The treasure hunt for V_{xb}
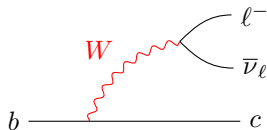
K. Keri Vos

Maastricht University & Nikhef
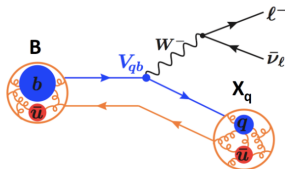
- Theory (Weak interaction): Transitions between quarks/partons

Exclusive versus Inclusive Theory
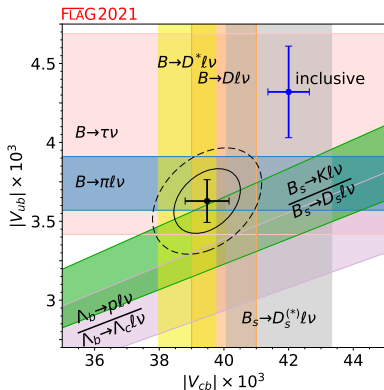
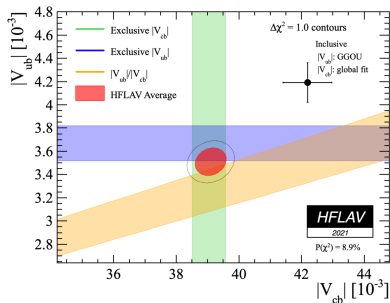- Theory (Weak interaction): Transitions between quarks/partons
- Observation: Transitions between hadrons

Challenge:

- Dealing with QCD at large distances/small scales
- Parametrize fundamental mismatch in non-perturbative objects
    - Calculable: Lattice or Light-cone sumrules
    - Measurable: from data

Longstanding Puzzle

2021 compilations. Here an 2023 update

Inclusive $B \to X_c$ decays

Inclusive Decays

Inclusive $B \to X_c \ell \nu$: Heavy Quark Expansion (HQE)

- b quark mass is large compared to Λ_{QCD}
- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Optical Theorem $\to$ (local) Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \dots \qquad d\Gamma_i = \sum_k C_i^{(k)} \left\langle B|O_i^{(k)}|B \right\rangle$$

- $C_i^{(k)}$ perturbative Wilson coefficients
- $\langle B| \dots |B \rangle$ non-perturbative matrix elements $\to$ string of iD
- operators contain chains of covariant derivatives

$$\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v (iD_\mu) \dots (iD_{\mu_n}) b_v|B\rangle$$

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- HQE parameters extracted from lepton energy, hadronic mass and q^2 moments
- Recent progress: ideas for the lattice Juetner et al. [202305.14092]

Decay rate

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 \cdots$$

- Γ_0: decay of the free quark (partonic contributions), $\Gamma_1 = 0$
- Γ_2: μ_π^2 kinetic term and the μ_G^2 chromomagnetic moment

$$2M_B\mu_\pi^2 = -\left\langle B|\bar{b}_v iD_\mu iD^\mu b_v|B\right\rangle$$
$$2M_B\mu_G^2 = \left\langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\right\rangle$$

- Γ_3: ρ_D^3 Darwin term and ρ_{LS}^3 spin-orbit term

$$2M_B\rho_D^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left[iD_\mu,[ivD,iD^\mu]\right]b_v|B\right\rangle$$
$$2M_B\rho_{LS}^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left\{iD_\mu,\left[ivD,iD_\nu\right]\right\}(-i\sigma^{\mu\nu})b_v|B\right\rangle$$

- Γ_4: 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109
- Γ_5: 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

Moments of the spectrum

Non-perturbative matrix elements obtained from moments of differential rate

Charged lepton energy

$$\langle E^n \rangle_{\mathrm{cut}} = \frac{\int_{E_\ell > E_{\mathrm{cut}}} dE_\ell \, E_\ell^n \, \frac{d\Gamma}{dE_\ell}}{\int_{E_\ell > E_{\mathrm{cut}}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

Hadronic invariant mass

$$\langle (M_X^2)^n \rangle_{\mathrm{cut}} = \frac{\int_{E_\ell > E_{\mathrm{cut}}} dM_X^2 \, (M_X^2)^n \, \frac{d\Gamma}{dM_X^2}}{\int_{E_\ell > E_{\mathrm{cut}}} dM_X^2 \, \frac{d\Gamma}{dM_X^2}}$$
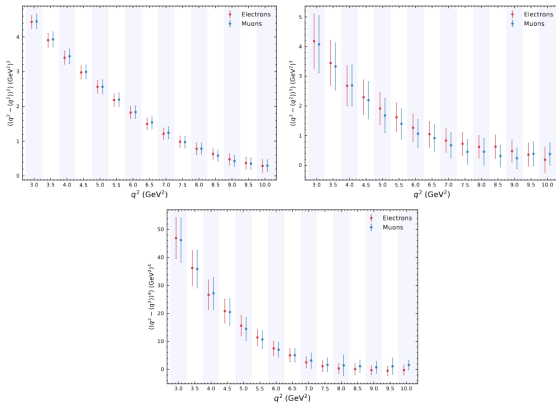
Dilepton momentum

$$\langle (q^2) \rangle_{\mathrm{cut}} = \frac{\int_{q^2 > q_{\mathrm{cut}}^2} dq^2 \, \frac{d\Gamma}{dq^2}}{\int_0 dq^2 \, \frac{d\Gamma}{dq^2}}$$

- Moments up to $n = 3, 4$ and with several energy cuts available
- Experimentally necessary to use some cut on the leptons

Belle Collaboration [2109.01685, 2105.08001]

Centralized moments as function of q^2_{cut}

$$\langle E_\ell^n \rangle, \langle (M_X^2)^n \rangle \quad \langle (q^2)^n \rangle_{\mathrm{cut}}$$
$$\downarrow$$
$$m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_d^3, r_E, r_G, s_E, s_B, s_{qB}, + \cdots$$

$$\downarrow$$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_{\mu_3} \mu_3 + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde{\rho}_D} \frac{\tilde{\rho}_D^3}{m_b^3} \right.$$
$$\left. + \Gamma_{r_E} \frac{r_E^4}{m_b^4} + \Gamma_{r_G} \frac{r_G^4}{m_b^4} + \Gamma_{s_B} \frac{s_B^4}{m_b^4} + \Gamma_{s_E} \frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}} \frac{s_{qB}^4}{m_b^4} \right]$$

$$\downarrow$$

$$V_{cb}$$

State-of-the-art in inclusive $b \to c$

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290; Fael, Schonwald, Steinhauser, Phys Rev. D 104 (2021) 016003; Fael, Schonwald, Steinhauser, Phys Rev. Lett. 125 (2020) 052003; Fael, Schonwald, Steinhauser, Phys Rev. D 103 (2021) 014005,

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi} \right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi} \right)^3 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi} \Gamma^{(\pi,1)} \right) \right.$$

$$\left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi} \Gamma^{(G,1)} \right) + \frac{\rho_D^3}{m_b^3} (\Gamma^{(D,0)} + \Gamma_0^{(1)} \left(\frac{\alpha_s}{\pi} \right)) + \mathcal{O} \left(\frac{1}{m_b^4} \right) + \cdots \right)$$

- Include terms up to $1/m_b^4 *$ see also Gambino, Healey, Turczyk [2016]
- α_s^3 to total rate and kinetic mass Fael, Schonwald, Steinhauser [2020, 2021]
- $\alpha_s \rho_D^3$ for total rate Mannel, Pivovarov [2020]
- Kinetic mass scheme 1411.6560,1107.3100; hep-ph/0401063

E_ℓ, M_X moments: q^2 moments*:

$|V_{cb}|_{\text{incl}}^{\text{BCG}} = (42.16 \pm 0.51) \times 10^{-3}$ $|V_{cb}|_{\text{incl}}^{q^2} = (41.69 \pm 0.63) \times 10^{-3}$

Gambino, Schwanda, PRD 89 (2014) 014022;
Alberti, Gambino et al, PRL 114 (2015) 061802;
Bordone, Capdevila, Gambino, Phys.Lett.B 822 (2021) 136679; Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi} \Gamma^{(\pi,1)} \right) \right.$$

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Challenges:

- Include higher-order $1/m_b$ and α_s corrections
- Proliferation of non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109

The advantage of q^2 moments

- Standard lepton energy and hadronic mass moments are not RPI quantities
- New q^2 moments are RPI!

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Reparametrization invariant quantities:

- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Choice of v not unique: Reparametrization invariance (RPI)

$$v_\mu \to v_\mu + \delta v_\mu$$

$$\delta_{RP} \, v_\mu = \delta v_\mu \; \text{ and } \; \delta_{RP} \, iD_\mu = -m_b \delta v_\mu$$

  - links different orders in $1/m_b \to$ reduction of parameters
  - up to $1/m_b^4$: 8 parameters (previous 13)

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- q^2 moments could enable a full extraction up to $1/m_b^4$

q^2 moments only analysis

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$|V_{cb}|_{\text{incl}}^{q^2} = (41.69 \pm 0.27|_{\mathcal{B}} \pm 0.31|_{\Gamma} \pm 0.18|_{\text{exp.}} \pm 0.17|_{\text{theo}} \pm 0.34|_{\text{const.}}) \times 10^{-3}$$

- First extraction using q^2 moments with $1/m_b^4$ terms
- Agreement with BCG extraction (differs due to branching ratio inputs)
  Bordone, Capdevila, Gambino [2021]

$$|V_{cb}|_{\text{incl}}^{\text{BCG}} = (42.00 \pm 0.51) \times 10^{-3}$$

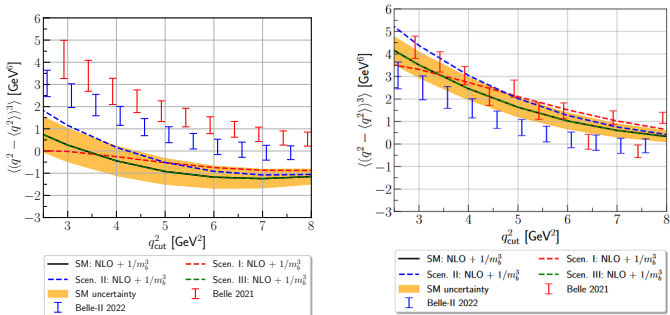- Higher order terms reduce value by 0.25%.

q^2 moments only analysis

$$|V_{cb}|_{\rm incl}^{q^2} = (41.69 \pm 0.63) \times 10^{-3}$$

- Higher order coefficients important to check convergence of the HQE

$$r_E^4 = (0.02 \pm 0.34) \cdot 10^{-1} \text{GeV}^4 \quad r_G^4 = (-0.21 \pm 0.69) \text{GeV}^4$$

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- Inputs for $B \to X_u \ell \nu$ Next, B lifetimes and $B \to X_s \ell \ell$ KKV, Huber, Lenz, Rusov, et al.
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• Additional 0.23 uncertainty due to missing higher orders
• In progress: Calculation of $1/m_c^2 1/m_b^3$ terms Mannel, Mulatin, KKV [in progress]

New Physics?

Fael, Rahimi, KKV [2208.04282]

- NP would also influence the moments of the spectrum
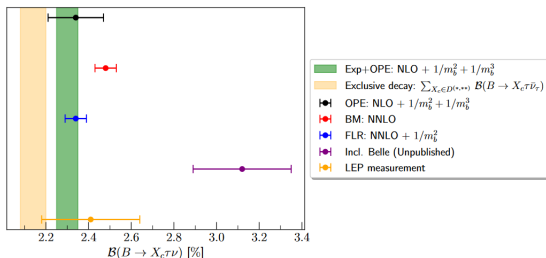- Requires a simultaneous fit of hadronic parameters and NP In progress..

KKV, Rahimi [2207.03432]

$$R_{e/\mu}(X) \equiv \frac{\Gamma(B \to X_c e \bar{\nu}_e)}{\Gamma(B \to X_c \mu \bar{\nu}_\mu)}$$

- Belle II result: $R(X_{e/\mu}) = 1.033 \pm 0.022$ with cut, see H. Junkerkalefeld [ICHEP]
- In agreement with new SM predictions: 1.006 ± 0.001 at 1.2σ

Interplay between electrons and muons

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- Next step ratios with τ! Need new measurements!

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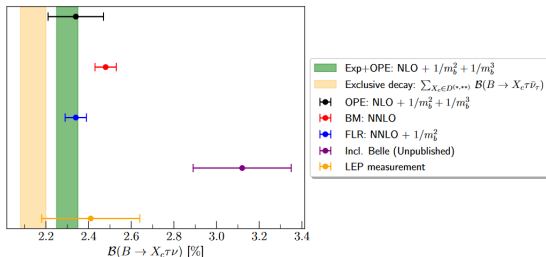
The challenge of exclusive V_{cb}

Exclusive V_{cb}

<u>$B \to D$ and $B \to D^*$</u>

- Form factors extracted from lattice, LC sumrules ($+$data)
- Knowledge on the q^2 dependence crucial
- BGL <small>Boyd, Grinstein,Lebed</small> or CLN/HQE <small>Caprini, Lellouch, Neubert</small> parametrization
    - Start of many discussions <small>Gambino, Jung, Schacht, Bordone, van Dyck, Gubernari, ...</small>
    - BGL: model independent parametrization using analyticity
    - CLN*: uses HQE at $1/m_b$ + assumptions <small>*justified at time of introduction</small>
- Improved HQE treatment including $1/m_c^2$ corrections <small>Bordone, van Dyk, Jung [1908.09398]</small>

$$|V_{cb}|_{\text{excl}} = (40.3 \pm 0.8) \times 10^{-3}$$

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$$|V_{cb}|_{excl} = (40.3 \pm 0.8) \times 10^{-3}$$

- $B \to D^*$ form factors at nonzero recoil Fermilab/MILC [2105.14019]
    - tension between the slope of the lattice and experimental data
- Same form factors determine SM predictions for $R_{D^{(*)}}$
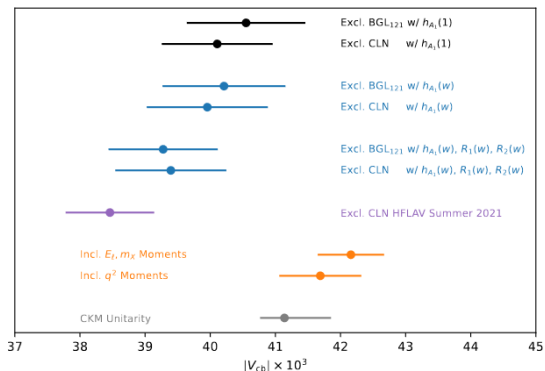- New experimental and lattice data needed!

$B \to D^*$ form factors
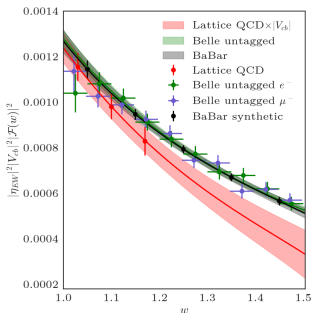
- Tension between the slope of the lattice and experimental data [Fermilab-MILC and HPQCD]
- JLQCD has different slope and agrees better with Belle 2019 data [?]
- See talk M.
- New experimental and lattice data needed!

Exclusive V_{cb}

Latest Belle analysis [2301.07529]. All with Fermilab/MILC form factors. See talk by Lu
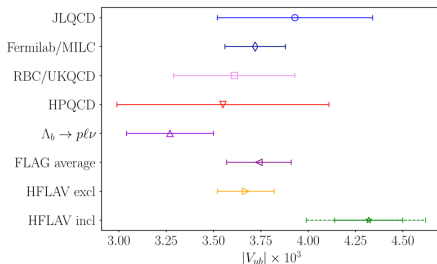

- Using form factors without shape information:

$$|V_{cb}|_{excl} = (40.6 \pm 0.9) \times 10^{-3}$$

- Also quotes CLN values $\rightarrow$ shows the 2023 V_{cb} puzzle

The challenge of V_{ub}

Exclusive V_{ub}

Exclusive $B \to \pi \ell \nu$

- Only one form factor
- Lattice QCD form factors largely agree with experimental shape [FNAL/MILC, RBC/UKQCD]
- New determination from JLQCD: $|V_{ub}|_{\text{excl, JLQCD}} = (3.93 \pm 0.41) \cdot 10^{-3}$ [PRD106 054502]

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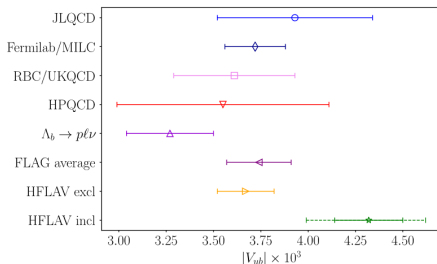
Combining with QCD sum rules

<div align="center">

Recent update:
Leljak, Melic, van Dyk [2102.07233]

$$|V_{ub}|_{\text{excl}} = (3.77 \pm 0.15) \cdot 10^{-3}$$

</div>

Inclusive V_{ub}

Inclusive $B \to X_u \ell \nu$

- Experimental cuts necessary to remove charm background
- Local OPE as in $b \to c$ cannot work
- Switch to different set-up using light-cone OPE
- Introduce non-perturbative shape functions ($\sim$ parton DAs in DIS)
- Different frameworks: BLNP, GGOU, DGE, ADFR

Recent update:

Belle [2102.00020]

$$|V_{ub}|_{\text{incl}} = (4.10 \pm 0.28) \cdot 10^{-3}$$

Inclusive determinations need to be scrutinized

Bosch, Lange, Neubert, Paz [2005]
Greub, Neubert, Pecjak [0909.1609]; Beneke, Huber, Li [0810.1230]; Becher, Neubert [2005]

Update of BLNP approach

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

- → H: Hard scattering kernel at $\mathcal{O}(m_b)$
- → J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\mathrm{QCD}}})$
- → S: Shape function at $\mathcal{O}(\Lambda_{\mathrm{QCD}})$

BLNP update - in progress -

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- $\rightarrow$ S: Shape function at $\mathcal{O}(\Lambda_{\mathrm{QCD}})$
- In progress: include known α_s^2 corrections
- Moments of shape functions can be linked to HQE parameters in $b \rightarrow c$
  - In progress: include higher-moments
  - kinetic mass scheme as in $b \rightarrow c$

BLNP update - in progress -

Bosch, Lange, Neubert, Paz [2005]
Greub, Neubert, Pecjak [0909.1609]; Beneke, Huber, Li [0810.1230]; Becher, Neubert [2005]

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  - $\rightarrow$ H: Hard scattering kernel at $\mathcal{O}(m_b)$
  - $\rightarrow$ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\mathrm{QCD}}})$
  - $\rightarrow$ S: Shape function at $\mathcal{O}(\Lambda_{\mathrm{QCD}})$

- In progress: include known α_s^2 corrections
- Moments of shape functions can be linked to HQE parameters in $b \rightarrow c$
  - In progress: include higher-moments
  - kinetic mass scheme as in $b \rightarrow c$
- Shape function is non-perturbative and cannot be computed
  - In progress: new flexible parametrization

Shape function parametrization
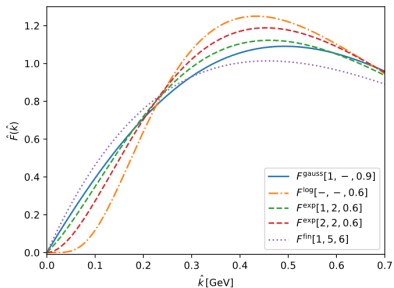
Olschewsky, Lange, Mannel, KKV [2306.xxxx]

- All moments of shape functions are linked to HQE parameters
- Allows for a range of different shapes $\rightarrow$ systematic uncertainty

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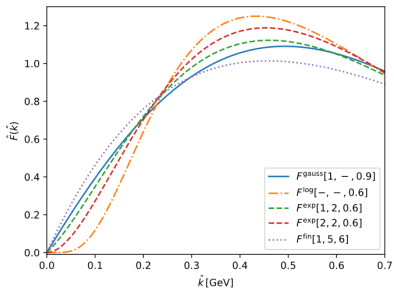
Olschewsky, Lange, Mannel, KKV [2306.xxxx]

- All moments of shape functions are linked to HQE parameters
- Allows for a range of different shapes $\rightarrow$ systematic uncertainty

In progress:

Lange, Mannel, Olschewsky, KKV [in progress]

$$|V_{ub}|_{\text{incl}} = \quad \text{Stay Tuned!}$$

Inclusive versus Exclusive semileptonic decays
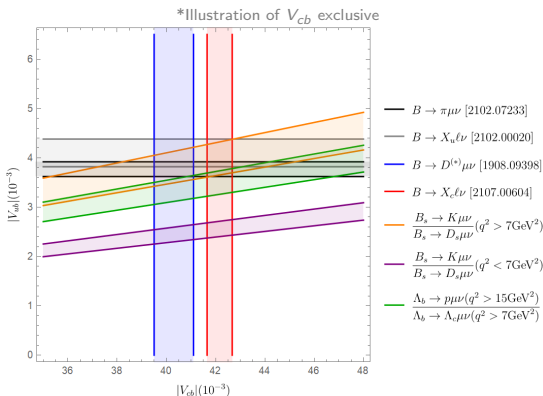
*Illustration of V_{cb} exclusive

- $B \to \pi\mu\nu$ [2102.07233]
- $B \to X_u\ell\nu$ [2102.00020]
- $B \to D^{(*)}\mu\nu$ [1908.09398]
- $B \to X_c\ell\nu$ [2107.00604]

- Recently a lot of attention for the V_{cb} puzzle! [Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Bordone, van Dyk, Gubernari]

*Illustration of V_{cb} exclusive

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- $B \to D^{(*)}\mu\nu$ [1908.09398]
- $B \to X_c\ell\nu$ [2107.00604]
- $\frac{B_s \to K\mu\nu}{B_s \to D_s\mu\nu}(q^2 > 7\text{GeV}^2)$
- $\frac{B_s \to K\mu\nu}{B_s \to D_s\mu\nu}(q^2 < 7\text{GeV}^2)$
- $\frac{\Lambda_b \to p\mu\nu(q^2 > 15\text{GeV}^2)}{\Lambda_b \to \Lambda_c\mu\nu(q^2 > 7\text{GeV}^2)}$

- Recently a lot of attention for the V_{cb} puzzle! [Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Bordone, van Dyk, Gubernari]

- Recent progress: V_{ub}/V_{cb} inclusive [See talk by Lu!]

- Unlikely to be due to NP [Jung, Straub [2018]]

- New data necessary: stay tuned!
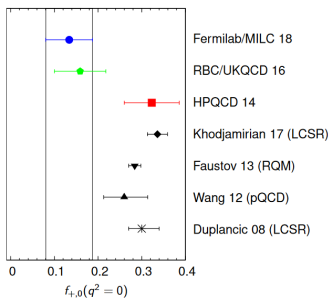
From $B_s \to K \mu \nu$ towards V_{ub}

Bolognani, van Dyk, KKV [in progress]

- Tensions for form factor at $q^2 = 0$
- Data from LHCb available for $q^2 < 7$ and $q^2 > 7$ GeV^2 LHCb [2012.05143]

Bolognani, van Dyk, KKV [in progress]
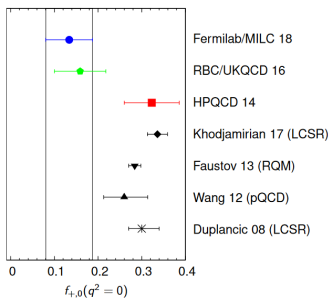
- Tensions for form factor at $q^2 = 0$
- Data from LHCb available for $q^2 < 7$ and $q^2 > 7$ GeV^2 LHCb [2012.05143]
- RBC/UKQCD 2023 update: Extrapolation to chiral continuum parametrization dependent! [Flynn et at. PRD 107 (2023) 114512]
- In progress: Combination of LCSR and Lattice

We are in the High-precision Era in Flavour Physics!

Outlook - inclusive decays

We are in the High-precision Era in Flavour Physics!

- Reached impressive precision

Outlook - inclusive decays

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- Still many things to work on!

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- Stay tuned for new data and updated theory predictions

Outlook - inclusive decays

We are in the High-precision Era in Flavour Physics!

- Reached impressive precision
- Still many things to work on!
- Stay tuned for new data and updated theory predictions

Close collaboration between theory and experiment necessary!

Backup

Shape functions

- Leading order shape functions

$$2m_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D)) b_v|B(v)\rangle$$

- Charged Lepton Energy Spectrum (at leading order)

$$\frac{d\Gamma}{dy} \sim \int d\omega \, \theta(m_b(1-y) - \omega) f(\omega)$$

- Moments of the shapefunction are related to HQE ($b \to c$) parameters:

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2} \delta''(\omega) - \frac{\rho_D^3}{m_b^3} \delta'''(\omega) + \cdots$$

- Shape function is non-perturbative

Shape function parametrization
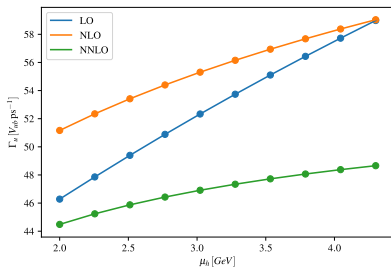
- α_s^2 corrections give large corrections [see also Pezcjak 2019]
- Required to make precision predictions

Contamination of the $B \to X_c \ell \nu$ signal

Avoid background subtraction by calculating the full inclusive width:

$$\mathrm{d}\Gamma(B \to X\ell) = \mathrm{d}\Gamma(B \to X_c\ell\bar{\nu}) + \mathrm{d}\Gamma(B \to X_u\ell\bar{\nu}) + \mathrm{d}\Gamma(B \to X_c(\tau \to \ell\bar{\nu}\nu)\bar{\nu})$$

- $\underline{b \to u\ell\nu \text{ contribution}}$: suppressed by V_{ub}/V_{cb}
- $\underline{b \to c(\tau \to \mu\nu\bar{\nu})\bar{\nu} \text{ contribution}}$: phase space suppressed
- QED effects
- Quark-hadron duality violation?

Goal:
provide theoretical description and compare with Monte-Carlo data used by Belle (II)

Challenge:
estimate how much this description would improve V_{cb} determination

$b \rightarrow u\ell\nu$ contribution: Local OPE

Neubert (1994); Bosch, Paz, Lange, Neubert (2004,2005)

- Can be analyzed in local OPE as $B \rightarrow X_c\ell\nu$ by taking $m_c \rightarrow 0$ limit

- For V_{ub} determination

    - large charm background requires experimental cuts
    - reduces the inclusivity and local OPE no longer converges
    - spectrum described by non-local OPE
    - convolution of pert. coefficients with shape function

Goal:

provide theoretical description and compare with Monte-Carlo data used by Belle (II)

- NLO + $1/m_b^2$+$1/m_b^3$

- In agreement with partonic calc of DFN De Fazio, Neubert (1999); Gambino, Ossola, Uraltsev (2005)

- First study: no α_s for $1/m_b^2$, no additional uncert. due to missing higher orders

- Inputs HQE parameters from $B \rightarrow X_c\ell\nu$ study Gambino, Schwanda [2014]; Gambino, Healey, Turczy [2016]

Monte Carlo versus HQE

Rahimi, Mannel, KKV [arXiv: 2105.02163]; De Fazio, Neubert 1999; Bosch, Lange, Neubert, Paz 2005

Compare local OPE with generator level Monte-Carlo data provided by Cao, Bernlochner

<u>Monte Carlo:</u>

- BLNP: specific shape function input parameters shape function parameters $b = 3.95$ and $\Lambda = 0.72$

- DFN: α_s corrections convoluted with the exponential shape function model
    - Inputs from $B \to X_c \ell \nu$ and $B \to X_s \gamma$ data using KN-scheme Kagan, Neubert 1998
    - $(\lambda_1^+, \lambda_2^+, \lambda_1^-, \lambda_2^-)$ are obtained by varying $\bar{\Lambda}$ and μ_π^2 within 1σ Buchmuller, Flacher, 2006

Hadronic contributions: "hybrid Monte Carlo" Belle Collabroation [arXiv:2102.00020.]

- convolution with hadronization simulation based on PYTHIA
- plus explicit resonances: $\bar{B} \to \pi \ell \bar{\nu}$ and $\bar{B} \to \rho \ell \bar{\nu}$

Monte Carlo versus HQE

MC-results are in good agreement with the HQE results

Monte Carlo versus HQE

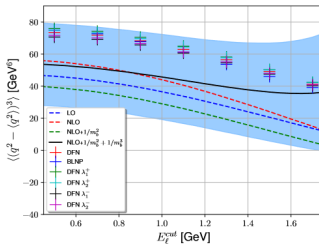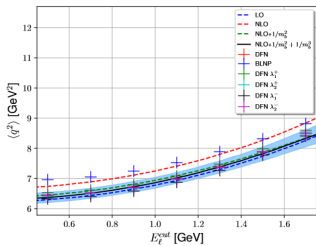Wide spread between MC for higher moments

Monte Carlo versus HQE

Rahimi, Mannel, KKV[arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner

Monte Carlo versus local OPE: remarks
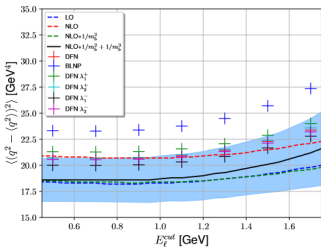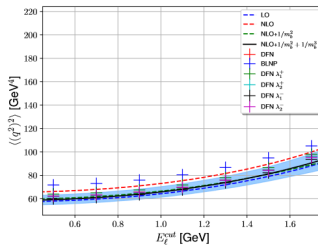
Rahimi, Mannel, KKV[arXiv: 2105.02163];

Remarks:

- DFN: Smearing corresponding to a shape function, mimicking some non-perturbative effects; may not capture all

- BLNP: should reproduce the HQE, with parameters adjusted to local HQE prediction
    - should include higher moments of the shape-function model?
    - include subleading shape functions?

- our HQE: interesting to include α_s to HQE parameters, α_s^2?

Five-body τ contribution

Contribution from five-body charm decay to $b \to c\ell\nu$ via

$$B(p_B) \to X_c(p_{X_c})(\tau(q_{[\tau]}) \to \mu(q_{[\mu]})\nu_\mu(q_{[\bar{\nu}_\mu]})\nu_\tau(q_{[\nu_\tau]}))\bar{\nu}_\tau(q_{[\bar{\nu}_\tau]})$$

:

- Phase space suppressed:

$$\frac{\Gamma_{\text{tot}}(b \to c\tau(\to \ell\bar{\nu}_\ell\nu_\tau)\bar{\nu}_\tau)}{\Gamma_{\text{tot}}(b \to c\ell\bar{\nu})} \sim 4.0\%$$

- Experimentally effects diminished by cutting on the invariant mass of the B
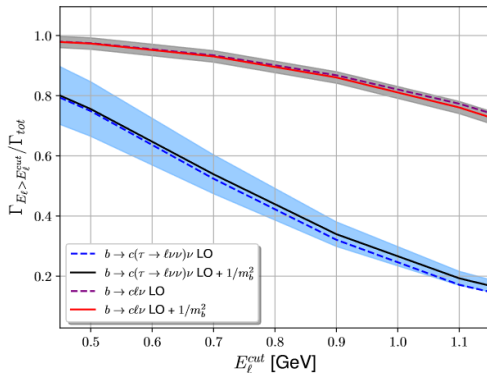
- Can be calculated exactly in the HQE

$$\frac{\mathrm{d}^8\Gamma}{\mathrm{d}q^2\,\mathrm{d}q^2_{\nu\bar{\nu}}\,\mathrm{d}p^2_{X_c}\,\mathrm{d}^2\Omega\,\mathrm{d}\Omega^*\,\mathrm{d}^2\Omega^{**}} = -\frac{3G_F^2|V_{cb}|^2\sqrt{\lambda}(q^2 - m_\tau^2)(m_\tau^2 - q^2_{\nu\bar{\nu}})\mathcal{B}(\tau \to \mu\nu\nu)}{2^{17}\pi^5 m_\tau^8 m_b^3 q^2} W_{\mu\nu} L^{\mu\nu}$$

  - $L_{\mu\nu}$ five-body leptonic tensor (narrow-width limit for τ)
  - $W_{\mu\nu}$ standard hadronic tensor including HQE parameters

- Interesting to search for new physics! Mannel, Rusov, Shahriaran (2017); Mannel, Rahimi, KKV [in progress]

Five-body τ contribution

Rahimi, Mannel, KKV[arXiv: 2105.02163];

No MC data available to test with

Theory guidance to include power corrections

Lowest State Saturation Approximation (LSSA)

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

$$\rho_D^3 = \varepsilon\mu_\pi^2, \quad \rho_{LS}^3 = -\varepsilon\mu_G^2, \quad \varepsilon \sim 0.4 \text{ GeV}$$

Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

- LSSA estimated as priors (60% gaussian uncertainty)
- $\mathcal{O}(1/m_b^4, 1/m_b^5)$ can then be included in fit Healey, Turczyk, Gambino, PLB 763 (2016) 60

$$|V_{cb}|_{\text{incl}} = (42.00 \pm 0.64) \times 10^{-3}$$

- -0.25% shift due to power corrections

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Towards the Ultimate Precision in $|V_{cb}|$

- Include α_s corrections to for ρ_D^3 Mannel, Pivovarov [in progress]; Gambino [in progress]
- Full determination up to $1/m_b^4$ from data possible?

Theory guidance to include power corrections

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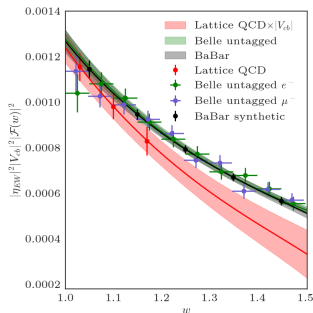
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Moments of the spectrum

Gambino, Schwanda Phys. Rev. D 89, 014022 (2014)
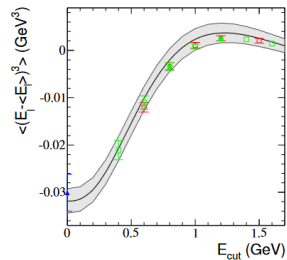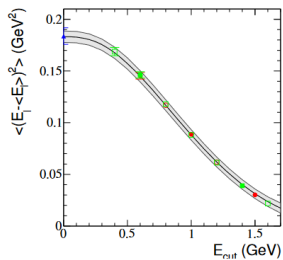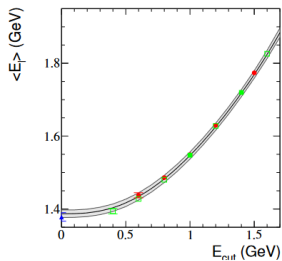
$B \to D^*$ form factors

- Tension between the slope of the lattice and experimental data
- Same form factors determine SM predictions for $R_{D^{(*)}}$
- New experimental and lattice data needed!

Exclusive $B \to D^{(*)}\ell\bar{\nu}$

- Form factor required (only for $B \to D$ available at different kinematic points)
- Different parametrizations for form factors: CLN Caprini, Lellouch, Neubert [1997] and BGL Boyd, Grinstein, Lebed [1995]
    - BGL: model independent based on unitarity and analyticity
    - CLN: Simple parametrization using HQE relations
- Some inconsistencies in the Belle data were pointed out see e.g. van Dyk, Jung, Bordone, Gubernari [2104.02094]
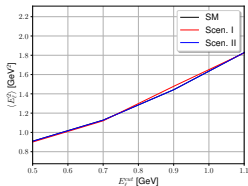
Inclusive $B \to X_c\ell\nu$

- Determined fully data driven including $1/m_b$ power corrections

Recently a lot of attention for the V_{cb} puzzle! Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Bordone, van Dyk, Gubernari

Stay tuned!

Inclusive $b \to c$

NP in the τ sector

- Affects also inclusive $B \to X_c \tau \nu$ Rusov, Mannel, Shahriaran [2017]
- Lepton and hadronic moments challenging to measure
- Recently moments of the five-body decay $B \to X_c \tau(\to \mu\nu\nu)\nu$ investigated Mannel, Rahimi, KKV [2105.02163]
- Would also be influenced by NP [in progress]
- Specific NP scenarios from global fit Mandal, Murgui, Penuela, Pich [2004.06726]

Preliminary!

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$$\frac{\mathrm{d}^8\Gamma}{\mathrm{d}q^2\,\mathrm{d}q^2_{\nu\bar{\nu}}\,\mathrm{d}p^2_{X_c}\,\mathrm{d}^2\Omega\,\mathrm{d}\Omega^*\,\mathrm{d}^2\Omega^{**}} = -\frac{3G_F^2|V_{cb}|^2\sqrt{\lambda}(q^2-m_\tau^2)(m_\tau^2-q_{\nu\bar{\nu}}^2)\mathcal{B}(\tau \to \mu\nu\nu)}{2^{17}\pi^5 m_\tau^8 m_b^3 q^2}W_{\mu\nu}L^{\mu\nu}$$

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Shape functions

- Leading order shape functions

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Shape functions

Lange, Neubert, Bosch, Paz

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

  - $\rightarrow$ H: Hard scattering kernel at $\mathcal{O}(m_b)$
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  - $\rightarrow$ S: Shape function at $\mathcal{O}(\Lambda_{\mathrm{QCD}})$
- Framework to include radiative corrections ($+$ NNLL resummation)
- Introduces 3 subleading shape functions

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- Framework to include radiative corrections (+ NNLL resummation)
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- Other approach: OPE with hard-cutoff μ Gambino, Giordano, Ossola, Uraltsev
    - Use pert. theory above cutoff and parametrize the infrared
    - Different definition of the shape functions
- Shape functions have to be parametrized and obtained from data

q^2 moments only analysis

$$|V_{cb}|^{q^2}_{\text{incl}} = (41.69 \pm 0.63) \times 10^{-3}$$

- Higher order coefficients important to check convergence of the HQE

$$r_E^4 = (0.02 \pm 0.34) \cdot 10^{-1} \text{GeV}^4 \quad r_G^4 = (-0.21 \pm 0.69) \text{GeV}^4$$

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- Inputs for $B \to X_u \ell \nu$ Next, B lifetimes and $B \to X_s \ell\ell$ KKV, Huber, Lenz, Rusov, et al.
- Additional 0.23 uncertainty due to missing higher orders

q^2 moments only analysis

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

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What about theory correlations?

- Flexible correlations between moments ρ_{mom} and different cuts ρ_{cut}
- Included by adding a penalty term to the χ^2
- Scan over large range of values
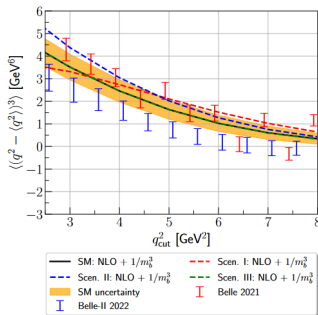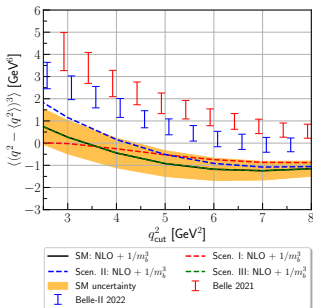- V_{cb} constant w.r.t. theory correlations
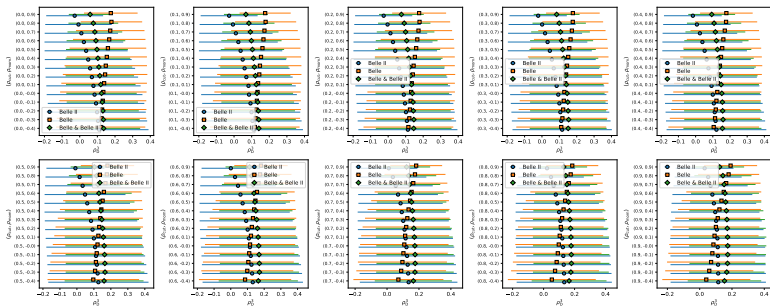
What about ρ_D^3?

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

- Large uncertainties on HQE elements
- Important: ρ_D^3 much smaller than previous!
- α_s^2 corrections to moments not yet included

Rahimi, Fael, Vos [2208.04282]

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- Large uncertainties on HQE elements
- **Important**: ρ_D^3 much smaller than previous!
- α_s^2 corrections to moments not yet included
- Corrections are negative Steinhauser, Fael, Schoenwald [2205.03410]
- Full analysis including all data is necessary! **Bernlochner, Fael, Prim, KKV [in progress]**
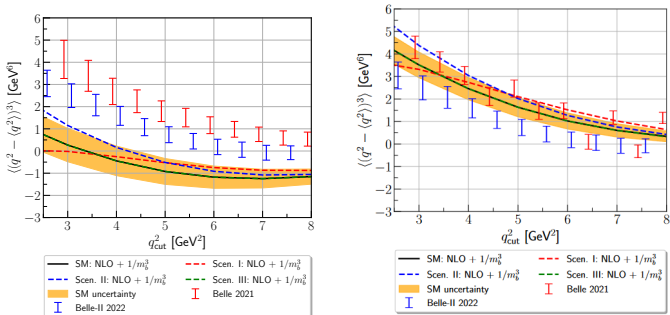
Rahimi, Fael, Vos [2208.04282]
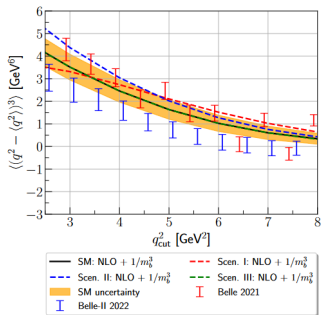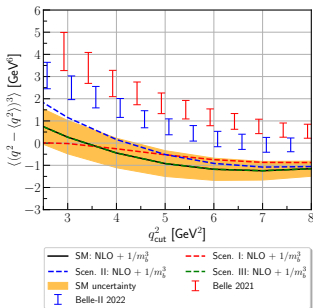
Even higher corrections?

- HQE set up with $m_c/m_b \sim \mathcal{O}(1)$
- IR sensitive terms for $m_c \to 0$ Bigi, Mannel, Turczyk, Uraltsev [0911.3322]
    - at dim-6: $1/m_b^3 \ln m_c^2$
    - at dim-8: $1/m_b^5 m_b^2/m_c^2 \sim 1/m_b^3 1/m_c^2$
- Numerically: $m_c^2 \sim m_b \Lambda_{\mathrm{QCD}}$
- in progress: Calculation and estimate of these effects

New Physics?

Fael, Rahimi, KKV [2208.04282]

- NP would also influence the moments of the spectrum
- Requires a simultaneous fit of hadronic parameters and NP In progress..