Moduli spaces: theory and coding

lundi 27 février 2023 - vendredi 3 mars 2023

Hotel Les Sources, Les Diablerets (CH)

Programme Scientifique

admcycles workshop
Johannes Schmitt
admcycles – the old and the new

Thomas Wennink
Computing the double ramification cycle with target variety

Henry Liu
Combinatorics of K-theoretic relative stable maps

Kai Hugtenburg
Gromov–Witten invariants of the intersection of quadrics in $\mathbb{P}^5$ via an R-matrix

Samir Canning
Vector bundles in admcycles
Moduli spaces conference

Luca Battistella
Curves of genus two in projective space
Much of classical algebraic geometry concerns embeddings of curves in projective space. Kontsevich's space of stable maps provides a compactification well-suited for enumerative geometry. Yet, in positive genus, the locus of smooth curves is usually not dense. I describe a smooth modular compactification in genus two that I have constructed with F. Carocci by harnessing the geometry of Gorenstein singularities and logarithmic techniques.

Xavier Blot
Hierarchies of meromorphic differentials
The goal of this talk is to present a new construction of integrable hierarchies based on the strata of meromorphic differentials. We will focus on the classical case for the trivial CohFT. We will explain how these hierarchies coincide with the DR hierarchies, resulting in a surprising equality between intersection numbers on the DR cycle and on the strata of meromorphic differentials. This is a work in progress with P. Rossi.

Alexandr Buryak
A proof of a noncommutative analog of Witten's conjecture
An explicit formula for the double ramification cycle on the moduli space of curves was proposed by Pixton several years ago and soon proved by Janda–Pandharipande–Pixton–Zvonkine. Actually, Pixton's formula gives a nonhomogeneous deformation of the double ramification cycle. In a joint work with P. Rossi we proved that the intersection numbers with this deformed class gives a solution of a noncommutative KdV hierarchy defined on the space of functions in two variables with the usual product replaced by the Moyal star-product. I will try to explain the main ideas of our proof.

Samir Canning
Pixton's conjecture, the Gorenstein question, and moduli of abelian varieties
I will explain some work in progress that reduces Pixton's conjecture for the moduli space of genus 6 curves of compact type to a calculation in admcycles. Then, I will turn to applications of this calculation to the Gorenstein question for the tautological ring of the moduli space of compact type

curves of arbitrary genus and number of marked points. This first part is joint with H. Larson and J. Schmitt. Finally, I will explain a curious and striking connection between the results in the first part of the talk and a seemingly unrelated question about tautological classes on moduli spaces of principally polarised abelian varieties, which again makes significant use of admcycles. This part is joint with D. Oprea and R. Pandharipande.

Francesca Carocci

Rubber tori and the boundary of expanded stable maps

Extending and generalising Jun Li's original approach to define relative GW invariants, Ranganathan constructs moduli spaces of (log) expanded stable maps. These spaces parametrise transverse stable maps to certain target expansions. In this talk, I will start by describing the geometry of the expansions that can appear as targets in the moduli space of expanded maps. I will then explain the identification of maps in the boundary induced by the action of rubber tori on the "higher levels" of the expanded target; such action can be satisfyingly described at the tropical level. In particular, I will explain the difficulties in obtaining a recursive description for the boundary of the moduli space of expanded maps. This is based on joint works with N. Nabijou.

Nitin Chidambaram

On the BGW tau function

One of the best studied KdV tau-functions is the so-called Kontsevich–Witten tau-function. In 1990, Witten conjectured that this tau-function has an algebro-geometric interpretation as the generating function of ψ-class intersections on the moduli space of curves. This conjecture was proved in 1991 by Kontsevich using matrix-model techniques. Another very well-known tau function (studied since the 90s) is the Brezin–Gross–Witten tau-function and it exhibits many properties analogous to the Kontsevich–Witten tau function. Up until recently however, the BGW tau function lacked an algebro-geometric interpretation. In 2017, Norbury conjectured that this tau-function is the descendant potential of a certain CohFT called the Theta class. In this talk, I'll explain the construction and properties of the Theta and talk about a proof of Norbury's conjecture. This is based on joint work with E. Garcia-Failde and A. Giacchetto.

Matteo Costantini

Chern classes of spaces of k-differentials and applications to ball quotients

Linear submanifolds are the most interesting and well-studied subvarieties of moduli spaces of abelian differentials. We present a formula for the Chern classes of their closure inside the compactification by multi-scale differentials. In particular, we compute a formula for the Chern classes of strata of k-differentials and apply it to test for Deligne–Mostow-type ball quotients. This is a joint work with M. Möller and J. Schwab.

David Holmes

Coefficients of higher powers of r in Chiodo classes

Let C/S be a family of curves, and L on C a line bundle. The double ramification (DR) cycle measures the locus $D \subset S$ over which L is fibrewise trivial. To say the same thing another way, the restriction of L to $C \times_S D$ is a pullback of some line bundle L' on D. Writing Z for the first Chern class of L', we can consider a sequence of cycles $\mathrm{DR}^0, \mathrm{DR}^1, \mathrm{DR}^2, \dots$ on the moduli space of curves, defined by taking the product $\mathrm{DR}^i := Z^i \mathrm{DR}$. For $i=0$ this recovers the usual DR cycle, for which a formula can be written by taking the coefficient of r^0 in a certain polynomial in r constructed from Chiodo classes. We will show (under some annoying hypotheses, hopefully to be removed soon) that the cycle DR^i is the coefficient of r^i in the same expression with Chiodo classes. This is joint work with D. Chen, S. Grushevsky, M. Möller, and J. Schmitt.

Martijn Kool

Counting surfaces on Calabi–Yau fourfolds
Building on work of Oh–Thomas, I will introduce invariants for counting surfaces on Calabi–Yau fourfolds. In a family, they are deformation invariant along Hodge loci. If non-zero, the variational Hodge conjecture for the family under consideration holds. Joint work with Y. Bae and H. Park.

Samouil Molcho
Twisting Mumford's Formula
The Brill–Noether classes are certain cycles on the Jacobian of the universal curve over $\mathcal{M}_{g,n}$ parametrizing loci of line bundles with excess sections. Given a section of the Jacobian, we can pull back the Brill–Noether classes to cycles on $\mathcal{M}_{g,n}$. An extension of these classes has been proposed by Pagani–Ricolfi–van Zelm. In this talk, I will explain how to get a certain refinement of these classes, and how to calculate this refinement and consequently the PRvZ classes. The key ingredient in the calculation is a twisted version of Mumford's formula. As an application, we obtain novel formulas for the double ramification cycle, and consequently (not known if novel) relations in the tautological ring. This is joint work in progress with A. Abreu and N. Pagani.

Sergej Monavari
Double nested Hilbert schemes and stable pair invariants
Hilbert schemes of points on a smooth projective curve are symmetric powers of the curve itself; they are smooth and we know essentially everything about them. We propose a variation by studying double nested Hilbert schemes of points, which parametrize flags of 0-dimensional subschemes satisfying certain nesting conditions dictated by Young diagrams. These moduli spaces are almost never smooth but admit a virtual structure à la Behrend–Fantechi. We explain how this virtual structure plays a key role in (re)proving the correspondence between Gromov–Witten invariants and stable pair invariants for local curves.

Rahul Pandharipande
Cycles on the moduli of curves via Torelli
I will present a new set of results and conjectures from the perspective of Noether–Lefschetz theory about cycles on the moduli space A_g of PPAVs. Pulling-back via Torelli leads to interesting cycles on the moduli of curves of compact type. Joint work with S. Canning and D. Oprea.

Adrien Sauvaget
A flat approach to Weil–Petersson polynomials
Moduli spaces of hyperbolic surfaces with geodesic boundaries carry a natural symplectic form: the Weil–Petersson (WP) form. Mirzakhani proved that the volumes of these spaces are polynomials in the lengths of the boundaries. Another interpretation of these WP polynomials was proposed by Norbury–Do as volumes of moduli spaces of surfaces with conical singularities. This point of view allowed them to produce a family of relations satisfied by WP polynomials. We will present a new family of relations generalising the one of Norbury–Do. The proof of these relations is based on the following heuristic: the WP symplectic form may be approximated by forms constructed on moduli spaces of flat surfaces with many small singularities.

Sergey Shadrin
Towards chain level GW theory: an algebraic toolkit
I want to explain some algebraic constructions that should be helpful in the understanding of the chain level lift of cohomological field theories. In particular, I want to show a remarkable and a very general method coming from the theory of graph complexes that in our situation allows to generate actions of Givental and Grothendieck–Teichmueller groups on (quantum) (homotopy) cohomological field theories within the same natural extension of Kontsevich's graph complex. A

joint work with V. Dotsenko, A. Vaintrob, and B. Vallette.

Martin Ulirsch
$P=W$ phenomena on abelian varieties
Let X be a complex abelian variety. In this talk we will explain and prove an analogue of both the (cohomological) $P=W$ conjecture and the geometric $P=W$ conjecture relating the topology of the Dolbeault moduli space of topologically trivial semistable Higgs bundles on X and the Betti moduli space of characters of the fundamental group of X. The geometric heart of our approach is a new kind of Hitchin fibration for Dolbeault moduli spaces on abelian varieties whose target is not an affine space of pluricanonical sections, but a suitable symmetric product. This is based on ongoing joint work with B. Bolognese and A. Küronya