# Kaons CP violation & rare decays

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### Strangeness

$$K^{+} \sim u\bar{s} \qquad \qquad K^{-} \sim \bar{u}s$$
$$K^{0} \sim d\bar{s} \qquad \qquad \overline{K}^{0} \sim \bar{d}s$$

#### **Differences to B physics**

- Less phase-space → less decays X
- Stronger CKM suppression of top contribution  $\rightarrow$  less SM background  $\checkmark$ Kaons:  $\lambda^5 \sim 0.0005$  B mesons:  $\lambda^3 \sim 0.01$
- Charm can *potentially* contribute → care is needed!

### Kaons: SM vs NP

**SM** loop induced, precision



**Heavy NP** virtual, indirect probe SUSY, Composite Higgs, Extra Dimensions, ...



**Light NP** decays to "invisible" Axions, Dark Photons, ...

light NP

# K- vs B/D-physics probes of NP

#### Complementarity

comparison only possible within specific models/setup → search in all sectors

Acquired theoretical control in Kaon modes
 Large sensitivity to high-scale dynamics

Non-MFV models / no-alignment of NP FV with SM FV
 → typically Kaon constraints supersede B/D probes
 Even in models with alignment Kaons important

 $\epsilon_K$  "excludes" composite-Higgs models partial compositeness and anarchic Yukawas

### Kaons in the era of the LHC

- If the LHC continues to reduce the parameter-space for TeV-scale NP
- the phenomenological **need** for flavour alignment is lifted (MFV, flavour symmetries, partial compositeness,...)
- ➔ "importance" of Kaon observables as probes of NP increases

There is a strong, ongoing and planned experimental program in place to take advantage of Kaon modes [recent review of Kaon community: Goudzovski et al., 2201.07805]

• situation similar for probes of light NPs (e.g., the QCD axion)

### Kaons and light-NP – example QCD Axion

$$\Gamma_K^{\rm tot} \sim M_K^5/M_W^4 \qquad \qquad \Gamma_B^{\rm tot} \sim M_B^5/M_W^4$$

• Dimension-5 QCD axion couplings:  $\frac{\partial^{\mu}a}{f_a} \bar{q}_i \gamma_{\mu}(\gamma_5) q_j$ 



[from R. Ziegler @ La Thuille]

### **Observables**

#### • CP violation in $K - \overline{K}$ mixing

- $\epsilon_K$ : indirect CP violation
- $\epsilon'/\epsilon$ : direct CP violation (hadronic uncertainties  $\rightarrow$  lattice, not today)

#### Rare FCNC decays

- $K \to \pi \nu \nu$  (the Kaon "golden" modes)
- $K_S 
  ightarrow \mu^+ \mu^-$  (NP sensitivity from time-dependent rates)
- $K_L 
  ightarrow \pi^0 \ell^+ \ell^-$  (sensitivity to tensors [Mescia, Smith, Trine 06], not today)
- non-FCNC decays, ...



# $\epsilon_K$ - disentagling long-distance charm contributions



- among the most stringent constraints of BSM with new FV (feeds back into predictions of clean BSM-sensitive rare decays)  $(K_L \rightarrow \pi \nu \nu$  [Buchalla, Buras 96;]  $K_S \rightarrow \mu \mu$  [Brod, ES 23])
- but prediction had reached an impasse

 $\epsilon_K$  seemed to be plagued by non-perturbative uncertainties associated to charm impeding further progress

 → solution "simple" (ct vs ut unitarity) modifying CKM-unitarity relation used decouples charm in ε<sub>K</sub>
 → opens path to permille-level accuracy [Brod, Gorbahn, ES 19)]



 $\lambda_u = -\lambda_t - \lambda_c$  ct-unitarity (old computation, 3 imaginary parts)  $\lambda_c = -\lambda_t - \lambda_u$  ut-unitarity (new computation, 2 imaginary parts)



- Beyond LO different conditions correspond to a rearrangement/reshuffling of the perturbative expansion (A priori it is not clear that results can be trivially translated into one another)
- It is always possible (phase-convention independent) to transparetly separate the long-distance charm contributions entering  $\Delta M_K$  from  $\epsilon_K$

[Brod, Gorbahn, ES 19]

# Phase-(in)dependent Hamiltonian

• Make convention-independence of  $\epsilon_K$  explicit (**Trick:** factor out  $1/(\lambda_u^*)^2$ )

$$\mathcal{H}^{\Delta S=2} = \frac{G_F^2 M_W^2}{4\pi^2} \frac{1}{(\lambda_u^*)^2} Q_{S2} \Big\{ f_1 \, \mathcal{C}_1 + i J \, [f_2 \, \mathcal{C}_2 + f_3 \, \mathcal{C}_3] \Big\} + \text{h.c.} + \dots$$

[Brod, Gorbahn, ES 19]

- $C_1, C_2, C_3$ : real Wilson coefficients
- $J, f_1, f_2, f_3$  real, phase-convention invariant CKM structures
- Real part of  $f_1 = |\lambda_u|^4$  is unique, splitting of  $f_2$  and  $f_3$  is not
- → One contribition to real part  $\propto C_1$  (relevant for  $\Delta M_K$ )
- → Only two independent imaginary pieces  $\propto Jf_1$  and  $Jf_2$

(relevant for  $\epsilon_K$ )

### Uncertainty estimates – $\epsilon_K$ without $\Delta M_K$ pollution

#### "Old" / ct-unitarity

$$|\epsilon_K| \propto |V_{cb}|^2 (1-\bar{\rho}) \hat{C}_{\mathrm{S2}}^{tt} + \hat{C}_{\mathrm{S2}}^{ct} - \hat{C}_{\mathrm{S2}}^{cc}$$

■ Do we vary scales indepedently? → large uncertainties

#### "New" / ut-unitarity

$$|\epsilon_K| \propto |V_{cb}|^2 (1-\bar{
ho}) \widehat{\mathscr{C}}_{\mathrm{S2}}^{tt} + \widehat{\mathscr{C}}_{\mathrm{S2}}^{ut}$$

● No ambiguity any more → clean/clear uncertainty estimation

$$|\epsilon_K| = \kappa_\epsilon C_\epsilon \widehat{B}_K |V_{cb}|^2 \lambda^2 ar\eta \Big( |V_{cb}|^2 (1-ar
ho) \eta_{tt} S(x_t) - \eta_{ut} S(x_c,x_t) \Big)$$



- $\hat{B}_K = 0.7625(97)$ [FLAG19, 1902.08191]
- $\eta_{ut} = 0.402(5)$ [Brod, Gorbahn, ES 19]

• 
$$|\epsilon_K|^{\text{SM}} = 2.16(18) \times 10^{-3}$$

• 
$$|\epsilon_K|^{\exp} = 2.228(11) \times 10^{-3}$$

# $\epsilon_K$ - disentagling charm contributions

- reshuffling the perturbative expansion  $\rightarrow \epsilon_K$  without  $\Delta M_K$  pollution
- opens path to permille level precision



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# Rare K decays

### Rare K decays: CKM structure in SM



With respect to CKM:

- CP conserving modes: potential top/charm competition X
- CP violating modes: free from long-distance up-quark (Λ<sub>QCD</sub>) contributions, suppressed, and dominated by short-distance dynamics ✓

### Rare K decays: GIM structure



➔ If photonic penguins contribute, long-distance "pollution"

→ Z penguins quadratically sensitive to x – hard quadratic GIM

# A tricky opportunity

$$K_S \to (\mu^+ \mu^-)_{\ell=0}$$

$$K_S o (\mu^+\mu^-)_{\ell=0}$$

CP structure depends on dimuon angular momentum,  $\ell$ 



- $K_L$ : long-distance dominated due to two-photon penguin BR $(K_L \rightarrow \mu \mu) = 6.84(11) \times 10^{-9}$
- $K_S: \ell = 0$  component, CP violating and top-quark dominated BR $(K_S \to \mu \mu) < 2.1 \times 10^{-10}$

**Challenges:** tag  $K_S$  (small lifetime) and access the  $(\mu\mu)_{\ell=0}$  state

# $\mathsf{BR}(K_S o \mu \mu)_{\ell=0}$ from time-dependent rate

Leverage intereference in **time-dependent** rate to access the short-distance dominated  $(\mu\mu)_{\ell=0}$  state [D'Ambrosio, Kitahara 17]

$$\frac{d\Gamma[K(t) \to (\mu\mu)_{\ell}]}{dt} \propto C_L^{\ell} e^{-\Gamma_L t} + C_S^{\ell} e^{-\Gamma_S t} + 2[C_{\sin}^{\ell} \sin(\Delta M t) - C_{\cos}^{\ell} \cos(\Delta M t)] e^{-\Gamma t}$$



$$\frac{\mathrm{BR}[K_S \to (\mu\mu)_{\ell=0}]}{\mathrm{BR}[K_L \to \mu\mu]} = \frac{\tau_S}{\tau_L} \left(\frac{C_{\mathrm{int}}}{C_L}\right)^2$$

[Dery et al 21]

[plot from J. Brod @ LPCP]

# $\mathsf{BR}(K_S o \mu \mu)_{\ell=0}$ SM prediction and NP sensitivity

- Top-quark dominated and  $\mathcal{CP} \rightarrow \sim 1\%$  theory uncertainty
- Sizable 3% component from CP-conserving amplitude  $imes \epsilon_K$
- → only rough estimate possible for relative strong phase [estimation from  $K_L \rightarrow \mu\mu K_L \rightarrow \gamma\gamma$  data] BR $(K_S \rightarrow \mu\mu)_{\ell=0} = 1.70(2)(1)(19) \times 10^{-13}$  [Brod, ES 22]
- Presently no heavy-NP bounds; not clear whether exp. feasible $\mathcal{L}_{\text{NP}} = \frac{C_{\text{NP}}}{\Lambda^2} (\bar{s}_L \gamma^\nu d_L) (\bar{\mu}_L \gamma_\nu \mu_L) + \text{hc}$
- ightarrow Measurement at 10% would probe  $\Lambda \sim 600~{
  m TeV}$  for Im $[C_{
  m NP}]$

### The golden modes

 $K_L \to \pi^0 \nu \bar{\nu}$ 

 $K^+ \to \pi^+ \nu \bar{\nu}$ 





• Top:  $\lambda_t oldsymbol{F}(oldsymbol{x_t}) \propto \lambda^5 m_t^2/M_W^2$ 

matching NLO@QCD [Misiak, Urban 99; Buchalla, Buras 99], NLO@EW [Brod, Gorbahn, ES 10], matching NNLO@QCD [in progress with Gorbahn, Yu]

- Charm:  $\lambda_c F(x_c) \propto \lambda m_c^2 / M_W^2 \log(m_c/M_W)$  obtained via operator mixing in EFT running NNLO@QCD [Buras et al 06] NLO@EW [Brod, Gorbahn 08]
- Up:  $\lambda_c F(x_u) \propto \lambda \Lambda_{QCD}^2 / M_W^2$  from  $\chi$ PT and lattice small, 10% of total charm contribution



 $K_L 
ightarrow \pi^0 
u ar{
u}$  CP violating mode (1% from CP-conserving amplitude  $imes \epsilon_K$ )

→ only top-quark contribution: short-dist. uncertainty of  $\mathcal{O}(2\%)$ 

$$\mathsf{BR}(K_L \to \pi^0 \nu \bar{\nu})^{\mathsf{SM}} = \kappa_L r_{\epsilon_K} \left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t\right)^2$$

#### $K^+ ightarrow \pi^+ u ar{ u}$ CP conserving mode

→ charm contributes:  $\mathcal{O}(3\%)$  long-distance theory uncertainties

$$\begin{split} \mathsf{BR}(K^+ \to \pi^+ \nu \bar{\nu})^{\mathsf{SM}} &= \kappa_+ (1 + \Delta_{\mathsf{EM}}) \bigg[ \bigg( \frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t \bigg)^2 \\ &+ \bigg( \frac{\mathrm{Re}\lambda_c}{\lambda} (P_c + \delta P_{cu}) + \bigg( \frac{\mathrm{Re}\lambda_t}{\lambda^5} X_t \bigg) \bigg)^2 \bigg] \end{split}$$

[details in Brod, Gorbahn, ES from BEAUTY2020]

### Top-contribution $X_t$ at NNLO in QCD



Reduces uncertainty from  $\pm 1\%$  (@NLO) down to  $\pm 0.1\%$  (@NNLO)

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[in progress with Gorbahn, Yu]

# **SM prediction using PDG input**

#### Preliminary numerics including X<sub>t</sub>@NNLO

$$BR(K^+ \to \pi^+ \nu \bar{\nu})^{SM} = 8.25(11)_{SD}(25)_{LD}(57)_{para} \times 10^{-11}$$
$$BR(K_L \to \pi^0 \nu \bar{\nu})^{SM} = 2.83(1)_{SD}(2)_{LD}(30)_{para} \times 10^{-11}$$

#### NA62 and KOTO measuments

$$\begin{split} &\mathsf{BR}(K^+ \to \pi^+ \nu \bar{\nu})^{\mathsf{NA62}} = (10.6^{+3.4}_{-3.4}|_{\mathsf{stat}} \pm 0.9|_{\mathsf{sys}) \times 10^{-11}} \\ &\mathsf{BR}(K_L \to \pi^0 \nu \bar{\nu})^{\mathsf{KOTO}} < 3.0 \times 10^{-9} \\ & \text{@90\% CL} \end{split}$$

Ratios of rates as clean SM tests

[Buras, Venturini 21]

$$R_{S} = \frac{\text{BR}(K_{S} \to \mu\mu)_{\ell=0}}{\text{BR}(K_{L} \to \pi^{0}\nu\bar{\nu})} \sim 1.55 \times 10^{-2} \frac{Y(x_{t})}{X(x_{t})}$$

Essentially CKM-parameter free (up to  $\mathcal{O}(3\%)$  correction)

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### Heavy NP in $K \rightarrow \pi \nu \nu$

powerful probes of NP (for Vector "SM-like" NP)

$$\mathscr{L}_{\rm NP} = \frac{C_{L/R}^V}{\Lambda^2} (\bar{s}\gamma^\mu P_{L/R} d) (\bar{\nu}_L \gamma_\mu \nu_L) + hc$$



### Lepton-Number-Violation in $K \rightarrow \pi \nu \nu$ distributions

• Lepton-number-violating (LNV) NP in  $s \rightarrow d\nu\nu$ 

$$\mathscr{L}_{\mathrm{LNV}} = \frac{C_{L/R}^S}{\Lambda^2} \underbrace{(\bar{s}P_{L/R}d)(\nu_L^T \mathcal{C}\nu_L)}_{\mathrm{scalar}} + \frac{C^T}{\Lambda^2} \underbrace{(\bar{s}_R \sigma_{\mu\nu} d_L)(\nu_L^T \mathcal{C}\sigma^{\mu\nu}\nu_L)}_{\mathrm{tensor/only LFV}} + hc$$

■ Modified vv distributions → probing LNV at NA62



[in progress with Sieja, Tabet; SMEFT/scalar-case Deppisch et al 20]

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### Lepton-Number-Violation in $K \rightarrow \pi \nu \nu$ distributions

$$\mathscr{L}_{\rm LNV} = \frac{C_{L/R}^S}{\Lambda^2} (\bar{s} P_{L/R} d) (\nu_L^T \mathcal{C} \nu_L) + \frac{C^T}{\Lambda^2} (\bar{s}_R \sigma_{\mu\nu} d_L) (\nu_L^T \mathcal{C} \sigma^{\mu\nu} \nu_L) + hc$$

• correlations/preliminary recast of NA62



- washout process, constrain Leptogenesis[Deppisch et al 15,18;Chun et al 17]
- Further options: sterile-ν signatures (K → πν<sub>l</sub>ν<sub>h</sub>, πν<sub>h</sub>ν<sub>h</sub>), SU(2) invariant models and LNV pheno, KOTO data, projections
   [in progress with Gorbahn and Moldanazarova]

## **Summary and conclusions**

- Kaons remain at the frontier of searches for heavy NP and light NP
- Precision predictions necessary to disentagle NP effects
- → Recent and expected improvements targeting 0.1%-level accuracy ( $\epsilon_K, K \to \pi \nu \bar{\nu}$ )
- New ideas to maximize impact of data
- → e.g., time-dependeet rate in  $K \rightarrow \mu\mu$  or  $K \rightarrow \pi\nu\bar{\nu}$ distributions

# backup

# $\epsilon_K$ : indirect CP violation

• If CP is conserved  $K_L \not\rightarrow \pi\pi$ , but mixing allows it:

$$\epsilon_K \equiv rac{\langle (\pi\pi)_{I=0} | K_L 
angle}{\langle (\pi\pi)_{I=0} | K_S 
angle} = rac{1-\lambda_0}{1+\lambda_0}$$

• With 
$$\lambda_0 = \frac{q}{p} \frac{\langle (\pi\pi)_{I=0} | \overline{K}^0 \rangle}{\langle (\pi\pi)_{I=0} | K^0 \rangle} = 1 - i\phi \underbrace{\frac{\Delta M_K}{\Delta M_K + i\Delta \Gamma_K/2}}_{\text{Take from experiment}}$$

• With the weak phase  $\phi = arg(-M_{12}/\Gamma_{12})$ 

Compute [Nierste in Anikeev et al; hep-ph/0201071]

# $\epsilon_K$ : indirect CP violation

In **PDG-phase convention**  $M_{12}$  and  $\Gamma_{12}$  nearly real

$$\epsilon_K = e^{i \phi_\epsilon} \sin \phi_\epsilon \left( rac{Im(M_{12})}{\Delta M_K} + \xi 
ight)$$

•  $\phi_{\epsilon} \equiv \arctan \frac{\Delta M_K}{\Delta \Gamma_K/2}$  and  $\xi = -\frac{Im(\Gamma_{12})}{2Re(\Gamma_{12})}$  (non-perturbative / long-distance)

(Estimates from:  $\epsilon'/\epsilon$  [Nierste 02, Buras et al 08], ChPT [Buras et al 10], lattice [Blum et al 15, Bai et al 15])

Im(M<sub>12</sub>): scale separation (EFT) factorizes computation

$$\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_{\mathsf{had}}) U(\mu_{\mathsf{had}}, \mu_c) U(\mu_c, \mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_{\mathsf{had}}) U(\mu_{\mathsf{had}}, \mu_c) U(\mu_c, \mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_{\mathsf{had}}) U(\mu_{\mathsf{had}}, \mu_c) U(\mu_c, \mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_{\mathsf{had}}) U(\mu_{\mathsf{had}}, \mu_c) U(\mu_c, \mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_{\mathsf{had}}) U(\mu_{\mathsf{had}}, \mu_c) U(\mu_c, \mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_{\mathsf{had}}) U(\mu_{\mathsf{had}}, \mu_c) U(\mu_c, \mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_{\mathsf{had}}) U(\mu_c, \mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_{\mathsf{had}}) U(\mu_{\mathsf{had}}, \mu_c) U(\mu_c, \mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_{\mathsf{had}}) U(\mu_c, \mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_{\mathsf{had}}) U(\mu_c, \mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_{\mathsf{had}}) U(\mu_c, \mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_{\mathsf{had}}) U(\mu_c, \mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_{\mathsf{had}}) U(\mu_c, \mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle (\mu_W) C(\mu_W)}_{\langle \mathcal{H}_{\mathsf{eff}} \rangle = \underbrace{\langle Q$$

hadronic matrix element,  $\hat{B}_K$  pertubative evolution of C

## **Traditional: ct-unitarity**

$$\mathcal{H}^{\Delta \mathrm{S}=2} = rac{G_F^2 M_W^2}{4\pi^2} Q_{\mathrm{S}2} \Big\{ \lambda_t^2 C^{tt} + \lambda_c^2 C^{cc} + \lambda_c \lambda_t C^{ct} \Big\} + \mathsf{h.c.} + \dots$$

Match to phase-convention "independent" Hamiltonian

• 
$$C^{cc} = C_1$$
  $C^{ct} = 2C_1 + C_3$   $C^{tt} = 2C_1 + C_2 + C_3$ 

•  $C_1$ , controlling  $\Delta M_K$ , enters all pieces

#### → <u>Bad</u> news

- $\Delta M_K$  is long-distance dominated [Brod, Gorbahn 11]
- → We artificially included  $C_1$  into the prediction of  $\epsilon_K$ !

#### We can do better.

### **New: ut-unitarity**

First proposed for a lattice computation [Christ et al 12, see also Barbieri 07]

$$\mathcal{H}^{\Delta \mathrm{S}=2} = rac{G_F^2 M_W^2}{4\pi^2} Q_{\mathrm{S}2} \Big\{ \lambda_t^2 C^{tt} + \lambda_u^2 C^{uu} + \lambda_u \lambda_t C^{ut} \Big\} + ext{h.c.}$$

Match to phase-convention "independent" Hamiltonian

• 
$$C^{uu} = \mathcal{C}_1$$
  $C^{tt} = \mathcal{C}_2$   $C^{ut} = \mathcal{C}_3$ 

• Disentangled  $Re(M_{12})$  and  $Im(M_{12})$ 

#### → <u>Good</u> news!, $Im(M_{12})$ without $\Delta M_K$ pollution

 It can be shown that Wilson coefficients and ADMs can be extracted from the "traditional" results

[Brod, Gorbahn, ES 19]

# **Comparison: ut VS ct**

Residual theory uncertainties from scale variation (μ<sub>t</sub>, μ<sub>c</sub>)



Miserable convergence behaviour of ct-unitarity (right panel)

• Excellent convergence behaviour of ut-unitarity (left panel)

$$\eta_{ut} = 0.402 \pm 0.005$$
 @ NNLO[Brod, Gorbahn, ES 19]  
 $\eta_{tt} = 0.55 \pm 0.02$  @ NLO [Buras et al 90 with our variation]

# **Comparison: ut VS ct**

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• Residual theory uncertainties from scale variation  $(\mu_t, \mu_c)$ 



Miserable convergence behaviour of ct-unitarity (right panel)

• Excellent convergence behaviour of ut-unitarity (left panel)

$$\eta_{ut} = 0.402 \pm 0.005$$
 @ NNLO[Brod, Gorbahn, ES 19]  
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#### Towards permille-level accuracy in $\epsilon_K$

 Charm no longer leading uncertainty

#### → Reassessment of uncertainties

- Reduce top uncertainty by NNLO QCD matching
   [in progress with Brod, Gorbahn, and Hang]
- Residual EW/QED uncertainties 2%? [path analogous to Brod, Gorbahn, ES 10; Brod, Gorbahn 08, Bobeth, Gorbahn, ES 14; Gambino, Kwiatkowski, Pott 98]
- Lattice target: ME with dynamical charm. Will allow to fully compute  $\epsilon_K$
- Reassess contributions from lon-local insertions

