Efimov States and Clusters

Bira van Kolck

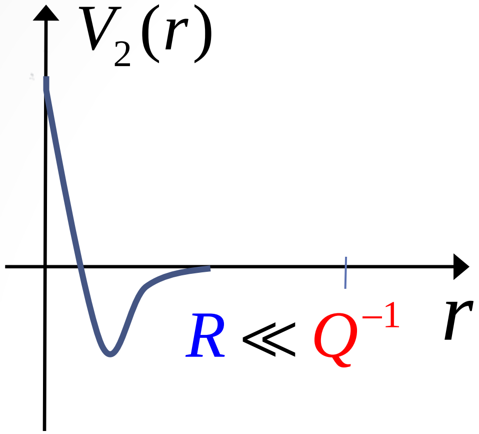
Outline

- Unitarity, why?
- Discrete scale invariance
- Clustering
- Halo/Cluster EFT
- Conclusion

Unitarity, why?

nonrelativistic,
short-range
interactions

$A=2$



singular interactions

regularization

$$V_2(\vec{r}) = \frac{4\pi}{m}\left[C_0(\Lambda)\,\delta^{(3)}_{\Lambda}(\vec{r}) + C_2(\Lambda)\,R^2\,\nabla^2\delta^{(3)}_{\Lambda}(\vec{r}) + \ldots\right]$$

quantum
multipole expansion

renormalization:
relative errors in observables $\sim Q/\Lambda \lesssim QR$
for $\Lambda \gtrsim 1/R$

order by order
in QR

vK '97,'99
Kaplan, Savage, Wise '98
...

$$T_2(k \ll R^{-1}) = \frac{4\pi}{m}\left(a_2^{-1} + ik - \frac{r_2}{2}k^2 + \ldots\right)^{-1} \quad [+\, l > 0]$$

Bethe '49

scattering
length

effective
range

unitarity limit $a_2^{-1} \approx \sqrt{mB_2} \to 0$ $\qquad r_2 \sim \ldots \sim R$ typically

Here:
$\hbar = 1, c = 1$
⇓
$[m] = [E] = [p]$
$= [r]^{-1} = [t]^{-1}$

Nucleons

$R \sim m_\pi^{-1}$

$^3S_1$

$\left(a_{2,I=0} m_\pi\right)^{-1} \simeq 0.26$

$r_{2,I=0} m_\pi \simeq 1.2$

$^1S_0$

$\left|a_{2,I=1,I_3=+1} m_\pi\right|^{-1} - \left|a_{2,I=1,I_3=0} m_\pi\right|^{-1} \simeq 0.12$

$\left|a_{2,I=1,I_3=0} m_\pi\right|^{-1} \simeq 0.06$

$\left|a_{2,I=1,I_3=-1} m_\pi\right|^{-1} - \left|a_{2,I=1,I_3=0} m_\pi\right|^{-1} \simeq 0.02$

$r_{2,I=1} m_\pi \simeq 1.9$

Atoms

$R \sim l_{\mathrm{vdW}}$

^{4}He

$l_{\mathrm{vdW}} / a_2 \approx 0.06$

$r_2 / l_{\mathrm{vdW}} \approx 1.3$

Near Feshbach resonances

$\left|l_{\mathrm{vdW}} / a_2\right| \to 0$

$r_2 / l_{\mathrm{vdW}} \sim 1$

$$\Rightarrow \quad T_2\left(\left|a_2^{-1}\right| \ll k \ll R^{-1}\right) = \frac{4\pi}{m}(ik)^{-1}\left(1 + \mathcal{O}\left(\frac{1}{ka_2}, kR\right)\right)$$

unitarity window

universality

no parameter!

details in (distorted-wave) perturbation th!!

cf. models

renormalization $\Rightarrow$ $C_0^{(0)}(\Lambda) = -\dfrac{1}{\theta_0 \Lambda}$

non-trivial fixed point

number depending on specific form of regulator (continuous) scale invariance

$$S=\int \frac{dt}{2m}\int d^3r\left\{\psi^\dagger\left(2m\frac{\partial}{\partial t}-\vec{\nabla}^2\right)\psi-4\pi C_0^{(0)}\left(\psi^\dagger\psi\right)^2+\ldots\right\}$$

invariant

$\updownarrow$ $\alpha \geq 0$

$r\to\alpha r \Leftrightarrow p\to\alpha^{-1}p \Leftrightarrow \Lambda\to\alpha^{-1}\Lambda$

$t/m\to\alpha^2\, t/m \Leftrightarrow mE\to\alpha^{-2}mE$

$\psi\to\alpha^{-3/2}\psi$

More bodies

Two-component fermions

e.g., neutrons

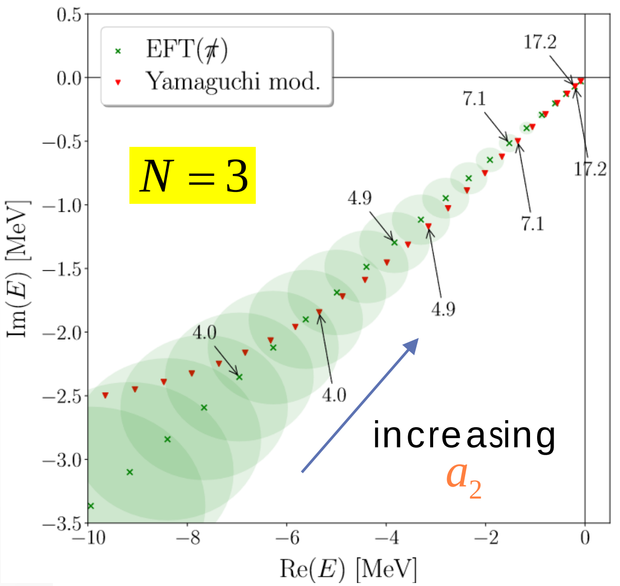
no isolated, finite-energy *S*-matrix poles

unless scale introduced by external interaction/trap

e.g. $$\left.\frac{E_N^{(0)}}{N}\right|_{N\to\infty}=\frac{3k_F^2}{10m}\left(\xi+\mathcal{O}\left(\frac{1}{k_F a_2},k_F r_2\right)\right)$$

$k_F=\left(3\pi^2\rho\right)^{1/3}$

universal number Bertsch '99

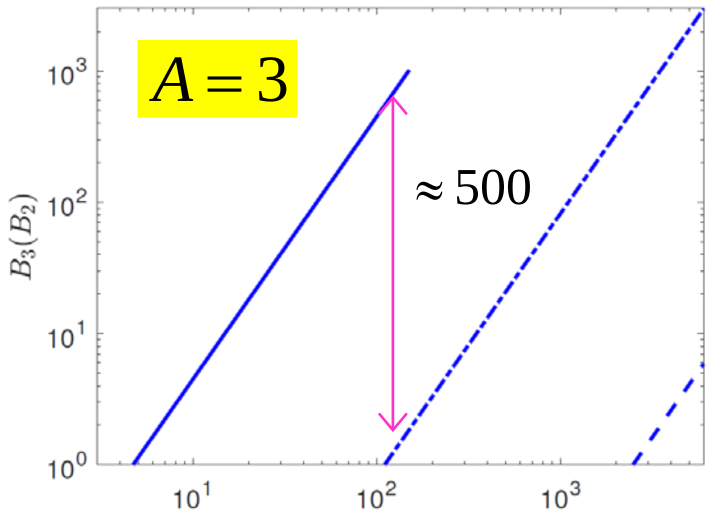


Dietz, Hammer, König, Schwenk, *Phys. Rev. C* **105** (2022) 064002

Discrete scale invariance

Bosons, multi-component fermions

e.g., nucleons



Feng Wu

$$\frac{B_3}{3} \propto \frac{\Lambda^2}{m}$$

Thomas collapse

Thomas '35

Bedaque, Hammer, vK '99 '00

...

For LO renormalization

$$V_3^{(0)}(\vec{r}_1-\vec{r}_2,\vec{r}_2-\vec{r}_3)=\frac{(4\pi)^2}{m}D_0^{(0)}(\Lambda)\,\delta_\Lambda^{(3)}(\vec{r}_1-\vec{r}_2)\,\delta_\Lambda^{(3)}(\vec{r}_2-\vec{r}_3)$$

$$D_0^{(0)}(\Lambda)\approx\frac{1}{\Lambda^4}\,\frac{\sin\left(s_0\ln(\Lambda_*/\Lambda)-\arctan(1/s_0)\right)}{\sin\left(s_0\ln(\Lambda_*/\Lambda)+\arctan(1/s_0)\right)}$$

limit cycle

cf. Wilson '71

dimensionful parameter $\quad s_0\simeq 1.00624$

$$S=\int\frac{dt}{2m}\int d^3r\left\{\psi^\dagger\left(2m\frac{\partial}{\partial t}-\vec{\nabla}^2\right)\psi-4\pi C_0^{(0)}\left(\psi^\dagger\psi\right)^2-(4\pi)^2D_0^{(0)}\left(\psi^\dagger\psi\right)^3+\ldots\right\}$$

invariant $\quad \alpha\to\alpha_n=\exp\left(n\pi/s_0\right)=(22.7)^n$

n integer

α_n ↕

for nucleons,
implies Wigner's SU(4)
but **much** stronger!

Two consequences

1) Towers of excited states

$$mB^{(0)}_{A,n} \to \alpha_l^{-2}\, mB^{(0)}_{A,n} = mB^{(0)}_{A,n+l} \quad \Rightarrow \quad mB^{(0)}_{A,n}(\Lambda_*) = mB^{(0)}_{A,0}(\Lambda_*)\exp\left(-2n\pi/s_0\right)$$

ground state

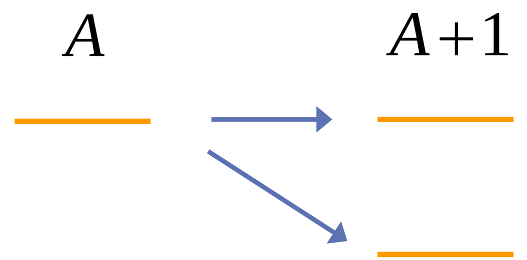
fixes tower position

$A=3$ Efimov '70

$A=4$ Hammer, Platter, '07

$A=5,6$ bosons — von Stecher '10'11; Gattobigio, Kievsky, Viviani '11'12

multiplication



Efimov state

Efimov descendants

Towers upon towers!

2) Ground-state correlations

single scale ➡ $$\frac{B_{A,0}^{(0)}(\Lambda_*)}{A} = \kappa_A \frac{B_{3,0}^{(0)}(\Lambda_*)}{3}$$

universal numbers

$\kappa_2 \equiv 0$
$\kappa_3 \equiv 1$
$\kappa_4 \simeq 3.5$ Hammer, Platter '07
$\kappa_{A\geq 5} \simeq ?$

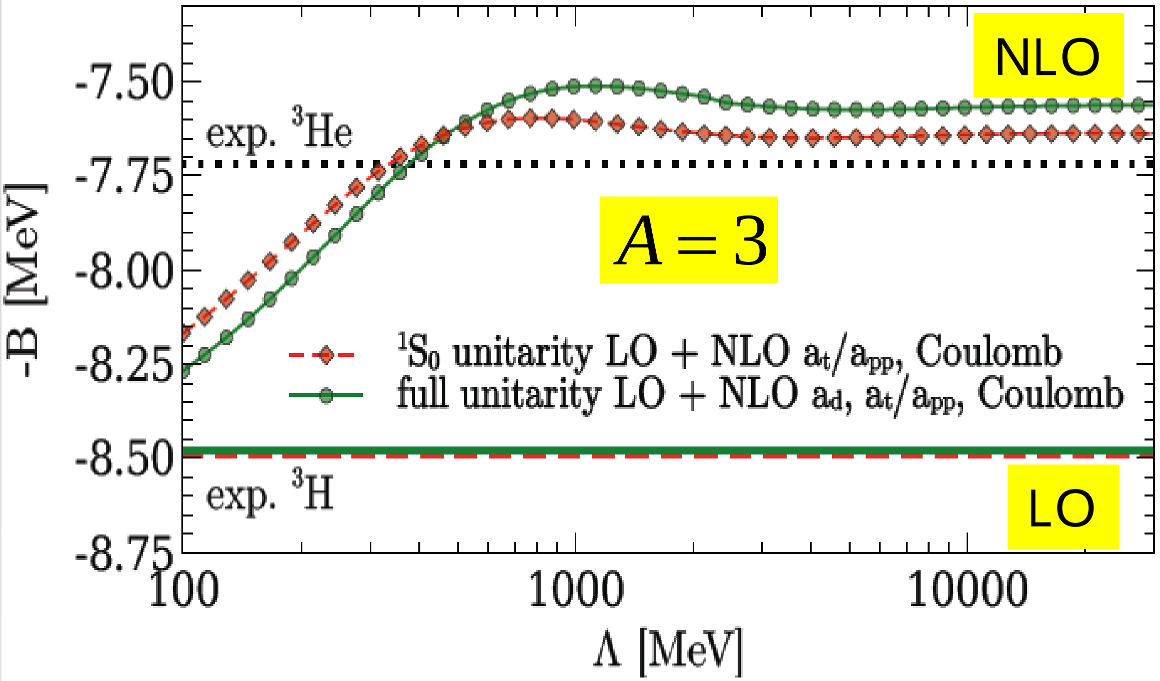
varying Λ_*

$A = 4$ — Tjon line — Platter, Hammer, Meißner '05

Tjon '75
Nakaichi, Akaishi, Tanaka, Lim '78

$A = 5,6$ bosons — Generalized Tjon lines — Bazak, Eliyahu + vK '16

Nakaichi, Akaishi, Tanaka, Lim '79'80



$$B_h^{(1)} - B_t \simeq -(0.92 \pm 0.18)\ \text{MeV}$$

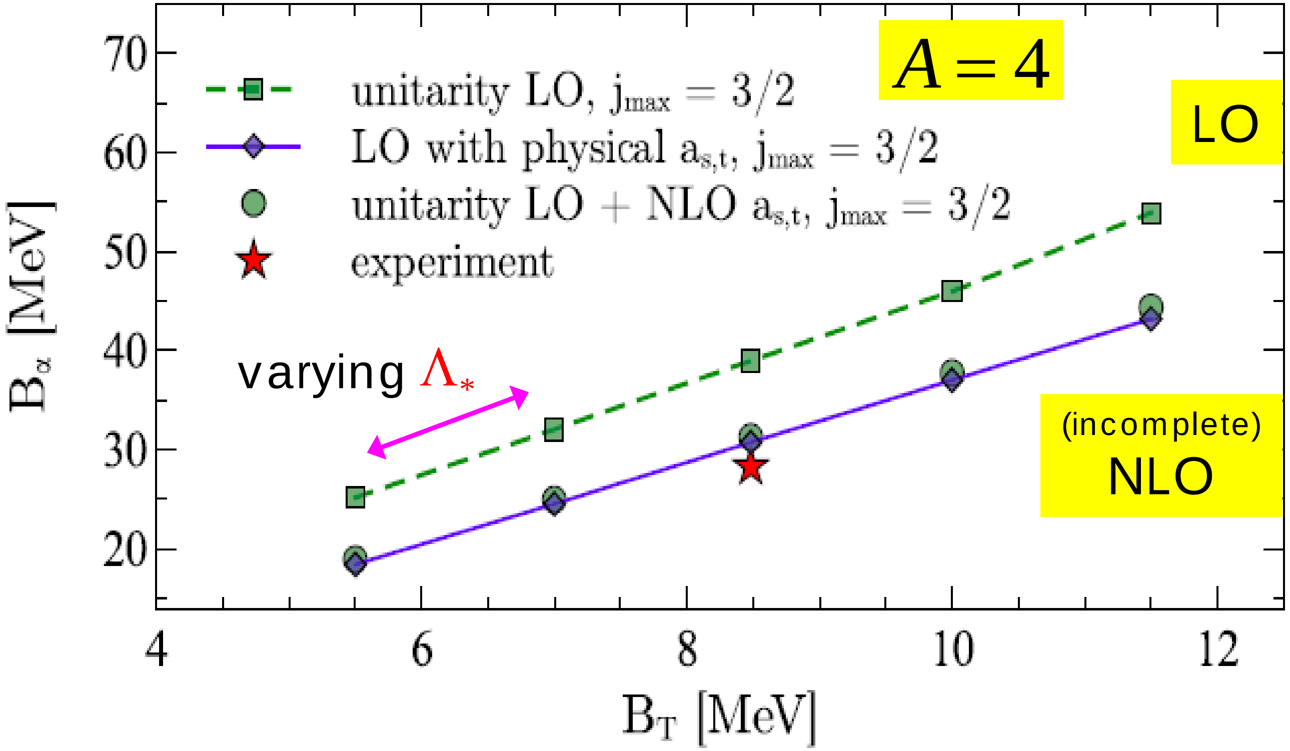
vs.

$$-0.764\ \text{MeV (exp)}$$

$$B_t = 8.48\ \text{MeV} \Rightarrow D_0^{(0)}(\Lambda)$$

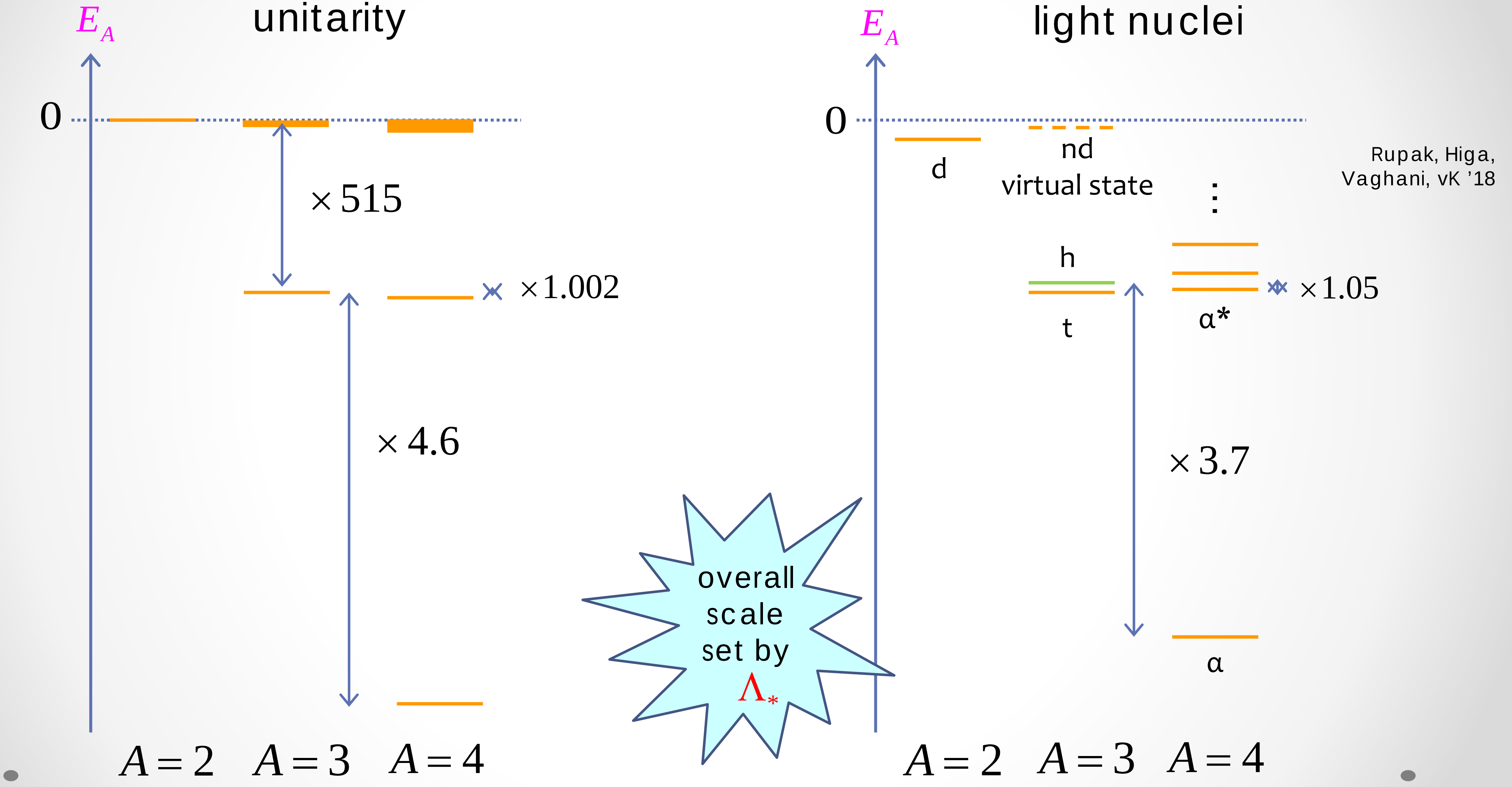
König, Grießhammer, Hammer, vK ’16

Nucleons



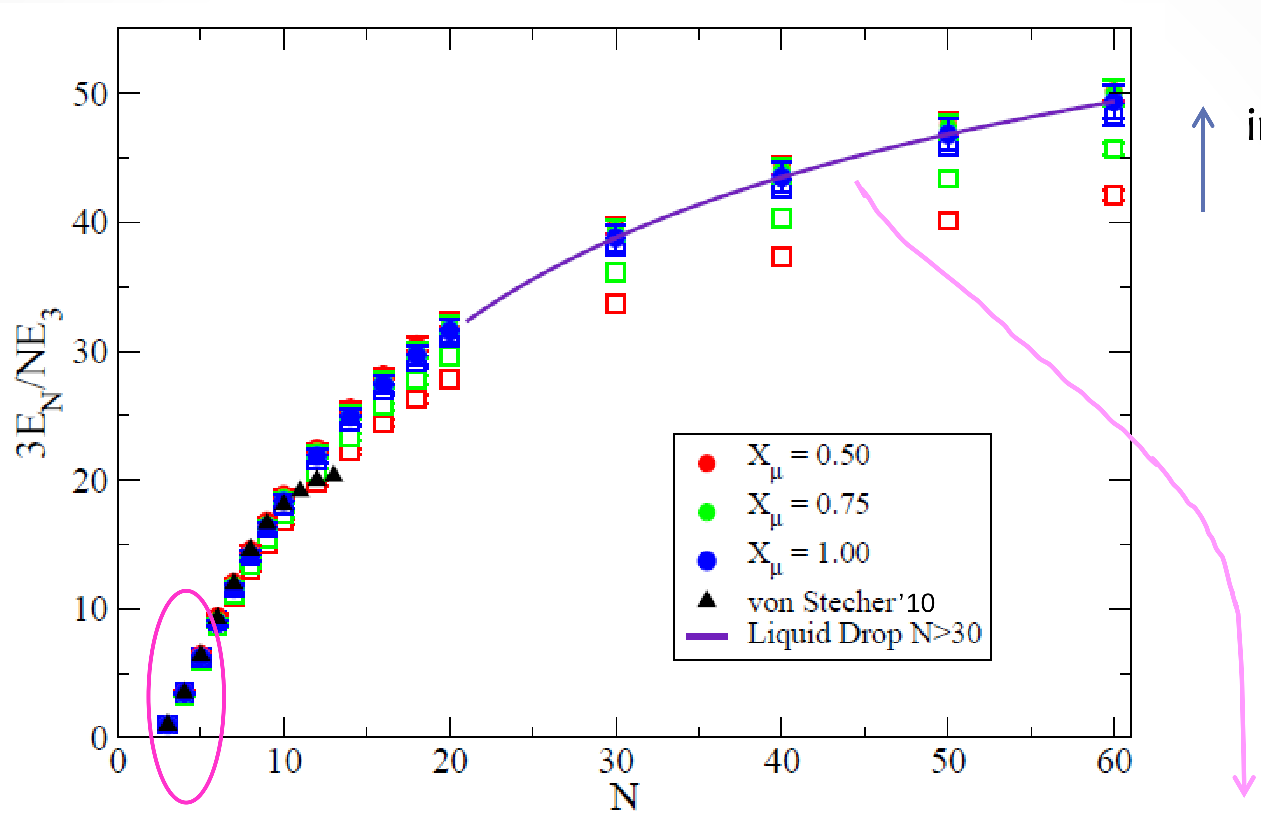
König, Grießhammer, Hammer, vK ’17

Schematically



Carlson, Gandolfi,
Vitiello, vK '17

Many bosons



κ_N

↑ increasing Λ

saturation!

cf. Piatecki, Krauth '14
Comparin, Krauth '16

Variational
and Diffusion
Monte Carlo

$$\kappa_N \approx \frac{3}{N}(N-2)^2$$

Bazak, Eliyahu + vK '16

$$\kappa_N = \kappa_\infty\left[1-\eta N^{-1/3}+\mathcal{O}\left(N^{-2/3}\right)\right]$$

$$\kappa_\infty = 90\pm 10 \quad \eta = 1.7\pm 0.3$$

cf. ^{4}He $\quad \kappa_\infty \approx 180 \quad \eta \approx 2.7$ Pandharipande *et al.* '83

Reproduced without unitarity expansion Pederiva, De-Leon '22

Dawkins, Carlson,
Gezerlis, vK ’20

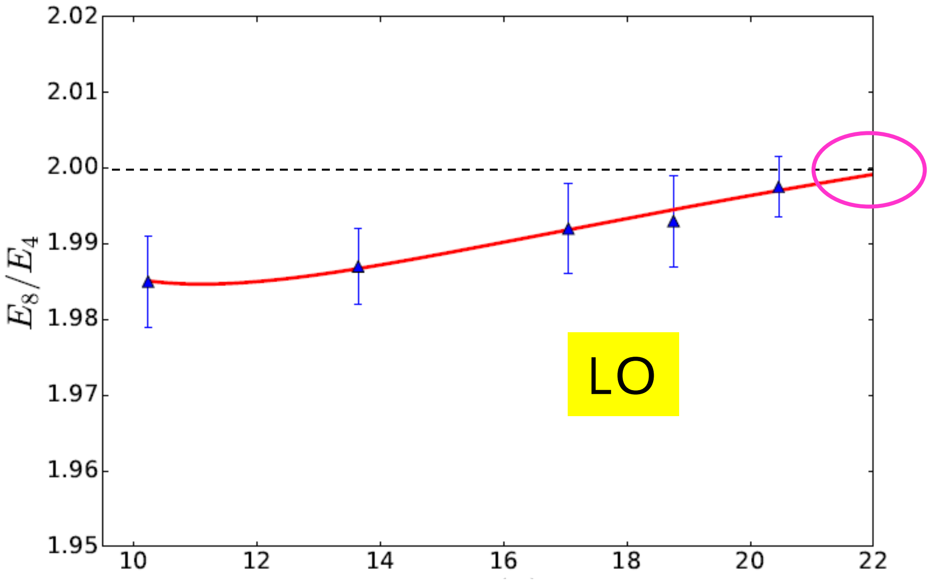
Clustering

Four-component fermions

$A=8$

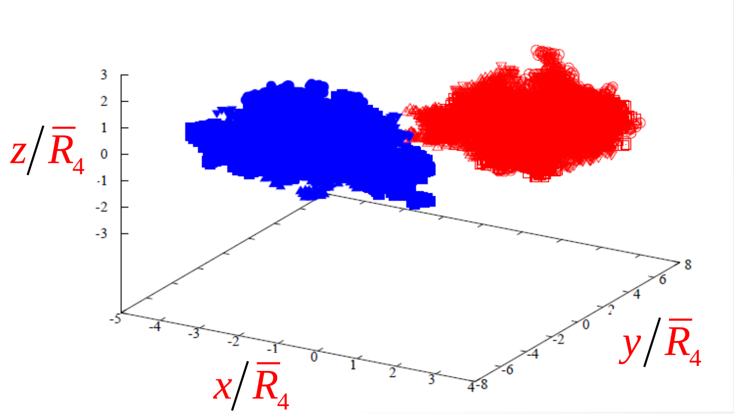
Variational and Diffusion Monte Carlo

$2\kappa_8/\kappa_4$



$\bar{R}_4\Lambda$

$\bar{R}_4 \equiv (2mB_4)^{-1/2}$



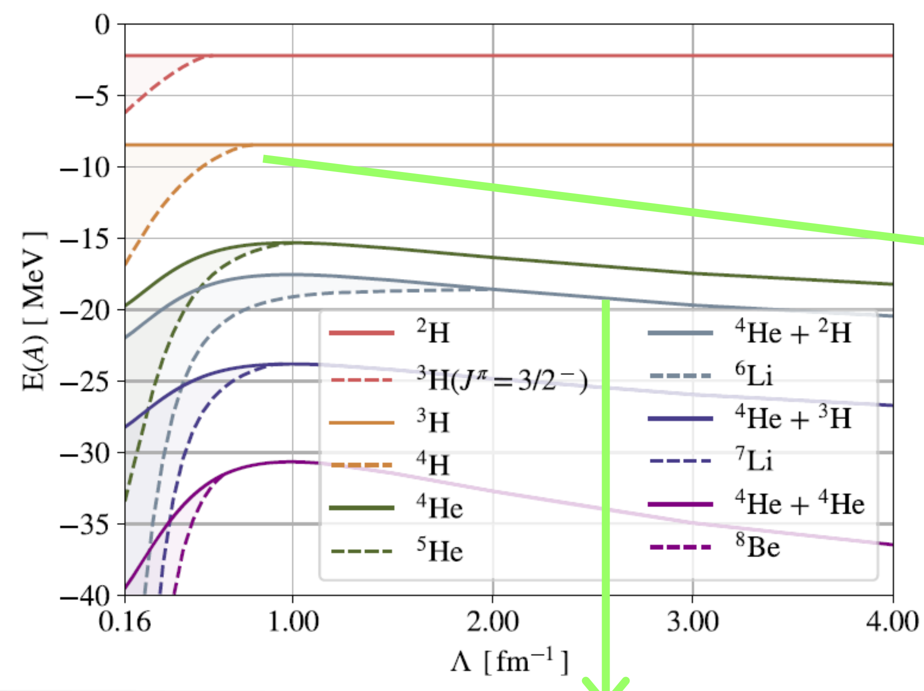
$\kappa_8 \simeq \kappa_4$?

consistent with ^{8}Be

Clustering a universal property of multi-component unitary fermions?

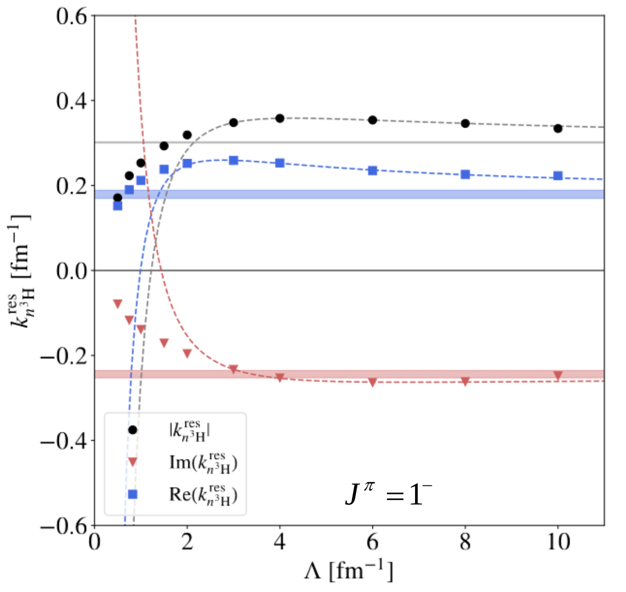
Similar away from unitarity

Schäfer, Contessi, Kirscher, Mareš, *Phys. Lett. B* **816** (2021) 136194

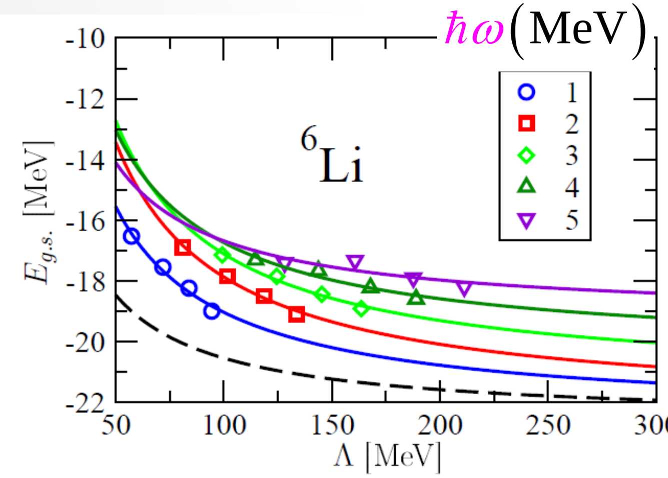


resonance

Contessi, Schäfer, Kirscher, Lazauskas, Carbonell, *Phys. Lett. B* **840** (2023) 137840



Stetcu, Barrett, vK '07



$E_\alpha + E_d \simeq -30$ MeV

E_α

Λ	m_π = 140 MeV
2 fm^{-1}	−23.17 ± 0.02
4 fm^{-1}	−23.63 ± 0.03
6 fm^{-1}	−25.06 ± 0.02
8 fm^{-1}	−26.04 ± 0.05
→ ∞	$-30^{\pm 0.3\,(sys)}_{\pm 2\,(stat)}$
Exp.	−28.30

$E_{^{16}O}$

Λ	m_π = 140 MeV
2 fm^{-1}	−97.19 ± 0.06
4 fm^{-1}	−92.23 ± 0.14
6 fm^{-1}	−97.51 ± 0.14
8 fm^{-1}	−100.97 ± 0.20
→ ∞	$-115^{\pm 1\,(sys)}_{\pm 8\,(stat)}$
Exp.	−127.62

Contessi, Lovato, Pederiva, Roggero, Kirscher, vK '17

Pionless EFT

nucleon → “elementary” field

no role for chiral symmetry

slow-moving nucleon

nonrelativistic expansion $\frac{Q}{m_N}$

$Q \sim \sqrt{2 m_N B_A / A} \ll m_\pi$

$\lambda \sim 1 \big/ \sqrt{2 m_N B_A / A}$

$1/m_\pi \cong 1.4 \text{ fm}$

e.g.

$1 \big/ \sqrt{2 m_N B_3 / 3} \approx 2.8 \,\text{fm}$

$1 \big/ \sqrt{2 m_N B_A / A}$

short-ranged

multipole expansion $\frac{Q}{m_\pi}, \ldots$

Halo/Cluster EFT

Bertulani, Hammer, vK '02
Bedaque, Hammer, vK '03
...

tight cluster → "elementary" field

$$Q \sim R_h^{-1} \ll A^{-1/3} m_\pi \sim R_c^{-1}$$

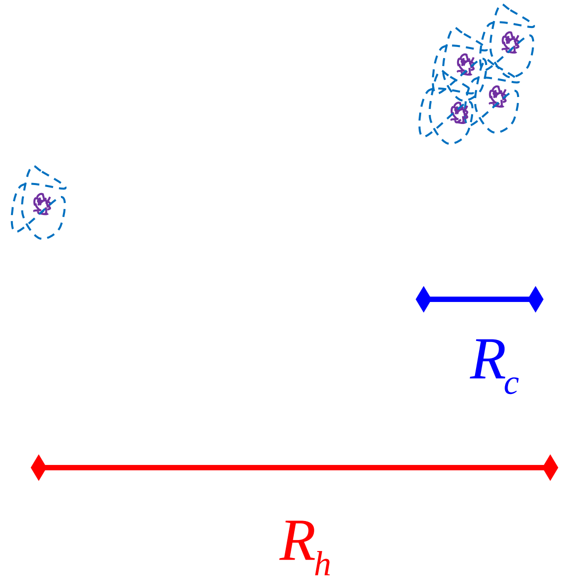
e.g.

$R_{^4\mathrm{He}} \approx 1.5\ \mathrm{fm}$

$R_{^6\mathrm{He}} \approx 2.4\ \mathrm{fm}$

slow-moving cluster

nonrelativistic expansion $\dfrac{Q}{A_c m_N}$



$\lambda \sim R_h$

short-ranged

multipole expansion $Q R_c, \ldots$

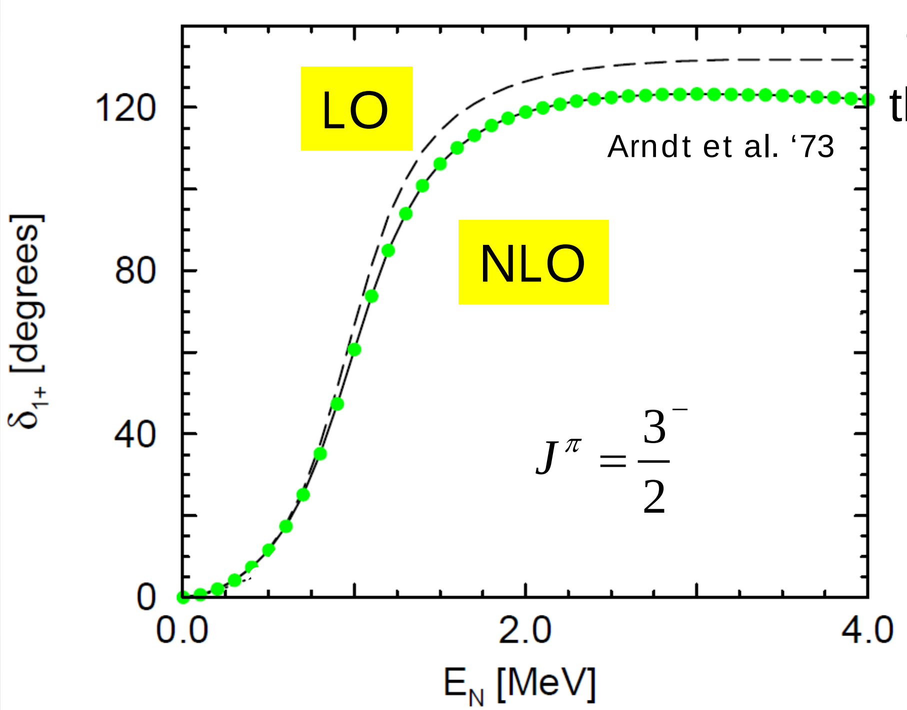
clustering near unitarity:

(light?) ground states = soups of alpha particles, trinucleons, and nucleons?

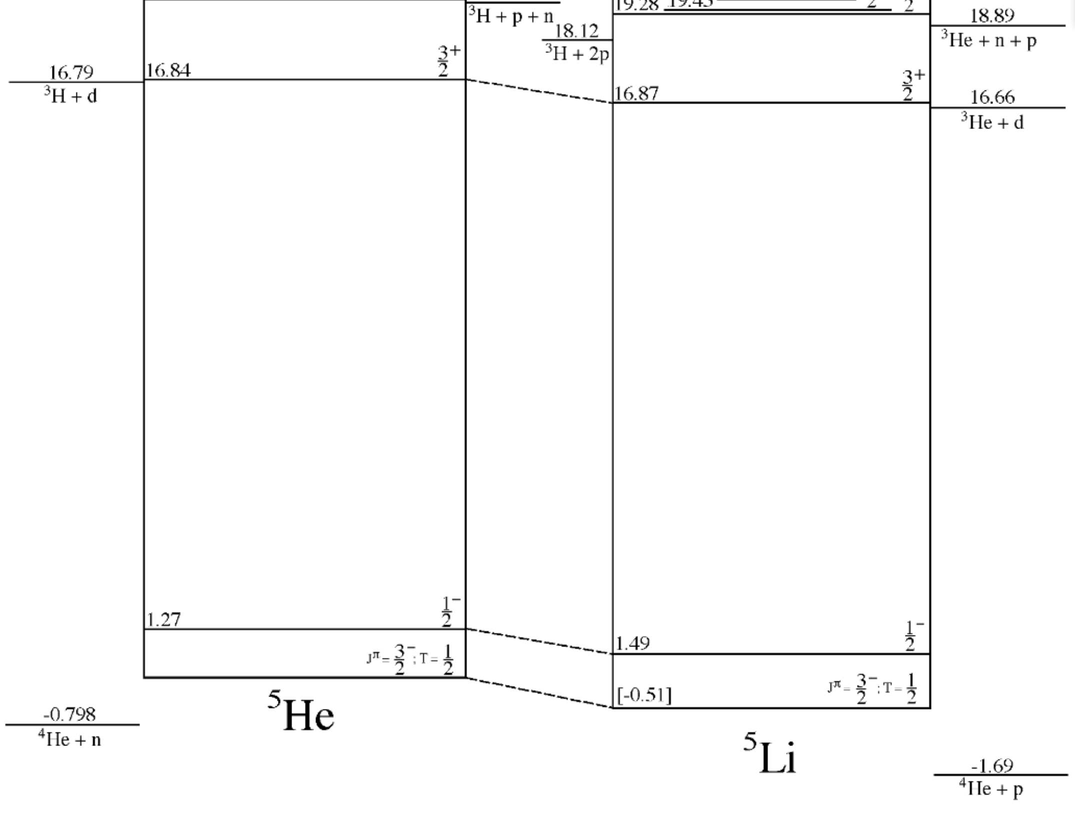
$A = 5$

nα scattering

Bertulani, Hammer, vK '03



two-parameter fit
three-parameter fit



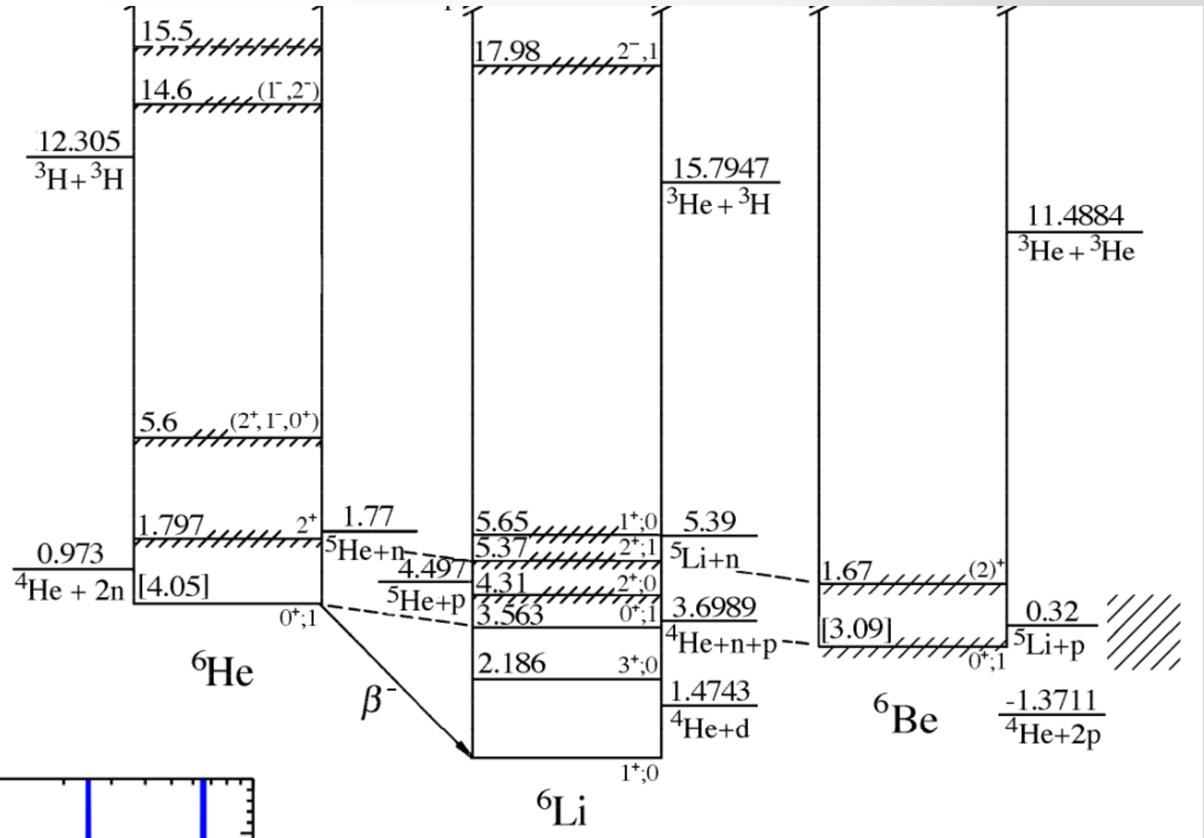
TUNL Nuclear Data Project

^{5}He $\quad E_R \simeq 0.8 \text{ MeV}$, $\quad \Gamma(E_R) \simeq 0.6 \text{ MeV}$ extracted

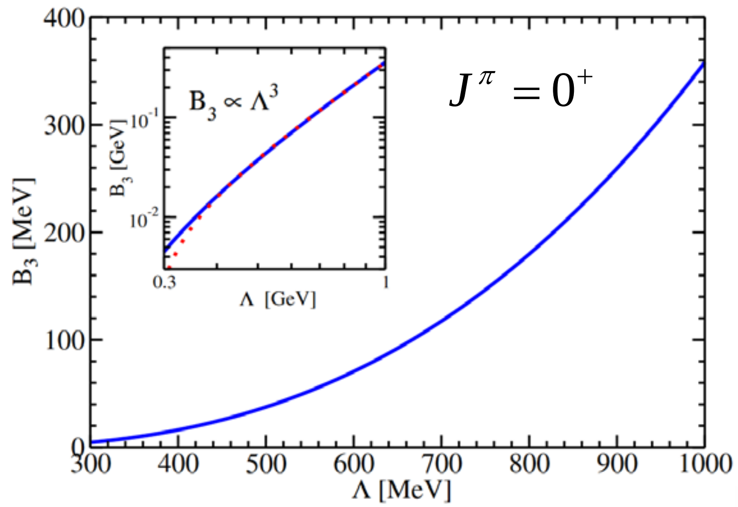
$A = 6$

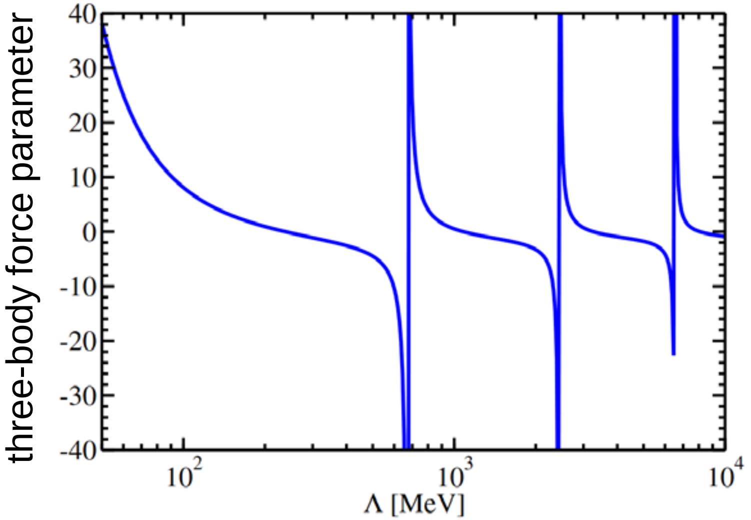
^{6}He

Rotureau, vK '13
Ji, Elster, Phillips '14
Ryberg, Forssén, Platter '17
...



TUNL Nuclear Data Project





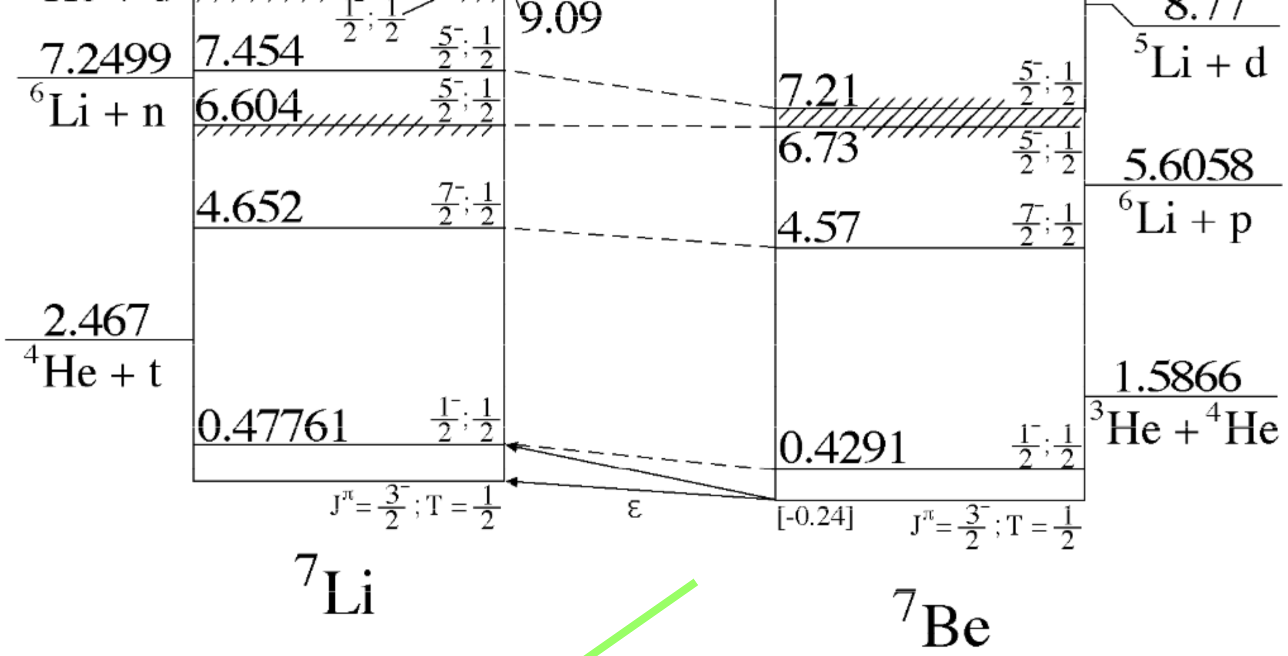
Ji, Elster, Phillips, *Phys. Rev. C* **90** (2014) 044004

LO input $B_{^6\mathrm{He}} \simeq 0.973\,\mathrm{MeV}$

$A = 7$

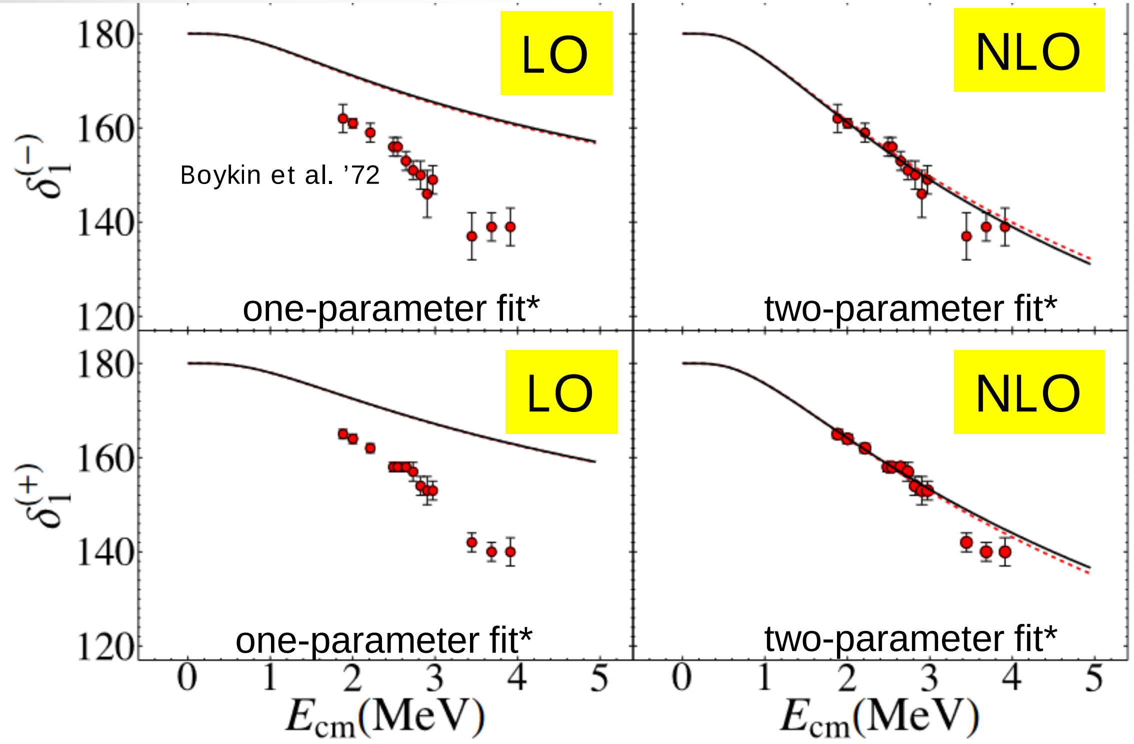
hα scattering

Higa, Rupak, Vaghani ’18
Premarathna, Rupak ’20
Zhang, Nollett, Phillips ’20
Poudel, Phillips ’22
...



TUNL Nuclear Data Project

Higa, Rupak, Vaghani, *Eur. Phys. J. A* **54** (2018) 89



$J^\pi = \frac{1}{2}^-$

LO input $\quad B_{^7\text{Be}^*} \simeq 1.16\ \text{MeV}$

$J^\pi = \frac{3}{2}^-$

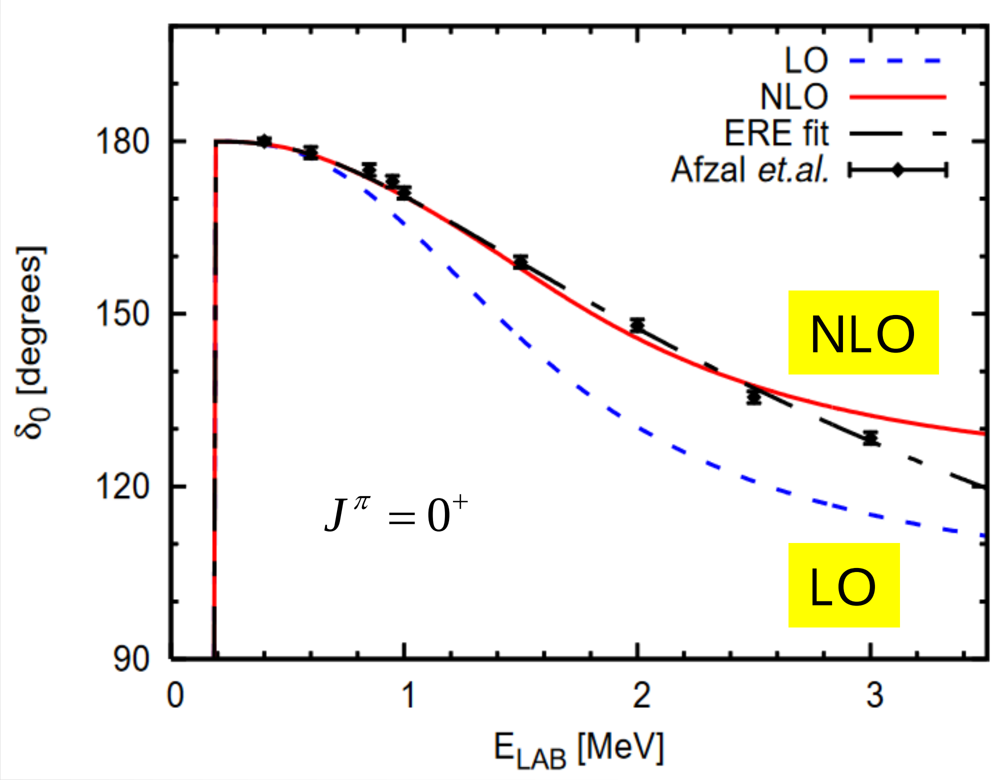
LO input $\quad B_{^7\text{Be}} \simeq 1.59\ \text{MeV}$

*fits include radiative capture data and parameters

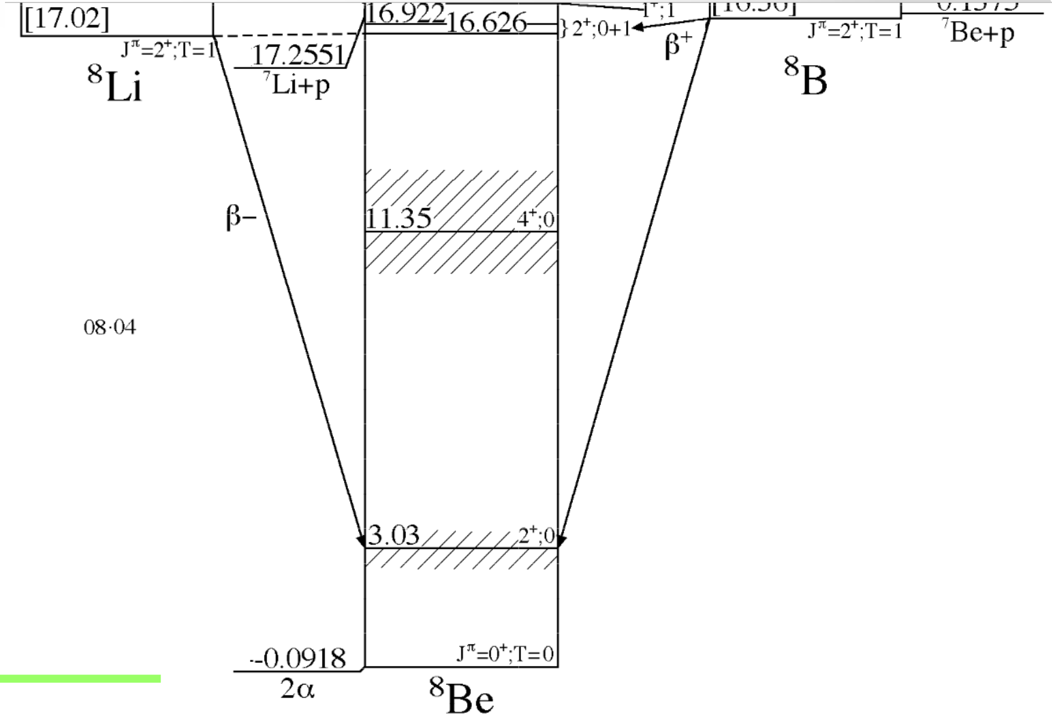
$A = 8$

αα scattering

Hammer, Higa, vK '08



one-parameter fit

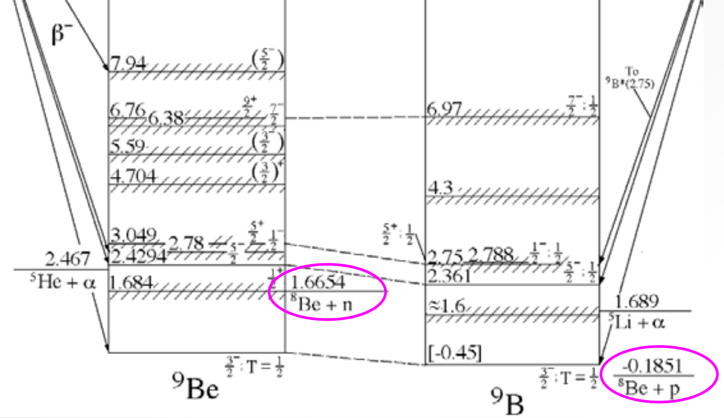


TUNL Nuclear Data Project

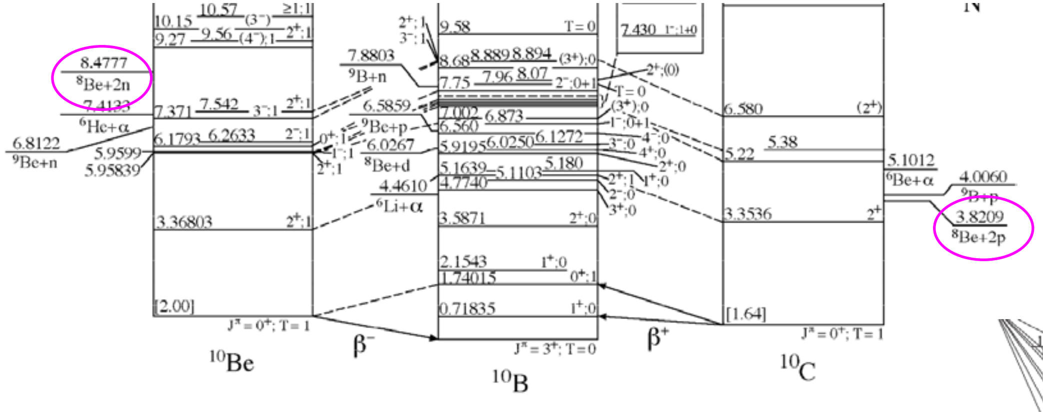
^{8}Be LO input

$$E_R = 184.15 \pm 0.07 \text{ keV}$$
$$\Gamma(E_R) = 11.14 \pm 0.50 \text{ eV}$$

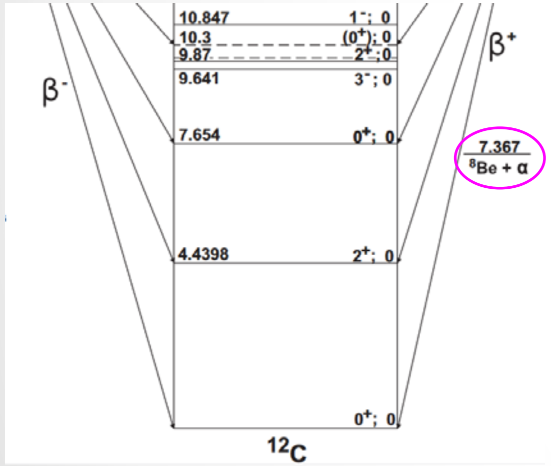
Benn et al. '67
Wüstenbecker '92



$\alpha + \alpha + N$

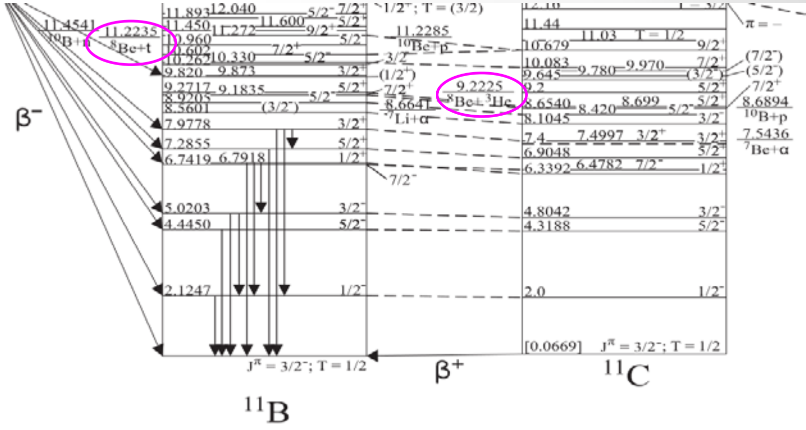


$\alpha + \alpha + N + N$



$\alpha + \alpha + \alpha$

...



$\alpha + \alpha + t / h$

How far can we go?

External probes

→Electromagnetic form factors

Canham, Hammer '08'10
Hammer, Phillips '11
...

→Radiative capture

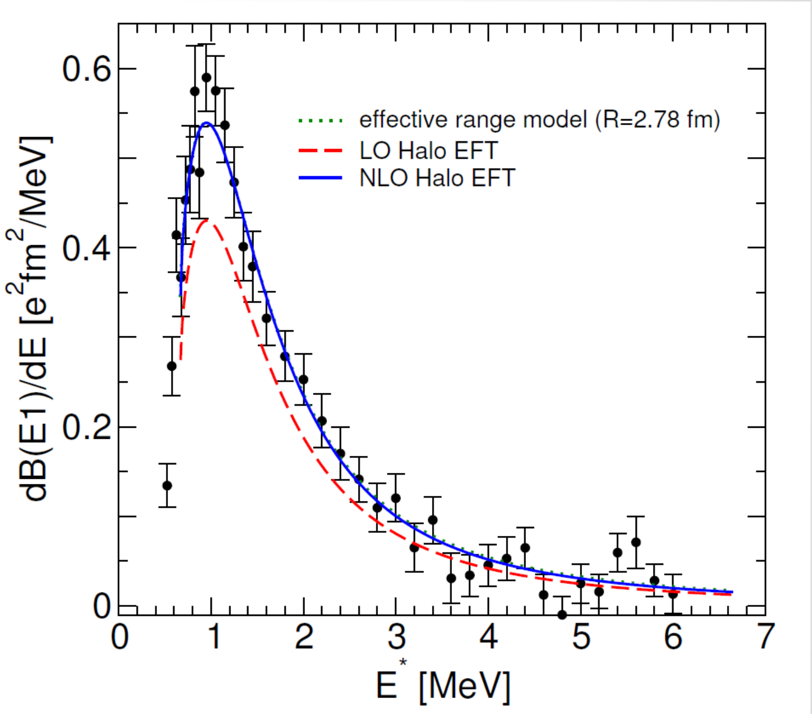
Rupak, Higa '11
Fernando, Higa, Rupak '12
...

→Electro/photodisintegration

Hammer, Phillips '11
Acharya, Phillips '13
...

→*etc.*

Example
^{11}Be Coulomb dissociation



Hammer, Phillips, *Nucl. Phys. A* **865** (2011) 17

Conclusions

Systems near unitarity can be described by essentially one parameter Λ_*

Renormalization leads to discrete scale invariance

Bosons saturate and form a quantum liquid

Multi-component fermions tend to clusterize

Expansion around unitarity works for light nuclei.
Beyond, can match onto Halo/Cluster EFT

Halo/Cluster EFT describes ground states and low-energy reactions

For discussion: the "reality" of clustering

Bottom-up logic here: proximity to threshold ⇒ clustering in space ⇒ clusters as "elementary" dofs

uncertainty principle *low resolution*

⇒ Halo/Cluster EFT ⇒ "reality" of clusters

most general form *where successful*

⇒ confirms proximity to threshold as the relevant criterion for clustering

experience
(also for other types of dofs)

Top-down approach: are there other criteria in "ab initio" nuclear EFT?

cf. Contessi's question to Ebran in day 1

cf. Duguet et al.'s non-observable nature of nuclear shell structure