The role of the nuclear radius in *Mirror* energy differences

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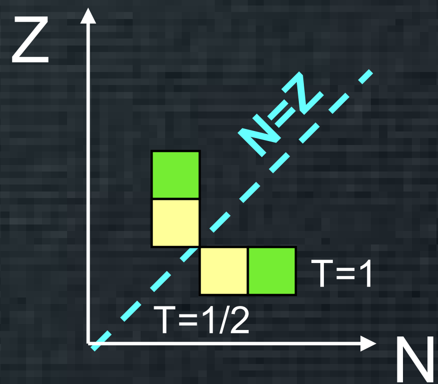
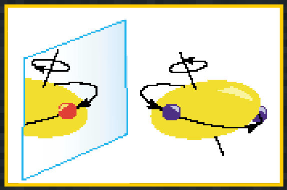
PhyNuBe
Clustering and symmetries in nuclear physics
Aussois, March 26-31, 2023

Mirror energy differences



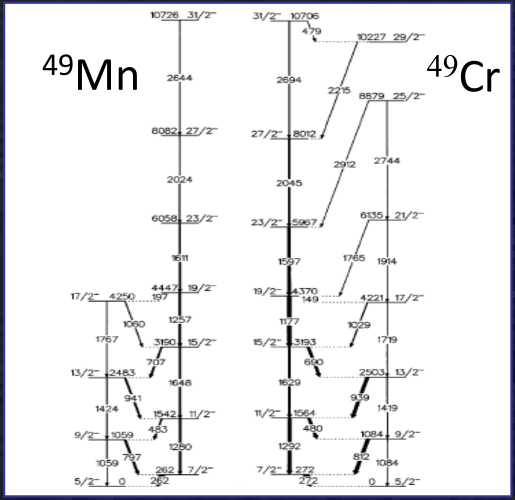


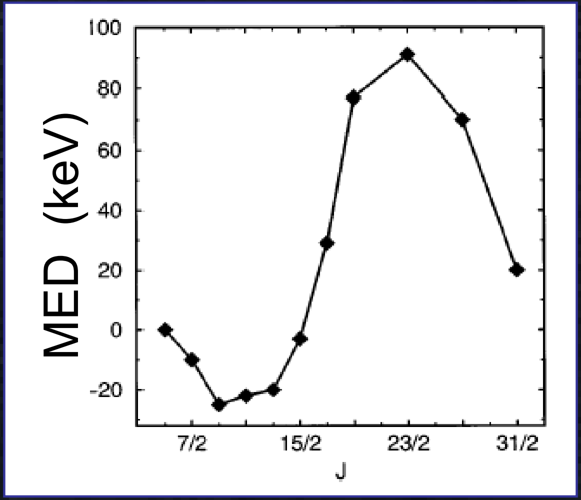
difference in excitation energies

$$MED_J = E^*_{J,-|T_z|} - E^*_{J,|T_z|}$$

Test the charge symmetry of the interaction

$V_{pp} = V_{nn}$





These (small) differences are mainly due to the Coulomb interaction

What can we learn from MED

They contain a richness of information
about spin-dependent structural phenomena

We can compute these energy differences
with the shell model and learn about
nuclear structure features:

- How the nucleus generates its angular momentum
- Isospin non-conserving terms of the interaction
- Evolution of radii (deformation) along a rotational band
- Estimate the neutron skin
- Learn about the configuration of the states

Outline

First part: show the effect of the nuclear radius changes as a function of the angular momentum in predicting the MED

Second part: use the measured MED to estimate the nucleon skin for every excited state

Third part: the MED of non-natural parity states as a test of the type of p-h excitations across the gap

Calculating the MED

We start from diagonalizing a nuclear hamiltonian that conserves isospin and treat Coulomb and other eventual isospin symmetry breaking (ISB) contributions perturbatively

$$H = H_{nuc} + V_C + V_B$$
$$V_C + V_B = V_{CM} + V_{Cm} + V_B$$

Multipole Coulomb V_{CM}

- correlations
- energy gains

V_B Isospin symmetry breaking term of non-Coulomb origin

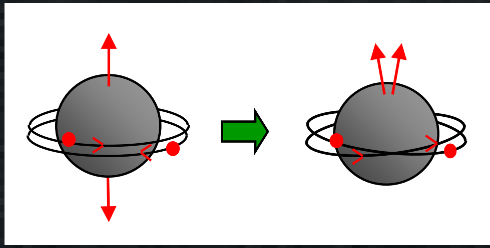
monopole Coulomb V_{Cm}

- represents a spherical mean field extracted from the interacting shell model
- determines the single-particle energies and the shell evolution

The different terms in the MED

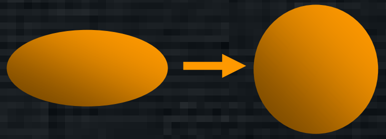
$$MED_J^{theo} = \Delta\langle V_{CM}\rangle_J + \Delta\langle V_{Cm}\rangle_J + \Delta\langle V_B\rangle_J$$

V_{CM} Multipole part of the Coulomb interaction → Between valence protons only



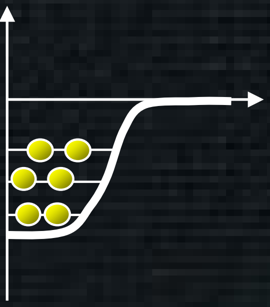
V_{Cm} monopole part of the Coulomb interaction:

- **radial effect: radius changes with J**



- $\ell\cdot\ell$ term to account for shell effects
- $\ell\cdot s$ electromagnetic spin-orbit term

→ **change the single-particle energies**



V_B isospin symmetry breaking term

$$V_{\pi\pi}^{J=0} - V_{\nu\nu}^{J=0} = -100\ \text{keV}$$

for all orbits

A.P. Zuker et al., PRL 89, 142502 (2002)
M.A. Bentley and S.M.L., PPNP 59, 497 (2007)

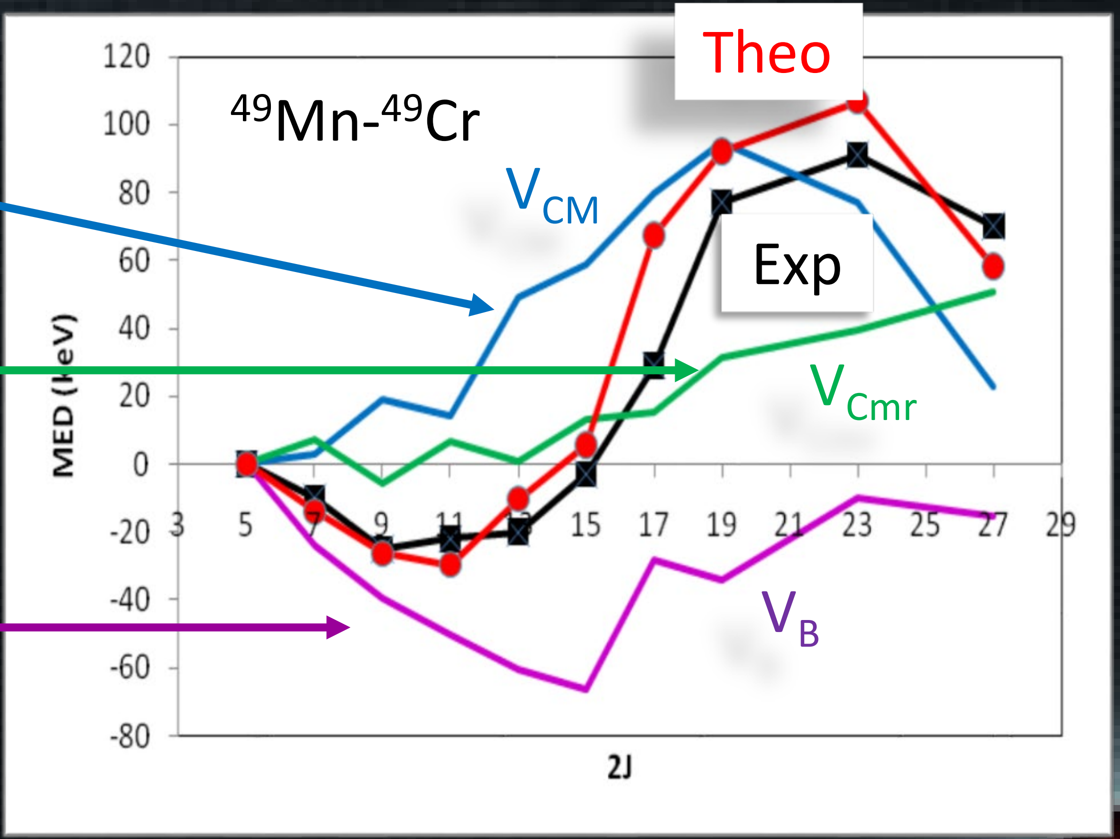
Calculating the MED with SM

$$MED_J^{theo} = \Delta\langle V_{CM}\rangle_J + \Delta\langle V_{Cm}\rangle_J + \Delta\langle V_B\rangle_J$$

VCM: gives information on the nucleon alignment or recoupling

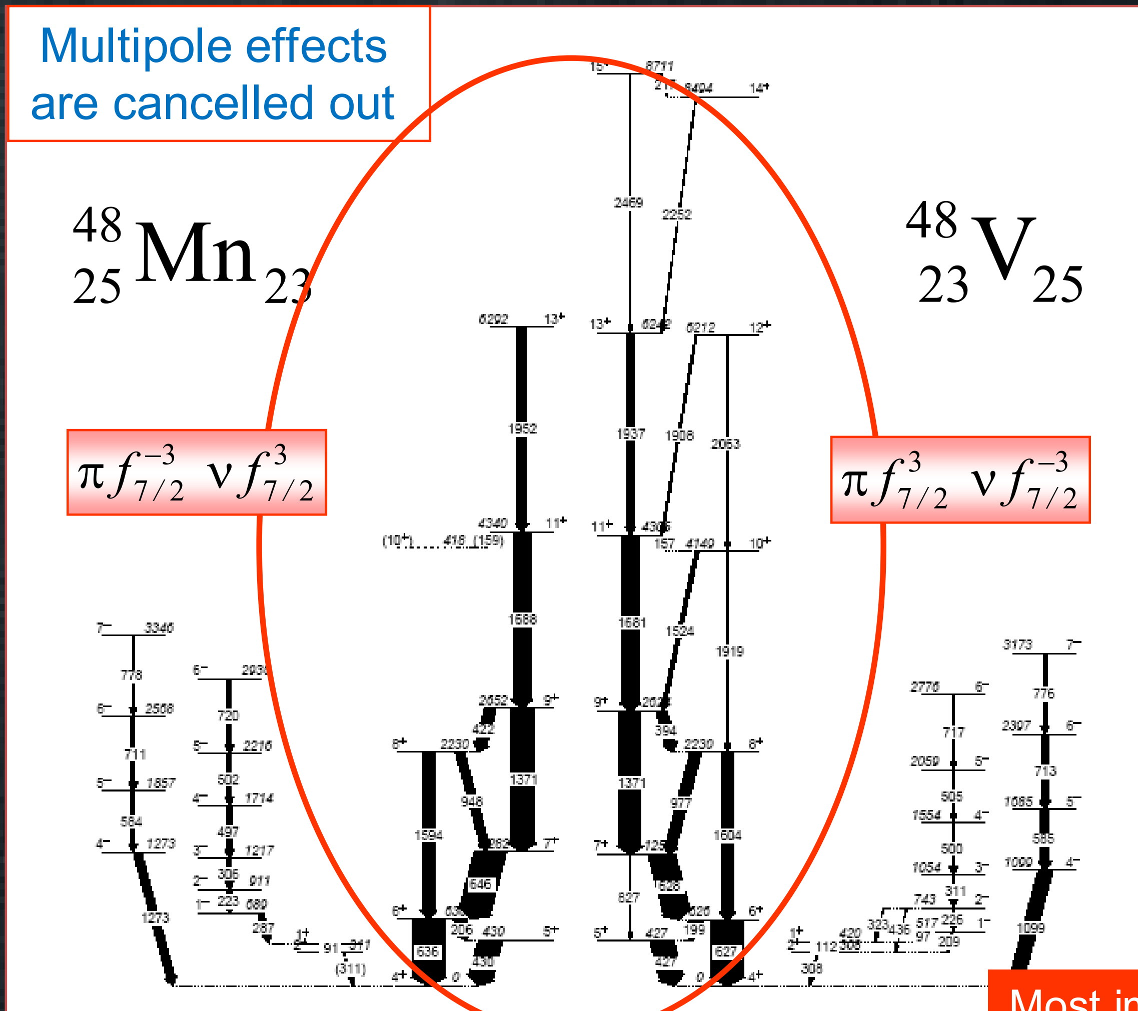
VCmr: gives information on changes in the nuclear radius

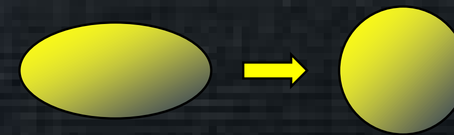
Important contribution from the ISB VB term:
of the same order as the Coulomb contributions

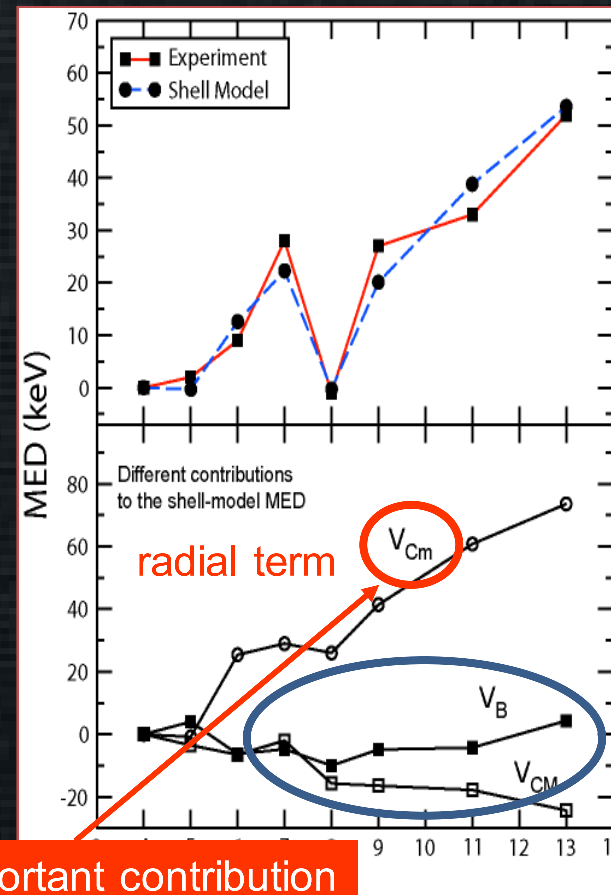


A. P. Zuker et al., PRL 89, 142502 (2002)

Evidence of the Coulomb radial effect







M.A. Bentley et al., PRL 97, 132501 (2006)

The nucleus changes shape towards band termination

The evolution of the radius

Coulomb energy of a charged sphere:

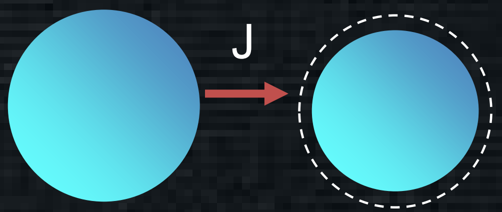
$$E_C = \frac{3Z(Z-1)e^2}{5R_C}$$

The difference between the energy of the ground states:

$$T_z = \pm\frac{n}{2}$$

$$\Delta E_C(J=0) = E_C(Z_>) - E_C(Z_<) = \frac{3n(2Z_> - n)e^2}{5R_C}$$

If R_C changes as a function of the angular momentum…



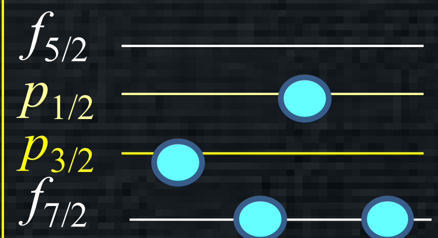
$$\Delta E_{Cr}(J) = \Delta E_C(J) - \Delta E_C(0) = \frac{3}{5}n(2Z_> - n)e^2\left(\frac{R_C(0) - R_C(J)}{R_C^2}\right)$$

$$= -\frac{3}{5}n(2Z_> - n)e^2\,\frac{\Delta R_C(J)}{R_C^2} = nC\cdot\Delta R_C(J)$$

Radial contribution to the MED

The radial effect with the shell model

The radius of a nucleus depends on the occupation of the different orbitals.
In the *fp* shell *p* orbits have larger radius than the *f*.
The radial term will depend on the change of occupation of the *p* orbitals as a function of *J*



$$V_{Cm,r}(J) = 2|T_z|\alpha \frac{(z_p + n_p)_J}{2}$$

z_p and n_p are the number of protons and neutrons in the p orbits, relative to the g.s. (J=0)

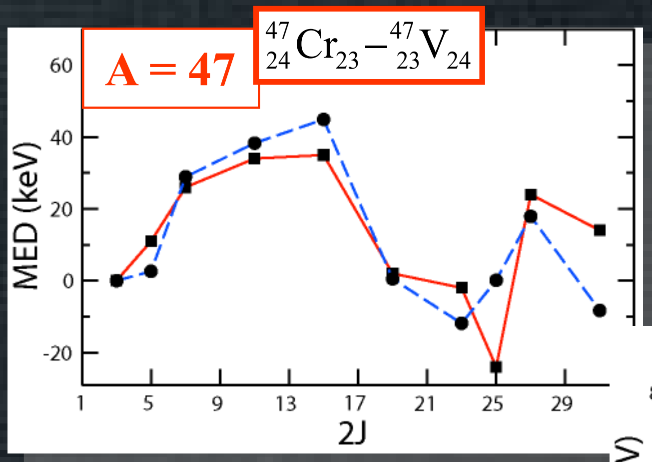
The radial monopole term depends on the occupation of the *p* orbits

α is not a free parameter but can be estimated from experimental data:

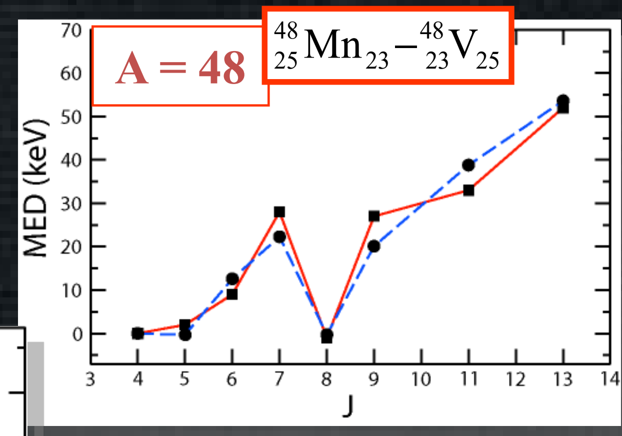
The radial parameter amounts to *α* ~ 200 keV for nuclei in the $f_{7/2}$ shell

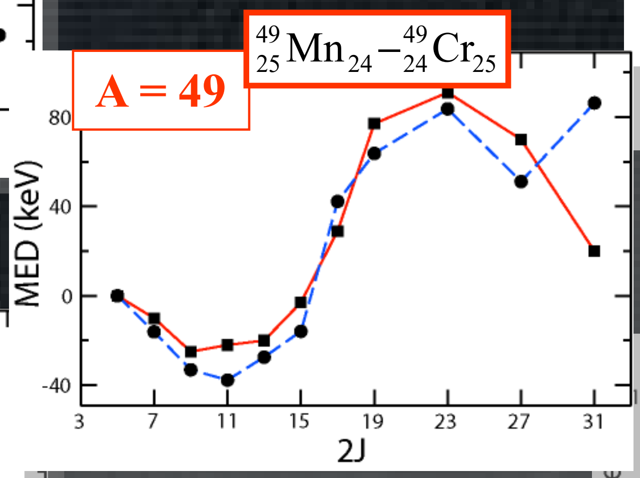
MED in the $f_{7/2}$ shell

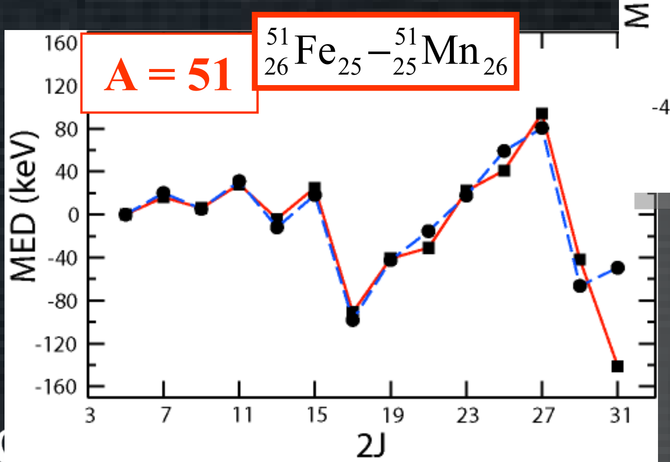
Very good quantitative description of data without free parameters



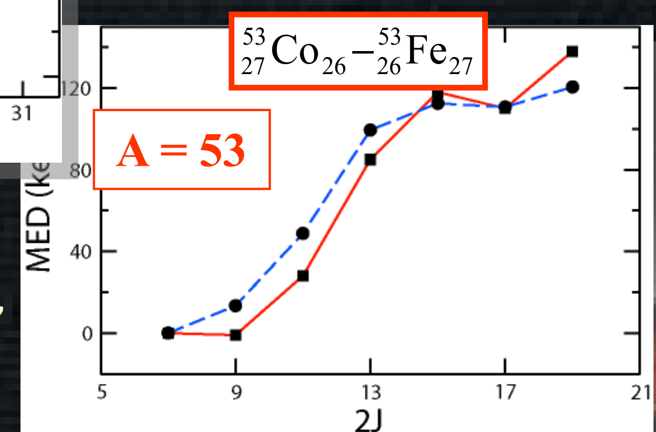
Experiment
Shell Model





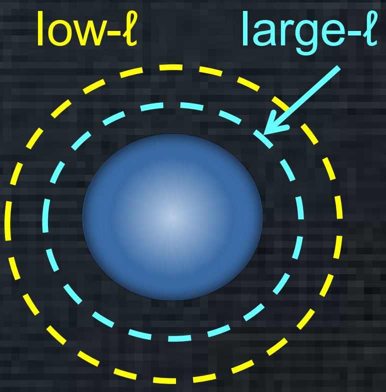


M.A. Bentley and SML, Prog. Part. Nucl. Phys. 59, 497-561 (2007)



The size of the orbital radius

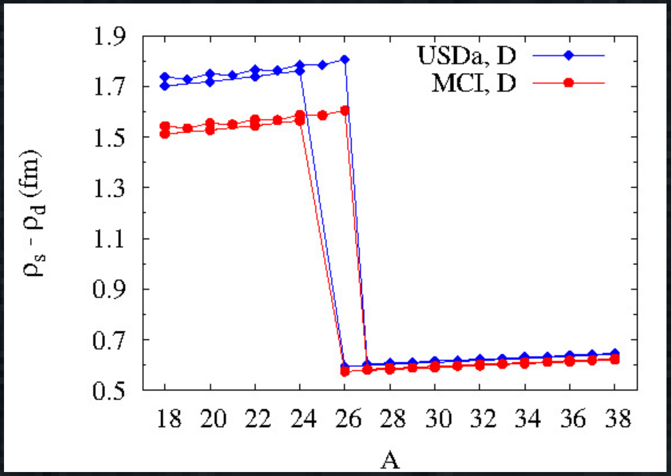
In a main shell, the radial extension of low-ℓ orbits is much larger that the others



In the sd shell Bonnard and Zuker have found a very peculiar behaviour of the $1s_{1/2}$ orbit:

$$r_s - r_d \approx 1.6 \text{ fm} \qquad Z, N \leq 14$$

$$r_s - r_d \approx 0.6 \text{ fm} \qquad Z, N > 14$$



A similar behavior is predicted in the *fp* shell: when the *p* orbits are occupied by one or more nucleons, the orbital radius decreasses.

$$V_{Cm,r}(J) = |T_z| \alpha \left(z_p + n_p\right)_J$$

$$\alpha \approx 200 \text{ keV for } (z_p + n_p) < 1$$

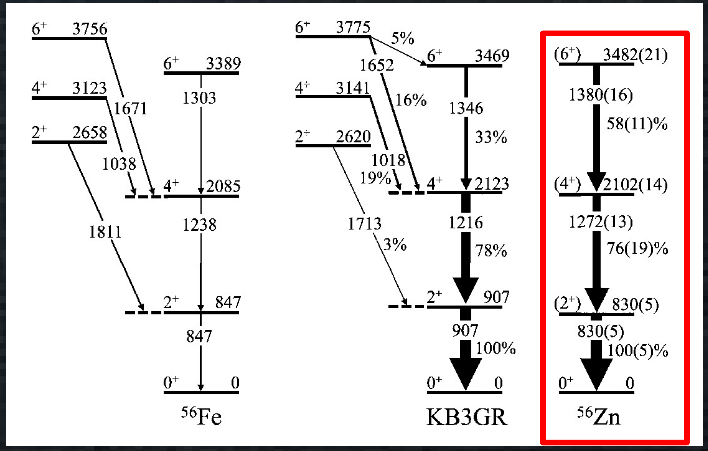
$$\alpha << 200 \text{ keV for } (z_p + n_p) \geq 1$$

J. Bonnard and A. P. Zuker,
JoP Conf. Series 1023 (2018) 012016

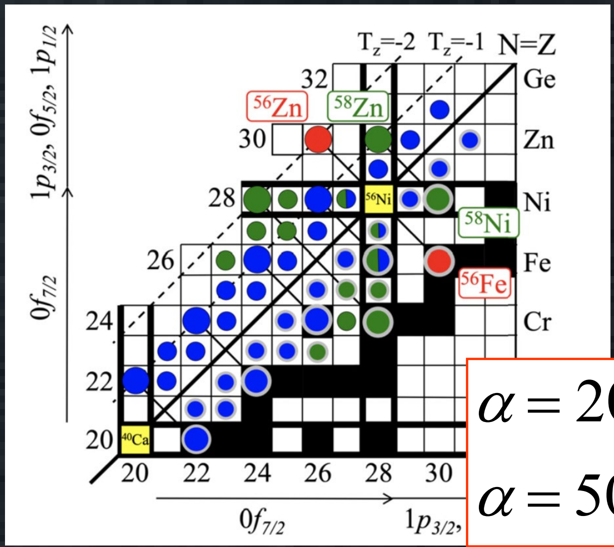
MED in T=2 A=56 mirrors

Recently, excited states in ^{56}Zn (T=2) have been observed for the first time in RIKEN

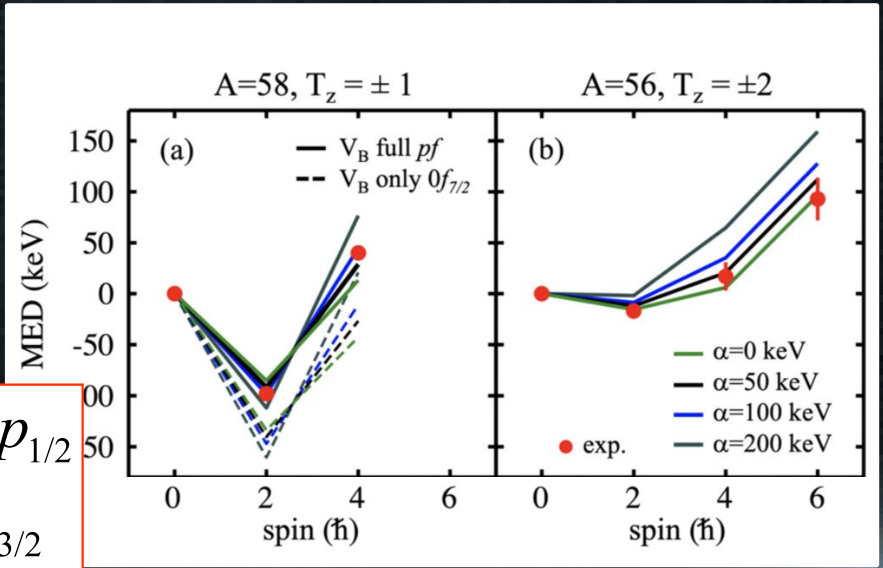
$$V_{Cm,r}(J) = |T_z|\,\alpha\left(z_p + n_p\right)_J$$



The role of the radial term increases with T

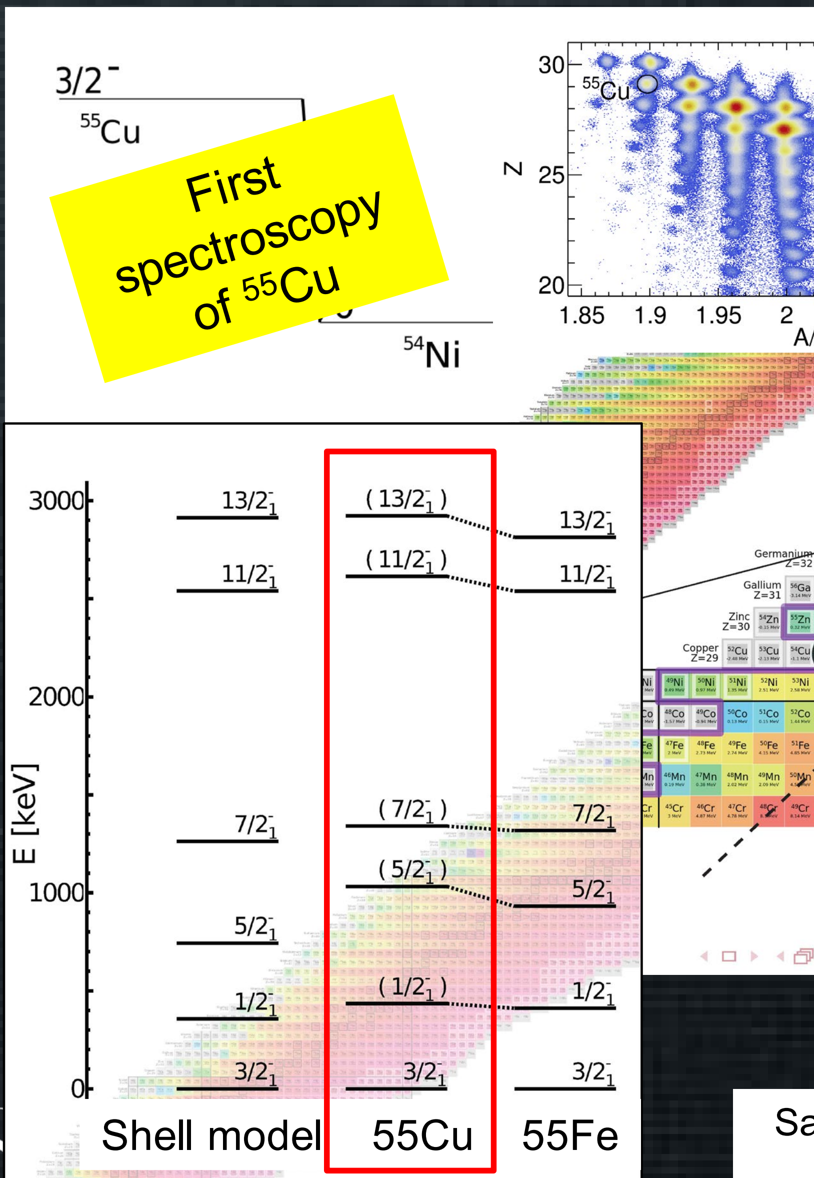


$$\alpha = 200 \text{ keV for } p_{1/2}$$
$$\alpha = 50 \text{ keV for } p_{3/2}$$

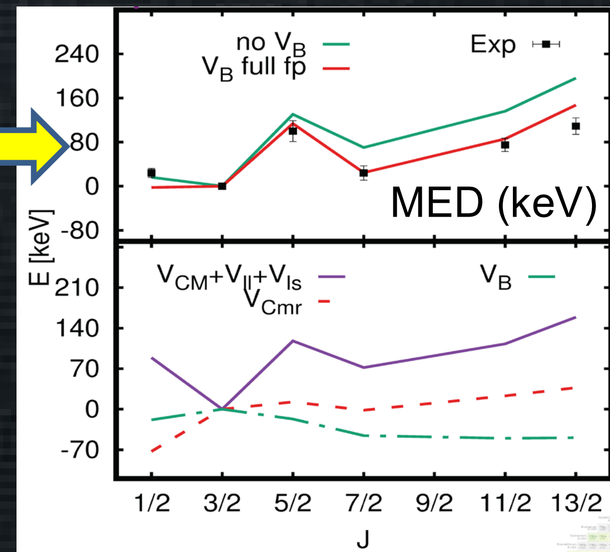


A. Fernandez et al., Phys. Lett. B 823 (2021) 136784

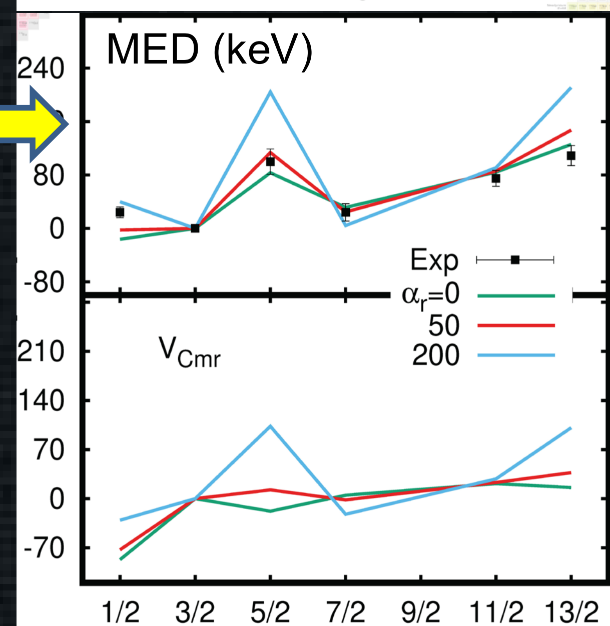
MED in T=3/2 A=55 mirrors



Importance of the contribution of the ISB V_B term for all orbits



Effect of the reduced radius of the $p_{3/2}$ orbit that is occupied by more than one nucleon



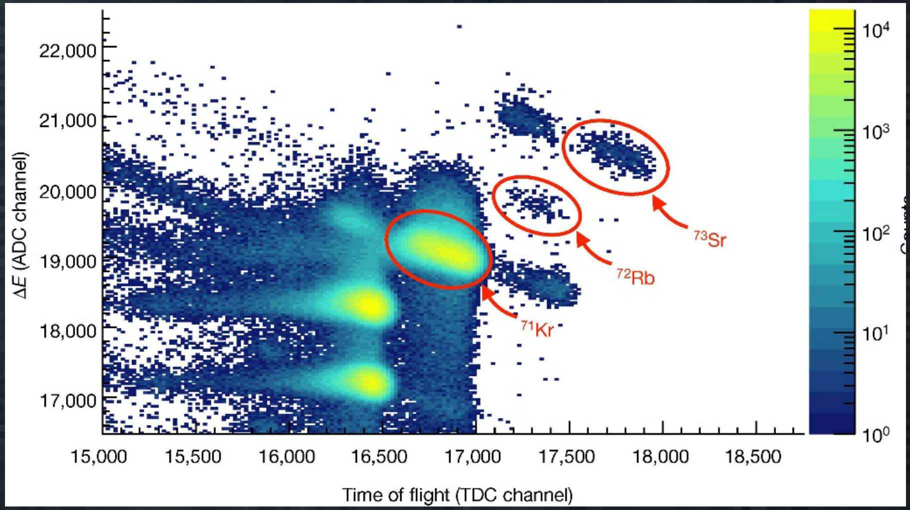
Sara Pigliapoco *et al.*, tbp (2023)

Take-home message

The radius of the low-ℓ orbit in a main shell decreases when is occupied by one or more nucleons

Recent MED data for T = 3/2 A = 61, 73 and T=1 A = 62, 70 confirm these conclusions

The case of ^{73}Sr-^{73}Br

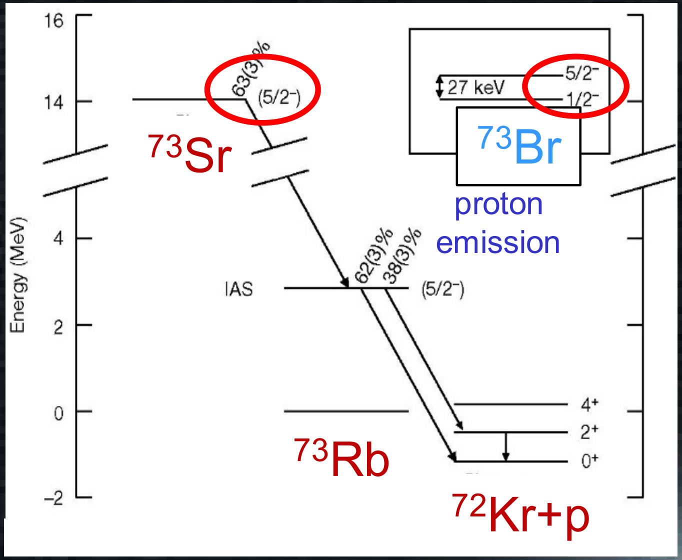


Study the decay of the T = 3/2, T_z=-3/2, ^{73}Sr

$^{73}_{38}$Sr $^{73}_{37}$Rb $^{73}_{36}$Kr $^{73}_{35}$Br

Isobaric multiplet

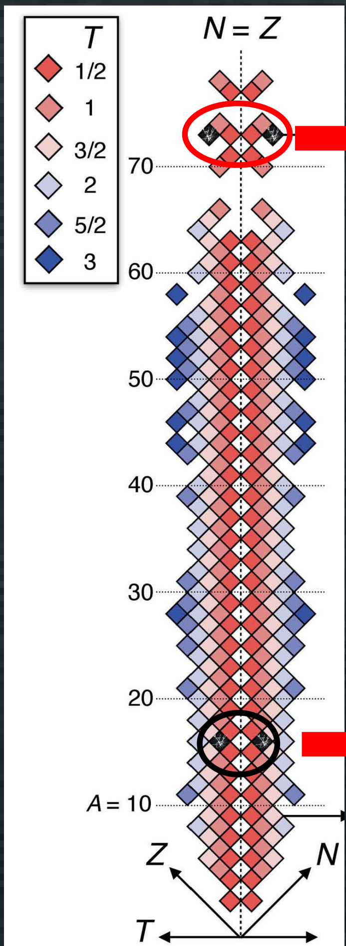
A $J^\pi = 5/2^-$ spin assignment to the g.s. of ^{73}Sr is needed to explain the proton-emission pattern observed from the T = 3/2 IAS in ^{73}Rb

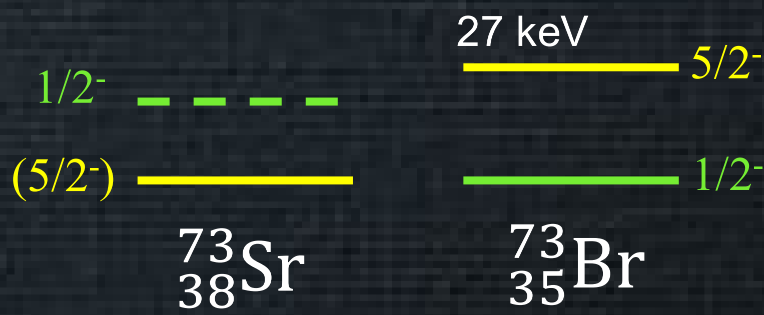


➡ the ground state of ^{73}Sr differs from that of its mirror ^{73}Br

E. M. Ho, A. M. Rogers *et al.*, Nature 580 52 (2020)

Interpretation of the data





Prolate and oblate shapes coexist in this mass region

Thomas-Ehrman shift

$^{16}_{9}F$ $^{16}_{7}N$

Using the effective interaction PFSDG-U (F. Nowacki et al. PRL 117, 272501 (2016), we compute the MED thus predicting the excitation energy of the 1/2- state at 16 keV in ^{73}Sr

S.M. Lenzi, A. Poves and A. O. Macchiavelli, PRC 031302(R) (2020)

MED and nuclear skin

Charge radii and nuclear skin

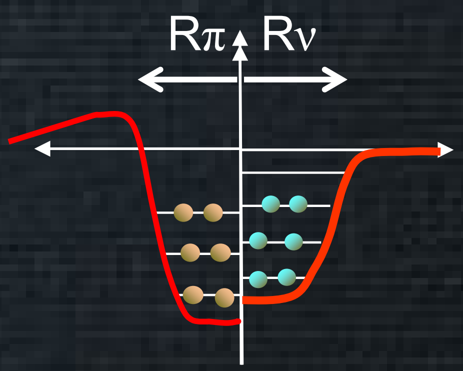
Charge radii can be measured via electron scattering

These measurements are limited to stable nuclei

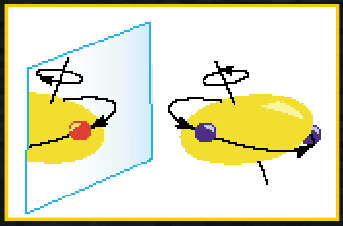
Neutron skin is still more difficult to measure

Laser spectroscopy allows to measure radial shifts along isotopic chains

This applies to ground states or isomeric states



Can we get any information on the evolution of radii in excited states and on the neutron skin?

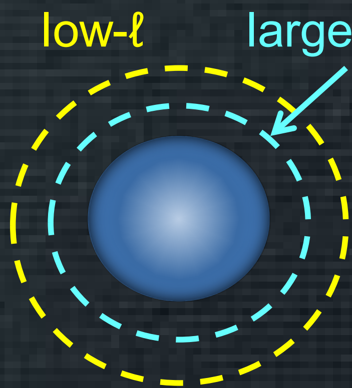


We will now use the MED data to deduce the nuclear skin

We will calculate the excitation energy of the levels in both mirror nuclei within the shell model framework using a realistic interaction in the *sd* shell that naturally includes all ISB terms.

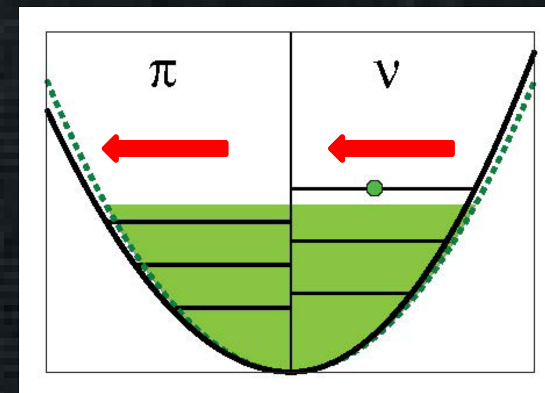
We will compute the MED and compare with data, varying the neutron skin in an iterative way, until we fit the experimental MED

Proton and neutron radii



Studying mirror energies in doubly-magic nuclei + 1 nucleon

Isovector monopole polarizability



A	J^{π}	$\Delta r_{\nu\pi}$ (fm)	
17	$5/2^{+}$	0.056	$d_{5/2}$
	$1/2^{+}$	**0.147**	$s_{1/2}$
41	$7/2^{-}$	0.015	$f_{7/2}$
	$5/2^{-}$	0.018	$f_{5/2}$
	$3/2^{-}$	**0.038**	$p_{3/2}$
	$1/2^{-}$	**0.037**	$p_{1/2}$

The addition of a nucleon induces changes in the potential wells of both protons and neutrons and tends **to equalize the radii**

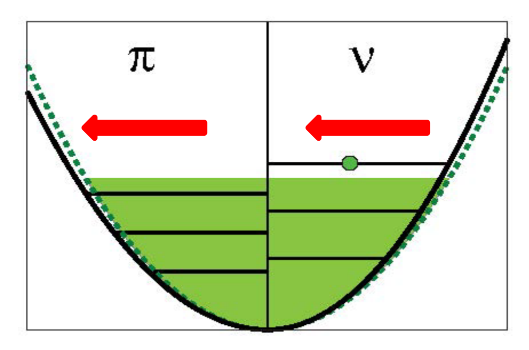
J. Bonnard, S.M.L. and A.P. Zuker, PRL 116, 212501 (2016)

Radii and MED

The size parameters are determined using:

- the (measured) charge radius of the neutron-rich partner
- the (measured) MED
- Isospin-symmetry arguments

$$\langle r_{\pi,\nu}{}^{2} \rangle \propto \frac{1}{\hbar\omega_{\pi,\nu}}$$



measured!

$$r_\pi(N > Z) = r_\nu(N < Z)$$

isospin symmetry

Due to the isovector monopole polarization, we need to determine the size of both potential wells to calculate the matrix elements of the effective interaction to obtain the MED. They are different for protons and neutrons!

The charge radius of the proton-rich partner is obtained from the MED

$$r_\pi(N < Z) = r_\nu(N > Z)$$

J. Bonnard et al., PRL 116, 212501 (2016)

MED and neutron skin in A=23

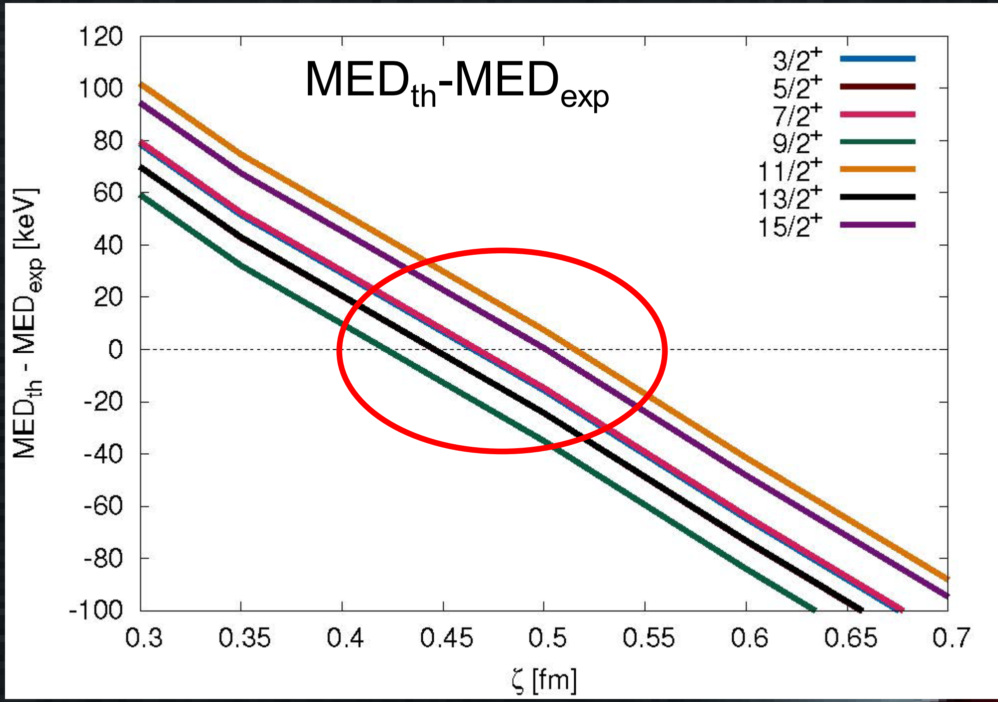
The MED depend linearly on the value of the neutron skin

neutron skin

$$\Delta r_{\nu\pi} = \sqrt{\langle r_\nu^2 \rangle} - \sqrt{\langle r_\pi^2 \rangle} = \zeta \cdot f(A,T)$$

J. Duflo, A. P. Zuker, PRC 66, 051304 (2002)

We vary ζ to match the experimental MED

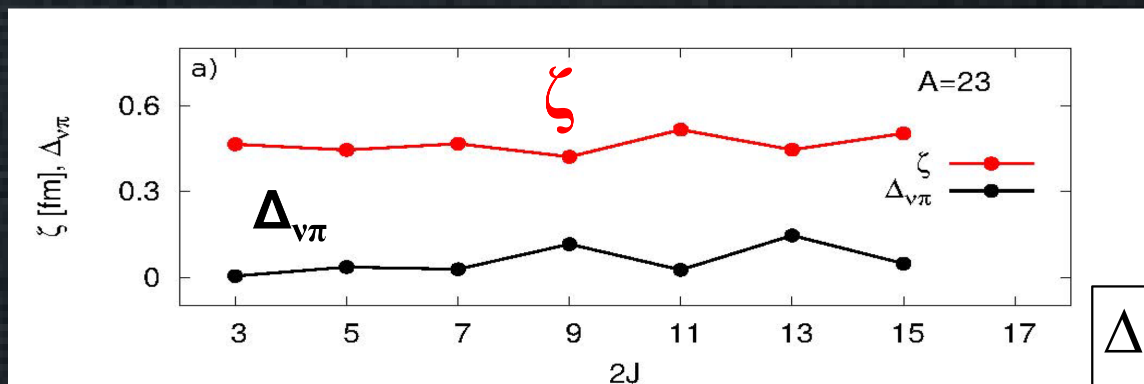


A. Boso *et al.*, Phys. Rev. Lett. 121, 032502 (2018)

MED and neutron skin

For ^{23}Na we obtain the neutron skin (in fm):

J	$3/2$	$5/2$	$7/2$	$9/2$	$11/2$	$13/2$	$15/2$
$\Delta r_{\nu\pi}$	0.0211	0.0202	0.0211	0.0192	0.0233	0.0202	0.0226



$$\Delta_{\nu\pi} = n_{s_{1/2}} - z_{s_{1/2}}$$

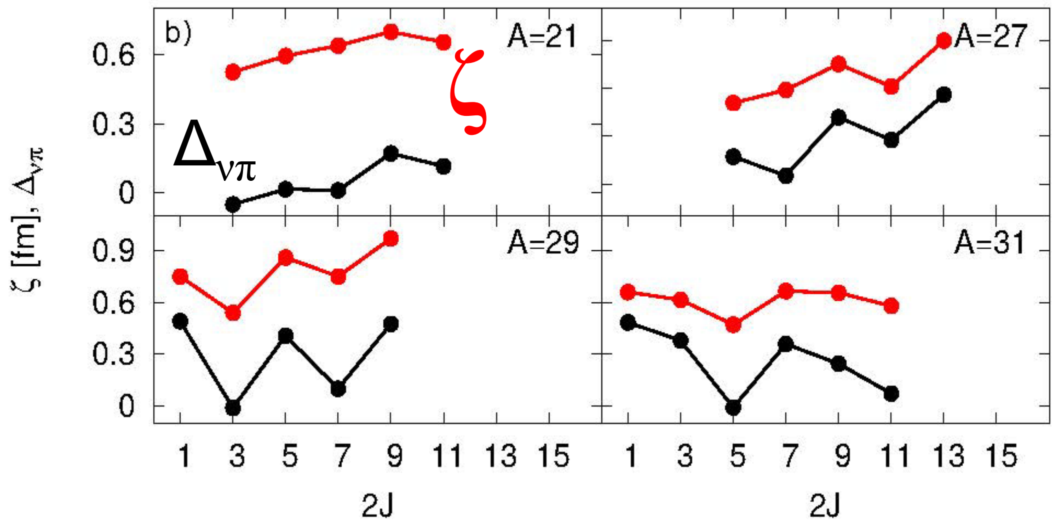
Interestingly, the skin is correlated with the difference of occupation number of neutrons and protons $\Delta_{\nu\pi}$ in the low-ℓ orbit $s_{1/2}$!

A. Boso *et al.*, Phys. Rev. Lett. 121, 032502 (2018)

Correlation between skin and difference of occupation numbers

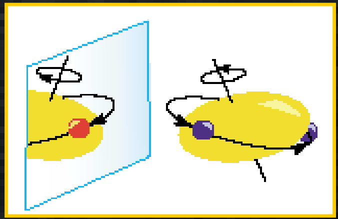
We apply this procedure to the MED for nuclei in the *sd* shell and deduce the value of the skin for each excited state

In all cases, the skin is correlated with the difference of occupation number of neutrons minus protons ($\Delta_{\nu\pi}$) in the $s_{1/2}$ orbit!



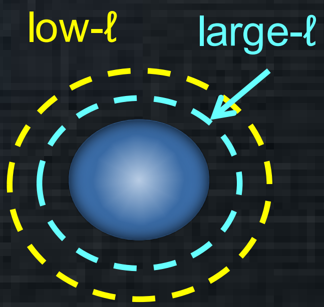
A. Boso *et al.*, Phys. Rev. Lett. 121, 032502 (2018)

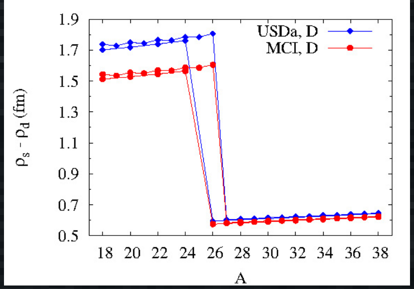
What do we learn from MED?

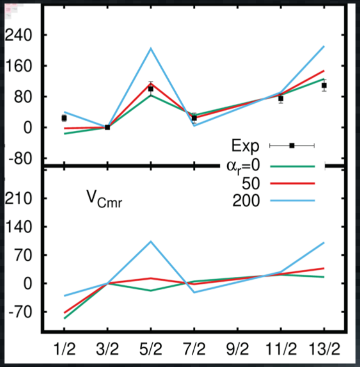


The MED are sensitive to the nuclear structure and therefore constitute a very powerful tool to understand several nuclear properties provided we use a non-free parameter, unique method for all mass regions

In particular, MED depend on the nuclear radius (average of protons and neutrons).
Low-ℓ orbits reduce their radius with occupancy.

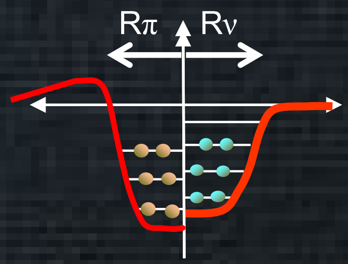


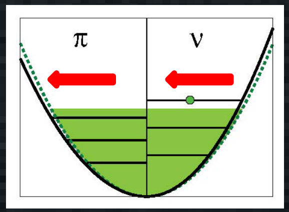


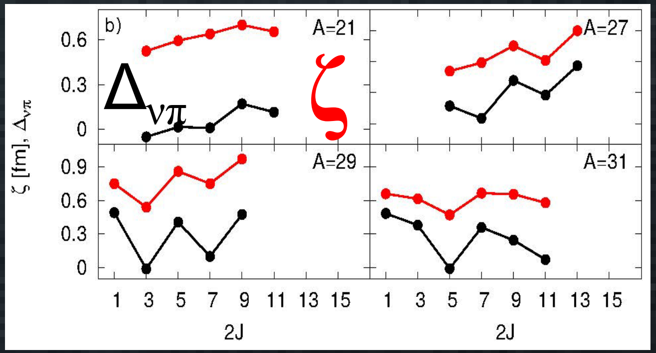


What do we learn from MED?

MED can give us information on the nuclear skin and its evolution with the spin.

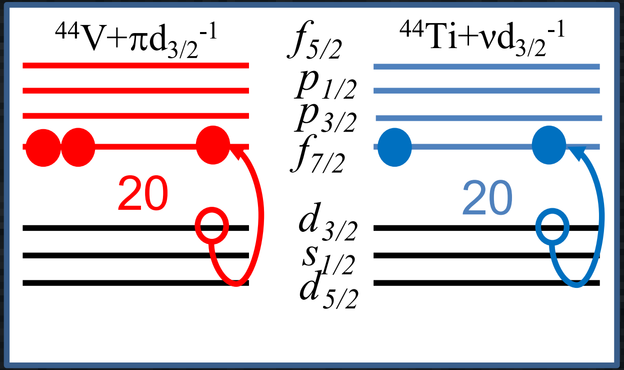






There is a clear correlation between the skin and the difference of occupation of neutrons and protons of the low-ℓ orbit.

The MED of non-natural parity give direct information on the character of p-h excitations in the wavefunction



Thank you for your attention

Backup slides

MED and neutron skin

Using a 5-parameter fit of measured charge radii for A<60

J. Duflo, A. P. Zuker, PRC 66, 051304 (2002)

$$\sqrt{\langle r_\pi^2\rangle} = A^{1/3}\left(\rho_0 + \frac{\zeta}{2}\frac{t_z}{A^{4/3}} - \frac{\upsilon}{8}t_z^2\right)e^{g/A} + \lambda D_{\pi\nu}$$

→ Obtain ħω → compute the matrix elements of a realistic CD interaction

→ Calculate the MED → Vary ζ to match the experimental MED

$$\Delta r_{\nu\pi} = \sqrt{\langle r_\nu^2\rangle} - \sqrt{\langle r_\pi^2\rangle} = \frac{\zeta\, t}{A} e^{g/A}$$

Obtain the neutron skin

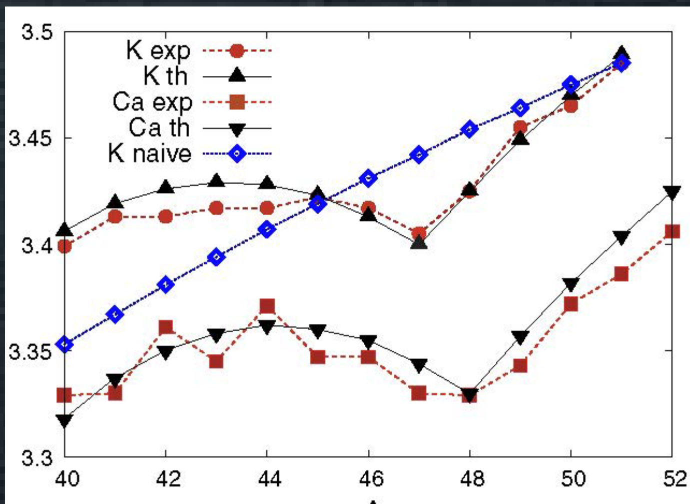
A	J^π	$\Delta r_{\nu\pi}$ (fm)	ζ (fm)	
17	$5/2^+$	0.056	0.90	*d*
	$1/2^+$	**0.147**	**2.37**	*s*
41	$7/2^-$	0.015	0.61	*f*
	$5/2^-$	0.018	0.71	*f*
	$3/2^-$	**0.038**	**1.50**	*p*
	$1/2^-$	**0.037**	**1.48**	*p*

Example: for nuclei with one nucleon over a doubly-closed shell nucleus, the neutron skin varies linearly with the ζ parameter

J. Bonnard et al., PRL 116, 212501 (2016)

Understanding ISB effects

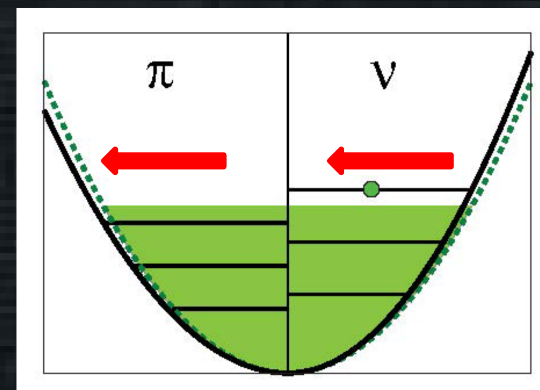
Charge radii in Ca and K isotopes



$\nu f_{7/2}$ $\nu p_{3/2}$

Proton radii increases when filling with neutrons the $p_{3/2}$ shell!

An effect to consider is the isovector polarizability



the addition of a nucleon induces changes in the potential wells of both protons and neutrons

An important conclusion from this work is that low-ℓ orbits have much larger radii than their partners in a main shell

J. Bonnard et al., PRL 116, 212501 (2016)