Evidence of nuclear deformation in low-energy experiments : methods, limitations, interpretation

- Nuclear deformation, where do
an historical approach to the conc

→ how this concept ha

- Nuclear deformation, can we ch

→ the network of (tru formation”

- concluding remarks

Indistinct and shadowy
Yet within it is an image;
Shadowy and indistinct,
Yet within it is a substance.
Dim and dark,
Yet within it is an essence.
The essence is quite genuine
And within it is something that can be tested.

Lao Tzu: Tao Te Ching XXI

This is called the shape that has no shape
The image that is without substance
This is called indistinct and shadowy.

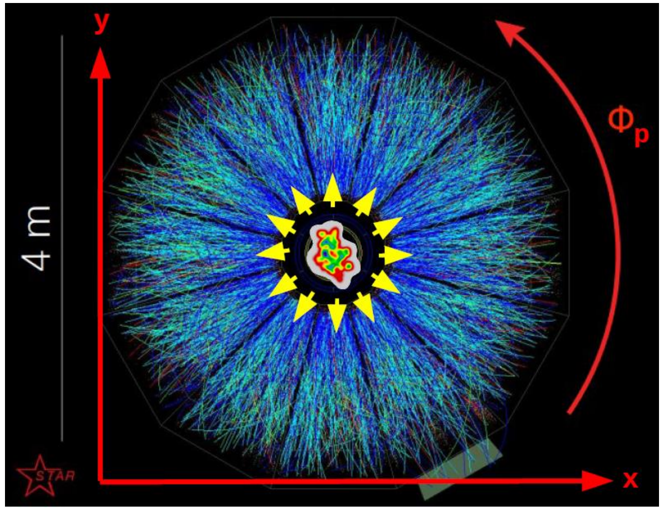
Lao Tzu: Tao Te Ching XIV

Mackintosh 1977 Rep. Prog. Phys. 40 731

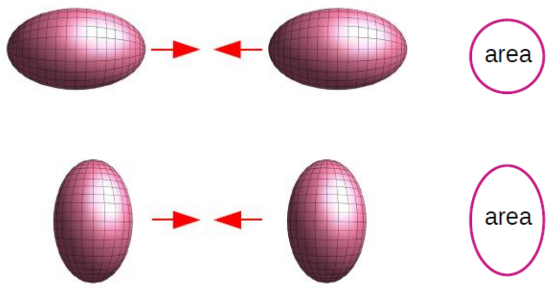
Context of this presentation

- Recycling a presentation made within ESNT workshop on "Deciphering nuclear phenomenology across energy scales" (20-23 September 2022)
- Are we observing the same things ?
- The unreasonable effectiveness of intrinsic nuclear shapes in high-energy collisions

adopted from Giuliano GIACALONE's talk



relativistic heavy ion collisions at RHIC and LHC (ALICE)



small v2
small area
large <pt>

large v2
large area
small <pt>

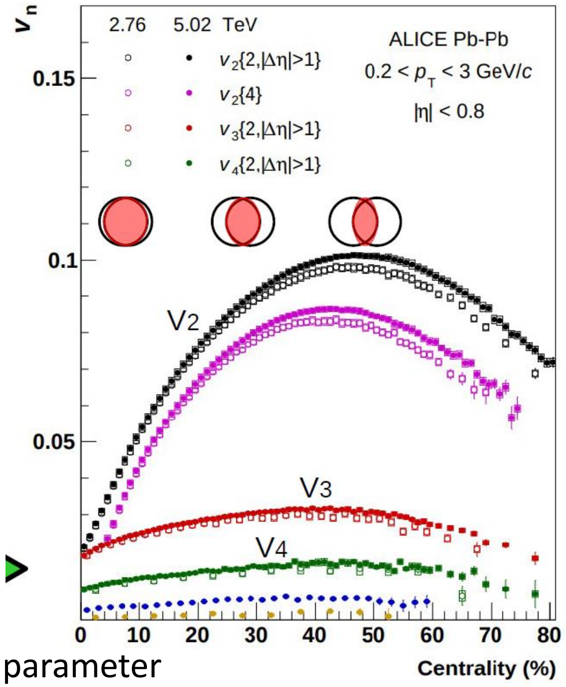
$$\rho_2 \equiv \rho(v_2^2, [p_t]) = \frac{\langle \delta v_2^2 \delta[p_t] \rangle}{\sqrt{\langle (\delta v_2^2)^2 \rangle \langle (\delta [p_t])^2 \rangle}}$$

becomes negative

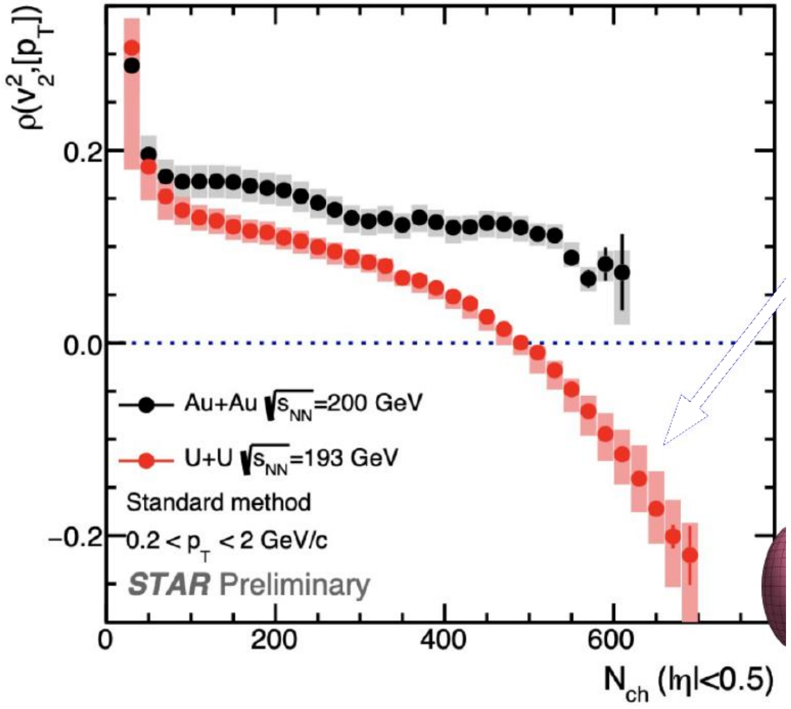
$$\frac{d^2N}{dp_T d\phi} = \frac{dN}{dp_T}\left(1 + 2\sum_{n=0}^{\infty} v_n \cos(\phi - \Phi_n)\right)$$

HOW EXPLOSIVE IS THE EXPANSION?

IS PARTICLE EMISSION ISOTROPIC IN AZIMUTH?



for n=2, "elliptic flow", due to impact parameter



Evidence of nuclear deformation in low-energy experiments : methods, limitations, interpretation

- Nuclear deformation, where does it come from ?

an historical approach to the concept of “nuclear deformation”

→ how this concept has "massively" imposed itself from the earliest measurements

- Nuclear deformation, can we characterize it ?

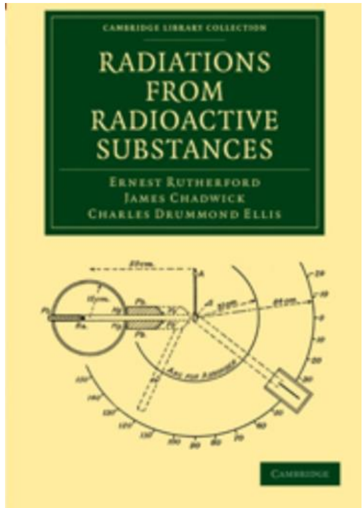
→ the network of (true) observables. NB it is improper to say that we “observe nuclear deformation”

- concluding remarks

1910

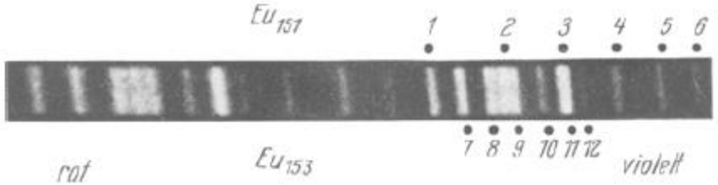
- Rutherford, Geiger, Marsden : α-scattering experiments
- the nucleus has a finite size
- it was natural to assume a spherical shape

(at that time)

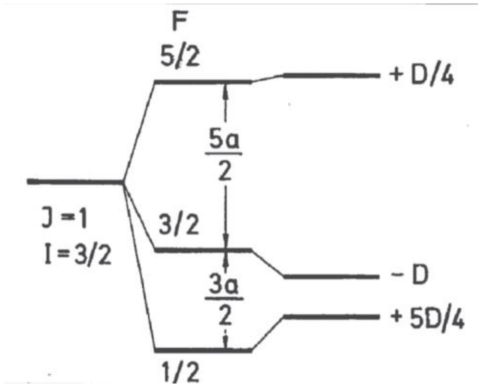


1935

Naturally it was the field of physics having the highest precision at the earliest time—atomic spectroscopy — which gave the first clear indications of nuclear electric quadrupole moments



Schüler and Schmidt
(atomic spectra of ^{151,153}Eu Z. Phys. 94 457)



hyperfine splitting : Landé's intervals are « perturbed »

H. Casimir, in Physica 2 (1935) suggestes that independent-particle motion characterizing the odd-nucleon in odd mass nuclei is influenced by the quadrupole deformed nuclear charge distribution
see [Heyde & Wood Phys. Scr. 91 083008 (2016)]

A surprise: experimentally first discovered at Columbia University in 1939 (Rabi *et al.*): the deuteron has a sizable quadrupole moment ! (now known to be 0.2860 ± 0.0015 e.fm^2)
(NB nuclear deformation has a lot to do with proton-neutron in medium interaction)

1944

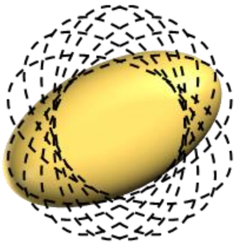
Brix & Kopfermann first suggested that some irregularities in isotope shifts between some rare-earth elements could be taken as an evidence of an intrinsic quadrupole moment for even-even nuclei with $I = 0$

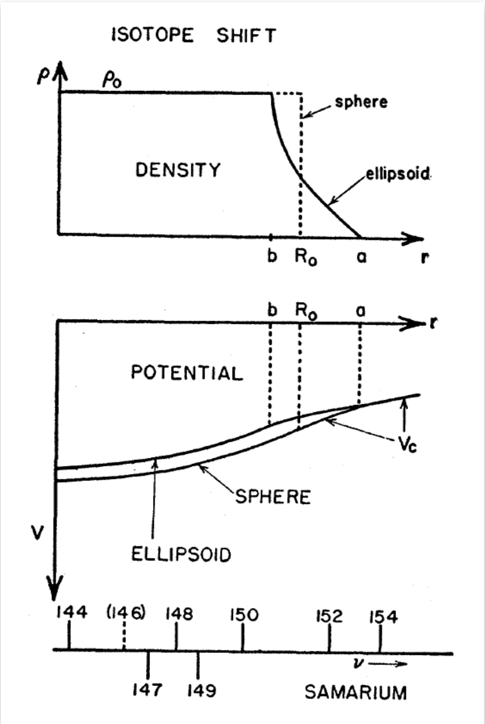
a revolution at that time !

$$Q_\lambda = e \sum_{i=1}^{Z} r_i^\lambda Y_{\lambda 0}(\Omega_i)$$

$$Q = (16\pi/5)^{1/2} \langle Q_2 \rangle_{M=J}$$

zero unless $I > {}^1/_2$

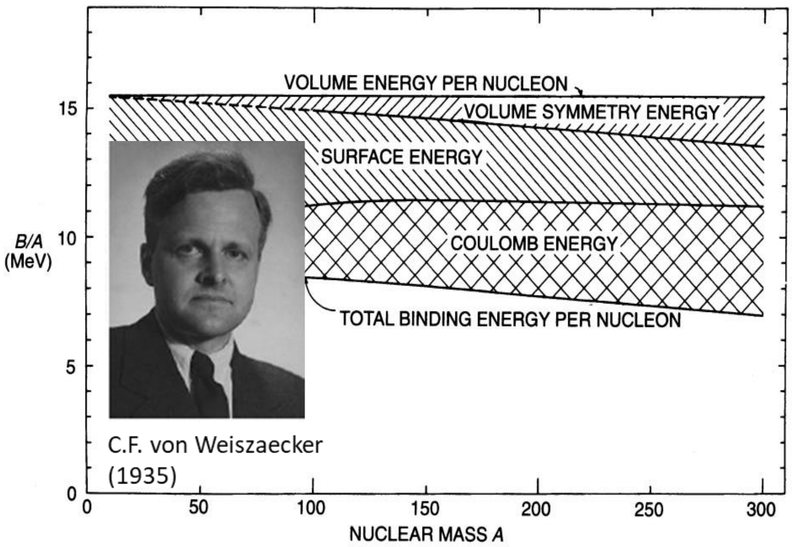




Temmer Rev. Mod. Phys. 30 (1958)

1930's : fission and neutron capture

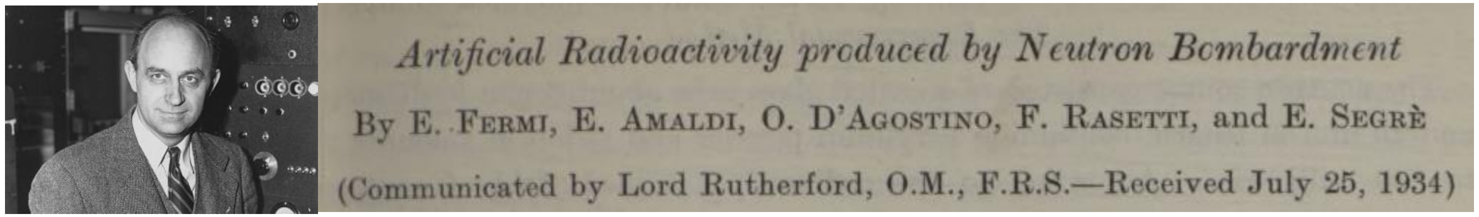
- liquid drop model



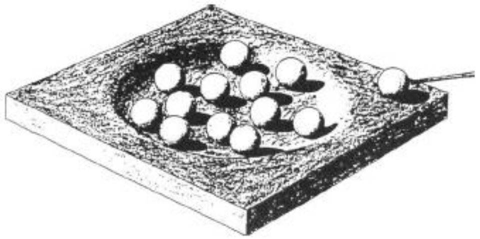
Numerical value of specific binding energy B/A, according to the semi-empirical mass formula. The constant volume energy enters with opposite sign to all the other contributions, which together reduce the binding down to the lower curve, fitted to empirical mass data.

- there is strictly nothing in this vision of nuclei than can lead to imagine permanent shape
- Deformation **requires** shell structures [as shown much later by Myers and Swiatecki (1966)]

- neutron capture/scattering on all available targets

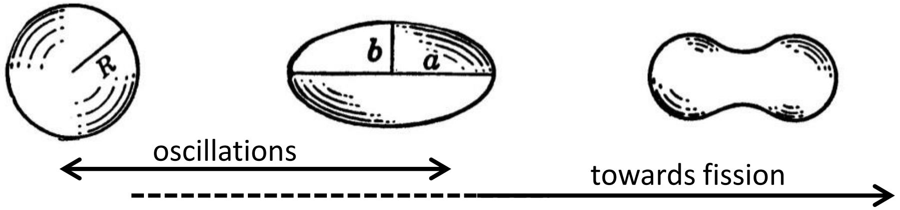

Artificial Radioactivity produced by Neutron Bombardment
By E. Fermi, E. Amaldi, O. D'Agostino, F. Rasetti, and E. Segrè
(Communicated by Lord Rutherford, O.M., F.R.S.—Received July 25, 1934)

- Niels Bohr's compound nucleus : neutron cross sections and statistical properties at high energy



- Bohr & Wheeler [Phys Rev 56 (1939)]: consider the liquid drop model as a dynamical model,

being able to exhibit vibrational and rotational collective modes of motion



1930's : growing evidence for individual particle motion

- foundation of the shell model

Maria Goeppert Mayer
(Nobel prize 1963)

Hans E. Suess

J. Hans D. Jensen
(Nobel prize 1963)

Otto Haxel

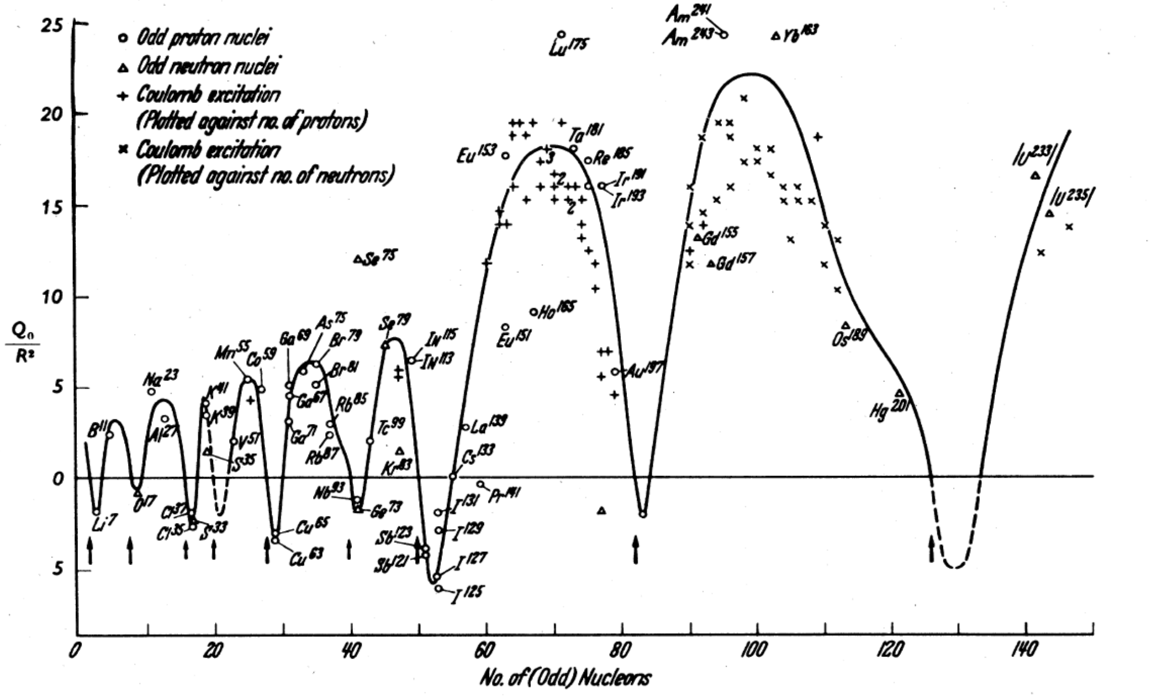
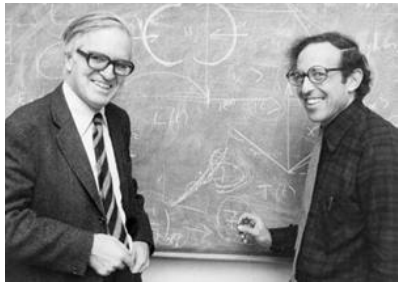


FIG. 4. A later plot of the intrinsic quadrupole moments, Q_0/R^2, prepared by C. H. Townes (1958), using $R=1.2\,A^{1/3}\times10^{-13}$ cm. This figure supercedes Fig. 3. It emphasizes the large size of the quadrupole moments relative to values $|Q_0/R^2|<1$ expected for a spherical nucleus shell model.



L. J. Rainwater
[Phys. Rev. 79 (1950) 432
shared Nobel Prize 1975 with A. Bohr and B.Mottelson

- As can be understood from Rainwater's Nobel Prize lecture notes :

the **real birth of the concept of nuclear deformation** came from the effort to reconcile two visions of the nucleus
i.e. to bring into the same description nuclear properties in apparent contradiction

"Dr. Bohr and I had many discussions of my concept. He was particularly interested in the dynamical aspects. The distortion bulge could in principle vibrate or move around to give the effect of rotational levels. The first result was his January 1951 paper "On the Quantization of Angular Momenta in Heavy Nuclei. "
The subsequent exploitation of the subject by Bohr, Mottelson and their colleagues is now history…"

around 1950 : the concept of nuclear deformation (i.e. intrinsic shape) is at the heart of all nuclear structure understanding within the "unified model" [Bohr & Mottelson Dan. Matt. Fys. Medd. 27 (1953)]

- one is led to describe the nucleus as a shell structure capable of performing oscillations in shape and size.
- The system exhibits many analogies to molecular structures with the interplay between electronic and nuclear motion

[Hill & Wheeler Phys Rev 89 (1953)]

adiabatic motion

Similarly the characteristic time of radial motion of a nucleon of average kinetic energy, $T=15$ Mev, is

$$t_{\text{nucleon}} \doteq \oint \left[\frac{2T}{M} - \frac{l(l+1)\hbar^2}{M^2r^2}\right]^{-\frac{1}{2}} dr$$

$$= \frac{2R}{(2T/M)^{\frac{1}{2}}}\left[1 - \frac{l(l+1)\hbar^2}{2MTR^2}\right]^{\frac{1}{2}}$$

$$< \frac{2R}{(2T/M)^{\frac{1}{2}}} = \frac{2R}{v} = \frac{2A^{\frac{1}{3}}r_0}{0.18c}$$

$$= 0.3\times 10^{-21} \text{ sec for } U^{236}, \quad (1)$$

an interval 15 times smaller than the estimated period,

$$t_2 = 2\pi\hbar/\hbar\omega_2$$
$$\doteq 2\pi\times 0.658\times 10^{-21} \text{ Mev sec}/0.8 \text{ Mev}$$
$$= 5\times 10^{-21} \text{ sec}, \quad (2)$$

of the lowest mode of capillary oscillation of the same nucleus.

adiabatic motion approximation allows to factorize the total wavefunction

describe the oscillations of the nucleus as a whole, specified by quantum numbers ν

shell model wave function for a fixed field specified by parameters α

$$\Psi_{n\nu}(x) = \phi_\nu(\alpha)\cdot\psi_n(x,\alpha)$$

But :

- the uncertainty principle has the consequence that fixing the intrinsic-state orientation entails indeterminancy in J
- the state is now more a wavepacket of states with different angular momentum values
- need to "project out", e.g. $|\Psi_{JM}{}^{(K)}\rangle = \frac{2J+1}{8\pi^2 N_J{}^{1/2}} \int d\Omega D_{MK}{}^J(\Omega) R(\Omega) |\phi_K(A)\rangle$
- introduces other issues (eg redundancy of coordinates etc)

Laboratoire de Physique des 2 infinis Irène Joliot-Curie

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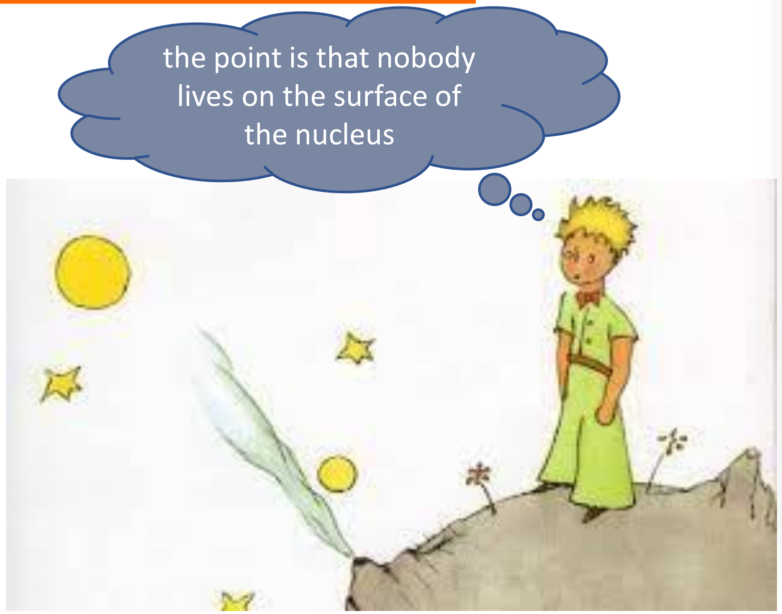

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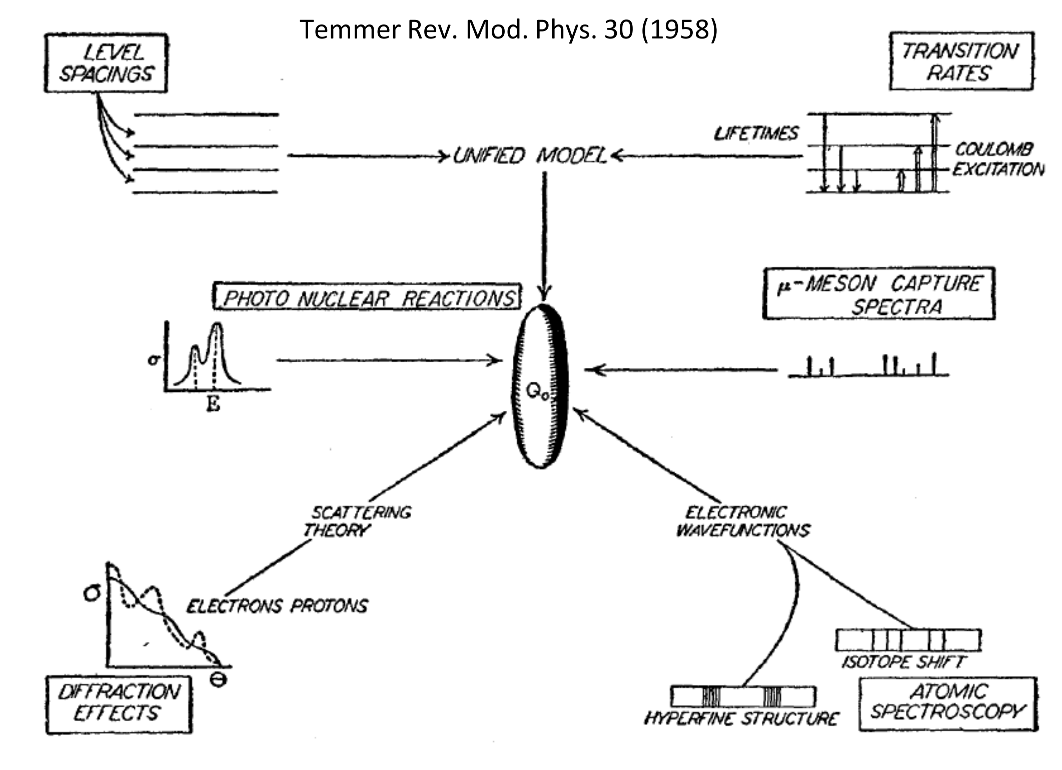
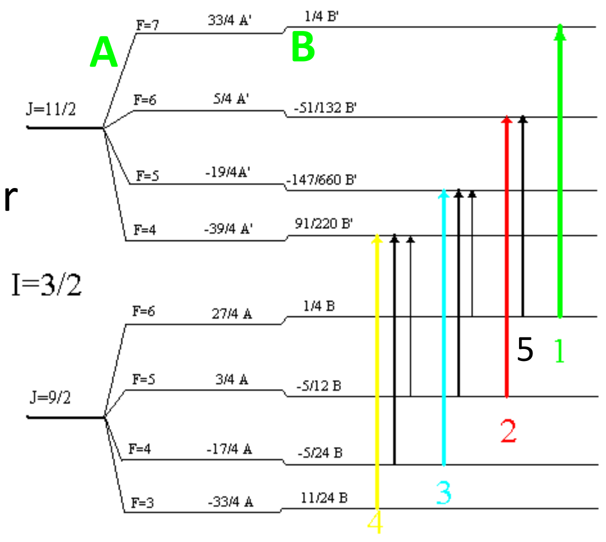
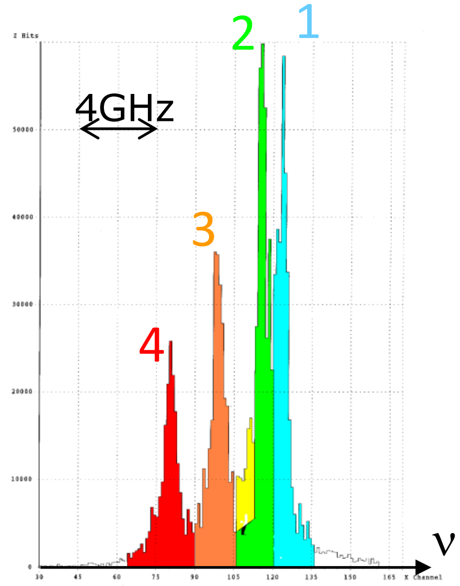


- interaction between em fields generated by the electronic cloud and those generated by the nucleus
- extract 2 hyperfine parameters (one selects atomic states with J suitable to get sufficient number of lines)

^{191}Ir



$\vec{F} = \vec{I} + \vec{J}$



λ=300 nm

$\nu \cong 10^6$ GHz

$h\nu_0 \cong 4$eV

$$\frac{\Delta E}{h\nu_0} \cong 10^{-6}$$

$$\mathbf{A} = \frac{\mu_I \bar{H}_0}{IJ}$$

μ_I → magnetic moment

$\bar{H}_0$ → magnetic field created by the motion of electrons at the nucleus

$$\mathbf{B} = e\, Q_s\, \bar{\Phi}_{JJ}(0)$$

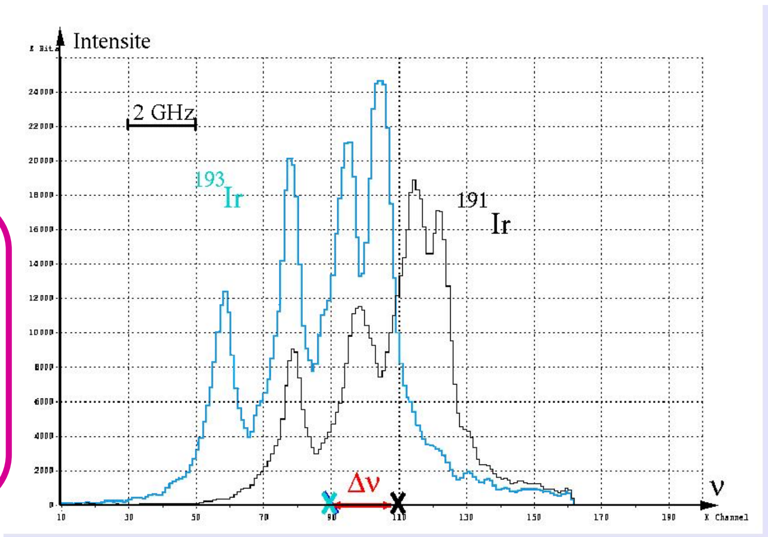
Q_s → "spectroscopic" quadrupole moment

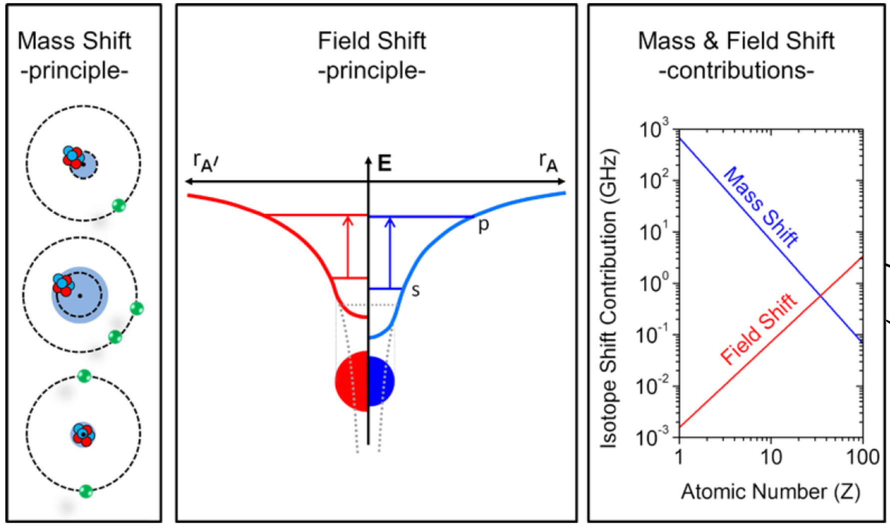
$\bar{\Phi}_{JJ}(0)$ → electric field gradient created by the motion of electrons at the nucleus

need a reference measurements for A and B on stable isotopes

- μ and Q can be extracted independently from any atomic or nuclear models (in particular no nuclear reaction/ scattering theory needed)
- but, strictly speaking, at that stage, no direct information on the nuclear deformation

- the isotope shift :
- the center of gravity of the hyperfine spectra moves with mass number





Nörtershäuser & Geppert Lecture Notes in Physics, Vol. 879

- Change of nuclear mass between isotopes⇒ nuclear recoil-energy contribution

MASS SHIFT

$$\Delta\nu_{iM}^{AA'} = (M_{iN} + M_{iS})\left(\frac{A'-A}{AA'}\right)$$

- Change of the nuclear charge density between isotopes :

VOLUME SHIFT

$$\Delta\nu_{Vol}^{AA'} = F_i \cdot \lambda^{AA'}$$

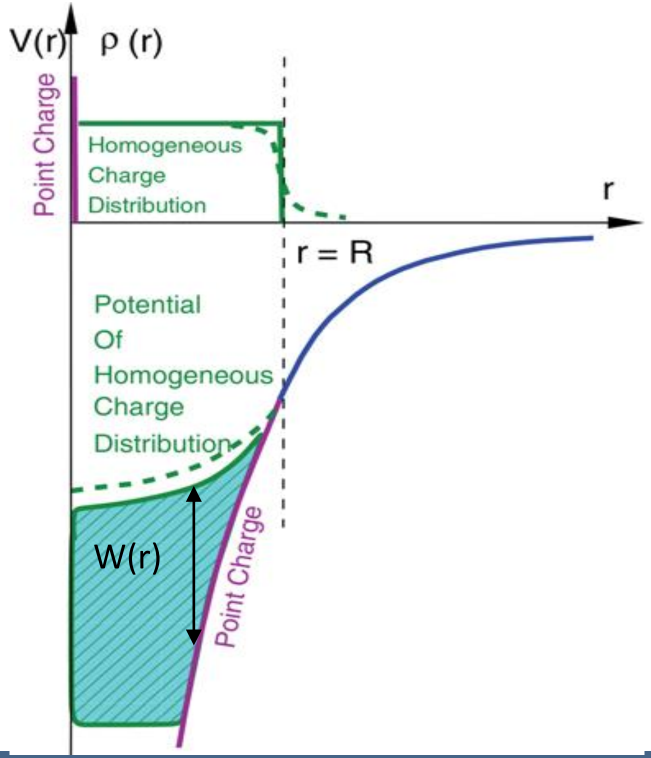
$$\Delta\nu_i^{AA'} = \Delta\nu_{iM}^{AA'} + \Delta\nu_{iVol}^{AA'}$$

Nuclear quantity

$$F_i = \frac{2\pi}{3}\frac{Ze^2}{h}\Delta|\psi(0)|^2$$

[GHz fm^{-2}]

Atomic quantity
(some s electronic wave components must be involved !)
most of the time has to be calculated
or empirical techniques (King plots)



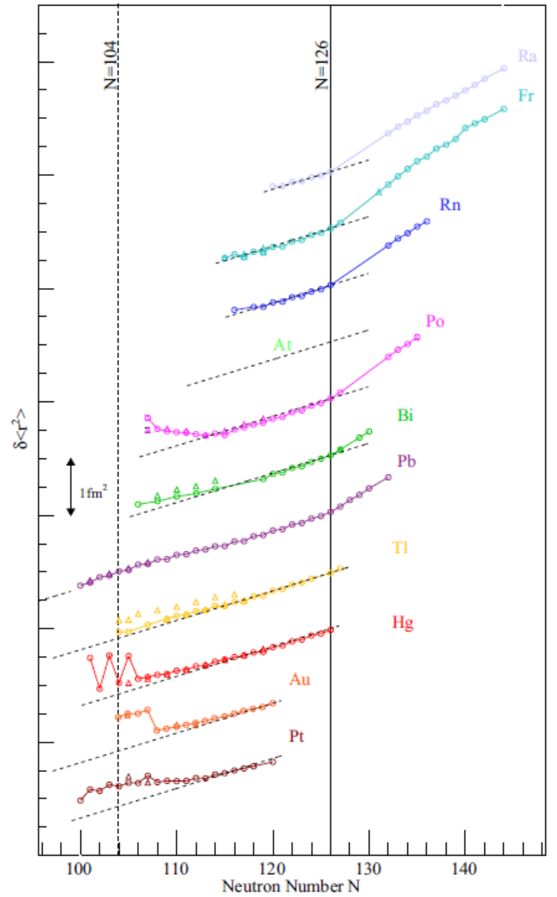
- the isotope shift :
- the center of gravity of the hyperfine spectra moves with mass number

Nuclear quantity $\lambda^{AA'}$ [fm^2] → **one single number** that encapsulates all effects leading to a change of the nuclear volume as seen from the electronic cloud

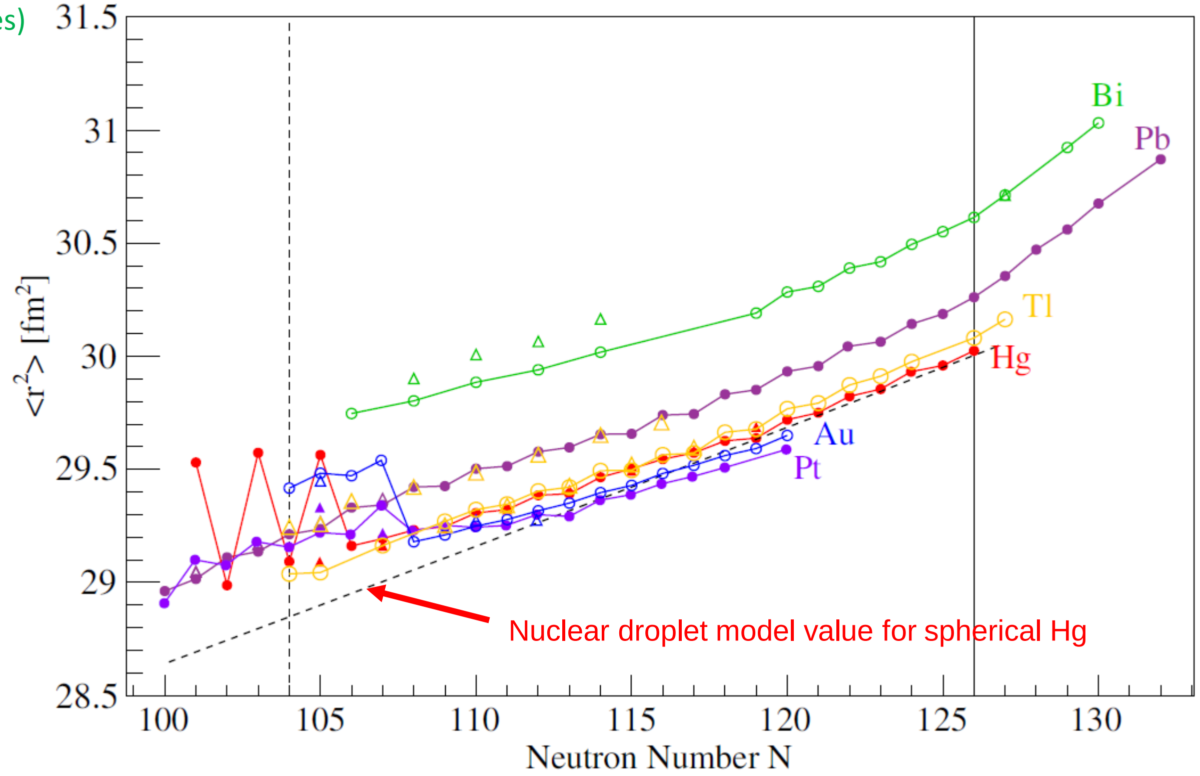
$$\lambda \approx \delta\langle r^2\rangle + \frac{C_2}{C_1}\delta\langle r^4\rangle + \frac{C_3}{C_1}\delta\langle r^6\rangle + \dots$$

Seltzer coefficients (calculated atomic quantities)

Nuclear droplet model $\Longrightarrow$ $\delta\langle r_c^2\rangle$ $\delta\langle \beta^2\rangle$



Cocolios Hyperfine Interact (2017) 238:16



Cocolios Hyperfine Interact (2017) 238:16

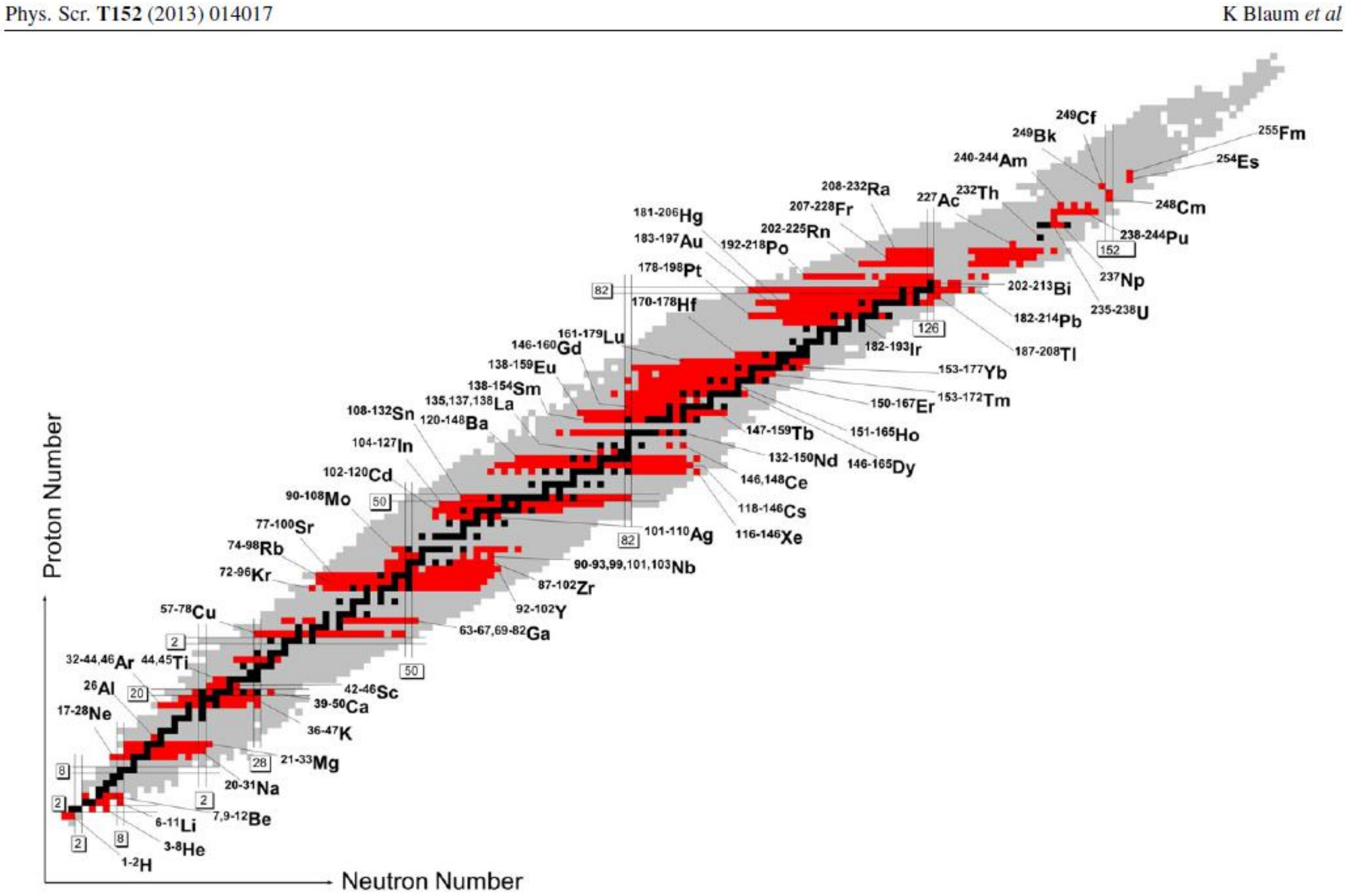
→ nobody dares to draw $\delta\langle\beta^2\rangle$ curves ! $\Rightarrow \langle\beta^2\rangle^{\frac{1}{2}}$ (if reference value available)

$$Q_S = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} Q_0 \Rightarrow \langle\beta\rangle^2$$

laboratory frame — rigid spheroid shape — intrinsic frame — "static" part of the deformation

$$\underbrace{\langle\beta^2\rangle}_{\text{charge radius}} = \underbrace{\langle\beta\rangle^2}_{\text{static } Q_0} + \underbrace{\langle\beta^2\rangle - \langle\beta\rangle^2}_{\text{dynamical effects}}$$

- a class of measurements that has reached an extraordinary level of refinement
- the only way to get an idea on $\langle R_c^2 \rangle$ of unstable nuclei, even far off β-stability, $\delta\langle r_c^2 \rangle$: propagate mean square radius change from isotopic shifts – propagation correct ? (deformation effects well taken into account ?) : complete mystery
- also μ and Q_s



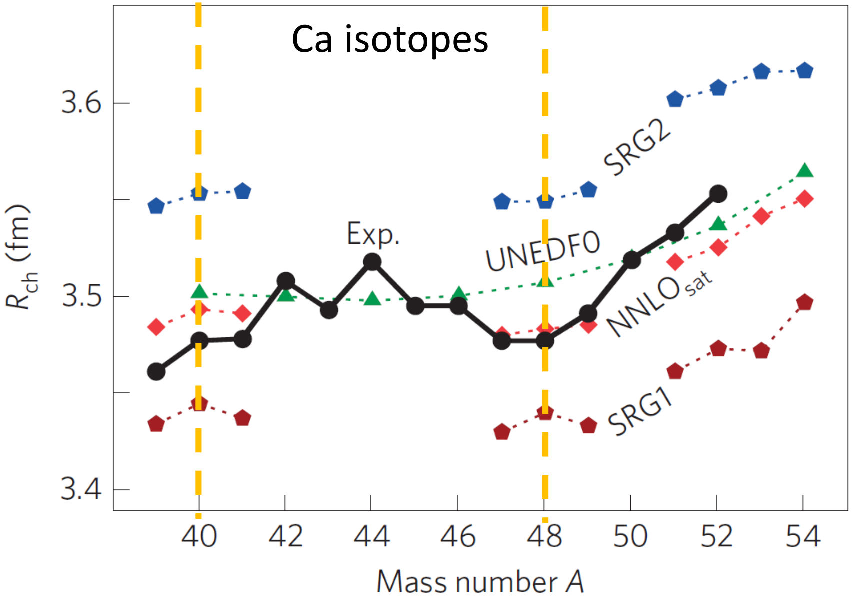
intermediate conclusions

- measurements which were the first historically to indicate the existence of a nuclear deformation (existence of a finite quadrupole moment of the charge distribution)

- we try to make say things about the nuclear deformation (the shape) to two quantities : λ (isotope shift) and Q_s
- and of course for all even-even nuclei ground state $I = 0 : Qs = 0$

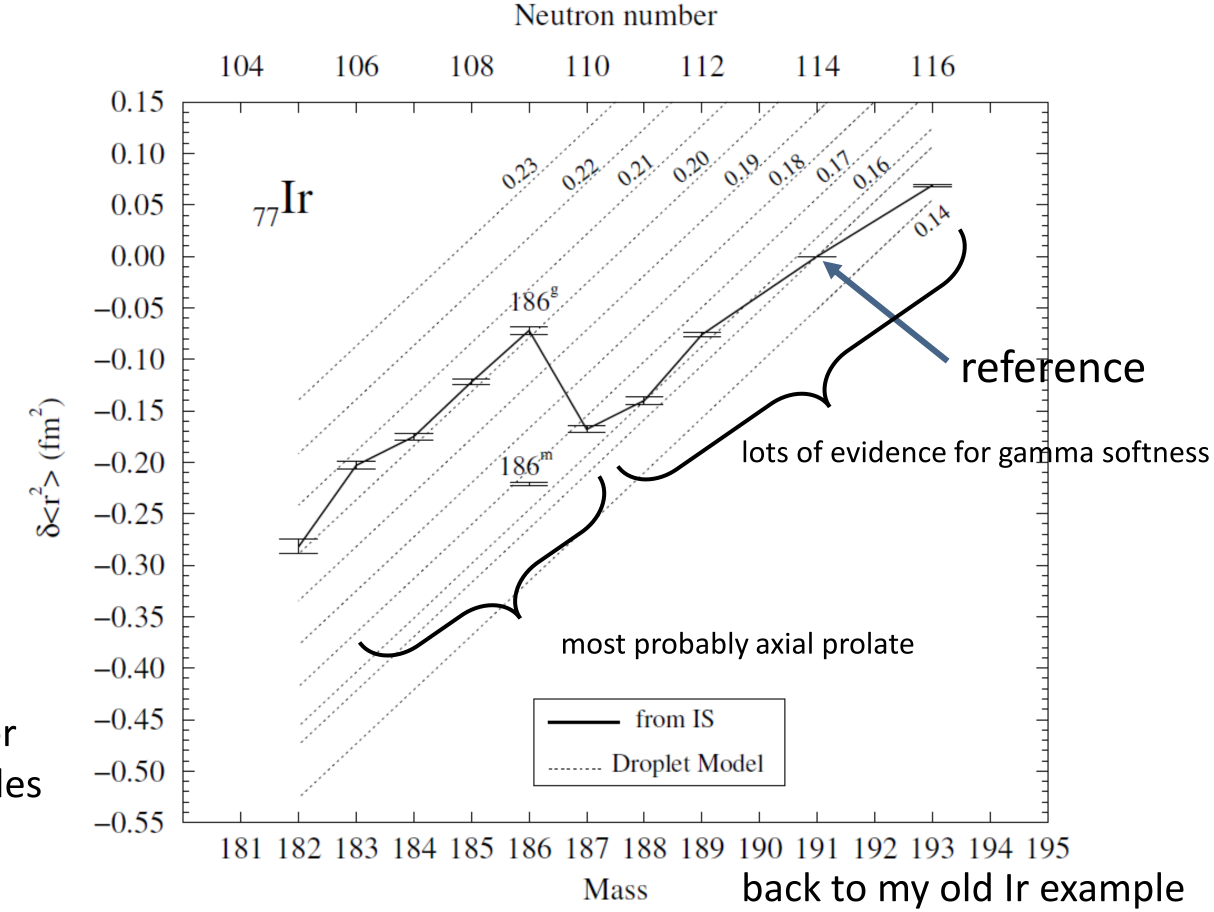
- to do so : assume homogeneously charged volume, axial shape

- remember : charge distribution only !



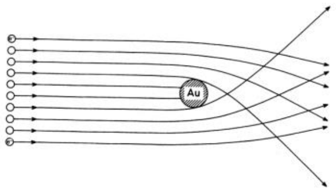
Garcia Ruiz *et al* NATURE PHYSICS 12 (2016)

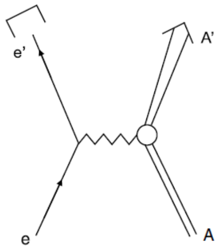
crying need for other observables

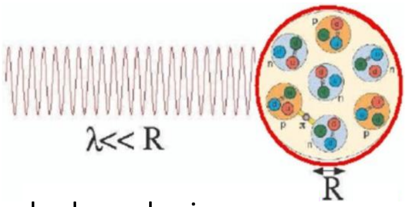


back to my old Ir example

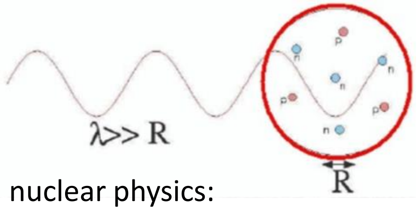
e momentum transfer $q \approx 1/\lambda$







hadron physics:
structure of the nucleon



nuclear physics:
internal structure of the nucleus
E_e= 500 MeV → ≈0.5 fm scale

Nobel price 1961

R. Hofstadter

1953 : e on Au Stanford

contrary to hadron probe, the only unknown in the reaction is the nuclear part

A(e,e) elastic cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{eA\to eA} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{1}{1+\frac{2E}{M}\sin^2(\theta/2)} |F(\vec{q})|^2$$

point charge nucleus

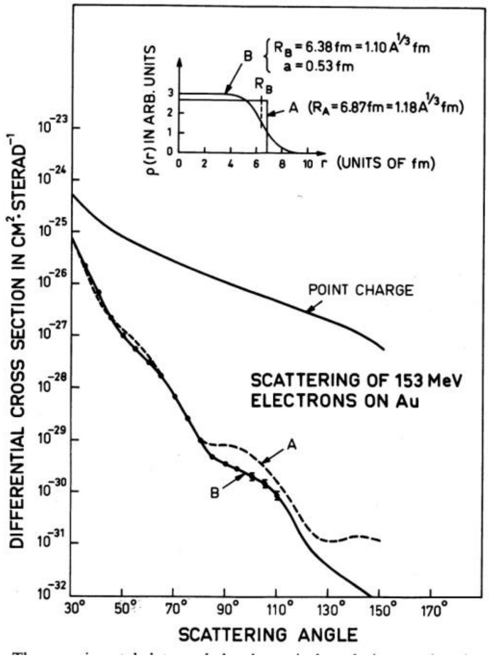
$$F(\vec{q}) = \frac{1}{Ze}\int \varrho(\vec{r})e^{i\vec{q}\cdot\vec{r}}d^3r$$

form factor

Fourier transform

$$\varrho(\vec{r}) = \frac{Ze}{(2\pi)^3}\int F(\vec{q})e^{-i\vec{q}\cdot\vec{r}}d^3q$$

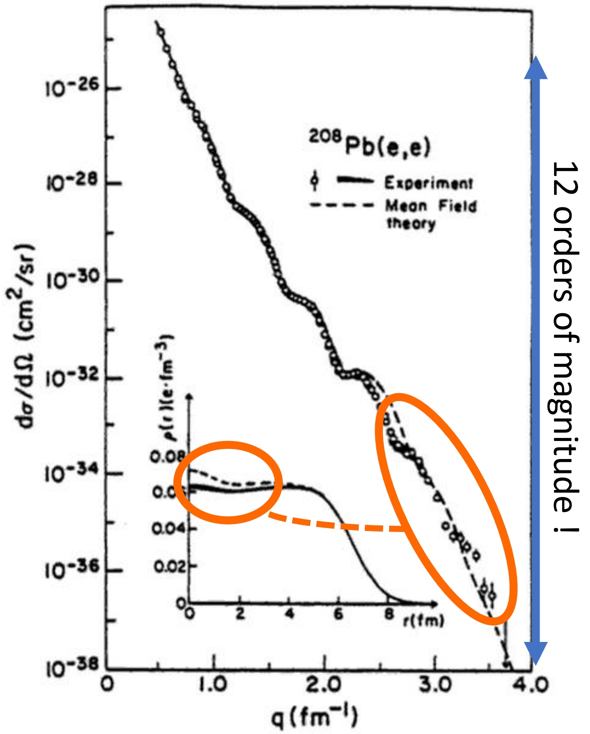
charge distribution
"model independent"

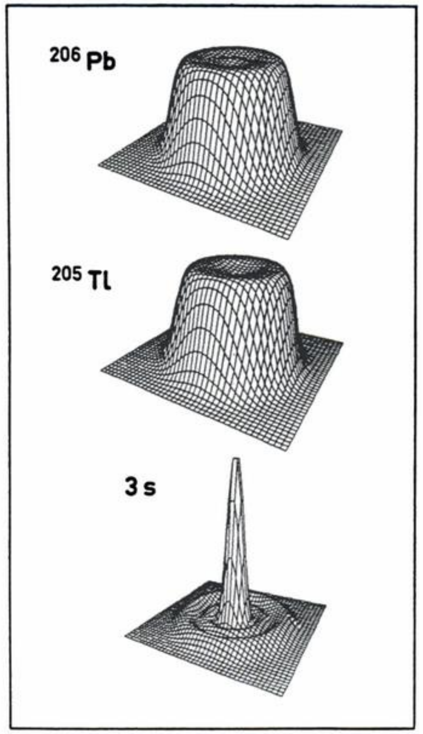


- T. Deforest, J.D. Walecka, Adv. Phys 15 (1966) 1.
- T.W. Donnelly, J.D. Walecka, Ann. Rev. Nucl. Part. Sci. 25 (1975) 329.
- B. Frois, C.N. Papanicolas, Ann. Rev Nucl. Part. Sci. 37 (1987) 133.

- perspectives with RIB

"Prospects for electron scattering on unstable, exotic nuclei"
Suda & Simon [Progress in Particle and Nuclear Physics 96 (2017)]



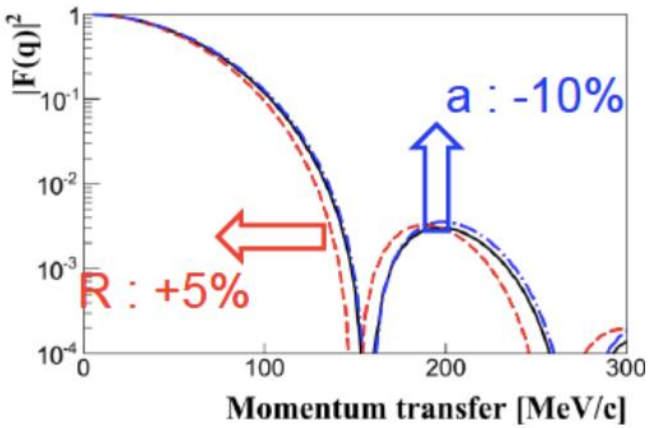


B. Frois et al
in Modern Topics in Electron Scattering
(World Scientific 1991)

- A(e,e) elastic scattering :

what we measure and analyze is essentially a diffraction pattern of the differential cross section as a function of the angle

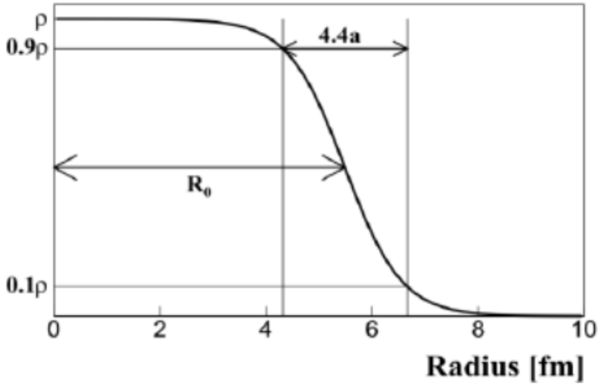
$$\left(\frac{d\sigma}{d\Omega}\right)_{eA\to eA} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{1}{1+\frac{2E}{M}\sin^2(\theta/2)}|F(\vec{q})|^2$$



$$\rho_C = \rho/(1+e^{(r-R)/a})$$

the usual 2-parameters radial charge distribution

rotation invariant (spherical)



but at the beginning others were tried :

Fermi:

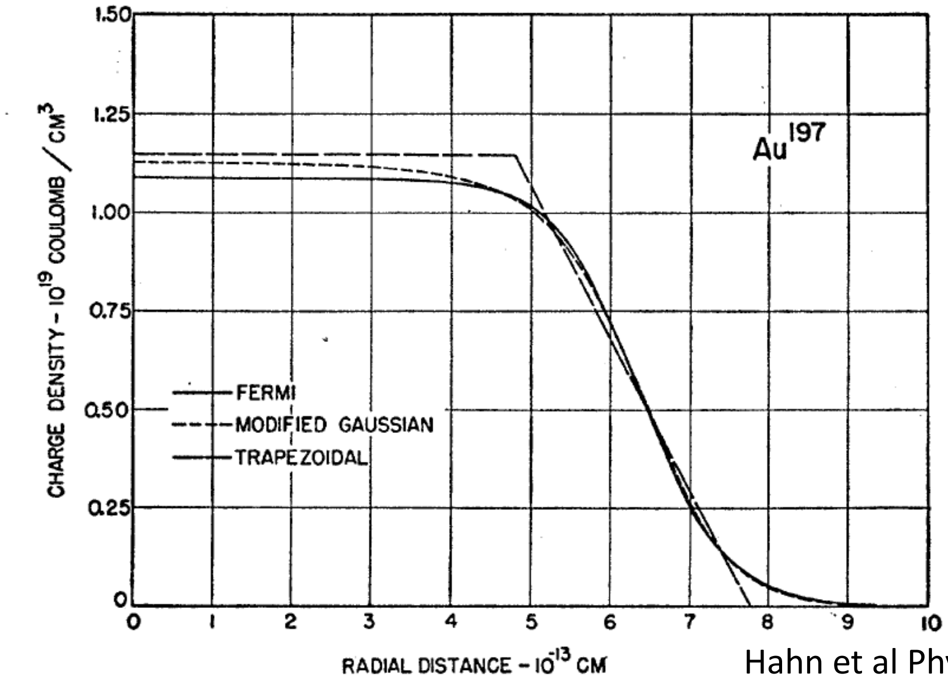
$$\rho(r)=\rho_1/\{\exp[(r-c)/z_1]+1\};$$

Modified Gaussian[22]:

$$\rho(r)=\rho_2/\{\exp[(r^2-c^2)/z_2^2]+1\};$$

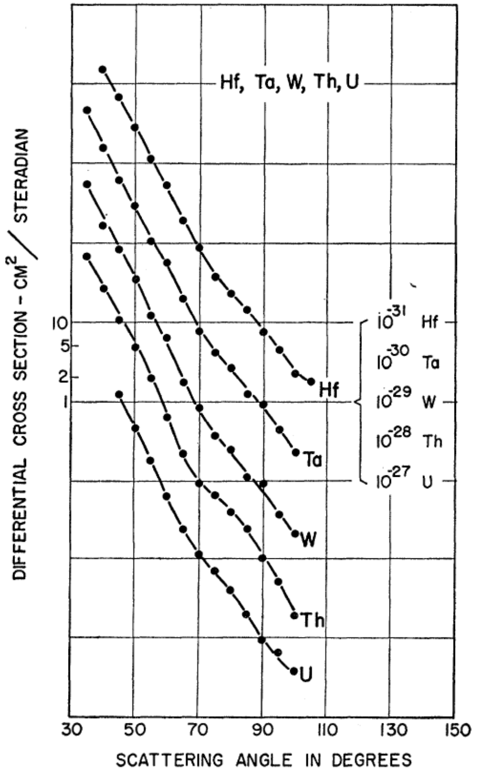
Trapezoidal:

$$\begin{aligned}\rho(r)&=\rho_3, \quad 0<r<c-z_3,\\ &=\rho_3(c+z_3-r)/2z_3, \quad c-z_3<r<c+z_3,\\ &=0, \quad r>c+z_3.\end{aligned}$$

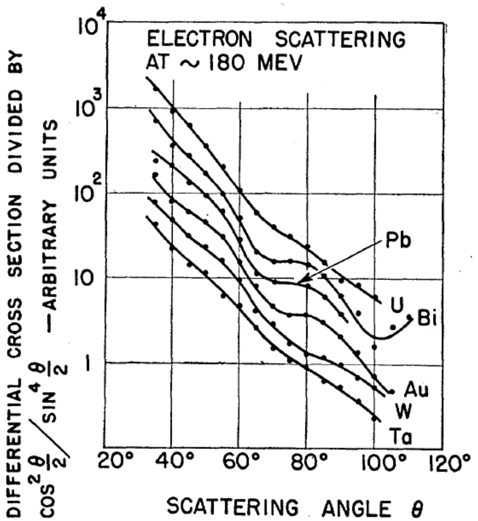


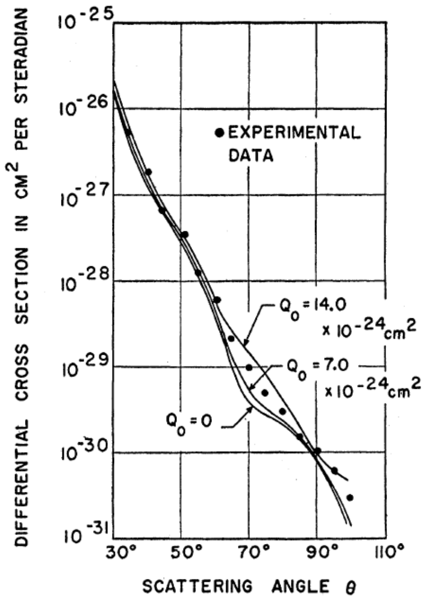
Hahn et al Phys Rev 101 (1956)

- the effect of quadrupole deformation on elastic scattering angular distribution : a « pattern killer »



Hahn et al Phys Rev 101 (1956)





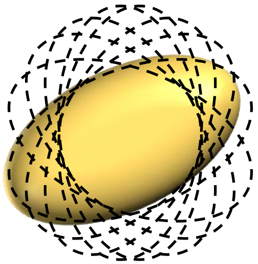
Downs et al Phys Rev 106 (1957)

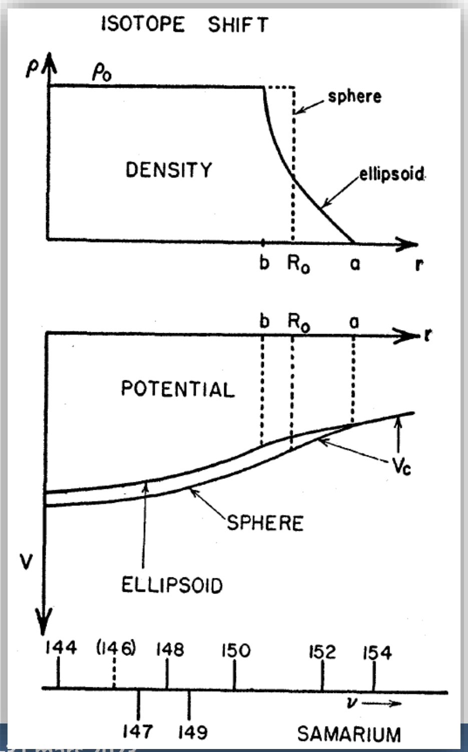
- two main reasons for that :

trivial : high resolution measurements are required as in deformed nuclei the elastic and inelastic components are very close to each other

not trivial : **nuclear translucency**

"In interpreting the experimental curves, it is necessary to include the effect of an ellipsoidal shape and to average it appropriately in all the aspects seen by approaching electrons. The averaging has the effect of rounding-off the nuclear surface and making the apparent surface thicker than it actually is." Hofstadter Rev. Mod. Phys. 1956





Temmer Rev. Mod. Phys. 30 (1958)

- adopted model-independ analysis :

Fourier-Bessel series expension [Dreher]

$$\rho(r) = \begin{cases} \sum_v a_v j_0(v\pi r/R) & \text{for} \quad r \leq R \\ & \text{(cut-off radius)} \\ 0 & \text{for} \quad r \geq R, \end{cases}$$

Sum of Gaussians [Sick]

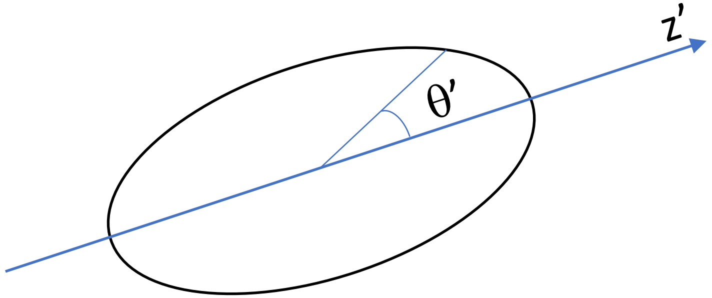
$$\rho(r) = \sum_i A_i\{\exp(-[(r-R_i)/\gamma]^2) + \exp(-[(r+R_i)/\gamma]^2)\},$$

- tabulated evaluated $\langle R_c \rangle$ At. Dat. Nucl. dat. tables 36 (1987) 60 (1995) etc
- advantage : the uncertainties in the charge distribution originating from the experimental errors and from the lack of knowledge about large-q behavior can be determined separately
- any reference/information to/on deformation is abandoned

- use of "deformed scattering models" exists : model dependent and relatively rare

$$\rho(\mathbf{r}') = \sum_L \rho_L(r') P_L(\cos\theta')$$ 2-L poles decomposition

$$F(\mathbf{q}) = \int \exp(i\mathbf{q}\cdot\mathbf{r})\,\rho(\mathbf{r})\,d^3r = \sum_L F_L(q^2)\,P_L(\theta') \qquad F_L(q^2) \equiv \int j_L(qr)\,\rho_L(r)\,d^3r$$



- $I = 0$ or ½ average the scattering amplitude over angles θ' so that it becomes proportional to $F_0(q^2)$
- $I \geq 1$

polarized nuclear population : interference between $F_L(q^2)$

unpolarized : interference terms vanish and $\frac{d\sigma}{d\Omega} \propto |F_0(q^2)|^2 + |F_2(q^2)|^2$

ANNALS OF PHYSICS 128, 286–297 (1980)

Theoretical Remarks for the Analysis of Electron Scattering Experiments on Rotational Nuclei*

E. MOYA DE GUERRA

PHYSICAL REVIEW C 98, 044310 (2018)

Elastic electron scattering form factors of deformed exotic Xe isotopes

combination of deformed Relativistic mean field and DWBA

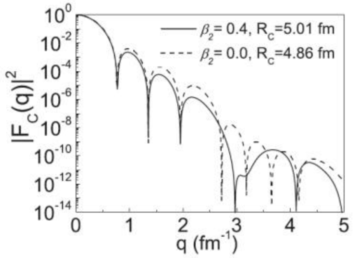


FIG. 9. The $|F_C(q)|^2$ of ^{132}Xe for $\beta_2 = 0.0$ and 0.4 calculated by PWBA method, where the corresponding $\rho_C(\mathbf{r})$ are obtained by the constrained RMF calculations with the NL3* parameter set.

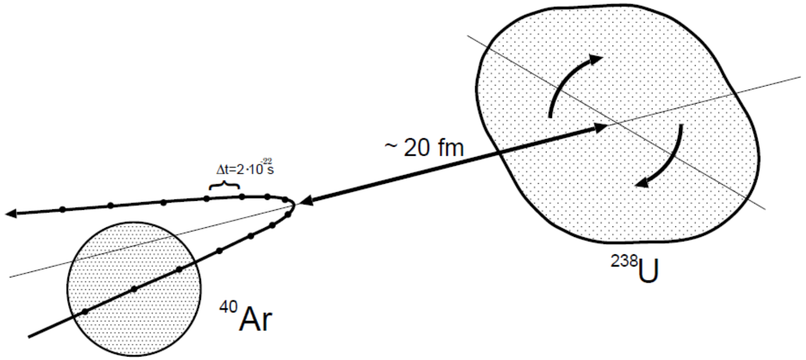
- as seen in introduction among the earliest evidence for nuclear deformation

“Aage Bohr pointed out to me at the time (1950) that if the nucleus is a spheroid with an "intrinsic“ quadrupole moment Q_0 relative to its distortion axis […] $Q_s = 0$ for $I = 0$ or $^1/_2$ but Q_0 may not be zero. Bohr, Mottelson and colleagues (Alder et al. , 1956) subsequently treated the situation for **Coulomb excitation** cross sections for low-lying rotational states. **The excitation cross sections uniquely establish the intrinsic quadrupole moment Q_0 for the ground states of distorted even-even nuclei as well as for odd A nuclei**. [L. J. Rainwater, Nobel Prize lecture (1975)]

- a tool of choice !
- a class of measurements that has reached a fantastic level of refinement – widely used with both stable and unstable nuclei

[see, e.g. Görgen and Korten J. Phys. G: Nucl. Part. Phys. 43 (2016) and M. Zielinska’s abundant literature]

- contrary to previous topic : here it’s all about **inelastic scattering**



Gaffney et al Nature 497 (2013)

Cline’s "safe energy" criterion –

$$D_{min} = 1.25 \cdot (A_p^{1/3} + A_t^{1/3}) + 5.0\ [fm]$$

the nuclear interaction is negligible

- model independent extraction of the shape (charge distribution) is possible using Kumar's quadrupole invariants
  - define a n-body quadrupole moment operator

(a scalar, can have non-vanishing matrix elements for any nuclear state)

$$P^{(n)} = ([P_2 \times P_2 \ldots \times P_2]_2 \cdot P_2)$$

where $P_{2\mu} = \sum_{i=1}^{A} e_i r_i^2 Y_{2\mu}(\Omega_i)$ is a 1-body electric quadrupole moment operator whose reduced matrix elements are related to the familiar quantities Q^S and B(E2)

$$M_{sr} = -\langle r \| P_2 \| s \rangle,$$

$$B(E2; s \rightarrow r) = (2I_s + 1)^{-1} M_{sr}^{\ 2},$$

$$Q_s^{\ S} = (16\pi/5)^{1/2} \langle s, M_s = I_s | P_{20} | s, M_s = I_s \rangle = -[16\pi I_s (2I_s - 1)/5(I_s + 1)(2I_s + 1)(2I_s + 3)]^{1/2} M_{ss}.$$

$$p_s^{(n)} = \langle s, M_s | P^{(n)} | s, M_s \rangle = (2I_s + 1)^{-1/2} \langle s \| P^{(n)} \| s \rangle$$

$$P_{2\mu} = \sum_{i=1}^{A} e_i r_i^2 Y_{2\mu}(\Omega_i)$$

- The trick is that the diagonal matrix elements $\langle s \| P^{(n)} \| s \rangle = M_{ss}$ can be written as a sum of products of n matrix elements of P_2 with the sum running over (n-1) intermediate states

examples :

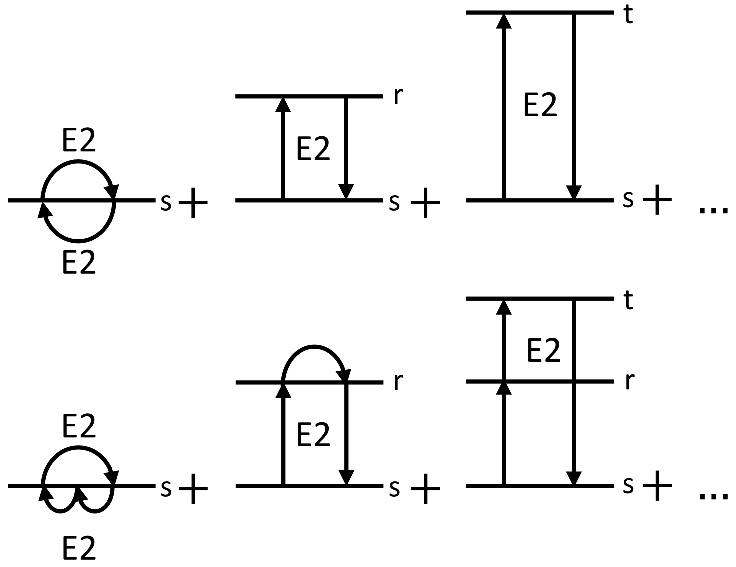
$$P_s^{(2)} = (2I_s + 1)^{-1} \sum_r M_{sr}^{\ 2}$$ 2-body moment

$$= \frac{5(I_s + 1)(2I_s + 3)}{16\pi I_s (2I_s - 1)} (Q_s^{\ S})^2 + \sum_{r \neq s} B(E2; s \rightarrow r)$$

$$P_s^{(3)} = -5^{1/2} (2I_s + 1)^{-1} (-1)^{2I_s} \sum_{rt} \begin{Bmatrix} 2 & 2 & 2 \\ I_s & I_r & I_t \end{Bmatrix} M_{sr} M_{rt} M_{ts}$$ 3-body moment

$p_s^{(2)} =$ … + …

$p_s^{(3)} =$ … + …

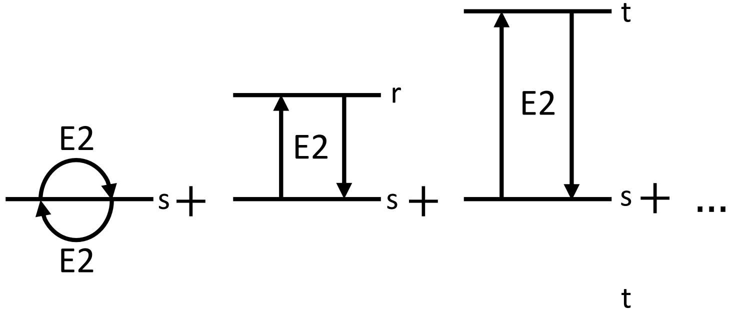


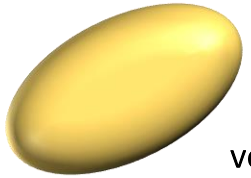
➢ how can we use these n-body moments to characterize the nuclear shape ?

$$P_s^{(2)} = (2I_s+1)^{-1}\sum_r M_{sr}^2$$

$$= \frac{5(I_s+1)(2I_s+3)}{16\pi I_s(2I_s-1)}(Q_s^{\,S})^2 + \sum_{r\neq s} B(E2; s\to r)$$

2-body moment : “a model-independent measure of the magnitude of intrinsic quadrupole moment or deformation” [Kumar 1975]

$p_s^{(2)} =$ 



- to relate these n-body moment to a nuclear shape

there is no choice but to use the concept of an equivalent ellipsoid

intrinsic

volume of an ellipsoid (with sharp border)

$P_{2\mu} = \sum_{i=1}^{A} e_i r_i^2 Y_{2\mu}(\Omega_i)$ replaced by a volume integral $Q_{s\mu}^{i} = (16\pi/5)^{1/2}\int \rho_s r^2 Y_{2\mu} dV$

this ellipsoid has same charge, volume $p_s^{(2)}$ and $p_s^{(3)}$ as the actual nucleus (but it is NOT the nucleus)

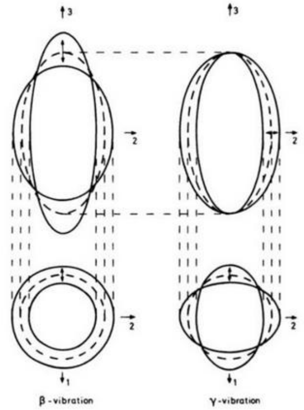
- then everything becomes “easy” :

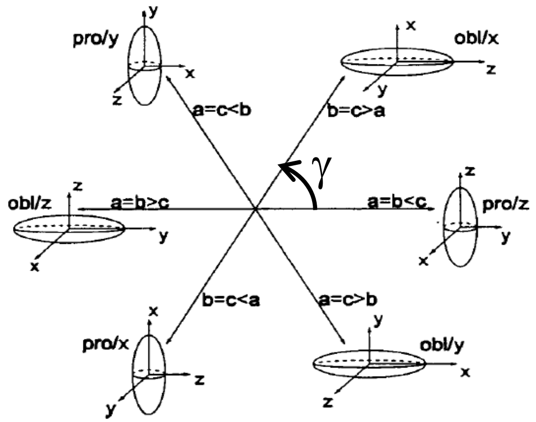
$Q_s^{\,i} = (16\pi/5)(p_s^{(2)})^{1/2},$ an intrinsic quadrupole moment for any state

$\cos 3\gamma_s = -(\tfrac{7}{2})^{1/2} p_s^{(3)} (p_s^{(2)})^{-3/2}.$ an asymmetry angle for any state

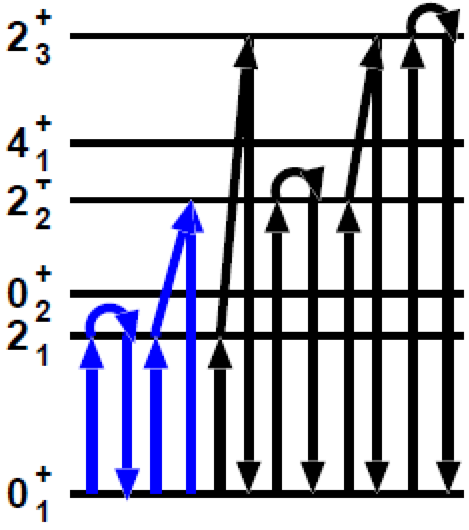
$\sigma_s(\beta) == \left[p_s^{(4)} - \left(p_s^{(2)}\right)^2\right]^{1/2}$ measures fluctuation in magnitude of nuclear deformation

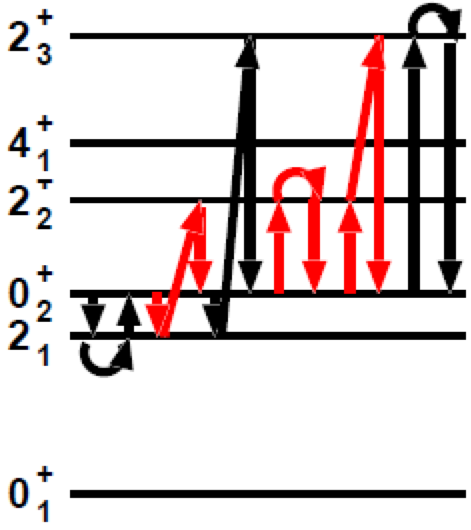
$\sigma_s(\gamma) == \left[p_s^{(6)} - \left(p_s^{(3)}\right)^2\right]^{1/2}$ measures fluctuation in asymmetry of nuclear deformation





- experimentally very challenging, but not impossible





- e.g. case of ^{100}Mo (stable)
- K. Wrzosek-Lipska et al. PRC 86, 064305 (2012)

TABLE IV. Reduced nondiagonal $E2$ matrix elements in ^{100}Mo obtained in the present work, compared to the $E2$ matrix elements determined previously [calculated from $B(E2)$ values from Refs. [8,9,11], assuming a positive sign].

$I_i \to I_f$	$\langle I_f \| E2 \| I_i \rangle$ (eb)			
	Present work	Previous measurements		
		[8]	[11]	[9]
$0_1^+ \to 2_1^+$	$0.68^{+0.01}_{-0.01}$	0.725(18)	0.689(17)	−0.725[a]
$0_1^+ \to 2_2^+$	$0.103^{+0.002}_{-0.001}$	0.106(4)	0.089(6)	0.097(4)
$0_1^+ \to 2_3^+$	$-0.016^{+0.003}_{-0.003}$			<0.03
$2_1^+ \to 0_2^+$	$0.513^{+0.009}_{-0.004}$	0.436(7)		−0.425(34)
$2_1^+ \to 2_2^+$	$0.94^{+0.02}_{-0.02}$	0.94(1)	0.83(6)	−0.86(4)
$2_1^+ \to 4_1^+$	$1.33^{+0.03}_{-0.02}$	1.325(1)	1.31(9)	1.38(5)
$2_1^+ \to 2_3^+$	$-0.070^{+0.007}_{-0.006}$			0.26(3)
$2_1^+ \to 4_2^+$	$0.063^{+0.025}_{-0.012}$			
$0_2^+ \to 2_2^+$	$-0.32^{+0.03}_{-0.02}$			<0.1
$0_2^+ \to 2_3^+$	$0.506^{+0.008}_{-0.006}$			0.47(5)
$2_2^+ \to 4_1^+$	$0.77^{+0.13}_{-0.10}$			0.1(1)
$2_2^+ \to 2_3^+$	$0.40^{+0.15}_{-0.13}$			0.3(3)
$2_2^+ \to 4_2^+$	$1.02^{+0.04}_{-0.03}$			0.89(7)
$4_1^+ \to 2_3^+$	$0.83^{+0.07}_{-0.04}$			−0.5(2)
$4_1^+ \to 4_2^+$	$0.99^{+0.05}_{-0.05}$			−0.87(7)
$4_1^+ \to 6_1^+$	$1.83^{+0.06}_{-0.06}$			−1.86(13)

[a]Taken from Ref. [8]. All signs in Ref. [9] are predicted by the IBM-2 model [10].

- determine all transition rates + spectroscopic quadrupole moments (including in excited states)
- need to isolated the pure E2 strength (know M1/E2 mixing coefficients)
- benefits from other measurements : lifetime measurement, static moments etc
- for determination : the relative sign of the matrix elements is required…

TABLE XIV. Experimental and theoretical mean values of the shape deformation parameters $\overline{\beta}$ and $\overline{\gamma}$. Experimental values were calculated from the mean values of the quadrupole invariants $\langle Q^2 \rangle$ and $\langle Q^3 \cos(3\delta) \rangle$. Theoretical results were obtained using the GBH model with the SIII and SLy4 variants of the interaction.

State	Shape parameter	GBH model SIII	GBH model SLy4	Experiment (present work)
0_1^+	$\overline{\beta}$	0.25	0.20	0.22 ± 0.01
	$\overline{\gamma}$ (deg)	22°	27°	$29° \pm 3°$
0_2^+	$\overline{\beta}$	0.30	0.24	0.25 ± 0.01
	$\overline{\gamma}$ (deg)	13°	18°	$10° \pm 3°$

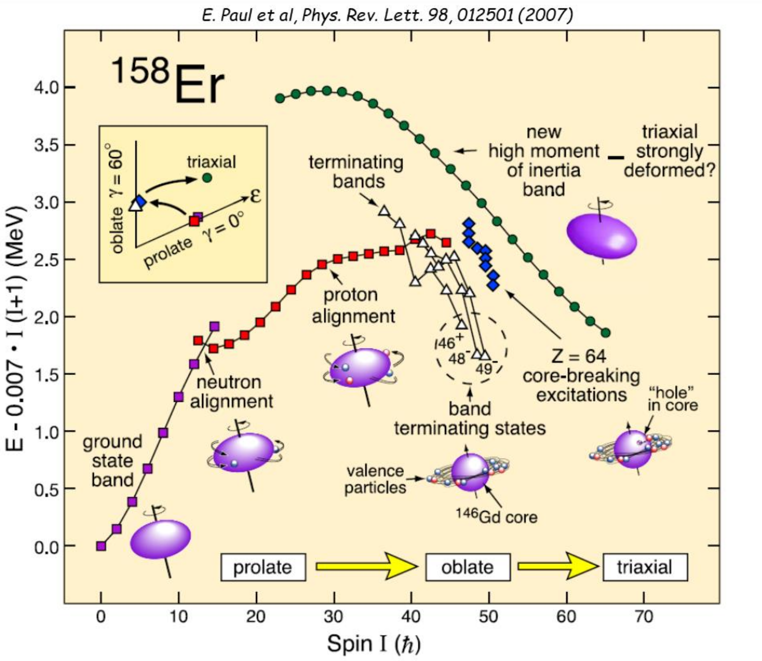
key tool :
GOSIA: Rochester - Warsaw semiclassical Coulomb excitation least-squares search code
Developed in early eighties by T. Czosnyka, D. Cline, C.Y. Wu (Bull. Am. Phys. Soc. 28 (1983) 745.) and continuously upgraded

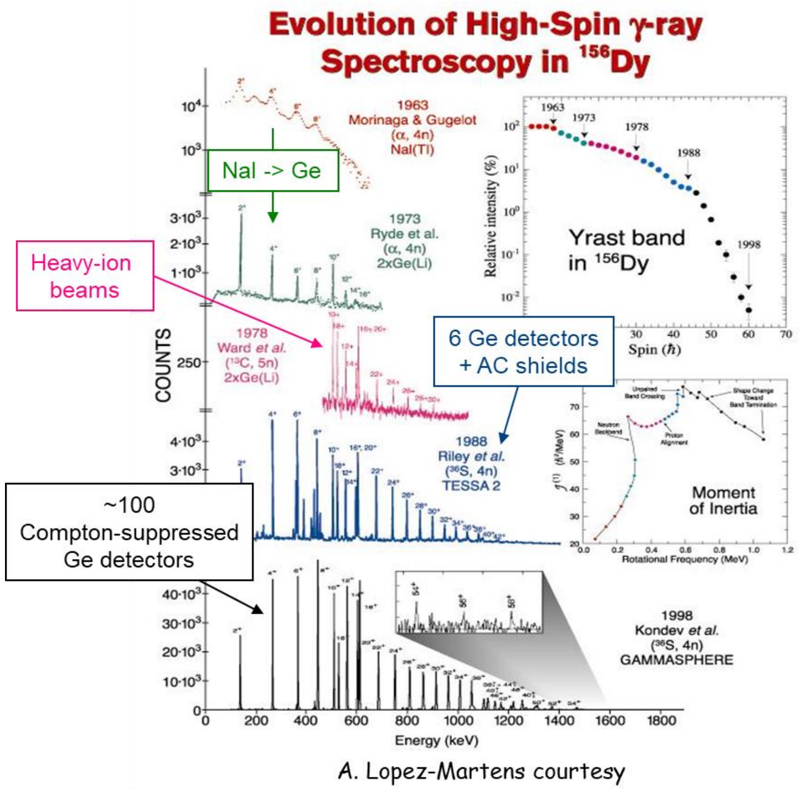
intermediate conclusions

- multi-step coulomb excitation : a high precision tool to investigate nuclear deformation
- model independence if data set rich enough, a few cases well characterized
- don't forget, though, that we interpret the measured quantities for **an equivalent ellipsoid (homogeneously charged, sharp border) with same charge, volume $p_s^{(2)}$ and $p_s^{(3)}$ as the actual nucleus (but it is NOT the nucleus)**
- remember : once again the probe is only sensitive to charge distribution

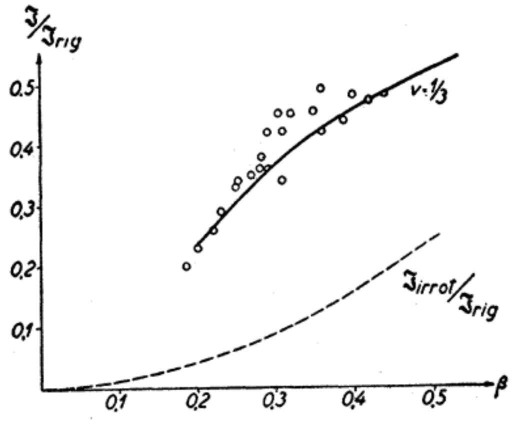
nuclear spectroscopy

- In the field of nuclear spectroscopy, the question is not so much to know if we are able to access the nuclear deformation (always in an indirect way via models).
- But in this field it seems that we are permanently affected by nuclear deformation and there are even some curious phenomena that demonstrate its tangibility
- The question of spin generation : eg without the notion of a deformed body, difficult to understand high-spin generation in nuclei





- not to mention :
  - Coriolis effects
  - shape coexistence
  - …



nuclear spectroscopy

- But in this field it seems that we are permanently affected by nuclear deformation and there are even some curious phenomena that demonstrate its tangibility

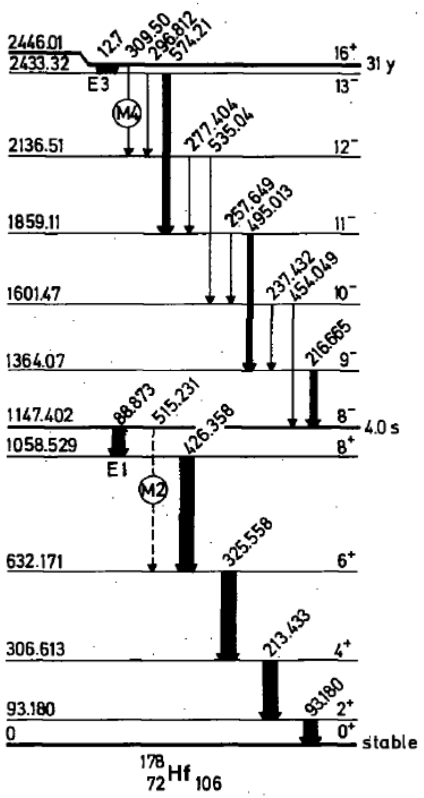


Fig. 1. Decay scheme.

VOLUME 72, NUMBER 17 PHYSICAL REVIEW LETTERS 25 APRIL 1994

Nuclear Properties of the Exotic High-Spin Isomer $^{178}Hf^{m2}$ from Collinear Laser Spectroscopy

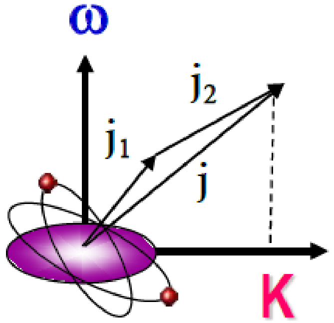
N. Boos,[1] F. Le Blanc,[2] M. Krieg,[1] J. Pinard,[3] G. Huber,[1] M. D. Lunney,[2] D. Le Du,[4] R. Meunier,[4] M. Hussonnois,[2] O. Constantinescu,[2] J. B. Kim,[2] Ch. Briançon,[4] J. E. Crawford,[6] H. T. Duong,[3] Y. P. Gangrski,[7] T. Kühl,[5] B. N. Markov,[7] Yu. Ts. Oganessian,[7] P. Quentin,[4] B. Roussière,[2] and J. Sauvage[2]

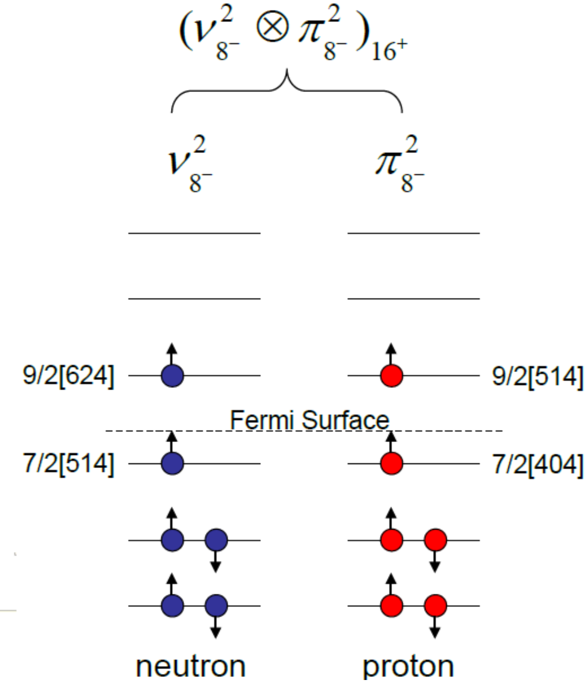
^{176}Yb(α,2n)^{178}Hf @ 36 MeV at Dubna
96% enriched Yb target
chemical separation at Orsay,
a sample of 6 ng of ^{178m2}Hf was prepared !

$$\delta\langle r^2\rangle^{178,178m2} = -0.059(9)\ \mathrm{fm}^2$$

$$\mu_I^{178m2} = +8.16(4)\mu_N$$

$$Q_s^{178m2} = +6.00(7)\ \mathrm{b}$$





Evidence of nuclear deformation in low-energy experiments : methods, limitations, interpretation

- Nuclear deformation, where does it come from ?

an historical approach to the concept of “nuclear deformation”

→ how this concept has "massively" imposed itself from the earliest measurements

- Nuclear deformation, can we characterize it ?

→ the network of (true) observables. NB it is improper to say that we “observe nuclear deformation”

- concluding remarks

the reality of nuclear deformation (a concept coined ~1950) has continuously eluded us, low energy nuclear physicists

and we continue to use and perfect the tools that the great founders already had at their disposal (even if it was still in very rudimentary forms),

the arrival of a new probe, a new approach, such as the one offered by high energy heavy ion collisions is therefore a major historical event

the concept of nuclear deformation (or shape, borrowed from our everyday life) is not without problems and sometimes creates paradoxes
and related to that last point, at the end of this exercise I will share with you some personal thoughts

→ the concept of nuclear deformation was introduced as an elegant way to reconcile very different properties (sometimes in apparent opposition) of atomic nuclei ... and certainly as a way to simplify wave functions and calculations. It was a time when nuclear spectroscopy was "enchanted“ with pictures, shapes and shells. That also explains the difficulties associated with it.

GUTH: I would like to quote Wigner:

"If I had a great calculating machine, I would perhaps apply it to the Schrodinger equation of each metal and obtain its cohesive energy, its lattice constant, etc. It is not clear, however, that I would gain a great deal by this. Presumably, all the results would agree with the experimental values and not much would be learned from the calculation. What would be preferable, instead, would be a vivid picture of the behavior of the wave function."*

BIEDENHARN: We always have time for a quotation from Wigner.

→ In fact nowadays our approach has become pragmatic: ask the best theories on the market to calculate the observables to which we have had access experimentally, compare the numbers, conclude which theory is the best and be happy about it.

→ We have already entered for ~2 decades in the era of spectroscopic disenchantment