Exotic Spectroscopy: A Lattice QCD perspective

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Clermont-Ferrand,
July 4, 2023

Based on material by R.J. Huspith

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GSI

Outline

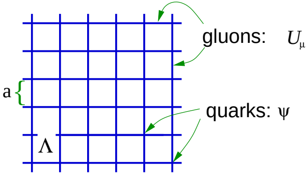
1. Introduction and Motivation
2. Four-quark states
3. B_{s0}^{*} and B_{s1}: Regular mesons or meson molecules/tetraquarks?
4. Beauty-full multi-quark states
5. Conclusions

What to call an exotic state in QCD ?

- Textbook: Quark-antiquark mesons and 3-quark baryons
- Historically, multiquark states and hybrids (made of quark and gluons) already suggested by Gell-Mann in addition
- We are now seeing some explicitly *exotic* states – in particular with heavy quarks
- Various possible structures: regular mesons/baryons; molecules; tetraquarks/pentaquarks; hybrid hadrons; glueballs; Di-Baryons
- For the purpose of this talk:
  I will also consider states with quantum numbers allowed by quark-antiquark states but unexpected properties as exotic
  Example: B_{s0}^* and B_{s1} mesons.

My method of choice: Lattice QCD

- Lattice QCD: Regularization of QCD by a 4-d Euclidean space-time lattice. Provides a calculational method.



Euclidean correlator of two Hilbert-space operators $\hat{O}_1$ and $\hat{O}_2$.

$$\left\langle \hat{O}_2(t)\hat{O}_1(0) \right\rangle = \sum_n e^{-t\Delta E_n} \langle 0|\hat{O}_2|n\rangle\langle n|\hat{O}_1|0\rangle$$
$$= \frac{1}{Z}\int \mathcal{D}[\psi,\bar{\psi},U] e^{-S_E} O_2[\psi,\bar{\psi},U] O_1[\psi,\bar{\psi},U]$$

- Path integral over the Euclidean action $S_{E,QCD}[\psi,\bar{\psi},U]$; (a sum over quantum fluctuations)
- Can be evaluated with *Markov Chain Monte Carlo* (using methods well established in statistical physics)

Systematic calculations and gauge field ensembles

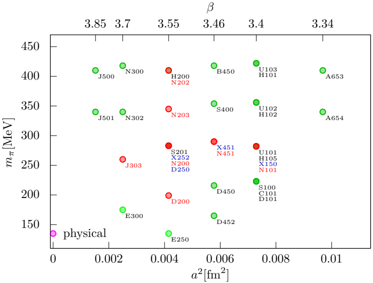
Important lattice systematics for bound-state calculations

- Taking the *continuum limit*: $a(g, m) \to 0$
- Taking the *infinite volume limit*: $L \to \infty$
- Calculation at (or extrapolation to) physical quark masses

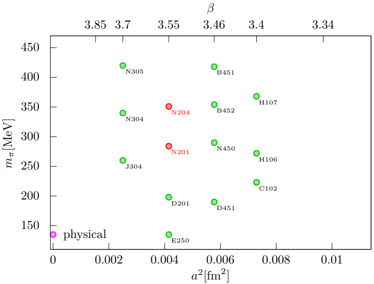
Example: CLS gauge-field library

`Bruno et al. JHEP 1502 043 (2015); Bali et al. PRD 94 074501 (2016)`

$Tr(M) = \text{const.}$



$m_s = \text{const.}$



`plot style by Jakob Simeth, RQCD`

Hierarchy of challenges on the lattice?

- **Relatively simple:** Masses of bound states; their quark mass-dependence, finite-volume dependence
  Caveats: signal to noise problems, computational cost
- **More difficult:** States close to threshold; QCD resonances; determination of scattering amplitudes through volume effects
- **Left for the future:** Structure of exotic states (through form factors, etc.)

- Hierarchy of projects:
  - Proof of principle (often single ensemble)
  - Explore quark mass dependence
  - Full spectroscopy calculation including continuum limit
  - Structure observables (transitions, form factors, . . .)

- Hierarchy of difficulties not the same as in experiment

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Tetraquarks - the T_{bb}

The $I(J^P) = 0(1^+)$ $ud\bar{b}\bar{b}$ tetraquark, T_{bb}, is the most concrete pure-tetraquark candidate phenomenologically and from the lattice in terms of being deeply-bound and strong-interaction-stable.
Cousin of the T_{cc} but likely has quite different physics,

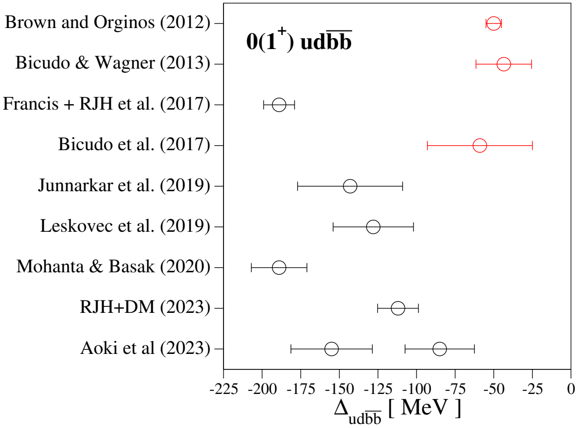
T_{bb} **bound by** ≈ 100 **MeV**, T_{cc} by 360 KeV

T_{bb} often described by the diquark picture:

- "Good" (attractive) light diquark $(u^T C\gamma_5 d)$ - lighter diquark increases binding
- Color-Coulomb heavy antidiquark $(\bar{b}C\gamma_i\bar{b}^T)$ - deeper binding as heavy mass gets heavier

No Wick-contractions with annihilation $\rightarrow$ easy to compute on the lattice!

Overview of Lattice $I(J^P) = 0(1^+)$ T_{bb} determinations



- Red: Static b-quarks; Black: Lattice NRQCD b quarks

An aside: tuning lattice NRQCD

R.J. Hudspith, DM, PRD 106, 034508 (2022)
R.J. Hudspith, DM, PRD 107, 114510 (2023)

The current state of the art in heavy-light multiquark states utilises lattice NRQCD for b-quarks

- Fully non-perturbative tuning of lattice NRQCD
- Runs with a random distribution for the action parameters
- Let the neural network make parameter predictions
- Due to additive mass we must only consider splittings
- 7-parameter tuning, bare mass aM_0 and corrections c_i
- Tuning precision is around 1%

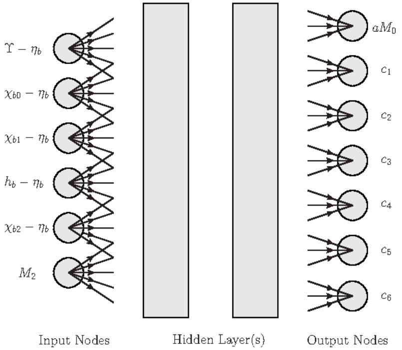
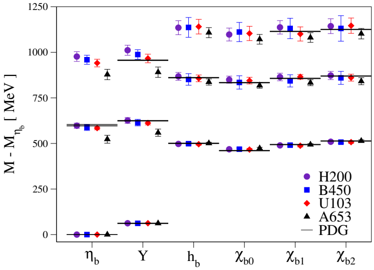


Figure: Schematic picture of our NRQCD setup

Excited bottomonium spectrum from our tuning



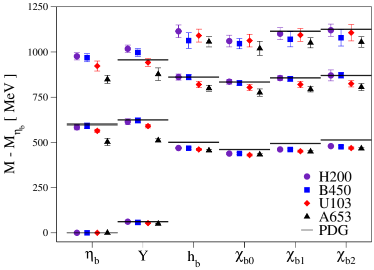


Figure: (Left) neural network tuning for excited bottomonia, (Right) tree-level tuning.

- Higher S- and P-wave states serve as a check whether our tuning leads to reasonable results
- Main results from the lattice spacing of U103; H200 used to estimate systematics

Our result - many configurations at many masses

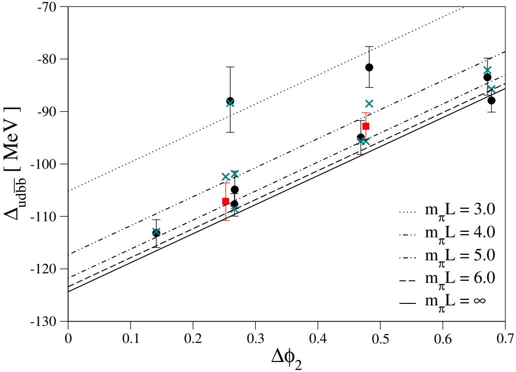


Figure: Mass and finite volume dependence of the binding energy of our T_{bb}

Heavy pion mass $\rightarrow$ shallower binding

Exponential finite volume effects $\rightarrow$ (deeply) bound state!

The sad aspect of T_{bb}: Difficult to see at the LHC

- T_{bb} is very heavy (≈ 10.5 GeV) and decays weakly
- A possible exemplary decay channel could be
  see Phys.Rev.Lett. 118 (2017) 14, 142001 - A. Francis, RJH et al.:

$$T_{bb} \to B^+ \bar{D}^0$$

- It is unlikely to be found anytime soon at the LHC

- Obvious next candidate 0^+ or 1^+ $ud\bar{c}\bar{b}$ "T_{cb}"
  potentially unbound or very weakly bound, due to the reduction of binding from the heavy antidiquark.
- Further exotic states $ud\bar{s}\bar{b}$ or $us\bar{c}\bar{b}$
  seem to be unlikely by diquark picture but worth investigating as some models predict these being deeply bound (mostly Chiral Quark models)

The $0^+/1^+$ T_{cb} - the jury is out!

- Could be shallow bound states or resonances.

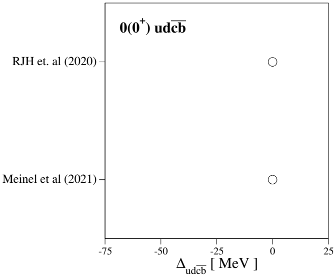


Figure: 0^+ and 1^+ $ud\bar{c}\bar{b}$ tetraquark binding energies

If bound, it is so shallow it will decay electromagnetically via $T_{bc} \to \bar{D}B\gamma$ (Phys.Rev.D 99 (2019) 5, 054505 - A. Francis, RJH, R. Lewis, K. Maltman). Errors for the "no-binding" findings maybe 10-20 MeV.

Ruling out some other deeply-bound states

R.J. Hudspith *et al.* PRD 102 114506 (2020)

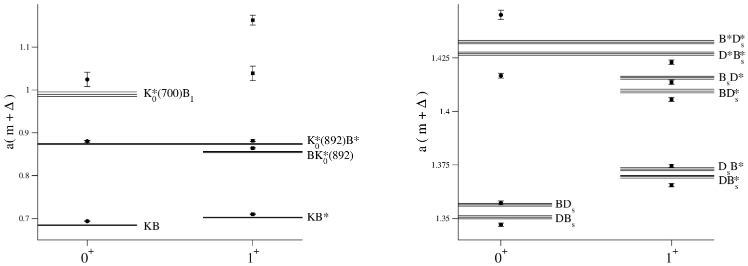
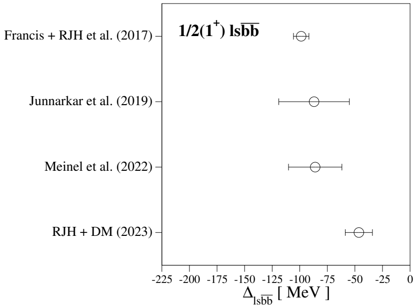


Figure: (Left) energies of 0^+ or 1^+ $ud\bar{s}\bar{b}$ states, (Right) similarly for a $\ell s\bar{b}\bar{c}$ tetraquark candidate.

- Energies suggest repulsion or only weak attractons (resonances?)
- Stark conflict with Chiral Quark models as no deep binding seen

The $\frac{1}{2}(1^+)$, T_{bbs}: Overview of binding energies



- Less bound than T_{bb} and heavier
- makes it even more difficult to detect experimentally, likely more interesting phenomenologically.

Note $(ud) \to (\ell s)$ gives $\approx$ 60 MeV reduction in binding energy.

The $\frac{1}{2}(1^+)$ $\ell c\bar{b}\bar{b}$ and $0(1^+)$ $sc\bar{b}\bar{b}$

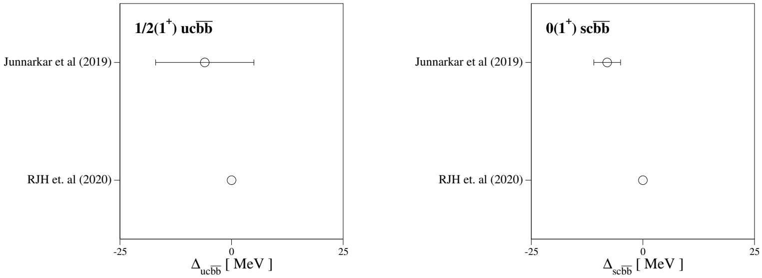


Figure: Binding energies of $\ell c\bar{b}\bar{b}$ (left) and $sc\bar{b}\bar{b}$ tetraquarks

Compatible with zero or very shallow binding

$\rightarrow$ more evidence that the simple diquark picture describes these states well

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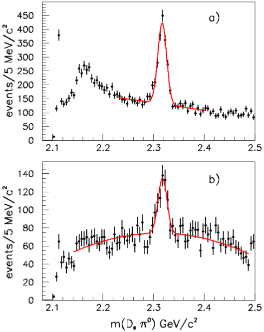
Exotic D_s and B_s candidates

Established s and p-wave hadrons:

D_s ($J^P = 0^-$) and D_s^* (1^-)
$D_{s0}^*(2317)$ (0^+), $D_{s1}(2460)$ (1^+),
$D_{s1}(2536)$ (1^+), $D_{s2}^*(2573)$ (2^+)

B_s ($J^P = 0^-$) and B_s^* (1^-)
?
$B_{s1}(5830)$ (1^+), $B_{s2}^*(5840)$ (2^+)

$D_{s0}^*(2317)$:
PRL 90 242001 (2003)



- Corresponding $D_0^*(2400)$ and $D_1(2430)$ are broad resonances
- Peculiarity: $M_{c\bar{s}} \approx M_{c\bar{d}}$ Is this really the case?
- Additional exotic states are expected (in the sextet representation)
- B_s cousins of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ not (yet) seen in experiment

Systematic uncertainties and final result

R.J. Hudspith, DM, PRD 107, 114510 (2023)

Resulting binding energies:

$$\Delta_{B_{s0}^*}(0,\infty,0) = -75.4(3.0)_{\text{Stat.}}(13.7)_{\text{a}}\ [\text{MeV}],$$
$$\Delta_{B_{s1}}(0,\infty,0) = -78.7(3.7)_{\text{Stat.}}(13.4)_{\text{a}}\ [\text{MeV}].$$

- Small uncertainty from statistics + combined extrapolation
- Largest systematics from usage of NRQCD/discretization effects
- Central value shifted by applying half the mass difference between two different lattice-spacings
- All other explored uncertainties (finite volume shapes, modified quark-mass dependence, etc.) small

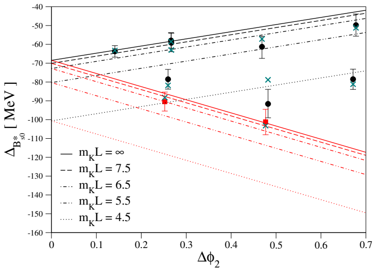
B_{s0}^* and B_{s1}: Chiral – infinite volume extrapolation

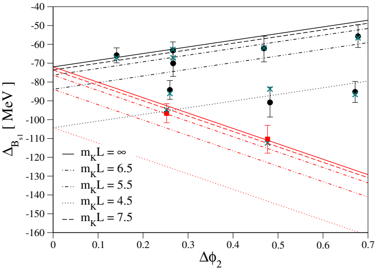
R.J. Hudspith, DM, PRD 107, 114510 (2023)

Combined extrapolation for the binding energy:

$$\Delta_{B_{s0}^*/B_{s1}}(\Delta\phi_2, m_K L, a) = \Delta_{B_{s0}^*/B_{s1}}(0, \infty, a)\left(1 + A\Delta\phi_2 + Be^{-m_K L}\right)$$

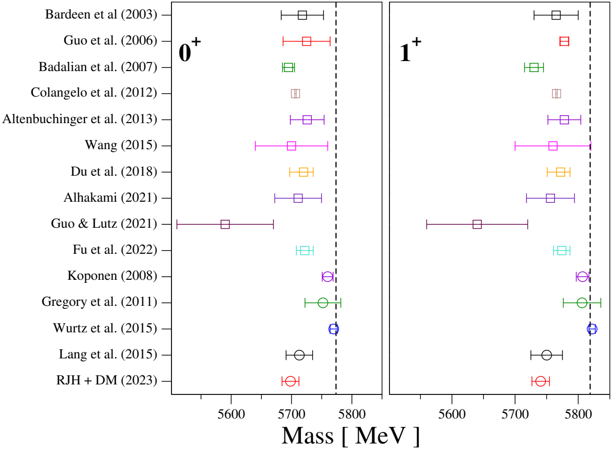
$$\Delta\phi_2 = \phi_2^{\mathrm{Lat}} - \phi_2^{\mathrm{Phys}} \qquad ; \qquad \phi_2 = 8t_0 m_\pi^2$$





- Two different am_s trajectories to control strange-quark dependence

Model and lattice results for the B_{s0} and B_{s1} mesons.



- Dominant uncertainty in our calculation from the use of Lattice NRQCD
- Could likely be improved by using an RHQ action for the b-quark

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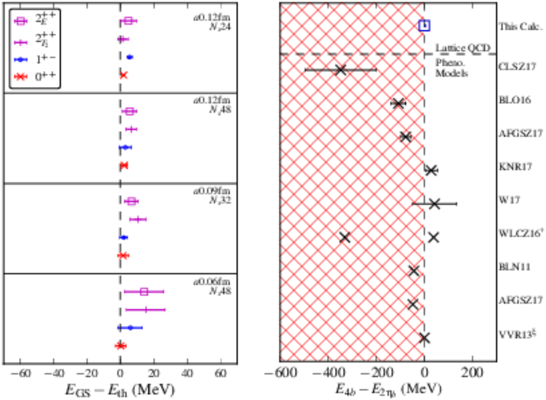
The T_{bbbb}: Comparing Lattice QCD and Models

C. Hughes and E. Eichten, PRD 97 054505 (2018)



- Several model predictions for a $bb\bar{b}\bar{b}$ tetraquark but emphatically ruled-out from being deeply bound from the lattice.

Dibaryons with beauty quarks

P. Junnarkar and N. Mathur, PRL 123 162003 (2019)

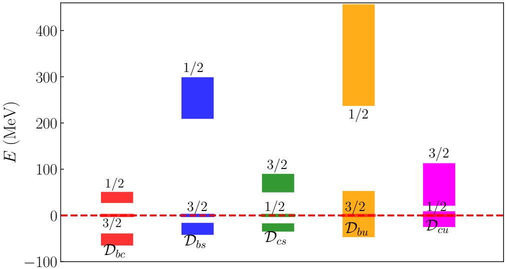


Figure: Binding energies of various deuteron-like dibaryons

- Studies D_{q_1,q_2} states made of 2 baryons with valence quarks $(q_1q_1q_2)$ and $(q_1q_2q_2)$
- Deeply bound deuteron-like dibaryons $\Omega_c\Omega_{cc}$, $\Omega_b\Omega_{bb}$, $\Omega_{ccb}\Omega_{cbb}$ states are seen to be strong-interaction stable

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Heavy-quark exotics from the lattice

- Lattice QCD is good at determining deeply-bound states and can rule out phenomenological models for states not yet observed in experiment
- The calculations are systematically-improvable and we are seeing convergence for the easiest-to-compute quantities such as the T_{bb}
- The smoking-gun tetraquark state T_{bb} is very difficult to see in current experiments; it is worth exploring weaker-bound candidates such as T_{bc}
- More and more indications that the multi-quark exotic spectrum at heavy masses is diverse
- Further insight can be gained from exploring the quark-mass dependence between charm and bottom.

Backup slides

CLS ensembles used for heavy-light mesons

R.J. Hudspith, DM, PRD 107, 114510 (2023)

Ensemble	Mass trajectory	$L^3 \times L_T$	$N_{\text{Conf}} \times N_{\text{Prop}}$
U103	$\text{Tr}[M] = C$	$24^3 \times 128$	1000×23
H101	$\text{Tr}[M] = C$	$32^3 \times 96$	500×12
U102	$\text{Tr}[M] = C$	$24^3 \times 128$	732×18
H102	$\text{Tr}[M] = C$	$32^3 \times 96$	500×16
U101	$\text{Tr}[M] = C$	$24^3 \times 128$	600×18
H105	$\text{Tr}[M] = C$	$32^3 \times 96$	500×16
N101	$\text{Tr}[M] = C$	$48^3 \times 128$	537×18
C101	$\text{Tr}[M] = C$	$48^3 \times 96$	400×16
H107	$\widetilde{m_s} = \widetilde{m_s}^{\text{Phys.}}$	$32^3 \times 96$	500×16
H106	$\widetilde{m_s} = \widetilde{m_s}^{\text{Phys.}}$	$32^3 \times 96$	500×16
H200	$\text{Tr}[M] = C$	$32^3 \times 96$	500×28

NRQCD action

Typical tadpole-improved NRQCD action (here we will use n=4)

`Lepage et al., PRD 46, 4052-4067 (1992)`

$$
\begin{aligned}
H_0 =& -\frac{1}{2aM_0}\Delta^2, \\
H_I =& \left(-c_1\frac{1}{8(aM_0)^2} - c_6\frac{1}{16n(aM_0)^2}\right)(\Delta^2)^2 + c_2\frac{i}{8(aM_0)^2}(\tilde{\Delta}\cdot\tilde{E} - \tilde{E}\cdot\tilde{\Delta}) + c_5\frac{\Delta^4}{24(aM_0)} \\
H_D =& -c_3\frac{1}{8(aM_0)^2}\sigma\cdot\left(\tilde{\Delta}\times\tilde{E} - \tilde{E}\times\tilde{\Delta}\right) - c_4\frac{1}{8(aM_0)}\sigma\cdot\tilde{B} \\
\delta H =& H_I + H_D.
\end{aligned}
$$

Propagators generated through symmetric evolution equation

$$
G(x,t+1) = \left(1-\frac{\delta H}{2}\right)\left(1-\frac{H_0}{2n}\right)^n \tilde{U}_t(x,t_0)^\dagger \left(1-\frac{H_0}{2n}\right)^n \left(1-\frac{\delta H}{2}\right) G(x,t).
$$

- We also tune a $\mathcal{O}(v^6)$ action with tree-level coefficients for the higher order terms

Input used for the tuning

Consider only quark-line connected parts of simple meson operators

$$O(x) = (\bar{b}\Gamma(x)b)(x),$$

State	PDG mass [GeV]	$\Gamma(x)$
$\eta_b(1S)$	9.3987(20)	γ_5
$\Upsilon(1S)$	9.4603(3)	γ_i
$\chi_{b0}(1P)$	9.8594(5)	$\sigma \cdot \Delta$
$\chi_{b1}(1P)$	9.8928(4)	$\sigma_j\Delta_i - \sigma_i\Delta_j \; (i \neq j)$
$\chi_{b2}(1P)$	9.9122(4)	$\sigma_j\Delta_i + \sigma_i\Delta_j \; (i \neq j)$
$h_b(1P)$	9.8993(8)	Δ_i

Table: Table of lattice operators used and their continuum analogs.