Lattice QCD w/ a focus on B and D form factors, $R(D^*)$, and quark masses

Andrew Lytle (FNAL-MILC collaboration)
University of Illinois @ Urbana-Champaign

04.07.23
Beauty 2023
Clermont-Ferrand, France

Introduction

- The LHC era has produced/is producing an incredible wealth of data
  - New processes
  - Ever increasing levels of precision
- Stringent Standard Model tests request corresponding levels of theoretical precision
  - $\rightarrow$ Tests of the CKM matrix
  - Detailed comparisons with observed data (R-ratios, shape data, ...)

Outline

- Lattice QCD - broad overview
- Leptonic decays (briefly)
- Semileptonic decays
  - $B_{(s)}$ decays
  - $D_{(s)}$ decays
- Quark masses
- Summary & Conclusion

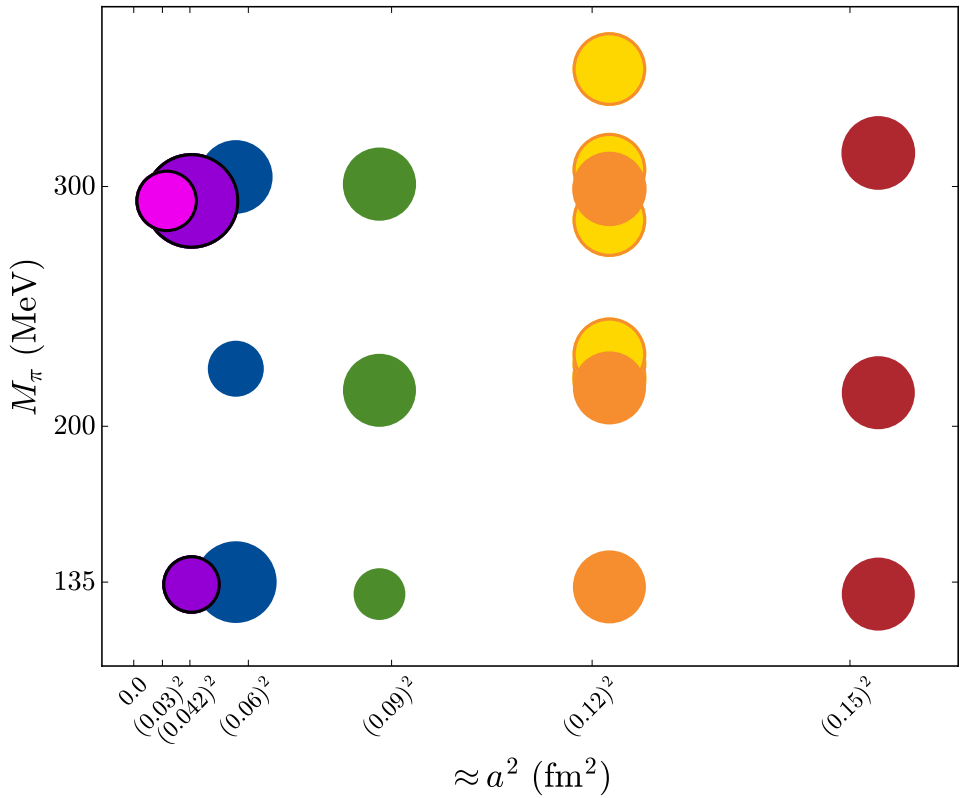
Lattice QCD - Broad overview

- From-first-principles approach to QCD/hadronic physics.
- Discretize the Euclidean time path integral $\rightarrow$ finite lattice spacing, a.
- Calculate hadronic observables using Monte Carlo & supercomputers, take $a \rightarrow 0$.
- Works well for
  - Leptonic decays (two-point functions)
  - Semileptonic decays (three-point functions)

  and is *systematically improvable*.

Lattice QCD - Broad overview

Example: MILC lattice ensembles 1712.09262



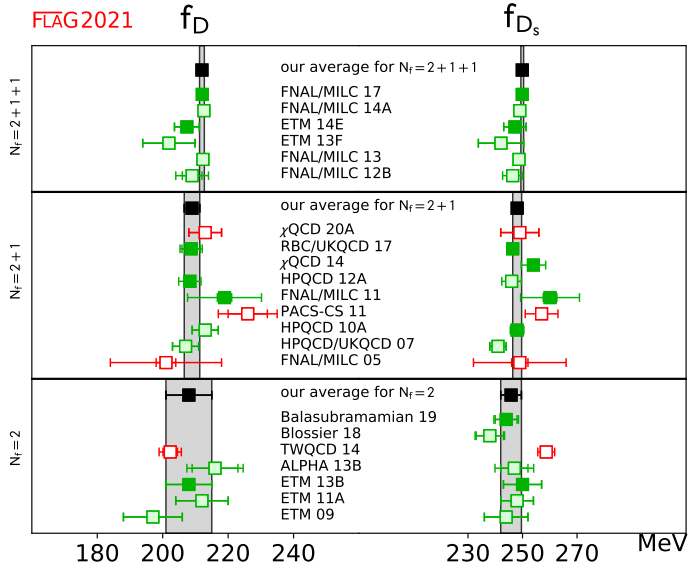
- 6 lattice spacings, finest $a \approx 0.03$ fm
- Physical pion masses at all but finest spacing

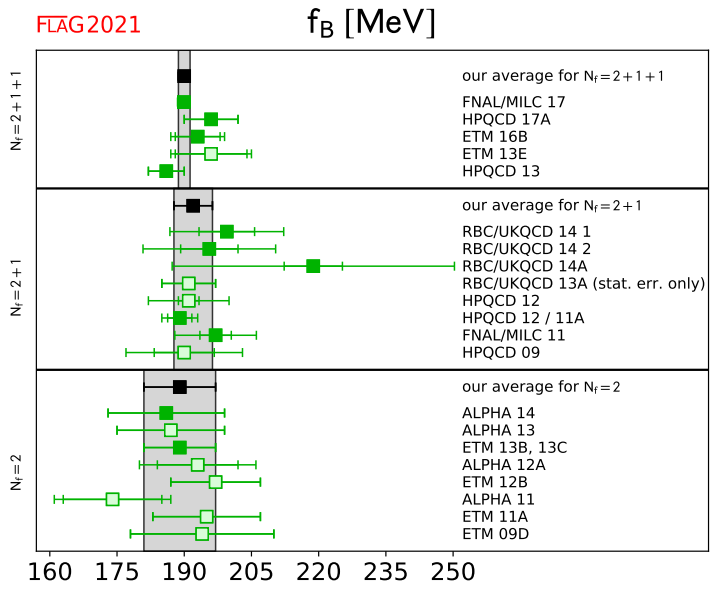
Leptonic decays

Leptonic decays - I

- Relatively straightforward to compute, with meson 2-point functions.
- Allow extraction of CKM matrix elements via leptonic decay measurements.
  - $f_D \to |V_{cd}| = 0.2173(47)_{\rm exp}(28)_{\rm em}(7)_{\rm latt}$
  - $f_{D_s} \to |V_{cs}|$ (sl decays now competitive)
  - $f_B \to |V_{ub}|$
- $f_{B_{(s)}}$ used for SM prediction of rare processes $B_{(s)} \to \mu^+\mu^-$
- In all cases lattice QCD uncertainties well below experimental uncertainties

Leptonic decays - II





- Sub-percent precision obtained, results from different collaborations in good agreement.
- To improve substantially requires including QED and strong-isospin breaking
- Substantial progress including QED corrections to these processes (cf radiative decays)

$B_{(s)} \to \mu^+\mu^-$ and $B \to \ell\nu$

$B_{(s)} \to \mu^+\mu^-$

- Highly suppressed FCNC process can give important constraints on new physics

- $\mathcal{B}(B_s \to \mu\mu)_{\rm SM} = 3.64(4)_{f_{B_s}}(8)_{\rm CKM}(7)_{\rm other} \times 10^{-9}$
  $\mathcal{B}(B_s \to \mu\mu)_{\rm exp} = 2.69(37) \times 10^{-9}$ ATLAS-CONF-2020-049

- $\mathcal{B}(B^0 \to \mu\mu)_{\rm SM} = 1.00(1)_{f_B}(2)_{\rm CKM}(2)_{\rm other} \times 10^{-10}$
  $\mathcal{B}(B^0 \to \mu\mu)_{\rm exp} < 1.6 \times 10^{-10}$

B leptonic decays

- $B \to \tau\nu$ may reach 3-5% precision at Belle II

- $B \to \mu\nu$ may reach $\sim 7\%$ precision at Belle II 1808.10567

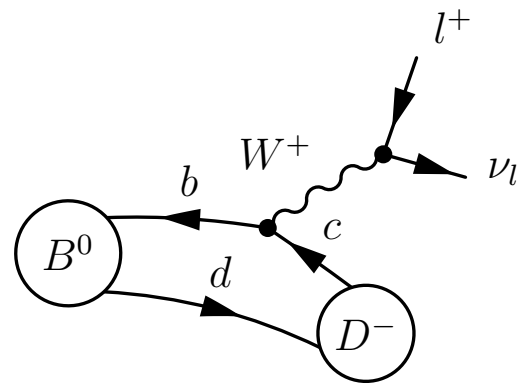
$B_{(s)}$-meson semileptonic decays

Semileptonic decays - I

SL Decay processes critical inputs for heavy flavor studies.

Lattice predictions needed for:

- Extracting CKM matrix elements from expt'l measurements
- Pure SM predictions of R-ratios
- SM predictions $\frac{d\Gamma}{dq^2}$, etc.



Lattice calculations based on 2- & 3-point correlators give matrix elements $\rightarrow f_i(q^2)$

Semileptonic decays - II

Tree level decays:

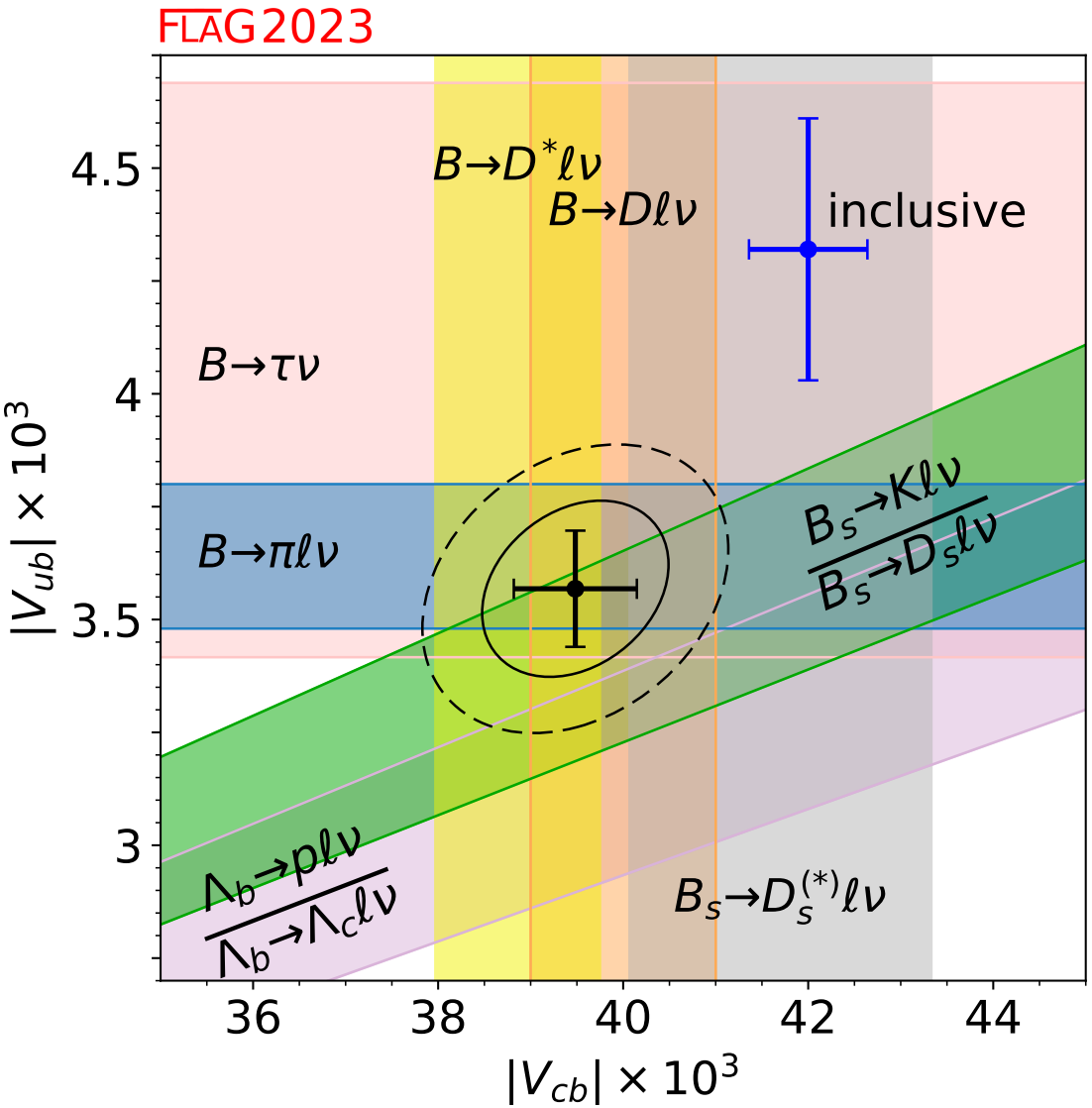
- $B \to D^{(*)} :\to |V_{cb}|$
- $B \to \pi :\to |V_{ub}|$
- $D \to K :\to |V_{cs}|$
- Baryon decays: $\to |V_{ub}|/|V_{cb}|$

Many different processes and results, can only highlight a few illustrative developments!

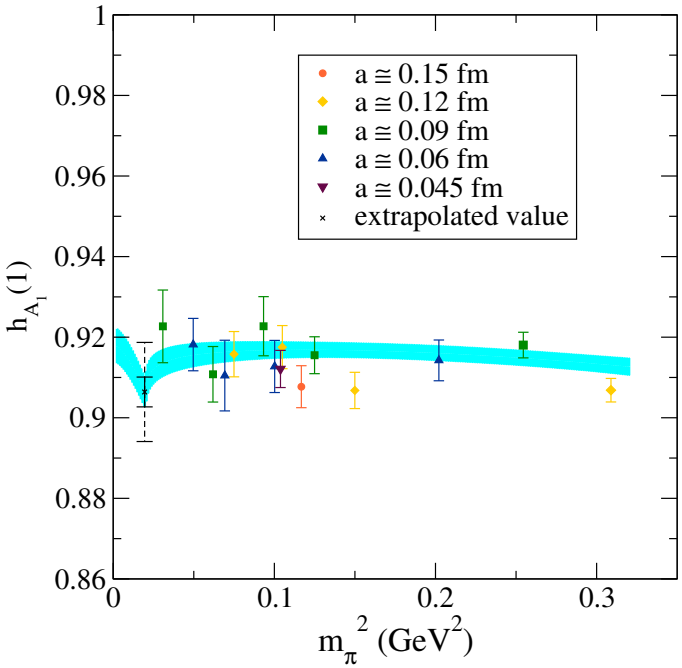
Loop level decays:

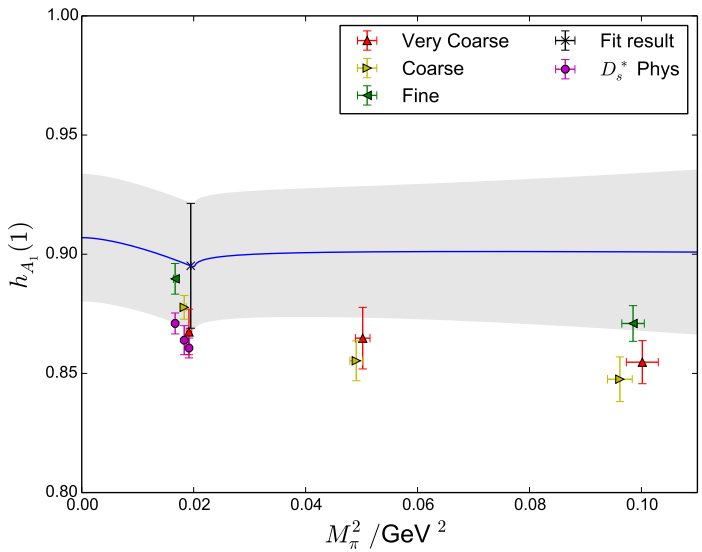
- Lattice form factors important for $B \to K\ell^+\ell^-$
- $B \to K^*\ell^+\ell^-$ requires recent theory developments in handling resonances/multi-hadron final states.

Semileptonic decays - III



$B \to D^*$ at zero recoil from LQCD





FNAL/MILC 1403.0635

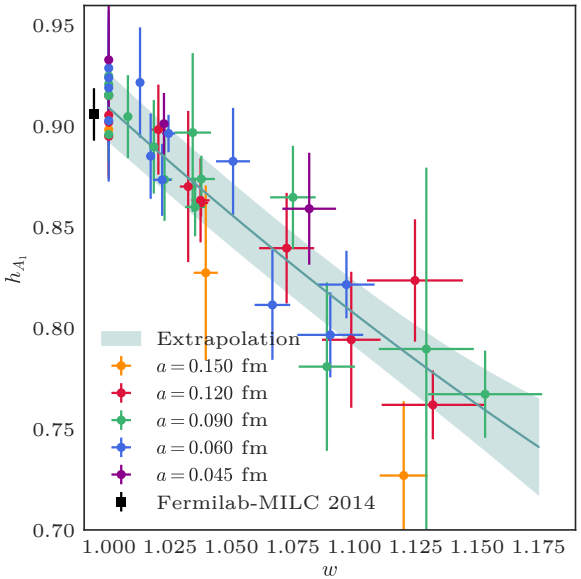
- $n_f = 2+1$ MILC asqtad ensembles
- Clover b with Fermilab interpretation
- $h_{A_1}(1) = 0.906(4)(12)$

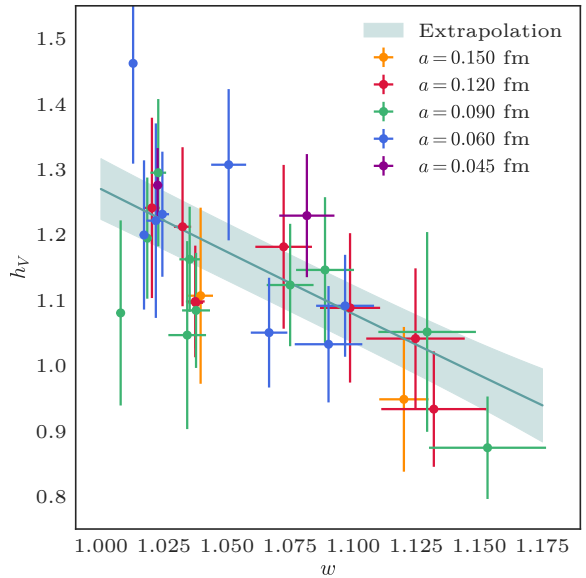
HPQCD 1711.11013

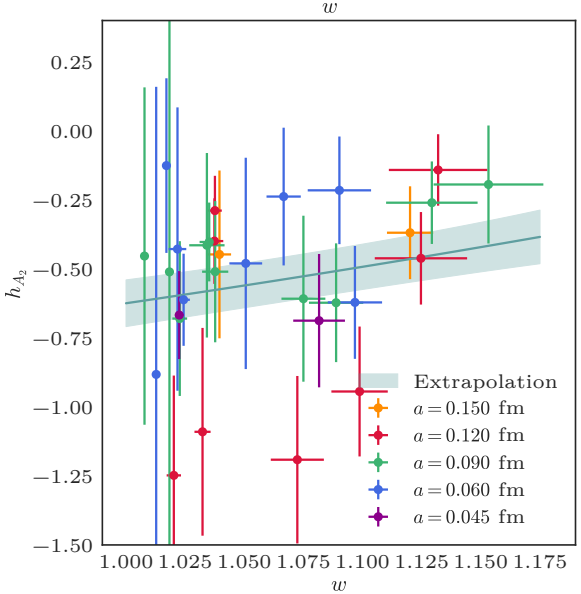
- $n_f = 2+1+1$ MILC HISQ ensembles
- NRQCD b quark
- $h_{A_1}(1) = 0.895(10)(24)$
- $h^s_{A_1}(1) = 0.883(12)(28)$

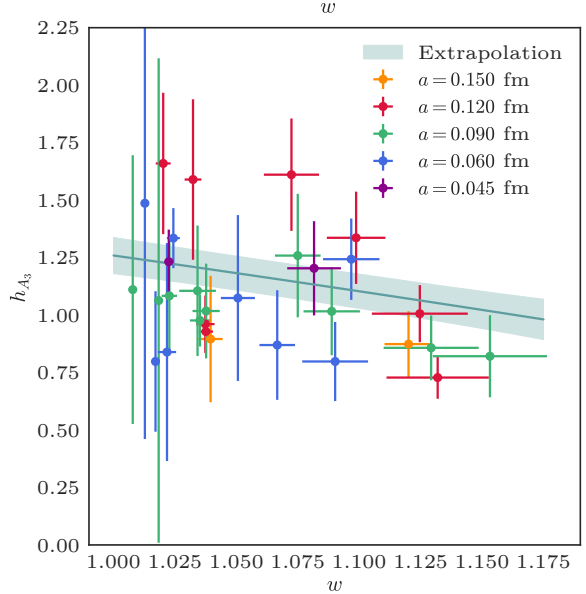
FNAL/MILC $B \to D^*$

2105.14019





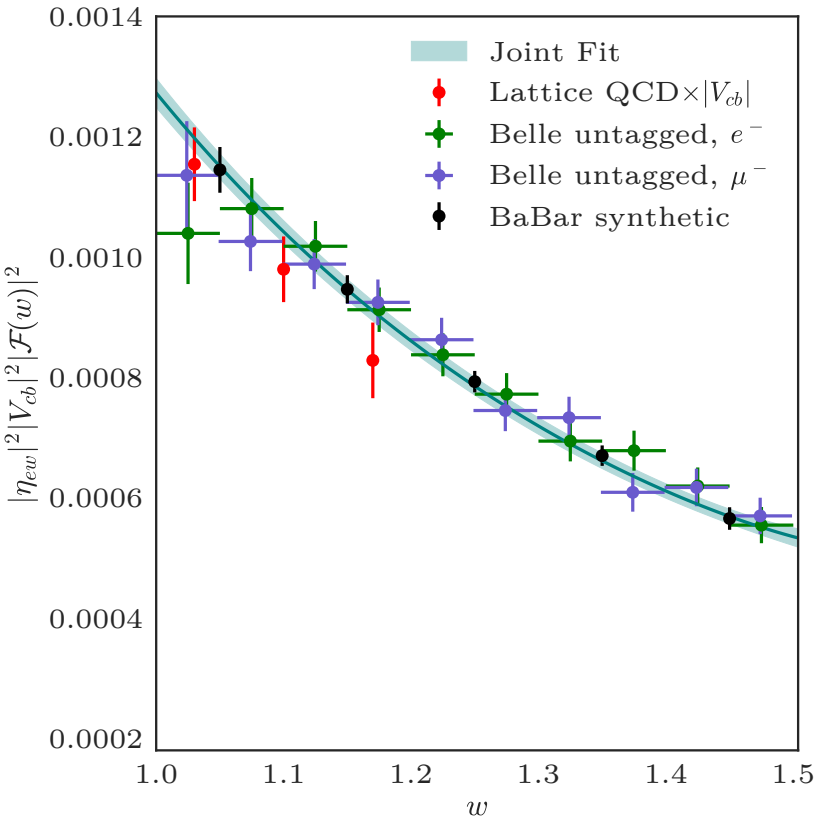


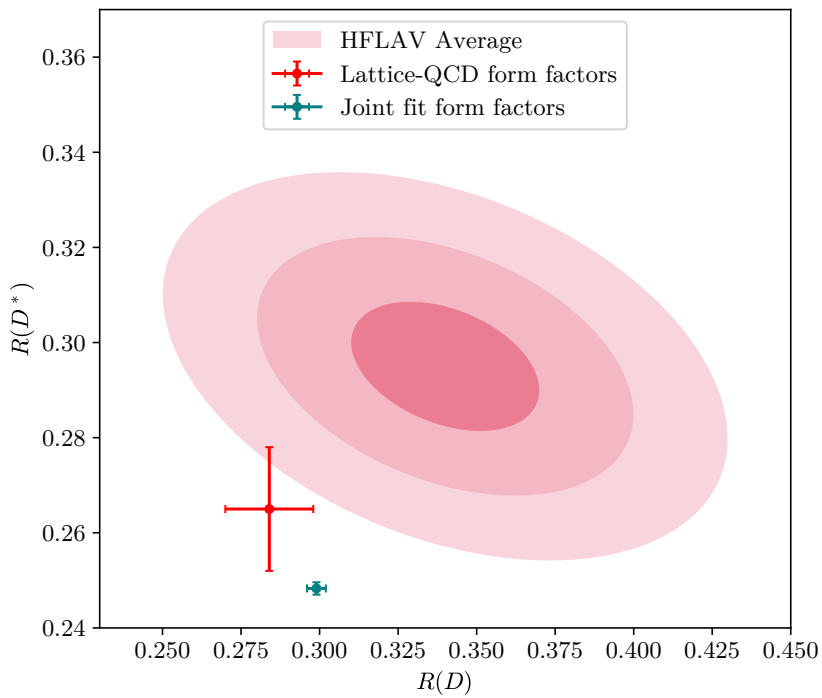


Figs. courtesy A. Vaquero

FNAL/MILC $B \to D^*$

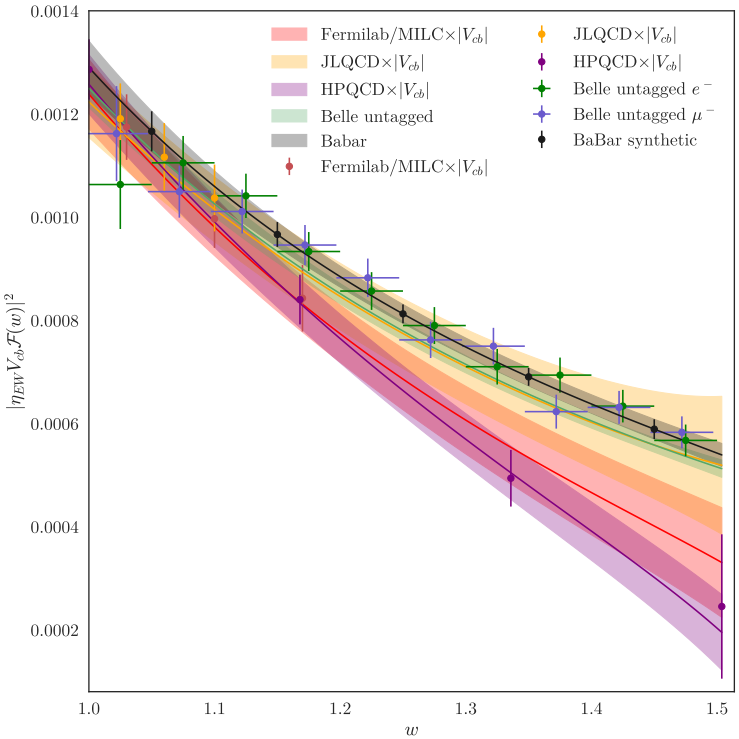
2105.14019

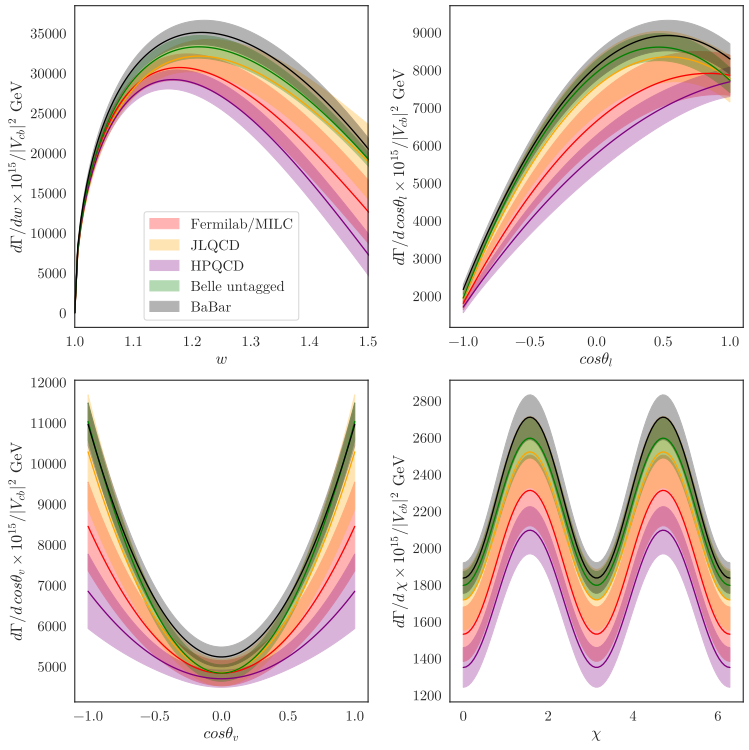




Figs. courtesy A. Vaquero

$B \to D^*$ comparisons





Figs. courtesy A. Vaquero

Lattice refs:

- JLQCD 2306.05657
- HPQCD 2304.03137
- FNAL/MILC 2105.14019

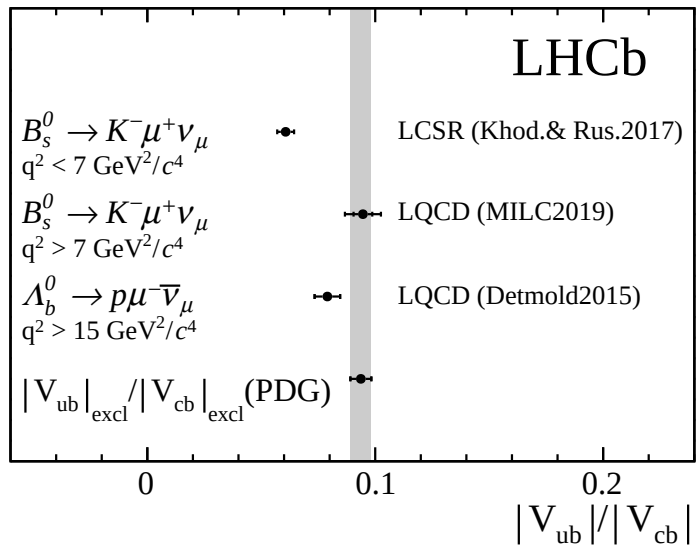
$B_s \to D_s^{(*)}\,\mu\,\nu_\mu$ at LHCb [2001.03225]

New set of measurements based on LHC Run 1 data.

- CLN: $|V_{cb}| = 41.4(6)_{\rm stat}(9)_{\rm syst}(12)_{\rm ext} \times 10^{-3}$
- BGL: $|V_{cb}| = 42.3(8)_{\rm stat}(9)_{\rm syst}(12)_{\rm ext} \times 10^{-3}$
- In this analysis, LQCD inputs improve statistical precision by 20% and 50% for CLN and BGL, respectively.

See also talk by Mirco Dorigo @LHCP2020
indico.cern.ch/event/856696/contributions/3742179/

New CKM constraints from LHCb. Based on baryon decays 1504.01568, and first msm't of $B_s \to K\mu\nu$ 2012.05143.

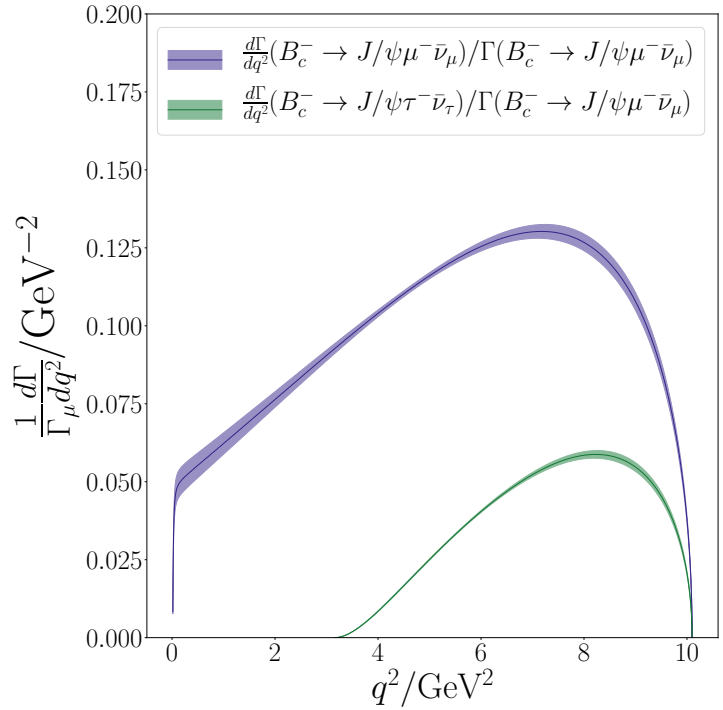


Extractions use lattice form factor calculations of baryon decays ($\Lambda_b \to \Lambda_c$ and $\Lambda_b \to p$ 1503.01421) and $B_s \to K$ and $B_s \to D_s$ form factors, respectively.

$R(B_c \to J/\psi)$

First lattice calculation of $B_c \to J/\psi$ 2007.06956
(All form factors across q^2 range)

- LHCb 1711.05623
  $R(J/\psi) = 0.71(17)_{\text{stat}}(18)_{\text{syst}}$
- w/in 2σ of theory range [0.25, 0.28]
- LQCD result: 0.2582(38)



FNAL-MILC all-HISQ semileptonic decays

Treatment of heavy quarks

Treatment of c and especially b quarks challenging in lattice simulation due to lattice artifacts which grow as $(am_h)^n$

- May use an effective theory framework to handle the b quark.
  - Fermilab method, RHQ, OK, NRQCD
  - Pros: Solves problem w/ am_h artifacts.
  - Cons: Requires matching, can still have ap artifacts.

- Also possible to use relativistic fermion provided a is sufficiently small $am_c \ll 1$, $am_b < 1$.
  - Use improved actions e.g. $\mathcal{O}(a^2) \to \mathcal{O}(\alpha_s a^2)$
  - Pros: Absolutely normalised current, straightforward continuum extrap.
  - Cons: Numerically expensive, extrapolate $m_h \to m_b$.

allhisq simulations

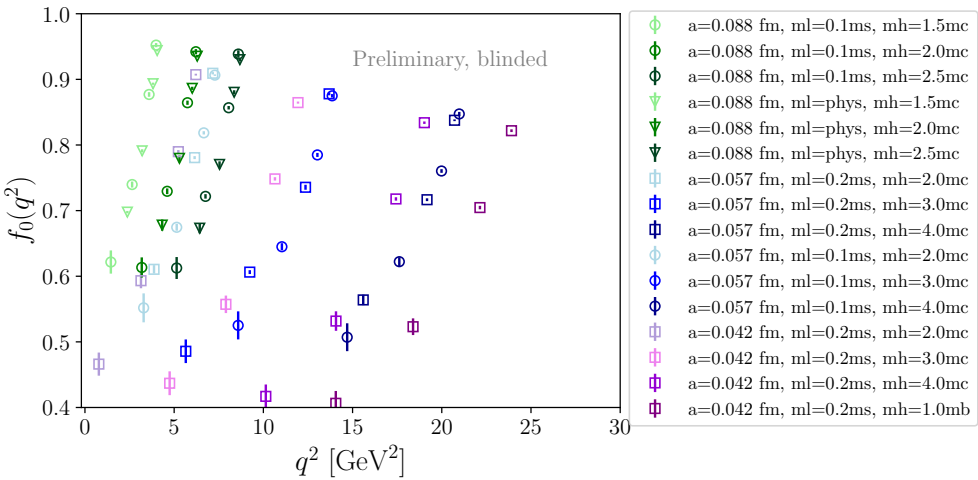
- Here we simulate *all* quarks with the HISQ action.
- Unified treatment for wide range of $B_{(s)}$ (and $D_{(s)}$) to pseudoscalar transitions
  - $B_{(s)} \to D_{(s)}$
  - $B_{(s)}/D_{(s)} \to K$
  - $B/D \to \pi$
- Ensembles with (HISQ) sea quarks down to physical at each lattice spacing.
- Enables correlated studies of ff *ratios*.

See 2022 Lattice proceeding for more details. 2301.09229

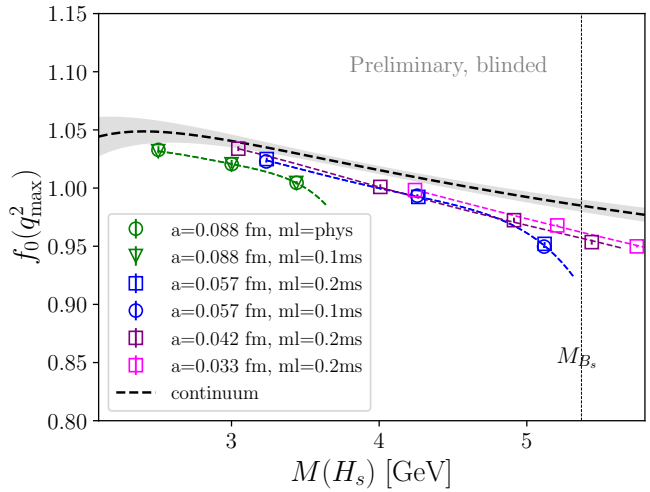
B-decays (preliminary)

2301.09229

$B_s \to K$



$B_s \to D_s$



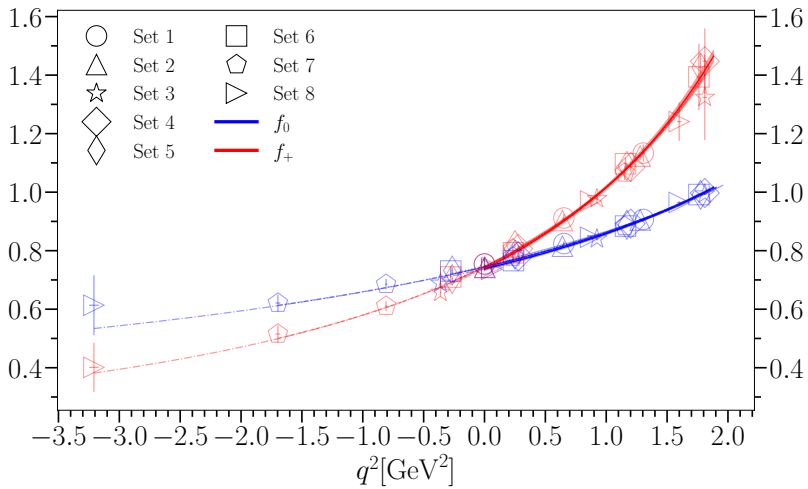
Preliminary results show

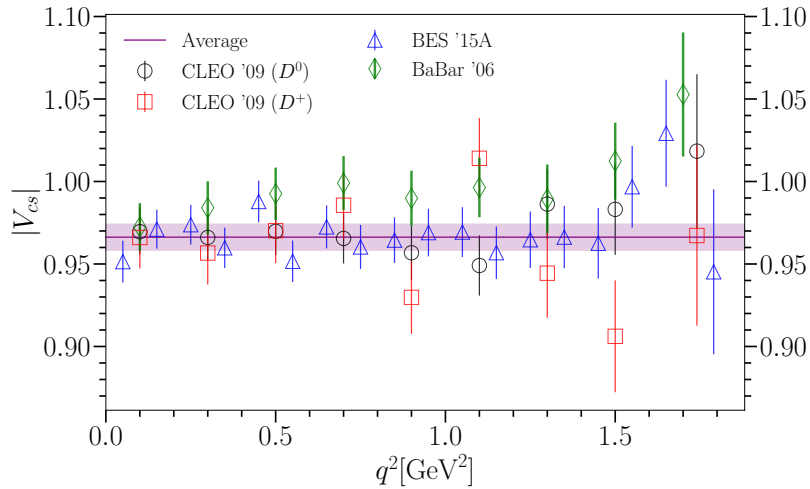
- Good q^2 coverage
- Good statistical precision
- Data at/beyond $M_{B_{(s)}} \to$ *interpolation* in heavy mass m_h

$D_{(s)}$-meson semileptonic decays

$|V_{cs}|$ from $D \to K$

New result from HPQCD 2104.09883. See also ETMC calculations 1706.03017, 1706.03657, 1803.04807.

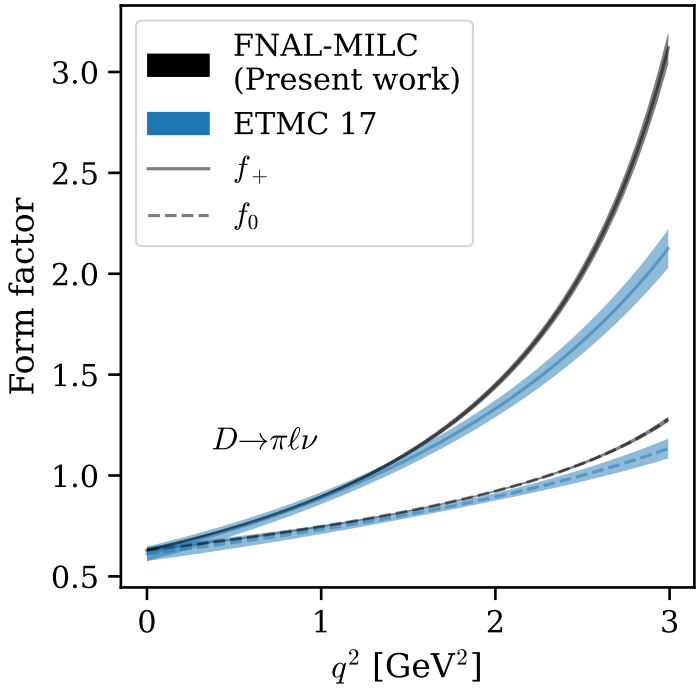


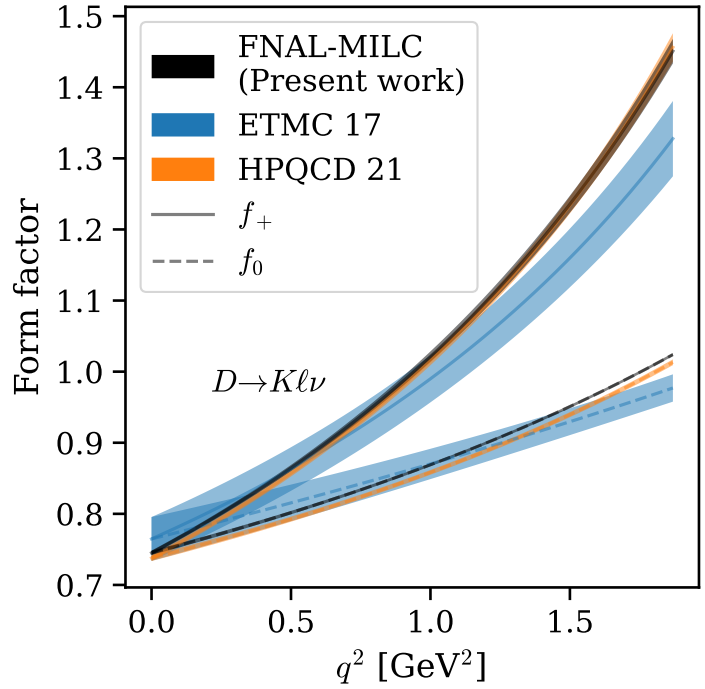


$$|V_{cs}| = 0.9663(53)_{\text{latt}}(39)_{\text{exp}}(19)_{\eta_{\text{EW}}}(40)_{\text{EM}}$$

Semileptonic determination of $|V_{cs}|$ now has smaller errors than leptonic determinations.

$D \to \pi, K$

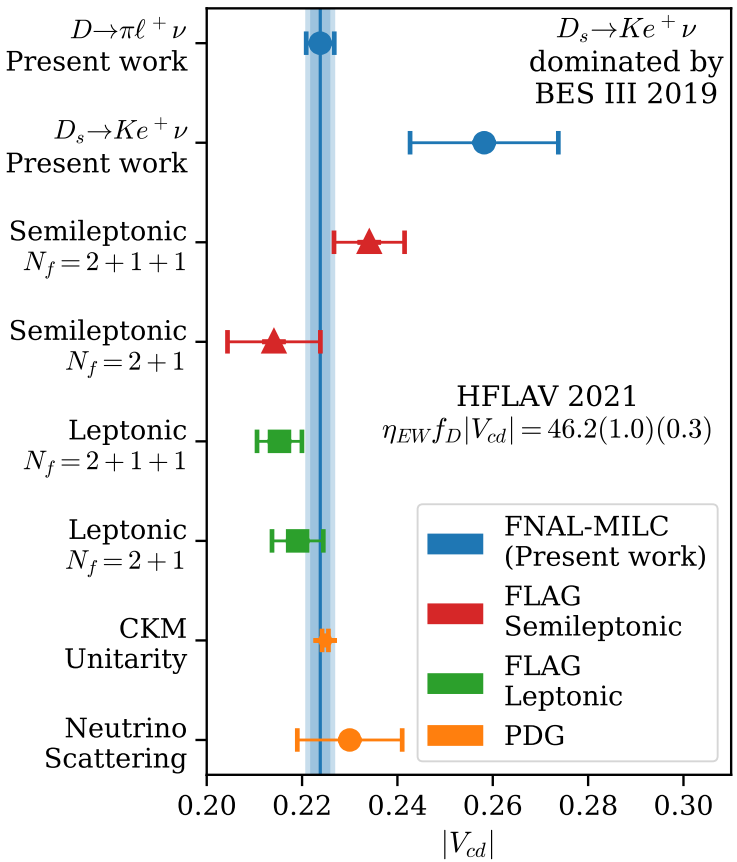


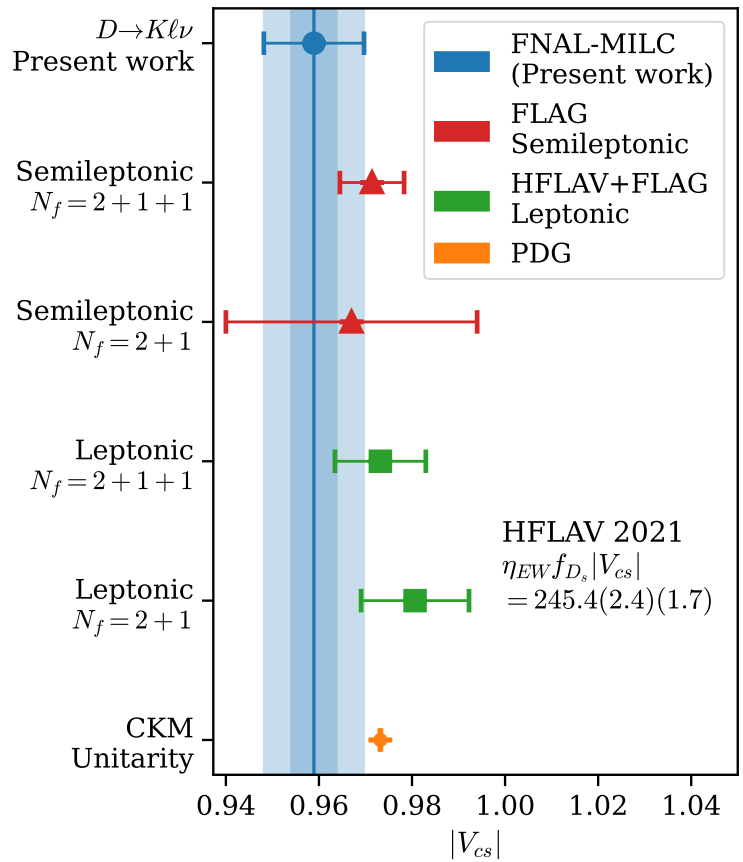


References:

- FNAL-MILC 2212.12648
- HPQCD 21 2104.09883
- ETMC 17 1706.03017

$D_{(s)} \to \pi, K$ $|V_{cd}|$ and $|V_{cs}|$ 2212.12648





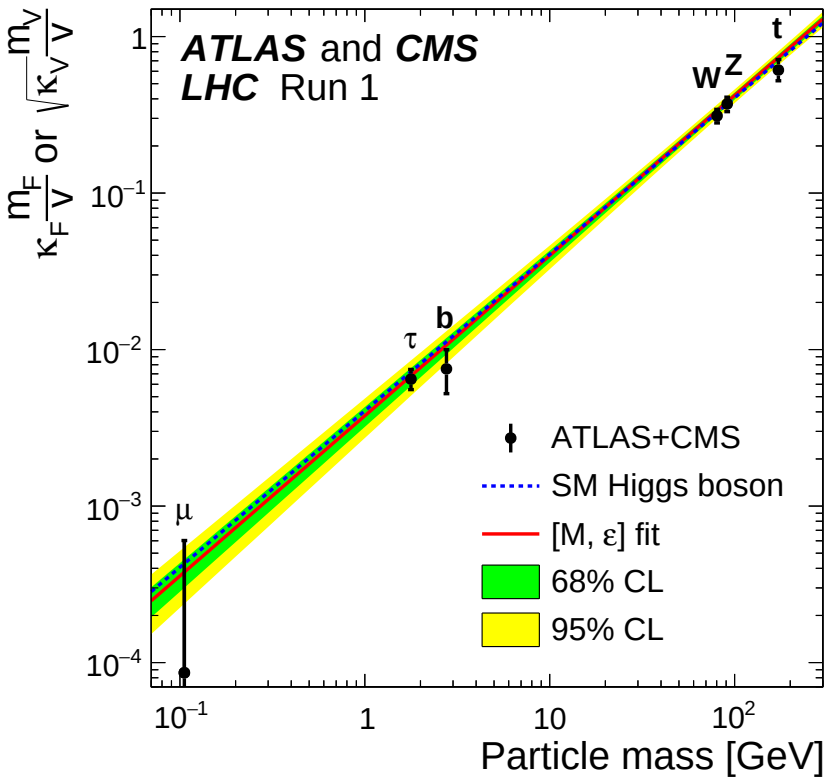
Semileptonic decays - Summary

- Thus far, successful collaboration with experiment in the LHC era
  - Theory predictions for new channels
  - Improved kinematic range
  - Improved precision
- Lattice errors roughly commensurate with experimental errors. In the next 5 years or so, these should continue to improve and lattice error may become sub-dominant.
- To go beyond this requires adding EM and strong isospin breaking effects.

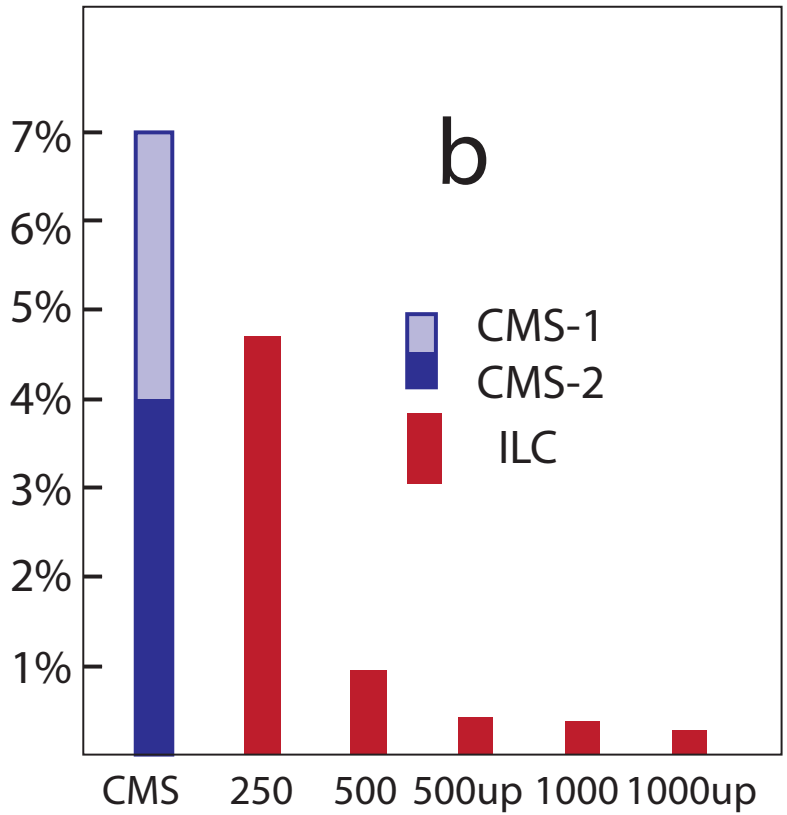
Quark masses

Quark masses - I

[1606.02266]



[1312.4974]



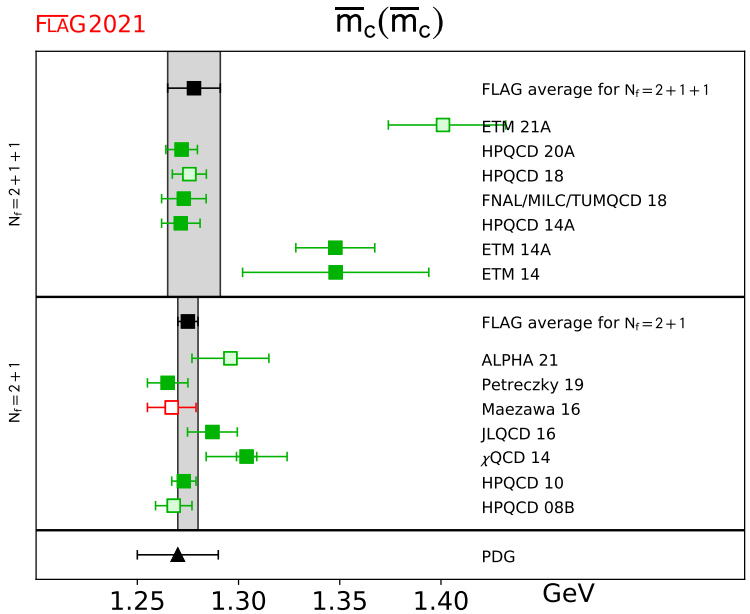
HL-LHC estimates $\sim 3\%$ $h b\bar{b}$, evidence for charm CMS:2022cju

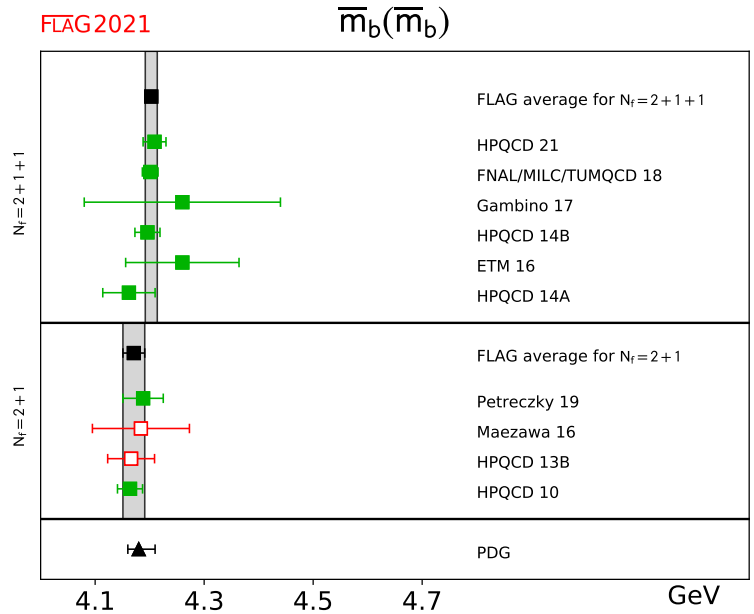
Est'd final ILC precision in $h b\bar{b}$ ($h c\bar{c}$) coupling: ~ 0.3 $(0.7)\%$.

Quark masses on the lattice

- Quarks are not asymptotic (physical) states due to confinement – mass cannot be measured directly.
- Quark masses are scheme and scale dependent, $m_q^{\text{scheme}}(\mu)$.
- Generally will quote results $m_q^{\overline{\text{MS}}}(\mu_{\text{ref}})$.
- Lattice input quark masses are non-universal (depend on discretisation), but can be connected to quark masses defined in a continuum scheme.

Quark masses - II



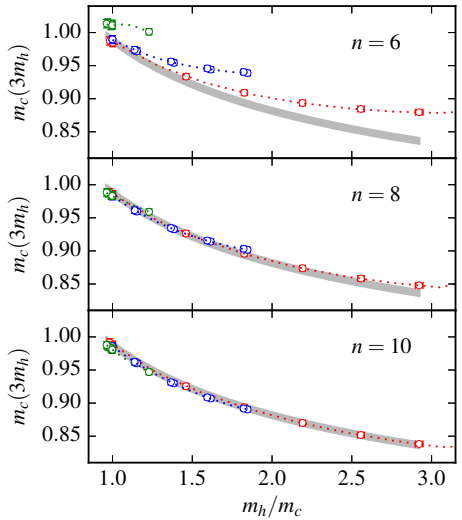


- A variety of different methods, with different systematics, have been developed. See 2110.15090 for a recent review.

- FLAG averages 2111.09849 $N_f = 2 + 1 + 1$
  $m_b(m_b) = 4.203(11)$ GeV $m_c(3\text{ GeV}) = 0.988(11)$

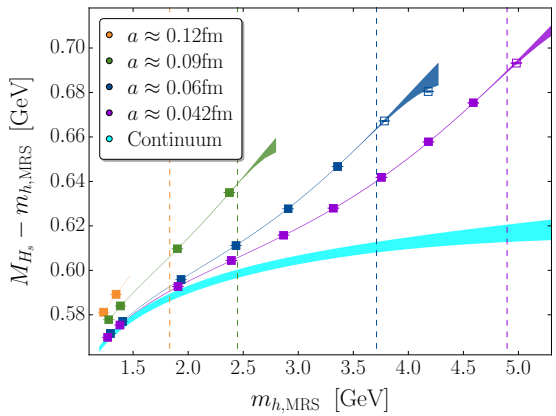
Quark masses - III

- Since CHARM 2015 - Highlights
  - Many new results: 13 since 2015, 4 since FLAG19.
    m_c: $6 \to 13$, $n_f = 3$: $5 \to 9$
    m_b: $4 \to 9$, $\quad n_f = 4$: $5 \to 13$
  - From more collaborations $\to$ differing discretizations/lattice artifacts under control.
  - New techniques with different sources of systematic uncertainty.
- QED corrections are small ($\lesssim 0.2\%$ for charm).
- Precision continues to improve.
- Complementary techniques - good agreement at sub-percent level.



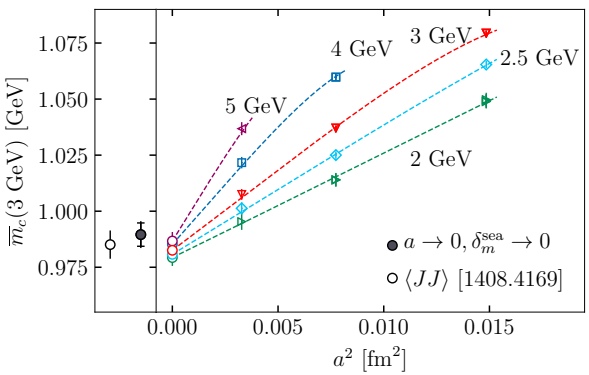
Time moments of charmonium correlators

1408.4169



Heavy-light meson masses and HQET

1802.04248



Quark-level diagrams on the lattice

1805.06225

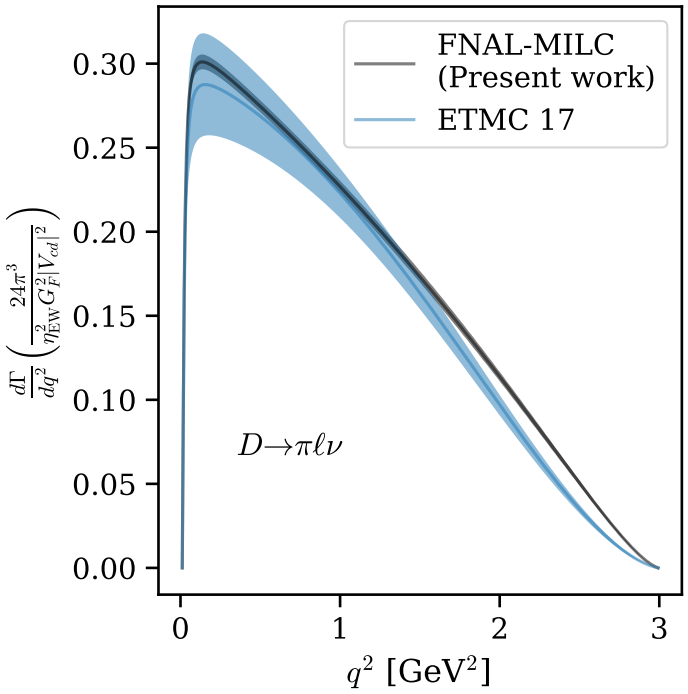
See also Schroedinger functional methods: 2101.02694 1802.05243

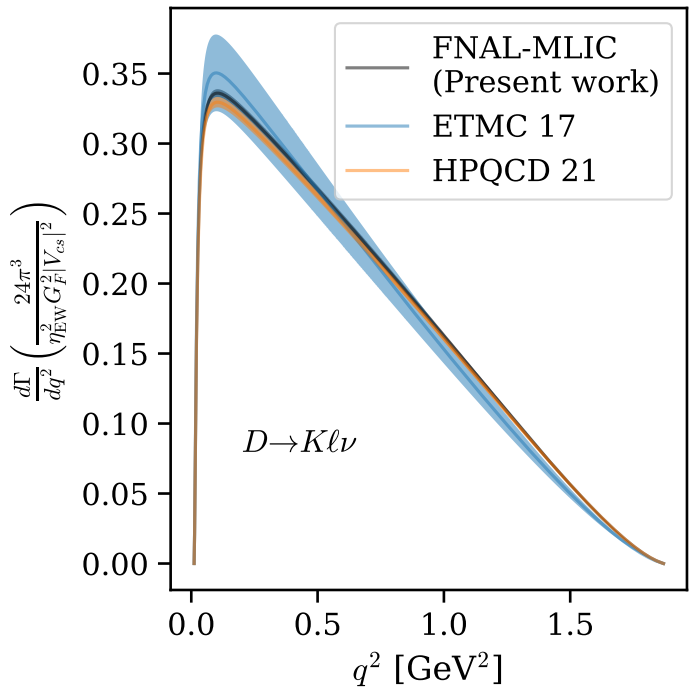
Summary

- We have looked at several places where lattice calculations impact precision SM tests
- Lattice QCD allows stringent comparisons of the Standard Model with experiment
  - Required for precision CKM studies
  - Rare processes and flavor anomalies
- Lattice theorists have kept pace with experimental advances at LHC, Belle II, and BES III
  - Calculations relevant for new msmt's
  - Improving precision

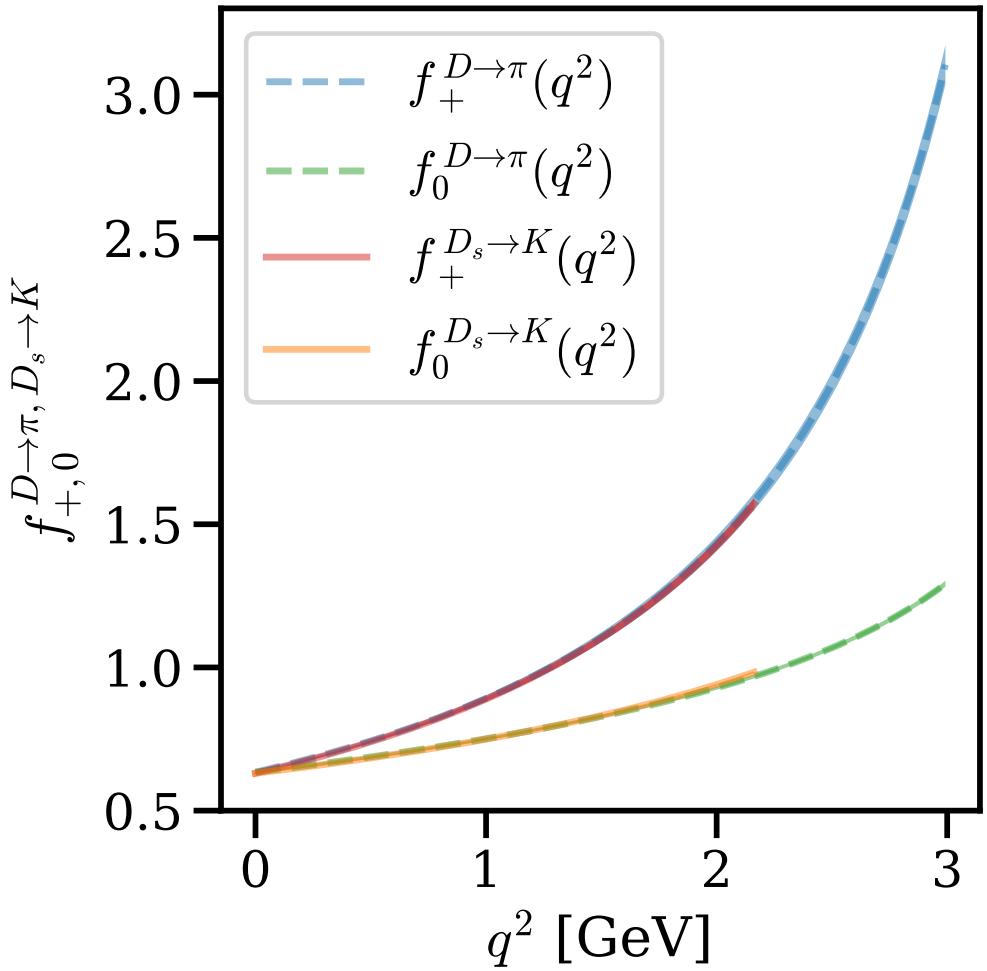
Thank you!

$D \to \pi, K$ rates from LQCD





References:

- FNAL-MILC 2212.12648
- HPQCD 21 2104.09883
- ETMC 17 1706.03017

$D_{(s)}$ decay — spectator quark dependence