



UNIVERSITÀ DEGLI STUDI DI NAPOLI
FEDERICO II



The Hoyle State in ^{12}C

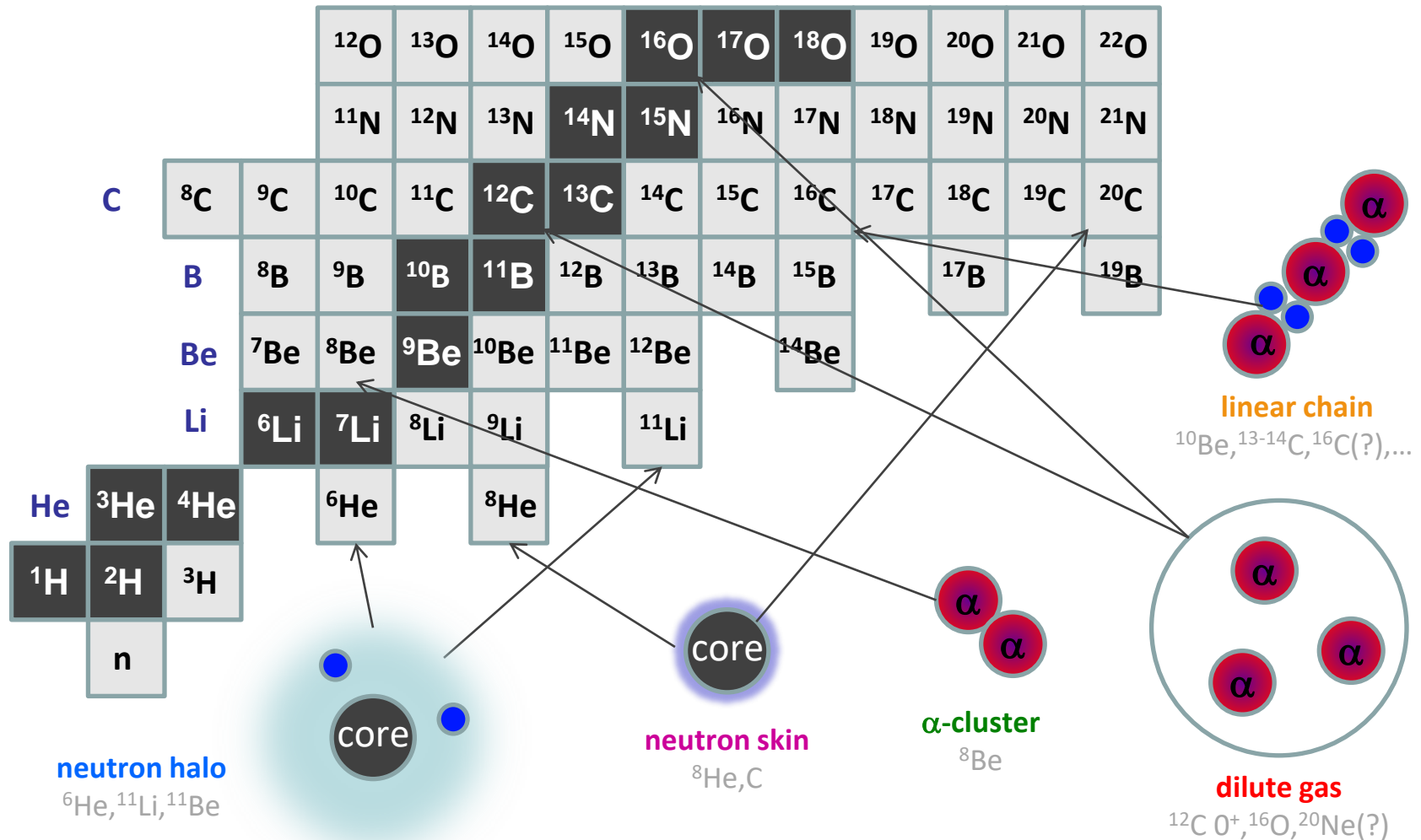
Daniele Dell'Aquila

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Napoli, Italy
INFN-Sezione di Napoli, Napoli, Italy**



- **Physics background**
 - Phenomenology of light nuclei → clustered states;
 - The Discovery of the Hoyle State in ^{12}C ;
 - Astrophysical implications;
 - The Structure of the Hoyle State;
- **Alpha-decay of the Hoyle State**
 - Direct vs Sequential decay;
 - Recent high-precision experiments;
 - Theoretical interpretations;
- **Radiative decay of the Hoyle State**
 - State-of-the-art;
 - New perspectives: the MORALIS experiment at INFN-LNL;
- **Conclusions**

Complexity of nuclear force → deviation from the **sphericity**: axial deformation (collective behaviours), spatial re-organization of nucleons in bounded **sub-units** (**cluster model**).

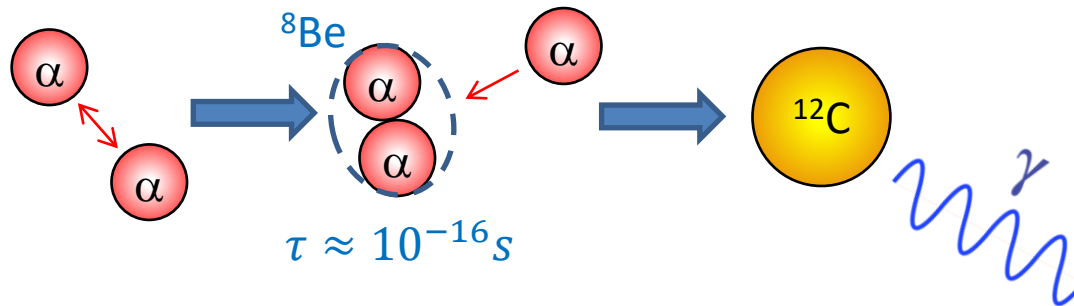
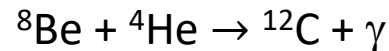


Helium burning in stars \rightarrow the discovery of the Hoyle state in ^{12}C

1939, Bethe \rightarrow formation of ^{12}C from the collisions of three α -particles improbable \rightarrow temperatures 50 times higher than the Sun required;

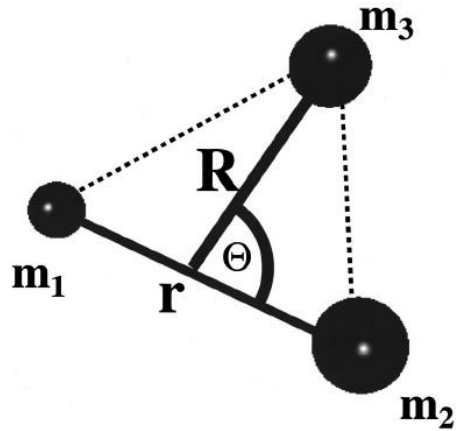
1951, Öpik \rightarrow the nucleosynthesis of carbon occurs in the Red Giant phase;

1952, Salpeter \rightarrow the nucleosynthesis of carbon is a *sequential* two-steps process;



1953, Hoyle \rightarrow to reproduce the observed C/O abundance \rightarrow the 3α process occurs via an s-wave resonance \rightarrow The Hoyle state (7.654 MeV, 0^+);

1957, Cook \rightarrow experimental confirmation of the existence of the Hoyle state.

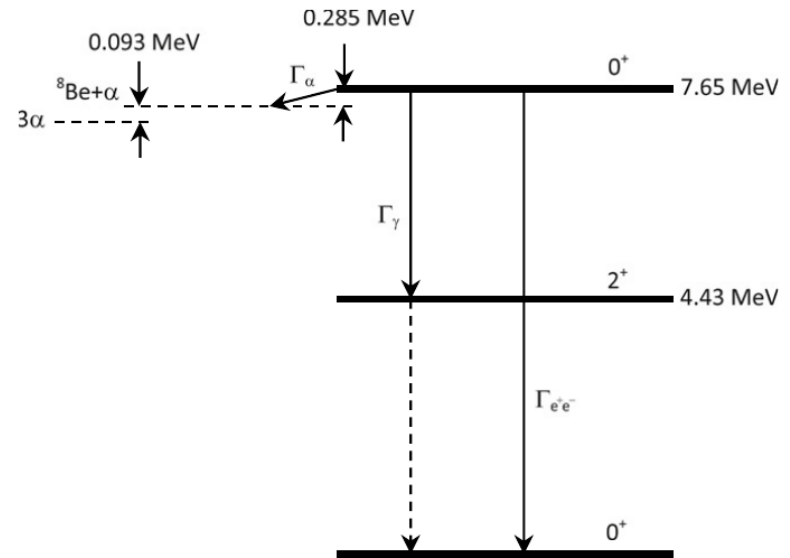
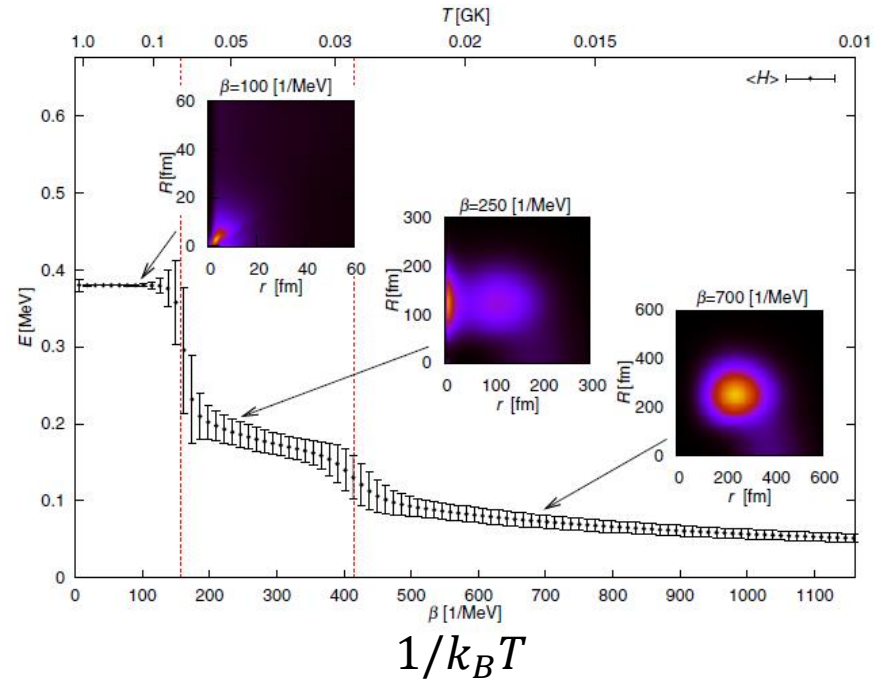


At stellar temperatures $T > 10^8$ K

The energy needed to form a ${}^8\text{Be}$ (92 keV) is extremely close to the *Gamow window* \rightarrow
 ex: $T = 10^8$ K \rightarrow Gamow peak $E_G = 85$ keV,
 $\Gamma_G = 60$ keV \rightarrow the process is resonant and proceeds through the Hoyle state in ${}^{12}\text{C}$.

the 3α reaction rate is related to the properties of the Hoyle state via:

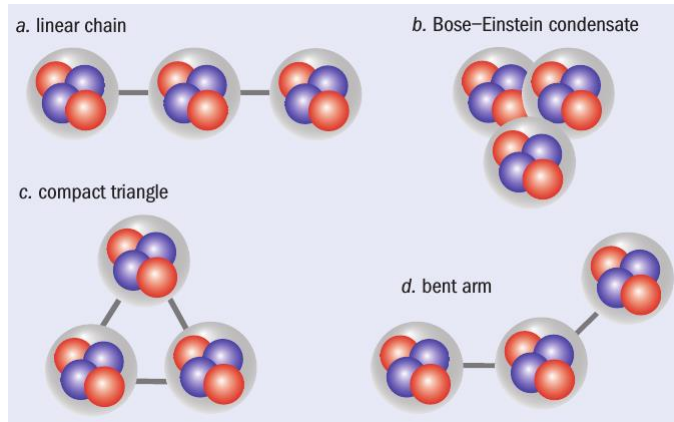
$$\langle \sigma v \rangle \propto \frac{\Gamma_\alpha \Gamma_{\text{rad}}}{\Gamma} e^{-\frac{E_R}{k_B T}}$$



Cluster state of ^{12}C located at 7.654 MeV (0^+) \rightarrow characterized by a pronounced cluster nature \rightarrow quite unusual and not well understood properties \rightarrow **challenging open question** in nuclear physics.

$$E_x = 7.654 \text{ MeV}$$

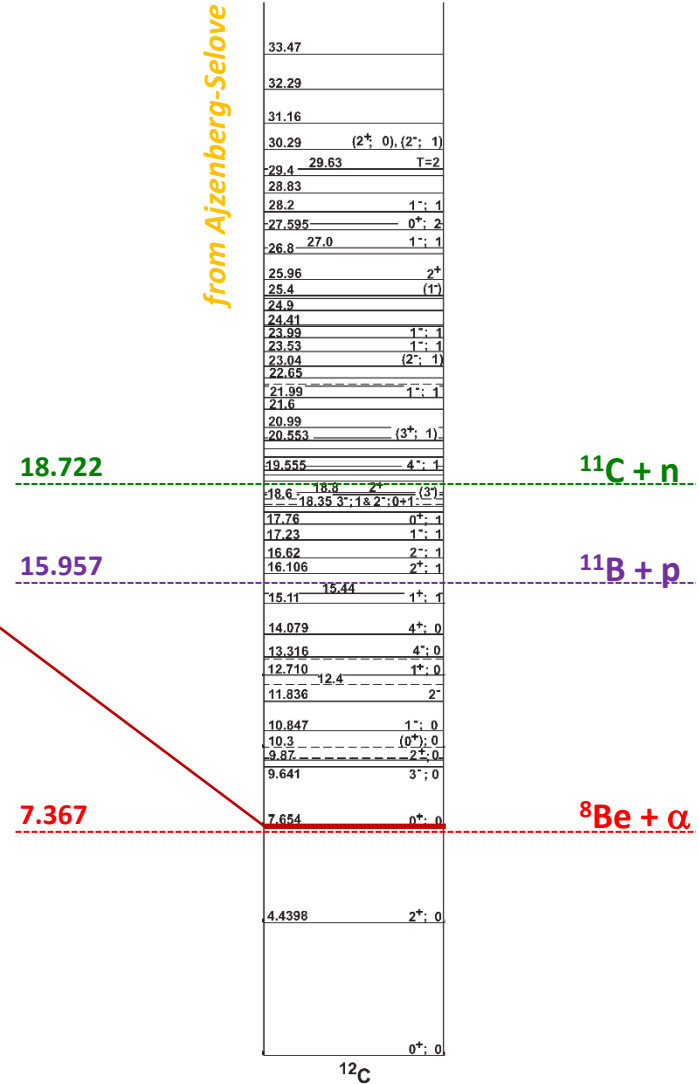
$$J\pi = 0^+$$



D. Jenkins and O. Kirsebom, The Secret of Life, Physics World Feb. 2013

- (a) linear chain: α -cluster model
- (b) Bose-Einstein condensate: microscopic models (GCM, RGM)
- (c) compact triangle: ACM
- (d) bent-arm: microscopic models (Faddeev three-body formalism)

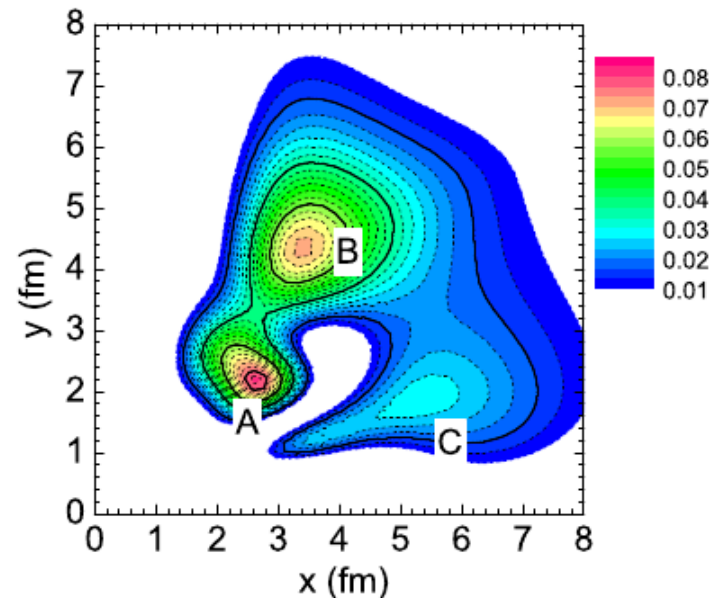
from Ajzenberg-Selove



Dynamical symmetries and rotational bands

H. Morinaga, Phys. Rev. 101 (1956) 254 → suggested that the Hoyle state in ^{12}C could be highly deformed → linear chain structure.

Modern microscopic calculations → density distribution $\rho(x,y)$ of the Hoyle state from microscopic calculations based on the Faddeev three-body formalism → three distinct peaks which correspond to an equilateral triangle configuration (A), or a bent-arm configuration (B), (C).



S. Ishikawa, Phys. Rev. C 90, 061604(R) (2014)

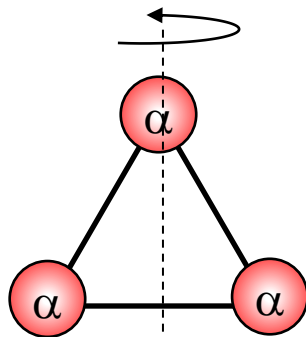
Dynamical symmetries and rotational bands

L.R. Hafstad, E. Teller, *Phys. Rev.* 54 (1938) 681 → suggested the role of dynamical symmetries in the 3-alpha cluster system → a simple picture can describe this system as a classical rotational top with a triangular symmetry → quantal rotational properties are given by the equation:

$$E_{J,K} = \frac{\hbar^2 J(J+1)}{2\mathcal{I}_{\text{Be}}} - \frac{\hbar^2 K^2}{4\mathcal{I}_{\text{Be}}}$$

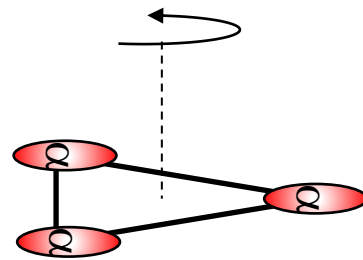
K: projection of the angular momentum onto the 3-fold symmetry axis.

One generates a number of rotational bands with different values of K.



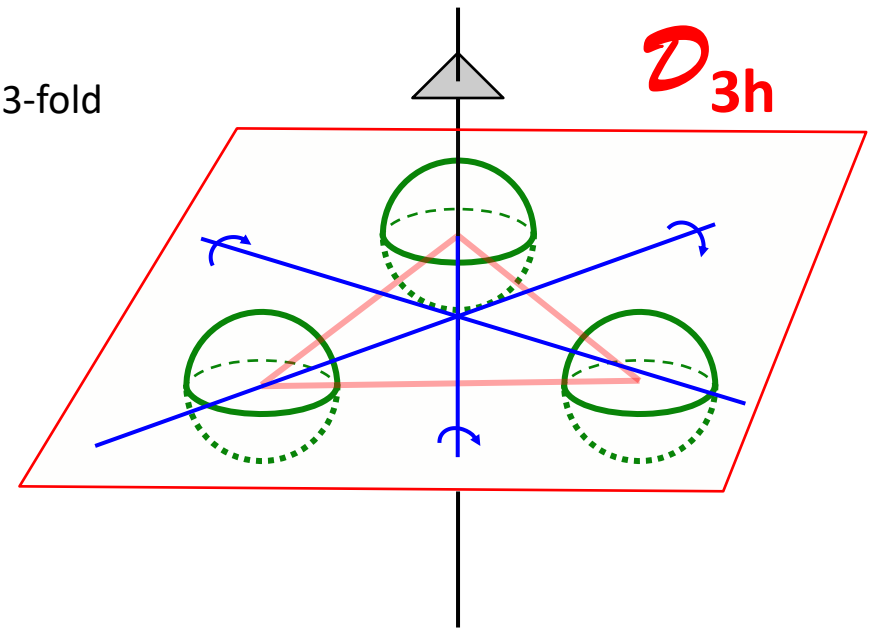
$K\pi=0^+$

$0^+, 2^+, 4^+, \dots$



$K\pi=3^-, 6^+$

$3^-, 4^-, 5^-, \dots$

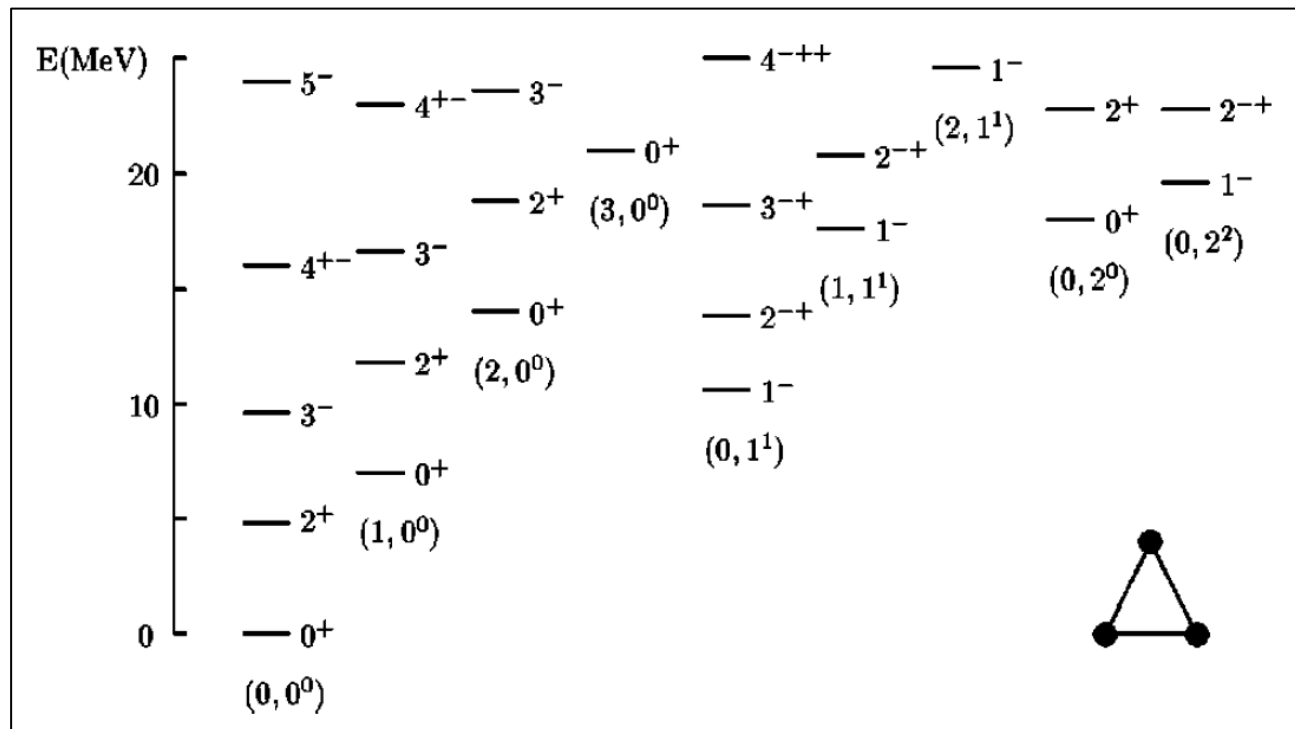


- **3-fold principal symmetry axis** (\mathcal{C}_3)
- **3 two-fold axes perpendicular to \mathcal{C}_3**
- **Horizontal mirror plane**

Dynamical symmetries and rotational bands

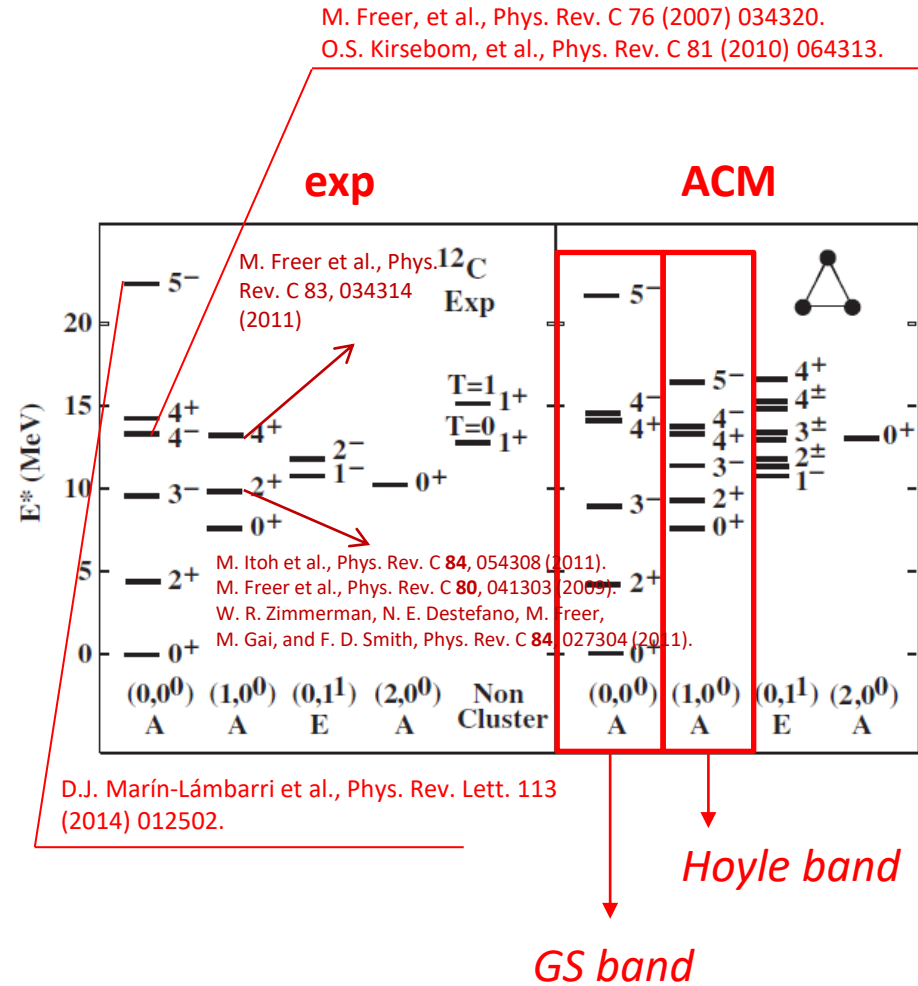
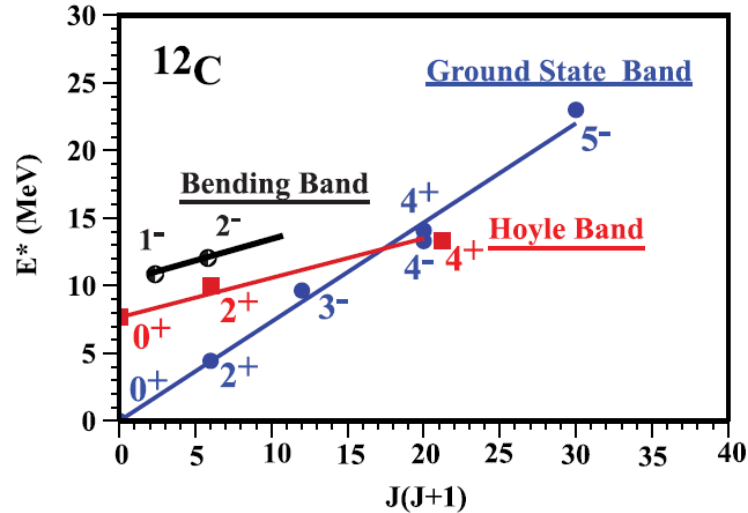
\mathcal{D}_{3h} point group symmetry \rightarrow can describe such rotations. The corresponding rotational-vibrational spectrum can be reproduced by the equation (see R. Bijker and F. Iachello, *Phys. Rev. C* **61** (2000) 067305):

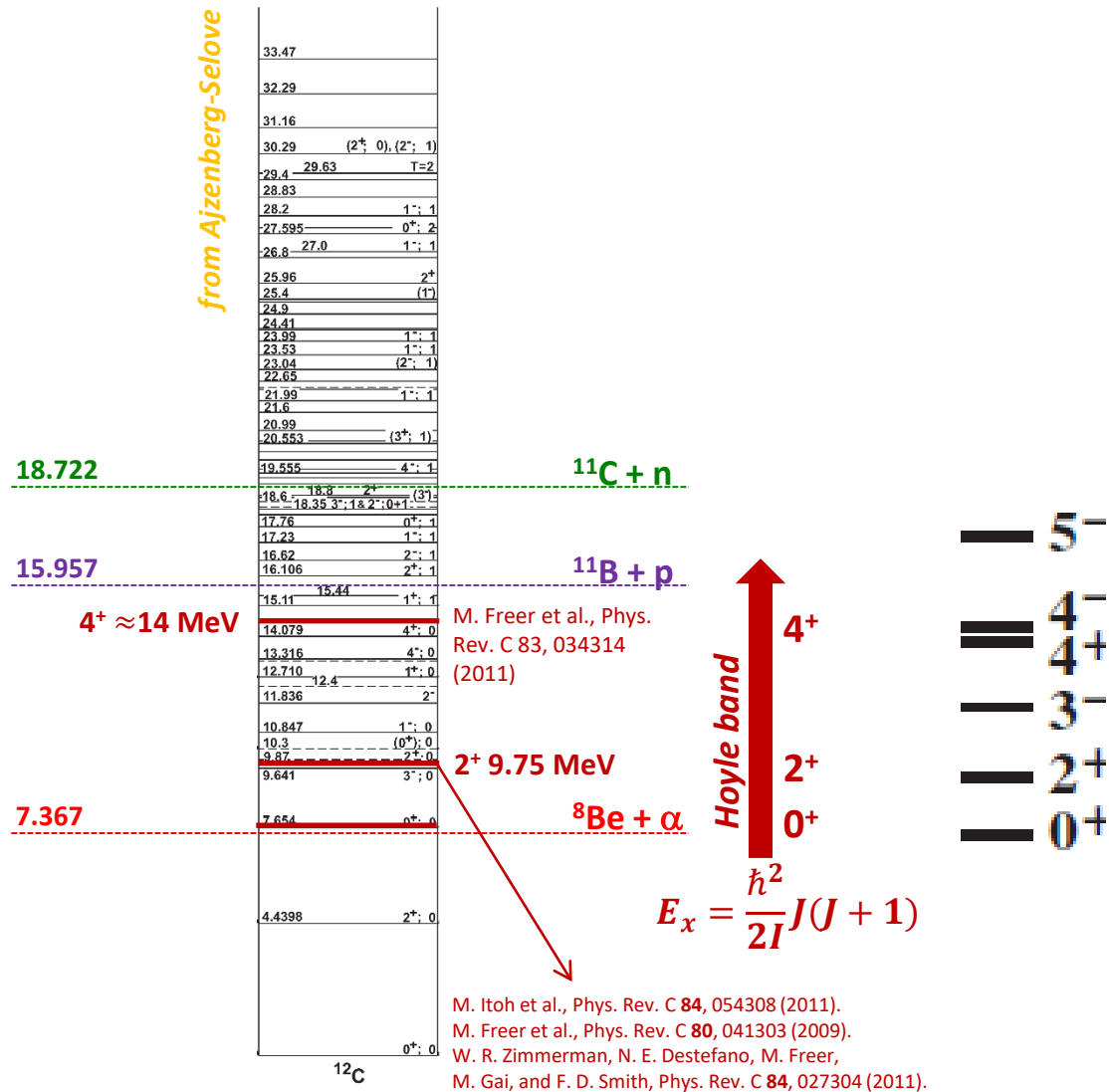
$$E = E_0 + Av_1 + Bv_2 + CL(L + 1) + D(K \pm 2l)^2$$



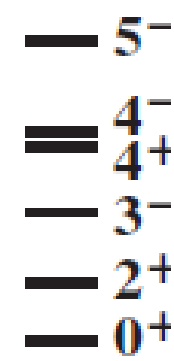
Dynamical symmetries and rotational bands

Marin-Lambarri, PRL (2014) →
first experimental evidence for \mathcal{D}_{3h}
symmetry in nuclei.





ACM from Marin-Lambarri, PRL (2014)

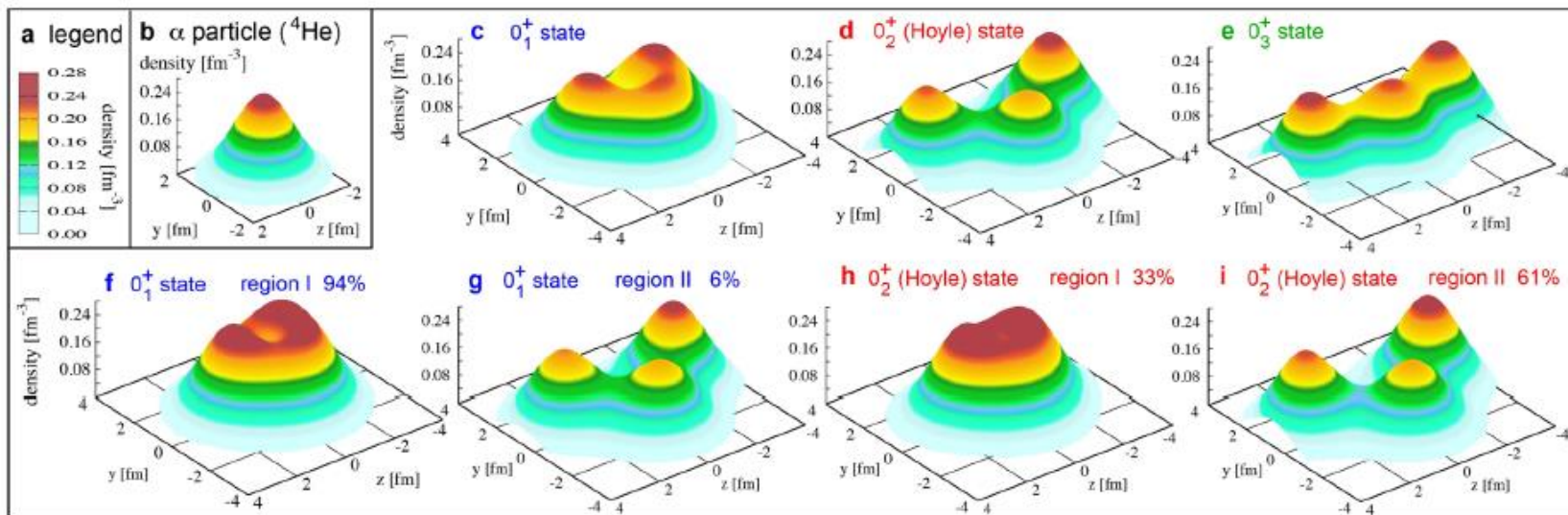


$$E_x = \frac{\hbar^2}{2I} J(J+1)$$

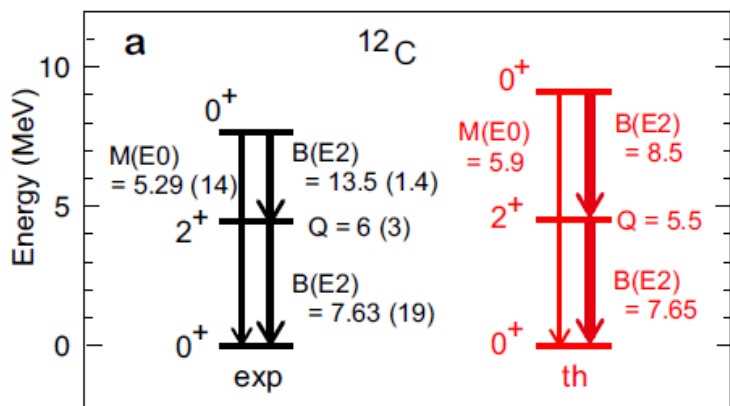
M. Itoh et al., Phys. Rev. C **84**, 054308 (2011).
 M. Freer et al., Phys. Rev. C **80**, 041303 (2009).
 W. R. Zimmerman, N. E. Destefano, M. Freer,
 M. Gai, and F. D. Smith, Phys. Rev. C **84**, 027304 (2011).

(1,0⁰)
A

Otsuka et al., Nature Comm. 2021; MCSM with statistical learning.

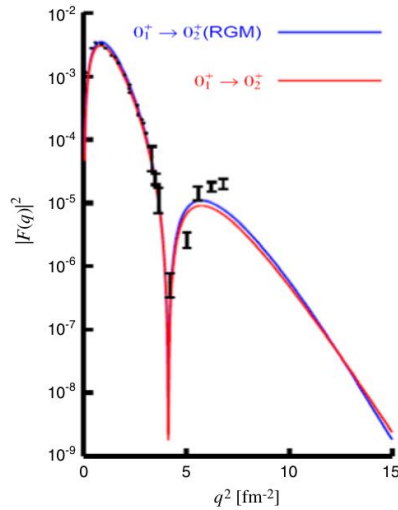


Reasonable reproduction of *transition strengths* and *quadrupole momenta*



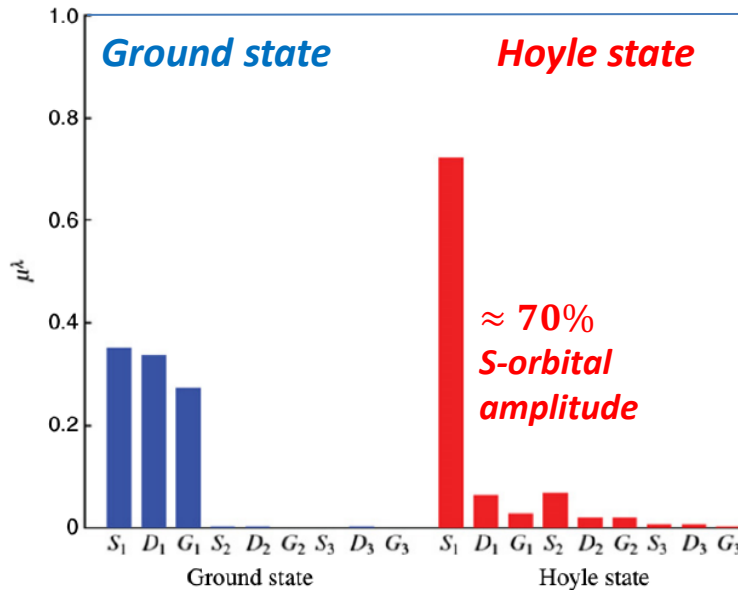
electron inelastic scattering

I. Sick, J.S. McCarthy, Nuclear Phys. A 150 (1970) 631
 A. Nakada et al., Phys. Rev. Lett. 27 (1971) 745
 P. Strehl, Th.H. Schucan, Phys. Lett. B 27 (1968) 641



Bose-Einstein condensation phenomena in nuclei

Weakly bound system → an α may tunnel significantly into the barrier → increasing of the volume → the internal structure of α -particles is no longer important (**antisymmetrization properties negligible**) → the bosonic system may condensate → *Bose Einstein Condensate* (BEC).



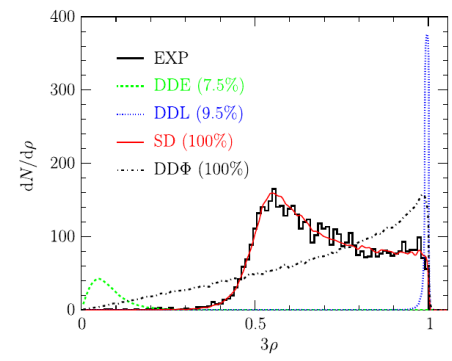
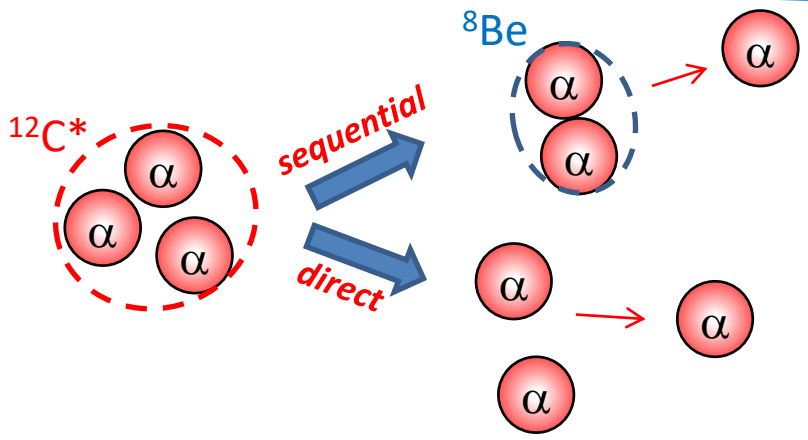
Tohsaki, Horiuchi, Schuck and Röpke (THSR) wave-function method → fit of the electron inelastic scattering $0^+ \rightarrow 0^+_2$ → Hoyle state radius 1.35 – 1.60 times the ground state one.

$$|\Psi_{THSR}\rangle = S_1 |\Psi_{S_1}\rangle + D_1 |\Psi_{D_1}\rangle + \dots$$

Dominant amplitude → BEC

- Taiichi Yamada and Peter Schuck, Eur. Phys. J. A26, 185 (2005)

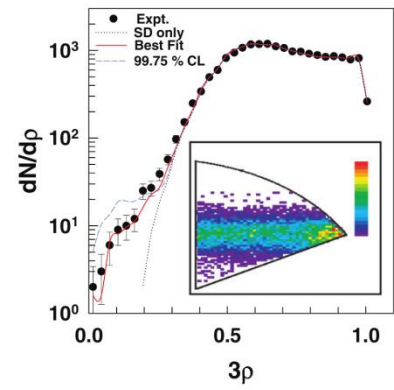
- Y. Funaki, H. Horiuchi, W. von Oertzen, G. Röpke, P. Schuck, A. Tohsaki, and T. Yamada, Phys. Rev. C 80, 064326 (2009)



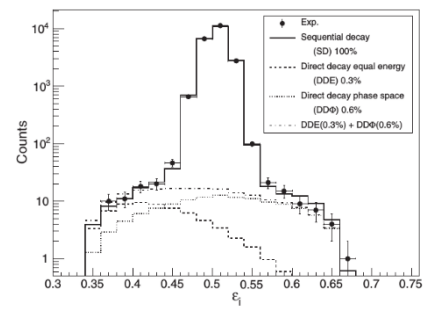
O.S. Kirsebom et al.
 ≈ 5000 events → no evidence of direct decays → upper limit of 0.5% to direct branching ratios.

Experiment	$\Gamma_{3\alpha}/\Gamma$ (%)
[1] M. Freer <i>et al.</i>	< 4
[2] Ad.R. Raduta <i>et al.</i>	17.0 ± 5.0
[3] J. Manfredi <i>et al.</i>	< 3.9
[4] O.S. Kirsebom <i>et al.</i>	< 0.5
[5] T.K. Rana <i>et al.</i>	0.91 ± 0.14
[6] M. Itoh <i>et al.</i>	< 0.2
[7] L. Morelli <i>et al.</i>	1.1 ± 0.8

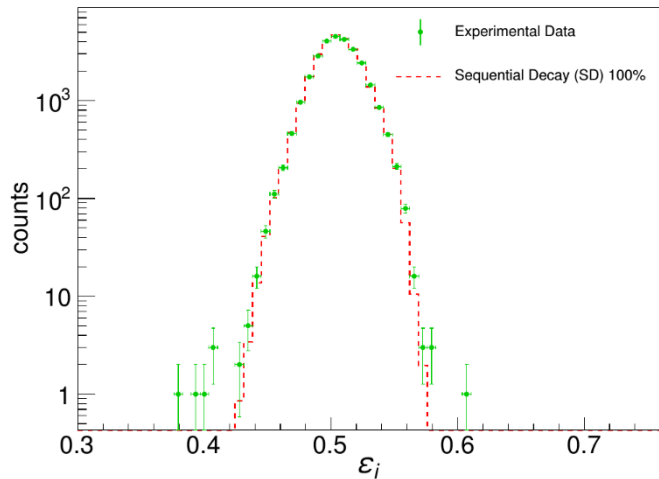
[1] M. Freer et al., Phys. Rev. C 49, R1751 (1994). [2] Ad. R. Raduta et al., Phys. Lett. B 705, 65 (2011). [3] J. Manfredi et al., Phys. Rev. C 85, 037603 (2012). [4] O. S. Kirsebom et al., Phys. Rev. Lett. 108, 202501 (2012). [5] T. K. Rana et al., Phys. Rev. C 88, 021601(R) (2013). [6] M. Itoh et al., Phys. Rev. Lett 113, 102501 (2014). [7] L. Morelli et al., J. Phys. G: Nucl. Part. Phys. 43, 045110 (2016).



T.K. Rana et al.
 ≈ 20000 events → non-vanishing direct decay branching ratio
 $\frac{\Gamma_{3\alpha}}{\Gamma} = 0.91 \pm 0.14$

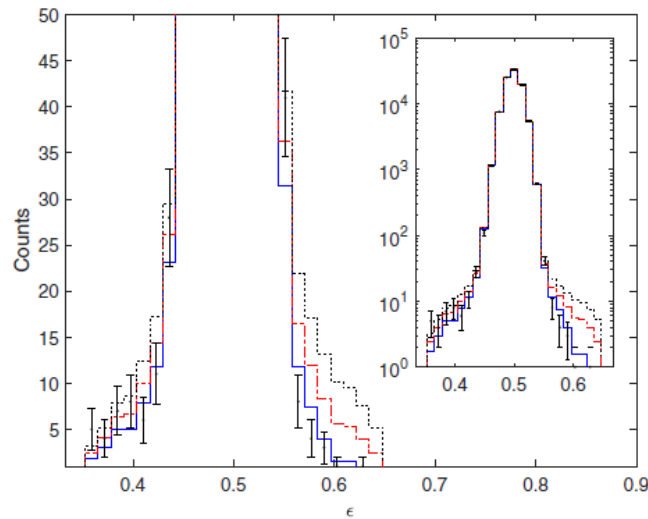


M. Itoh et al.
 ≈ 20000 events → direct decay under the sensitivity of the experimental sensitivity → <0.2%



D. Dell'Aquila et al., *Phys. Rev. Lett.* (2017)
 $^{14}\text{N}(d,a)^{12}\text{C}$ with multi-particle coincidence techniques and a high-resolution hodoscope of silicon detectors.

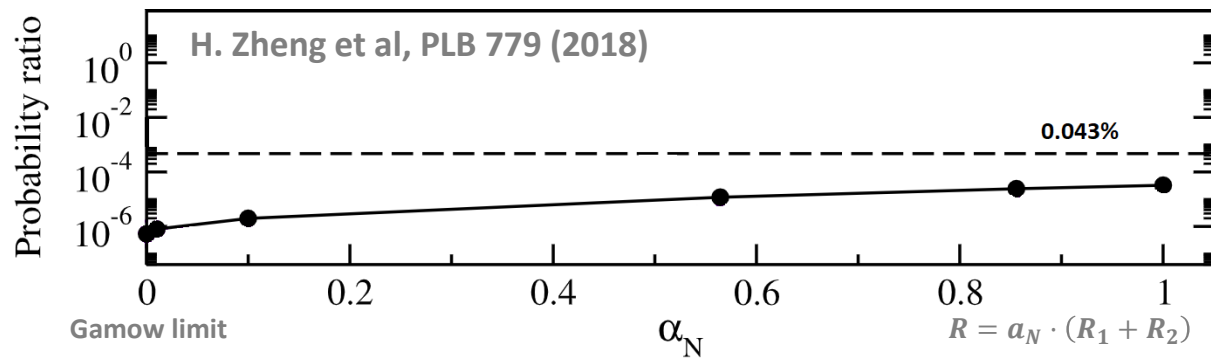
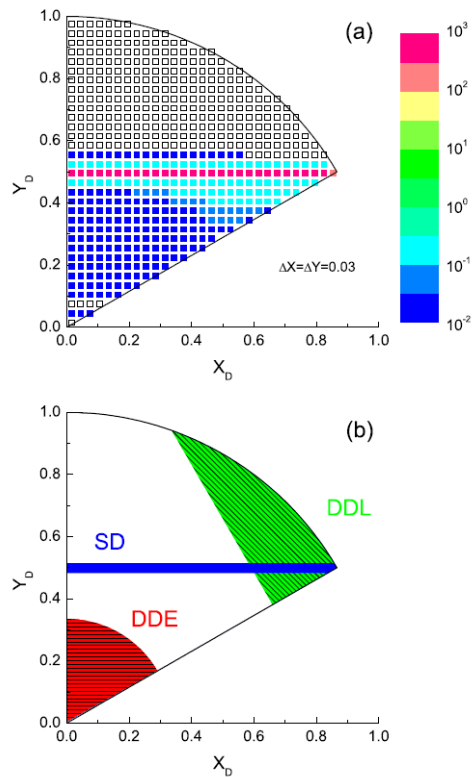
$$\frac{\Gamma_{3\alpha}}{\Gamma} < 4.3 \cdot 10^{-4}$$

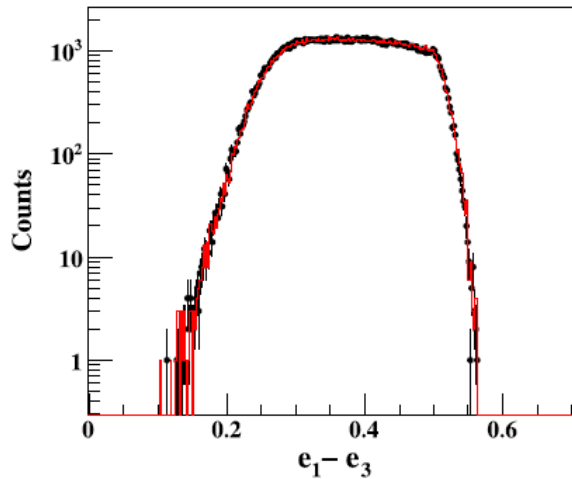


R. Smith et al., *Phys. Rev. Lett.* (2017)
 Alpha-inelastic scattering: $^{12}\text{C}(4\text{He}, 3\alpha)\alpha$ using an array of silicon strip detectors.

$$\frac{\Gamma_{3\alpha}}{\Gamma} < 4.7 \cdot 10^{-4}$$

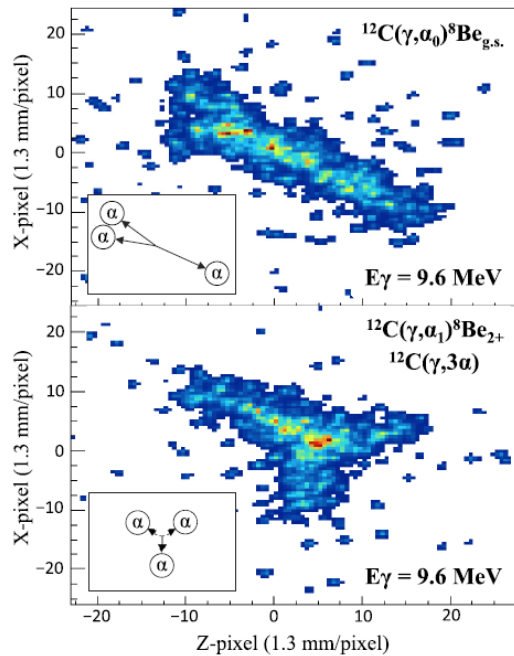
S. Ishikawa, PRC 90 (2014)





T.K. Rana et al., Phys. Lett. B (2017)
Alpha-inelastic scattering: $^{12}\text{C}(^4\text{He}, ^3\alpha)\alpha$ using an array of silicon strip detectors.

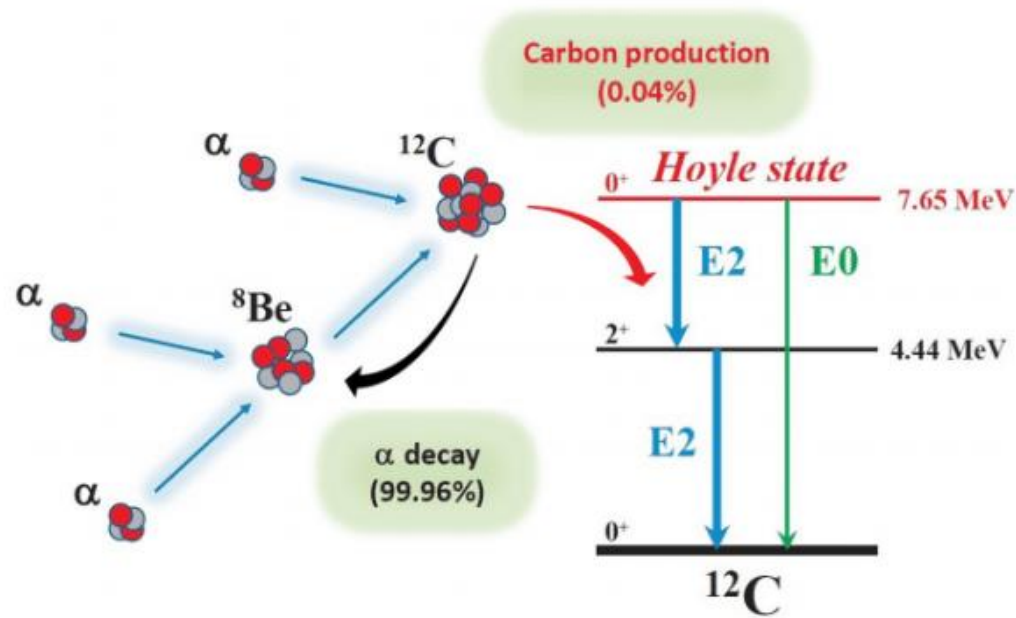
$$\frac{\Gamma_{3\alpha}}{\Gamma} < 1.9 \cdot 10^{-4}$$



R. Smith et al., Phys. Rev. C (2020)
Gamma-induced dissociation of $^{12}\text{C} \rightarrow \Gamma_{3\alpha}$ for the 2+ collective excitation of the Hoyle state investigated with a TPC \rightarrow decay penetrabilities are used to determine a stringent upper limit to the direct 3-alpha decay of the Hoyle state.

$$\frac{\Gamma_{3\alpha}}{\Gamma} < 5.7 \cdot 10^{-6}$$

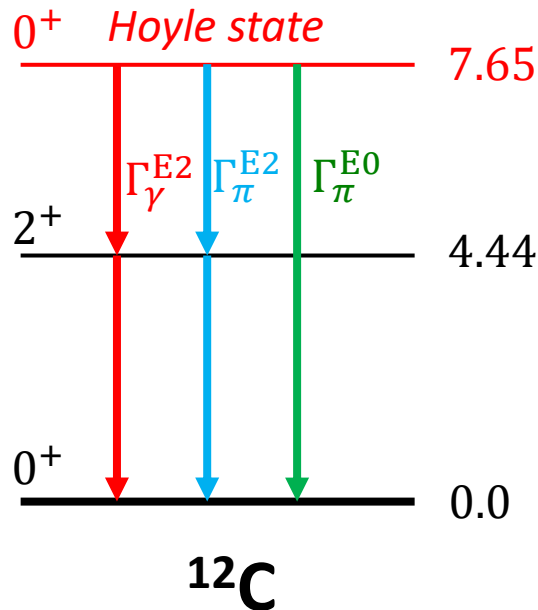
Experiment	Γ_{DD}/Γ (%)	type	year
M. Freer et al.	< 4	L.I.	1994
Ad.R. Raduta <i>et al.</i>	17.0 ± 5.0	H.I.	2011
J. Manfredi et al.	< 3.9	H.I.	2012
O.S. Kirsebom et al.	< 0.5	L.I.	2012
<i>T.K. Rana et al.</i>	0.91 ± 0.14	L.I.	2013
M. Itoh et al.	< 0.2	L.I.	2014
L. Morelli et al.	1.1 ± 0.8	H.I.	2016
D. Dell'Aquila et al.	< 0.043	L.I.	2017
R. Smith et al.	< 0.047	L.I.	2017
T.K. Rana et al.	< 0.019	L.I.	2019
J. Bishop et al.	< 0.043	beta	2020
<i>R. Smith et al.</i>	$< 5.7 \cdot 10^{-4}$	ind.	2020



Reaction rate of the three-alpha process in stars

→ spectroscopic properties of the Hoyle state in ^{12}C :

$$\langle \sigma v \rangle \propto \frac{1}{T^3} \Gamma_{rad} e^{-\frac{E_R}{k_B T}}$$



$$\Gamma_{rad} = \Gamma_{\pi}^{E0} + \Gamma_{\pi}^{E2} + \Gamma_{\gamma}^{E2}$$

$$\Gamma_{rad} = \Gamma_{\pi}^{E0} \left[\frac{\Gamma}{\Gamma_{\pi}^{E0}} \right] \left[\frac{\Gamma_{rad}}{\Gamma} \right]$$

$$\Gamma_{\pi}^{E0} = 63.3(20) \mu\text{eV}$$

M. Freer and H. O. U. Fynbo, *Prog. Part. Nucl. Phys.* 78, 1(2014).

$$\Gamma_{\pi}^{E0}/\Gamma = 7.6(4) \cdot 10^{-6}$$

T. K. Eriksen et al., *Phys. Rev. C* 102, 024320 (2020).

$$\Gamma_{rad}/\Gamma = 4.19(11) \cdot 10^{-4}$$

D. E. Alburger, *Phys. Rev.* 124, 193 (1961).

P. A. Seeger and R. W. Kavanagh, *Nucl. Phys.* 46, 577 (1963).

I. Hall and N. W. Tanner, *Nucl. Phys.* 53, 673 (1964).

D. Chamberlin et al., *Phys. Rev. C* 9, 69 (1974).

C. N. Davids, et al., *Phys. Rev. C* 11, 2063 (1975).

H.-B. Mak et al., *Phys. Rev. C* 12, 1158 (1975).

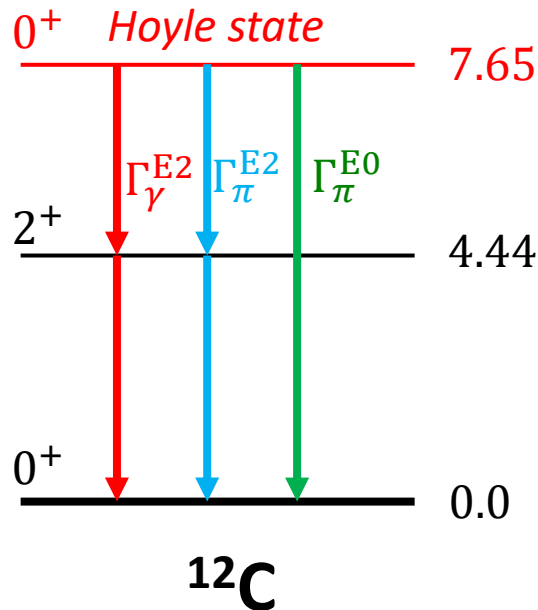
R. G. Markham et al., *Nucl. Phys. A* 270, 489 (1976).

A. W. Obst and W. J. Braithwaite, *Phys. Rev. C* 13, 2033(1976)

Reaction rate of the three-alpha process in stars

→ *spectroscopic properties of the Hoyle state in ^{12}C :*

$$\langle \sigma v \rangle \propto \frac{1}{T^3} \Gamma_{rad} e^{-\frac{E_R}{k_B T}}$$



→ *state-of-the-art*

$$\Gamma_{rad}/\Gamma = 4.19(11) \cdot 10^{-4}$$

D. E. Alburger, Phys. Rev. 124, 193 (1961).

P. A. Seeger and R. W. Kavanagh, Nucl. Phys. 46, 577 (1963).

I. Hall and N. W. Tanner, Nucl. Phys. 53, 673 (1964).

D. Chamberlin et al., Phys. Rev. C9, 69 (1974).

C. N. Davids, et al., Phys. Rev. C11, 2063 (1975).

H.-B. Mak et al., Phys. Rev. C12, 1158 (1975).

R. G. Markham et al., Nucl. Phys. A270, 489 (1976).

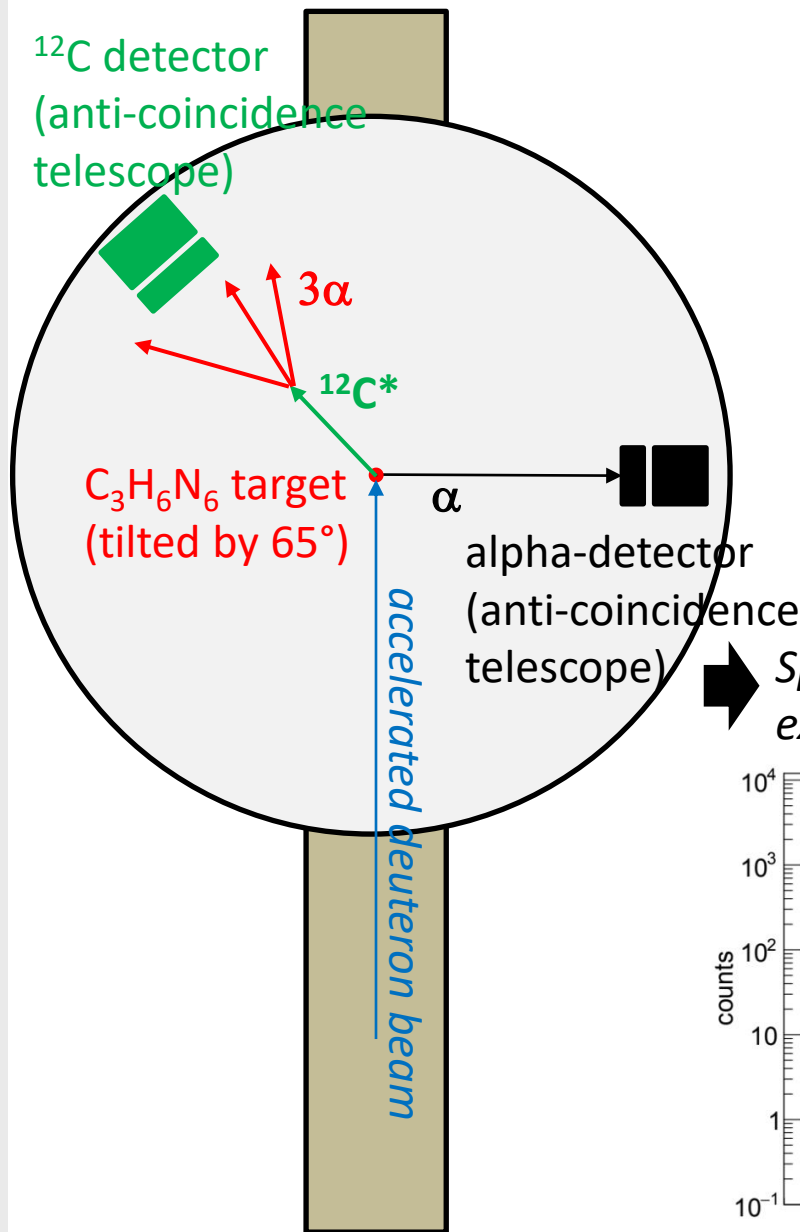
A. W. Obst and W. J. Braithwaite, Phys. Rev. C13, 2033 (1976).

→ *recent improved gamma-ray spectroscopy experiment*

$$\Gamma_{rad}/\Gamma = 6.2(6) \cdot 10^{-4}$$

T. Kibédi et al., Phys. Rev. Lett. 125 (2020) 182701.

→ *Tremendous impact on the three-alpha process in stars → about 34%!!!*



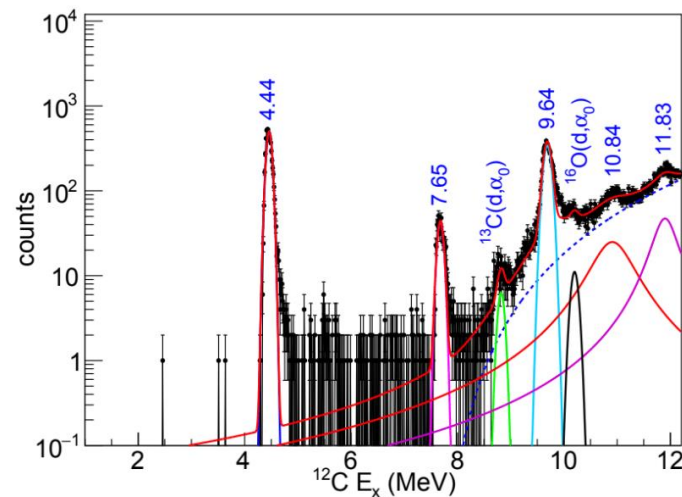
Use the $^{14}\text{N}(d,\alpha)^{12}\text{C}$ reaction to populate ^{12}C in its Hoyle state and then measure the fraction of its radiative decays by deducing the $^{12}\text{C}-\alpha$ coincidence rate.

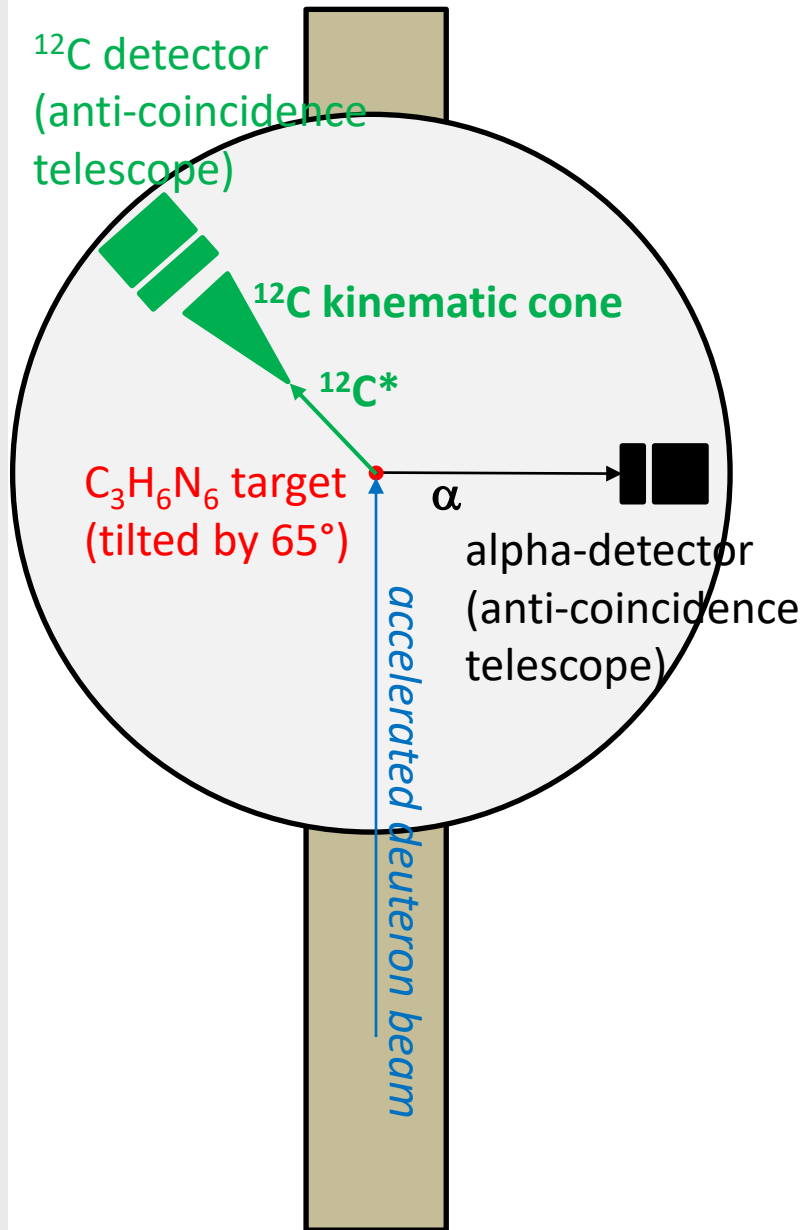
Beam: d @ 2.7 MeV (CN accelerator)

Target: $\text{C}_3\text{H}_6\text{N}_6 + \text{C}$ backing

Detectors: Anti-coincidence telescope (90°) + Anti-coincidence telescope (64.8°).

Spectrum from the GHOST test experiment at the CN accelerator (LNL)



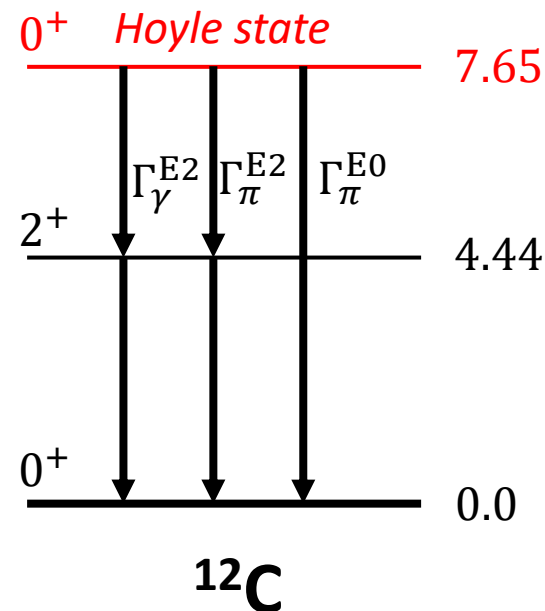


Use the $^{14}\text{N}(d,\alpha)^{12}\text{C}$ reaction to produce ^{12}C in its Hoyle state and then measure the fraction of its radiative decays by deducing the $^{12}\text{C}-\alpha$ coincidence rate.

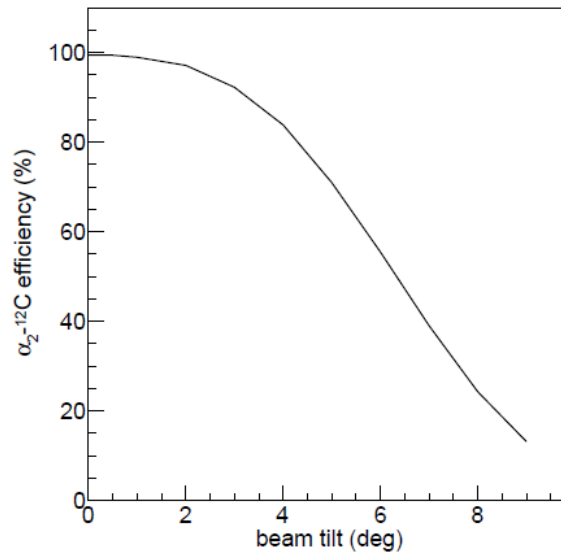
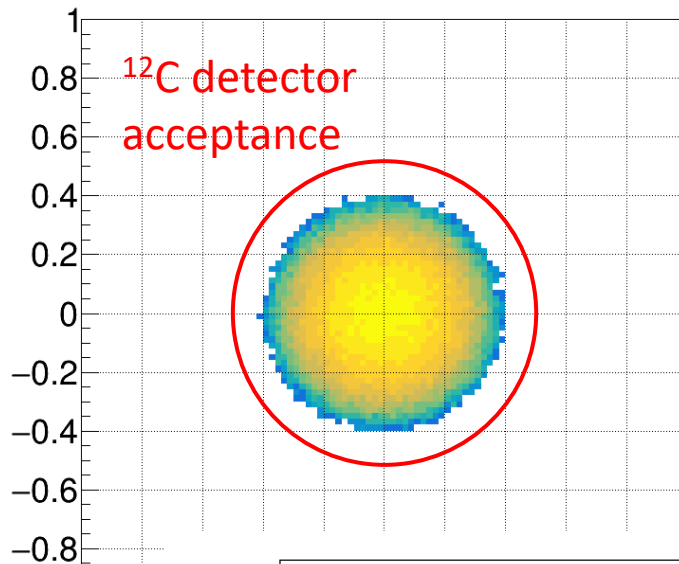
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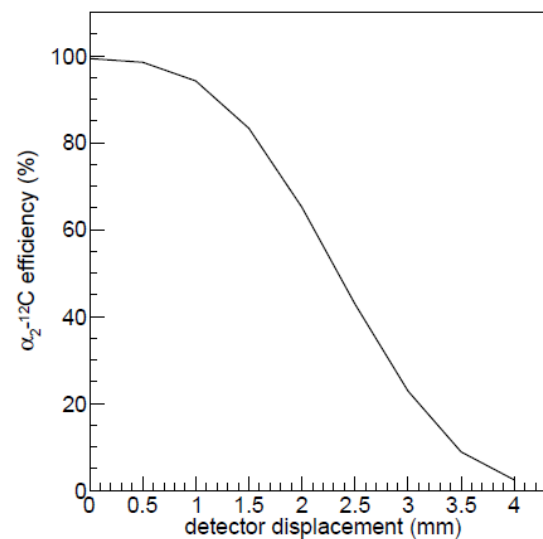
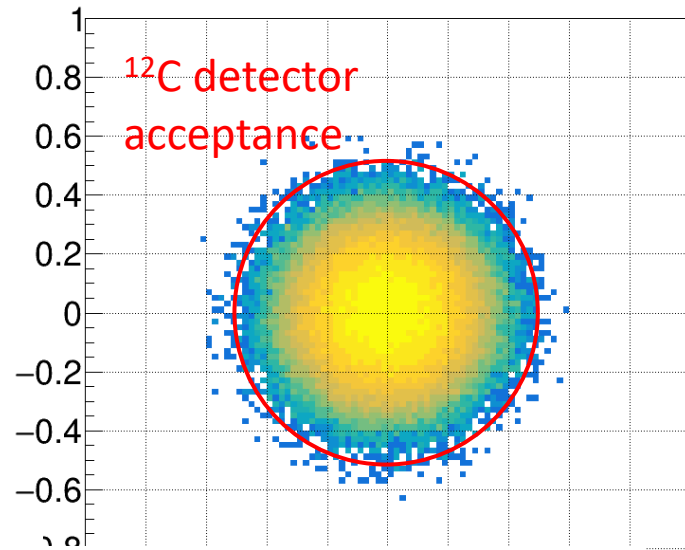
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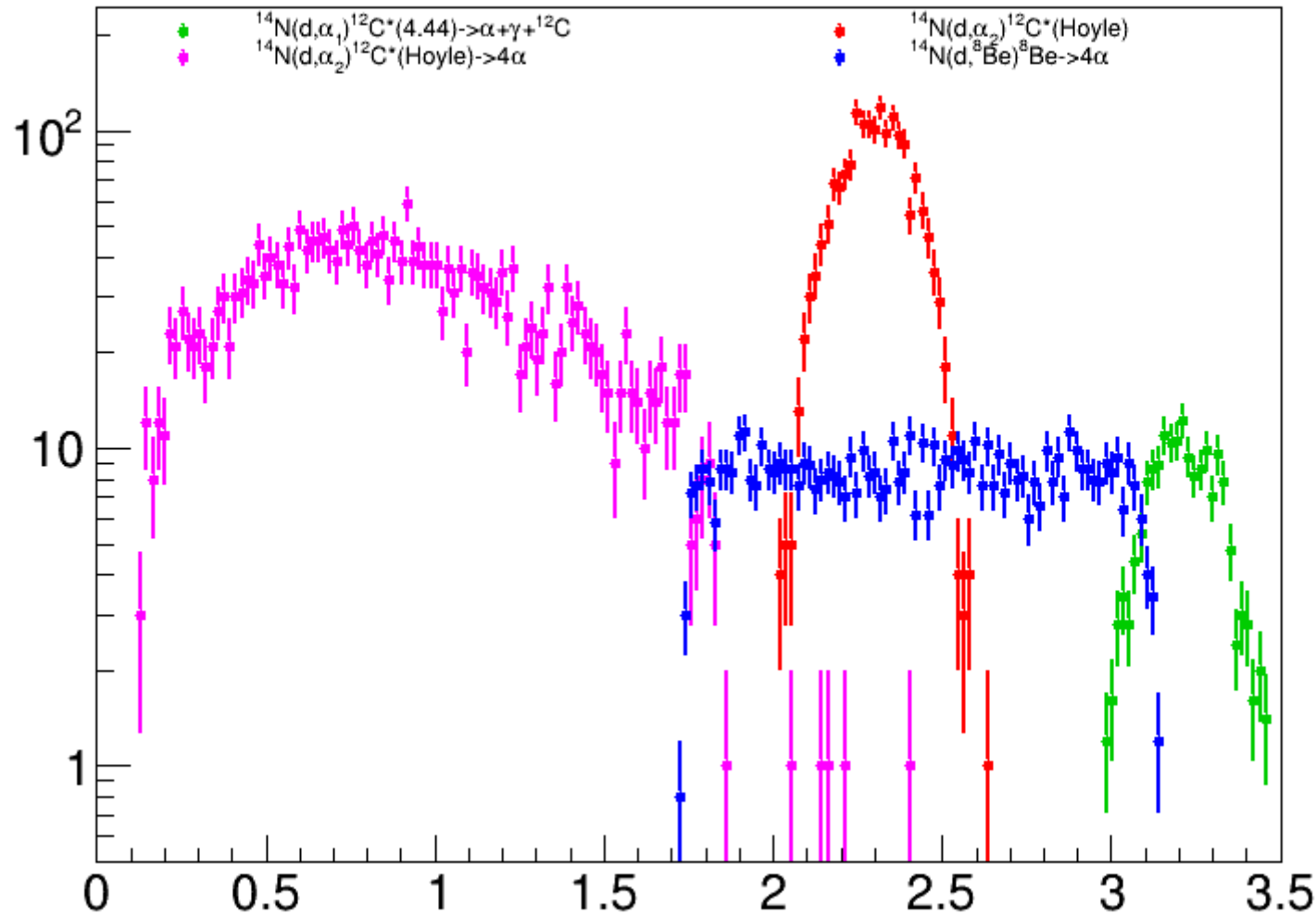
➔ Without straggling effects in the target



➔ With straggling effects in the target



^{12}C spectrum seen at the forward detector (full simulation with energy loss and energy and angular straggling + beam energy and angular straggling)



- Carbon isotopes are key nuclei to investigate clustering aspects as they are the simplest systems candidate to form linear chain structures;
- The Hoyle state in ^{12}C has a pronounced cluster nature that has a deep impact in nuclear astrophysics and nuclear structure;
- The alpha-decay of the Hoyle state in ^{12}C has been for a long time an open question in nuclear physics due to the numerous contrasting experiments reported in the literature;
- Modern high-precision experiments allowed to explore with enhanced sensitivity the direct decay of the Hoyle state, which seem to agree with theoretical investigations;
- Another open issue relates to the radiative decay width of the Hoyle state, which directly impacts the rate of the three-alpha process in stars → new experiment at INFN-LNL in the near future.

Thank you for your attention!