Seniority in atomic nuclei

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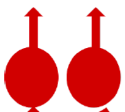
Overview

- The seniority concept and its implications
- Seniority in the neutron-rich Pb istopes beyond N=126
- Do Sn istopes present a seniority scheme? Caveat!
- The N=50 isotones and the Z=28 isotopes: Valence Mirror Symmetry Partners -- $g_{9/2}$
- Manifestation of the Berry phase in the mid-shell $g_{9/2}$ nucleus: ^{213}Pb
- Summary
- Future perspectives

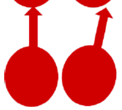
What is seniority?

Seniority υ is the number of unpaired identical nucleons to J=0 in a $|j^n\rangle$ configuration → Lecture by P. Van Isacker.

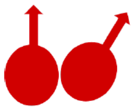
Id est → $g_{9/2}$



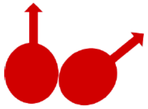
Seniority?



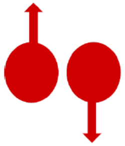
Seniority?



Seniority?



Seniority?



Seniority?

What is seniority?

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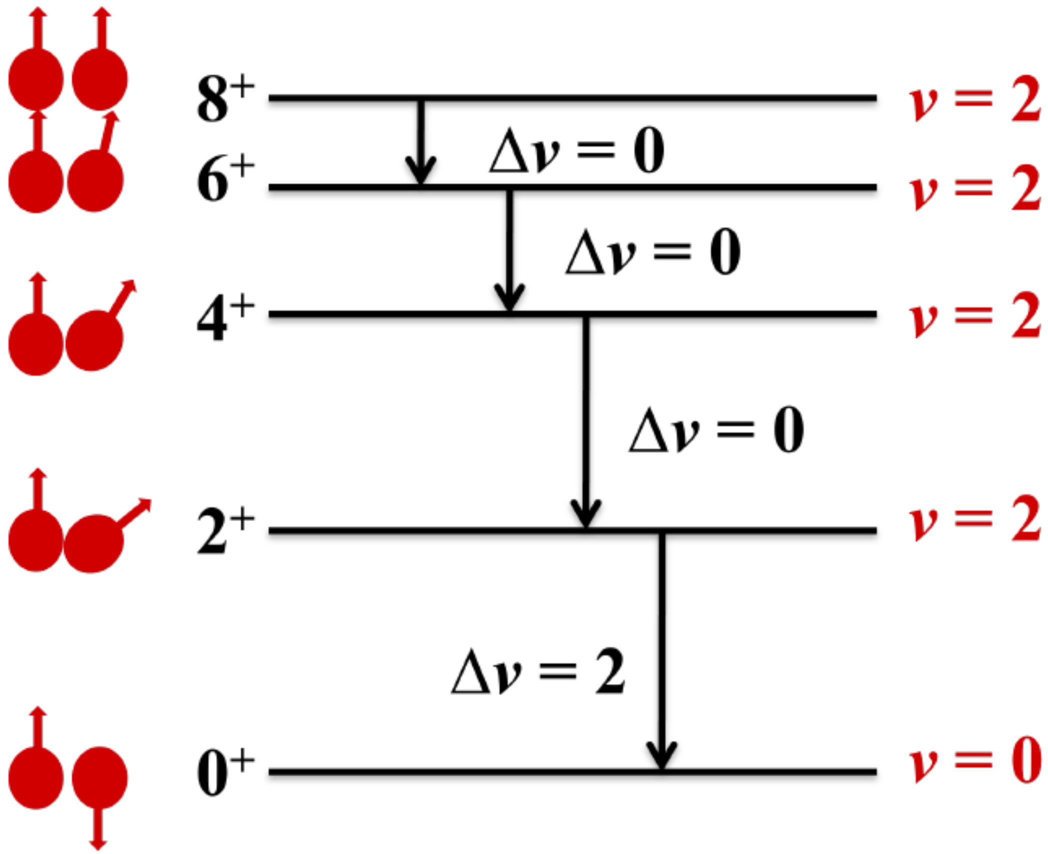
Id est for $g_{9/2}$

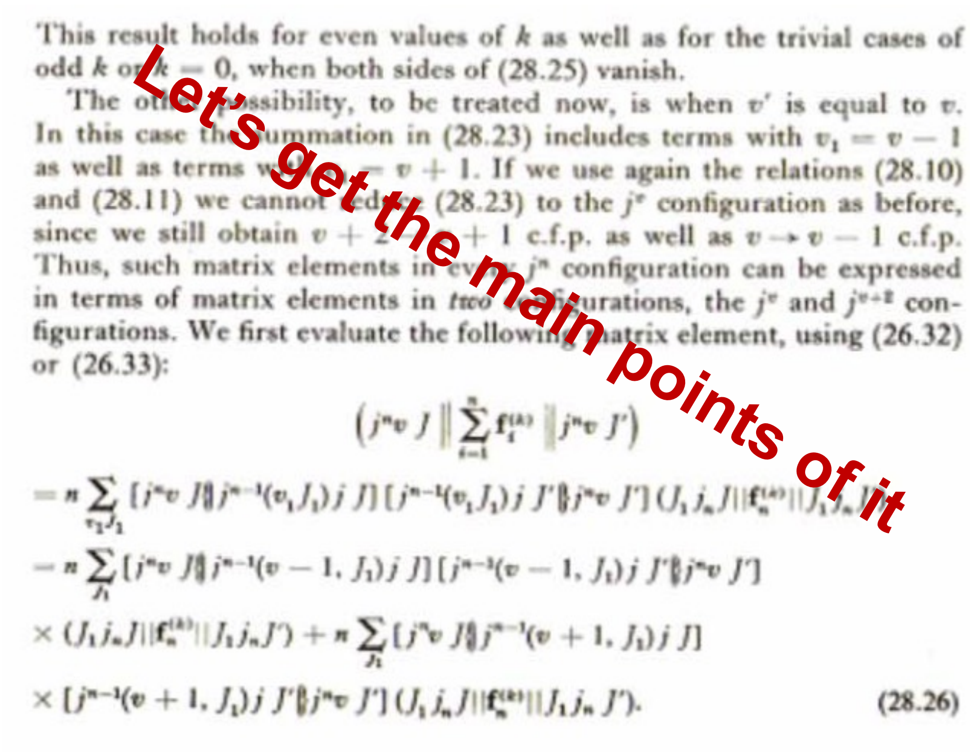


Is J=8 the largest angular momentum $g_{9/2}$ υ=2 configuration?

The seniority concept

- It leads to many simple poweful results under very general conditions. Matrix elements can be classified in terms of whether or not they conserve seniority
- Many realistic residual interactions seem to conserve seniority. So this scheme gives reaonable predictions for actual nuclei

Talmi de-Shalit



This result holds for even values of k as well as for the trivial cases of odd k or $k = 0$, when both sides of (28.25) vanish.

The other possibility, to be treated now, is when v' is equal to v. In this case the summation in (28.23) includes terms with $v_1 = v - 1$ as well as terms with $v_1 = v + 1$. If we use again the relations (28.10) and (28.11) we cannot reduce (28.23) to the j^v configuration as before, since we still obtain $v + 2 \to v + 1$ c.f.p. as well as $v \to v - 1$ c.f.p. Thus, such matrix elements in every j^n configuration can be expressed in terms of matrix elements in *two* configurations, the j^v and j^{v+2} configurations. We first evaluate the following matrix element, using (26.32) or (26.33):

$$\left(j^n v J \left\| \sum_{i=1}^{n} \mathbf{f}_i^{(k)} \right\| j^n v J'\right)$$

$$= n \sum_{v_1 J_1} [j^n v J \{| j^{n-1}(v_1 J_1) j J] [j^{n-1}(v_1 J_1) j J' |\} j^n v J'] (J_1 j_n J \| \mathbf{f}_n^{(k)} \| J_1 j_n J')$$

$$= n \sum_{J_1} [j^n v J \{| j^{n-1}(v - 1, J_1) j J] [j^{n-1}(v - 1, J_1) j J' |\} j^n v J']$$

$$\times (J_1 j_n J \| \mathbf{f}_n^{(k)} \| J_1 j_n J') + n \sum_{J_1} [j^n v J \{| j^{n-1}(v + 1, J_1) j J]$$

$$\times [j^{n-1}(v + 1, J_1) j J' |\} j^n v J'] (J_1 j_n J \| \mathbf{f}_n^{(k)} \| J_1 j_n J'). \qquad (28.26)$$

Seniority conservation

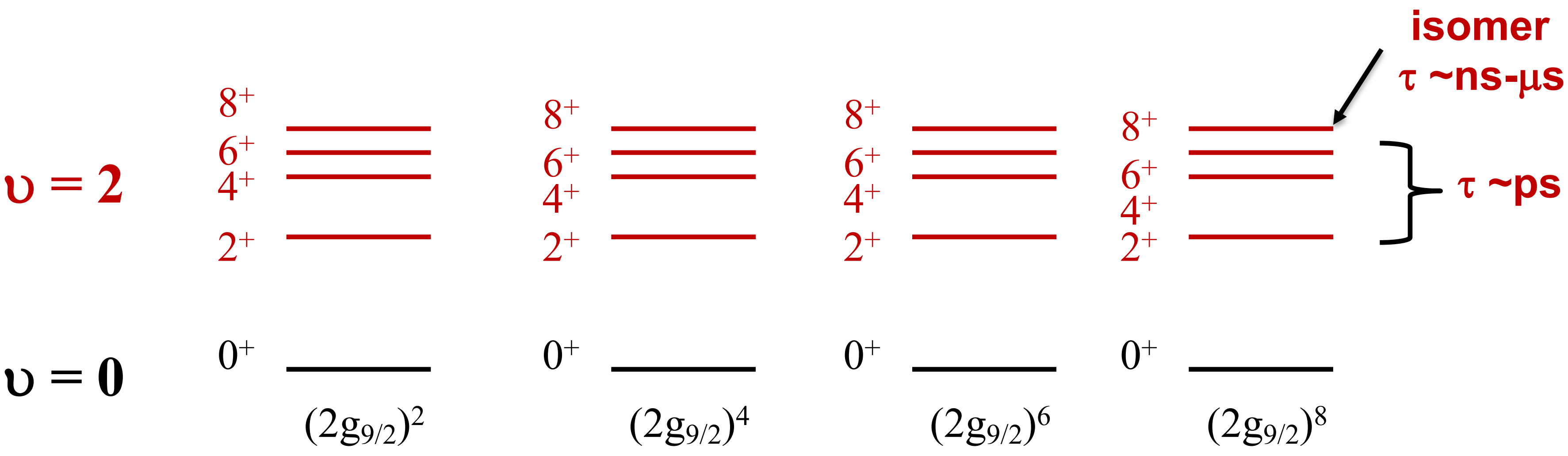
- **Seniority is conserved by any two-body interaction in the j^n configuration for $j \leq 7/2$**

  - Theorem: any two-body interaction in the j^n configuration is diagonal in the seniority scheme, provided it is diagonal in the j^3 configuration, that is, if there are no finite matrix elements connecting $\upsilon = 3$ with $\upsilon = 1$ states.
  - two-body interactions only connect states of the same J (only states of equal J can mix). This condition is automatically satisfied for any J value that is not common to both the $| j^3 \upsilon=3 \rangle$ $| j^3 \upsilon=1 \rangle$ states.
  - For $j = 1/2$ and $3/2$ it is trivial: they have no $\upsilon = 3$ states since they become maximally filled (midshell) at $n = 1$ and $n = 2$, respectively.
  - *Can you derive the rule for $j = 5/2$ and $7/2$?*

- **For $j > 7/2$ seniority is not necessarily conserved**. Concept of partial seniority conservation → P. Van Isacker PRL 100, 052501 (2008).

The seniority scheme: energies

Nucleons in a valence j^n configuration behave according to a seniority scheme: the states can be labelled by their seniority υ



In a pure seniority scheme, the relative level energies do not depend on the number of particles in the shell j

The 8^+ state will be an isomer due to the small energy separation

Why such energy level separation?

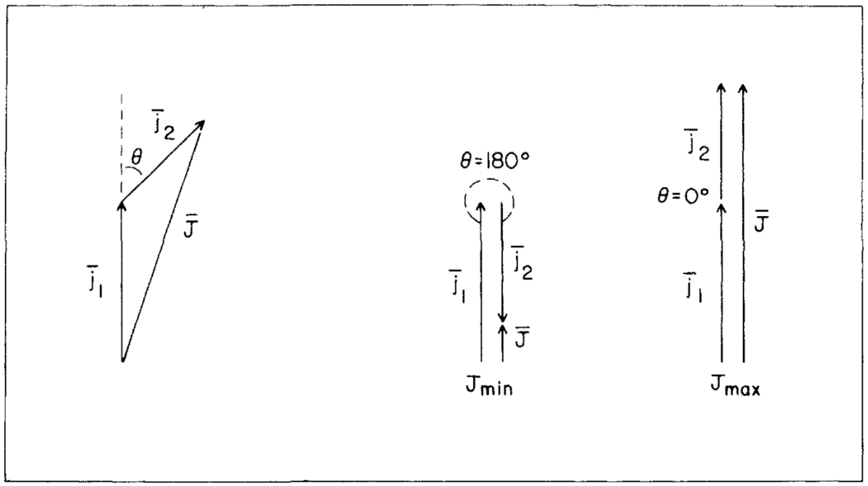
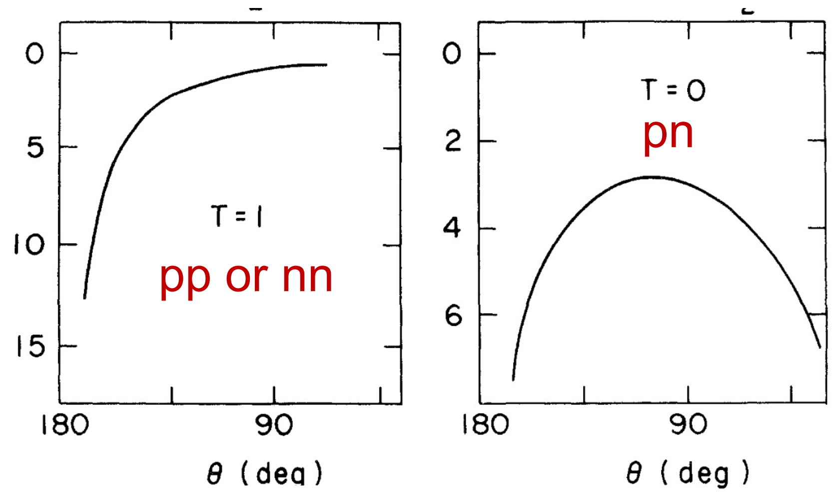


Fig. 4.8. Definition and schematic illustration of some of the ideas used in the geometrical analysis of short-range residual interactions.



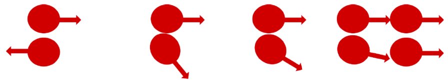
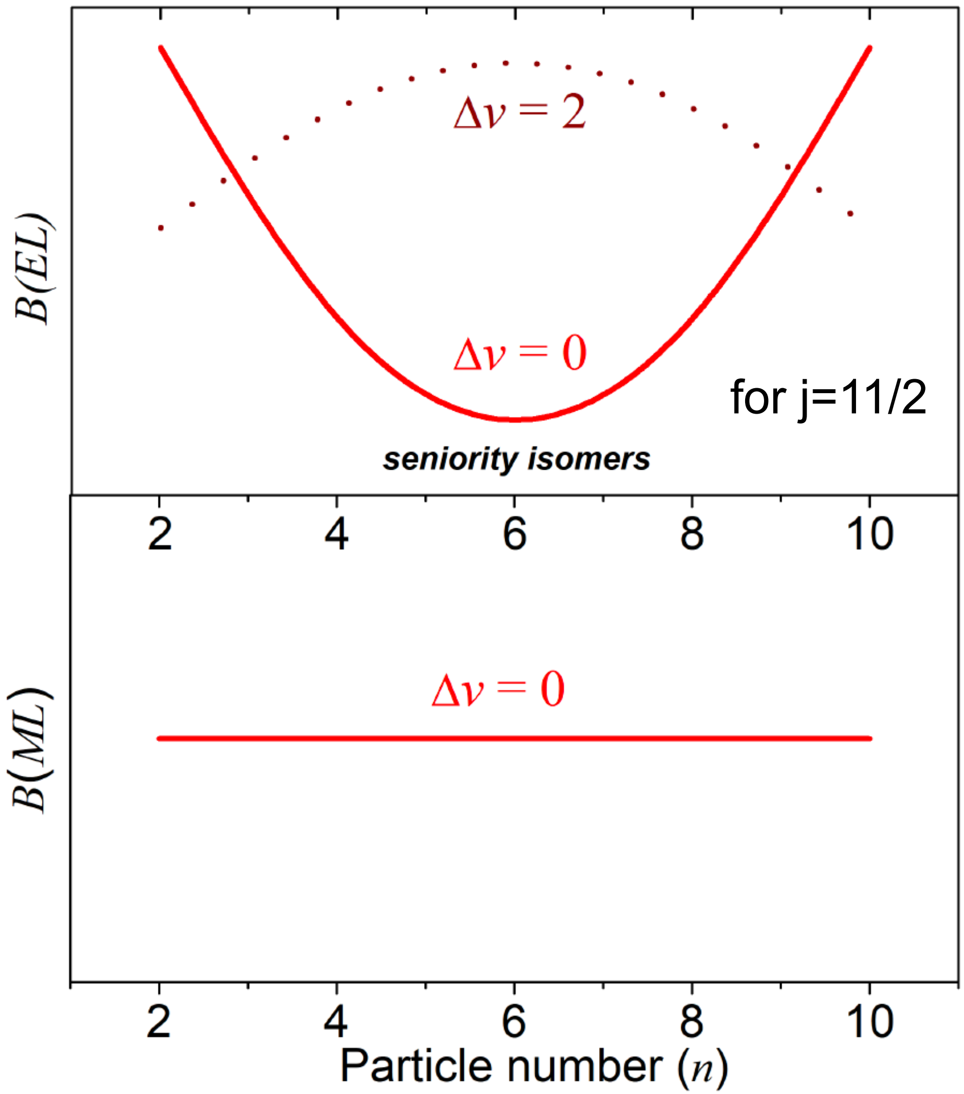


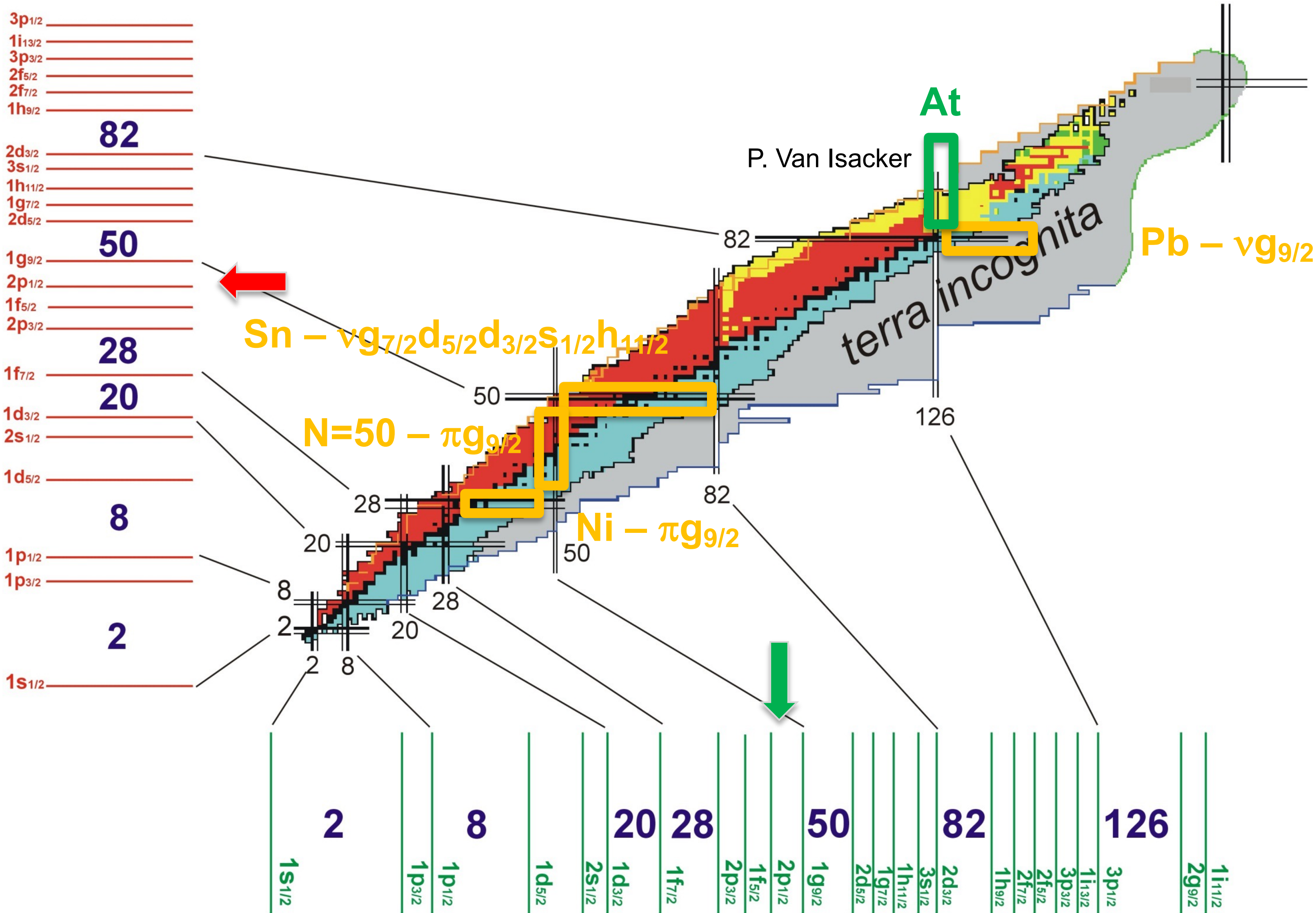
Fig. 4.9 more att The T = ngth (lower values correspond to (Left) The T=1 (J even) states. (Right) l above their respective plots.

- The interaction should be small for theta θ= 90 for both T = 0 and T = 1, since the particles are orbiting in nearly perpendicular planes and are seldom close enough to interact.
- **For T =1**, the interaction is strong when the two nucleons orbit in opposite directions (J = 0 θ = 180). It vanishes when they orbit in the same direction (J_{max}, θ = 0) two particles have identical quantum numbers and the spatial wave function is required to be antisymmetric.
- For the T= 0, particles are distinct and, for both the small and large / extremes, the orbits are nearly coplanar. Since we need not worry about antisymmetry.

The seniority scheme: B(EL), B(ML)

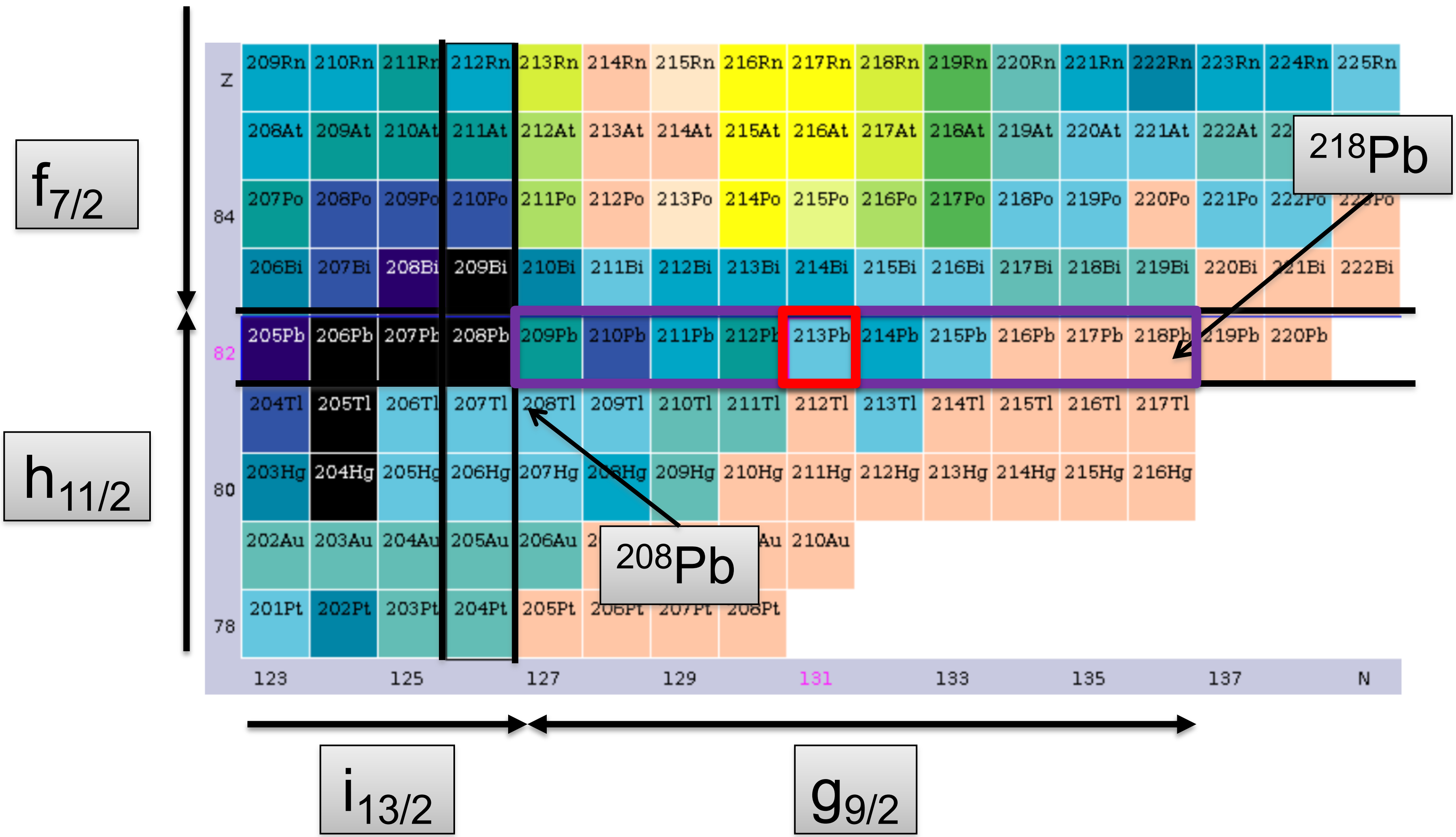


Where υ is to be conserved?

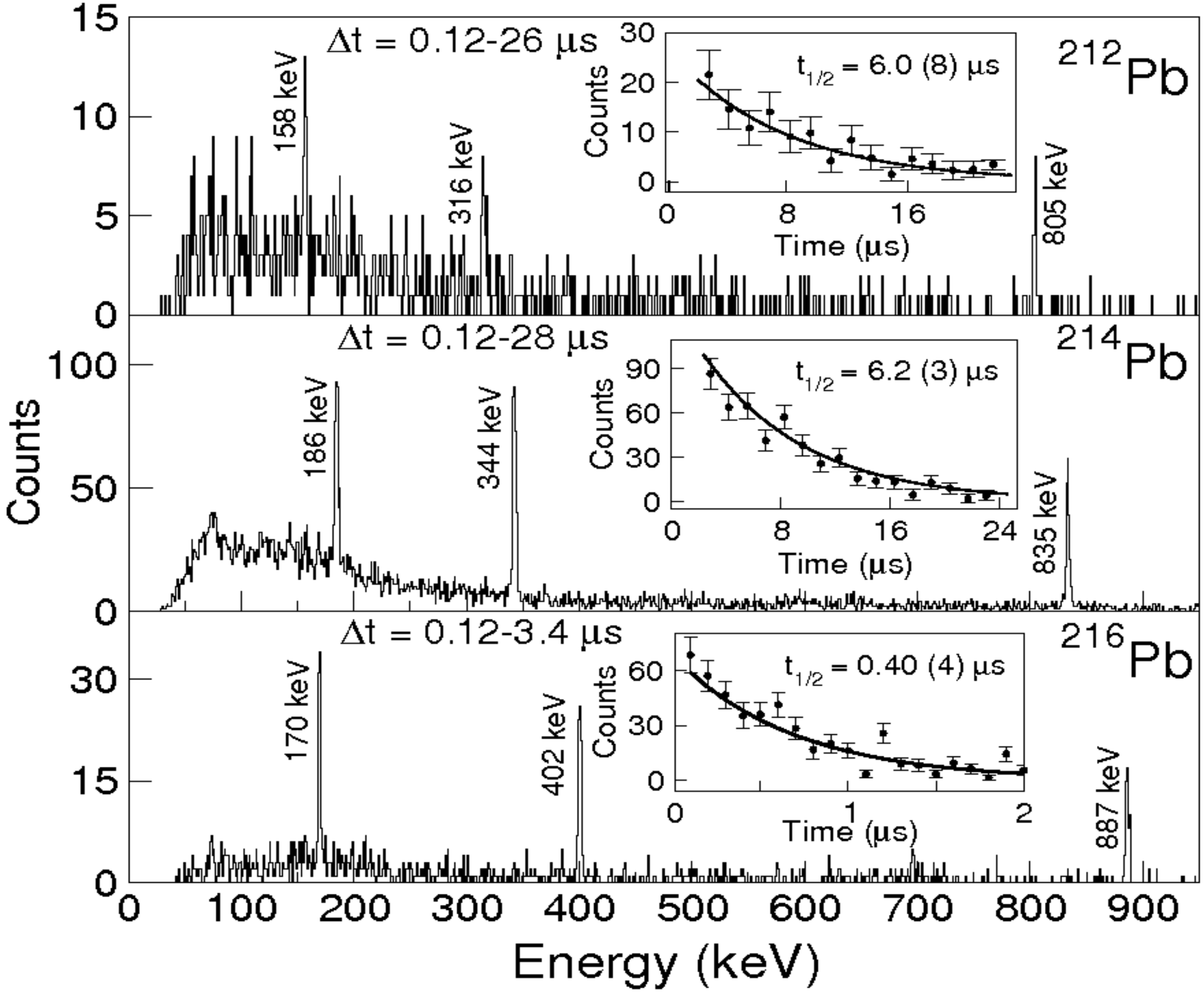


Example: neutron-rich lead isotopes – the $g_{9/2}$ orbital

The Z=82 and beyond N=126

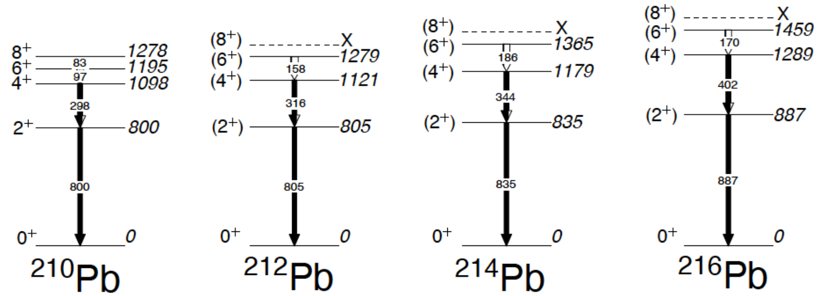


$^{212,214,216}Pb$: 8^+ isomer

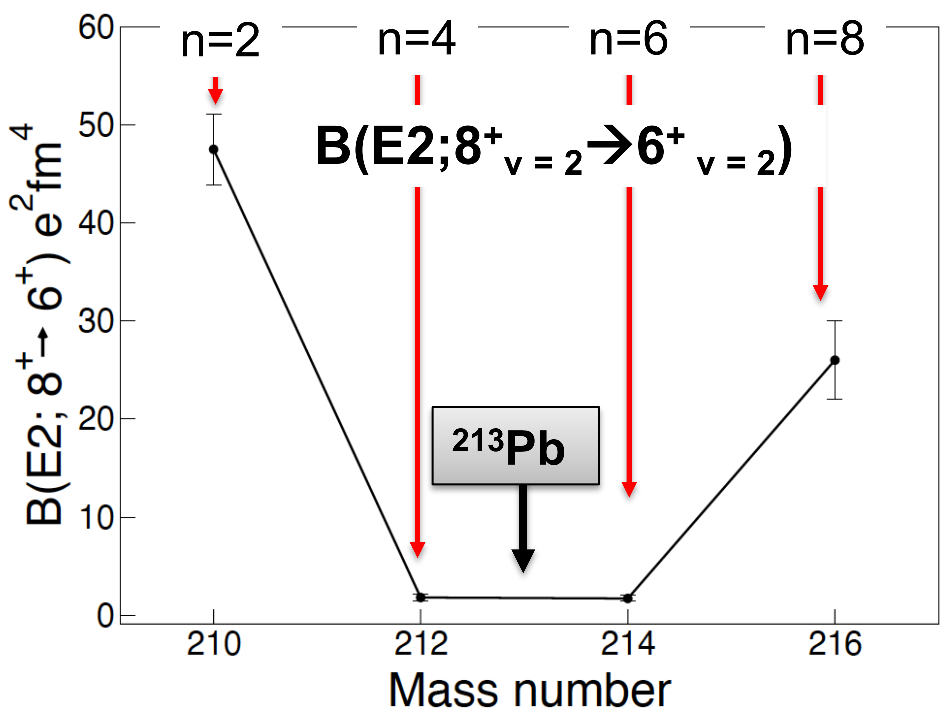


Reduced transition prob. B(E2)

B(E2) calculated considering internal conversion coefficients, and a 20-90 keV energy interval for unknown transitions.



$B(E2) \sim E_{\gamma}^{-5}\,(1+\alpha)^{-1}\tau^{-1}$



Filling of the $g_{9/2}$

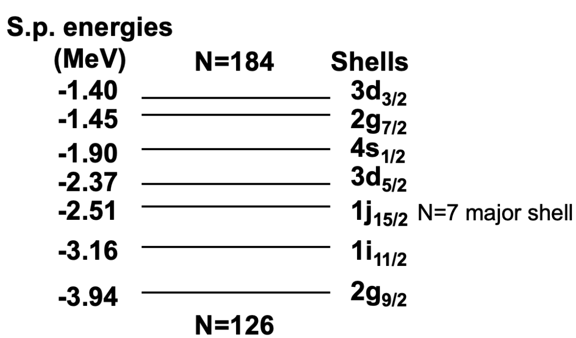
Shell Model calculations Kuo-Herling

Calculations with Antoine and Nathan codes and K-H interaction

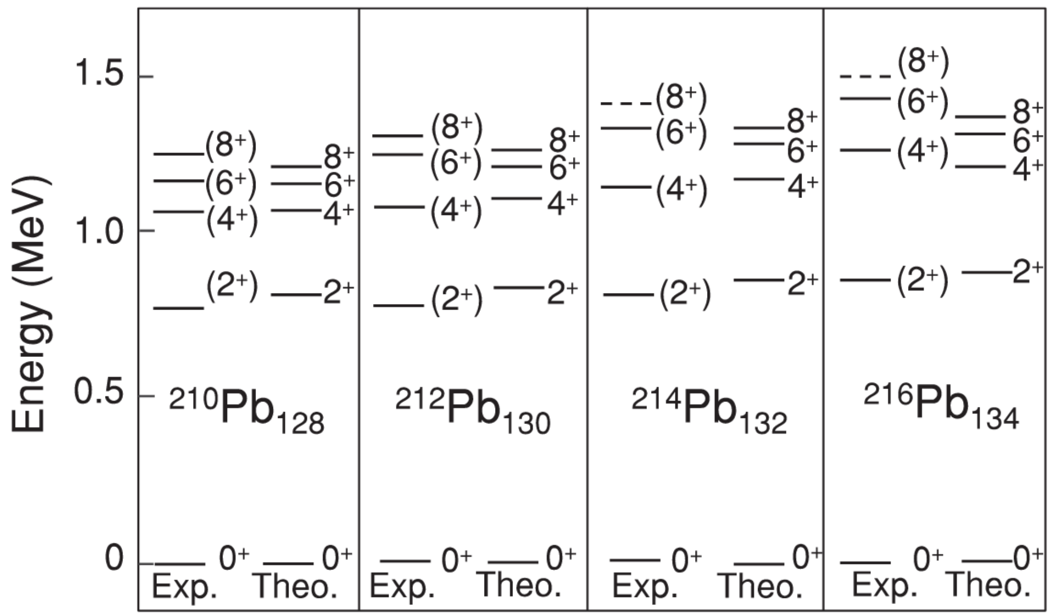
E.K. Warburton and B.A. Brown PRC43, 602 (1991).

^{208}Pb is the core (Z=82, N=126).

- For neutron-rich Lead isotopes, the N=6 major shell is involved



E.K. Warburton and B.A. Brown PRC43, 602 (1991).



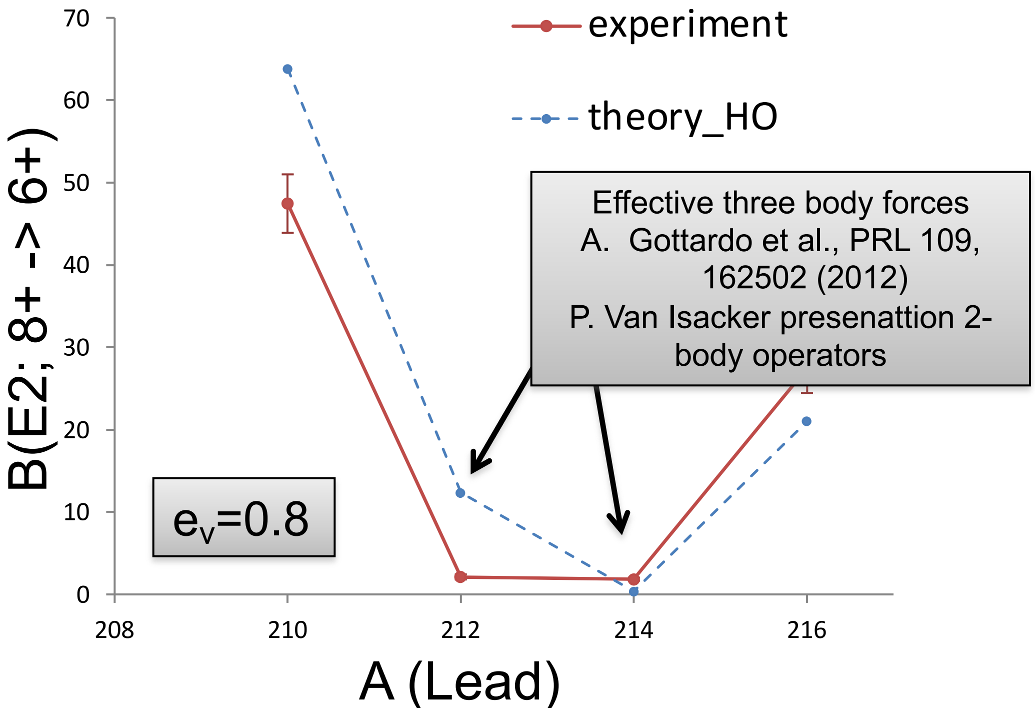
Reduced transition prob. B(E2)

B(E2) calculated considering internal conversion coefficients, and a 20-90 keV energy interval for unknown transitions.

	^{210}Pb	^{212}Pb	^{214}Pb	^{216}Pb
Isomer $t_{1/2}$ (μs)	**0.20 (2)**	**6.0 (8)**	**6.2 (3)**	**0.40 (4)**
B(E2) e^2fm^4 Exp.	**47(4)**	**1.8(3)**	**1.4-1.9**	**24.7-30.5**
B(E2) e^2fm^4 KH	**41**	**8**	**0.26**	**16.4**

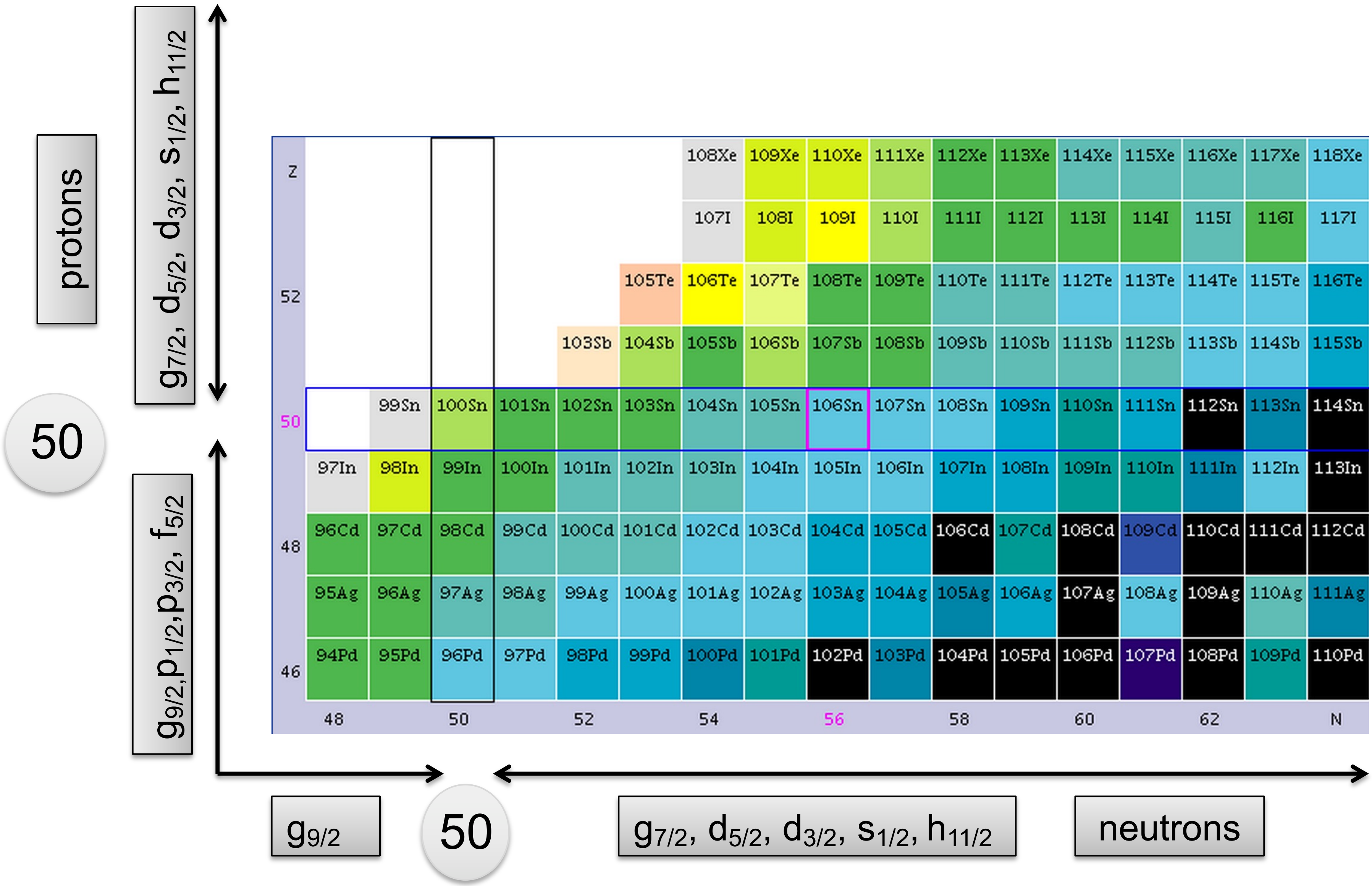
Upper limit 90 keV based on K_α X rays intensity (K electrons bound ~88 keV)

$$B(E2) \sim E_\gamma^{-5} (1+\alpha)^{-1} \tau^{-1}$$

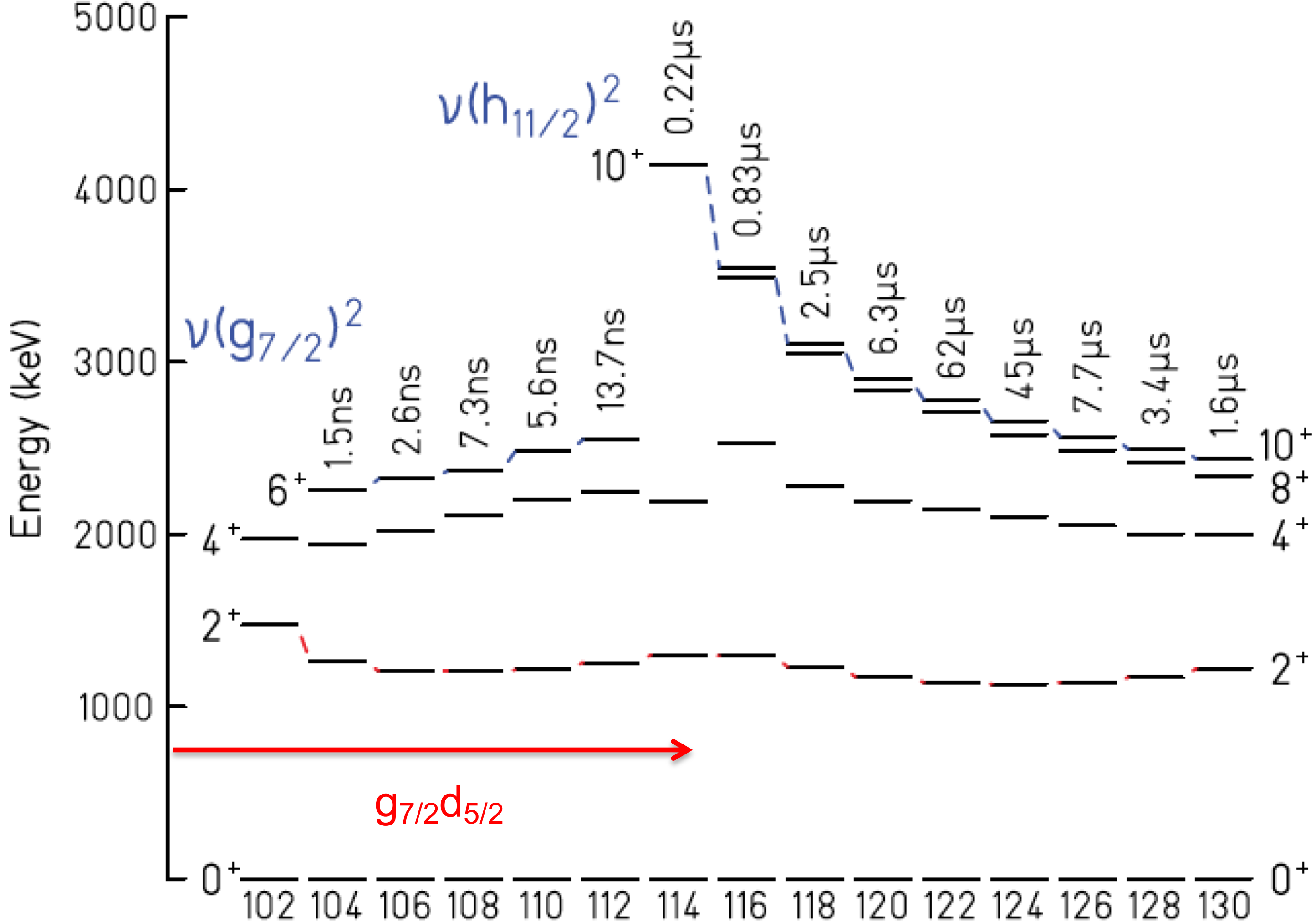


Caveat: neutron-deficient Sn isotopes

Light Sn isotopes



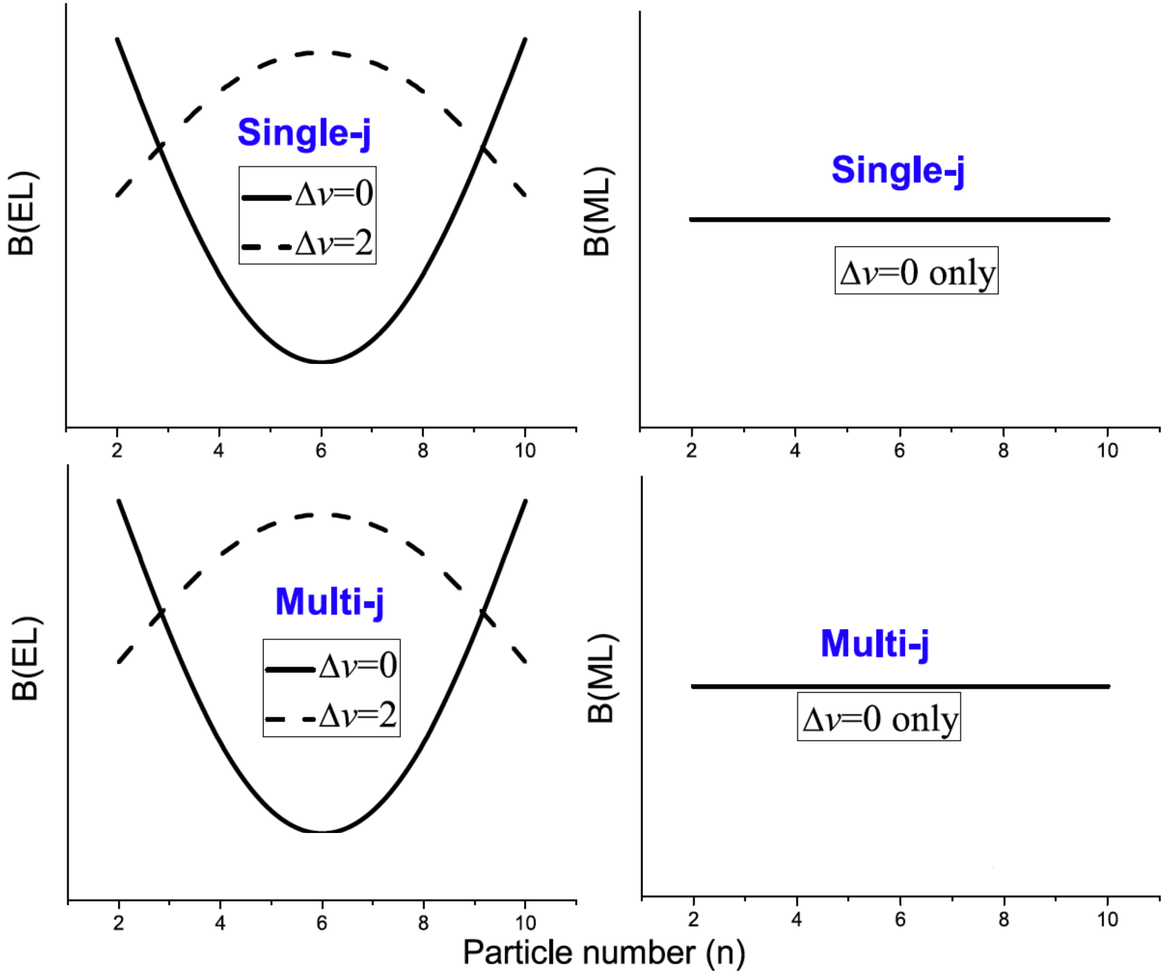
Excitation energies in Sn isotopes



R. Kumar et al., PRC **81**, 024306 (2010)

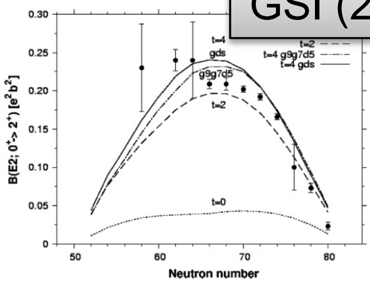
Generalized seniority scheme

Schematic variation of reduced transition probabilities for electric and magnetic transitions in both a single-j shell and a multi-j shell, respectively, by using the electromagnetic and seniority selection rules.



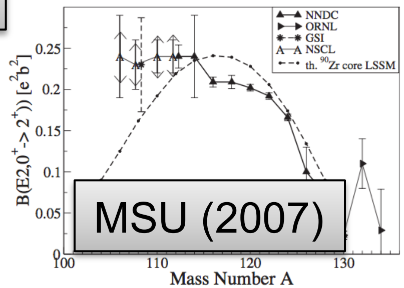
Many worldwide measurements

GSI (2005)



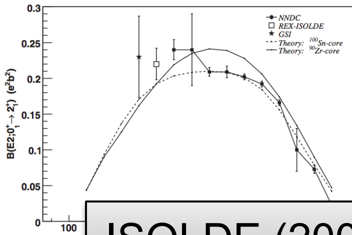
A. Banu et al., PRC **72**, 061305(R) (2005)

MSU (2007)



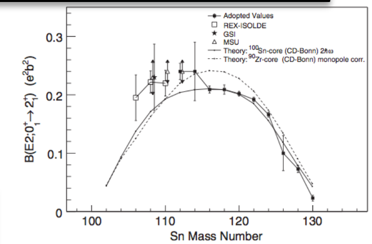
C. Vaman et al. PRL **99,** 162501 (2007)

ISOLDE (2007)



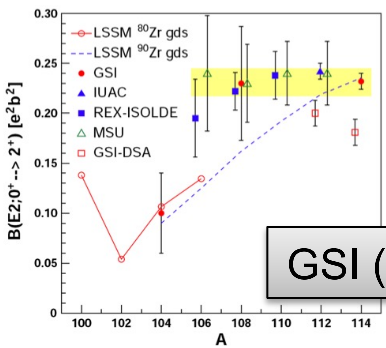
J. Cederkall et al., PRL 98 (2007) 172501.

ISOLDE (2008)



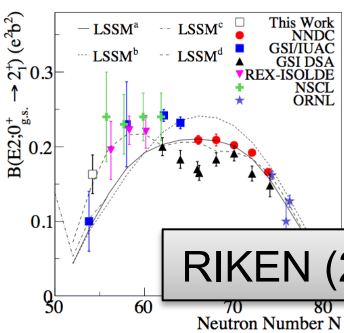
A.Ekstrom et al., PRL 101 (2008) 012502.

GSI (2013)



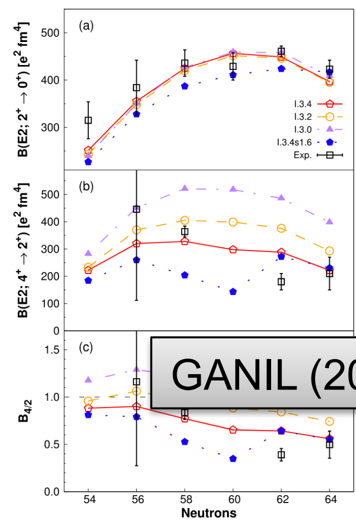
G. Guastalla et al., PRL 110 (2013) 172501.

RIKEN (2013)



P. Doornenbal et al., PRC90, 061302(R) (2014)

GANIL (2020)



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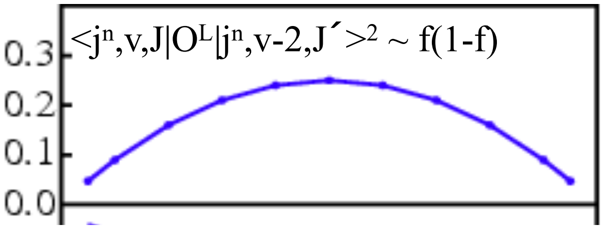
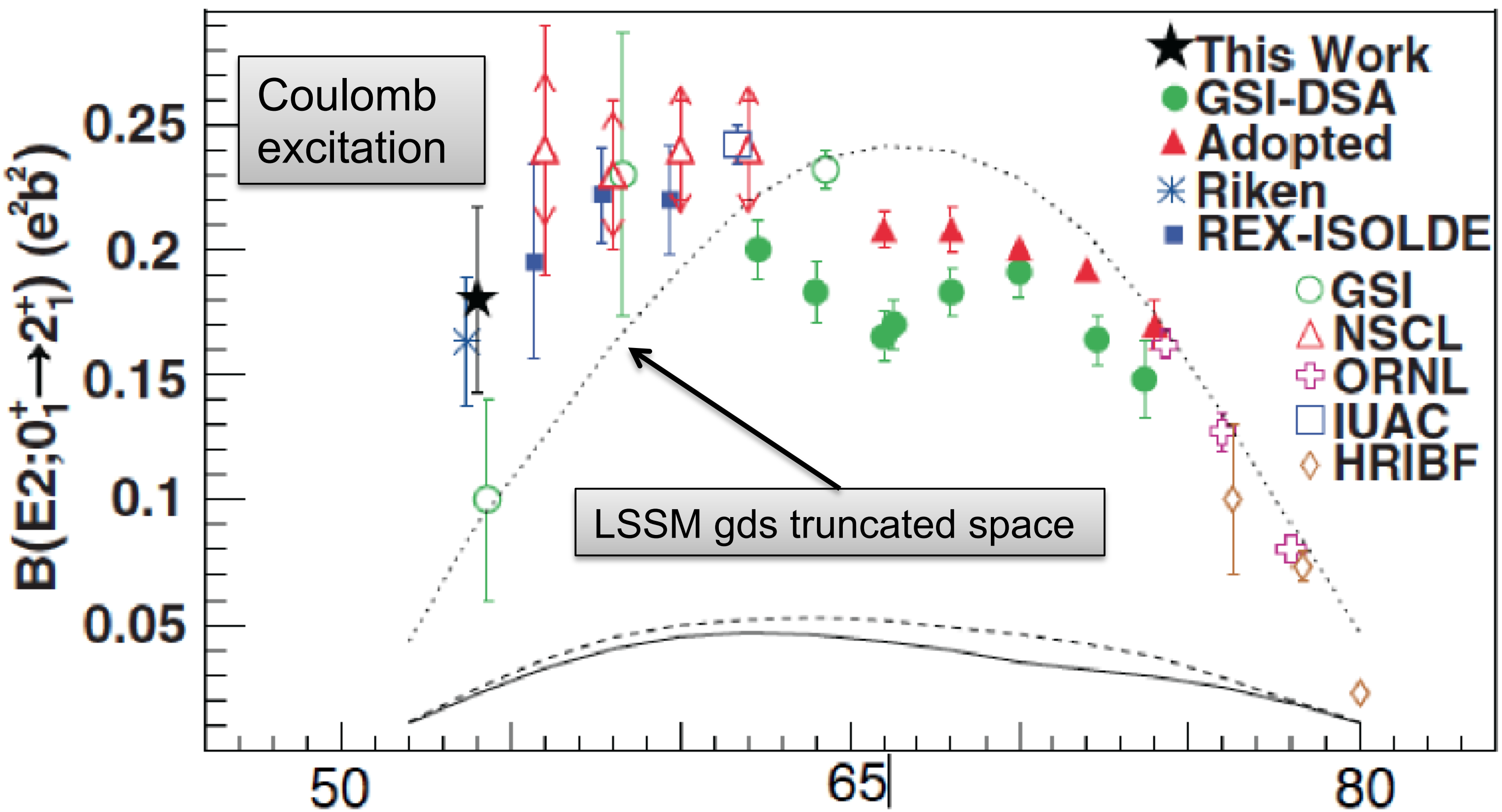
M. Siciliano et al., PLB806, 135474 (2020)

Coulomb excitation measurements
(safe or relativistic)

B(E2) transition probabilities in Sn





V.M. Bader et al., PRC **88**, 051301(R) (2013)

Proton excitations in Sn isotopes

PHYSICAL REVIEW C **72**, 061305(R) (2005)

^{108}Sn studied with intermediate-energy Coulomb excitation

A. Banu,[1,2,*] J. Gerl,[1] C. Fahlander,[3] M. Górska,[1] H. Grawe,[1] T. R. Saito,[1] H.-J. Wollersheim,[1] E. Caurier,[4] T. Engeland,[5] A. Gniady,[4] M. Hjorth-Jensen,[5] F. Nowacki,[4] T. Beck,[1] F. Becker,[1] P. Bednarczyk,[1,6] M. A. Bentley,[7] A. Bürger,[8] F. Cristancho,[3,†] G. de Angelis,[9] Zs. Dombrádi,[10] P. Doornenbal,[1,11] H. Geissel,[1] J. Grębosz,[1,6] G. Hammond,[12,‡] M. Hellström,[1,§] J. Jolie,[11] I. Kojouharov,[1] N. Kurz,[1] R. Lozeva,[1,||] S. Mandal,[1,¶] N. Mărginean,[9] S. Muralithar,[1,**] J. Nyberg,[13] J. Pochodzalla,[2] W. Prokopowicz,[1,6] P. Reiter,[11] D. Rudolph,[3] C. Rusu,[9] N. Saito,[1] H. Schaffner,[1] D. Sohler,[10] H. Weick,[1] C. Wheldon,[1,††] and M. Winkler[1]

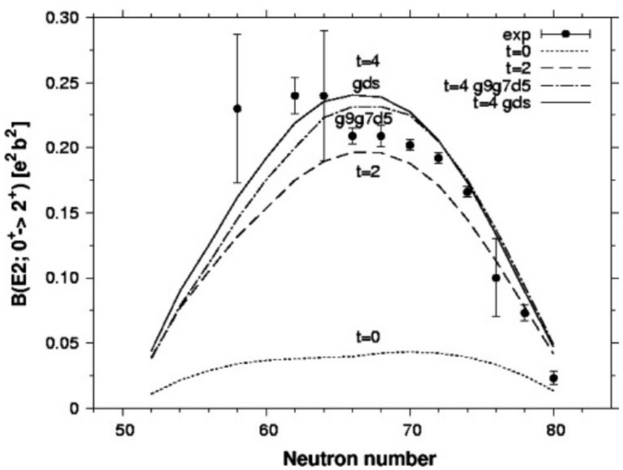
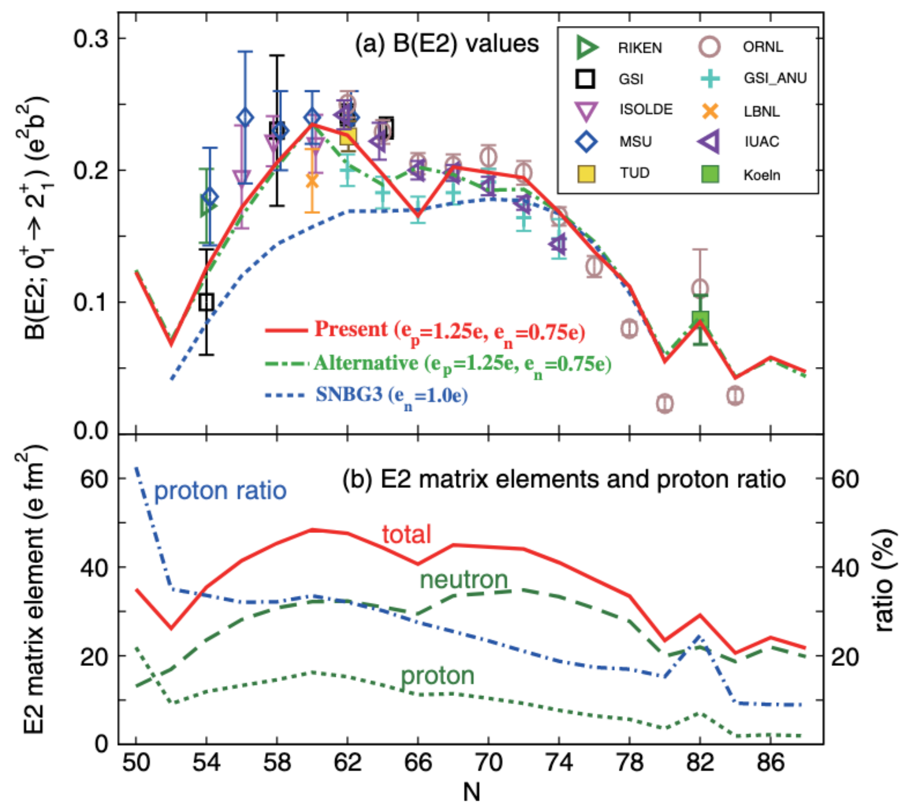


FIG. 3. Comparison of measured $B(E2\uparrow)$ values with LSSM predictions by taking into account ph core excitations. The $t = 0$ curve corresponds to calculations only for the valence neutrons as active particles. The $t = 2$ curve shows the major contribution as given by proton core excitations. The $t = 4$ curves are shown for the whole tin chain in a truncated proton model space and untruncated in the *gds* major shell.

PHYSICAL REVIEW LETTERS **121,** 062501 (2018)

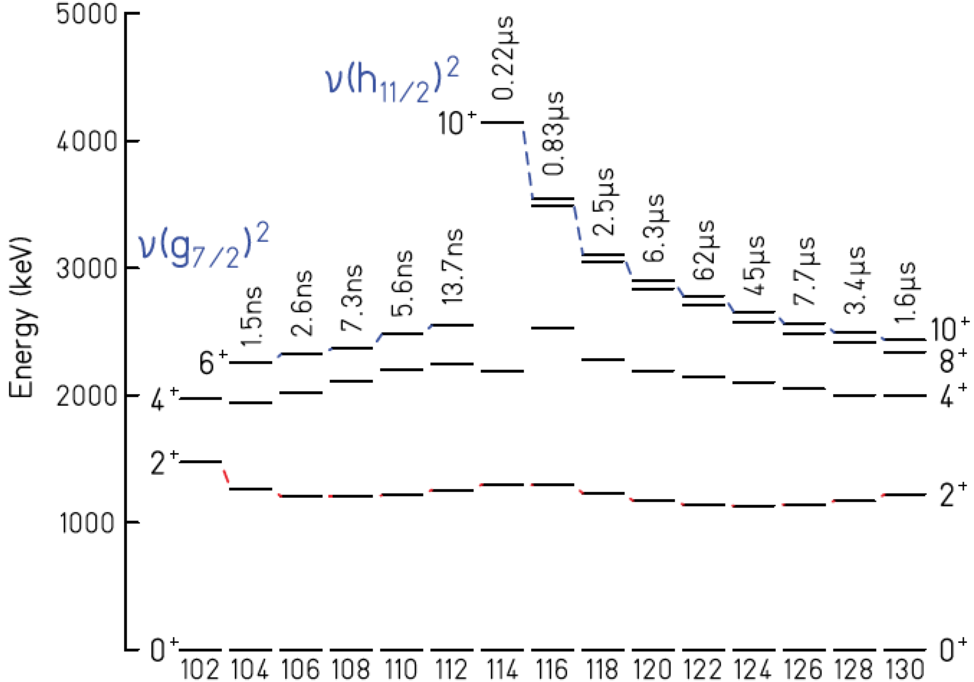
Novel Shape Evolution in Sn Isotopes from Magic Numbers 50 to 82

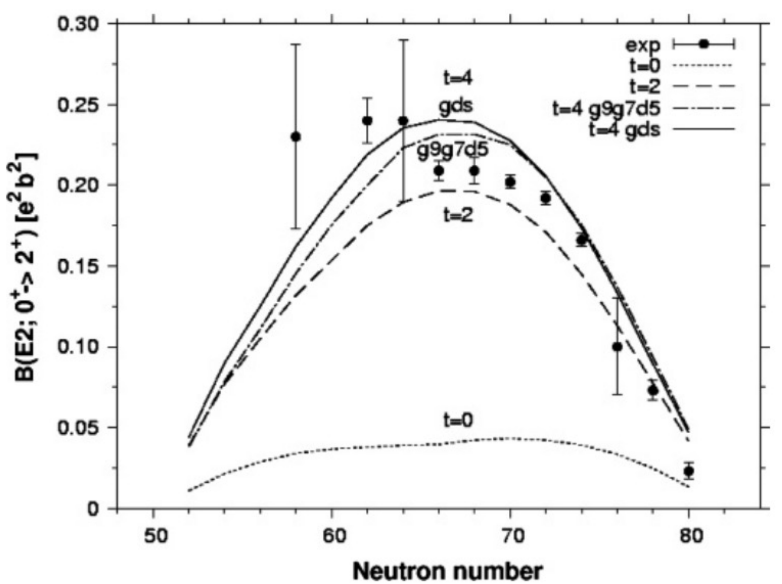
Tomoaki Togashi,[1] Yusuke Tsunoda,[1] Takaharu Otsuka,[2,1,3,4,5,*] Noritaka Shimizu,[1] and Michio Honma[6]



Caveat: seniority seeming like

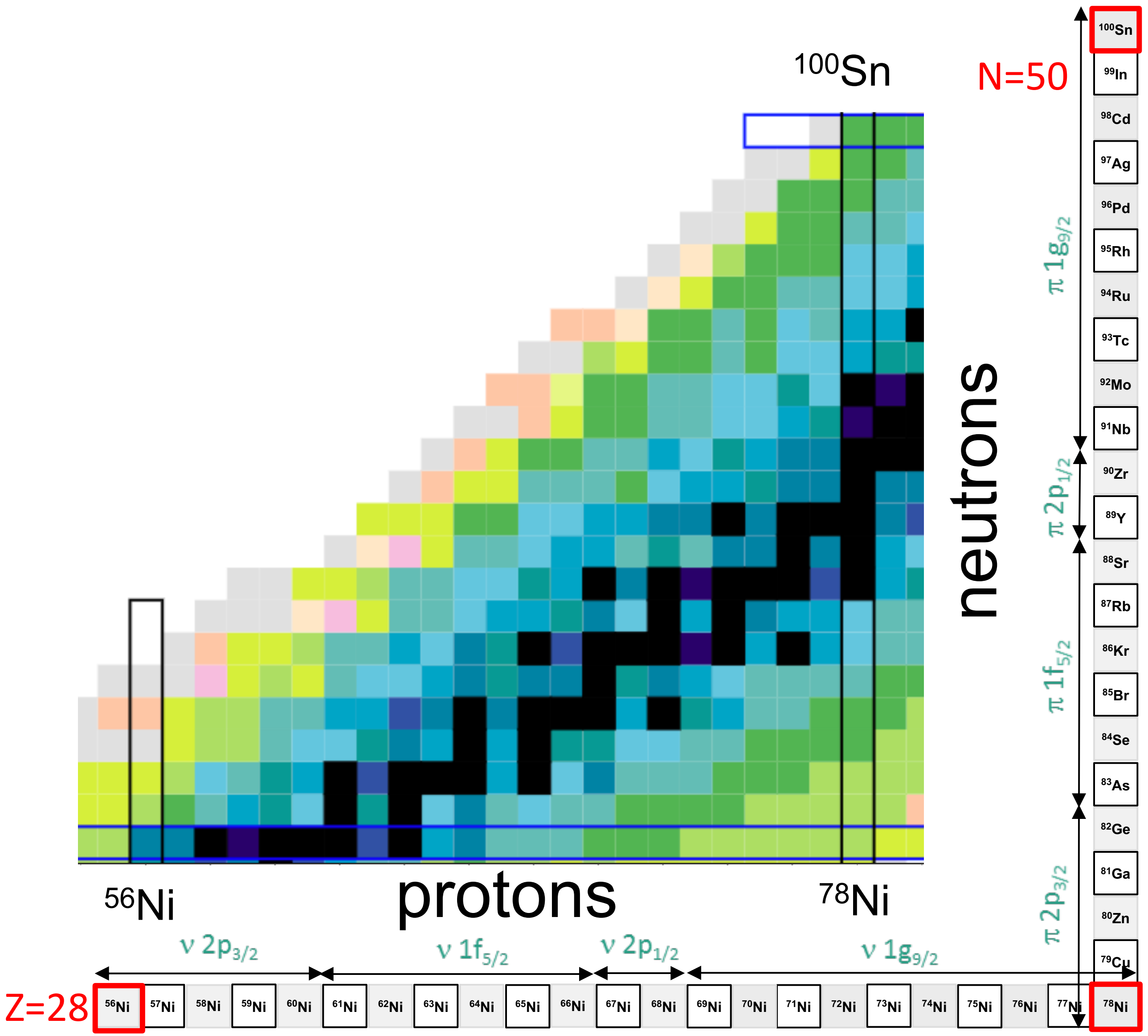
Even if energies seem rather constant 2^+
Even if B(E2: 2+ → 0+) transition probabilities seem to follow υ
→ we have protons and neutrons and these nuclei seem deformed so seniority can not be applied.



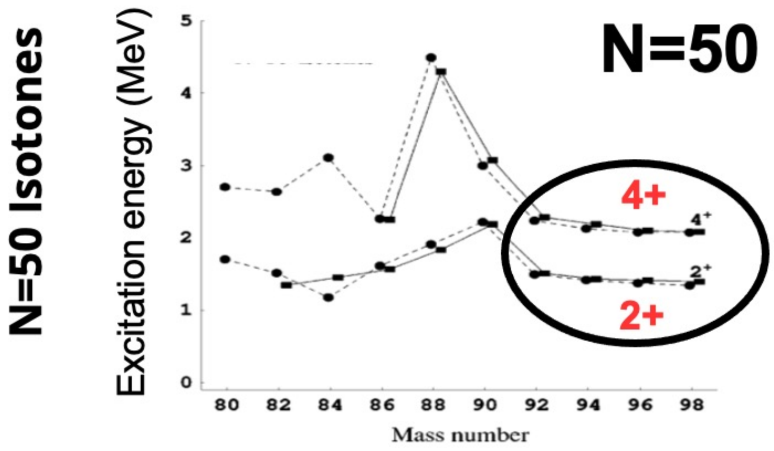


The N=50 isotones and the Z=28 isotopes

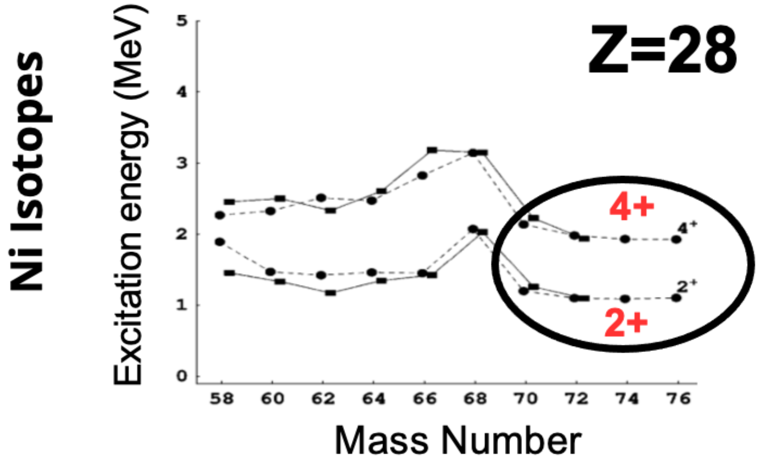
The first $g_{9/2}$ proton and neutron orbital



The first $g_{9/2}$ orbital



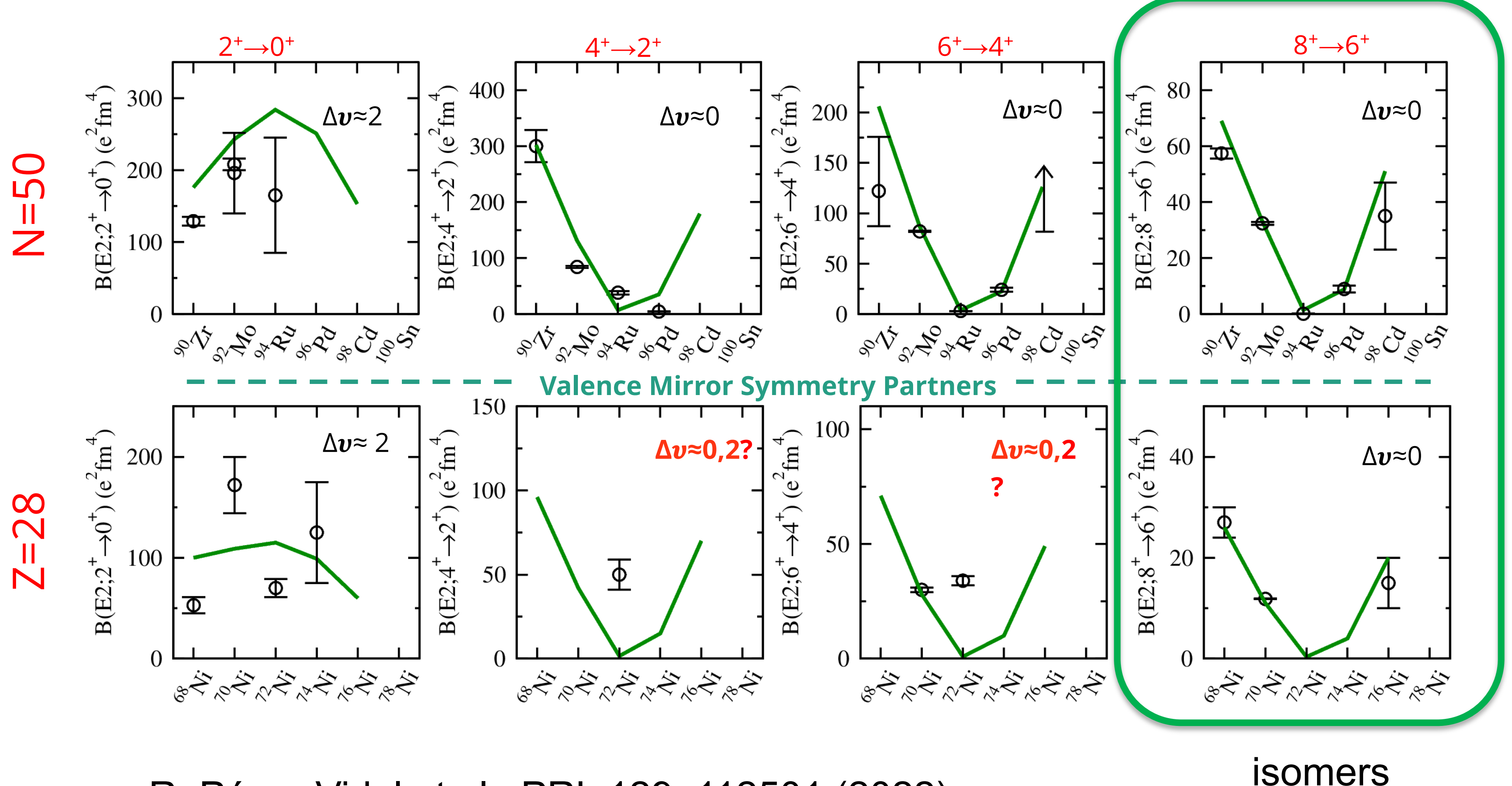
Valence Mirror Symmetry Partners



Seniority scheme in the $g_{9/2}$ shell

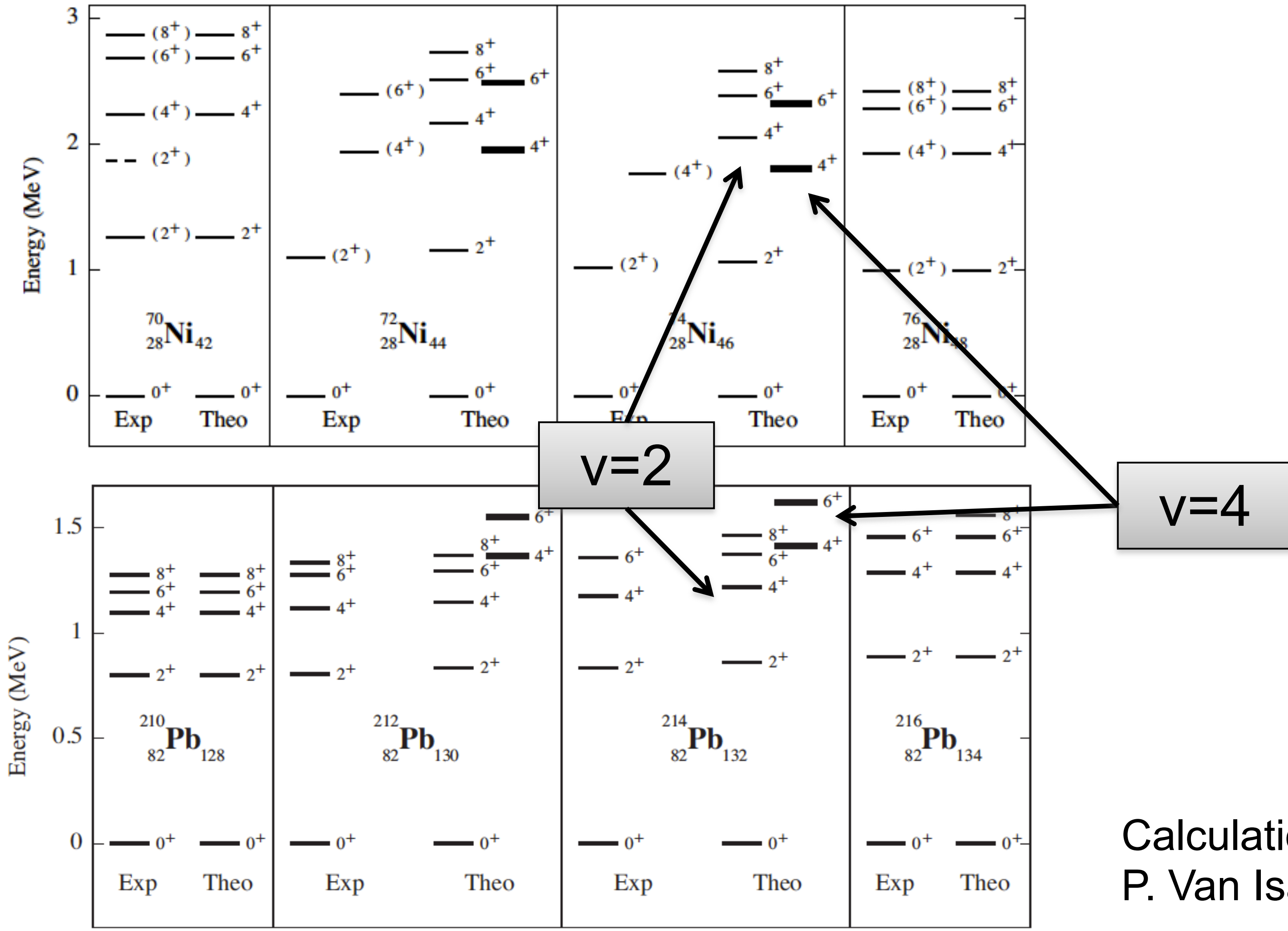
N=50 isotones R.M. Pérez-Vidal et al. Submitted to PRL
Z=28 isotopes[6] T. Marchi et al PRL 2014, K. Kolos et al PRL 2016, C.J. Chiara et al PRC 2015, A.I. Morales et al. PRL 2018, M. Sawicka et al PRC 2003, A.I. Morales et al. PRC 2016

Shell model icalculations for the Z=28 isotopes (N=50 isotones) performed in the 0f5/2, 1p3/2, 1p1/2, and 0g9/2 neutron (proton) model space and with the neutron (proton) effective charge derived from microscopic calculations. **A. Gargano and G. di Gregorio Private communcation.**



R. Pérez-Vidal et al., PRL 129, 112501 (2022)

Seniority Mixing

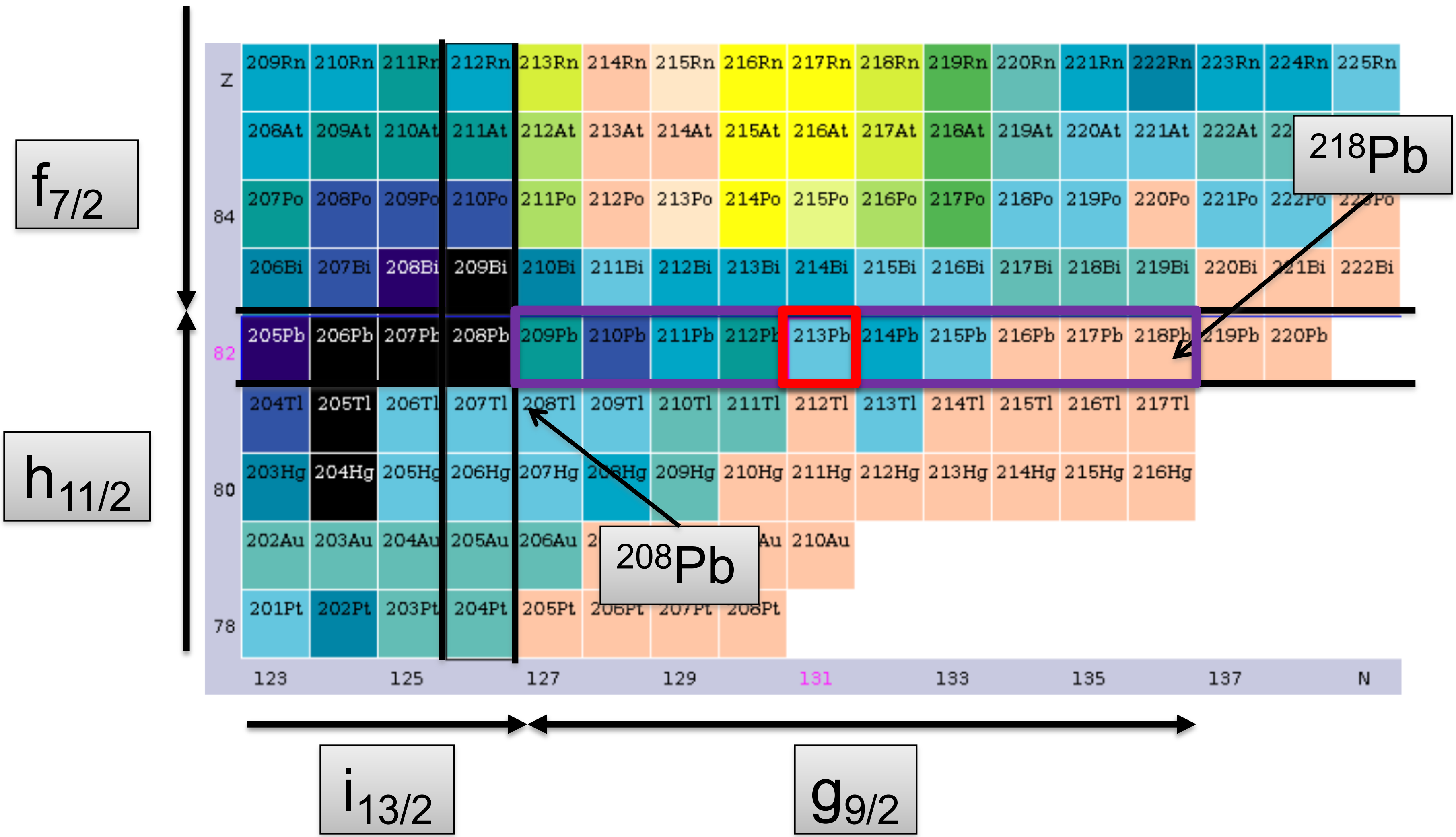


Calculations by
P. Van Isacker

In the middle of the $2g_{9/2}$ shell: ^{213}Pb

The conservation of seniority is a consequence of a geometric phase associated with particle-hole conjugation, which becomes observable in semi-magic nuclei where nucleons half-fill the valence shell.

The Z=82 and beyond N=126



Berry phase

Hamiltonian depends on a set of parameters ζ, then the **phase change of an eigenstate** of **H**(ζ) over a **closed path** in the parameter space is a gauge invariant quantity, and as such **observable**.

Proc. R. Soc. Lond. A **392**, 45–57 (1984)
Printed in Great Britain

Quantal phase factors accompanying adiabatic changes

By M. V. Berry, F.R.S.
H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol BS8 1TL, U.K.

(*Received* 13 *June* 1983)

A quantal system in an eigenstate, slowly transpor… by varying parameters $\boldsymbol{R}$ in its Hamiltonian $\hat{H}$… geo-metrical phase factor $\exp\{i\gamma(C)\}$ in additio… namical phase factor. An explicit general formula … erms of the spectrum and eigenstates of $\hat{H}(\boldsymbol{R})$ over… C. If C lies near a degeneracy of $\hat{H}$, $\gamma(C)$ takes a simple … ncludes as a special case the sign change of eigenfunctions of r… metric matrices round a degeneracy. As an illustration $\gamma(C)$ is calcu… ed for spinning particles in slowly-changing magnetic fields; although the sign reversal of spinors on rotation is a special case, the effect is predicted to occur for bosons as well as fermions, and a method for observing it is proposed. It is shown that the Aharonov–Bohm effect can be interpreted as a geometrical phase factor.

Optics, condensed matter, quantum electrodynamics, …

REVIEWS

Geometric phase from Aharonov–Bohm to Pancharatnam–Berry and beyond

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…t | Whenever a quantum system undergoes a cyclic evolution governed by a slow change …ters, it acquires a phase factor: the geometric phase. Its most common formulations are …e Aharonov–Bohm phase and the Pancharatnam and Berry phase, but both earlier …ifestations exist. Although traditionally attributed to the foundations of quantum … geometric phase has been generalized and become increasingly influential in … condensed-matter physics and optics to high-energy and particle physics …d mechanics to gravity and cosmology. Interestingly, the geometric phase also …unique opportunities for quantum information and computation. In this Review, we first …troduce the Aharonov–Bohm effect as an important realization of the geometric phase. Then, we discuss in detail the broader meaning, consequences and realizations of the geometric phase, emphasizing the most important mathematical methods and experimental techniques used in the study of the geometric phase, in particular those related to recent works in optics and condensed-matter physics.

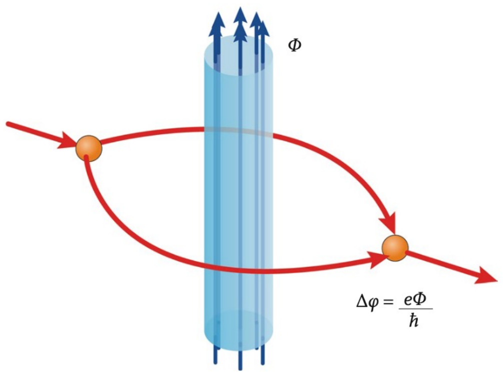


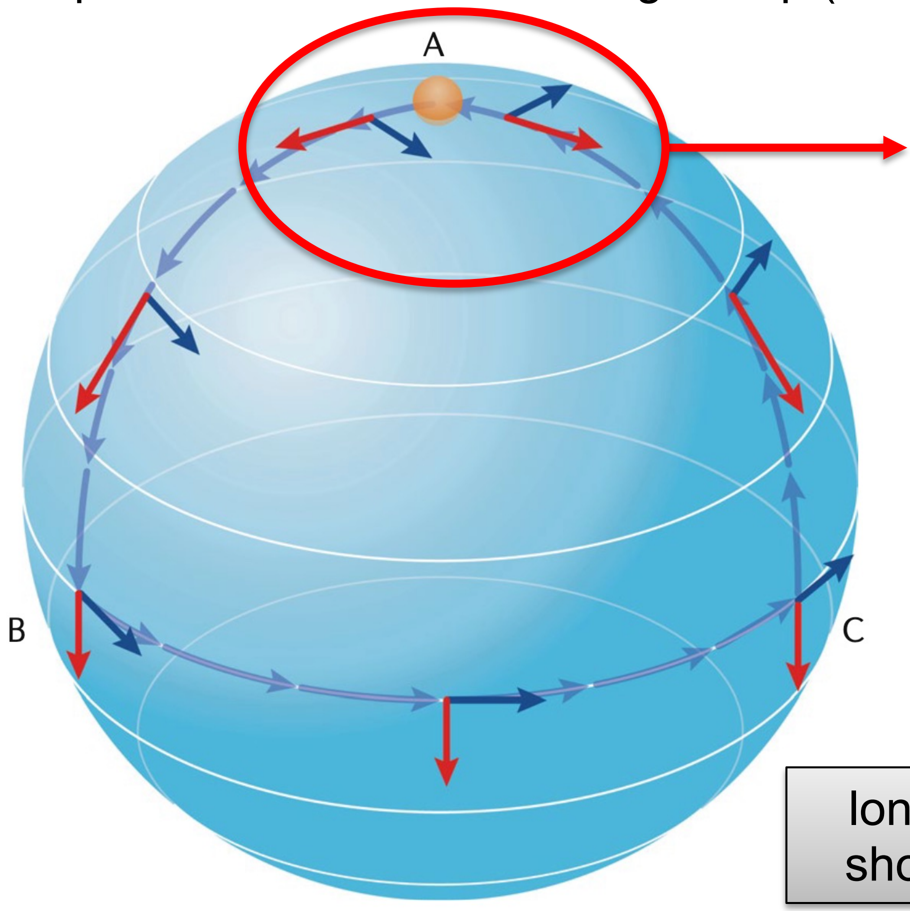
Fig. 1 | **The Aharonov–Bohm effect.** An electron is encircling a magnetic flux Φ (vertical blue arrows) confined to a thin, long solenoid. Although the magnetic field is zero in the vicinity of the superposed wavepackets, the vector potential is non-zero outside the solenoid. Thus, the electronic wavepackets acquire a relative phase of $\exp(ie\Phi/\hbar)$, which causes their interference pattern to change.

The Berry phase

An intuitive classic example of transport

B. Goss Levi, Phys. Today **46**, 17 (1993)

Displacement of a vector along a loop (closed path) drawn on a sphere



the vectors rotate

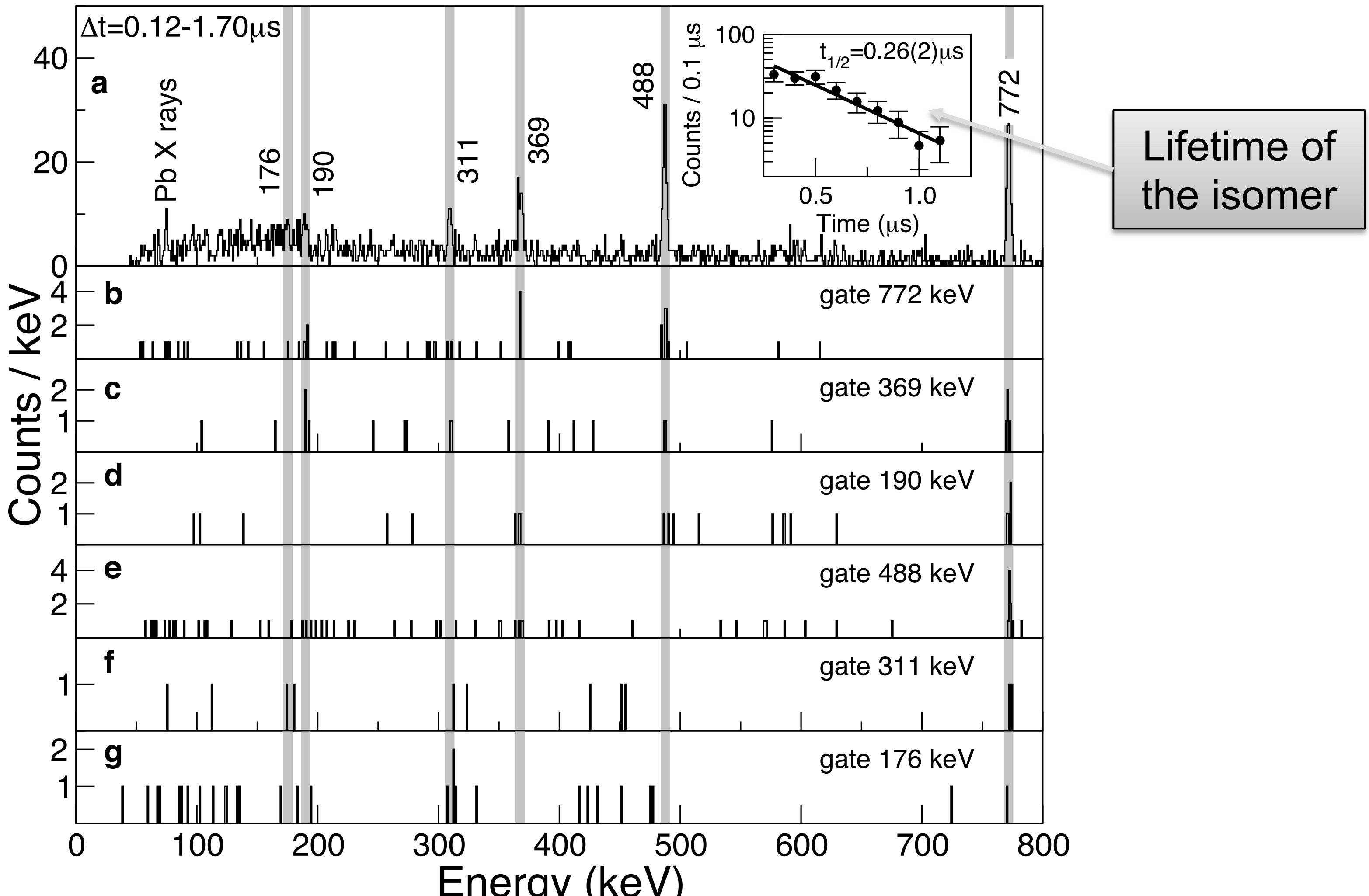
- here 90 degrees, but
- it will depend on the path

Geometric phase affects all areas of physics

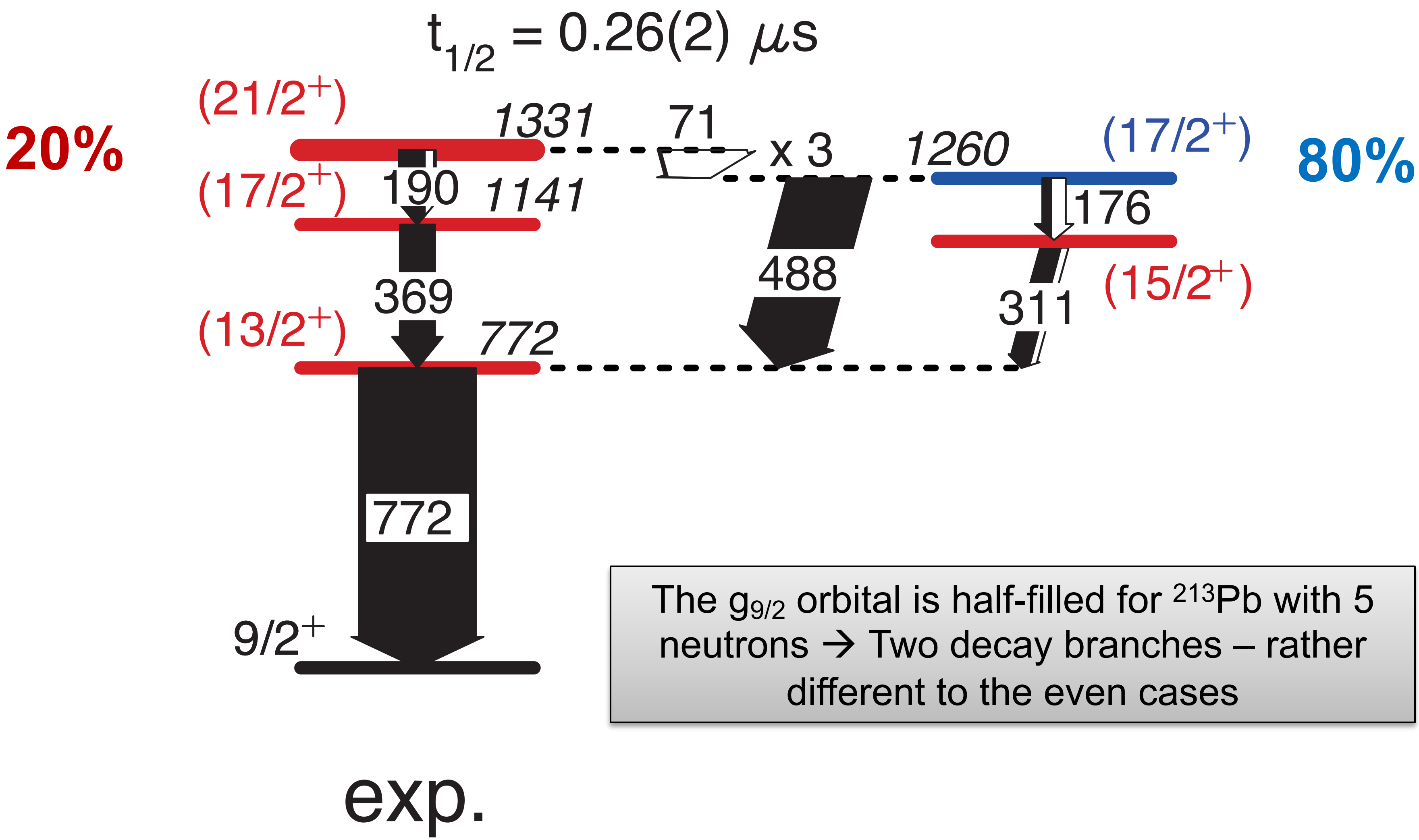
long vector points "South" and the short one points "East" all the time

Gamma spectrum of ^{213}Pb

spectrum in delayed coincidence with about 2100 ^{213}Pb ions

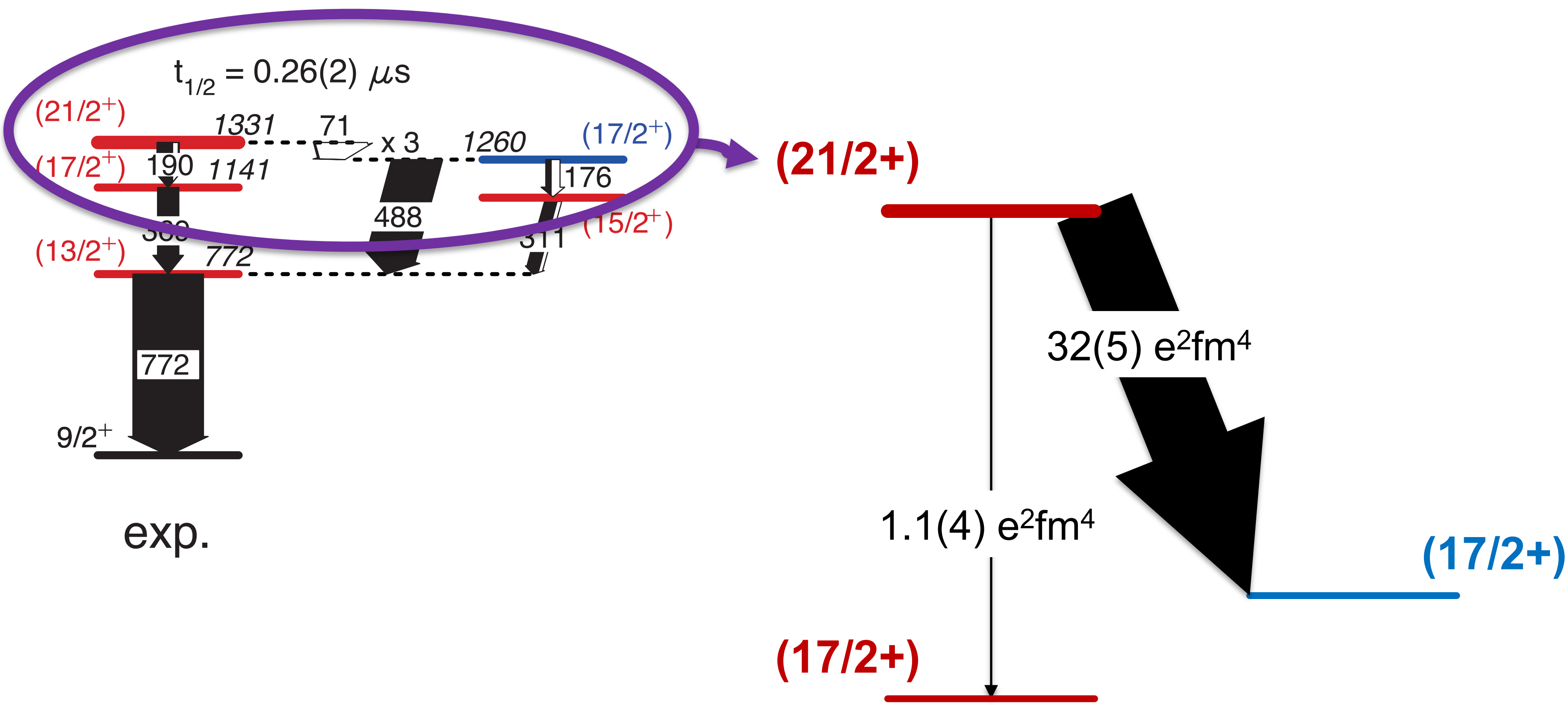


Level scheme of ^{213}Pb



The $g_{9/2}$ orbital is half-filled for ^{213}Pb with 5 neutrons → Two decay branches – rather different to the even cases

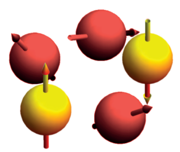
Reduced transition probabilities ^{213}Pb



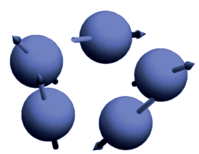
SM calculations ^{213}Pb

Although the $g_{9/2}$ shell is not isolated in energy, it is found to carry the dominant component of the wave function

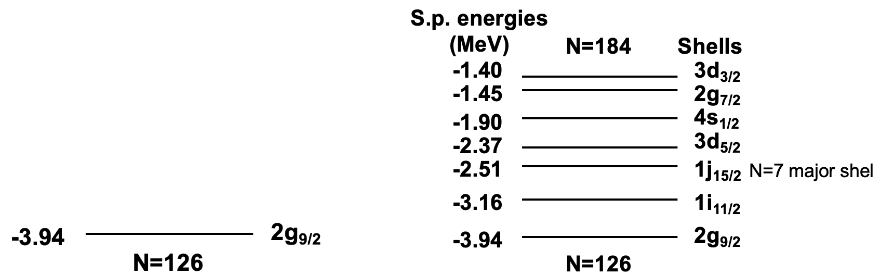
υ=3

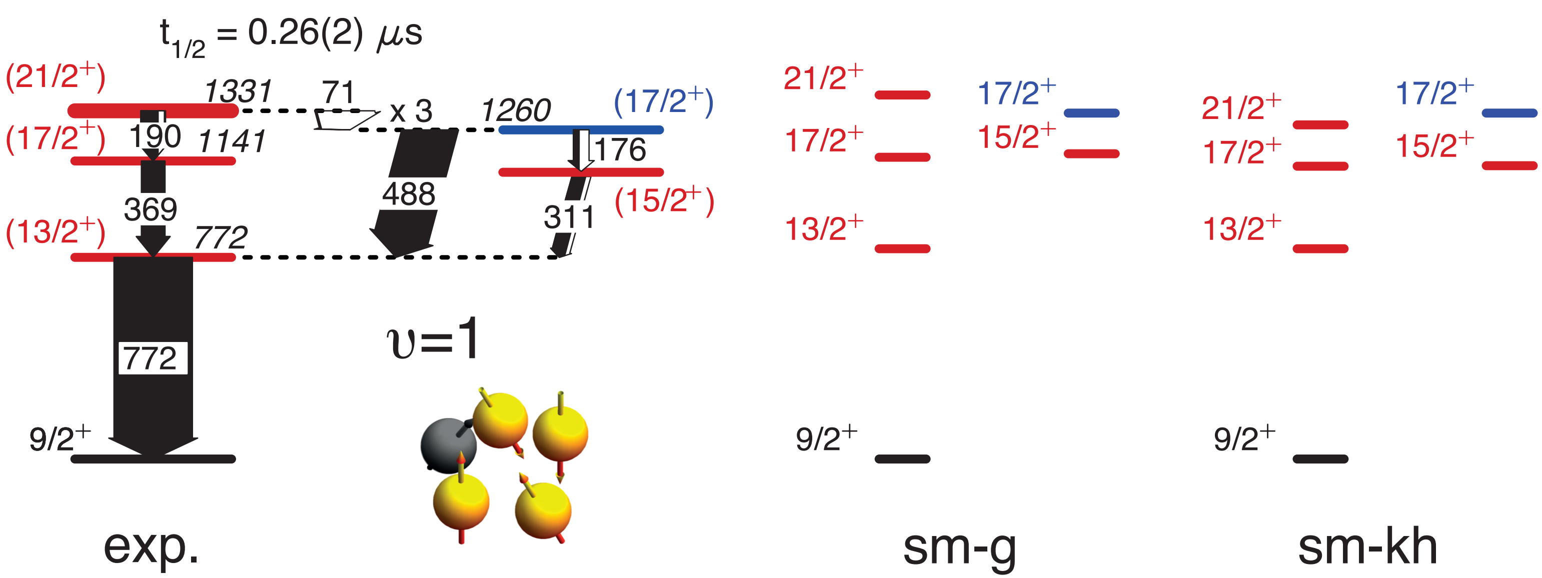


υ=5



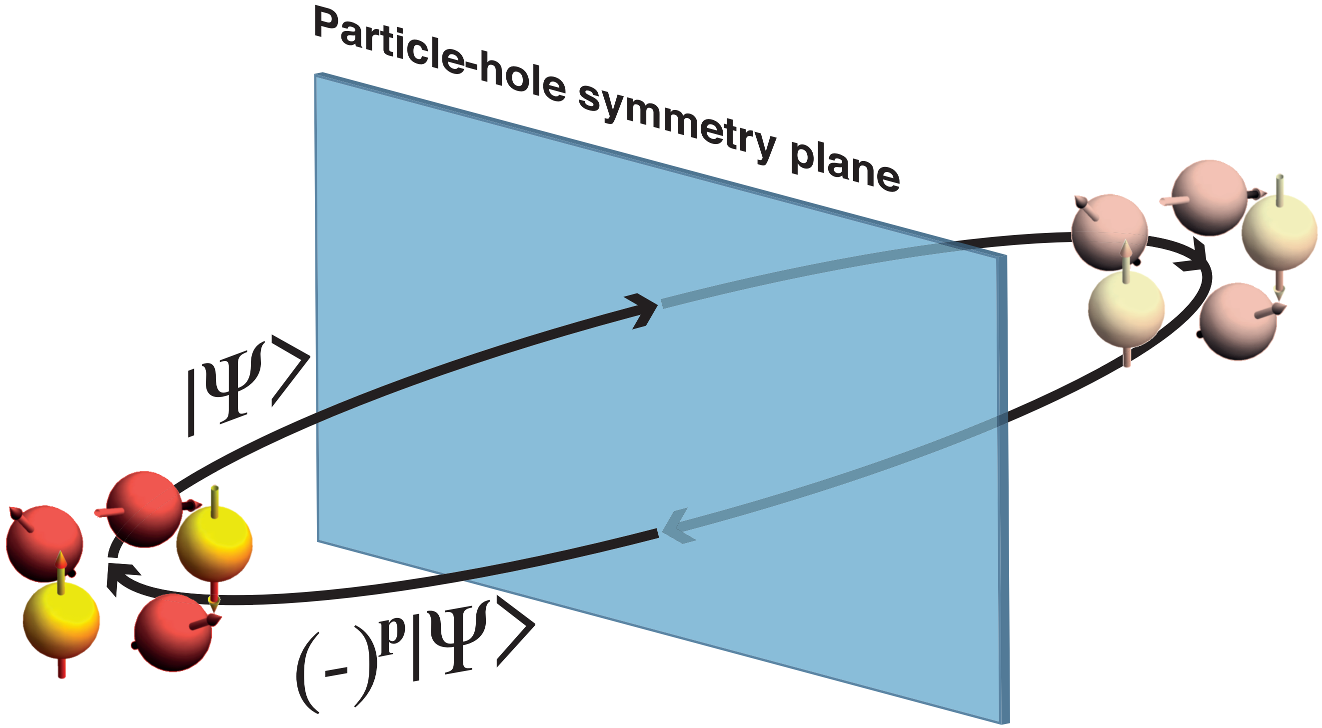
SM calculations: valence space





Berry phase

In the middle of the $g_{9/2}$ shell → a **particle-hole transformation** transforms the nucleus in **itself** but with a **phase**

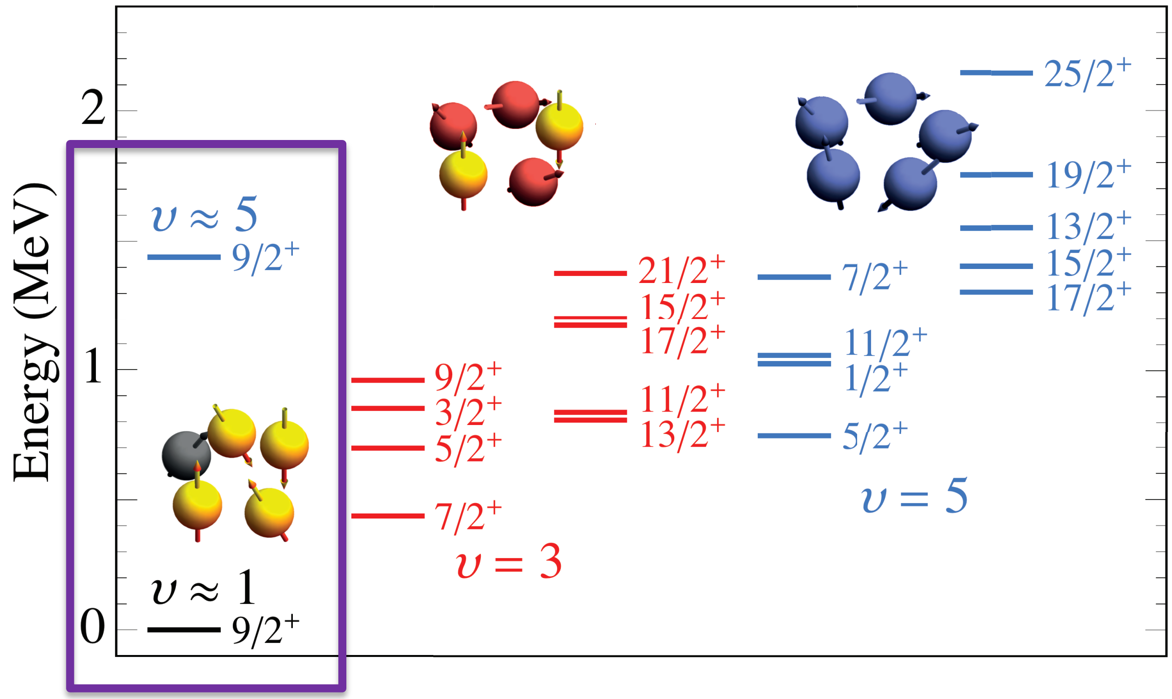


Original and transformed state is the same besides a phase $(-)^p$

R. D. Lawson, Theory of the Nuclear Shell Model (Clarendon Press, Oxford, 1980).

Manifestation of the Berry phase

For any two-body interaction, of all possible $g_{9/2}^5$ states only two can mix, namely those with J = 9/2⁺ and seniorities υ = 1 and υ = 5.

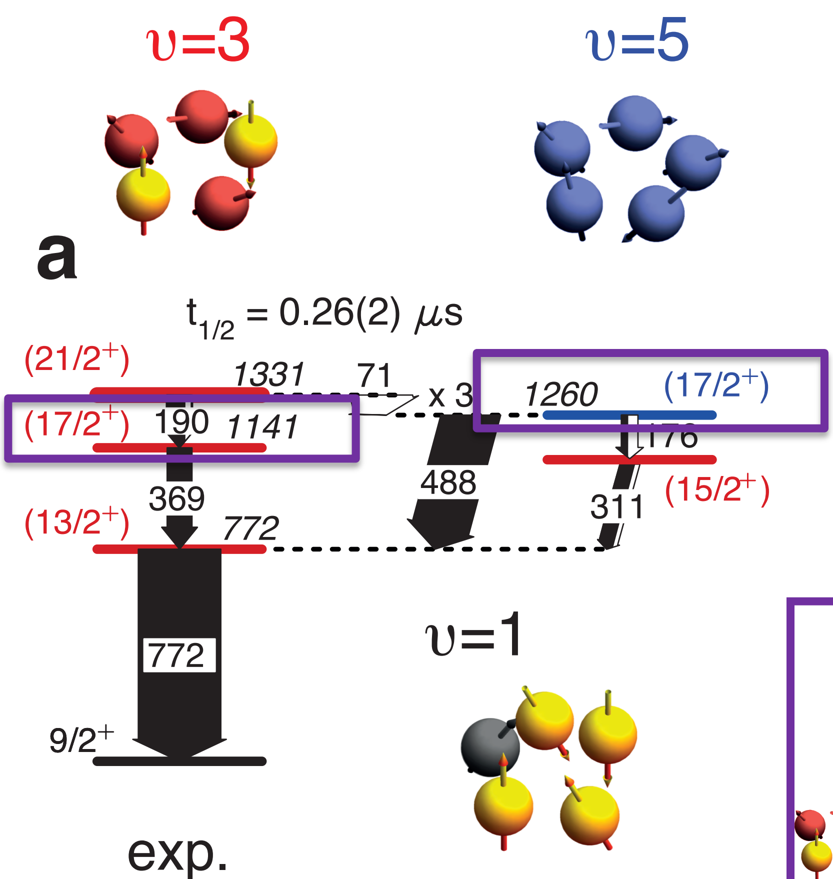


As a result, each $J = 0$ pair in an n-particle state induces a minus sign under particle–hole conjugation such that

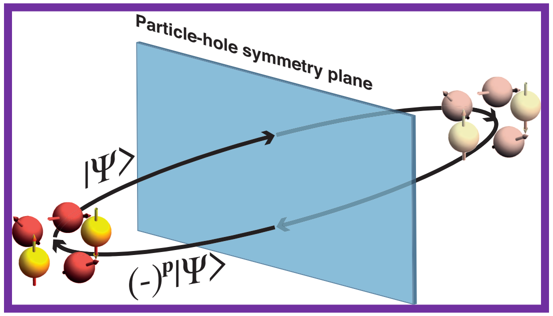
$$\Gamma|j^n \upsilon J_t\rangle = (-)^{(n-\upsilon)/2}|j^{2j+1-n}\upsilon J_t\rangle,$$

$$\langle j^n \upsilon J_t|\hat{V}|j^n \upsilon' J_t\rangle = (-)^{(\upsilon-\upsilon')/2}\langle j^n \upsilon J_t|\hat{V}|j^n \upsilon' J_t\rangle,$$

Manifestation of the Berry phase



- The E2 transitions satisfy the selection rule Δυ = +/- 2
- The purity of seniority in the $17/2^+$ states follows from the self-conjugate character of ^{213}Pb
- **The particle-hole symmetry prevents Δυ = 2 mixing as a consequence of an observable Berry phase.**



The Berry phase gives us a link between seniority conservation and universal quantum mechanical features.

Summary

- Seniority is conserved by any two-body interaction in the j^n configuration for $j \leq 7/2$
- For j >7/2 seniority is not necessarily conserved.
- Seniority can be a good description of actual nuclei when we have no proton-neutron interaction → deformed nuclei. Caveat Sn isotopes.
- Examples in neutron-rich Pb and Ni and N=50 isotones
- Apperance of a geometrical phase associated to particle hole conjugation → understanding of seniority selection rules. The Berry phase gives us a link between seniority conservation and universal quantum mechanical features.

Future perspectives

- Need accurate measurements of the transitions probabilities of the states below the isomers. We need systematics.
  - Lifetimes (RDDS or fast timing)
  - Coulex

- Going beyond j=9/2 shell, e.g. j=11/2, …

- Other mid-shell cases to look into the Berry phase:
  - ^{73}Ni with five neutrons in $0g_{9/2}$
  - ^{95}Rh N=50 mid-shell in $0g_{9/2}$
  - ^{213}Fr with five protons in $0h_{9/2}$

- How to further look into seniority: Transfer reactions: complications with the reaction dynamics …

END