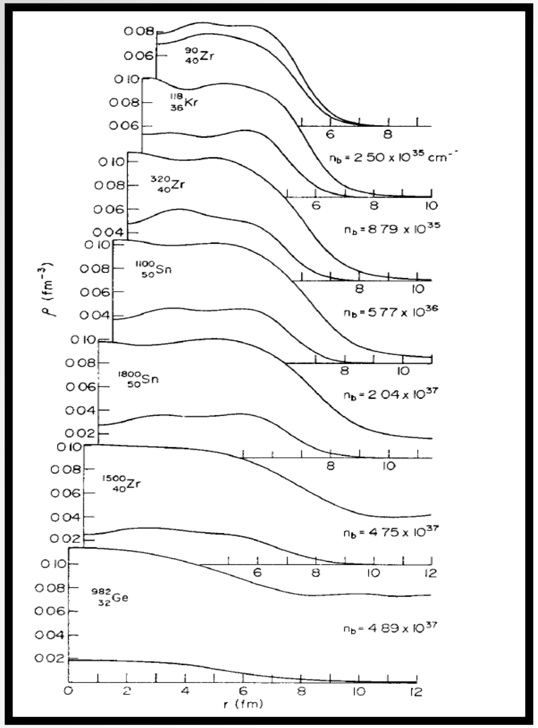
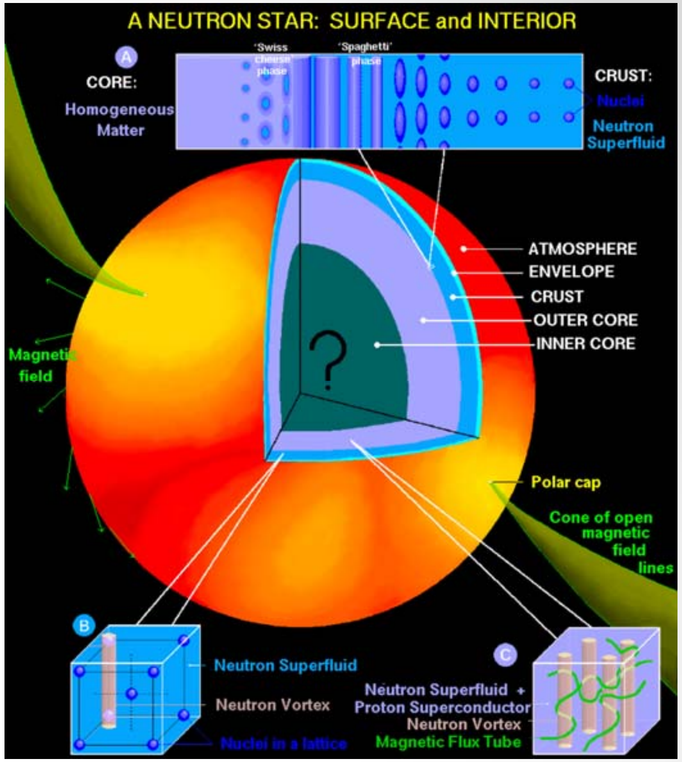




Clustering in neutron star matter

F.Gulminelli, (LPC and Normandie Université, Caen)

Transport properties in compact stars

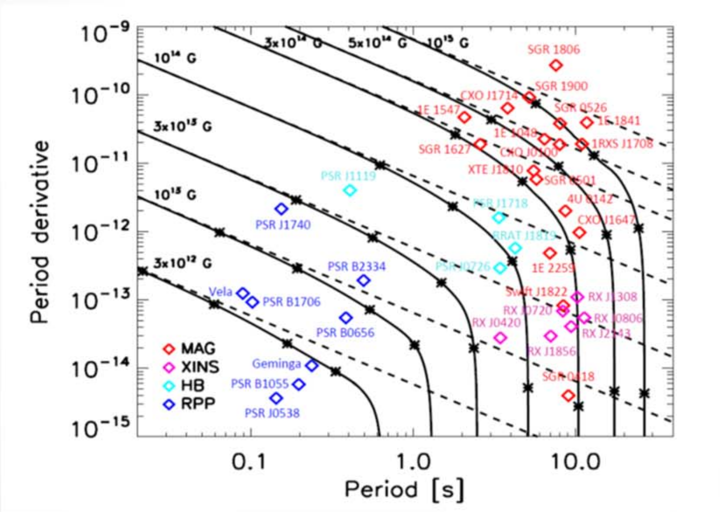
- Most signals from CS involve transport properties
- Some of them mainly concern $\rho \lesssim \rho_0$ matter ≡ clusters
  - NS cooling: B-thermal evolution
  - Relaxation after accretion & deep crust heating
  - CC, PNS cooling & mergers

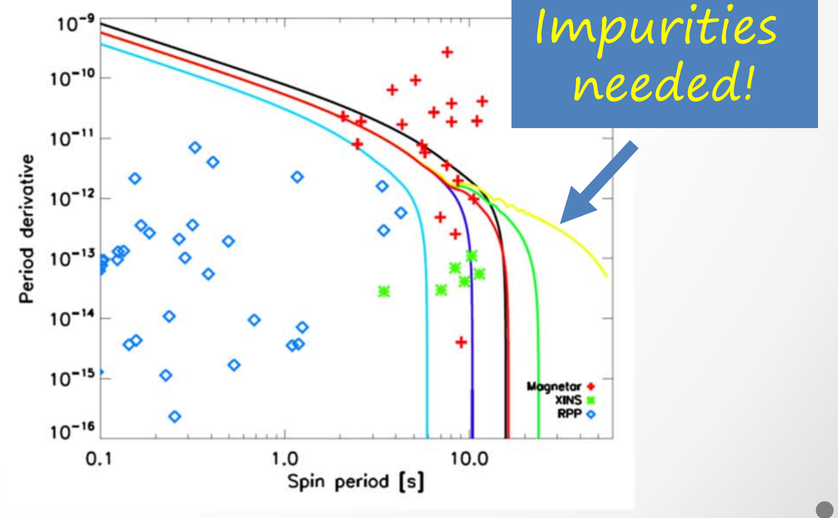
} e-Z, $T \approx 10^8 K$

ν-Z $T > 10^{10} K$

Schmitt&Shternin Springer 2018

- Key concept: quantify disorder => resistivity
  - T≫0 : distribution of nuclei (or pasta)
  - True even for a catalyzed crust: **impurities**





J.Pons et al. Nat.Phys. 2013

Transport properties in compact stars

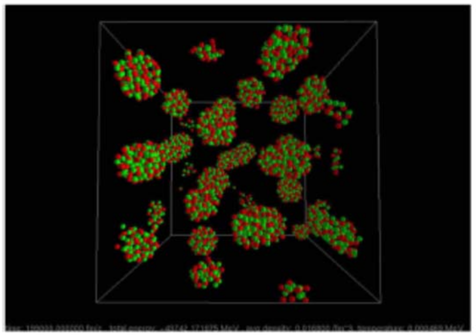
- **T<T$_m$** $\nu_{tot} = \nu_{e,i} + \nu_{e,imp}$ $Z^2 \leftrightarrow \boldsymbol{Q} = \sum_j \boldsymbol{n_j}\left(Z_j - \langle Z \rangle\right)^2$ Impurity factor
- **T>T$_m$** $\nu_{e(\nu),i} \rightarrow \sum_j \boldsymbol{n_j} \nu^j_{e(\nu),i}$ $\nu^j_{e(\nu),i} \propto S^j(k)$ Static structure factor

Present situation:

- n_j from Saha equations (Nuclear Statistical Equilibrium) or classical MD simulations Z.Lin et al,PRC **102**(2020)045801
- Q taken as a free parameter in cooling and relaxation simulations A.Deibel et al.ApJ839(2017)

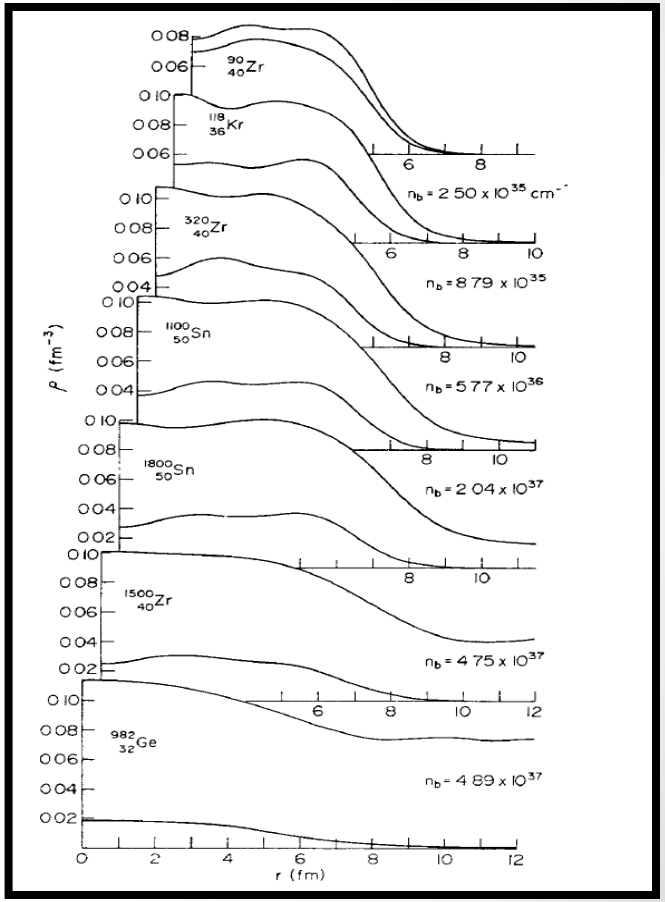
$$n_{AZ} = g^T_{AZ}\left(\frac{M_{AZ}T}{2\pi\hbar^2}\right)^{3/2} exp\left[\frac{N\mu_n + Z\mu_p - M_{AZ}}{T}\right]$$

THE ASTROPHYSICAL JOURNAL, 852:135 (16pp), 2018



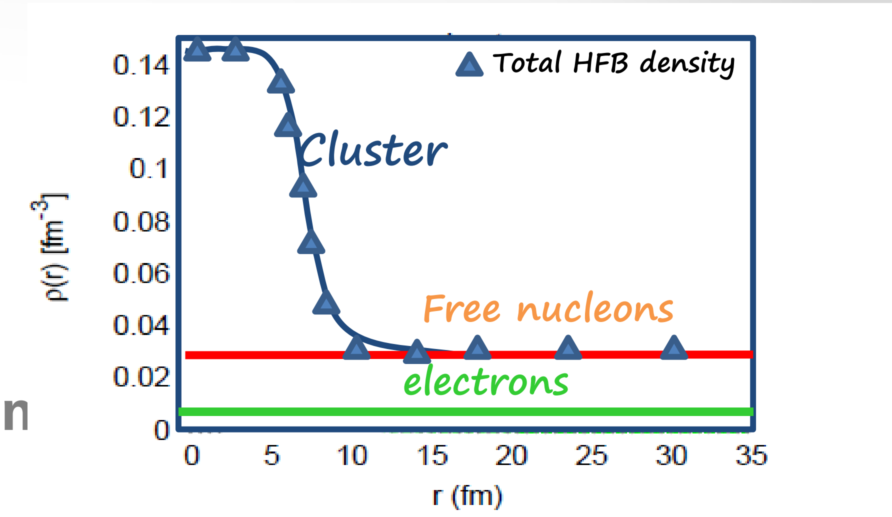
Towards a more controlled theoretical treatment

- Aim: having the nuclear functional as unique uncertainty ⇔ **unified** treatment at all ρ and T
- Let us start from what we know: variational calculations in the WS cell
- From WS cell to Multi-Component (liquid or solid) plasma: **cluster DoF**



J. W. Negele and D. Vautherin, NPA 207, 298 (1973)

From WS to MCP: mapping in 2 steps



- **WS cell with a microscopic function**

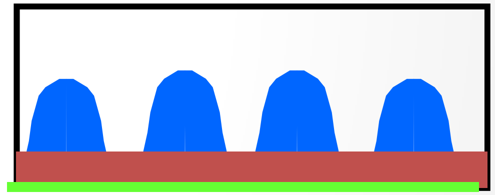
$$\mathcal{F}_{WS}\left(\hat{\rho}_q,\hat{\kappa}_q\right)=\mathcal{E}_{micro}-TS_{micro}=min$$

$$F_{AZ}=V_{WS}\left(\mathcal{F}_{WS}-\mathcal{F}_g\right)+\delta F$$

=> Optimal particle (and pairing) densities

1. **OCP with cluster DoF**

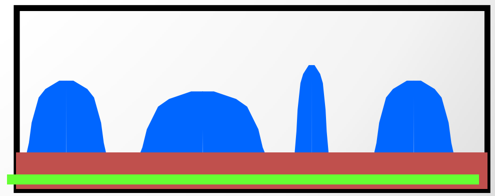
$$\mathcal{F}_{WS}\equiv\mathcal{F}^{OCP}(A,Z,\rho_{gq})=\mathcal{F}(\rho_{gq})+\frac{F_{AZ}}{V_{WS}}=min$$



=> Optimal cluster

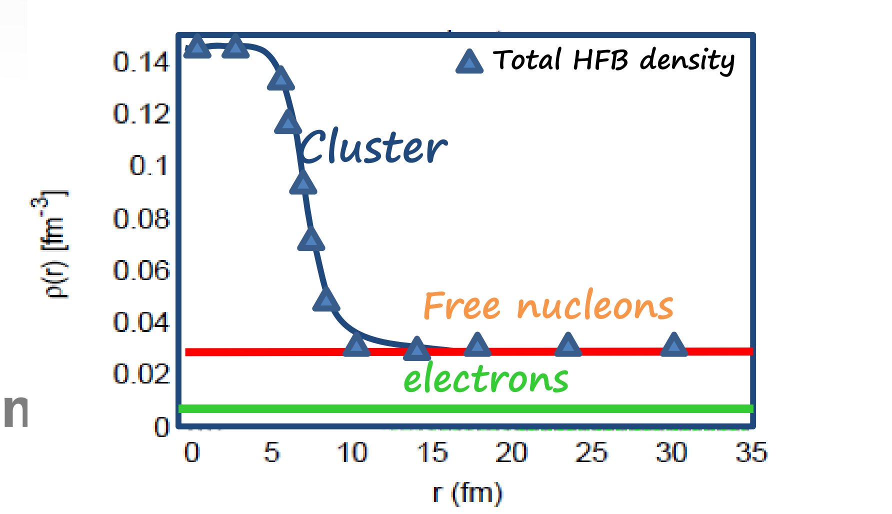
2. **MCP with cluster DoF**

$$\mathcal{F}^{MCP}\left(\{n_{AZ}\},\rho_{gq}\right)=\mathcal{F}(\rho_{gq})+\sum_{A,Z}n_{AZ}\,F_{AZ}+\delta F=min$$



=> Optimal distribution

From WS to MCP: mapping in 2 steps



- **WS cell with a microscopic function**

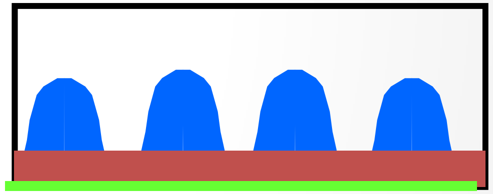
$$\mathcal{F}_{WS}\left(\hat{\rho}_q, \hat{\kappa}_q\right) = \mathcal{E}_{micro} - TS_{micro} = min$$

$$F_{AZ} = V_{WS}\left(\mathcal{F}_{WS} - \mathcal{F}_g\right) + \delta F$$

=> Optimal particle (and pairing) densities

1. OCP with cluster DoF

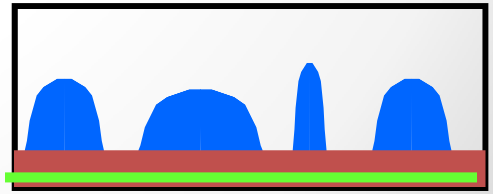
$$\mathcal{F}_{WS} \equiv \mathcal{F}^{OCP}\left(A, Z, \rho_{gq}\right) = \mathcal{F}\left(\rho_{gq}\right) + \frac{F_{AZ}}{V_{WS}} = min$$



=> Optimal cluster

2. MCP with cluster DoF

$$\mathcal{F}^{MCP}\left(\{n_{AZ}\}, \rho_{gq}\right) = \mathcal{F}\left(\rho_{gq}\right) + \sum_{A,Z} n_{AZ} F_{AZ} + \delta F = min$$



=> Optimal distribution

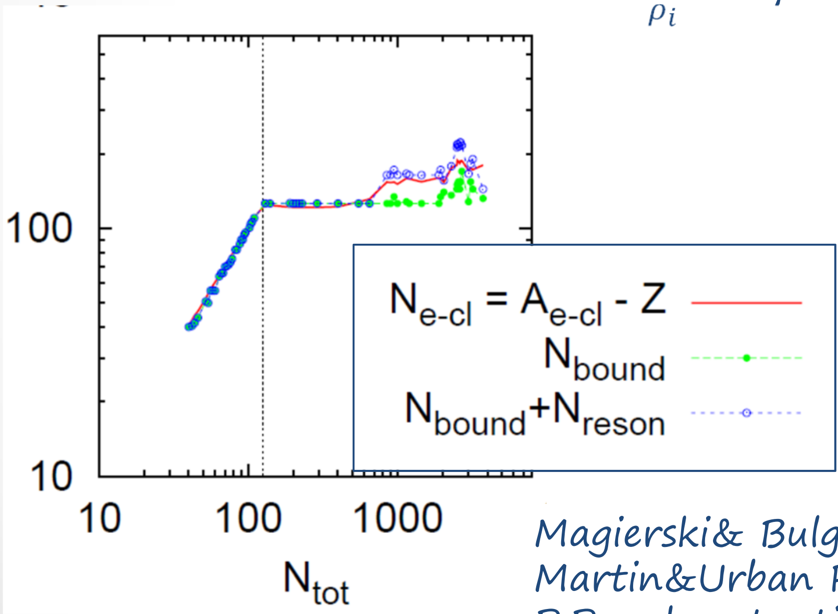
OCP with cluster DoF

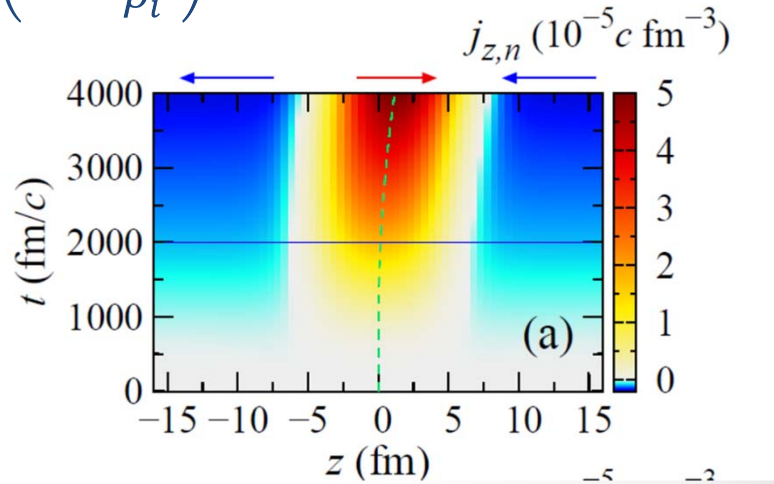
$$F_{AZ} = F_{AZ}^0 + T\left(ln\frac{\lambda_{AZ}^3(M^*)}{gV_f} - 1\right)$$

- CM degree of freedom: vibrations & translations ($T>T_m$)
- n-p interaction: (only) bound neutrons are entrained by the ion

$$\nabla^2\Phi = 0 \;\Leftrightarrow\; M^* = M\left(1 - \delta^f + \frac{(\delta^f - \gamma)^2}{\delta^f + 2\gamma}\right) \approx M\left(1 - \frac{\rho_{gn}}{\rho_i}\right)$$

$$\delta^f = \frac{\rho_n^f}{\rho_i} \qquad \gamma = \frac{\rho_{gn}}{\rho_i}$$



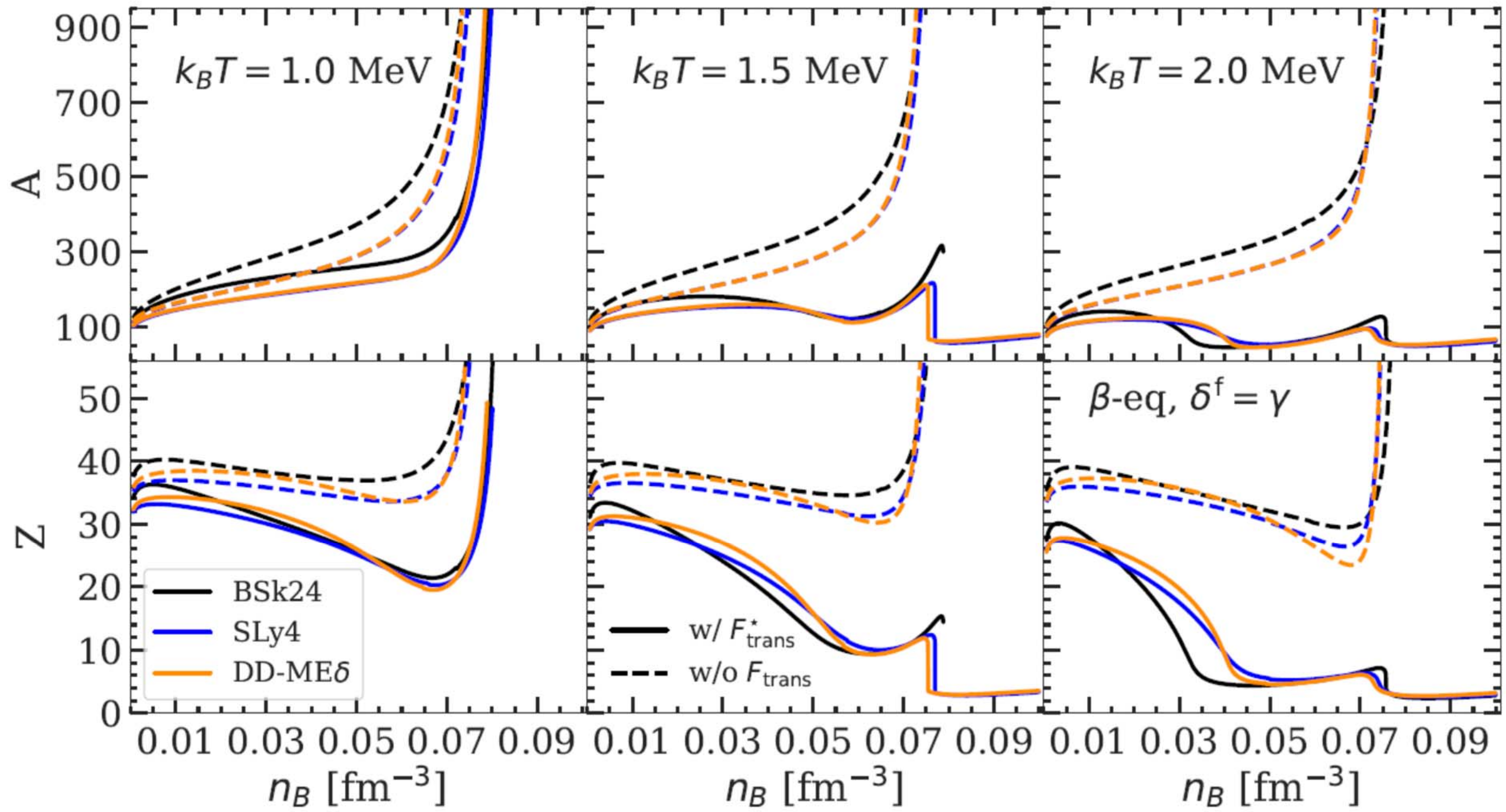


K.Sekizawa et al PRC 2022

Magierski& Bulgac NPA 2004
Martin&Urban PRC 2016
P.Papakonstantinou et al, PRC 2013

OCP with cluster DoF

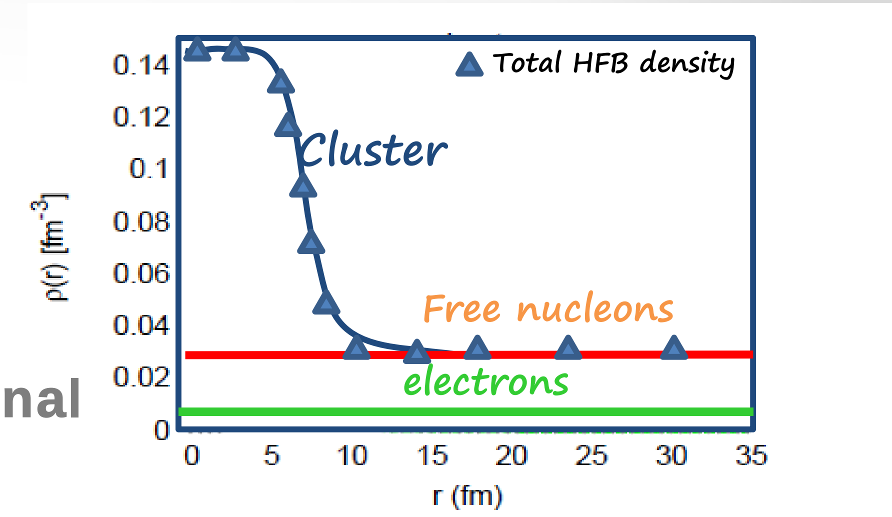
$$F_{AZ} = F_{AZ}^{0} + T\left(ln\frac{\lambda_{AZ}^{3}(M^{*})}{gV_{f}} - 1\right)$$



Essentially He clusters
@high density !

H.DinhThi et al, A&A in press

From WS to MCP: mapping in 2 steps



$$F_{AZ} = V_{WS}\left(\mathcal{F}_{WS} - \mathcal{F}_g\right) + \mathcal{F}_g V_{AZ} + \delta F$$

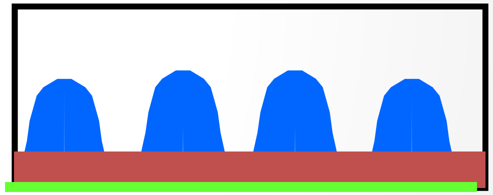
- **WS cell with a microscopic functional** $@(\rho_B, Y_p, T)$:

$$\mathcal{F}_{WS}\left(\hat{\rho}_q, \hat{\kappa}_q\right) = \mathcal{E}_{micro} - TS_{micro} = min$$

=> Optimal particle (and pairing) densities

1. **OCP with cluster DoF**

$$\mathcal{F}^{OCP}\left(A, Z, \rho_{gq}\right) = \mathcal{F}\left(\rho_{gq}\right) + \frac{F_{AZ}}{V_{WS}} = min$$

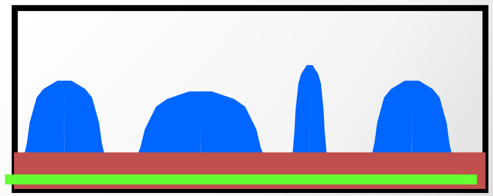


=> Optimal cluster

2. **MCP with cluster DoF**

$$\mathcal{F}^{MCP}\left(\{n_{AZ}\}, \rho_{gq}\right) = \mathcal{F}\left(\rho_{gq}\right) + \sum_{A,Z} n_{AZ} F_{AZ} = min$$

=> Optimal distribution



The cluster distribution

Grams 2018, PRC, 97, 035807
Fantina 2020, A&A, 633, A149
Carreau 2020, A&A, 640, A77
Dinh-Thi 2023, to be submitted

- $d\mathcal{F}_{MCP}(\{n_{AZ}\}) = 0$ leads to:

Continuum subtracted & microscopic level density

$$n_{AZ} = \left(\frac{M^*_{AZ} T}{2\pi\hbar^2}\right)^{3/2} \exp\beta[N\mu_n + Z\mu_p - F_i + R_{AZ}(n_e)]$$

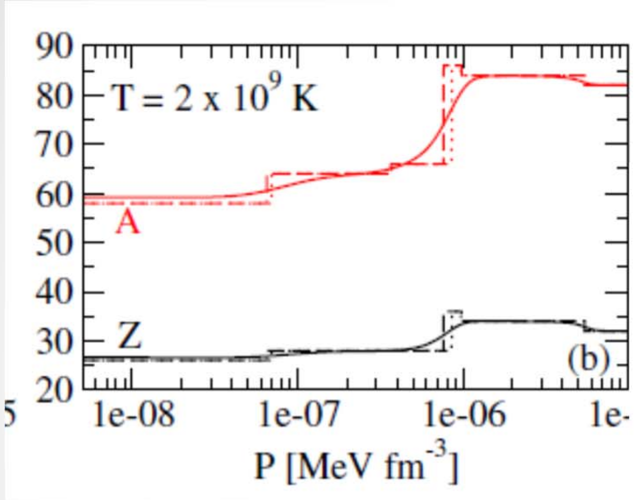
Rearrangement ($n_e = \sum_{AZ} Z n_{AZ}$)

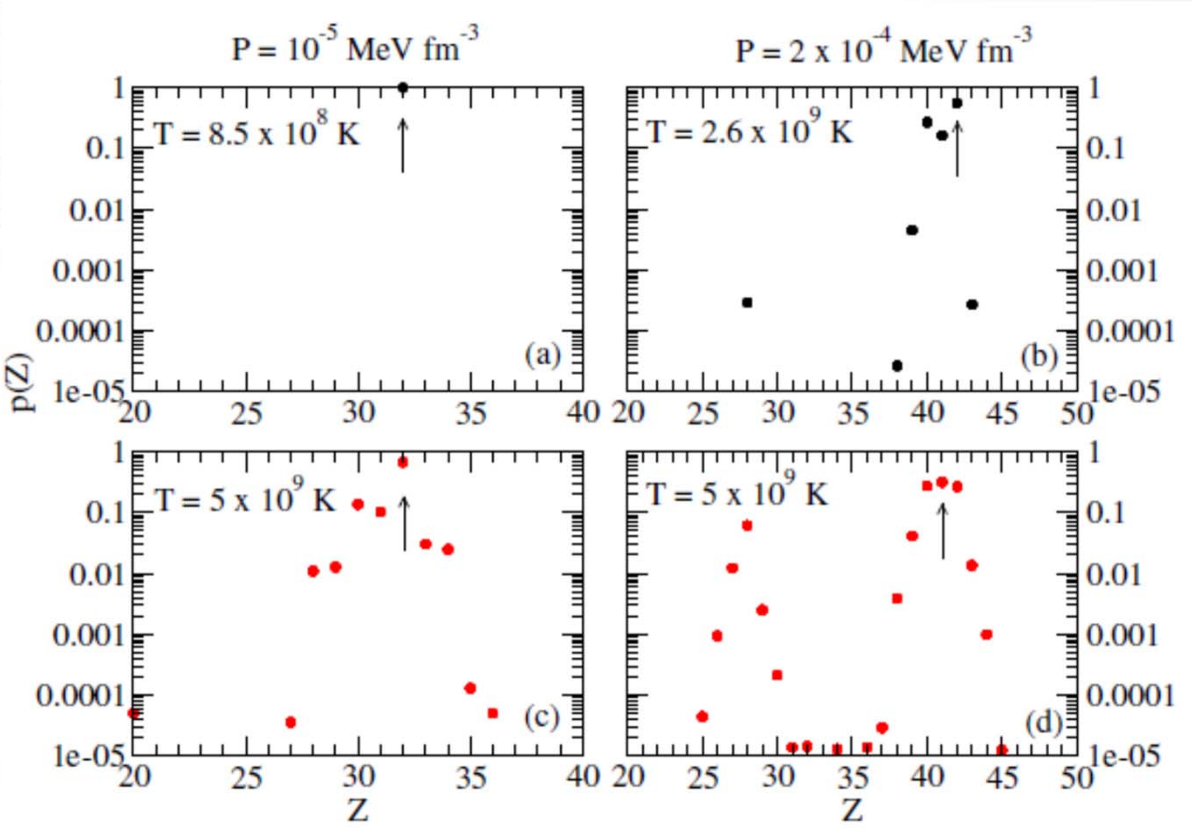
$$\mu_q = \frac{\partial \mathcal{F}_\mu}{\partial \rho_{gq}} + \sum_{AZ} n_{AZ} \frac{\partial F_{AZ}}{\partial \rho_{gq}} \left(1 - \sum_{AZ} n_{AZ} V_{AZ}\right)^{-1} \approx \frac{\partial \mathcal{F}_\mu}{\partial \rho_{gq}} + \frac{1}{V_{WS}^{OCP}} \frac{\partial F_{AZ}^{OCP}}{\partial \rho_{gq}} \left(1 - u_{AZ}^{OCP}\right)^{-1}$$
$$= \mu_q^{OCP}$$

Self-consistent μ(ρ)

Perturbation 1st order

Nuclear distribution in the outer crust

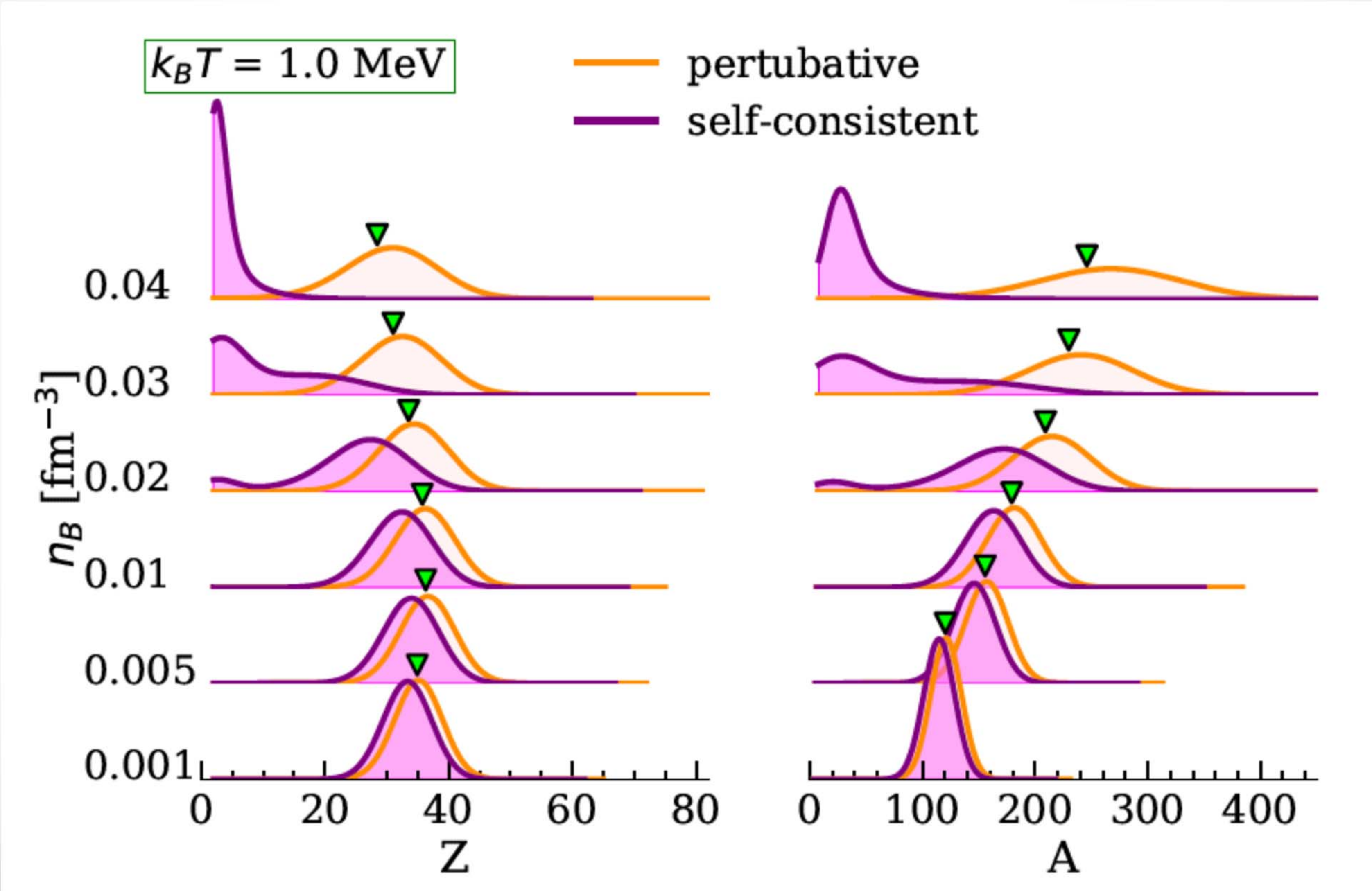




- Fantina 2020, A&A, 633, A149

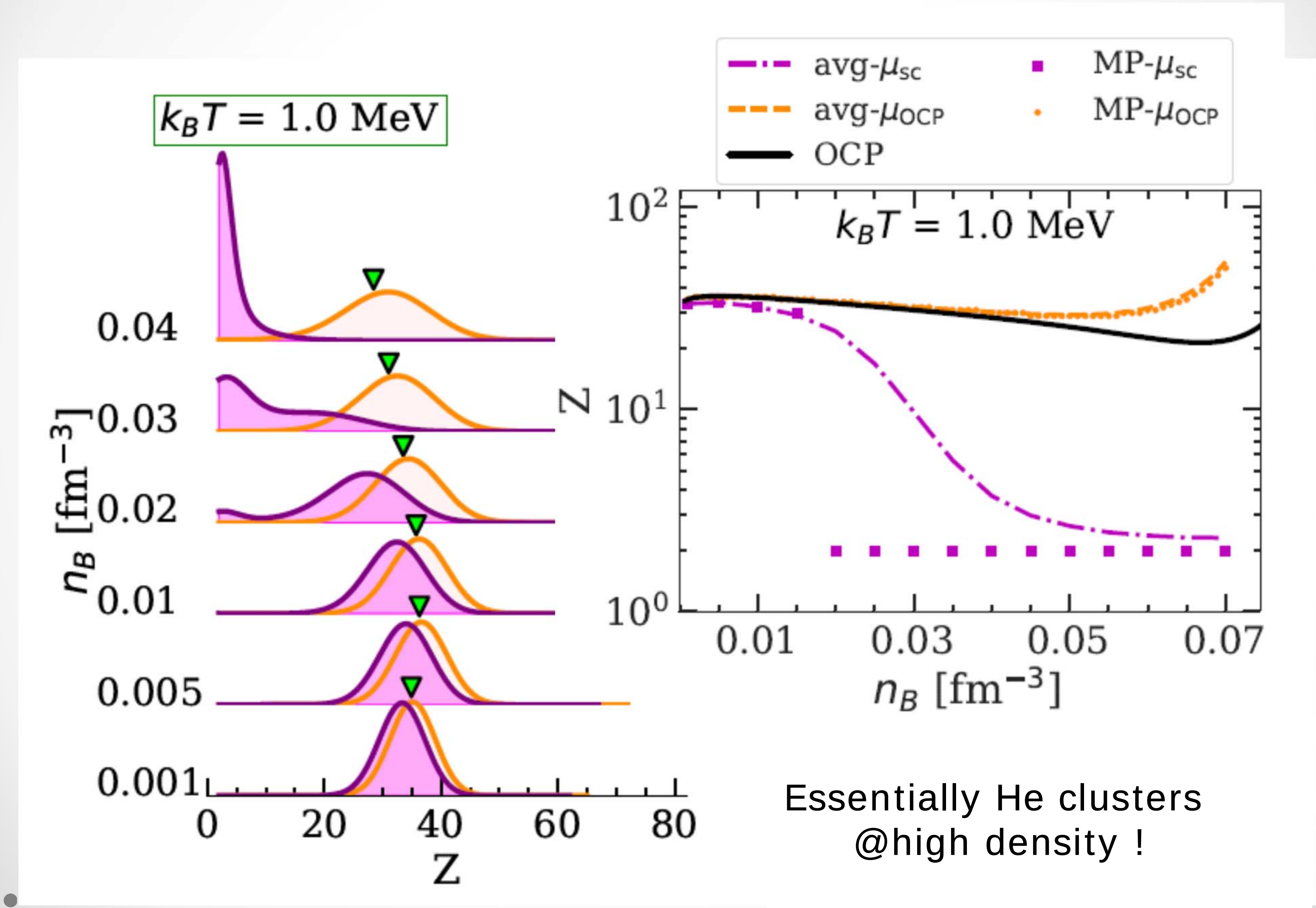
The cluster distribution

Dinh-Thi 2023, to be submitted



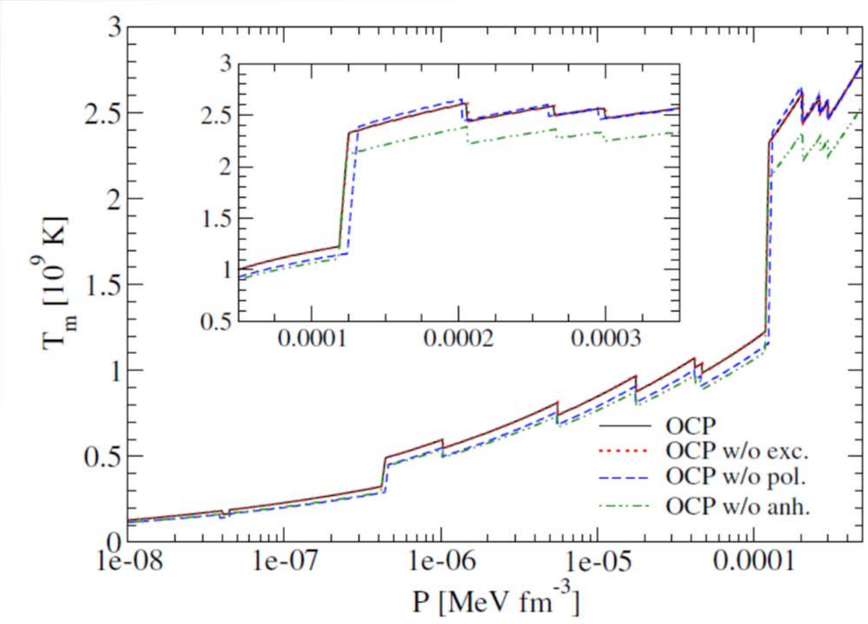
The cluster distribution

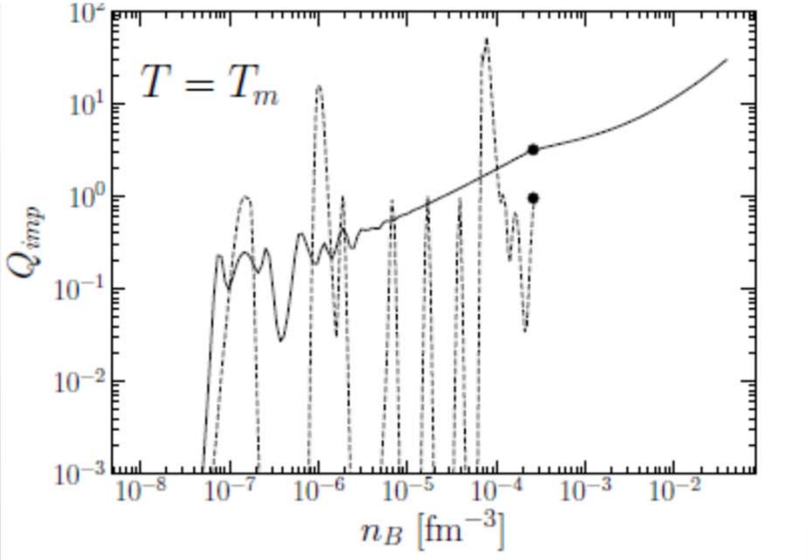
Dinh-Thi 2023, to be submitted

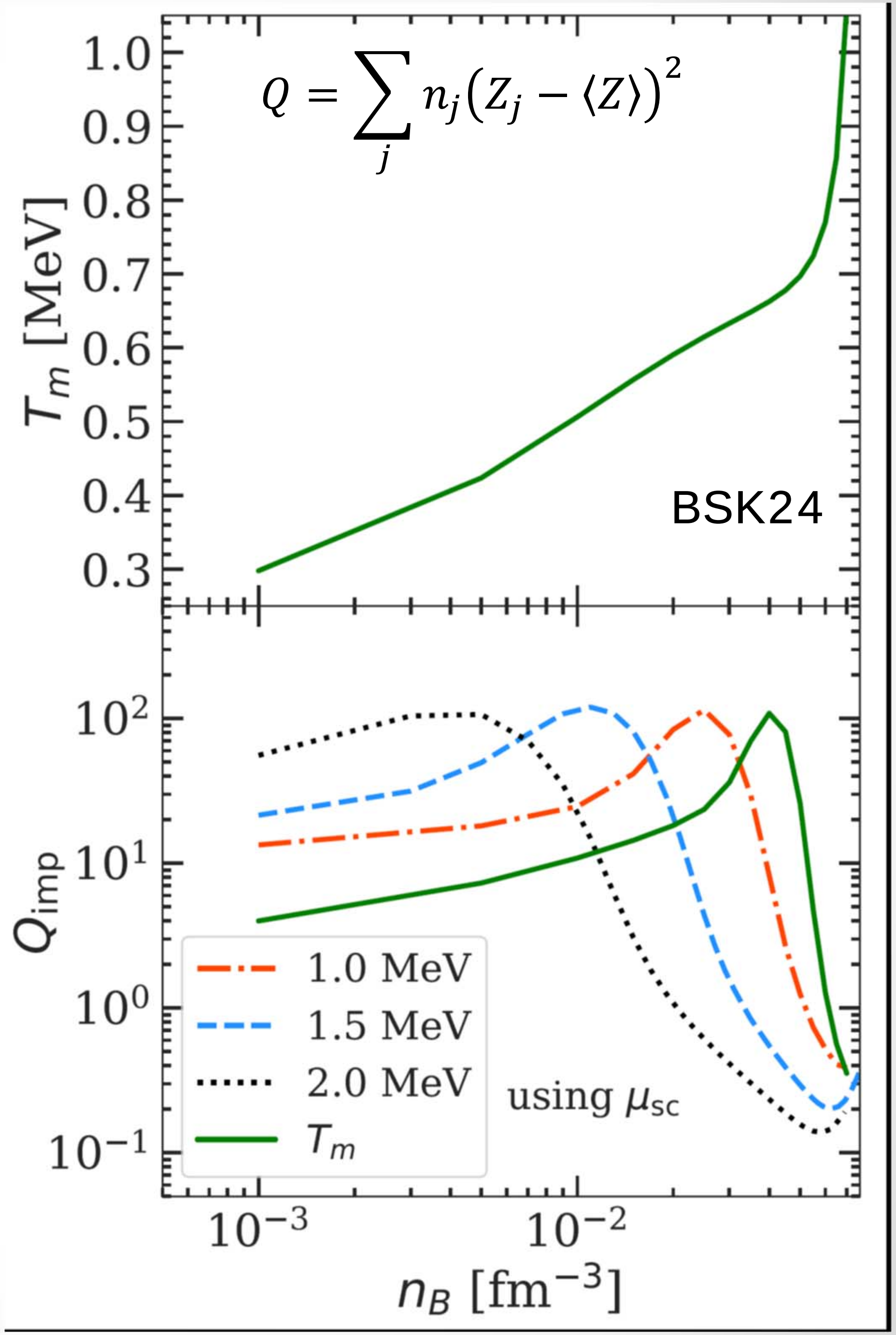


Essentially He clusters
@high density !

Impurity factor

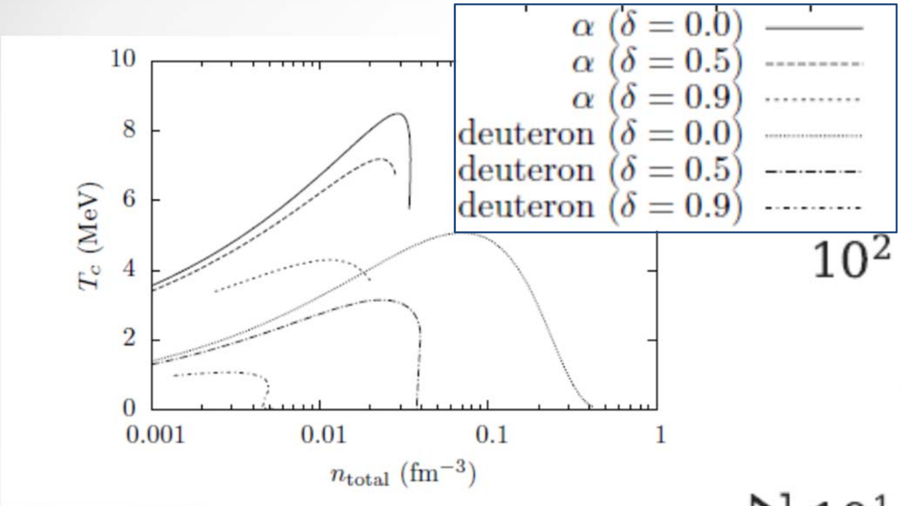




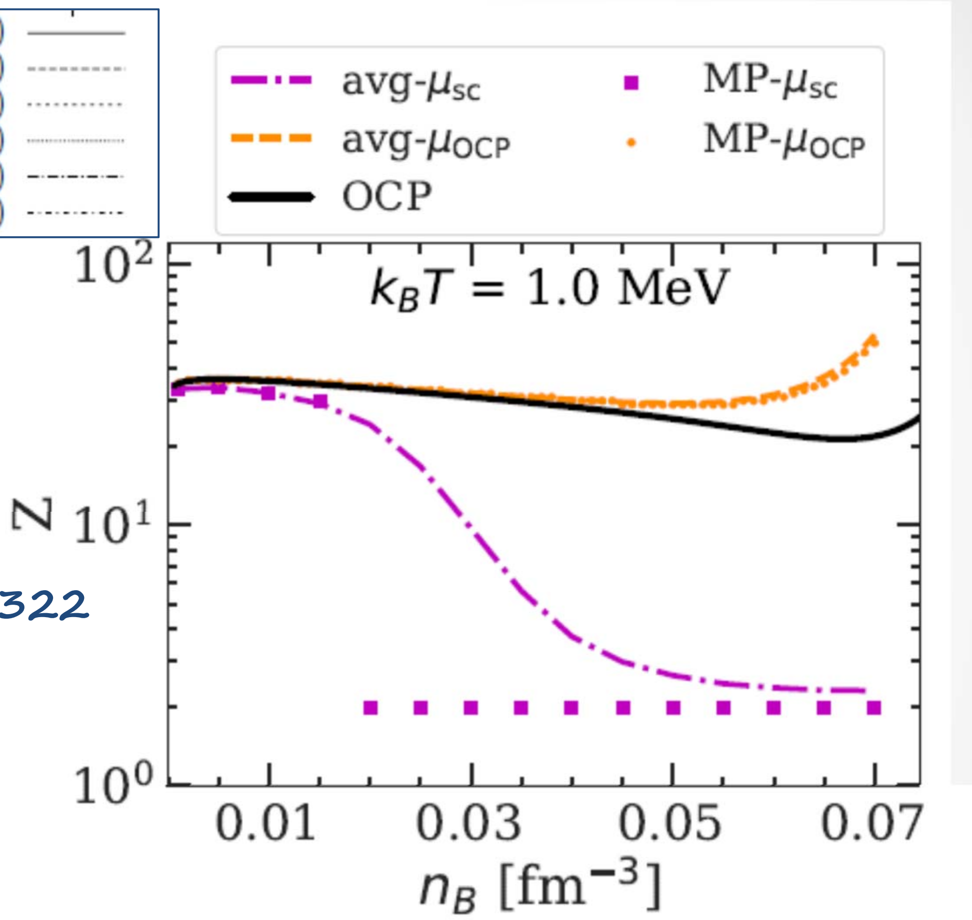


The cluster distribution

Dinh-Thi 2022, to be submitted



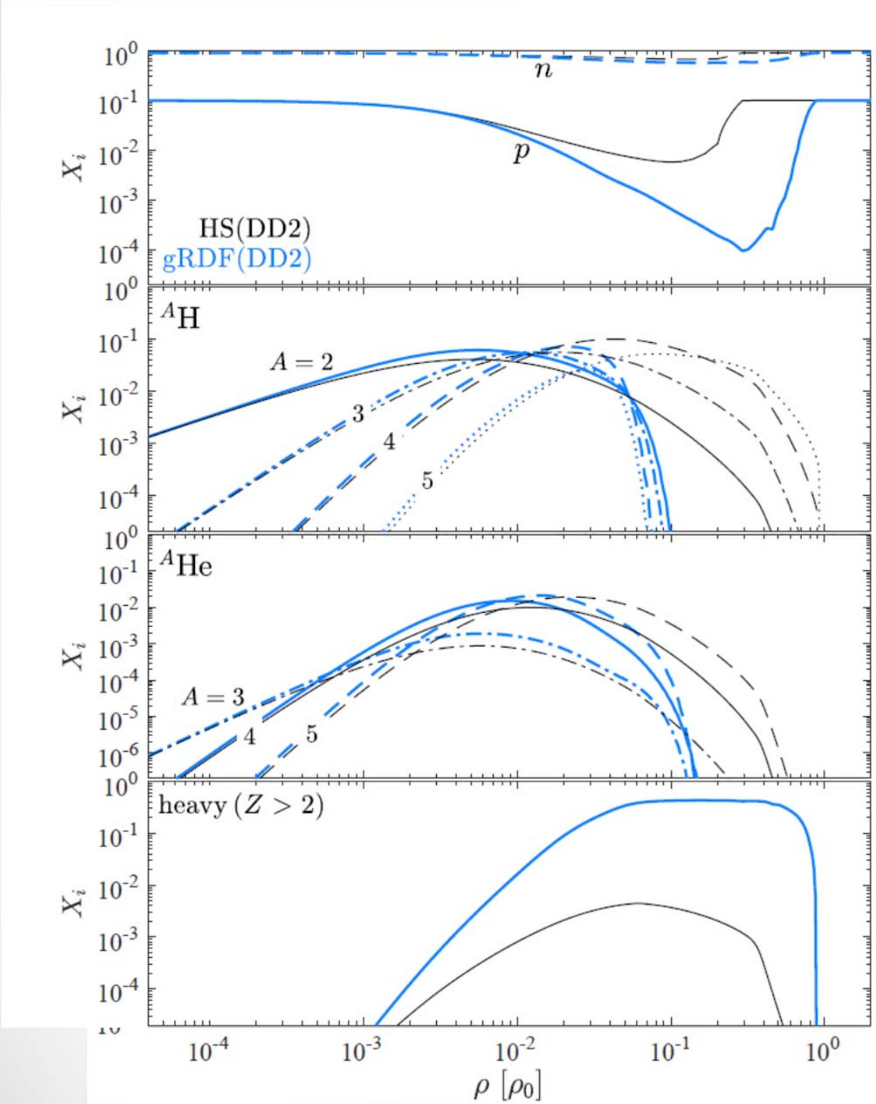
Takaaki Sogo, Gerd Röpke, and Peter Schuck, Phys. Rev. C 82, 034322



Is the cluster dominance at high density (=> Q decrease) realistic ?

Essentially He clusters @high density !

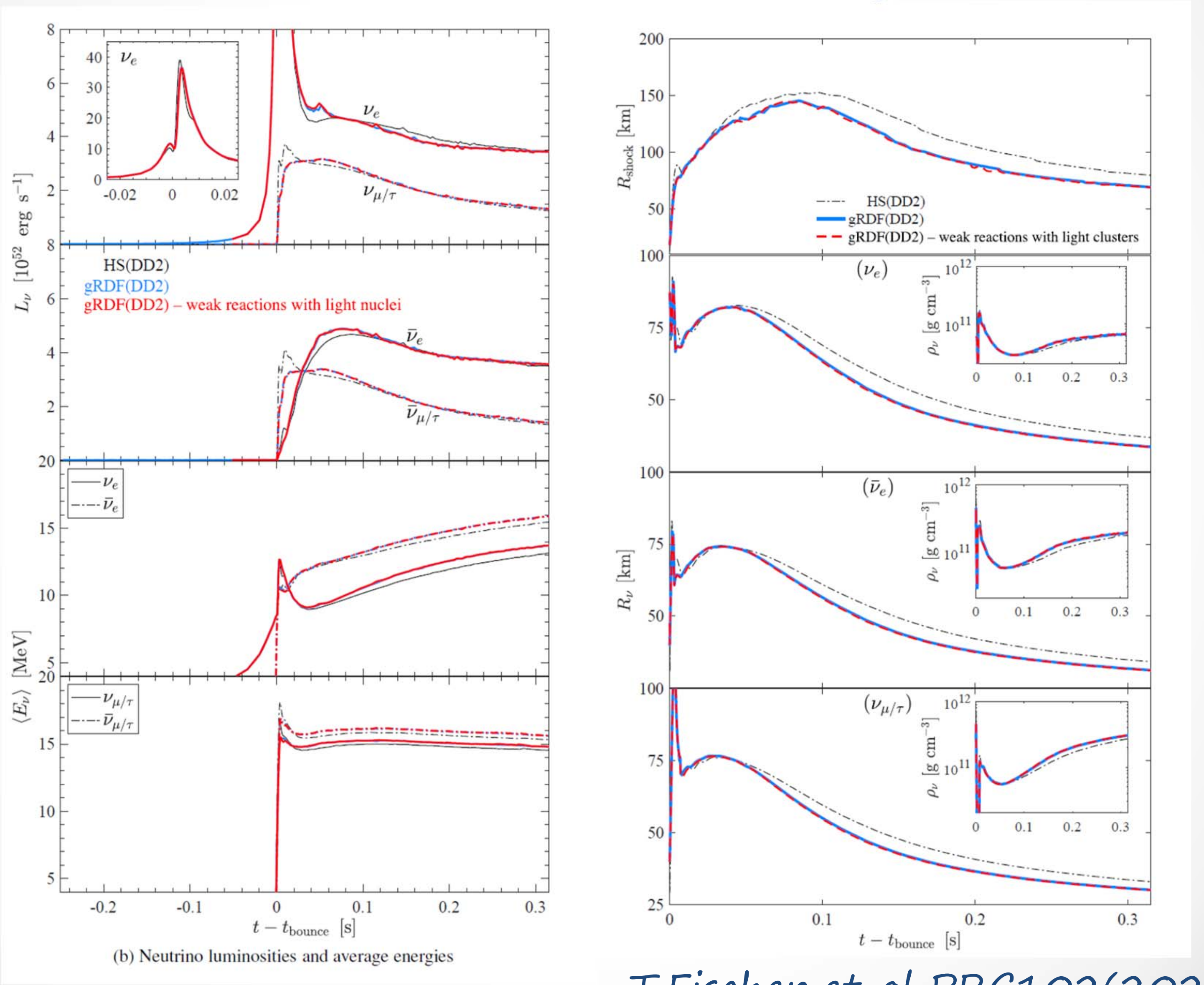
Clusters in-medium effects



T.Fischer et al PRC102(2020)

- the ETF approach is not very realistic for light clusters!
- Alternative approaches: in-medium modified meson couplings H.Pais, FG PRC 97(2018)045805; quasi-particle virial expansion G.Roepke, PRC101 (2020) 064310
- Constraining the in-medium modifications: see Alex talk!

Effect on the CCSN dynamics



T.Fischer et al PRC102(2020)

Conclusions

- A thermodynamically consistent formalism to calculate matter composition from a given microscopic energy functional: a unified treatment for neutron stars and supernova matter

⇒ First microscopic evaluation of the impurity factor
⇒ Possible implications for CCSN

- Important differences wrt Saha equation
  - Subtraction of continuum states: reduced partition sum
  - In-medium modified cluster energies (ETF)
  - Rearrangement terms modify even the average quantities
- Important differences wrt calculations in the WS cell
  - Center of mass motion favours the appearence of light clusters
  - Bimodal cluster distributions => increase of Qimp!
  - Cluster melting => Z=2 dominance close to the core at high temperature
  - Light cluster functional still to be improved