^{\lambda\tau} \right]^{00}$$

$$W_{ikjl}^{\lambda\tau} \propto \sum_{JT} V_{ijkl}^{JT} \left\{ \begin{array}{ccc} i & k & \lambda \\ j & l & \lambda \\ J & J & 0 \end{array} \right\} \left\{ \begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & \tau \\ \frac{1}{2} & \frac{1}{2} & \tau \\ T & T & 0 \end{array} \right\}$$

$\mathcal{H}_m$ corresponds only to the terms $\lambda\tau$ =00 and 01 which implies that $i = j$ and $k = l$ and writes as

$$\mathcal{H}_m = \sum_i n_i \epsilon_i + \sum_{i \leq j} n_i . n_j \, V_{ij}$$

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$$\mathcal{H}_M = \mathcal{H} - \mathcal{H}_m$$

Multipole Hamiltonian

$H_{multipole}$ can be written in two representations, particle-particle and particle-hole. Both can be brought into a diagonal form. When this is done, it comes out that only a few terms are coherent, and those are the simplest ones:

- $L = 0$ isovector and isoscalar pairing
- Elliott's quadrupole
- $\vec{\sigma}\vec{\tau} \cdot \vec{\sigma}\vec{\tau}$
- Octupole and hexadecapole terms of the type $r^{\lambda} Y_{\lambda} \cdot r^{\lambda} Y_{\lambda}$

Besides, they are universal (all the realistic interactions give similar values) and scale simply with the mass number

	pp(JT)			ph($\lambda\tau$)			
	10	01	21	20	40	10	11
KB	-5.83	-4.96	-3.21	-3.53	-1.38	+1.61	+3.00
USD-A	-5.62	-5.50	-3.17	-3.24	-1.60	+1.56	+2.99
CCEI	-6.79	-4.68	-2.93	-3.40	-1.39	+1.21	+2.83
NN+NNN-MBPT	-6.40	-4.36	-2.91	-3.28	-1.23	+1.10	+2.43
NN-MBPT	-6.06	-4.38	-2.92	-3.35	-1.31	+1.03	+2.49

Multipole Hamiltonian

$H_{multipole}$ ... le
and pa
When
cohere

- L =
- Ell
- $\vec{\sigma}\vec{\tau}$
- Oc

Beside
similar

KB	0
USD	9
CCE	8
NN+	8
NN-	9

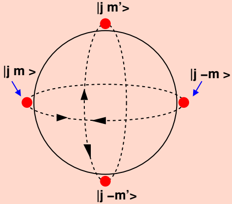
- **Pairing regime: spherical nuclei**

  ground state = pairs of like-particles coupled at J=0 (seniority v=0)

  2^+ state (break of pair; v=2) at high energy

  Underlying SU2 symmetry

  superfluid nucleus:

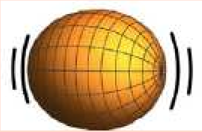


  Typical example: **Tin isotopes**

- **Quadrupole regime: deformed nuclei**

  Underlying SU3 symmetry

  prolate nucleus:



  Typical example: **open shell N=Z nuclei**

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-5.48	-6.24	BONNB	-2.82	-1.39	+3.64
-5.69	-5.90	USD	-3.18	-1.60	+3.08
-4.75	-4.46	KB3	-2.79	-1.39	+2.46
-5.06	-5.08	FPD6	-3.11	-1.67	+3.17
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Elliott's SU(3) model

see Eisenberg and Greiner, Nuclear Theory volume 3, Microscopic Theory of the nucleus (chapter on Nuclear rotations)

- Assuming spin-isospin SU(4) symmetry (no spin-orbit)
- and a quadrupole-quadrupole residual interaction:

$$\mathcal{H} = \sum_k \left(\frac{p_k^2}{2m} + \frac{1}{2} m\omega^2 r_k^2 \right) + \kappa Q.Q$$

which can be rewritten

$$\mathcal{H} = \sum_k \left(\frac{p_k^2}{2m} + \frac{1}{2} m\omega^2 r_k^2 \right) + 4\kappa C_{SU3} - 3\kappa(\vec{L}.\vec{L})$$

the eigenenergies have the following form:

$$E = \hbar\omega \left(N + \frac{3}{2} \right) + 4\kappa(\lambda^2 + \lambda\mu + \mu^2 + 3(\lambda + \mu)) - 3\kappa L(L + 1)$$

where λ and μ are the labels of the SU3 irrep. and L the angular momentum. Therefore it gives a description of deformation via a rotationally invariant mixing of spherical orbits.

Spherical Shell Model and Deformation

Consider the quadrupole force alone, taken to act in the *p*-th oscillator shell. It will tend to maximize the quadrupole moment, which means filling the lowest orbits obtained by diagonalizing the operator

$$Q_0 \;=\; 2z^2 - x^2 - y^2$$

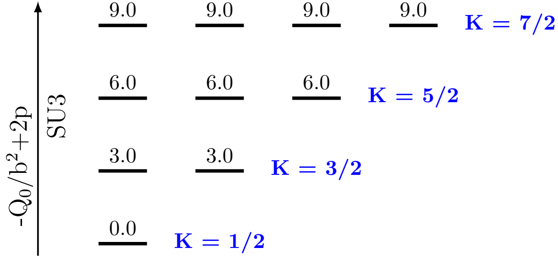
Using the cartesian representation,
$Q_0 = 2n_z - n_x - n_y = 3n_z - N$, we find eigenvalues $2p$, $2p - 3, \ldots$, etc.

By filling the orbits orderly we obtain the intrinsic states for the lowest SU(3) representations:

- $(\lambda, 0)$ if all states are occupied up to a given level
- (λ, μ) otherwise

Spherical Shell Model and Deformation

Diagonalization of the operator $Q_0 = 2z^2 - x^2 - y^2$ in a major HO-shell without spin-orbit (SU3-Nilsson-like single particle levels)



π, ν

k=2p

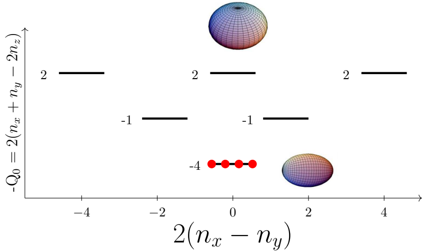
Example in the *sd* shell

In the *sd* shell, N=2

$$N = n_x + n_y + n_z$$

There are 6 possible (n_x, n_y, n_z):
(2,0,0) (0,2,0) (0,0,2)
(1,1,0) (1,0,1) (0,1,1)

$$Q_0 = 2n_z - n_x - n_y = (4, 1, -2)$$



Starting filling from below → prolate deformation

Starting filling from above → oblate deformation

Deformed harmonic oscillator and SU3 model

$$h = -\frac{\hbar^2}{2m}\Delta + \frac{1}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

spherical orbits $|nljm\tau\rangle$ → (n_x, n_y, n_z)

due to σ and τ each (nx, ny, nz) state is 4 fold degenerate

Correspondance between intrinsic states of harmonic oscillator and SU3 states:

Highest Weight state (λ, μ) build on (n_x, n_y, n_z) state at the spherical limit

$$\begin{cases} N = n_x + n_y + n_z \\ \lambda = n_z - n_x \\ \mu = n_x - n_y \end{cases}$$

Spherical Shell Model and Deformation

Nucleus	configuration	Intrinsic state	$(\lambda\mu)$	shape
^{4}He	0p0h	$(000)^4$	(0,0)	spherical
^{8}Be	0p0h	$(000)^4(001)^4$	(4,0)	prolate
^{10}Be	0p0h	$(000)^4(001)^4(100)^2$	(4,2)	prolate
	2p2h	$(000)^4(001)^4(002)^2$	(6,0)	prolate
^{12}C	0p0h	$(000)^4(100)^4(010)^4$	(0,4)	oblate
	4p4h	$(000)^4(001)^4(002)^4$	(12,0)	prolate
^{16}O	0p0h	$(000)^4(100)^4(010)^4(001)^4$	(0,0)	spherical
	4p4h	$(000)^4(100)^4(001)^4(002)^4$	(8,4)	triaxial
	8p8h	$(000)^4(001)^4(002)^4(003)^4$	(24,0)	prolate
^{20}Ne	0p0h	$(000)^4(100)^4(010)^4(001)^4(002)^4$	(8,0)	prolate
^{24}Mg	0p0h	$(000)^4(100)^4(010)^4(001)^4(002)^4(101)^4$	(8,4)	triaxial
^{28}Si	0p0h	$(000)^4(100)^4(010)^4(001)^4(200)^4(020)^4(110)^4$	(0,12)	oblate
	0p0h	$(000)^4(100)^4(010)^4(001)^4(002)^4(101)^4(011)^4$	(12,0)	prolate

Deformed harmonic oscillator, even if inadequate for a detailled description contains already many clusterization effects in light nuclei

- Y. Abgrall, G. Baron, E. Caurier and G. Monsonego
  Nuc. Phys. **A131** (1969) 609
- Y. Abgrall, B. Morand, and E. Caurier
  Nuc. Phys. **A192** (1972) 372

Spherical Shell Model and Deformation

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JOURNAL DE PHYSIQUE *Colloque C6, supplément au n° 11-12, Tome 32, Novembre-Décembre 1971, page C6-63*

DEFORMED STRUCTURES AND ALPHA-PARTICLE DESCRIPTION OF LIGHT NUCLEI

Y. ABGRALL

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and

E. CAURIER

Laboratoire de Physique Théorique, Centre de Recherches Nucléaires de Strasbourg, France

- Y. Abgrall, B. Morand, and E. Caurier
  Nuc. Phys. **A192** (1972) 372

Spherical Shell Model and Deformation

Moreover, it can be shown to be a limit of the Brink α model when distances go to zero:

Case of ^{8}Be:

$$\Psi = \mathcal{A}|\phi(r_1 r_2 r_3 r_4; d_1)\phi(r_5 r_6 r_7 r_8; d_2)\rangle \text{ with } \phi(r_k, d_i) = Nexp - \frac{(r-d_i)^2}{2b^2},$$

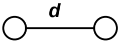
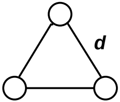
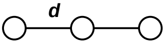
$$\phi_1 = e^{-\frac{(x-d)^2}{2b^2}} \text{ and } \phi_2 = e^{-\frac{(x+d)^2}{2b^2}}$$

develop when $d \to 0$ as

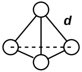
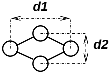
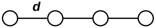
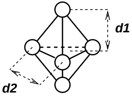
$$e^{-\frac{x^2}{2b^2}}(1 - \frac{dx}{2b^2} + \frac{1}{2}(\frac{dx}{2b^2})^2 - ...) \text{ and } e^{-\frac{x^2}{2b^2}}(1 + \frac{dx}{2b^2} + \frac{1}{2}(\frac{dx}{2b^2})^2 + ...)$$

After orthogonalization one gets $\boldsymbol{e^{-\frac{x^2}{2b^2}}}$ and $\boldsymbol{xe^{-\frac{x^2}{2b^2}}}$,
so the Slater Determinant built on these functions corresponds to the $(000)^4(001)^4$ state of the deformed harmonic oscillator

Spherical Shell Model and Deformation

Nucleus	configuration	limit	Intrinsic state	$(\lambda\mu)$
^{8}Be		$d \to 0$	$(000)^4(001)^4$	(4,0)
^{12}C		$d \to 0$	$(000)^4(100)^4(010)^4$	(0,4)
		$d \to 0$	$(000)^4(001)^4(002)^4$	(12,0)

Spherical Shell Model and Deformation

Nucleus	configuration	limit	Intrinsic state	$(\lambda\mu)$
^{16}O		$d \to 0$	$(000)^4(100)^4(010)^4(001)^4$	(0,0)
		$d_1 \to 0$, $\frac{d_1}{d_2} \to \infty$	$(000)^4(100)^4(001)^4(002)^4$	(8,4)
		$d \to 0$	$(000)^4(001)^4(002)^4(003)^4$	(24,0)
^{20}Ne		$d_1 \to 0$, $\frac{d_1}{d_2} \to \infty$	$(000)^4(100)^4(010)^4(001)^4(002)^4$	(8,0)

Brink α cluster model $\to$ Deformed H.O. $\to$ SU3 eigenstates

Spherical Shell Model and Deformation

- nuclear shell model often considered to be applied only when nuclear manifestations are dominated by single particle degrees of freedom
- BUT work of Elliot: deformation in light nuclei explained by algebraic SU3 model

QUESTIONS:

- Is it possible to describe rotationnal motion within the spherical shell model, beyond *sd* shell where SU3 applies ?
- which are the minimal valence spaces containing the relevant degrees of freedom ?
- Is there anything like an intrinsic state in the shell model wavefunctions ?

Spherical Shell Model and Deformation

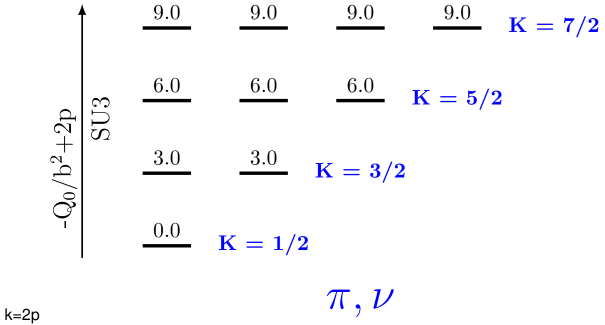
- Limitations of SU3 model:
  - as the spin orbit term becomes rapidly important its applicability stops at the sd shell
  - but can be recovered approximately as in the **pseudo-SU3** or **quasi-SU3** schemes.

See:

- A. P. Zuker, J. Retamosa, A. Poves, and E. Caurier Phys. Rev. bf C52, R1741 (1995)
- A. P. Zuker, A. Poves, F. Nowacki, and S. Lenzi Phys. Rev. bf C92, 024320 (2015)

Spherical Shell Model and Deformation

The resulting "quasi SU(3)" quadrupole operator respects SU(3) relationships, except for $m = 0$, where the correspondence breaks down. The resulting spectrum is a quasi-2q_{20} (to be compared with the SU3 one). The result is not exact for the $K = 1/2$ orbits but a very good approximation.



Diagonalization of the operator $Q_0 = 2z^2 - x^2 - y^2$ in a major HO-shell without spin-orbit in the subspace of the aligned orbits (Quasi-SU3)

Spherical Shell Model and Deformation

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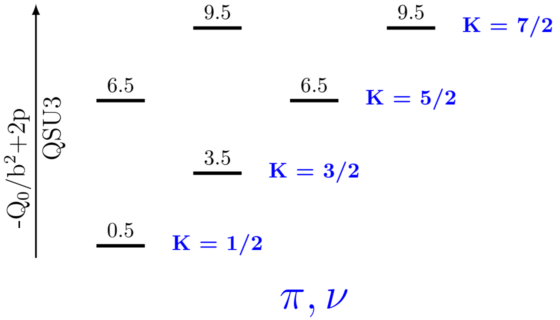


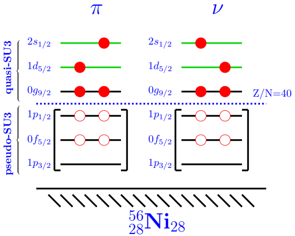
k=2p-1/2

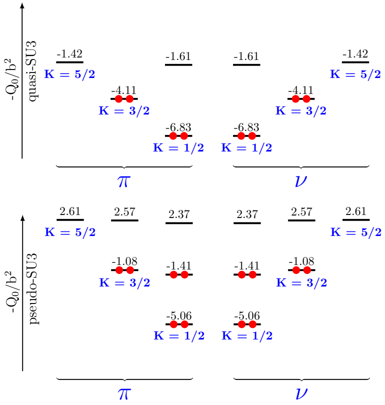
Diagonalization of the operator $Q_0 = 2z^2 - x^2 - y^2$ in a major HO-shell without spin-orbit in the subspace of the aligned orbits (Quasi-SU3)

Island of Inversion at the N=Z line

◇ Strongly deformed states at $N = Z$:

- Shape transition between ^{84}Mo and ^{86}Mo
- Configuration mixing in ^{72}Kr
- Most deformed cases for ^{76}Sr, ^{80}Zr





nucleus	NpNh*	ZRP	PHF	DNO-SM	Exp.
		B(E2)(e^2.fm^4)			
^{76}Se	4p-4h	924	806	1847	2220
	8p-8h	2189	2101		
	12p-12h	2316	-		
^{80}Zr	4p-4h	587	637	2325	1910
	8p-8h	1713	1509		
	12p-12h	2663	2396		

Island of Inversion at the N=Z line

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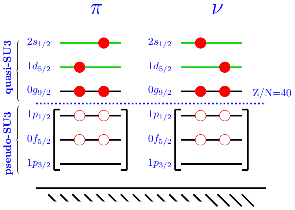


R.D.O. Llewellyn *et al.*, Phys. Rev. Lett. **124**, 152501 (2020)

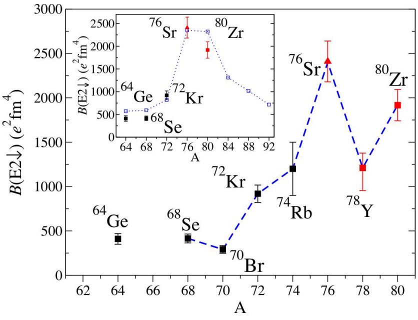
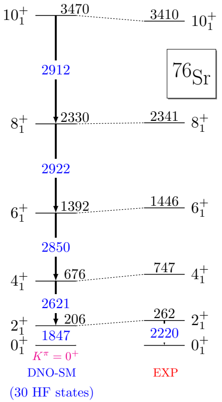


FIG. 3. Schematics of the $B(E2\downarrow)$ values for the $N = Z$ nuclei



SU2 Quasi-Spin for Shell-Model

The wave function is written as successive coupling of one shell wave functions (c. f. p. 's) defined by $|(j_i)^{n_i} v_i \gamma_i x_i\rangle$:

$$\Big[\; \big[\; |(j_1)^{n_1} v_1 \gamma_1 x_1\rangle \; |(j_2)^{n_2} v_2 \gamma_2 x_2\rangle \; \big]^{\Gamma_2} \; ... \; |(j_k)^{n_k} v_k \gamma_k x_k\rangle \Big]^{\Gamma_k}$$

Single shell matrix elements calculation can be simplified with the quasi-spin formalism

$SU(2)$ Algrebra for Quasi-Spin:

$$S^+ = -\sqrt{\frac{\Omega}{2}}(\tilde{a}_j \tilde{a}_j)^0 \quad S^- = -\sqrt{\frac{\Omega}{2}}(a_j^\dagger a_j^\dagger)^0$$

$$S_z = \frac{\Omega - n}{2} \qquad S = \frac{\Omega - v}{2}$$

with $\Omega = (2j+1)/2$ and n, number of particles in the shell j

SU2 Quasi-Spin for Shell-Model

From $|nv\rangle$ to $|SS_z\rangle$ representation and operators expressed in the two body interaction are spherical tensors in the Quasi-spin space:

- $A \equiv \begin{pmatrix} \tilde{a}_j \\ a_j^\dagger \end{pmatrix}$ tensor of rank $\frac{1}{2}$
- $(AA)^\omega$
  - ω even : tensor of rank 1
  - ω odd : tensor of rank 0
- $[(AA)^\omega A]^j$ mixed tensors of rank $\frac{1}{2}$ et $\frac{3}{2}$
- $[(AA)^\omega (AA)^\omega]^0$ mixed tensors of rank 0, 1 and 2

SU2 Quasi-Spin for Shell-Model

Two main advantages:

- easy C.F.P. calculation:

  $\langle n v \gamma x || \mathcal{O} || n' v' \gamma' x' \rangle = \langle S \gamma x ||| \mathcal{O} ||| S' \gamma' x' \rangle \times$Clebsh-Gordan coeff.

  Matrix elements are doubly reduced in spin and Quasi-spin

- easier calculation of $\mathcal{N}$ body matrix elements :

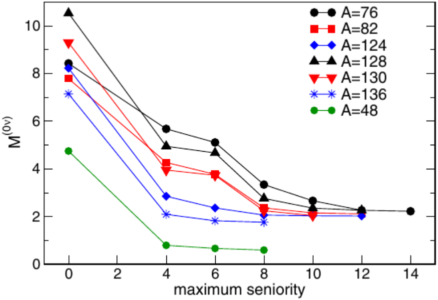
  $$\begin{aligned} & \langle S S_z \gamma \Gamma || \mathcal{O} || S' S'_z \gamma' \Gamma' \rangle \\ = & \langle S \gamma \Gamma || \mathcal{O}_1 || S' \gamma' \Gamma' \rangle \, \langle S S_z | \mathcal{O}_2 | S' S'_z \rangle \end{aligned}$$

  Instead of computing one large matrix $\langle q \times r| \ |q' \times r' \rangle$,
  one is left with two smaller matrices $\langle q| \ |q' \rangle$ et $\langle r| \ |r' \rangle$

Pairing correlations and $0\nu\beta\beta$ decay

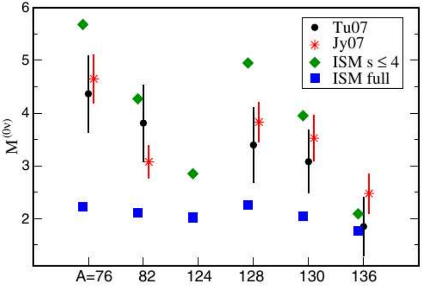
$0\nu\beta\beta$ decay favoured by proton-proton, neutron-neutron pairing, but it is disfavored by proton-neutron pairing

Ideal case: superfluid nuclei reduced with high-seniorities



E. Caurier et al., PRL100 052503 (2008)

Addition of isoscalar pairing reduces matrix element value



E. Caurier et al., PRL100 052503 (2008)

Related to approximate SU(4) symmetry of the $\sum H(r)\sigma_i\sigma_j\tau_i\tau_j$ operator

Summary

- Multipole Hamiltonian decomposition shows leading Pairing and Quadrupole terms
- These terms carry underlying SU2/SU3 symmetries
- Intimate natural relation between
  Brink α cluster model $\rightarrow$ Deformed H.O. $\rightarrow$ SU3 eigenstates
- SU3 variants (pseudo/quasi) still at play and very efficient for development of deformation in heavier mass regions

Quasi-SU3

In *jj* coupling the angular part of the quadrupole operator $q^{20} = r^2 C^{20}$ has matrix elements

$$\langle j\, m | C^2 | j+2\, m \rangle \approx \frac{3[(j+3/2)^2 - m^2]}{2(2j+3)^2},$$

$$\langle j\, m | C^2 | j+1\, m \rangle = -\frac{3m[(j+1)^2 - m^2]^{1/2}}{2j(2j+2)(2j+4)}$$

The $\Delta j = 2$ numbers are—within the approximation made—identical to those in *LS* scheme, obtained by replacing *j* by *l*. The $\Delta j = 1$ matrix elements are small, both for large and small *m*, corresponding to the lowest oblate and prolate deformed orbits respectively.

Spherical Shell Model and Deformation

		LS coupling		$l \gg \|m\|$
$\langle lm\|\tilde{Q}_{20}\|lm\rangle$	=	$\dfrac{l(l+1)-3m^2}{(2l-1)(2l+3)}$	$\rightarrow$	$\dfrac{1}{4}$
$\langle lm\|\tilde{Q}_{20}\|l+2\ m\rangle$	=	$\dfrac{3[(l+1)^2-m^2]^{1/2}[(l+2)^2-m^2]^{1/2}}{2(2l+1)^{1/2}(2l+3)(2l+5)^{1/2}}$	$\rightarrow$	$\dfrac{3}{8}$
		jj coupling		$j \gg \|m\|$
$\langle jm\|\tilde{Q}_{20}\|jm\rangle$	=	$\dfrac{j(j+1)-3m^2}{4j(j+1)}$	$\rightarrow$	$\dfrac{1}{4}$
$\langle jm\|\tilde{Q}_{20}\|j+1\ m\rangle$	=	$\dfrac{3m[(j+1)^2-m^2]^{1/2}}{4j(j+1)(j+2)}$	$\rightarrow$	$\dfrac{3m}{4} \sim 0$
$\langle jm\|\tilde{Q}_{20}\|j+2\ m\rangle$	=	$\dfrac{3[(j+1)^2-m^2]^{1/2}[(j+2)^2-m^2]^{1/2}}{8(j+1)(j+2)}$	$\rightarrow$	$\dfrac{3}{8}$

Quasi-SU3

If the spherical j-orbits are degenerate, the $\Delta j = 1$ couplings, though small, will mix strongly the two $\Delta j = 2$ sequences (e.g., $(f_{7/2}p_{3/2})$ and $(f_{5/2}p_{1/2})$). The spin-orbit splittings will break the degeneracies and favour the decoupling of the two sequences. Hence the idea of neglecting the $\Delta j = 1$ matrix elements and exploit the correspondence

$$l \longrightarrow j = l + 1/2 \quad m \longrightarrow m + 1/2 \text{ and } -m \longrightarrow -m - 1/2$$

which is one-to-one (except for $m = 0$).